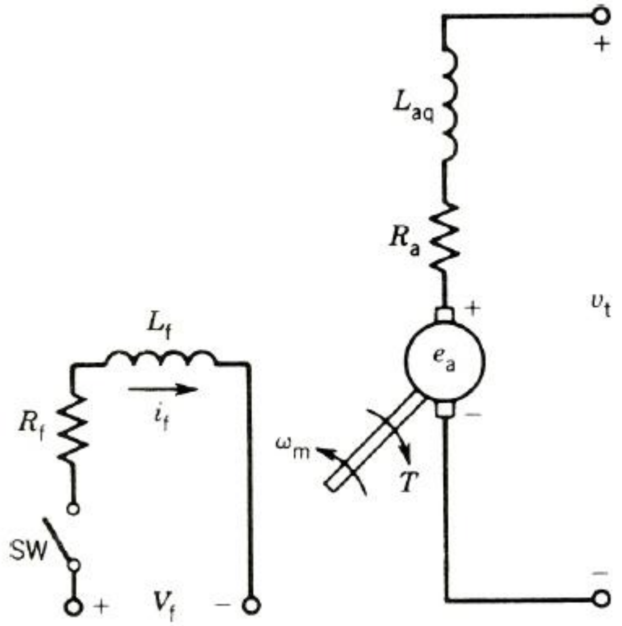
Chapter 5 Transients and Dynamics

Learning Objectives

- Understand transient function of generator
- Know dynamics of motoring mode
- Classify different generator/motor conditions

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.1 Separately Excited DC generator



- Armature inductance is represented by q-axis inductance
- Assume magnetic linearity is held

$$e_a = K_a \Phi \omega_m = K_f i_f \omega_m$$

$$T = K_a \Phi i_a = K_f i_f i_a$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.1 Separately Excited DC generator

- If armature is open-circuited, the generator can be running at a constant speed

$$V_f = R_f i_f + L_f \frac{di_f}{dt}$$

- The Laplace transform with zero initial conditions

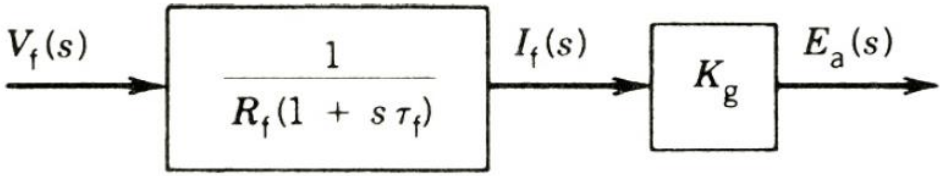
$$V_f(s) = R_f I_f(s) + L_f s I_f(s) = I_f(s)\left(R_f + sL_f\right)$$

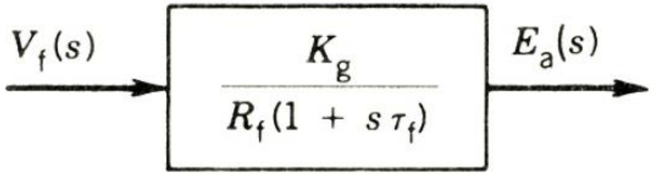
$$\frac{I_f(s)}{V_f(s)} = \frac{1}{R_f + sL_f} = \frac{1}{R_f\left(1 + s\tau_f\right)}$$

where $\tau_f = L_f / R_f$ is the time constant of the field circuit

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.1 Separately Excited DC generator





