PART C

TRANSIENT STABILITY MODELS AND CONTROL

1. SYNCHRONOUS GENERATOR

Materials of this section are taken from parts of Chapter 12, Elgerd.

Under this chapter, we will be concerned with the behavior of synchronous generators under large disturbance conditions. In most cases, this so-called transient stability study would have to be carried out using digital computer simulation program, in which the models of generator and its excitation and speed-governor control systems would be included. The key issue is the determination of the rotor angle accurately and this would require accurate models of the various network components, of which the generator is only one.

1.1. Basic Assumptions:

1. In transient stability study, often we will have to represent the generators individually, unless one is certain a group of generators are behaving in the same way. In which case, an equivalent model is acceptable;
2. If the generators behave in a coherent manner, then the use of a common area frequency would be reasonable. However, if the generators do not behave in a coherent manner, then it is more practical to use the rotor angle position δ_i to convey the stability state of the i^{th} machine, rather than based on the rotor speed alone;
3. Due to the high inertia of generators, usually the individual rotor speed deviations are small relative to the synchronous speed. So one could assume all the static portion of the network to be in the power frequency (50Hz or 60Hz) steady-state. All the voltages and currents and power will therefore be computed from the algebraic equations, and for balanced network, these are shown under the load flow formulation. Individual generators will be described by the differential equations, and the machines are mutually coupled via the network algebraic equations of lines and transformers;

4. Under unbalanced conditions, we will only consider the positive-phase sequence currents and voltages, because only the positive-phase sequence components would result in the synchronizing torques in the generators.

1.2. The Swing Equation

Consider the ith generator in a general network. Its turbine power is denoted as P_{Ti}, (which could be the same as the mechanical power P_{Mi} considered later). Let the output generator power be P_{Gi}. If there is a mismatch in the two powers, the difference will

1. change the stored kinetic energy, or rotor speed, of the unit;

2. to overcome the damping torque that develops in the damper windings.

So, mathematically,

$$P_{Ti} - P_{Gi} = \frac{d}{dt}(W_{kin,i}) + P_{\text{damper}} \qquad (12\text{-}1)$$

***The kinetic energy term*:**

$W_{kin,i}$ represents the kinetic energy in the generator-turbine system, then the 1st term on the RHS of (12.1) becomes

$$\frac{d}{dt}W_{\text{kin},i} = \frac{W^0_{\text{kin},i}}{\pi f^0}\frac{d^2\delta_i}{dt^2} \qquad \text{MW} \qquad (12\text{-}2)$$

because

$f = (1/2\pi)\,(d\delta/dt)$

$dW_{kin,i}/dt = (2W_{kin,i}^0/f_0)\,(df/dt)$

$W_{kin,i}^0$ is the kinetic energy of unit *i* measured in mega-joules at rated frequency. The angle δ_i is the rotor position of unit *i* with respect to the synchronous frame of reference.

The damping term:

As the speed of the rotor deviates from the synchronous speed, currents will be induced in the damper windings, and a retarding torque will be produced to oppose the motion. This effect can be approximated by the equation

$$P_{\text{damper}} \approx D_i \frac{d\delta_i}{dt} \qquad \text{MW} \qquad (12\text{-}3)$$

where D_i is a positive machine parameter measured in MW/electrical degree per sec. So from (12.2) and (12.3), we obtain

$$P_{Ti} - P_{Gi} = \frac{W^0_{\text{kin},i}}{\pi f^0} \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} \qquad \text{MW} \qquad (12\text{-}4)$$

The swing equation in p.u. form:

Divide (12.4) by the base MVA, we have

$$P_{Ti} - P_{Gi} = \frac{H_i}{\pi f^0} \frac{d^2\delta_i}{dt^2} + D_i \frac{d\delta_i}{dt} \qquad \text{per unit MW} \qquad (12\text{-}5)$$

where H_i is the p.u. inertia constant of unit *i*.

Usually the practical way to find the solution of the swing equation is through numerical means. Interested readers may wish to consult Elgerd or Kundur for some numerical examples.

1.3. The Transient Turbine Power P_T

P_T changes are entirely dependent upon of the control actions of the speed-governor which will be discussed in another chapter. Normally in the initial first second or so, P_T is almost constant and hence, one can assume that $P_T = P_T^0$ = constant.

1.4. Transient generator power P_G.

One has to derive expression relating P_G with δ during the transient swings. During a disturbance, terminal voltage will change from its pr-disturbance value V_0 to post-disturbance value V^f, as shown in Fig 12.3. The change in voltage will cause a corresponding change in current and also the real power.

