PART B

STEADY-STATE POWER SYSTEM NETWORKS

Chapter 1

POWER FLOW MODELLING

1.1 INTRODUCTION

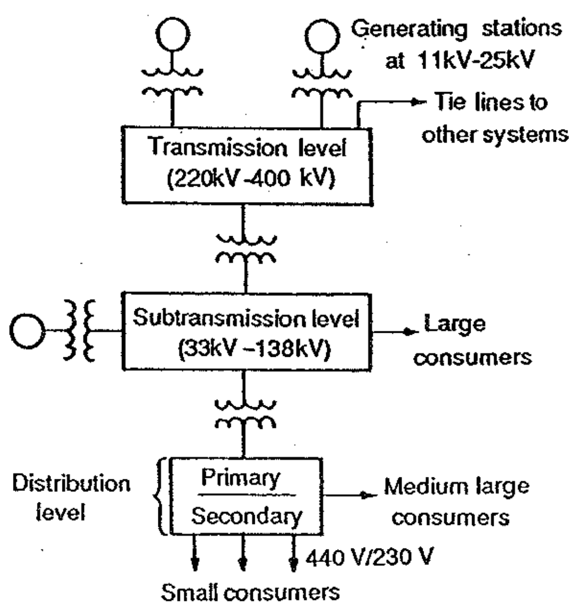


Figure 1.1 Structure of a power system

Generating stations, transmission lines (cables) and the distribution systems are the main components of an electric power system. Generating stations and a distribution system are connected through transmission lines (cables), which also connect one power system (grid, area) to another. A distribution system connects all the loads in an area to the transmission lines (cables). Figure 1.1 shows the structure of a power system.

Power flow studies are of great importance in planning and designing the future expansion of power system networks as well as in determining the best operation of existing systems. Power flow studies in a power system are to obtain the steady state solutions of the power system network. The power system is modelled by an electric network (power flow equations) and solved for the steady state powers (real and reactive powers) and voltages (the magnitudes and phase angles) at various buses. The analysis methods of circuit theory are not possible, as the loads in a power system are given in terms of complex powers rather than impendances, and the generators behave more like power sources than voltage sources.

In power flow analysis, we are mainly interested in voltages at various buses and power injection into the transmission system. Figure 1.2 shows the one-line diagram of a power system having five buses.

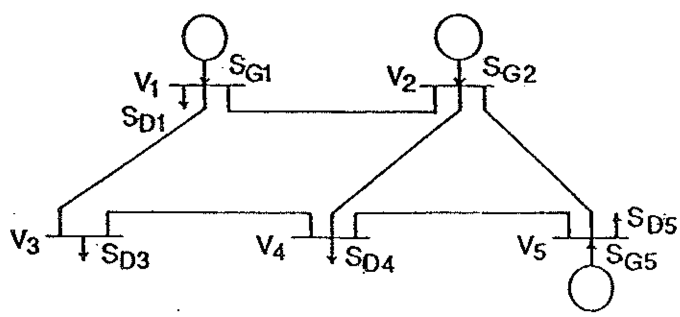


Figure 1.2 One-line diagram of a five-bus system

Here S_G's and S_D's represent the complex powers injected by generators and complex powers drawn by the loads, and V's represent the complex voltages at the various buses.

1.2 POWER FLOW EQUATIONS

1.2.1 CLASSIFICATION OF SYSTEM BUSES

An electric power system is an AC network generating and transferring electrical energy. Every bus (node) of the system will be characterised with active and reactive powers P and Q and a complex voltage $\bar{V}$ which actually means two variables – magnitude V and phase angle δ. So every bus of the power system is associated with four variables – P, Q, V and δ.

For power flow studies, buses can be classified into three categories:

1) *Generator bus:* P and V known; Q and δ unknown.

At each generator bus, the generating unit supplies energy to local loads as well as the other loads along the transmission lines as shown in Figure 1.3.

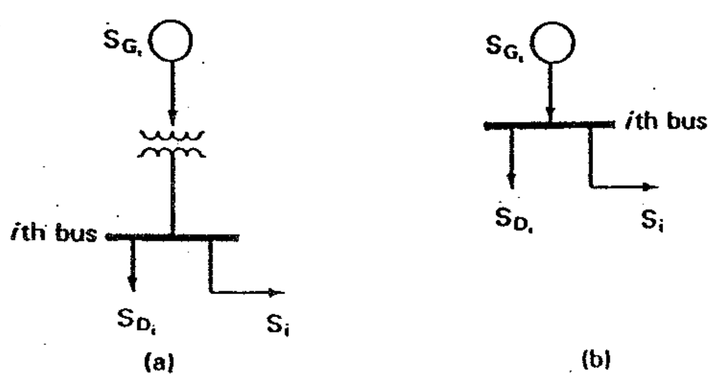


Figure 1.3 Generator bus: (a) Symbolic representation; (b) Simplified version with transformer symbol removed.

The net outflow of complex power from the ith bus is represented by

$$S_i = S_{Gi} - S_{Di}$$

A generator bus is also known as the P-V bus.

2) *Load bus:* P and Q known; V and δ unknown

There is no generating source at a load bus; instead real and reactive powers are continuously drawn from the bus to meet the needs of customers. The symbolic representation of a load bus is shown in Figure 1.4.

Figure 1.4 Symbolic representation of a load bus

Here it is instructive to note that the complex power injected into transmission network at the ith load bus is

$$S_i = 0 - S_{Di} = -S_{Di}$$

A load bus is also known as P-Q bus.

3) *Slack bus:* V and δ known; P and Q unknown

Besides generator and load buses, the power balance condition that must prevail throughout the electric power system calls for yet another bus type. Generally, a slack is a generating unit, which can be adjusted to take up whatever is needed to ensure power balance. The slack bus is usually identified as bus 1.

1.2.2 POWER FLOW EQUATIONS

Consider an ith bus of an n-bus power system as shown in Figure 1.5.

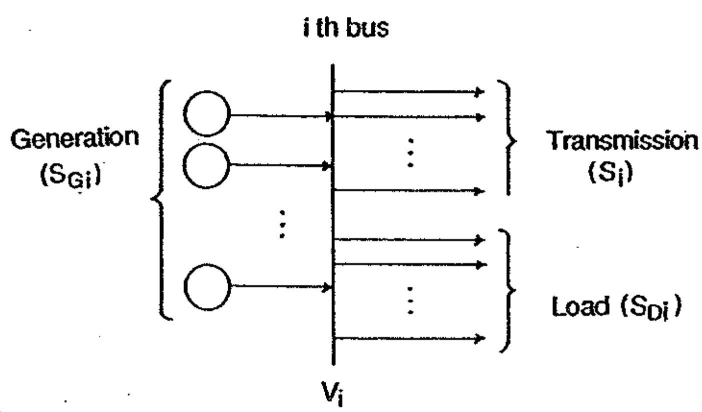


Figure 1.5 ith bus in general connected to generators, loads and transmission

The power at each bus injected into the transmission system is called the bus power, S_i. The ith bus power is defined as

$$S_i = S_{Gi} - S_{Di}$$

Writing the complex powers in terms of real and reactive powers, we have

$$S_{Gi} = P_{Gi} + jQ_{Gi}$$

$$S_{Di} = P_{Di} + jQ_{Di}$$

$$S_i = P_i + jQ_i$$

where $i = 1, 2, \ldots$

It follows that

$$S_i = S_{Gi} - S_{Di} = (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di})$$

The bus current at the ith bus $\bar{I}_i$ is defined as

$$\bar{I}_i = \bar{I}_{Gi} - \bar{I}_{Di}$$

The *power flow* equations are a set of equations which can be formulated from the transmission network model of the electric power system employing Kirchhoff's laws.

n-bus case:

Consider the ith bus in an n-bus system, as shown in Figure 1.6.

