Problem Set 5 Transient and Dynamics

1. A separately excited DC motor consists of the following parameters

   $R_a = 0.5\ \Omega \qquad L_{aq} = 0 \qquad B = 0 \qquad J = 0.1\ \text{kgm}^2$

   The rotational loss is negligible. The motor is used to drive an inertia load of 1.0 kgm^2. With the rated field current and an armature terminal voltage of 100 V, the motor and the load consist of a steady-state speed of 1500 rpm. At a certain time, the armature terminal voltage is suddenly increased to 120 V.

   a. Obtain an expression for the speed as a function of time.

   b. Find the speed 1 second after the step increase in the terminal voltage.

   c. Find the final steady-state speed of the motor.

2. A separately excited DC motor consists of the following parameters

   $R_a = 0.4\ \Omega \qquad L_{aq} = 0 \qquad K_f = 1 \qquad B = 0 \qquad J = 4.5\ \text{kgm}^2$

   The motor operates at no-load with terminal voltage of 220 V and field current of 2 A. Rotational losses are negligible. The motor is intended to be stopped by plugging, i.e., by reversal of its armature terminal voltage ($V_t = -220$ V)

   a. Find the no-load speed of the motor.

   b. Obtain an expression for the motor speed after plugging.

   c. Find the time taken for the motor to reach zero speed.

3. A separately excited DC motor consists of the following parameters

   $R_a = 0.4\ \Omega \qquad K_f = 1 \qquad B = 0.1\ \text{kgm}^2/\text{s} \qquad J = 2.0\ \text{kgm}^2$

   The motor drives a constant load torque. With field current of 2 A and armature terminals connected to a 100 V DC source, the motor rotates at 450 rpm.

   a. Find the motor current.

   b. Find the friction torque ($B\omega_m$) and load torque.

   c. The motor is now disconnected from the DC supply. Obtain an expression for speed as a function of time.

   d. Following (c), the load torque remains on the motor shaft after the motor is disconnected from the supply. Find the new steady-state speed.

4. A separately excited DC motor consists of the following parameters

   $R_a = 0.5\ \Omega \qquad K_f = 1 \qquad B = 0.1\ \text{kgm}^2/\text{s} \qquad J = 2.0\ \text{kgm}^2$

With field current of 2 A and the motor terminals connected to a 100 V DC supply, the motor rotates at no-load and draws an armature current of 2.469 A.

a. Find the motor speed and developed torque.

b. A load of constant torque of 10 Nm is now applied. Obtain an expression for speed as a function of time.

c. Following (b), find the new steady-state speed, motor current and developed torque.

5. Consider a separately excited DC generator with following parameters

R_f = 100 Ω L_f = 40 H R_a = 0.2 Ω L_{aq} = 10 mH

K_g = 100 V per field ampere at 1000 rpm

The generator is operated at rated speed of 1200 rpm with field current as 2 A. The armature is suddenly connected to a load with 1.8 Ω and 10 m H in series. Find

a. Load terminal voltage as a function of time.

b. Steady-stage value of the load terminal voltage.

c. Torque as a function of time.

6. Consider a separately excited DC generator with following parameters

R_a = 0.5 Ω K_f = 1 B = 0.1 kgm^2/s J = 2.0 kgm^2

With field current of 2 A, the motor is connected to a 100 V DC supply. It rotates at no-load with speed of 471.569 rpm.

a. Find the motor current and developed torque.

b. If the field current is reduced to 1 A, derive an expression for speed as a function of time.

c. Find the new steady-state speed, motor current and developed torque.

Answers

1.