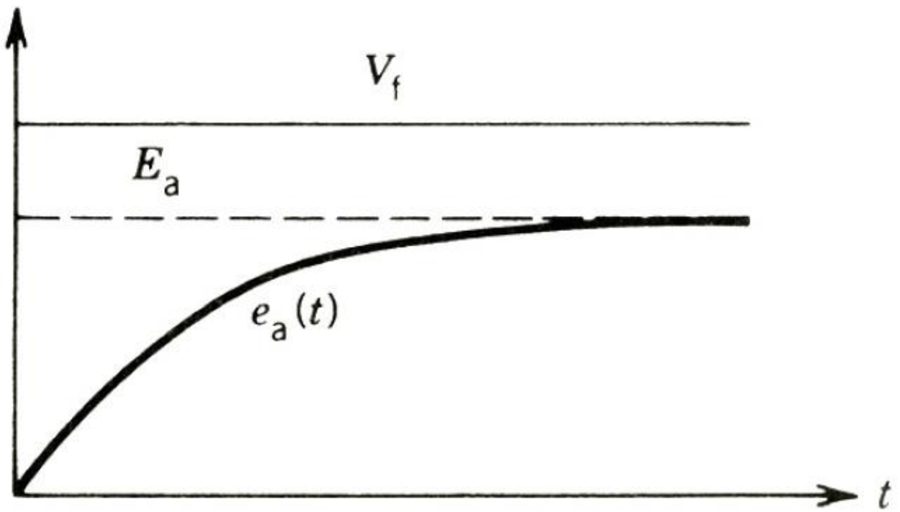
- The generated voltage in the armature circuit

$$e_a = K_f i_f \omega_m = K_a \Phi \omega_m$$

$$E_a(s) = K_g I_f(s)$$

$$\frac{E_a(s)}{V_f(s)} = \frac{E_a(s)}{I_f(s)} \cdot \frac{I_f(s)}{V_f(s)} = \frac{K_g}{R_f\left(1 + s\tau_f\right)}$$

5.1 Separately Excited DC generator



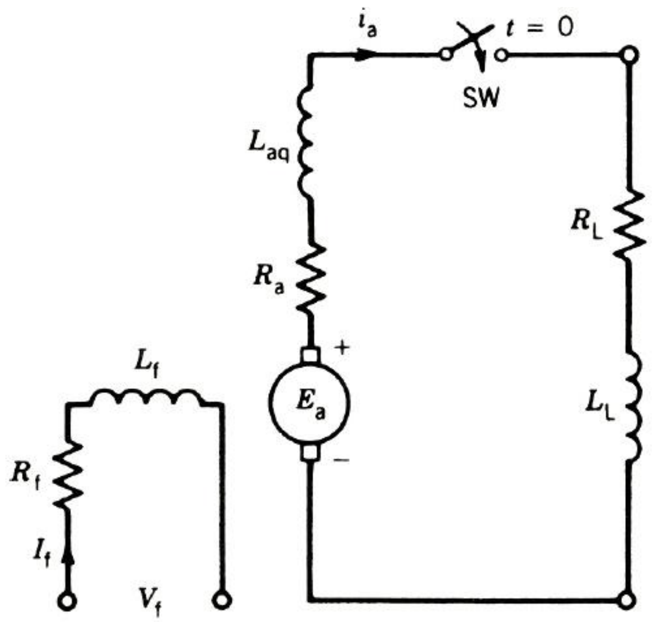
- Time domain response

$$e_a(t) = \frac{K_g V_f}{R_f}\left(1 - e^{-t/\tau_f}\right)$$

$$= E_a\left(1 - e^{-t/\tau_f}\right)$$

where $E_a = e_a(\infty) = K_a V_f / R_f = K_g I_f$ at steady-stage

5.1 Separately Excited DC generator



- If the load is connected to armature terminal

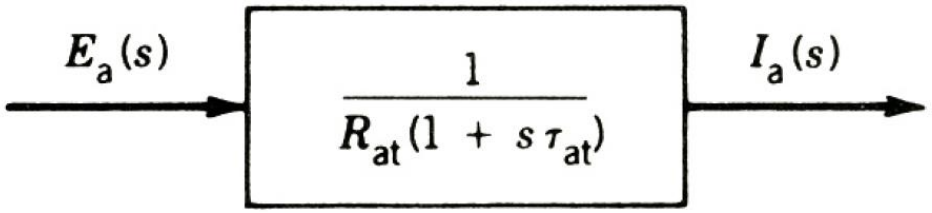
$$E_a = R_a i_a + L_{aq}\frac{di_a}{dt} + R_L i_a + L_L\frac{di_a}{dt}$$