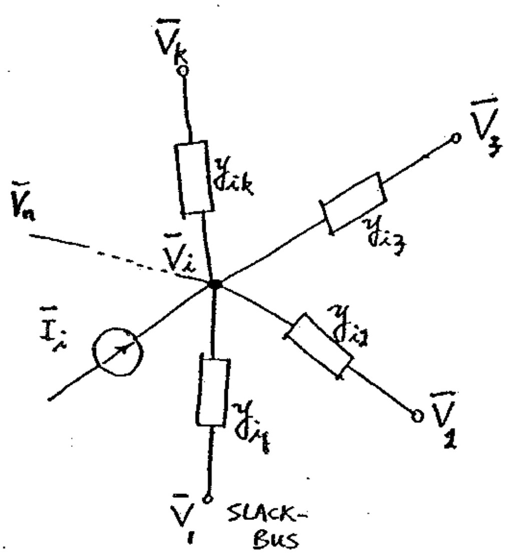


Figure 1.6 ith bus in an n-bus system

For an n-bus electric power system with slack bus at Bus 1:

$$\bar{I}_i = (\bar{V}_i - \bar{V}_1)y_{i1} + (\bar{V}_i - \bar{V}_2)y_{i2} + ... + (\bar{V}_i - \bar{V}_n)y_{in}$$

where $i = 1, 2, ..., n$. Then we have

$$\bar{I}_i = \bar{V}_i(y_{i1} + y_{i2} + ... + y_{in}) - \bar{V}_1 y_{i1} - \bar{V}_2 y_{i2} - ... - \bar{V}_n y_{in}$$

i.e.

$$\bar{I}_i = \sum_{k=1}^{n} \bar{Y}_{ik}\bar{V}_k \qquad i = 1, 2, ..., n \tag{1.1}$$

where

$\bar{I}_i$ = complex current entering in the ith bus

$\bar{V}_k$ = complex voltage to ground of the bus k

$\bar{Y}_{ik}$ = complex admittance between buses i and k

$\bar{Y}_{ii}$ = the sum of all self-admittance connected to the ith bus

$$\bar{Y}_{ii} = \sum_{k=1}^{n} y_{ik} \qquad i \neq k$$

$\bar{Y}_{ik}$ = the negative transfer admittance connected between buses i and k – mutual admittance

$$\bar{Y}_{ik} = -y_{ik}$$

In matrix form, (1.1) becomes

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\mathbf{V}_{bus} \tag{1.2}$$

where

$$\mathbf{I}_{bus} = [\bar{I}_1,\ \bar{I}_2,\ ...,\ \bar{I}_n]^T$$

$$\mathbf{V}_{bus} = [\bar{V}_1,\ \bar{V}_2,\ ...,\ \bar{V}_n]^T$$

$$\mathbf{Y}_{bus} = \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} & \dots & \bar{Y}_{1n} \\ \bar{Y}_{21} & \bar{Y}_{22} & \dots & \bar{Y}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{Y}_{n1} & \bar{Y}_{n2} & \dots & \bar{Y}_{nn} \end{bmatrix}$$

In a power system, complex power is a more important quantity than the current. The complex power into a bus can be expressed as

$$P_i + jQ_i = \bar{V}_i\bar{I}_i^* \tag{1.3}$$

Superscript * in equation (1.3) indicates the conjugate of a complex number. Substituting (1.3) into (1.1) gives

$$P_i + jQ_i = \bar{V}_i \sum_{k=1}^{n} \bar{Y}_{ik}^* \bar{V}_k^* \qquad i = 1, 2, ..., n \tag{1.4}$$

Knowing that

$$\bar{V}_i = e_i + jf_i = V_i \angle \delta_i$$

$$\bar{V}_k^* = e_k - jf_k = V_k\angle - \delta_k$$

$$\bar{Y}_{ik}^* = G_{ik} - jB_{ik} = Y_{ik}\angle - \theta_{ik}$$

Then the equation (1.4) can be expressed in the following polar form

$$P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik} V_k e^{j(\delta_i - \delta_k - \theta_{ik})} \qquad i = 1, 2, ..., n \tag{1.5}$$

or equivalently in rectangular form

$$P_i + jQ_i = (e_i + jf_i) \sum_{k=1}^{n} (G_{ik} - jB_{ik})(e_k - jf_k) \qquad i = 1, 2, ..., n \tag{1.6}$$

Remember that for each bus there are two unknowns. For a system of n buses, there are $2n$ unknowns and $2n$ real equations which result from the set of complex equations (1.4). Hence the solution is feasible. It is clear that the determination of unknowns for the slack bus does not require any extra equation. The unknowns of the slack bus are automatically determined when the unknowns of the other buses are found out. Hence the equation for the slack bus can be deleted and thus $(n-1)$ equations of the type shown in (1.4)-(1.6) are needed for the power flow study.

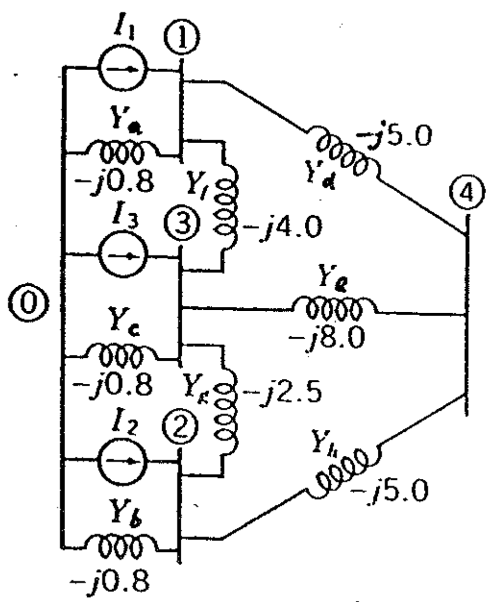


Figure 1.7

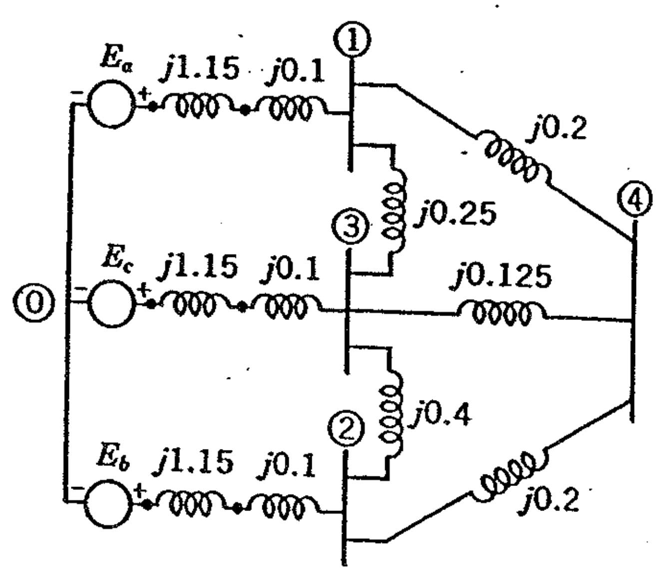


Figure 1.8

Example 1.1:

Write in matrix form the node equations necessary to solve for the voltage of the numbered buses of Figure 1.7. The network is equivalent to that of Figure 1.8. The emfs shown in Figure 1.8 are $\bar{E}_a = 1.5\angle 0^0$, $\bar{E}_b = 1.5\angle -36.87^0$ and $\bar{E}_c = 1.5\angle 0^0$, all in per unit. Also solve the node equations to find the bus voltages.

