



EE6508 - Power Quality

Capacitor Switching Overvoltages

presented by

Asst. Prof. Amer M. Y. M. Ghias

School of Electrical and Electronic Engineering

Office: S2-B2C-109

Consulting Hours: Thursday 3pm-5pm or by appointments

Email: amer.ghias@ntu.edu.sg

Tel: 6790 5631

S2 AY2019-2020

Acknowledgement: these lectures are based on materials obtained from Prof. Abhisek Ukil and Dr. Tan Kuan Tak, whose help is hereby gratefully acknowledged.

Revised Jan 2019.

Lecture Outline

- Capacitor Switching Overvoltages
 - LC circuit and utilization
 - RLC circuit: damping
 - Capacitor switching transients
 - Managing effects of capacitor switching transients

References

Texts:

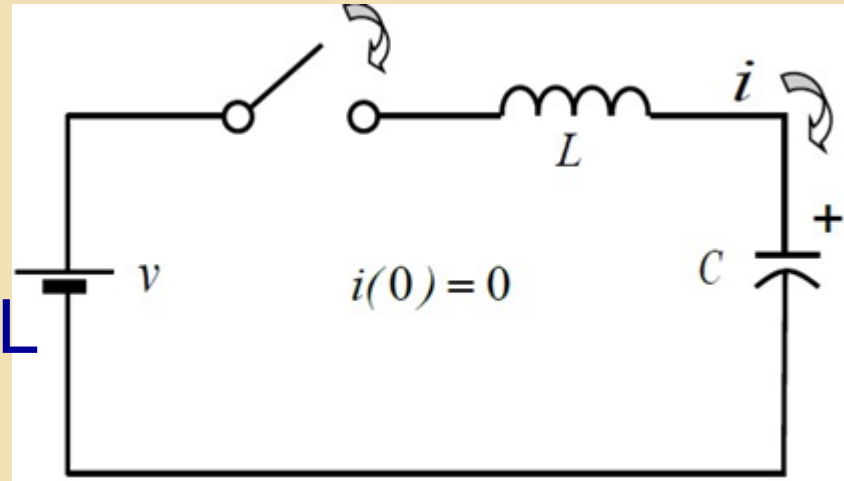
1. Dugan R C, McGranaghan M F, Santoso S, and Beaty H W, Electrical Power Systems Quality, Second Edition, McGraw-Hill, 2002. 2.
2. Kennedy B W, Power Quality Primer, First Edition, McGraw-Hill, 2000.

Reference:

3. Allan Greenwood, “Electrical Transients in Power Systems”, 2nd Edition, John Wiley & Sons Inc., 1991.

The LC Circuit – Current Response

- Consider the LC circuit as shown in the figure
- When the switch is closed, the circuit equation using KVL can be written as:



$$v = L \frac{di}{dt} + v_c, \text{ where } i = C \frac{dv_c}{dt}$$

- Differentiating the equation we get:

$$L \frac{d^2 i}{dt^2} + \frac{dv_c}{dt} = 0 \Rightarrow L \frac{d^2 i}{dt^2} + \frac{i}{C} = 0 \Rightarrow \frac{d^2 i}{dt^2} + \frac{i}{LC} = 0$$

- Apply Laplace transformation:

$$s^2 i(s) - si(0) - i'(0) + \frac{i(s)}{LC} = 0 \text{ ---- (A)}$$

The LC Circuit – Current Response

- As the initial current before and the moment when the switching is closed is $i(0) = 0$, and assuming the initial voltage of the capacitor is zero. The inductor voltage at $t = 0$, can be written as:

$$v_L(0) = L \frac{di(0)}{dt} = v - v_c(0) = v \Rightarrow i'(0) = \frac{v}{L} \text{ ----- (B)}$$

- Using equation B in A, we get:

$$s^2 i(s) - \frac{v}{L} + \frac{i(s)}{LC} = 0 \Rightarrow (s^2 + \frac{1}{LC})i(s) = \frac{v}{L} \Rightarrow i(s) = \frac{v/L}{s^2 + (1/LC)}$$

- Rearranging the above Laplace current equation as:

$$i(s) = \frac{v/L}{s^2 + (1/LC)} = \frac{v}{\sqrt{L/C}} \frac{\sqrt{1/LC}}{s^2 + (\sqrt{1/LC})^2}$$

- Transforming the above equation to time domain using, $\sin(\omega t) = \frac{\omega}{s^2 + (\omega)^2}$

$$i(t) = \frac{v}{\sqrt{L/C}} \sin(\sqrt{1/LC} t)$$

The LC Circuit – Current Response

- The current time response of the LC circuit is $i(t) = \frac{v}{\sqrt{L/C}} \sin(\sqrt{1/LC} t)$
- Note the two significant parameters:
 - $\sqrt{L/C}$ behaves as an impedance and it is so-called surge impedance of the circuit, which is denoted by Z_o
 - $\sqrt{1/LC}$ behaves as a angular frequency. It is called natural frequency and is denoted by ω_o
 - Hence the current $i(t) = \frac{v}{Z_o} \sin(\omega_o t)$
- **Note:** The parameters, Z_o and ω_o , determine the magnitude and the frequency of the transient current. Hence they play a vital role in transient analysis. Where:
 - $Z_o = \sqrt{L/C}$ (Ω) is the surge impedance of the circuit
 - $\omega_o = \sqrt{1/LC}$ (rad/sec) = angular natural frequency or
 $f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$ (Hz) = natural frequency

The LC Circuit –Capacitor Voltage Response

- For the Capacitor voltage, Consider the LC circuit as shown in the figure
- When the switch is closed, the circuit equation using KVL can be written as:

$$v = L \frac{di}{dt} + v_c \text{ --- (C)}$$

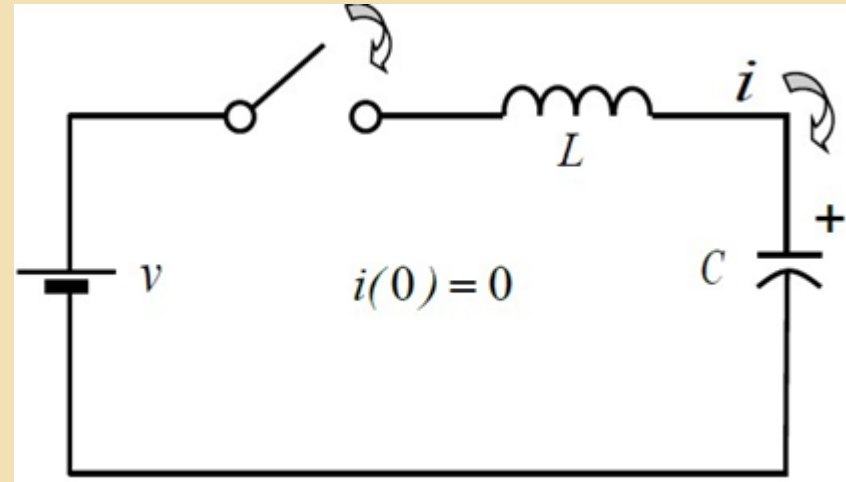
$$i = C \frac{dv_c}{dt} \text{ --- (D)}$$