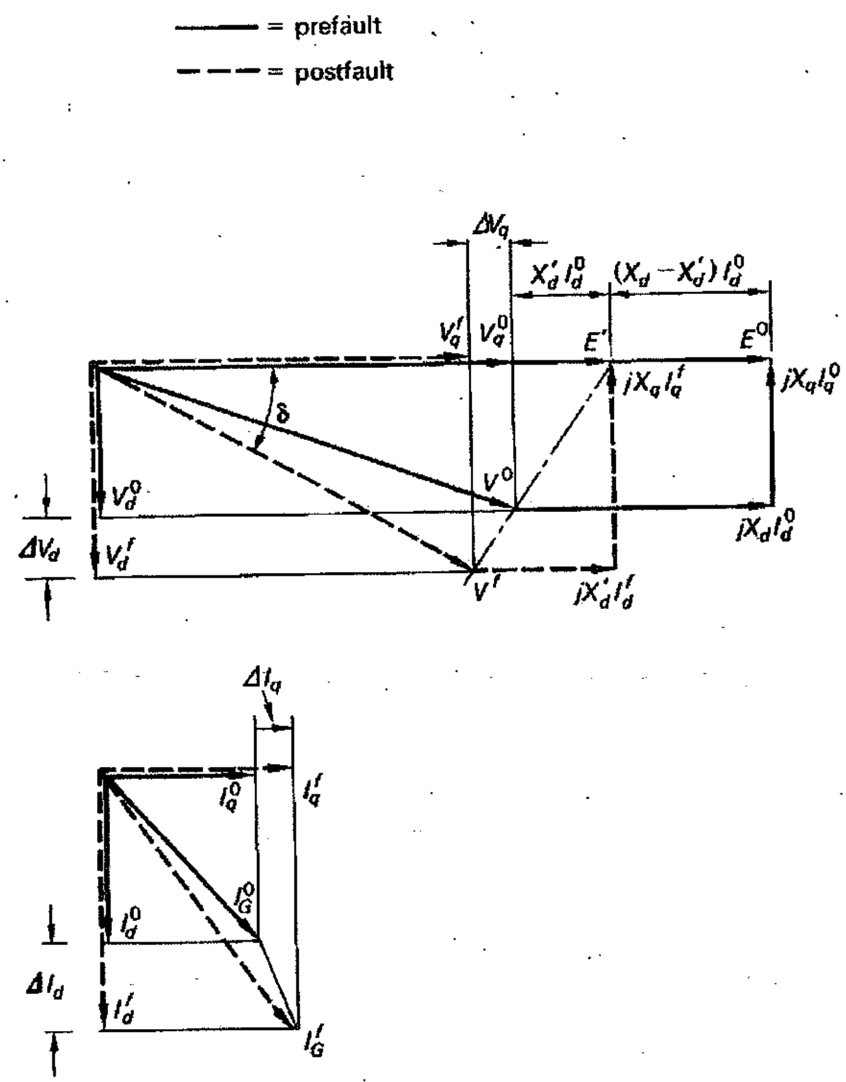


Fig. 12-3 Phasor diagram showing the relationships between prefault and postfault voltages and currents.

The pre-disturbance state:

Fig 12.3 shows the phasor diagram before (solid lines) and immediately after the disturbance (dotted lines).

Pre-disturbance steady-state current I_G^0 d and q –axis currents are

$$|I_q^0| = \frac{|V_d^0|}{X_q} \tag{12-7}$$

$$|I_d^0| = \frac{|E^0| - |V_q^0|}{X_d} \tag{12-8}$$

The pre-disturbance real power is the real part of $V_0(I_G^0)^*$ and is equal to equation (4.65) described earlier.

The post-disturbance state:

As a result of the disturbance, the d and q axis components of the voltage and current changes by the amounts ΔV_d, ΔV_q, ΔI_d and ΔI_q respectively.

Also from earlier discussion of generator under sudden short-circuit conditions, any changes in i_d brings proportional changes in i_r. The effect of this is to reduce the effective reactance from the steady-state value X_d to the transient value X_d'. A gradual change in the voltage difference $|E| - |V_q|$ would, from equation (12.8) brings about a change determined by X_d. A very sudden change, $|V_q|$ will on the contrary cause a much larger current change, which can be computed from

$$|\Delta I_d| = \frac{|\Delta V_q|}{X_d'} \tag{12-9}$$

In the q-axis, the flux associated with i_q does not link with the rotor field winding. Except for the damper winding actions which would be over in a couple of cycles, therefore one can use

$$|\Delta I_q| = \frac{|\Delta V_d|}{X_q} \tag{12-10}$$

with confidence.

Now consider the situation when the generator is suddenly disconnected from the network. The sudden current change would be I_d^0 and I_q^0 and from equations (12.9) and (12.10), the sudden change in the terminal voltage components are $X_d'I_d^0$ and

$X_qI_q^0$. The generator terminal voltage will suddenly change to the new value E' shown in Fig 12.3.

From Fig 12.3, E' is given by

$$|E'| = |E^0| - (X_d - X_d')\,|I_d^0| \qquad (12\text{-}11)$$

Substitute I_d^0 from (12.8) into the last equation, we have

$$|E'| = \frac{X_d'\,|E^0| + (X_d - X_d')\,|V_q^0|}{X_d} \qquad (12\text{-}12)$$

where E' is expressed in terms of the pre-disturbance voltages.

Transient power formula:

Fig 12.3 clearly indicates that the transient post-disturbance voltage phasors and E' are related to each other in a similar manner as the static pre-disturbance voltage phasors and E^0. In fact, the static and transient phasor diagrams become identical is we make the substitutions

$$E^0 \rightarrow E'$$
$$X_d \rightarrow X_d'$$

Due to the above identity, we can derive the transient power expression by making the above substitution to equation (4.65) to give

$$P_G = \frac{|V^f|\,|E'|}{X_d'}\sin\delta + \frac{|V^f|^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d'}\right)\sin 2\delta \qquad (12\text{-}13)$$

where $\delta = \angle E' - \angle V^f$ and is not to be confused with the angle δ_i in equation (12.2).

Transient power formula, neglecting saliency:

P_G contains a sin 2δ term but usually it is rather small. So if we neglect it, in effect we say

$$X_q = X_d' \qquad (12\text{-}16)$$

and the transient power becomes

$$P_G \approx \frac{|V^f|\,|E'|}{X_d'} \sin\delta \qquad (12\text{-}17)$$

If equation (12.16) applies, then the phasor diagram of Fig 12.3 simplifies to Fig 12.5.