Solution:

The current sources are

$$\bar{I}_1 = \bar{I}_3 = \frac{1.5\angle 0^0}{j1.25} = 1.2\angle -90^0 = 0 - j1.2 \; p.u.$$

$$\bar{I}_2 = \frac{1.5\angle -36.87^0}{j1.25} = 1.2\angle -126.87^0 = -0.72 - j0.96 \; p.u.$$

Self-admittance in per unit are

$$\bar{Y}_{11} = -j5.0 - j4.0 - j0.8 = -j9.8$$

$$\bar{Y}_{22} = -j5.0 - j2.5 - j0.8 = -j8.3$$

$$\bar{Y}_{33} = -j4.0 - j2.5 - j8.0 - j0.8 = -j15.3$$

$$\bar{Y}_{44} = -j5.0 - j5.0 - j8.0 = -j18.0$$

and the mutual admittance in per unit are

$$\bar{Y}_{12} = \bar{Y}_{21} = 0 \qquad \bar{Y}_{23} = \bar{Y}_{32} = j2.5$$

$$\bar{Y}_{13} = \bar{Y}_{31} = j4.0 \qquad \bar{Y}_{24} = \bar{Y}_{42} = j5.0$$

$$\bar{Y}_{14} = \bar{Y}_{41} = j5.0 \qquad \bar{Y}_{34} = \bar{Y}_{43} = j8.0$$

The node equations in matrix form are

$$\mathbf{I}_{bus} = \mathbf{Y}_{bus}\mathbf{V}_{bus}$$

i.e

$$\begin{bmatrix} -j1.2 \\ -0.72 - j0.96 \\ -j1.2 \\ 0 \end{bmatrix} = \begin{bmatrix} -j9.8 & 0 & j4.0 & j5.0 \\ 0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j15.3 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \\ \bar{V}_3 \\ \bar{V}_4 \end{bmatrix}$$

We know that

$$\mathbf{Y}_{bus}^{-1} = \mathbf{Z}_{bus}$$

where $\mathbf{Z}_{bus}$ is the bus impedance matrix.

$$\mathbf{Z}_{bus} = \begin{bmatrix} j0.4774 & j0.3706 & j0.4020 & j0.4142 \\ j0.3706 & j0.4872 & j0.3922 & j0.4126 \\ j0.4020 & j0.3922 & j0.4558 & j0.4232 \\ j0.4142 & j0.4126 & j0.4232 & j0.4733 \end{bmatrix}$$

$$\mathbf{V}_{bus} = \mathbf{Z}_{bus}\mathbf{I}_{bus}$$

Then we have

$$\mathbf{V}_{bus} = \begin{bmatrix} 1.4111 - j0.2668 \\ 1.3830 - j0.3508 \\ 1.4059 - j0.2824 \\ 1.4009 - j0.2971 \end{bmatrix} = \begin{bmatrix} 1.436\angle - 10.71^0 \\ 1.427\angle - 14.24^0 \\ 1.434\angle 11.36^0 \\ 1.432\angle - 11.97^0 \end{bmatrix} p.u.$$

□

1.2.3 SOLUTION METHODS

Equations (1.4)-(1.6) represent a set of nonlinear simultaneous algebraic equations for which there is no general solutions until now. These are to be solved iteratively by some numerical method. The following two methods have been extensively used in power system analysis:

1. *Gauss-Seidel* (G-S) method
2. *Newton-Raphson* (N-R) method

The G-S method is simple and takes less computation time per iteration but requires a large number of iterations for obtaining the solution. On the other hand, the N-R method involves more calculation per iteration but only a few iterations are required for obtaining the solution.

1.3 NETWORK CALCULATION

1.3.1 MATRIX PARTITIONING

A useful method of matrix manipulation, called *partitioning*, consists in recognizing various parts of a matrix as submatrices which are treated as single elements in applying the usual rules of multiplication and addition. Consider two matrices A and B

$$\mathrm{A} = \begin{bmatrix} \mathrm{D} & \mathrm{E} \\ \mathrm{F} & \mathrm{G} \end{bmatrix}$$

$$\mathrm{B} = \begin{bmatrix} \mathrm{H} \\ \mathrm{J} \end{bmatrix}$$

Then the product is

$$\mathrm{C} = \mathrm{AB} = \begin{bmatrix} \mathrm{D} & \mathrm{E} \\ \mathrm{F} & \mathrm{G} \end{bmatrix} \begin{bmatrix} \mathrm{H} \\ \mathrm{J} \end{bmatrix}$$

The submatrices are treated as single elements to obtain

$$\mathrm{C} = \begin{bmatrix} \mathrm{DH} + \mathrm{EJ} \\ \mathrm{FH} + \mathrm{GJ} \end{bmatrix}$$

The product is finally determined by performing the indicated multiplication and addition of the submatrices. The submatrices to be multiplied must be compatible originally.

1.3.2 NODE ELIMINATION BY MATRIX ALGEBRA

Nodes may be eliminated by matrix manipulation of the standard node equations. However, only those nodes at which current does not enter or leave the network can be eliminated.

We rewrite the node equation (1.2), in matrix form

$$\mathrm{I}_{bus} = \mathrm{Y}_{bus}\mathrm{V}_{bus}$$

To eliminate nodes which have no injected or outgoing current (i.e no generating source or load connected), the above equation must be so arranged that elements associated with nodes to be eliminated are in the lower rows of the matrices. When partitioned according to this rules, the node equation (1.2) becomes

$$\begin{bmatrix} \mathbf{I}_A \\ \mathbf{I}_X \end{bmatrix} = \begin{bmatrix} \mathbf{K} & \mathbf{L} \\ \mathbf{L}^T & \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{V}_A \\ \mathbf{V}_X \end{bmatrix} \tag{1.7}$$

where $\mathbf{I}_X$ is the submatrix composed of the currents entering the nodes to be eliminated and $\mathbf{V}_X$ is the submatrix composed of the voltages of these nodes. The self- and mutual-admittances composing $\mathbf{K}$ are those identified only with nodes to be retained. $\mathbf{M}$ is the self- and mutual- admittances identified only with nodes to be eliminated. $\mathbf{L}$ and its transpose $\mathbf{L}^T$ are composed of only those mutual-admittances common to nodes to be retained and those to be eliminated.

Performing the multiplication indicated in (1.7) gives

$$\mathbf{I}_A = \mathbf{K}\mathbf{V}_A + \mathbf{L}\mathbf{V}_X \tag{1.8}$$

and

$$\mathbf{I}_X = \mathbf{L}^{\mathbf{T}}\mathbf{V}_A + \mathbf{M}\mathbf{V}_X \tag{1.9}$$

Since $\mathbf{I}_X = 0$, we have

$$\mathbf{V}_X = -\mathbf{M}^{-1}\mathbf{L}^{\mathbf{T}}\mathbf{V}_A \tag{1.10}$$

This expression for $\mathbf{V}_X$, (1.10), when substituted in (1.8) gives

$$\begin{aligned} \mathbf{I}_A &= \mathbf{K}\mathbf{V}_A - \mathbf{L}\mathbf{M}^{-1}\mathbf{L}^{\mathbf{T}}\mathbf{V}_A \\ &= \{\mathbf{K} - \mathbf{L}\mathbf{M}^{-1}\mathbf{L}^T\}\mathbf{V}_A \\ &= \mathbf{Y}'_{bus}\mathbf{V}_A \end{aligned} \tag{1.11}$$

where $\mathbf{Y}'_{bus}$ is the modified admittance matrix with reduced dimensions

$$\mathbf{Y}'_{bus} = \mathbf{K} - \mathbf{L}\mathbf{M}^{-1}\mathbf{L}^T \tag{1.12}$$

Example 1.2:

If the generator and transformer connected at bus 3 are removed from the circuit of Figure 1.8, then nodes 3 and 4 will be eliminated by the matrix-algebra procedure just described. Find the equivalent circuit with these nodes eliminated, and find the complex power transferred into or out of the network at nodes 1 and 2. Also find the voltage at node 1.