(a) Because rotational losses are neglected, in steady state, motor does not produce any torque. Therefore, before the voltage was changed, $I_a = 0 \quad E_a = V_t$

$K_m \omega_m = 100V$

$$K_m = \frac{100}{\frac{1500}{60} \times 2\pi} = 0.637\, V/rad \cdot sec^{-1}$$

After the voltage was changed,

$V_t = e_a + R_a i_a = K_m \omega_m + R_a i_a$

Also, $T = K_m i_a = J \frac{d\omega_m}{dt}$

or $V_t = K_m \omega_m + R_a \frac{J}{K_m} \frac{d\omega_m}{dt}$

$$= 0.637\omega_m + \frac{0.5 \times 1.1}{0.637} \frac{d\omega_m}{dt}$$

So, $V_t = 0.64\omega_m + 0.86 \frac{d\omega_m}{dt}$

$V_t(s) = 0.64\omega_m(s) + 0.86(s\omega_m(s) - \omega_{m0})$

Where $\omega_{m0} = \frac{1500}{60} \times 2\pi = 157.1\, rad/sec$

$$\frac{120}{s} = 0.64\omega_m(s) + 0.86s\omega_m(s) - 0.86 \times 157.1$$

$$\Rightarrow \omega_m(s) = \frac{120 + 135.11s}{s(0.64 + 0.86s)}$$

$$= \frac{120 + 135.11s}{0.86s\left(s + \frac{0.64}{0.86}\right)} = \frac{139.1 + 157.1s}{s(s + 0.744)}$$

$$= \frac{A}{s} + \frac{B}{s + 0.744}$$

$A = 187, B = -29.9$

$\therefore \omega_m(t) = \mathcal{L}^{-1}\{\omega_m(s)\} = 187 - 29.9e^{-0.744t}\, rad/sec$

(b) $\omega_m|_{t=1} = 187 - 29.9e^{-0.744}$

$= 172.95\, rad/sec$

(c) $\omega_m(\infty) = 187\, rad/sec$

2.

(a) $K_m = K_f i_f = 1 \times 2 = 2$

No-load $\longrightarrow$ $V_t = E_a = 220\ V = K_m \omega_{m0}$

$$\omega_{m0} = \frac{220}{2} = 110\ rad/sec$$

(b) After voltage reversal $\longrightarrow$ $V_t = -220\ V$

$$V_t = e_a + R_a i_a = K_m \omega_m + R_a i_a$$

$$T = K_m i_a = J \frac{d\omega_m}{dt}$$

$$V_t = K_m \omega_m + R_a \frac{J}{K_m} \frac{d\omega_m}{dt}$$

$$= 2\omega_m + \frac{0.4 \times 4.5}{2} \frac{d\omega_m}{dt}$$

$$= 2\omega_m + 0.9 \frac{d\omega_m}{dt}$$

$$V_t(s) = 2\omega_m(s) + 0.9(s\omega_m(s) - \omega_{m0})$$

$$\frac{-220}{s} = 2\omega_m(s) + 0.9s\omega_m(s) - 0.9 \times 110$$

$$\Rightarrow \omega_m(s) = \frac{-244.44 + 110s}{s(s + 2.222)}$$

$$= \frac{A}{s} + \frac{B}{s + 2.222}$$

Where $A = -110, B = 220$

$$\omega_m(t) = \mathcal{L}^{-1}\{\omega_m(s)\} = -110 + 220e^{-2.222t}\ rad/sec$$

(c) $0 = -110 + 220e^{-2.222t}$

$$t = 0.315\ sec$$

3.

(a) $K_m = K_f i_f = 1 \times 2 = 2\,V/rad \cdot sec^{-1}$

$$\omega_{m0} = \frac{450}{60} \times 2\pi = 47.124\,rad/sec$$

$$E_a = K_m \omega_m = 2 \times 47.124 = 94.248\,V$$

$$I_a = \frac{100 - 94.248}{0.5} = 11.504\,A$$

(b) $T = K_m I_a = 2 \times 11.504 = 23\,N \cdot m$

$$T_B = 0.1 \times 47.124 = 4.7124\,N \cdot m$$

$$T_L = 23 - 4.7124 = 18.2876\,N \cdot m$$

(c) $T = K_m i_a = 0 = J\frac{d\omega_m}{dt} + B_m \omega_m + T_L$

$$J(s\omega_m(s) - \omega_{m0}) + B\omega_m(s) + \frac{T_L}{s} = 0$$

$$Js\omega_m(s) - J\omega_{m0} + B\omega_m(s) + \frac{18.2876}{s} = 0$$

$$\omega_m(s) = \frac{J\omega_{m0} - \frac{18.2876}{s}}{B + Js} = \frac{94.248s - 18.2876}{2(s + 0.05)s}$$