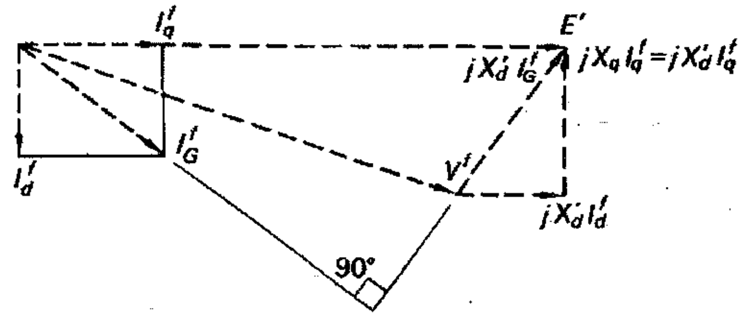


Fig. 12-5 If saliency is neglected, i.e., if $X_q = X_d'$, the post-fault portion of Fig. 12-3 looks like this.

Note that the phasor voltage difference E' – V^f is 90° ahead of I_G^f. That is

$$V^f = E' - jI_G{}^f X_d' \qquad (12\text{-}18)$$

In this case, one can represent the generator with the equivalent diagram Fig 12.6 to represent the generator under this condition. *E'* is the emf behind the transient reactance X_d'.

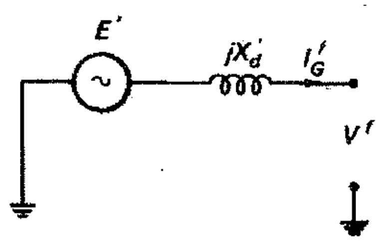


Fig. 12-6 Equivalent network representation of synchronous generator for the nonsalient case.

In summary, *E'* can be treated as a constant only if the field winding time constant is infinite. As the transient dies out gradually, over several seconds, the static steady-state equation (4.76) becomes valid. However, over this period, the actions of the automatic voltage regulator superimpose on the response. This will be discussed later.

Chapter 2

ACTIVE POWER AND FREQUENCY CONTROL

2.1 INTRODUCTION

The function of an electric power system is to convert energy from one of the naturally available forms to the electrical form and to transport it to the points of consumption. A properly designed and operated power system should meet following fundamental requirements:

1. The system must be able to meet the continually changing load demand for active and reactive power.
2. The system should support energy at minimum cost and with minimum ecological impact.
3. The "quality" of power supply must meet certain minimum standards with regard to the following factors:
   (a) Constancy of frequency;
   (b) Constancy of voltage; and
   (c) Level of reliability.

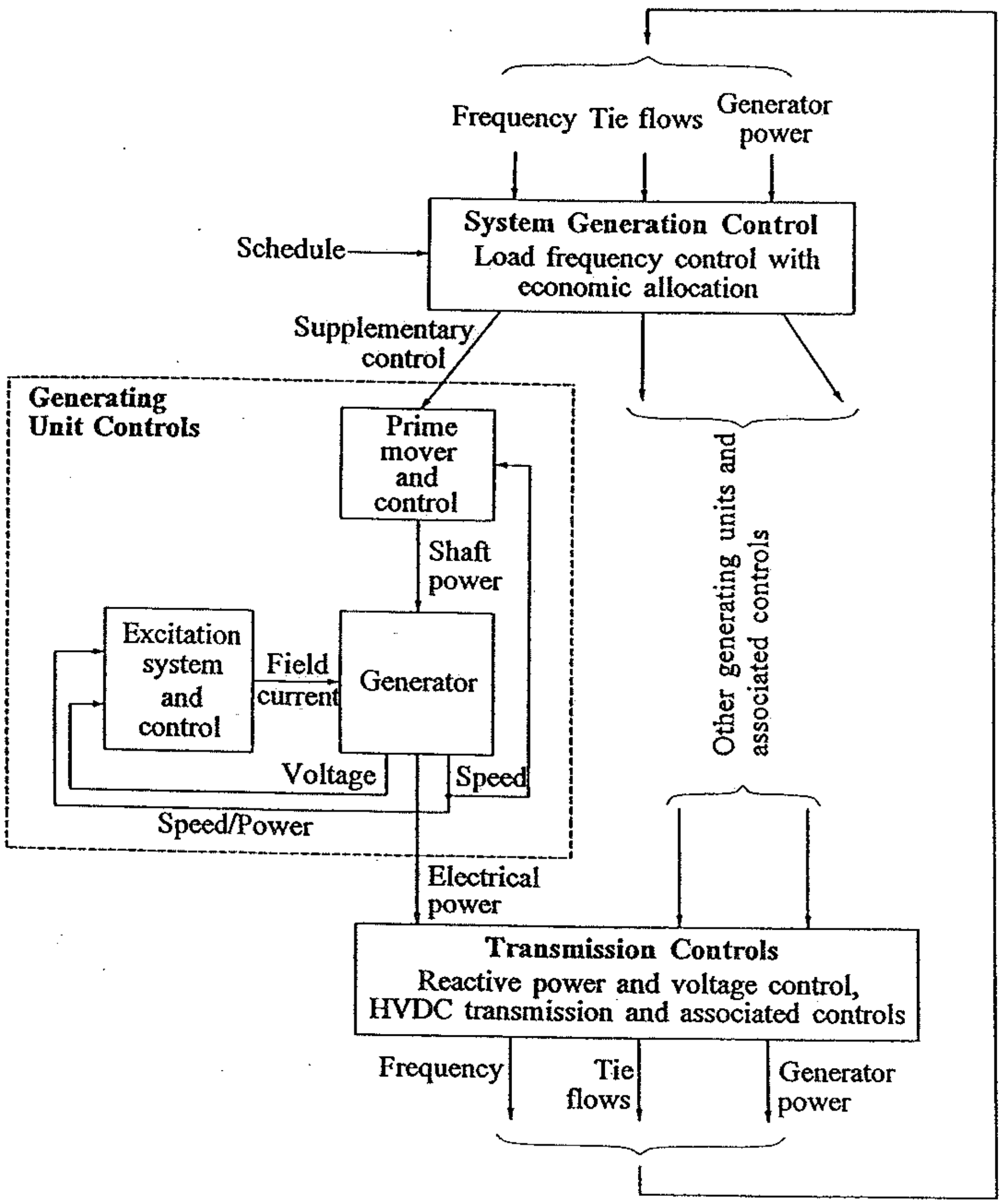


Fig. 2.1 Control System Structure of a Power Network

Fig 2.1 shows the various control system components of a power network. In this overall structure, there are controllers operating directly on individual system elements. For example, those pertaining to the generating unit, there are the prime mover and excitation controls. The primary purpose of the system generation control is to balance the total system generation against load demands and losses, so that the desired frequency and power interchange with neighboring systems (tie flows) is maintained. More will be said about the generator and system generation control later.