Solution:

The bus admittance matrix of the circuit partitioned for elimination of nodes 3 and 4 is

$$\mathbf{Y}_{bus} = \begin{bmatrix} \mathbf{K} & \mathbf{L} \\ \mathbf{L}^T & \mathbf{M} \end{bmatrix}$$

where

$$\mathbf{K} = \begin{bmatrix} -j9.8 & 0 \\ 0 & -j8.3 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} j4.0 & j5.0 \\ j2.5 & j5.0 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} -j14.5 & j8.0 \\ j8.0 & -j18.0 \end{bmatrix}$$

The inverse of $\mathbf{M}$ is

$$\mathbf{M}^{-1} = \frac{1}{-197} \begin{bmatrix} -j18.0 & j8.0 \\ j8.0 & -j14.5 \end{bmatrix} = \begin{bmatrix} j0.0914 & j0.0406 \\ j0.0406 & j0.0736 \end{bmatrix}$$

Then

$$\mathbf{L}\mathbf{M}^{-1}\mathbf{L}^T = - \begin{bmatrix} j4.9264 & j4.0736 \\ j4.0736 & j3.4264 \end{bmatrix}$$

$$\mathbf{Y}'_{bus} = \mathbf{K} - \mathbf{L}\mathbf{M}^{-1}\mathbf{L}^T = \begin{bmatrix} -j4.8736 & j4.0736 \\ j4.0736 & -j4.8736 \end{bmatrix}$$

The equivalent circuits are shown in Figure 1.9 and Figure 1.10.

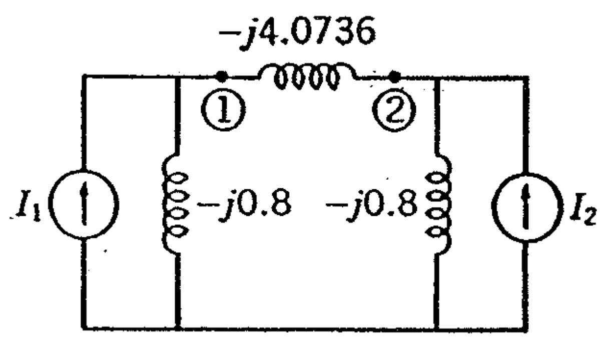


Figure 1.9 The circuit of Figure 1.8

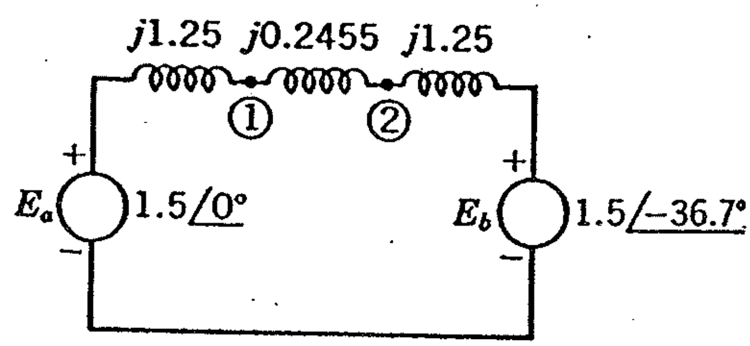


Figure 1.10 The circuit of Figure 1.8

Examination of the matrix shows that the admittance between the two remaining buses 1 and 2 is $-j4.0736$, and the reciprocal of which is the per-unit impedance between these buses. The admittance between each of these buses and reference bus is

$$-j4.8736 - (-j4.0736) = -j0.8 \; p.u.$$

The circulating current is

$$I = \frac{1.5\angle 0^0 - 1.5\angle - 36.87^0}{j(1.25 + 1.25 + 0.2455)} = \frac{1.5 - 1.2 + j0.9}{j2.7455}$$

$$= 0.3278 - j0.1093 = 0.3455\angle - 18.44^0 \; p.u.$$

The power out of source a is

$$1.5\angle 0^0 \times 0.3455\angle 18.44^0 = 0.492 + j0.164\ p.u.$$

And the power into source b is

$$1.5\angle -36.87^0 \times 0.3455\angle 18.44^0 = 0.492 - j0.164\ p.u.$$

Total reactance X in the circuit is $j2.7455$.

The reactance volt-amperes in the circuit is

$$I^2X = 0.3455^2 \times 2.7455 = 0.328 = 0.164 + 0.164$$

The voltage at node 1 is

$$1.5 - j1.25(0.3278 - j0.1093) = 1.363 - j0.410\ p.u.$$

□

The matrix partitioning method is a general method which is more suitable for computer solutions. However, for the elimination of a large number of nodes, the matrix **M** whose inverse must be found will be large.

Inverting a matrix is avoided by eliminating one node at a time, and the process is very simple. The node to be eliminated must be the highest numbered node, and renumbering may be required. The matrix **M** becomes a single element and $\mathbf{M}^{-1}$ is the reciprocal of the element. The original admittance matrix partitioned into submatrices **K**, **L**, $\mathbf{L}^T$ and **M** is

$$\mathbf{Y}_{bus} = \begin{bmatrix} \bar{Y}_{11} & \dots & \bar{Y}_{1(n-1)} & \bar{Y}_{1n} \\ \vdots & \ddots & \vdots & \vdots \\ \bar{Y}_{(n-1)1} & \dots & \bar{Y}_{(n-1)(n-1)} & \bar{Y}_{(n-1)n} \\ \bar{Y}_{n1} & \dots & \bar{Y}_{n(n-1)} & \bar{Y}_{nn} \end{bmatrix}$$

the reduced $(n-1) \times (n-1)$ matrix will be, according to equation (1.12),

$$\mathbf{Y}'_{bus} = \begin{bmatrix} \bar{Y}_{11} & \dots & \bar{Y}_{1(n-1)} \\ \vdots & \ddots & \vdots \\ \bar{Y}_{(n-1)1} & \dots & \bar{Y}_{(n-1)(n-1)} \end{bmatrix} - \frac{1}{\bar{Y}_{nn}} \begin{bmatrix} \bar{Y}_{1n} \\ \vdots \\ \bar{Y}_{(n-1)n} \end{bmatrix} [\bar{Y}_{n1} \ \dots \ \bar{Y}_{n(n-1)}] \tag{1.13}$$

and when the indicated manipulation of the matrices is accomplished, the element in row k and column j of the resulting $(n-1) \times (n-1)$ matrix will be

$$\bar{Y}_{kj(new)} = \bar{Y}_{kj(orig)} - \frac{1}{\bar{Y}_{nn}}\bar{Y}_{kn}\bar{Y}_{nj} \tag{1.14}$$

Example 1.3:

Perform the node elimination of Example 1.2 by first removing node 4 and then by removing node 3.