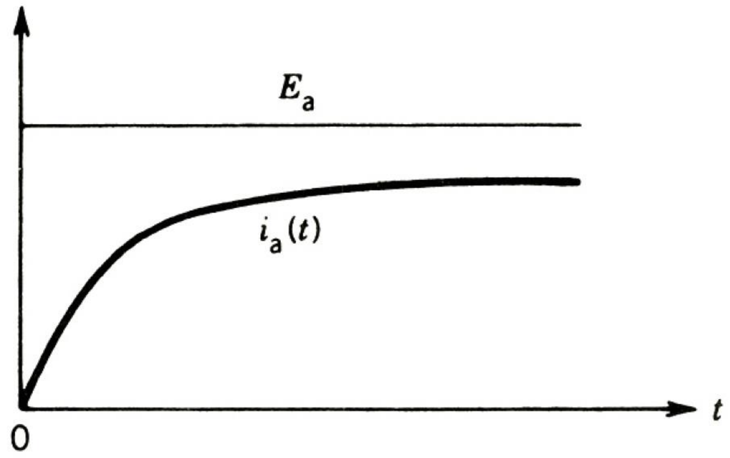
$$= (R_a + R_L)i_a + \left(L_{aq} + L_L\right)\frac{di_a}{dt}$$

$$= R_{at} i_a + L_{at}\frac{di_a}{dt}$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.1 Separately Excited DC generator





- The Laplace transform

$$E_a = R_{at}I_a(s) + L_{at}sI_a(s)$$

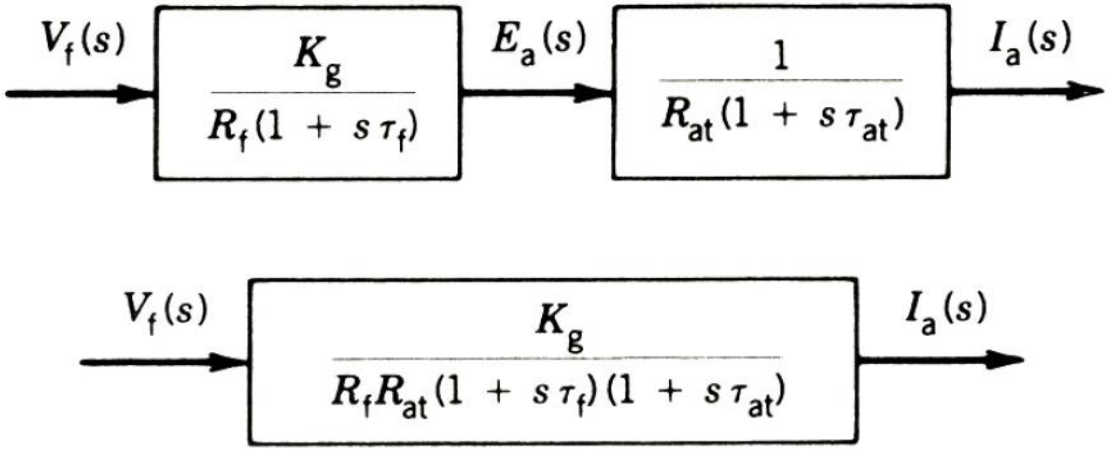
$$\frac{I_a(s)}{E_a(s)} = \frac{1}{R_{at}(1 + s\tau_{at})}$$

- Time domain response

$$i_a(t) = \frac{E_a}{R_{at}}\left(1 - e^{-t/\tau_{at}}\right)$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.1 Separately Excited DC generator