- Differentiating equation D will give,

$$\frac{di}{dt} = C \frac{d^2 v_c}{dt^2} \text{ --- (E)}$$

- Rearranging equation C and using in equation E will give

$$C \frac{d^2 v_c}{dt^2} = \frac{di}{dt} = \frac{v}{L} - \frac{v_c}{L} \Rightarrow C \frac{d^2 v_c}{dt^2} + \frac{v_c}{L} = \frac{v}{L} \Rightarrow \frac{d^2 v_c}{dt^2} + \frac{v_c}{LC} = \frac{v}{LC} \text{ ---(F)}$$



The LC Circuit –Capacitor Voltage Response

- Let say $\omega_o^2 = 1/LC$, then the equation F can be written as:

$$\frac{d^2 v_c}{dt^2} + \omega_o^2 v_c = \omega_o^2 v \text{ ---(G)}$$

- Using Laplace transformation of equation G, we get:

$$s^2 v_c(s) - s v_c(0) - v_c'(0) + \omega_o^2 v_c(s) = \frac{v \omega_o^2}{s} \text{ ---(H)}$$

- Assuming the capacitor is initialling charge to give the initial voltage of $v_c(0)$, then equation H can we written as:

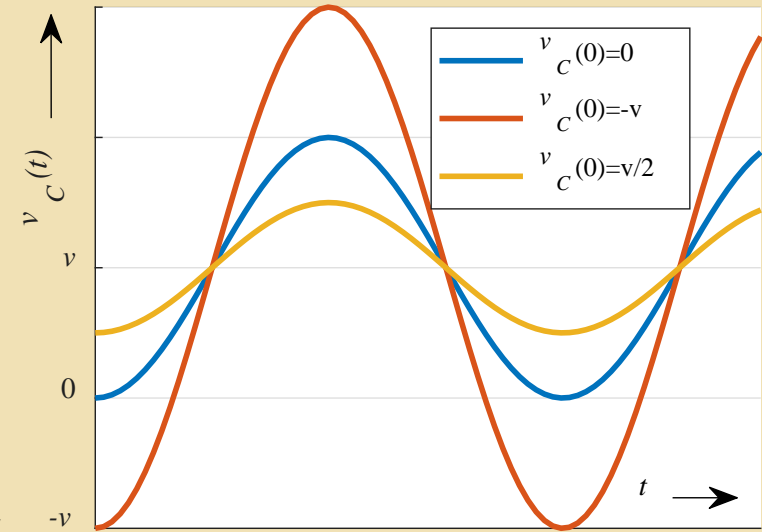
$$s^2 v_c(s) + \omega_o^2 v_c(s) = \frac{v \omega_o^2}{s} + s v_c(0) \Rightarrow (s^2 + \omega_o^2) v_c(s) = \frac{v \omega_o^2}{s} + s v_c(0),$$
$$v_c(s) = \frac{v \omega_o^2}{s(s^2 + \omega_o^2)} + \frac{s v_c(0)}{(s^2 + \omega_o^2)} = v \left[\frac{1}{s} - \frac{s}{(s^2 + \omega_o^2)} \right] + v_c(0) \left[\frac{s}{(s^2 + \omega_o^2)} \right]$$

- Transforming the above equation to time domain using: $\cos(\omega t) = \frac{s}{s^2 + (\omega)^2}$

$$v_c(t) = v[1 - \cos \omega_o t] + v_c(0)[\cos \omega_o t] = v - [v - v_c(0)] \cos \omega_o t$$

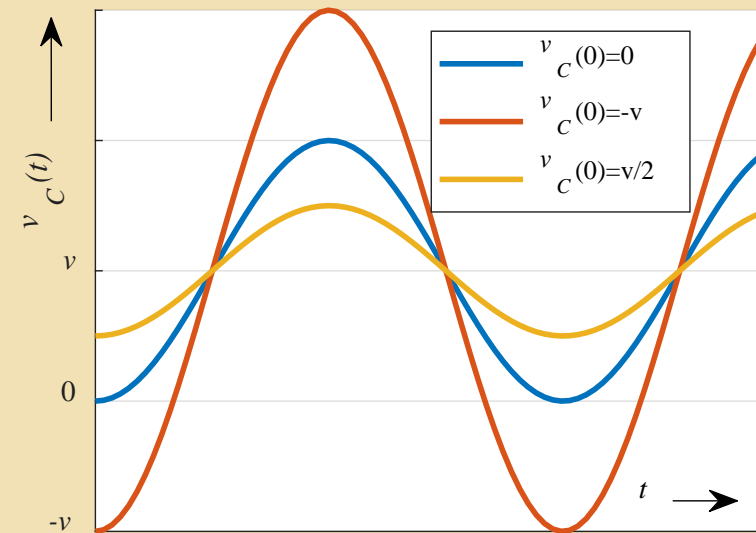
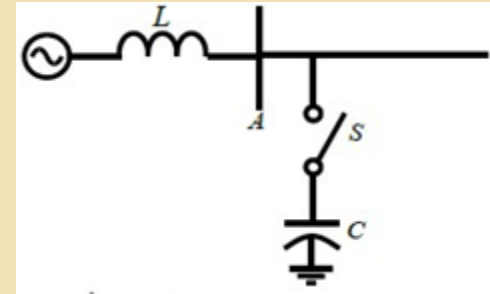
The LC Circuit –Capacitor Voltage Response

- The behaviour of the capacitor voltage, v_c is graphically shown in the figure for different values of $v_c(0)$.
- It can be observed that v_c can exceed the exciting voltage v .
 - Its should be noted that the v_c oscillates about v , which is the steady state value of v_c .
 - For no initial charge on the capacitor, the capacitor voltage can reach a value of $2v$. For certain values of initial charge voltage $v_c(0)$, the capacitor voltage can exceed the voltage value of $2v$, i.e. For $v_c(0) = -v$, the capacitor voltage oscillates between $(-v$ and $3v)$.
 - The capacitor voltage oscillation continues because of the undamped system with no resistance.
- In practical/real systems with resistance in the circuit, the oscillations will be damped out and the peak value usually does not reach $2v$.



The LC Circuit in Power Systems

- LC circuits are very common in power systems.
- For example the shunt capacitor shown in the diagram becomes a LC circuit whenever, the resistance is ignored.
- The switching of the capacitor can produces over voltages in the system, which are dependant on:
 - The initial charge on the capacitor during the switching, and
 - The instantaneous voltage during the switching
- The inclusion of the resistance R in the analysis is important for the realistic evaluation of the transient overvoltage.

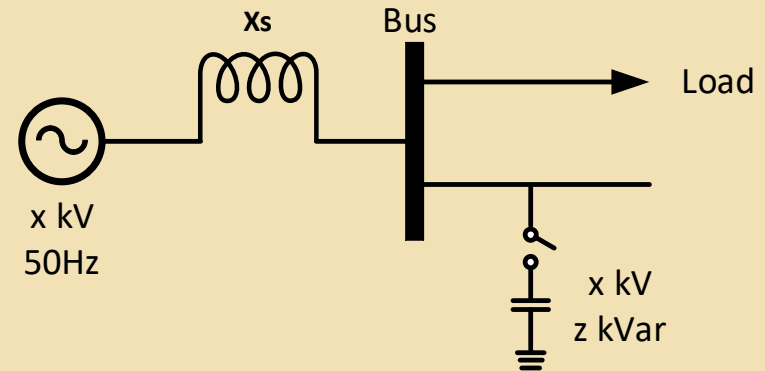


The LC Circuit - Exercise

Figure shows a single-phase, 50-Hz system with $X_s = 0.2\Omega$. The capacitor bank was fully discharged. When the switch is closed, the maximum transient voltage across the capacitor bank reaches 200kV with a frequency of 400 Hz.