Solution:

As in Example 1.2, the original matrix now partitioned for removal of one node is

$$\mathbf{Y}_{bus} = \begin{bmatrix} -j9.8 & 0 & j4.0 & j5.0 \\ 0 & -j8.3 & j2.5 & j5.0 \\ j4.0 & j2.5 & -j14.5 & j8.0 \\ j5.0 & j5.0 & j8.0 & -j18.0 \end{bmatrix}$$

To modify the element $j2.5$ in row 3, column 2 subtract from it the product of the elements enclosed by rectangles and divided by the element in the lower right corner. We find the modified element

$$\bar{Y}'_{32} = j2.5 - \frac{j8.0 \times j5.0}{-j18.0} = j4.7222$$

Similarly the new element in row 1, column 1 is

$$\bar{Y}'_{11} = -j9.8 - \frac{j5.0 \times j5.0}{-j18.0} = -j8.4111$$

Other elements are found in the same manner to yield

$$\mathbf{Y}'_{bus} = \begin{bmatrix} -j8.4111 & j1.3889 & j6.2222 \\ j1.3889 & -j6.9111 & j4.7222 \\ j6.2222 & j4.7222 & -j10.9444 \end{bmatrix}$$

Reducing the above matrix to remove node 3 yields

$$\mathbf{Y}''_{bus} = \begin{bmatrix} -j4.8736 & j4.0736 \\ j4.0736 & -j4.8736 \end{bmatrix}$$

which is identical to the matrix found by the matrix-partitioning method where two nodes were removed at the same time. □

2. CONTROL OF VOLTAGE AND REACTIVE POWER

The materials of this chapter are taken from parts of Chapters 2, 3 and 5, Weedy.

2.1. Some Basic Voltage-Reactive Power Relationships

Consider the simple transmission link shown on Fig 2.22 (a). Suppose one wish to establish the equations of E, V and δ using the phasor diagram shown on Fig 2.22(b).

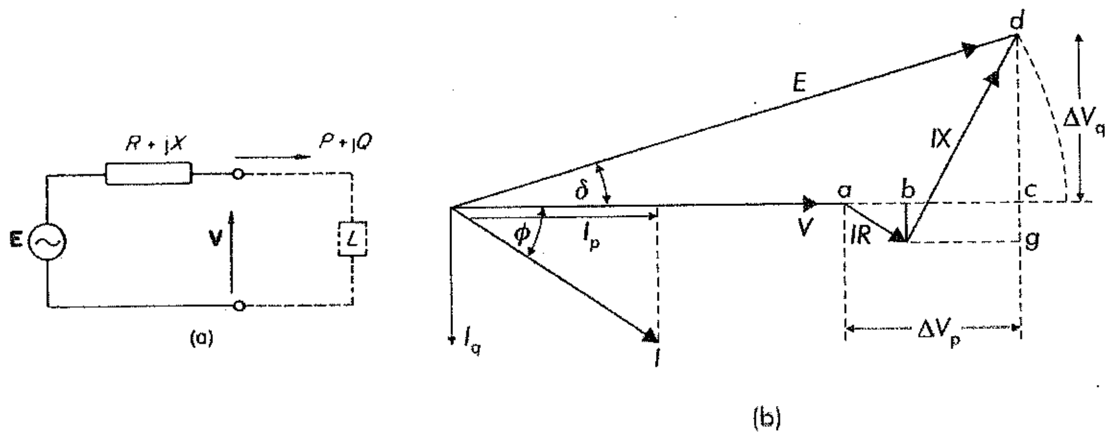


Figure 2.22 Phasor diagram for transmission of power through a series impedance

Then one obtains

$$
\begin{aligned}
E^2 &= (V + \Delta V_p)^2 + \Delta V_q^2 \\
&= (V + RI\cos\phi + XI\sin\phi)^2 + (XI\cos\phi - RI\sin\phi)^2
\end{aligned}
$$

$$
\therefore\ E^2 = \left(V + \frac{RP}{V} + \frac{XQ}{V}\right)^2 + \left(\frac{XP}{V} - \frac{RQ}{V}\right)^2 \tag{2.8}
$$

Hence

$$
\Delta V_p = \frac{RP + XQ}{V} \tag{2.9}
$$

and

$$
\Delta V_q = \frac{XP - RQ}{V} \tag{2.10}
$$

If

$$
\Delta V_q \ll V + \Delta V_p
$$

then

$$E^2 = \left(V + \frac{RP + XQ)}{V}\right)^2$$

and

$$E - V = \frac{RP + XQ}{V} = \Delta V_{\mathrm{p}}$$

The last equation gives the approximate arithmetic difference between the voltages. In many cases when

R = 0,

Then

$$E - V = \frac{XQ}{V} \qquad (2.11)$$

So one notes that

- **Voltage depends only on Q, and from (2.8), this is valid if PX << V^2 + QX.**
- **The angle of transmission δ is obtained from sin $^{-1}$(ΔV_q/E) and depends only on P.**

2.2. Generation and Absorption of Reactive Power

The characteristics of the various power system components, from the point of view of reactive power, will now be examined.

2.2.1 Synchronous generators

The generators can be used to generate or absorb reactive power. See a typical example, Fig 3.14.

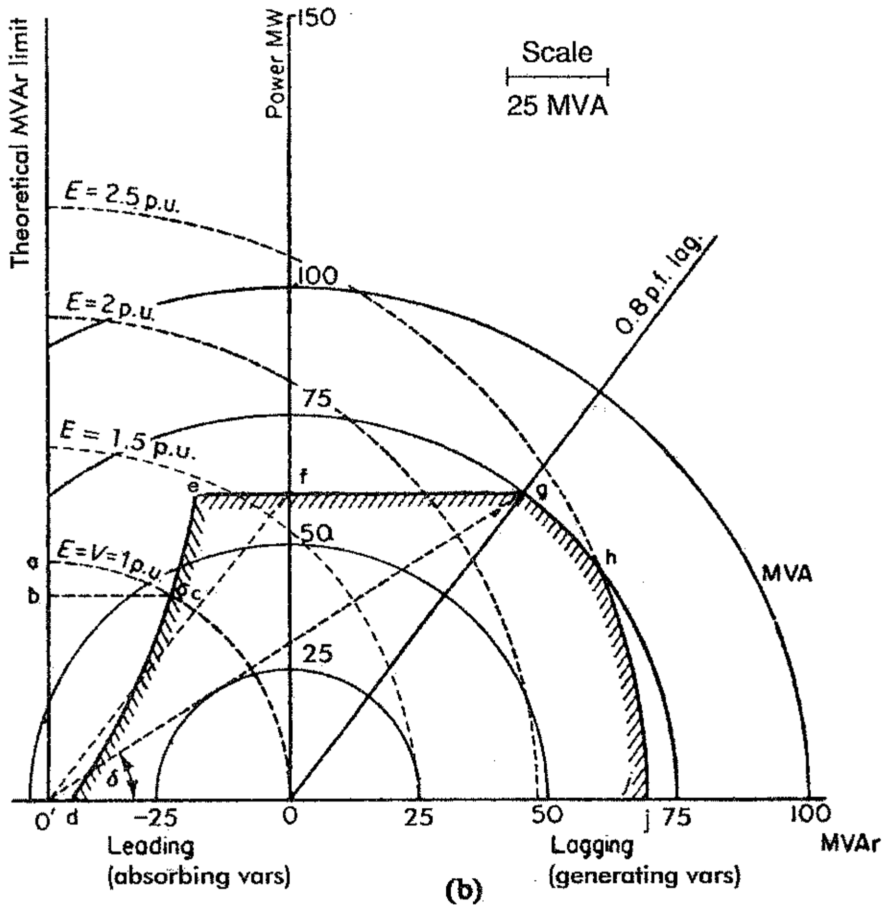


Figure 3.14 Performance chart of a synchronous generator

The ability to supply and absorb Q is determined by the short circuit ratio $= 1/X_d$. Modern generators have low SCR for economic reason, hence the inherent ability of generators to operate at leading power factor is not large.

