Algebra II

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Wintersemester 2017/18

This text consists of notes of the lecture Algebra II taught at the University of Bonn by Professor Jens Franke in the winter term (Wintersemester) 2017/18.

Please report bugs, typos etc. through the *Issues* feature of github.

Contents

Introduction

After a slight delay due to the Professor being confused by the large attendance to his lecture, Franke briefly recaps the contents of his lecture course Algebra I. Our notes to this lecture can be found here[1]. He mentions specifically

- Hilbert's Basissatz and Nullstellensatz,
- the Noether Normalization Theorem,
- the Zariski-topology on k^n,
- irreducible topological spaces and their correspondence to the prime ideals of $k[X_1, \dots, X_n]$,
- Noetherian topological spaces and their unique decomposition into irreducible subsets,
- the dimension of topological spaces and codimension of their irreducible subsets,
- catenary topological spaces,
- the fact that k^n is catenary and $\dim(k^n) = n$,
- quasi-affine varieties,
- structure sheaves,
- the fact that quasi-affine varieties X are catenary and $\dim(X) = \deg\operatorname{tr}(K(X)/k)$, where $K(X)$ is the quotient field of $\mathcal{O}(X)$. By the way, there is a nice alternative characterization as a direct limit (or colimit)

$$K(X) = \varinjlim_{\substack{\emptyset \neq U \subseteq X \\ U \text{ open}}} \mathcal{O}(U)\ .$$

[1] https://github.com/Nicholas42/AlgebraFranke/tree/master/AlgebraI

- going up and going down for integral ring extensions,
- localizations.

The following definition won't appear in the lecture, but in Algebraic Geometry I instead.

Definition 1. A **(pre)scheme** is a *locally ringed space* X with a structure sheaf $\mathcal{O}_X$ such that each $x \in X$ has an open neighbourhood U which is *isomorphic* to $\operatorname{Spec} R$ for some ring R. That is, $U \cong \operatorname{Spec} R$ as topological spaces and $\mathcal{O}_X|_U$ restricts to $\mathcal{O}_{\operatorname{Spec} R}$.

Exercises will be held on Wednesday from 16 to 18 and Friday from 12 to 14 in Room 0.008. It is necessary to have achieved at least half the points on the exercise sheets in order to attend the exams.