- Calculate the unknown voltage (x) in kV.
- Calculate the unknown (z) in kVar and the surge impedance.

Solution will be solved in the class



The RLC Circuit: Damping

- Consider the RLC circuit as shown in the figure
- When the switch is closed, the circuit equation using KVL can be written as:

$$v = Ri + L \frac{di}{dt} + v_c \text{ --- (I)}$$

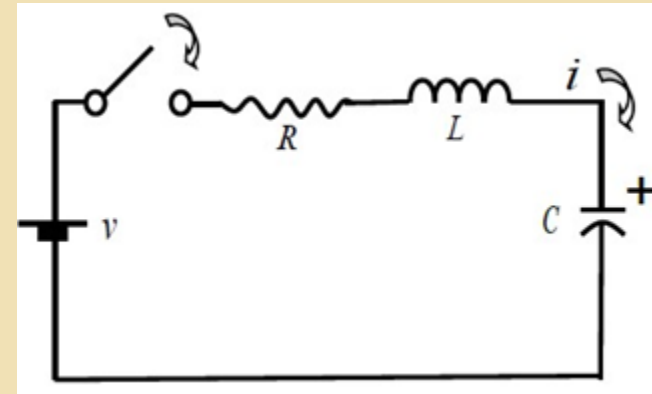
$$i = C \frac{dv_c}{dt} \text{ --- (J)}$$

- Differentiating equation J will give,

$$\frac{di}{dt} = C \frac{d^2v_c}{dt^2} \text{ --- (K)}$$

- Rearranging equation I and using in equation K will give

$$C \frac{d^2v_c}{dt^2} = \frac{di}{dt} = \frac{v}{L} - \frac{v_c}{L} - i \frac{R}{L} \Rightarrow C \frac{d^2v_c}{dt^2} + \frac{v_c}{L} + i \frac{R}{L} = \frac{v}{L} \Rightarrow \frac{d^2v_c}{dt^2} + \frac{v_c}{LC} + i \frac{R}{LC} = \frac{v}{LC}$$
$$\frac{d^2v_c}{dt^2} + \frac{v_c}{LC} + \frac{R}{L} \frac{dv_c}{dt} = \frac{v}{LC} \text{ --- (L)}$$



The RLC Circuit: Damping

- Using Laplace transformation of equation L, we get:

$$s^2 v_c(s) - s v_c(0) - v_c'(0) + \frac{v_c(s)}{LC} + \frac{R}{L} s v_c(s) - \frac{R}{L} v_c(0) = \frac{v}{sLC} \text{ ---(M)}$$

- Assuming the capacitor initial voltage is zero ($v_c(0)=0$), then equation M can be written as:

$$s^2 v_c(s) + \frac{v_c(s)}{LC} + \frac{R}{L} s v_c(s) = \frac{v}{sLC} \Rightarrow (s^2 + \frac{1}{LC} + \frac{1}{L/R} s) v_c(s) = \frac{v}{sLC},$$

$$v_c(s) = \frac{v}{sLC(s^2 + \frac{1}{LC} + \frac{1}{L/R} s)} \text{ ---(N)}$$

- Lets say $T_\phi = \frac{L}{R}$ and $T = \frac{1}{\omega_o} = \sqrt{LC}$, then equation N can be written as:

$$v_c(s) = \frac{v}{sT^2(s^2 + \frac{1}{T^2} + \frac{1}{T_\phi} s)} = \frac{v}{T^2} \frac{1}{s(s^2 + \frac{1}{T_\phi} s + \frac{1}{T^2})}$$

The RLC Circuit: Damping

- The solution requires:

$$\mathcal{L}^{-1} \left\{ \frac{v}{T^2} \frac{1}{s(s^2 + \frac{1}{T_\phi} s + \frac{1}{T^2})} \right\}$$

which can be shown to be

$$f(\eta) = v \left[1 - e^{\frac{-t'}{2\eta}} \left(\frac{\sin(4\eta^2 - 1)^{\frac{1}{2}} \frac{t'}{2\eta}}{(4\eta^2 - 1)^{\frac{1}{2}}} + \cos(4\eta^2 - 1)^{\frac{1}{2}} \frac{t'}{2\eta} \right) \right]$$

where,

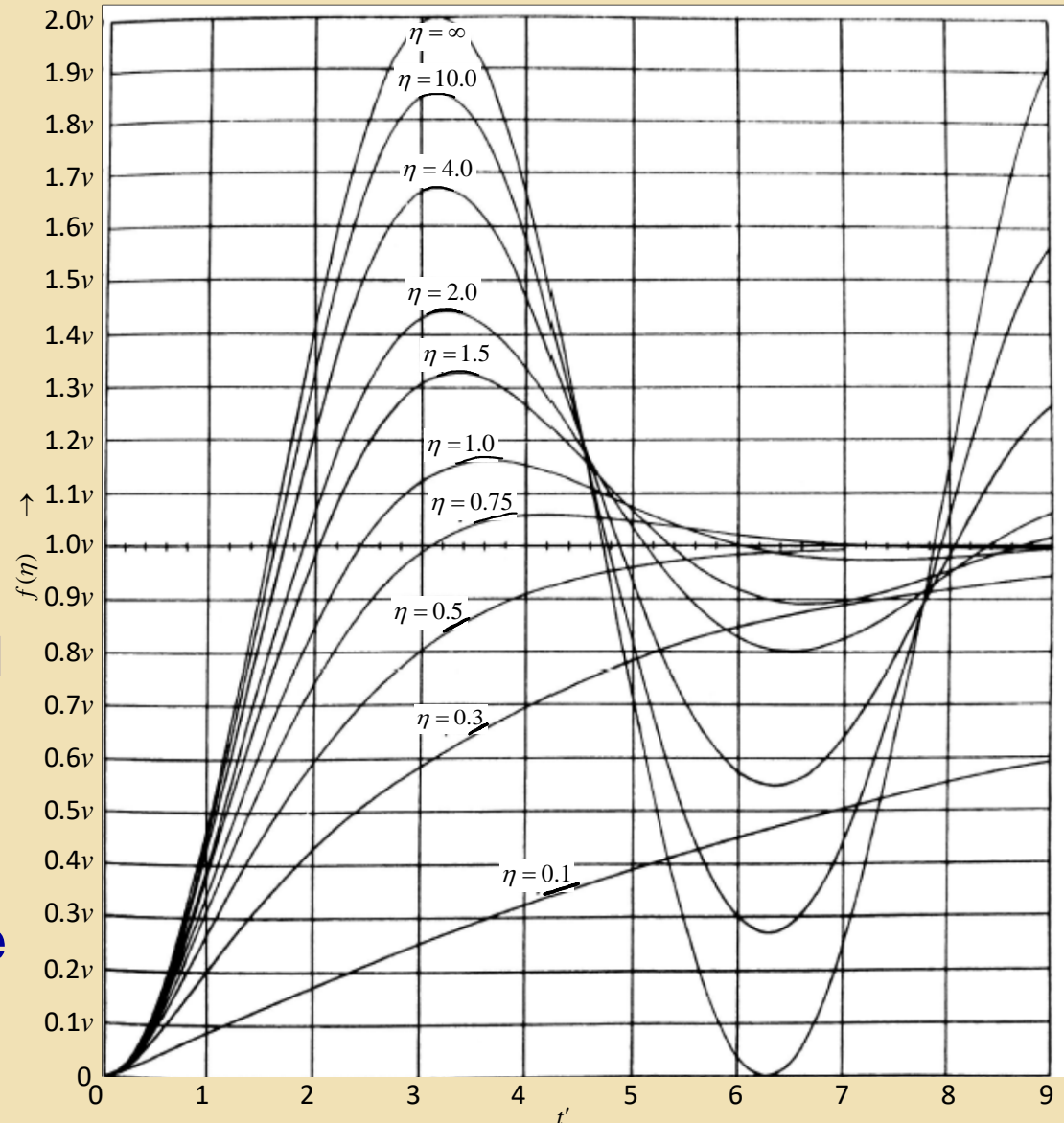
$$\eta = \frac{T_\phi}{T} = \frac{L/R}{\sqrt{LC}} = \frac{Z_o}{R}, \text{ is called the damping constant, and } t' = \frac{t}{T}$$

