

# **Algebra II**

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This text consists of notes of the lecture Algebra II taught at the University of Bonn by Professor Jens Franke in the winter term (Wintersemester) 2017/18.

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# 1. Introduction

After a slight delay due to the Professor being confused by the large attendance to his lecture, Franke recaps briefly the topics of his lecture Algebra I. Our notes to this lecture can be found [here](https://github.com/Nicholas42/AlgebraFranke/tree/master/AlgebraI)<sup>1</sup>. He mentions specifically

- Hilbert's Basissatz and Nullstellensatz,
- the Noether Normalization Theorem,
- the Zariski-topology on  $k^n$ ,
- irreducible topological spaces and their correspondence to the prime ideals of  $k[X_1, \dots, X_n]$ ,
- Noetherian topological spaces and their unique decomposition into irreducible subsets,
- the dimension of topological spaces and codimension of their irreducible subsets,
- catenary topological spaces,
- the fact that  $k^n$  is catenary and  $\dim(k^n) = n$ ,
- quasi-affine varieties,
- structure sheaves,
- the fact that quasi-affine varieties  $X$  are catenary and  $\dim(X) = \deg \operatorname{tr}(K(X)/k)$ , where  $K(X)$  is the quotient field of  $\mathcal{O}(X)$ ,
- going up and going down for integral ring extensions,
- localizations.

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<sup>1</sup><https://github.com/Nicholas42/AlgebraFranke/tree/master/AlgebraI>

**Definition 1** (Scheme). A *(pre)scheme* is a *locally ringed space*  $X$  with a structure sheaf  $\mathcal{O}_X$  such that we have locally  $X \simeq \operatorname{Spec} R$  for some ring  $R$ .