Fig 2.2 shows the structure of a power network control operating under a so-called power pool arrangement.

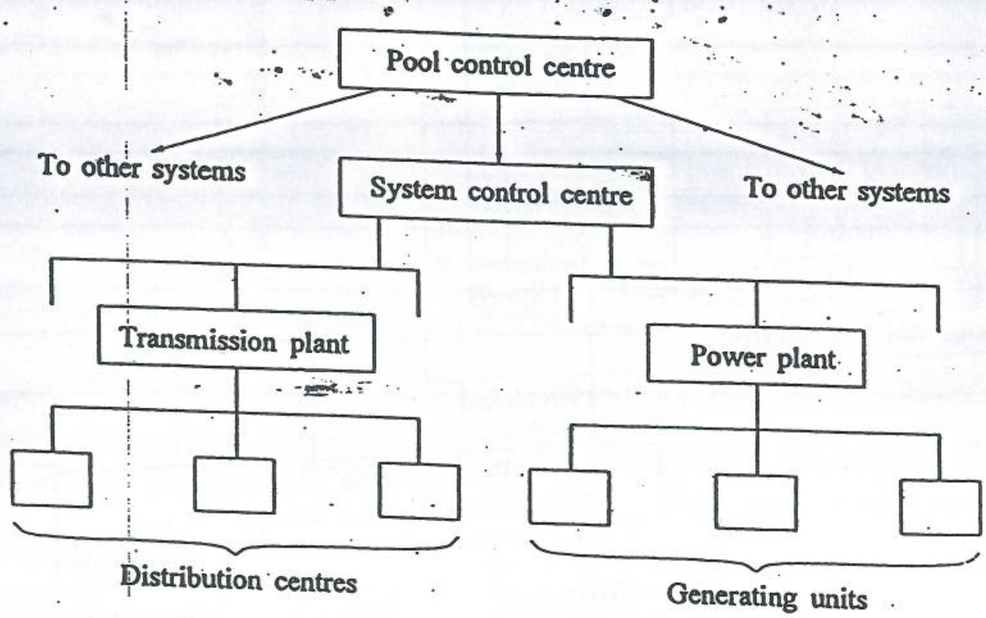


Fig 2.2 Structure of a power pool network control scheme

In this structure, in addition to controllers operating directly on individual system elements such as the generators, there is usually some form of overall plant controller that coordinates the controls of closely linked elements. The plant controllers are in turn supervised by system controllers at the operating centers. The system controller actions are coordinated by pool-level master controllers. The overall control system is thus highly distributed, and relies on many different types of telemetering and control signals. Supervisory Control and Data Acquisition (SCADA) systems provide information to indicate the system status.

Figure 2.3 shows the typical function performed at a energy control center and The diagram showing the data collection from a power system is shown in Figure 2.4.