- The nature of the response depends on the value of η or the degree of damping. The response may be plotted for various values of η , as shown in the figure in the next slide. Which can be easily used to find the peak value and other characteristics of the response.
- It can be seen that the damping decreases as η increases.

The RLC Circuit: Damping

$$f(\eta) = v \left[1 - \varepsilon \frac{t'}{2\eta} \left(\frac{\sin(4\eta^2 - 1)^{\frac{1}{2}} \frac{t'}{2\eta}}{(4\eta^2 - 1)^{\frac{1}{2}}} \right) + \cos(4\eta^2 - 1)^{\frac{1}{2}} \frac{t'}{2\eta} \right] \quad [3]$$

- It is seen that the system is undamped and the maximum voltage is $2v$, as expected for $\eta = \infty$ ($R \approx 0$).
- For η between (0.5 and ∞), the response is oscillatory.
- At $\eta = 0.5$, the system is critically damped and the response ceases to be oscillatory.
- For $\eta < 0.5$, the system is over damped and the response is quite slow.



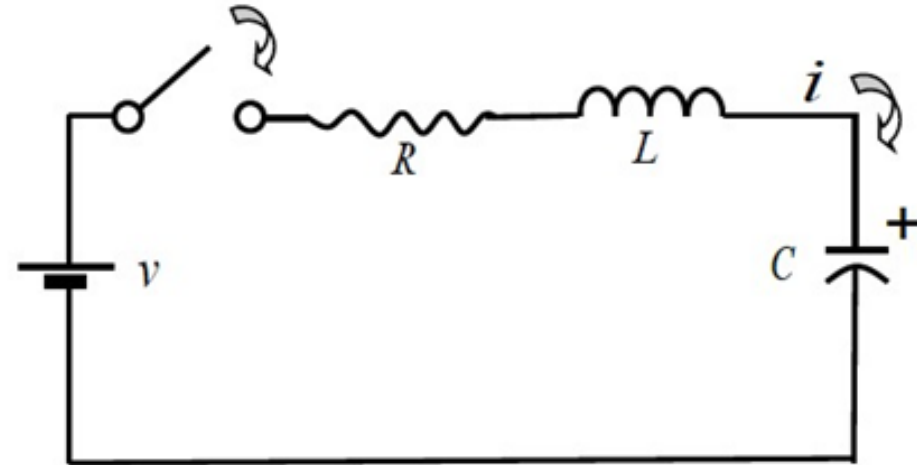
Exercise

It is given for the circuit that:

$R = 4.7 \, \Omega$, $L = 1.96 \, \text{mH}$,

$C = 10 \, \mu\text{F}$, and $V = 11 \, \text{kV}$

Estimate the maximum voltage across C when the switch is closed.



As above,

$$v_C(s) = v \frac{1}{T^2} \frac{1}{s \left[s^2 + s/T_\phi + 1/T^2 \right]}$$

$$Z_0 = \sqrt{L/C} = \sqrt{1960/10} = 14 \, \Omega \quad \text{and} \quad \eta = Z_0/R = 14/4.7 = 2.98$$

From the figure, the multiplying factor ≈ 1.55

Therefore, maximum voltage $= 1.55 \times 11 \approx 17 \, \text{kV}$

This maximum voltage occurs at : $t' \approx 3.1$, or $t \approx 3.1 \times T$
($T = \sqrt{LC}$)

Why is LC circuit important?

- LC circuits are encountered everywhere in power systems.
 - Systems, e.g., lines and capacitors
 - Power system components, e.g., transformers.
 - All power components have capacitance. For example, transformers have capacitance between turns, from conductors to core and to tank. All are important different transient conditions.

