Solutions to Week 2 Lecture Notes
Exercise 3: VQ

1 (a)

Main steps involved in a VQ-based image compression scheme:

(i) A codebook consisting of codevectors is generated by using training methods such as LBG algorithm. Both the encoder and decoder should have the same codebook, which is used during the encoding and decoding processes.

(ii) During the encoding process, an image is partitioned into multiple $n \times n$ blocks (subimages). For each image block, the closest codevector with the smallest Euclidean distance in the codebook is determined. The index of this codevector is then encoded, in this case, using Huffman coding.

(iii) During the decoding process, the index of the encoded codevector can be used to determine the codevector through table lookup of the codebook. The codevectors are then re-assembled to form a reconstructed image.

Some selected issues involved in generation of codebook:

(i) Training data

The training data set (images) should be representative of the class of images to be compressed.

(ii) Block size

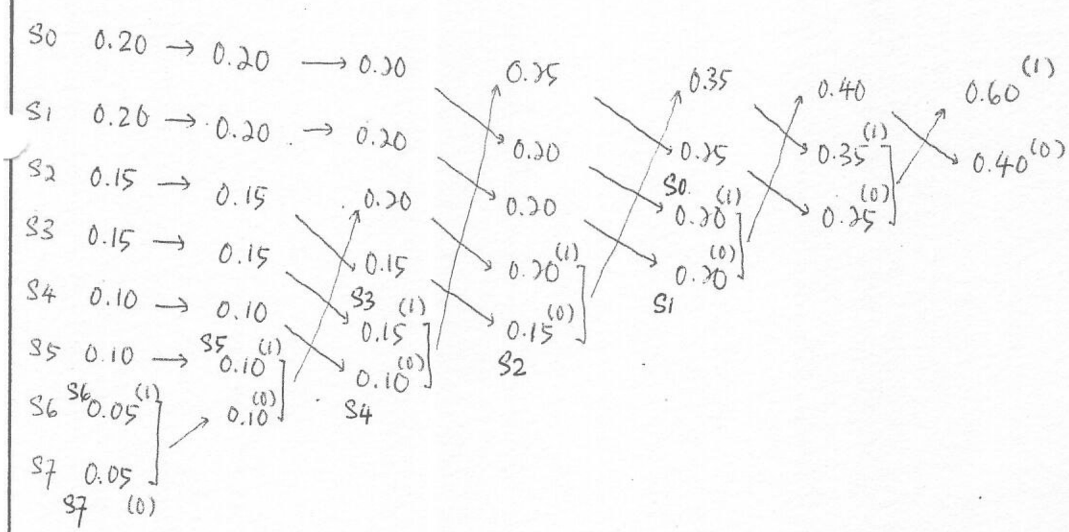
If we use a larger block size while keeping the number of codevectors constant, the data rate will decrease while the reconstruction error will increase.

(iii) Number of codevectors

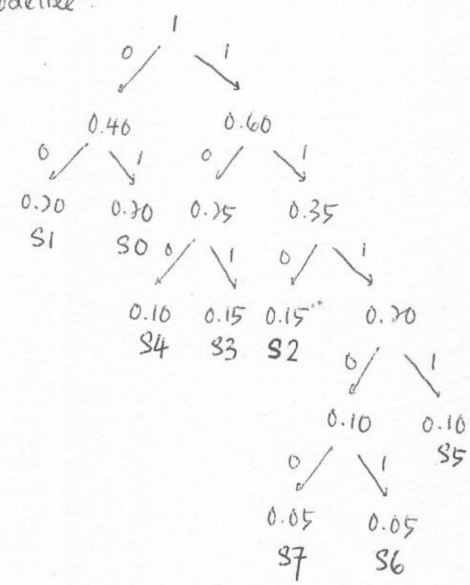
If we use a larger number of codevectors while keeping the block size constant, the data rate will increase while the reconstruction error will decrease.

(iv) Other issues include the training time, training algorithms, distance measure, and parameters affecting the trade-off between data rate and distortion.

1 (b) To design a set of Huffman codewords :



Huffman codetree :



1 (b)

Designed Huffman code set :

s_0 : 01

s_1 : 00

s_2 : 110

s_3 : 101

s_4 : 100

s_5 : 1111

s_6 : 11101

s_7 : 11100

Other Huffman codesets are also possible.

1 (c) Average number of bits/symbol

$= 2 \times 2 \times 0.2 + 2 \times 3 \times 0.15 + 3 \times 0.1 + 4 \times 0.1 + 2 \times 5 \times 0.05$

$= 2.9$ bits/symbol

For each 2×2 block :

Number of bits required for uncompressed block $= 4 \times 2 = 8$ bits

" " " " " symbol $= 2.9$ bits

$\therefore$ Compression ratio $= \dfrac{8}{2.9}$

$= 2.76$

1 (d)

original image (f) $\longrightarrow$ reconstructed image ($\hat{f}$)

$$
\left.\begin{array}{cc|cc|cc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\hline
0 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 2 \\
\hline
1 & 2 & 2 & 2 & 3 & 3 \\
2 & 2 & 2 & 2 & 2 & 3
\end{array}\right.
\quad \longrightarrow \quad
\left.\begin{array}{cc|cc|cc}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 \\
\hline
2 & 2 & 2 & 2 & 3 & 3 \\
2 & 2 & 2 & 2 & 3 & 3
\end{array}\right.
$$

square error

$= \|f - \hat{f}\|^2$

$= 1 + 1 + 1 + 1$

$= 4$