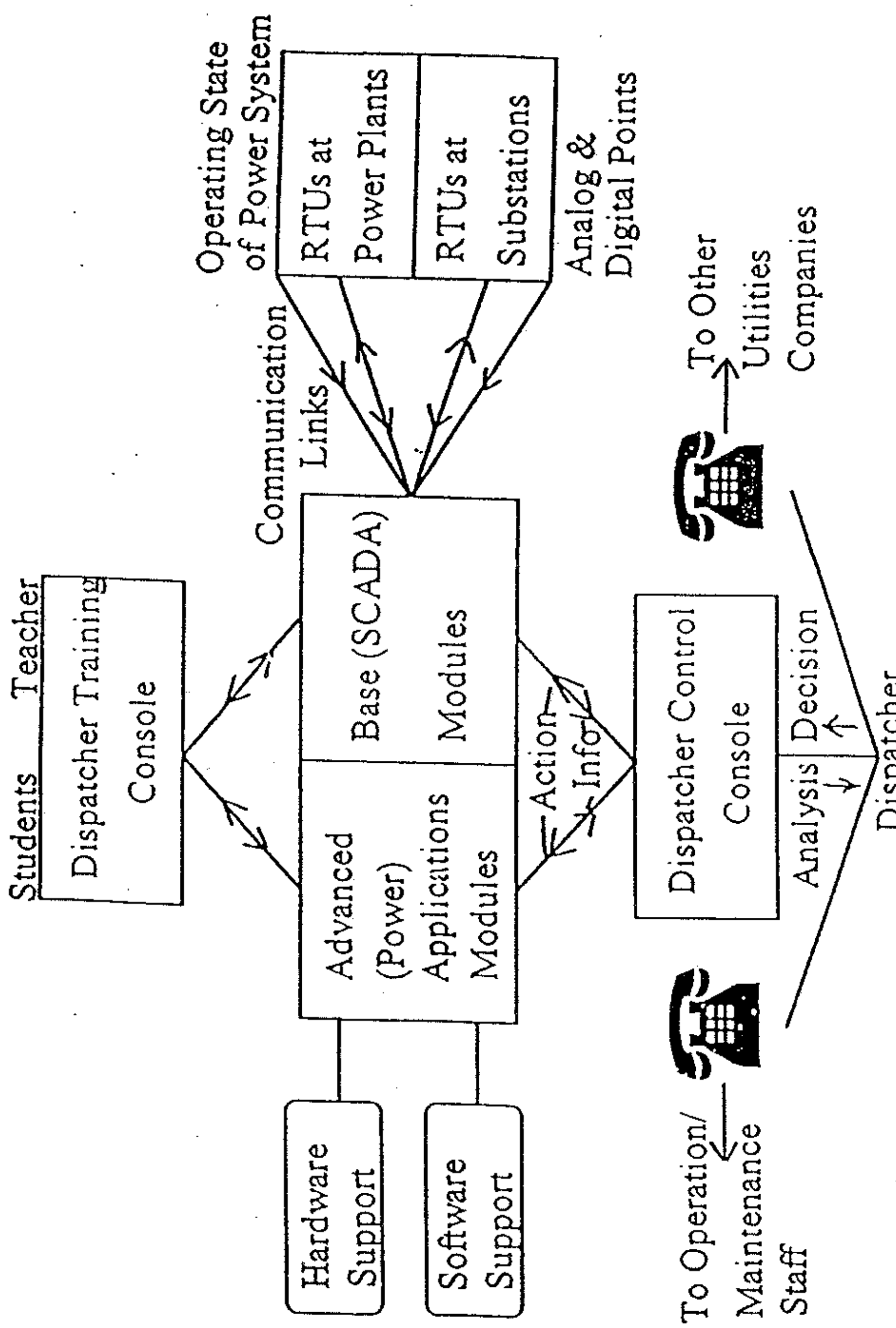


Figure 2.3 Typical function performed at a energy control center

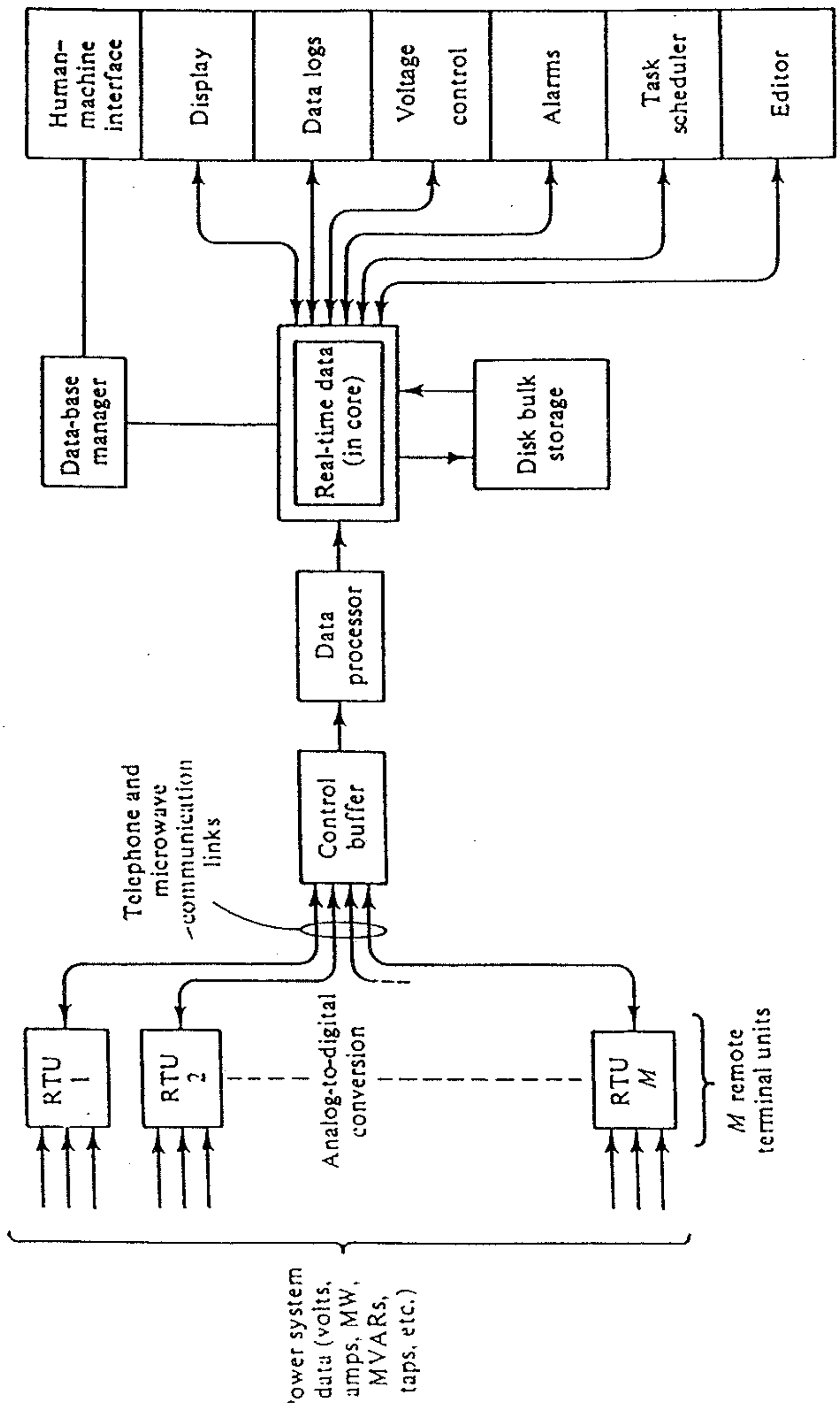


Figure 2.4 Data collection from a power system

The overview diagram of power application functions is given in Figure 2.5.