- Total transfer function

$$\frac{I_a(s)}{V_f(s)} = \frac{I_a(s)}{E_a(s)} \cdot \frac{E_a(s)}{V_f(s)} = \frac{K_g}{R_f R_{at}(1+s\tau_f)(1+s\tau_{at})}$$

$$V_f(s) = \frac{V_f}{s}$$

5.1 Separately Excited DC generator

- Combining the equations

$$I_a(s) = \frac{K_g V_f}{R_f R_{at}\left(1 + s\tau_f\right)\left(1 + s\tau_{at}\right)}$$

$$= \frac{K_g V_f}{R_f R_{at}\tau_f \tau_{at} s\left(s + 1/\tau_f\right)\left(s + 1/\tau_{at}\right)}$$

$$= \frac{A}{s\left(s + 1/\tau_f\right)\left(s + 1/\tau_{at}\right)}$$

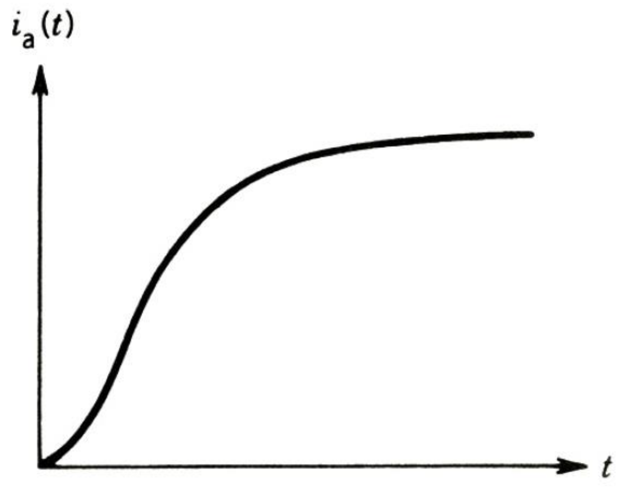
$$= \frac{A_1}{s} + \frac{A_2}{s + 1/\tau_f} + \frac{A_3}{s + + 1/\tau_{at}}$$

where $A_1 = \left.\frac{A}{\left(s + 1/\tau_f\right)\left(s + 1/\tau_{at}\right)}\right|_{s=0} = A\tau_f \tau_{at}$ $A_2 = \left.\frac{A}{s\left(s + 1/\tau_{at}\right)}\right|_{s=-1/\tau_f}$

$A_3 = \left.\frac{A}{s\left(s + 1/\tau_f\right)}\right|_{s=-1/\tau_{at}}$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.1 Separately Excited DC generator



- Time domain response

$$i_a(t) = A_1 + A_2 e^{-t/\tau_f} + A_3 e^{-t/\tau_{at}}$$

- A_1 is known as the steady-state value as $A_1 = i_a(\infty) = \left(K_g V_f\right)/\left(R_f R_{at}\right) = K_g I_f / R_{at} = E_a / R_{at}$

Concept Check 5.1

Consider a separately excited DC generator with following parameters

$R_f = 100\ \Omega$ $\qquad$ $L_f = 25$ H

$R_a = 0.25\ \Omega$ $\qquad$ $L_{aq} = 0.02$ H

$K_g = 100$ V $\quad$ per field ampere at rated speed

(a) The generator is operated at rated speed with a field circuit voltage of 200 V is suddenly supplied to the field winding, find

I. Armature-generated voltage as a function of time (Ans: $e_a(t) = 200(1 - e^{-t/0.25})$)
II. Steady-state armature voltage (Ans: 200 V)
III. Time required for armature voltage to rise to 90 % of its steady-stage (Ans: 0.575 s)

(b) The generator is operated at rated speed with load of 1 Ω and 0.15 H in series to the armature terminals. A field circuit of 200 V is suddenly supplied to the field winding. Find the armature current as a function of time (Ans: $i_a(t) = 160 - 351e^{-4t} + 191e^{-7.35t}$)