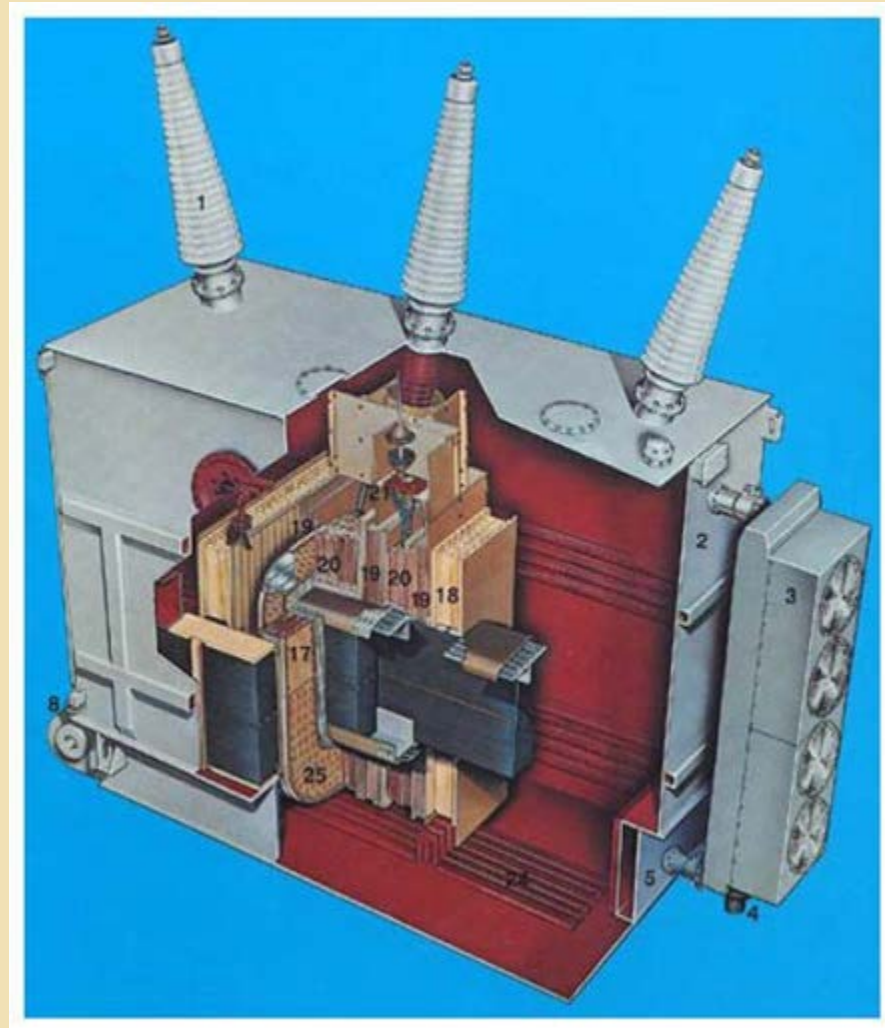
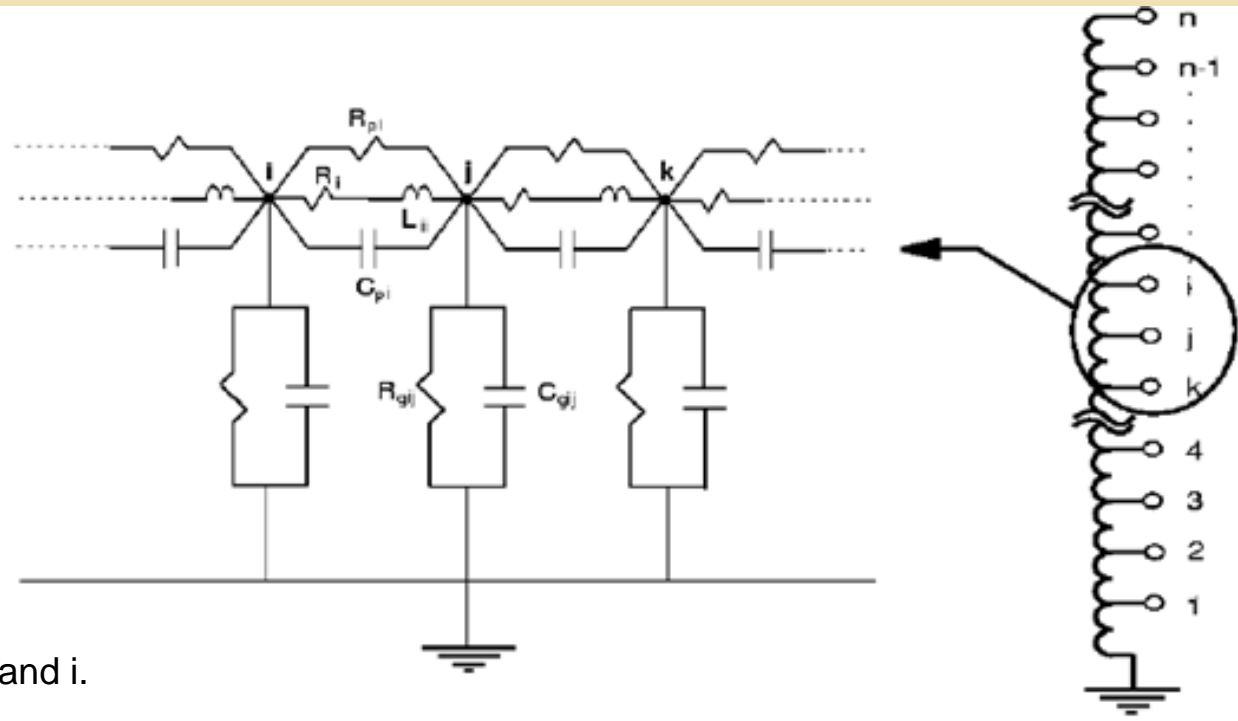


Figure: Transformer cut away view

Why is LC circuit important?

Section of Model Coil

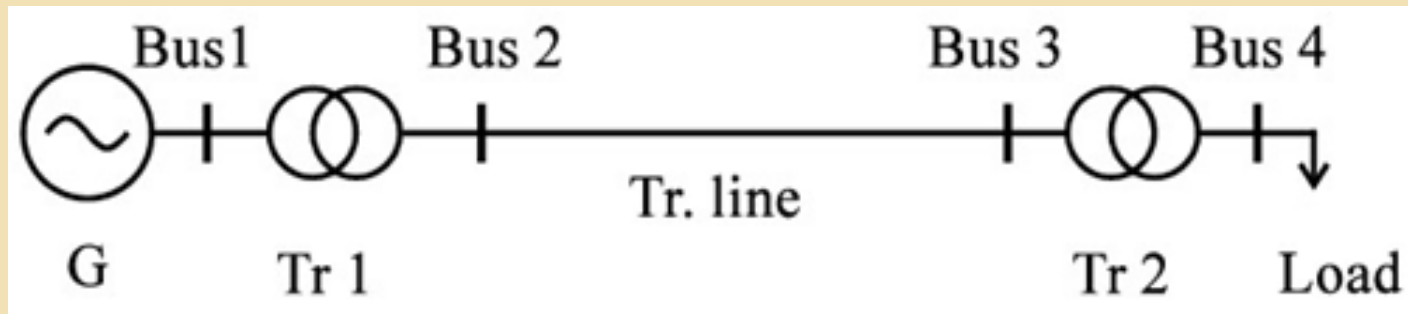


Mutual inductances should be considered, i.e., M_{ji} is between j and i .

- The details needed or desired for the model depend on specific problems.
- So is the importance or significance of resistance (R).

Power System Components: Models & Parameters

- Power system performance is commonly analysed by using the equivalent circuit of one phase of the circuit in actual units or in per unit values.
- The component parameters are obtained from the rating or name plate of the component.



- Star phase analysis: For a balanced three-phase system, star phase of the system may be obtained by:
 - Reducing line voltage by a factor of $\frac{1}{\sqrt{3}}$: $V_{ph} = \frac{V_L}{\sqrt{3}}$
 - Reducing delta connected impedances by a factor of $\frac{1}{3}$: $Z_Y = \frac{Z_{\Delta}}{3}$

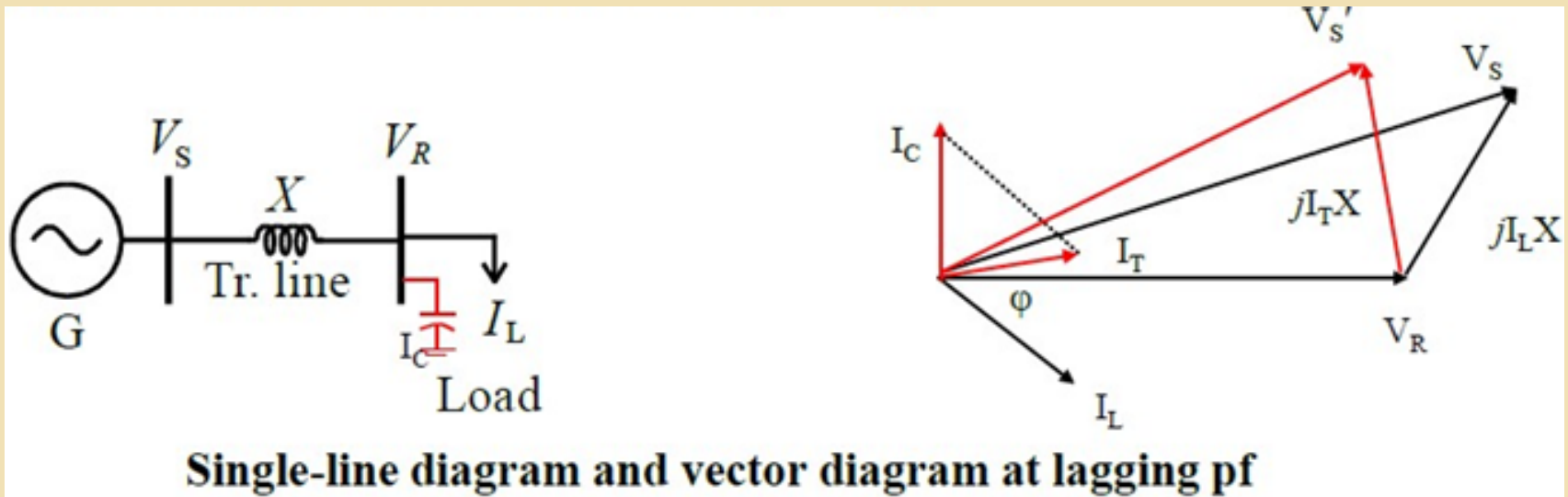
Per Unit (pu) System

- A very convenient technique especially when several voltage levels are involved in a system.
- A proper and consistent base must be specified a different sections of the system to obtain appropriate pu values.