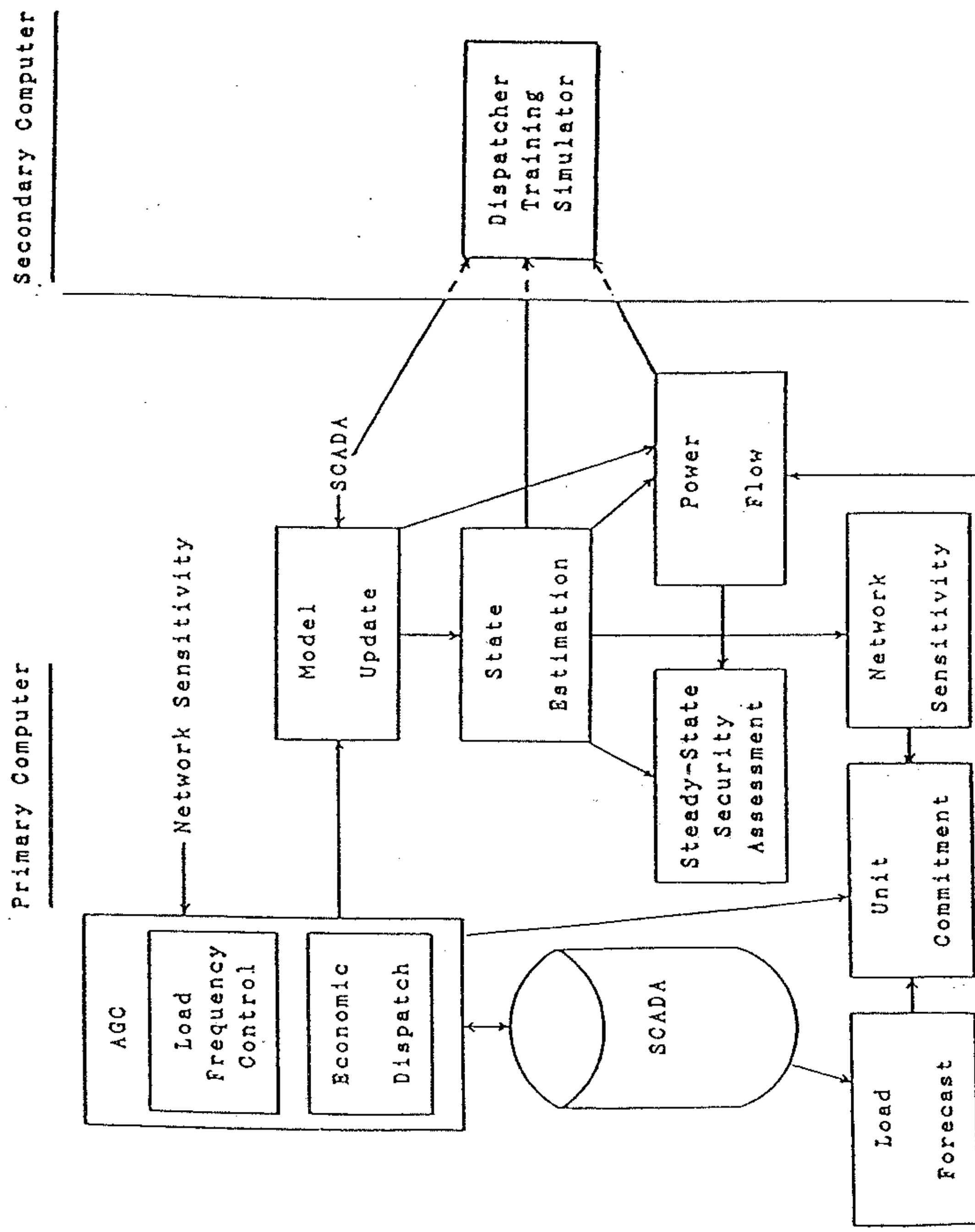


Figure 2.5 Overview diagram of power application functions

2.2 CONTROL OF POWER AND FREQUENCY

2.2.1 The Turbine Speed-Governor Control Loop

A simplified schematic diagram of a traditional governor system is shown in Fig. 4.1 below.

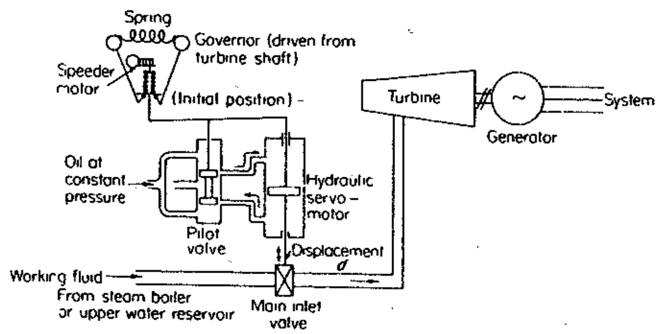


Figure 4.1 Governor control system employing the Watt governor as sensing device and a hydraulic servo-system to operate main supply valve. Speeder-motor gear determines the initial setting of the governor position

Frequency sensing device is shown as the time-honored Watt centrifugal governor, although modern generators use electro-hydrau-toothed wheel and magnetic-probe pickup. Same principle applies, in which movement of the level due to speed changes is transmitted to move the main inlet valve. In this way, energy input to the turbine can be regulated. An approximate governor characteristic of a large steam turbo-generator is as shown in Fig 4.2.