$$= \frac{A_1}{s} + \frac{A_2}{s + 0.05}$$

$$A_1 = s\omega_m(s)|_{s=0} = \left.\frac{94.248s - 18.2876}{2(s + 0.05)}\right|_{s=0} = -182.876$$

$$A_2 = (s + 0.05)\omega_m(s)|_{s=0} = \left.\frac{94.248s - 18.2876}{2s}\right|_{s=-0.05} = 230$$

$$\omega_m(s) = -\frac{182.876}{s} + \frac{230}{s + 0.05}$$

$$\omega_m(t) = -182.876 + 230e^{-0.05t}\,rad/sec$$

$$\omega_m(\infty) = -182.876\,rad/sec$$

CHECK: $\omega_m(0) = -182.876 + 230 = 47.124 = \omega_{m0}$

$$T_B = 0.1 \times 182.876 = 18.2876 = T_L$$

4.

(a) $K_f I_f = 1 \times 2 = 2$

$$100 = 2\omega_0 + 0.5 \times 2.469 = E_a + I_a R_a$$

$$\omega_0 = \frac{100 - 1.24}{2} = 49.38\, rad/sec = \frac{49.38}{2\pi} \times 60 = 471.54\, rpm$$

$$T = K_f I_f I_a = 2 \times 2.469 = 4.938\, N \cdot m$$

(b) $100 = K_m \omega_m + R_a i_a$

$$K_m i_a = J \frac{d\omega_m}{dt} + B\omega_m + T_L$$

$$100 = K_m \omega_m + R_a \left[\frac{J}{K_m}\frac{d\omega_m}{dt} + \frac{B\omega_m}{K_m} + \frac{T_L}{K_m}\right]$$

$$= 2\omega_m + 0.5 \left[\frac{2}{2}\frac{d\omega_m}{dt} + \frac{0.1}{2}\omega_m + \frac{10}{2}\right] = 2\omega_m + 0.5\frac{d\omega_m}{dt} + 0.025\omega_m + 2.5$$

$$97.5 = 0.5\frac{d\omega_m}{dt} + 2.025\omega_m$$

In S domain,

$$\frac{97.5}{s} = 0.5[s\omega_m(s) - \omega_0] + 2.025\omega_m(s) = 0.5[s\omega_m(s) - 49.38] + 2.025\omega_m(s)$$

$$\omega_m(s) = \frac{\frac{97.5}{s} + 24.69}{0.5s + 2.025} = \frac{195 + 49.38s}{s(s + 4.05)} = \frac{A_1}{s} + \frac{A_2}{s + 4.05} = \frac{48.15}{s} + \frac{1.2319}{s + 4.05}$$

$$\omega_m(t) = 48.15 + 1.2319e^{-4.05t}$$

$$\omega_m|_{ss} = \omega_m(\infty) = 48.15\, rad/s \longrightarrow 459.8\, rpm$$

$$I_a|_{ss} = \frac{V_t - K_m\omega_m(\infty)}{R_a} = \frac{100 - 2 \times 48.15}{0.5} = 7.4\, A$$

$$T|_{ss} = K_m I_a|_{ss} = 2 \times 7.4 = 14.8\, N \cdot m$$

5.