1-phase system or per phase basis	3-phase system
base S (kVA) $S_b = \text{Rated MVA}$	base S (kVA) $S_b = \text{Rated 3-ph MVA}$
base kV $V_b = \text{Rated kV}$	base kV $kV_b = \text{Rated line kV}$
Then,	Then,
Base current (A) $I_b = \frac{kVA_b}{kV_b}$	Base current (A) $I_b = \frac{kVA_b}{\sqrt{3} kV_b}$
Base impedance $Z_b = \frac{kV_b \times 1000}{I_b}$ $= \frac{(kV_b)^2 \times 1000}{kVA_b} = \frac{(kV_b)^2}{MVA_b}$	Base impedance $Z_b = \frac{(kV_b / \sqrt{3}) \times 1000}{I_b}$ $= \frac{(kV_b)^2 \times 1000}{kVA_b} = \frac{(kV_b)^2}{MVA_b}$
Then pu values are calculated as:	Then pu values are calculated as:
pu value $= \frac{\text{Actual Value}}{\text{Base Value}}$	pu value $= \frac{\text{Actual Value}}{\text{Base Value}}$

Shunt Compensation – for System Voltage Control

- The variation in voltage levels between different locations can be quite significant depending up the load level and the power factor
- For lagging pf loads, it is obvious that $V_S \gg V_R$



- If $V_R = 1\text{pu}$, $V_S > 1\text{pu}$, and
If $V_S = 1\text{pu}$, $V_R < 1\text{pu}$
- Shunt compensation consisting of capacitors banks are commonly utilized to control the power factor to maintain V_R and V_S within acceptable range around 1pu.
- After the compensation, $V_S' \approx V_R$

Capacitor Switching Transients

- Consider a solidly grounded 60-Hz, 34.5kV distribution system:
 - Fault current at the bus = 25kA
 - C_1 is 18MVA bank,
 - C_2 is 10MVA bank.

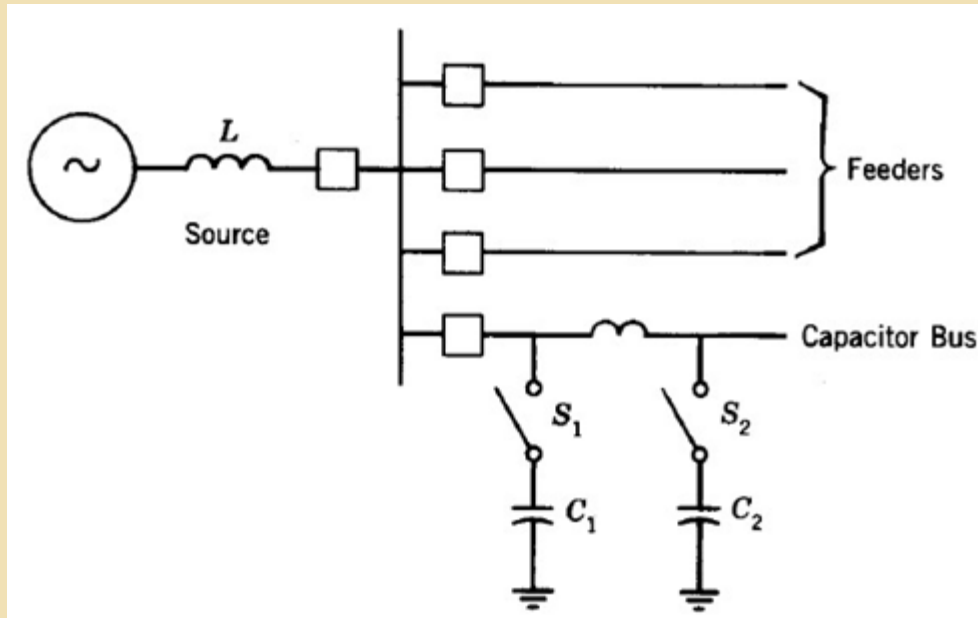
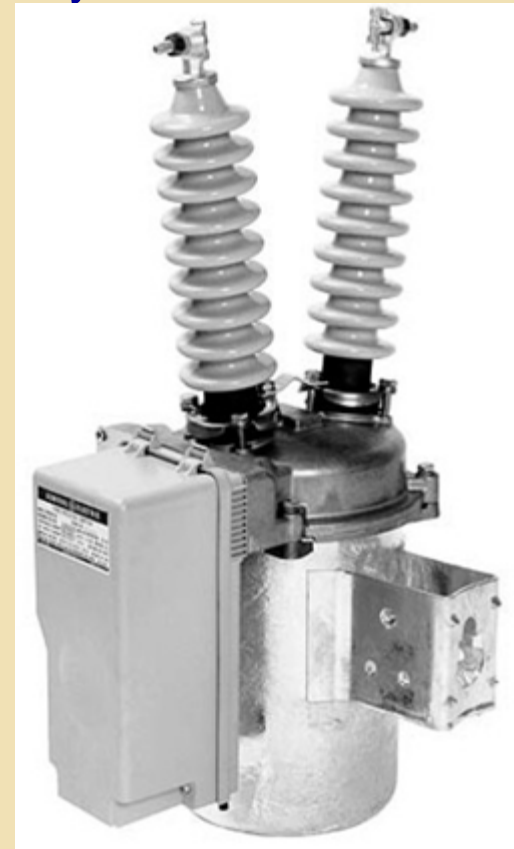
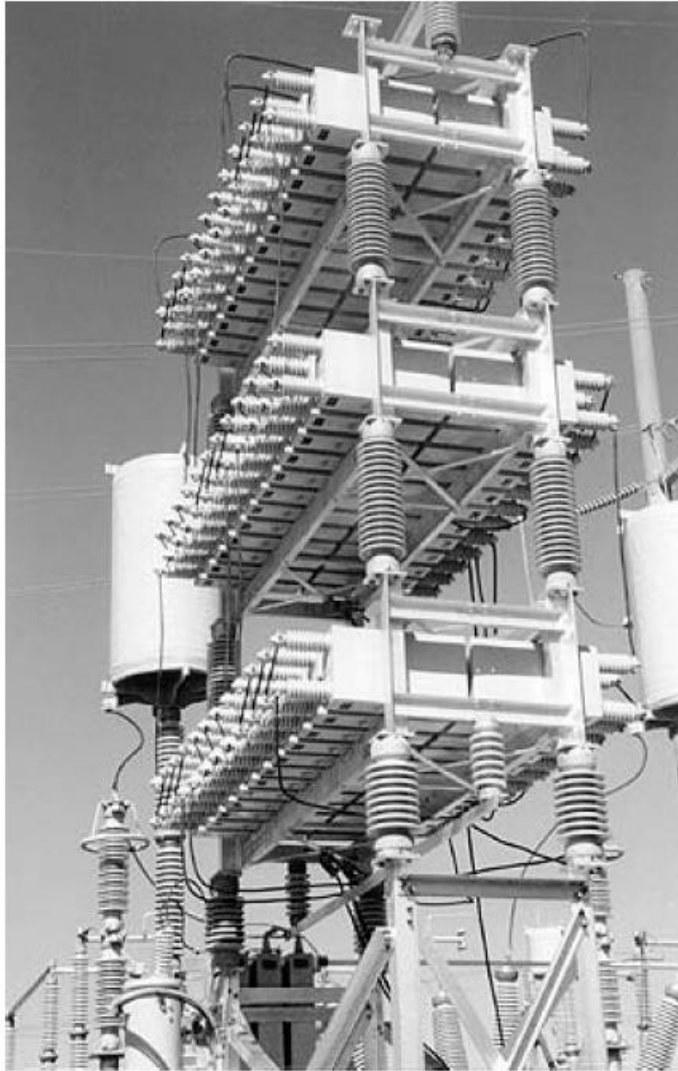