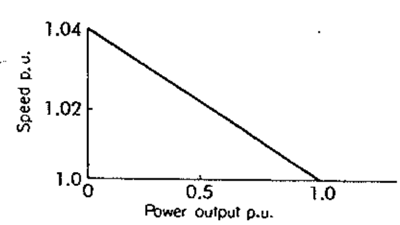


Figure 4.2 Idealized governor characteristic of a turboalternator with 4 per cent droop from zero to full load

An important feature of the governor system is the mechanism by which the main-valve positions can be adjusted through the speeder motor, in addition to that due to the speed change. This will result in a family of curve, such as that shown in Fig 4.3. By adjusting the speeder motor, one can use this to achieve operating conditions which will be economical, a topic to be described later.

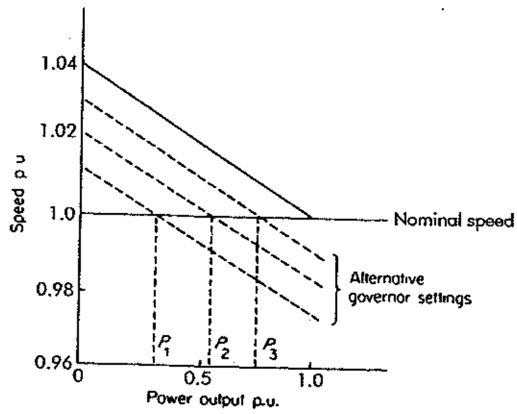


Figure 4.3 Effect of speeder gear on governor characteristics P_1, P_2, and P_3 = outputs at various settings but at same speed

The turbine torque and speed may be expressed approximately as

$$T = T_0 (1 - kN)$$

where T_0 is the torque at speed N_0, k is a constant for the governor system.

Note in practice, speed must change by a certain amount before the valve commences to operate. Hence there is always a dead-band in the mechanism. Also, delays in the hydraulic pilot-valve and servo-motor systems mean that a typical delay of some 0.2-03 s is inevitable.

Fig 4.4 shows typical response of a generator in practice, due to

- **A sudden decrease in electrical output power caused by e.g. an external fault**
- **Delay due to governor-steam system to adjust generator input torque (energy)**
- **Dead-time**

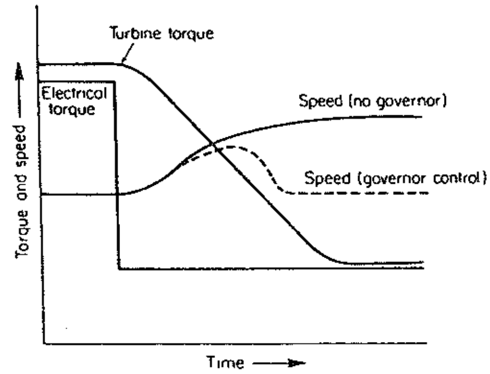


Figure 4.4 Graphs of turbine torque, electrical torque, and speed against time when the load on a generator suddenly falls

Fig 4.5 shows the speed-governor and voltage-regulator control systems of a typical generator unit.

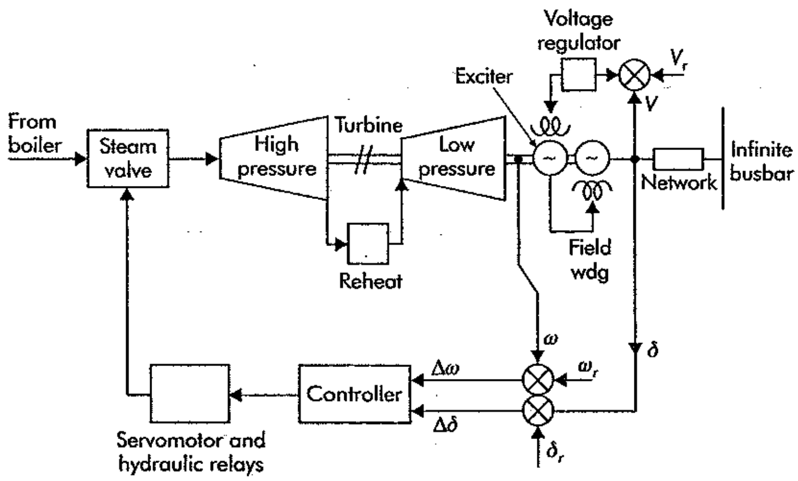


Figure 4.5 Block diagram of complete turboalternator control systems.

Consider only the speed-governor control loop. Two factors influence the dynamic characteristic of the prime mover system:

1. entrained steam between the inlet valve and the 1st stag of the turbine

2. the storage action in the reheater which causes the output of the low-pressure turbine to lag that of the high-pressure side.

The transfer function:

Prime Mover Torque/valve opening

could be represented mathematically by taking into account of the two effects as

$G_1G_2/(1+s\tau_1)(1+s\tau_2)$

where G_1 and G_2 are the entrained steam and reheater constants and τ_1 and τ_2 are the respective time-constant. Note that these are ideal linear models of describing the effects.

Similarly the transfer function between the valve opening to a change in speed can be represented by

$G_3G_4G_5 / (1+s\tau_3)(1+s\tau_4)(1+s\tau_5)$

where τ_3, τ_4 and τ_5 are the governor-relay, primary-relay and secondary relay time constants. Again these are ideal representations of the equipment. One may find typical parameters describing these transfer functions in the IEEE Standards 122-1991 and 125-1998.

In order to obtain the complete control loop, one would need to include the synchronous generator transfer function which would be described next.

2.2.2 Fundamentals of Speed Governing

The objective of the speed control is to maintain the network frequency constant. The simplest way to illustrate how this can be achieved is to consider the isolated system supplying a load, shown in Fig. 2.8.