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

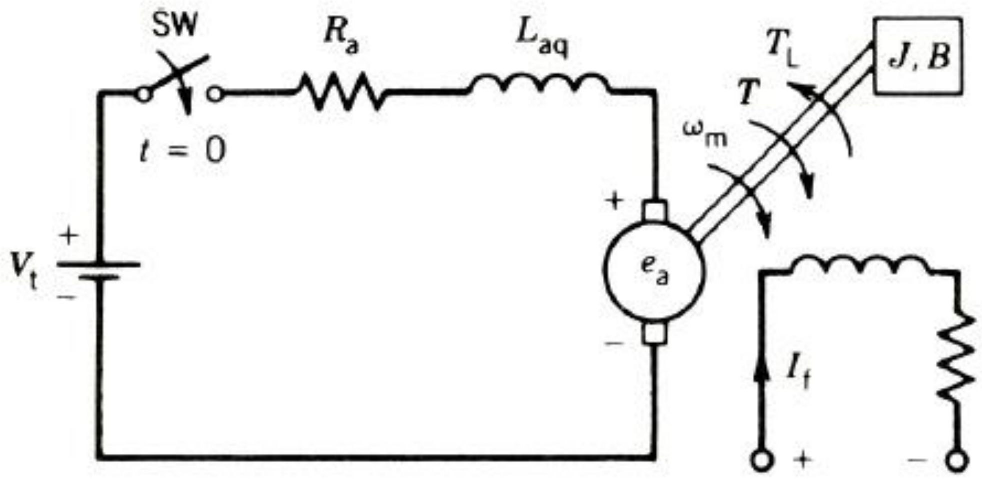
Concept Check 5.1

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Concept Check 5.1

Concept Check 5.1

5.2 DC Motor Dynamics



- DC motor is usually controlled with constant field excitation and varied voltage
- Assume magnetic linearity is held

$$T = K_f i_f i_a = K_m i_a$$

$$e_a = K_f i_f \omega_m = K_m \omega_m$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.2 DC Motor Dynamics

- The Laplace transform

$$T(s) = K_m I_a(s)$$

$$E_a(s) = K_m \omega_m(s)$$

- After the switch is closed

$$V_t = e_a + R_a i_a + L_{aq}\frac{di_a}{dt}$$

$$= K_m \omega_m + R_a i_a + L_{aq}\frac{di_a}{dt}$$

- The Laplace transform with zero initial conditions

$$V_t(s) = K_m \omega_m(s) + R_a I_a(s) + L_{aq} s I_a(s)$$

$$= K_m \omega_m(s) + I_a(s) R_a (1 + s\tau_a)$$

where $\tau_a = L_{aq} / R_a$ is the time constant of the armature

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.2 DC Motor Dynamics

- Dynamic equation for the mechanical system

$$T = K_m i_a = J\frac{d\omega_m}{dt} + B\omega_m + T_L$$

- The Laplace transform

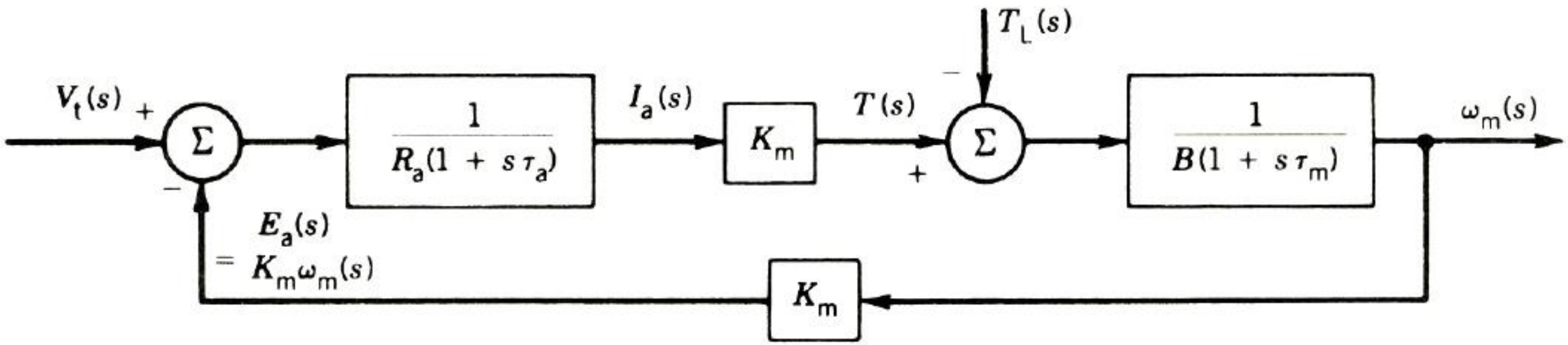
$$T(s) = K_m I_a(s) = Js\omega_m(s) + B\omega_m(s) + T_L(s)$$