By operating the generators at over-excited mode, the machines can generate Q, as shown in the figure.

Generators are usually the main source of supply to system of Q, and can operate at leading power factor if the so-called ~~rotor~~ stator end-region thermal limit and stability limit are not breached.

2.2.2 Overhead Lines, transformers and cables

When fully loaded, lines absorb Q in the amount I^2X where X is the line reactance. On lightly loaded lines, the shunt capacitance of longer lines become predominant and the lines act as Q generators.

Transformers always absorb Q.

Cables are generators of Q due to their high shunt capacitance.

2.2.3 Loads

Usually loads are absorbers of Q.

2.3. Methods of Voltage Control – some traditional methods

2.3.1 Injection of Q

2.3.1.1 Shunt capacitors or reactors

Shunt capacitors are used for lagging power factor circuits, whereas reactors (L) are used on those with leading power factors e.g. cable circuits.

Capacitors are connected either directly to busbars or to the tertiary winding of transformer. Disadvantage is when the voltage drops, Q produced is reduced by the V^2 effect. So when needed, its effectiveness decreases.

2.3.1.2 Series capacitors

Series capacitors are used to reduce the series reactance of line conductors, so that the effective reactance between source and load is reduced. One major drawback is the large short-circuit current during fault, and special protection devices are needed to alleviate the problem. Fig 5.6 shows a series capacitor installed on a line, and the phasor diagram describing the voltage and current relationship.

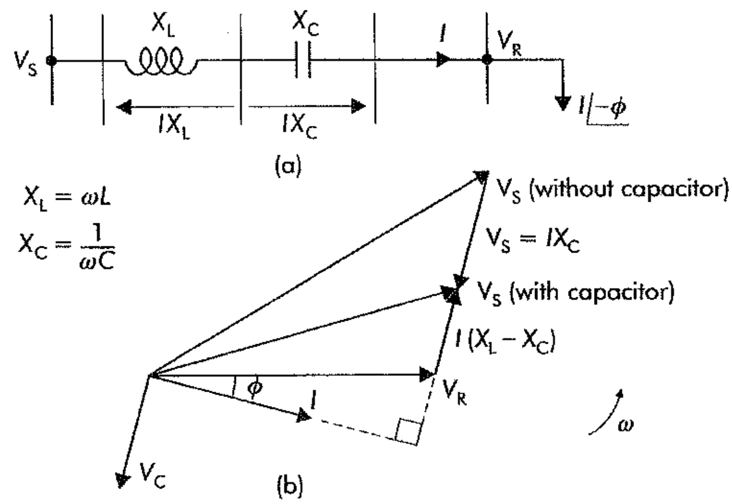


Figure 5.6 (a) Line with series capacitor, *C* load. (b) Phasor diagram for fixed V_R

2.3.1.3 Synchronous condensers

A synchronous condenser is a synchronous motor without a mechanical load. Depending on the excitation level, it can absorb or generate Q. Due to motor losses, the input power factor is not zero. The losses can be high compared to a shunt capacitor.

A great advantage is the flexibility of operation for all load conditions. Also as it is rotating machinery, its stored kinetic energy is useful for riding through transient disturbances.

2.3.2 Tap-changing Transformers

By changing the tap ratios of transformers, the voltage on the secondary side can be varied. A typical example of such application on a radial line is shown on Fig 5.10.

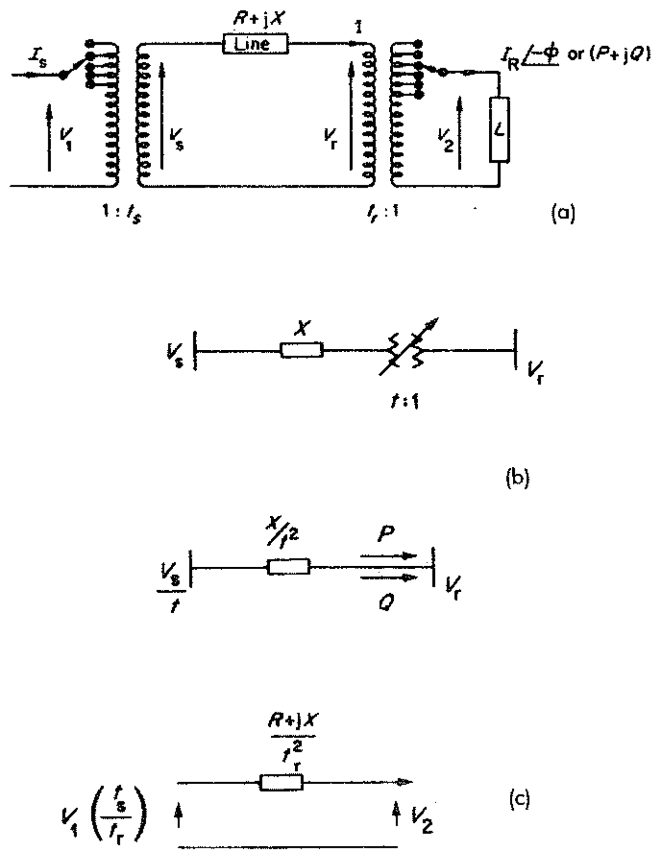


Figure 5.10 (a) Coordination of two tap-changing transformers in a radial transmission link (b) and (c) Equivalent circuits for dealing with off-nominal tap ratio. (b) Single transformer. (c) Two transformers

Let t_s and t_r be the fractions of the nominal transformation ratios, i.e. tap ratio/nominal ratio. For example, a transformer of nominal voltage ratio 6.6 to 33 kV when tapped to 6.6kV to 36 kV has a t_s value of 36/33 or 1.09.

As shown, suppose V_1 and V_2 are the nominal voltages. So the actual sending-end voltage of the line is $V_s=t_sV_1$ whereas the receiving-end of the line has a voltage value $V_r=t_rV_2$. In order to keep the tap range minimal and the overall voltage level remains in the same order, also assume that the produce

$$t_s t_r = 1.$$

Suppose the load at the line receiving end is P+jQ.

So across the line impedance, one can make use of the approximate voltage difference equation (2.9) to give

$$V_1 t_s - V_2 t_r \approx (RP + XQ) / t_r V_2$$

Re-arrange terms, and substitute $t_r = 1/t_s$, one obtains

$$V_2 = \tfrac{1}{2}[t_s^2 V_1 \pm t_s(t_s^2 V_1^2 - 4(RP + XQ))^{\frac{1}{2}} \qquad (5.7)$$

So one could use the last equation to determine the tap setting, once V_2 is given.

3.3 Booster Transformers

Used when it is desirable to increase the voltage at an intermediate point of a line rather than at the line ends. It is cheaper because it would be of smaller capacity than a transformer.

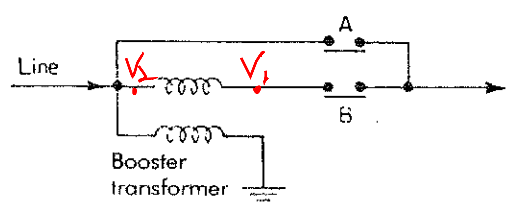


Figure 5.15 Connection of in-phase booster transformer. One phase only shown

Fig 5.15 shows how the booster can be brought into service by the closure of the relay B and the opening of A. The relay operations can be effected through the change in current level: for example, at high load, a heavy current could require a boost in line voltage, so B is closed and A is open. Effectively it provides an in-phase boost to the voltage.