Figure 3.4 Switching capacitors on a substation bus [3].



- We investigate the transients following the closing of switch S_1 (picking up a capacitor bank)

Capacitors



Capacitor Switching Transients

- The effective circuit for closing the C_1 is as shown:

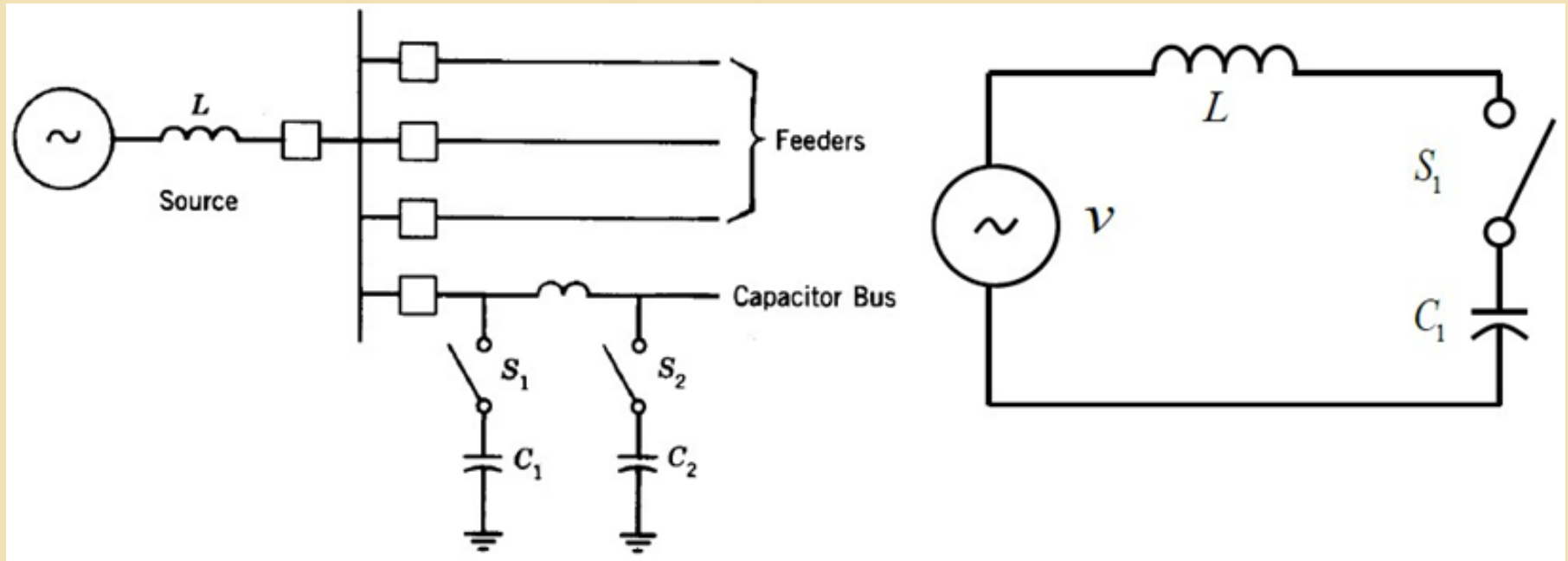


Figure 3.4 Switching capacitors on a substation bus [3].

- Find the parameter of the circuit

– Find the value of L from the fault level specified at the bus:

$$V_{LN} = \frac{34.5kV}{\sqrt{3}}, I_f = 25kA \Rightarrow Z_f = \frac{34.5kV}{\sqrt{3} \times 25kA} = 0.797\Omega = 2 \times \pi \times f \times L,$$

$$L = \frac{0.797\Omega}{2 \times \pi \times 60} = 0.0021H$$

Capacitor Switching Transients

- Find the value of C_1 from the capacitor bank rating:

$$S_{C1} = 18MVA \Rightarrow 18 \times 10^6 = \sqrt{3}VI \Rightarrow I = \frac{18 \times 10^6}{\sqrt{3} \times 34.5 \times 10^3} = 301.2A$$

$$X_C = \frac{V_{LN}}{I} = \frac{34500}{\sqrt{3} \times 301.2} = 66.1\Omega = \frac{1}{2 \times \pi \times f \times C_1} \Rightarrow C_1 = \frac{1}{2 \times \pi \times f \times 66.1} = 40.1\mu F$$

- V is the instantaneous voltage across switch S_1 at the time of closing.
- If C_1 is charged at the time of closing, the initial voltage has to be accounted.
- Closing Switch S_1
 - Two characteristic parameters are:

$$\checkmark \text{ Surge impedance: } Z_o = \left(\frac{L}{C}\right)^{1/2} = \left(\frac{2.1 \times 10^{-3}}{40.1 \times 10^{-6}}\right)^{1/2} = 7.237\Omega$$

$$\checkmark \text{ Natural frequency: } \omega_o = \left(\frac{1}{LC}\right)^{1/2} = \left(\frac{1}{2.1 \times 10^{-3} \times 40.1 \times 10^{-6}}\right)^{1/2} = \frac{3446 \text{ rad}}{\text{s}}$$

$$f_o = \frac{3446}{2\pi} = 548.4 \text{ Hz}$$

Capacitor Switching Transients

- Inrush current from source in C_1 :

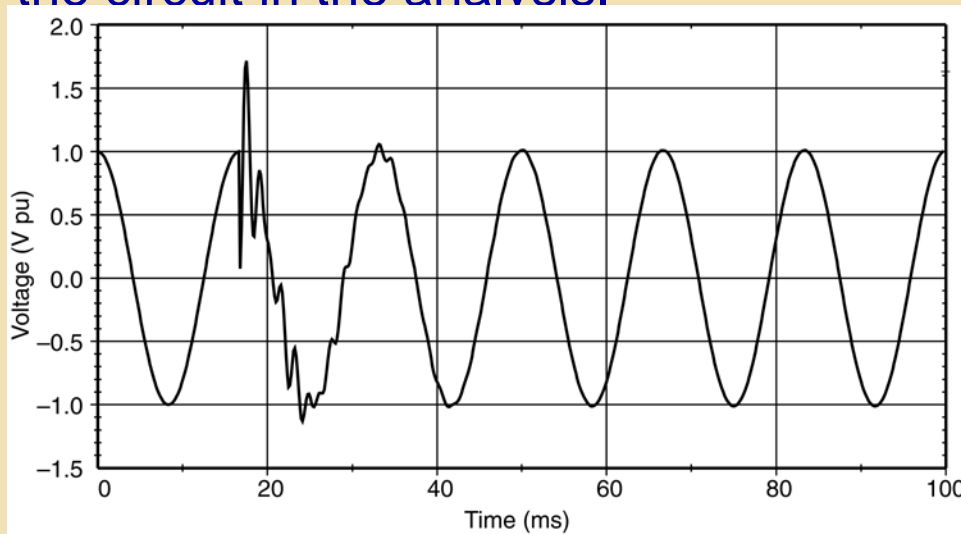
$$I_1 = \frac{V(0)}{Z_o} \sin \omega_o t$$

- Peak inrush current if the switch closes at the voltage peak:

$$i_{peak} = \frac{V_{peak}}{Z_o} = \frac{34.5\sqrt{2}}{\sqrt{3} \times 7.237} = 3.893\text{kA}$$

Which is considerably greater than the steady state current.