$$\omega_m(s) = \frac{T(s) - T_L(s)}{B(1 + sJ/B)} = \frac{K_m I_a(s) - T_L(s)}{B(1 + s\tau_m)}$$

where $\tau_m = J / B$ is the mechanical time constant of the system

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.2 DC Motor Dynamics



- Combining equations

$$I_a(s) = \frac{V_t(s) - E_a(s)}{R_a(1 + s\tau_a)} = \frac{V_t(s) - K_m\omega_m(s)}{R_a(1 + s\tau_a)}$$

5.2 DC Motor Dynamics

- If load torque is proportional to speed

$$T_L = B_L \omega_m$$

$$J = J_{motor} + J_{load}$$

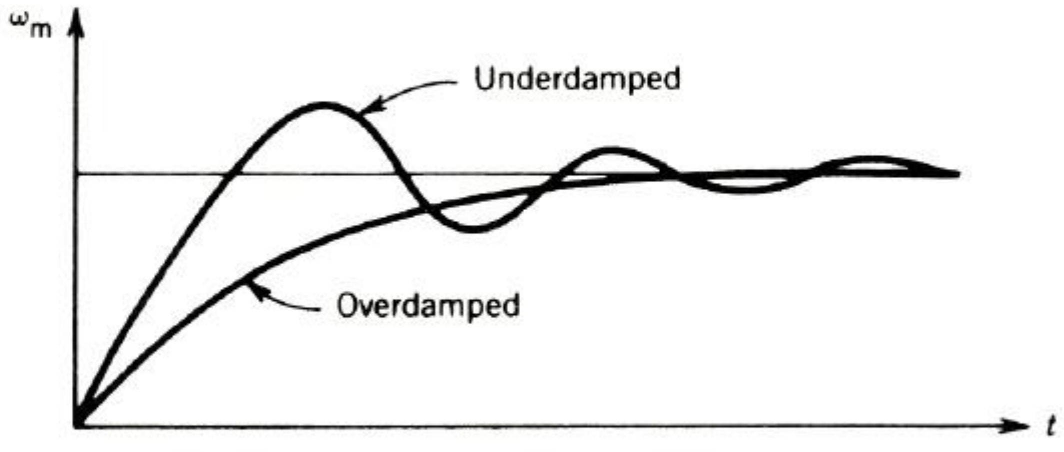
$$K_m I_a(s) = J s \omega_m(s) + B_m \omega_m(s) + B_L \omega_m(s)$$

$$= J s \omega_m(s) + (B_m + B_L)\omega_m(s)$$

$$= J s \omega_m(s) + B \omega_m(s)$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.2 DC Motor Dynamics



- Load increases the viscous friction of the system

$$V_t(s) = K_m \omega_m(s) + \frac{BR_a}{K_m}(1 + s\tau_m)(1 + s\tau_a)\omega_m(s)$$

$$\frac{\omega_m(s)}{V_t(s)} = \frac{1}{K_m + (B\,R_a/K_m)(1 + s\tau_m)(1 + s\tau_a)}$$

- The response can be underdamped or overdamped based on the values of time constants and other parameters