3.4 Phase-shift Transformers

A quadrature phase shift can be achieved, as shown in Fig. 5.17. The phasor diagram shows that by arranging the use of the tapping of the energized transformer, several values of phase shift may be obtained.

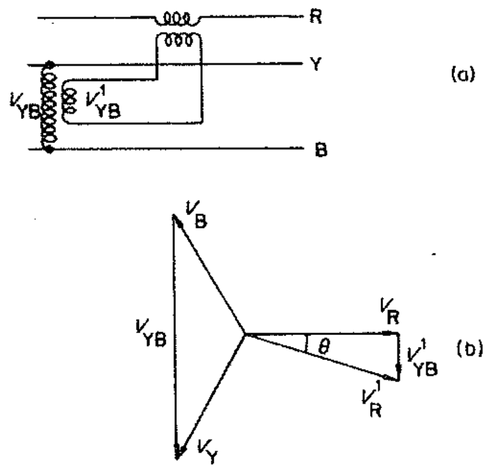


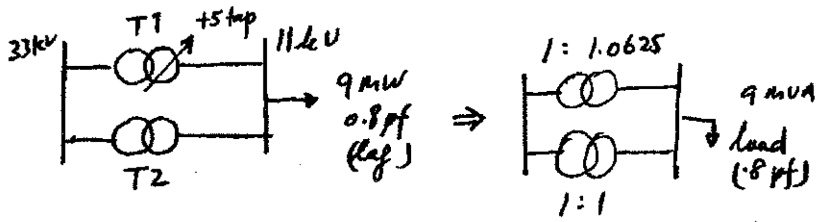
Figure 5.17 (a) Connections for one phase of a phase-shifting transformer. Similar connections for the other two phases. (b) Corresponding phasor diagram

Other more modern techniques will be described later by Prof. ~~Haque~~. Wang Youyi

WORKED EXAMPLES

3.13 Two identical transformers each have a nominal or no-load ratio of 33/11 kV and a leakage reactance of 2 Ω referred to the 11 kV side; resistance may be neglected. The transformers operate in parallel and supply a load of 9 MVA, 0.8 p.f. lagging. Calculate the current taken by each transformer when they operate five tap steps apart (each step is 1.25 per cent of the nominal voltage). Also calculate the kVAr absorbed by this tap setting.

Solution:



Let the two transformers be denoted as T_1 and T_2.

Before the tap change, calculate the load current through each transformer as follows:

Use 9 MVA and 11 kV as base values, we have

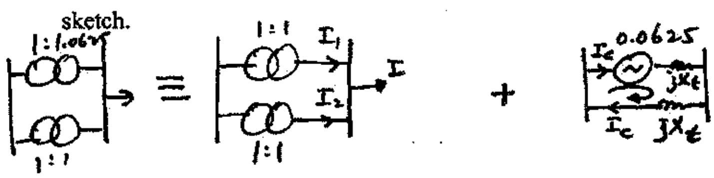
Base Z = 11x11/9 = 13.44 Ω, base current = 9/(√3x11) kA or 472 Amps.

The transformer leakage reactance becomes $jX_t = j2/13.44$ p.u. or j0.1487 p.u.

So load power demand is 1 p.u., 0.8 pf lag, load voltage is also 1 p.u., thus

$I = 1 \angle -\cos 0.8 = 1 \angle -36.87°$ p.u.

Hence, the current through each transformer is I/2 or $I_1 = I_2 = 0.5\angle -36.87°$ p.u. See the sketch.



Assume that the ~~load~~ source side voltage remains constant at 33 kV after the tap change. This is a reasonable assumption because usually the source side fault level is much higher than the load capacity. Hence the source side voltage is not sensitive to the tap change. We shall also assume that the load demand remains constant. This may be a reasonable assumption if loads are consisting of constant-power motor drives. Also, if the 11 kV bus voltage does not vary significantly, then a constant impedance load will also result in the load demand being constant.

Next, since the transformers operate at 5 tap apart, each step is of 1.25%, therefore, we may assume that T_1 has its tap position raised. We will also assume that the tap is on the load side of T_1. Then T_1 effective turns ratio is 1: 1.0625, as 5 steps correspond to 6.25%. (We could try other combinations of tap positions, but this can be left as another exercise).

Notice that with the new tap positions, there is a circulating current component I_c. The appearance of I_c is solely due to the difference in the transformers turns ratio. In this case, one can represent this difference as an equivalent voltage source, equals to 0.0625 p.u. but without any phase shift. The situation is given by the sketch ~~below~~ above. This current can be calculated as follows:

$I_c = 0.0625 / (2 \times j0.1487) = -j0.21$ p.u.

By superposition, the resulting currents in T_1 and T_2 become

$I_1' = I_1 + I_c = 0.5\angle{-36.87^\circ} - j0.21 = 0.648 \angle{-51.84^\circ}$ p.u. or 306 A (Ans)

$I_2' = I_2 - I_c = 0.4 + j0.3 - j0.21 = 0.41 \angle{-12.68^\circ}$ or 194 A. (Ans).

Take note of the direction of I_c in the above calculation.

The reactive power consumed by the transformers due to the tap change is

$3 (I_c)^2 \times (X_t + X_t) = 3\times (0.21 \times 472)(2 + 2) = 118$ kVAr (3 phase).

(One could calculate what the increase in the reactive power loss in the circuit is. This is left as an exercise.)

One could also estimate the actual voltage (V) at the 11-kV bus in the following way. On the T_2 branch, suppose the 33kV bus voltage remains unchanged for the reason explained earlier, therefore

$1 \angle\delta = V\angle 0 + jX_t \times I_2' = V\angle 0 + j0.1487 \times (0.41 \angle{-12.68^\circ})$

Solving the above, one obtains

$V = 0.9848$ p.u.

And the angle

$\delta = 3.44^\circ$.

Hence, one observes that the 11-kV bus voltage changes slightly (about 1.5% lower) and the source side voltage is now 3.44° leading the load-side voltage.

(One can calculate the pre-tap change δ but this is left as an exercise for you.)

5.11 Explain the limitations of tap-changing transformers. A transmission link (Figure 5.25(a) connects an infinite busbar supply of 400 kV to a load busbar supplying 1000 MW, 400 MVAr. The link consists of lines of effective impedance $(7 + j70)\ \Omega$ feeding the load busbar via a transformer with a maximum tap ratio of 0.9:1. Connected to the load busbar is a compensator. If the maximum overall voltage drop is to be 10 per cent with the transformer taps fully utilized, calculate the reactive power requirement from the compensator.