- The peak voltage across $C_1 = 2 \times 34.5 \times \sqrt{2/3} = 56.34\text{kV}$
- This peak voltage will not be realized if we include the resistance inherently present in the circuit in the analysis.



Notable Observations

- Remarks:

- Time of closing switch is important – it determines the initial voltage impressed.
- Voltage or charge on the capacitor at the instant of switching is important, since it will modify the initial voltage.
- Frequency of inrush is of the order of 548.45Hz, which is very high compared to the system frequency of 60Hz. For this purpose of the transient analysis, the system voltage may be assumed to remain constant at its initial value.
- The effect of damping has not been included in the above analysis. Some resistance is inherent in the system. The maximum over voltage of 200% is rarely observed. A peak value of between 130% to 170% is more common.

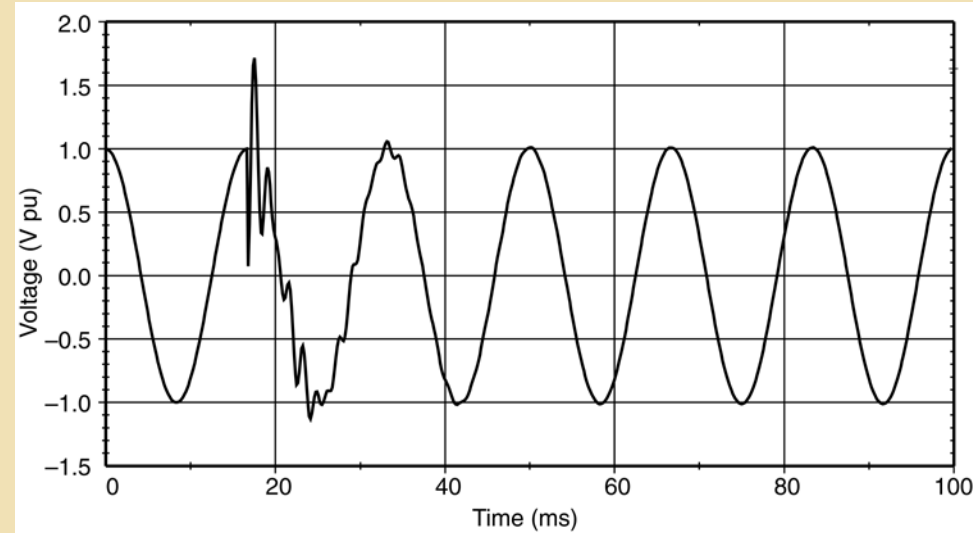
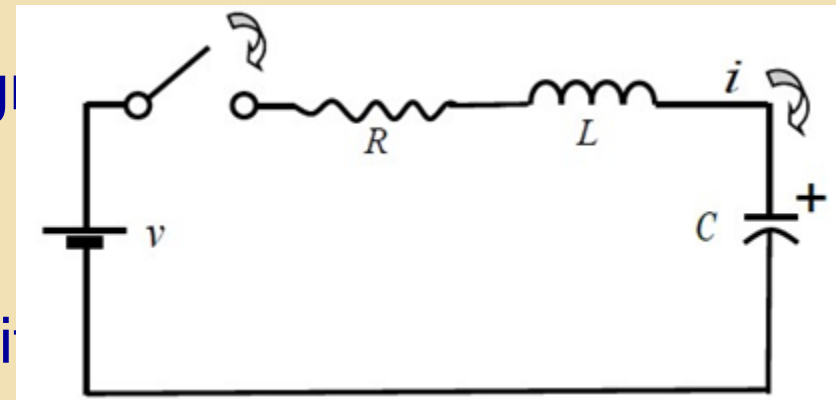


Figure 2.3: Low frequency oscillatory transient caused by capacitor bank energization, 34.5-kV bus voltage [3].

Exercise

A 13.8-kV substation is supplied from a three-phase, 50-Hz, 10-MVA source with equivalent reactance and resistance of 2pu and 0.5pu respectively. The substation is provided with shunt compensation of 1-MVA, 13.8-kV capacitor bank.

- Draw the equivalent circuit to analyse the switching of the capacitor bank
(practice drawing single line diagram and equivalent circuits).
- Determine the values of the circuit components.
- Practice applying pu systems.



$$[L = 121.2\text{mH}, R = 9.52\Omega, \text{ and } C = 16.71\mu]$$

Exercise – Previous Continue

- Derive the equation for the overvoltage during the switching process and determine the maximum expected peak voltage in each phase.
(hint: Make use of the RLC plots discussed earlier)

Practice deriving
$$v_c(s) = \frac{v}{T^2} \frac{1}{s(s^2 + \frac{1}{T} s + \frac{1}{T^2})}$$

Then, $Z_o = \sqrt{L/C} = \sqrt{1000 \left(\frac{121.2}{16.71} \right)} = 85.16\Omega$ and $\eta = \frac{Z_o}{R} = \frac{85.16}{9.52} = 8.95$

from slide 15 figure, about 1.83 is the multiplying factor,

therefore. The maximum voltage $V_{\text{peak}} \approx 1.83 \times 13.8 \times \sqrt{2/3} = 20.6\text{kv}$

- Calculate T and use the figure to find out how long after the switching does this maximum voltage occur, i.e., t and t' .

Managing Effects of Capacitor Switching Transients

- Switching times:
 - Avoid a match between capacitor switching time and load pick up time. Nuisance tripping's of Adjustable Speed Drives which are sensitive to over voltages are avoided.
- Insertion of switching resistances
 - Well established technique to insert resistances in series with the switch during switching times to increase damping to reduce the peak voltage.
- Synchronous closing
 - Its is a relatively new technique which attempts to switch on capacitors. When the instantaneous system voltage is approximately the same as the voltage across the capacitor.
- Adjusting capacitor location
 - The over voltage is maximum at the vicinity of the capacitor location. The effects may be reduced by moving the capacitor bank away from sensitive loads.

Managing the Effects of Magnification of Voltage

- Common way to manage/alleviate the effects of such overvoltages are:
 - Control the feeder capacitance switching transient. The methods have been discussed earlier
 - Provide suitable voltage arresters at customer's end.
 - Use of harmonic filters in place of capacitors at the customers end.
 - The presence of inductor improves the performance.
 - Provide suitable reactance's in series of sensitive loads such as variable speed drives.