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.2 DC Motor Dynamics

- If armature inductance is neglected

$$\frac{\omega_m(s)}{V_t(s)} = \frac{1}{K_m + (B\,R_a/K_m)(1 + s\tau_m)}$$

$$= \frac{K_m}{K_m^2 + R_a B} \cdot \frac{1}{1 + s\tau'_m}$$

where $\tau'_m = \frac{R_a B}{K_m^2 + R_a B}\tau_m < \tau_m$

- If viscous fiction is zero

$$K_m I_a(s) = J s \omega_m(s)$$

$$V_t(s) = K_m \omega_m(s) + \frac{J\omega_m(s) R_a (1 + s\tau_a)}{K_m}$$

$$\frac{\omega_m(s)}{V_t(s)} = \frac{1}{K_m + (J\,R_a/K_m)s(1 + s\tau_a)}$$

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

5.2 DC Motor Dynamics

- If the supply is suddenly disconnected

$$T = K_m i_a = J\frac{d\omega_m}{dt} + B\omega_m = 0$$

$$B\omega_m = -J\frac{d\omega_m}{dt}$$

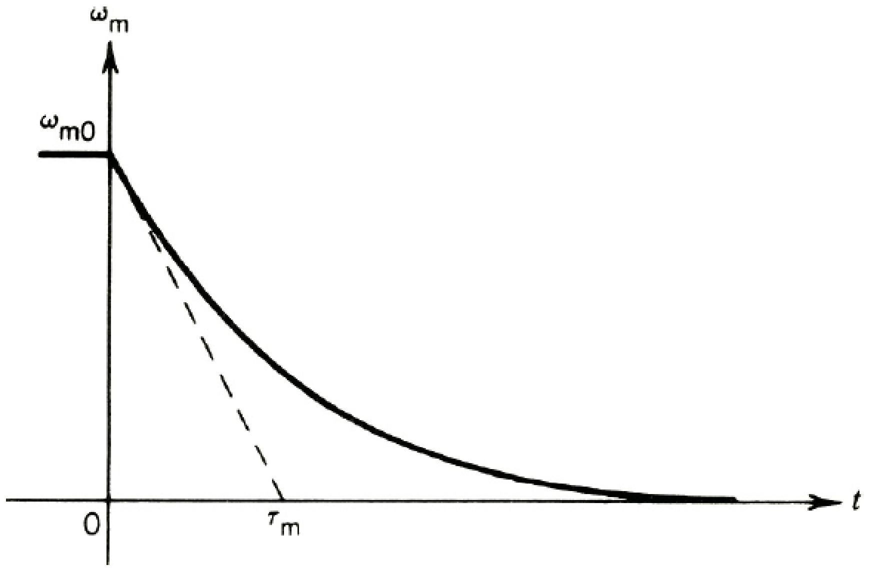
- The Laplace transform

$$B\omega_m(s) = -J[s\omega_m(s) - \omega_{m0}]$$

$$\omega_m(s) = \frac{J\omega_{m0}}{B + sJ} = \frac{\omega_{m0}}{s + B/J}$$

$$= \frac{\omega_{m0}}{s + 1/\tau_m}$$

5.2 DC Motor Dynamics



- The time domain response of speed

$$\omega_m(t) = \omega_{m0} e^{-t/\tau_m}$$

- It decreases exponentially with time constant τ_m
- The intersection of the initial slop represents mechanical time constant

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Concept Check 5.2

Consider a separately excited DC motor with following parameters

$$R_a = 0.5\ \Omega \qquad L_{aq} \simeq 0 \qquad B \simeq 0$$

With a field current of 1 A, the motor can generate an open-circuit armature voltage of 220 V at 2000 rpm. The motor drives a constant load torque of 25 Nm. The combined inertia of motor and load is 2.5 kgm^2.

(a) Derive expression for speeds and armature current as a function of time (Ans: $\omega_m(t) = 198.2(1 - e^{-0.882t})$; $i_a(t) = 23.8 + 416.2e^{-882t}$)

(b) Find the steady-state values of the speed and armature current (Ans: 198.2 rad/s; 23.8 A)

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Concept Check 5.2

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Concept Check 5.2

NANYANG TECHNOLOGICAL UNIVERSITY SINGAPORE

Concept Check 5.2