(a) $K_g\big|_{1200\,rpm} = 100 \times \frac{1200}{1000} = 120\,V/A$

$@\,1200\,rpm, I_f = 2\,A$

$\therefore\ E_a|_{1200} = K_g\big|_{1200} \times\ I_f = 120 \times 2 = 240\,V$

$$E_a(t) = (\underbrace{R_a + R_L}_{R_T})i_a(t) + (\underbrace{L_{ag} + L_L}_{L_T})\frac{d}{dt}i_a(t)$$

Take Laplace Transformation

$$I_a(s) = \frac{E_a|_{1200}}{R_T} \times \frac{1}{s\left(1 + \frac{L_T}{R_T}s\right)}$$

$$i_a(t) = \mathcal{L}^{-1}\{I_a(s)\} = \frac{240}{2}(1 - e^{-t/\tau_{at}})$$

$$where\ \tau_{at} = \frac{L_T}{R_T} = \frac{(10+10)\times 10^{-3}}{0.2 + 1.8} = 0.01\,sec$$

$$V_t(t) = R_L i_a + L_L\frac{di_a}{dt} = 1.8\ x\ 120(1 - e^{-100t}) + 10\ x\ 10^{-3}(120\ x\ 100e^{-100t})$$

$$= 216 - 96e^{-100t}$$

(b) $V_T(\infty) = 216\,V$

(c) $T = K_f I_f I_a$

$E_a = K_f I_f \omega_m = K_g I_f$

$K_f \omega_m = K_g$

$$\therefore K_f = \frac{K_g}{\omega_m} = \frac{100}{1000 \times 2\pi/60} = \frac{3}{\pi}$$

$$\therefore T(t) = \frac{3}{\pi} \times 2 \times i_a(t) = 229.2(1 - e^{-100t})\ N\cdot m$$

Or, $T(t) = \frac{E_a i_a(t)}{\omega_m} = \frac{240}{1200\times 2\pi/60} \times 120(1 - e^{-100t})$

$$= 229.2(1 - e^{-100t})\ N\cdot m$$

6.

(a) $\omega_{m0} = \frac{471.569}{60} \times 2\pi = 49.3827\, rad/sec$

$$I_a = \frac{100 - 2 \times 49.3827}{0.5} = 2.48\ A$$

$$K_m = K_f I_f = 2$$

$$T = K_m I_a = 2 \times 2.48 = 4.96\ N \cdot m$$

(b) I_f reduced to 1 A, find $\omega_m(t)$

$$K_m = K_f I_f = 1 \times 1 = 1$$

$$100 = K_m \omega_m + R_a i_a$$

$$K_m i_a = J \frac{d\omega_m}{dt} + B\omega_m$$

$$100 = K_m \omega_m + R_a \left(\frac{J}{K_m} \frac{d\omega_m}{dt} + \frac{B}{K_m} \omega_m \right) = K_m \omega_m + \frac{R_a J}{K_m} \frac{d\omega_m}{dt} + \frac{R_a B}{K_m} \omega_m$$

$$= 1 \times \omega_m + \frac{0.5 \times 2}{1} \frac{d\omega_m}{dt} + \frac{0.5 \times 0.1}{1} \omega_m = \omega_m + \frac{d\omega_m}{dt} + 0.05\omega_m$$

$$\frac{100}{s} = \omega_m(s) + s\omega_m(s) - \omega_{m0} + 0.05\omega_m(s)$$

$$\frac{100}{s} + 49.38 = \omega_m(s)(1.05 + s)$$

$$\omega_m(s) = \frac{100 + 49.38s}{s(s + 1.05)} = \frac{A_1}{s} + \frac{A_2}{s + 1.05}$$

$$A_1 = \frac{100}{1.05} = 95.24$$

$$A_2 = \frac{100 + 49.38(-1.05)}{-(1.05)} = -45.86$$

$$\omega_m(t) = 95.24 - 45.86e^{-1.05t}$$

$$\omega_m|_0 = 95.24 - 45.86 = 49.38\, rad/sec$$

CHECK: $\omega_m|_\infty = 95.24\, rad/sec = 909.4729\, rpm$

$$I_a|_\infty = \frac{100 - 1 \times 95.24}{0.5} = 9.52\ A$$

$$T = K_m I_a = 9.52\ N \cdot m$$