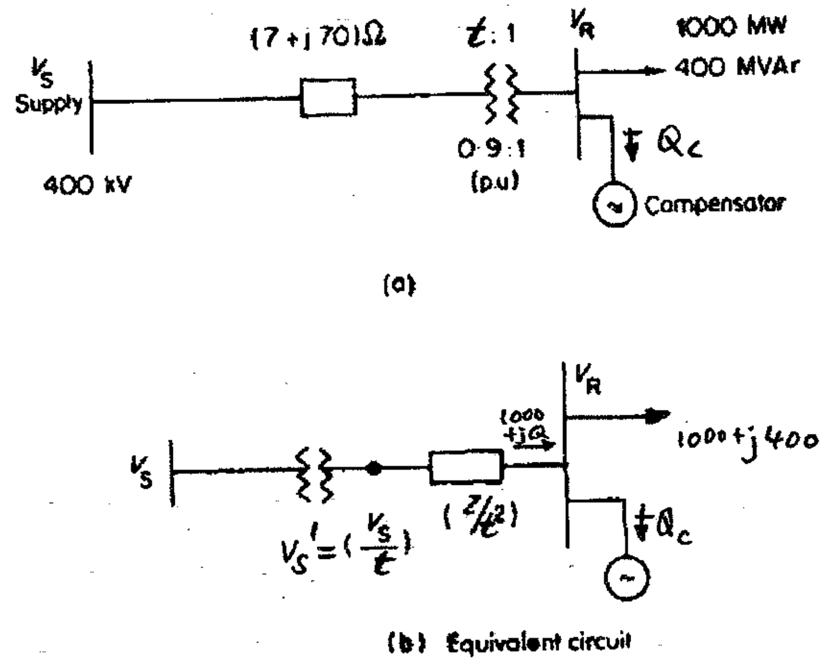


Figure 5.25 Circuits for Problem 5.11

Solution:

Use 1000 MVA, 400kV as base values.

Therefore, impedance base = 400x400/1000 = 160 ohms. Z = 7 + j70 ohms/phase = 0.04375 + j0.4375 p.u.

Refer Z to the load side, it becomes Z/t^2, $\mathbf{V_s}$ becomes $\mathbf{V_s}' = \mathbf{V_s}/t$ where t:1 is the transformer turns ratio. (The bold letters indicate these are phasor quantities. * indicates complex conjugate.)

Load demand is 1000 + j 400 MVA or 1 + j 0.4 p.u. Suppose the reactive power compensator output is Q_c p.u. Therefore, the net demand at the receiving end of the link is 1 + j (0.4+ Q_c) p.u., with the compensator assumed to draw Q_c p.u., as shown.

As it is required that the voltage drop across the link is to be no more than 10%, and as the infinite bus voltage is 400 kV, therefore at the maximum drop, |**Vr**| = 0.9 p.u. and with the transformer at the maximum boost condition, i.e. t = 0.9. (Tap position above this value will not give maximum voltage boost to **Vr** for a fixed **Vs**).

From the circuit diagram, one can write the voltage equation

$$\mathbf{Vs'} = \mathbf{Vr} + (Z/t^2)\,\mathbf{I} \qquad (1)$$

Where

$$1 + j\,(0.4 + Q_c) = \mathbf{Vr} \times \mathbf{I}^* \qquad (2)$$

Substitute **I** from (2) into (1), and recognize **Vr = 0.9** $\angle 0$, t = 0.9, **Vs' = 1** $\angle\ \delta$ /t and Z = 0.04375 + j0.4375 p.u. into (1), we have

$$(1/0.9) \angle\ \delta = 0.9 \angle 0 + [1 + j\,(0.4 + Q_c)]^* / (0.9 \angle 0) \times (0.04375 + j0.4375)/(0.9 \times 0.9)$$

$\delta = 32.8°$

with $Q = -0.04368$

For ease of manipulation, let Q = 0.4 + Q_c. After a few algebraic steps, we should obtain

$$1 = 1.0376 + 0.875Q + 0.2945Q^2 \qquad (3)$$

Solving for Q, we should obtain

Q = -0.00436 or -2.92 p.u.

The second solution is too large as to be practical. So we accept the first solution.

Q = -0.00436 p.u. or -43.6 MVAr

Since Q = 0.4 + Q_c p.u., therefore

Qc = -0.443.6 p.u. or -443.6 MVAr

That is the compensator has to generate 443.6 MVAr (capacitive) in order to support Vr = 0.9 p.u.

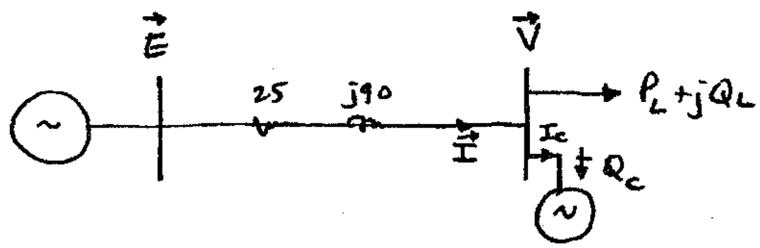
One note that the compensator injects a 43.6 MVAr back into the link, having met all the load reactive power requirement. Reactive power is also imported from the infinite bus into the link, and you can work out the power flow using **Vs' x I***.

5.5 A three-phase transmission line has resistance and inductive reactance of 25 Ω and 90 Ω, respectively. With no load at the receiving end, a synchronous compensator there takes a current lagging by 90°; the voltage is 145 kV at the sending end and 132 kV at the receiving end. Calculate the value of the current taken by the compensator.

When the load at the receiving end is 50 MW, it is found that the line can operate with unchanged voltages at the sending and receiving ends, provided that the compensator takes the same current as before, but now leading by 90°.

Calculate the reactive power of the load.

Solution



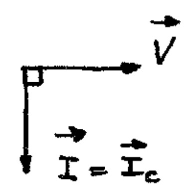
Use 100 MVA and 132kV as base values. Therefore, base current = 100/(√3 x 132) kA = 437.2 A, base impedance = 132x132/100 ohms = 174.24 ohms.

(i). On no load, $P_L + j\,Q_L = 0$.

E = (145/132) ∟δ p.u. = 1.098 ∟ δ p.u., **V** = 1 ∟ 0° (reference phasor),

I = |**I** |∟-90° p.u. since the compensator current lags **V**. We assume the compensator is lossless.

Line impedance Z_l = (25 + j 90) /174.24 p.u. = 0.536∟74.5° p.u..



Therefore, from

E = **V** + **I** Z_l

One obtains

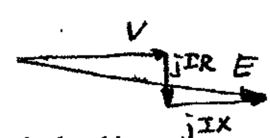
1.098 ∟δ = 1 ∟ 0° + |**I** |∟-90° x (0.536∟74.5°)

Solving, we have a quadratic equation in |**I** |. After a few steps of calculations, the solutions are

|**I** | = 0.188 p.u. and – 3.8 p.u.

Negative value has no meaning, so

|**I** | = 0.188 p.u. or 82.2 A (Ans.)



It is lagging power factor at the line end. In other word, the compensator is absorbing reactive power (Q_c) of +0.188 p.u. or 18.8 MVAr.

(ii) On load, $P_L = 0.5$ p.u., $Q_L = ?$.

However now the compensator is operating at the leading power factor and has the same current as before. So,

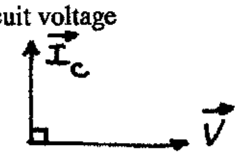
$Qc = -0.188$ p.u. or the compensator is generating reactive power (capacitive).

Let the reactive power at the receiving end be $Q = Q_L - 0.188$ p.u. Thus, the complex power there is $P_L + jQ$ or $0.5 + jQ$ p.u.

The line current **I** is

$\mathbf{I} = (0.5 + j\,Q)^*/\mathbf{V}$

We still let $\mathbf{V} = 1 \angle 0°$ (reference phasor). Therefore, from the same circuit voltage equation



$\mathbf{E} = \mathbf{V} + \mathbf{I}\,Z_l$

We have

$1.098 \angle \delta' = 1 \angle 0° + (0.5 + j\,Q)^* \mathrm{x}\ (0.536 \angle 74.5°)$

for which the solutions are

$Q = -.0087$ or -3.61 p.u.

The second solution is too large as to be practical.

Hence, $Q_L = -0.0087 + 0.188 = 0.179$ p.u. or 17.9MVAr (ans)