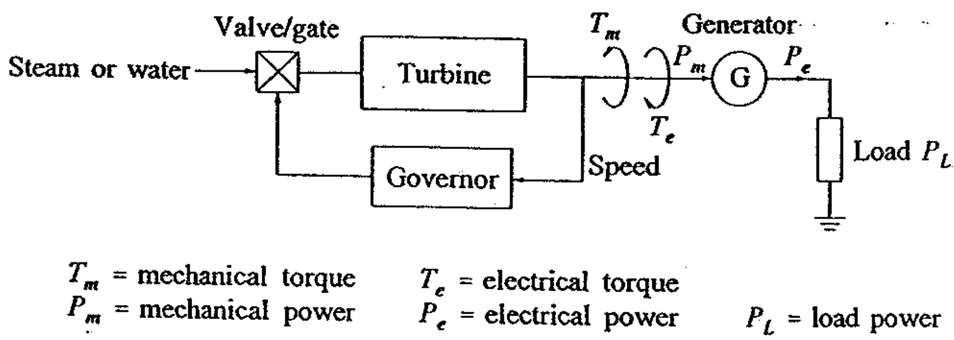


Figure 2.8 Single area system diagram for LFC

GENERATOR MODEL:

Consider the small deviation in the speed of a generator, we have

$$\frac{d\triangle\frac{\omega}{\omega_s}}{dt} = \frac{1}{2H}(\triangle P_m - \triangle P_e) \tag{2.1}$$

where $\omega = 2\pi f$, f denotes the frequency, in radian, $\omega_s = 2\pi f_s$, $f_s = 50Hz$; H is the per unit inertia constant, in seconds, P_m and P_e are the per unit mechanical and electrical power, respectively.

With speed expressed in per unit, we have

$$\frac{d\triangle\omega}{dt} = \frac{1}{2H}(\triangle P_m - \triangle P_e) \tag{2.2}$$

Taking Laplace transform, it gives that

$$\triangle\Omega(s) = \frac{1}{2Hs}(\triangle P_m(s) - \triangle P_e(s)) \tag{2.3}$$

Figure 2.9 shows the block diagram of the generator.

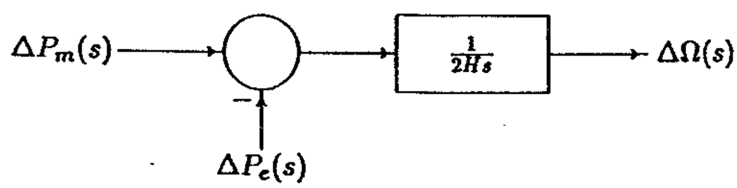


Figure 2.9 Generator block diagram

LOAD MODEL:

The load on a power system consists of a variety of electrical devices. For resistive loads, such as lighting and heating loads, the electrical power is independent of frequency. However, a lot of loads, such as motors, are sensitive to changes in frequency. The speed-load characteristic of a composite load is approximated by

$$\Delta P_e = \Delta P_L + D \triangle \omega \tag{2.4}$$

where ΔP_L is the nonfrequency-sensitive load change, and $D \Delta\omega$ is the frequency-sensitive load change. D is expressed as percent change in load divided by percent change in frequency. For example, if a load is changed by 1.6 percent for a 1 percent change in frequency, then $D = 1.6$. Including the load model in the generator block diagram, results in the block diagram of Figure 2.10.

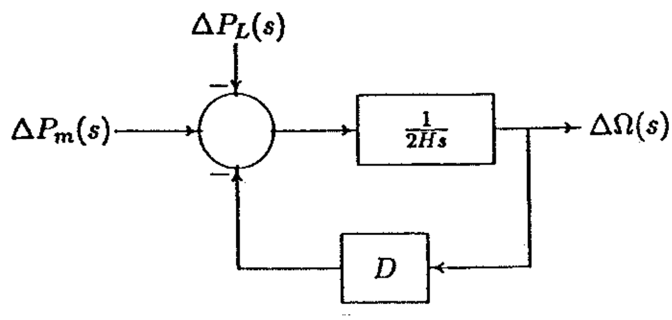
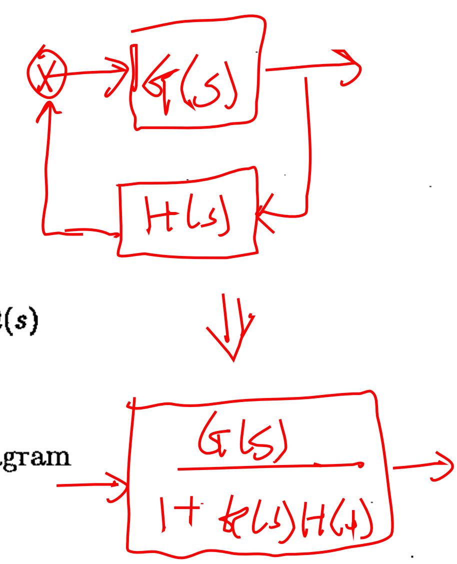


Figure 2.10 Generator and load block diagram

Figure 2.11 shows that the generator and load block diagram eliminating the simple feedback loop.



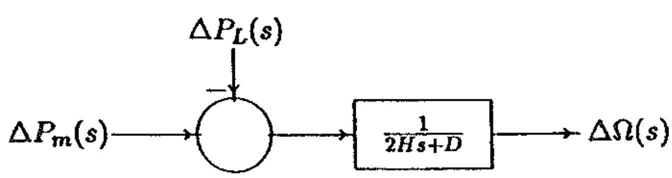


Figure 2.11 Generator and load block diagram

PRIME MOVER MODEL:

The source of mechanical power, commonly known as the *prime mover*, may be hydraulic turbines, steam turbines and gas turbines. The model of the turbine relates changes in mechanical power output ΔP_m to changes in valve position ΔP_v. Different types of turbines vary widely in characteristics. The simplest prime mover model for the nonreheat steam turbine is

$$G_T(s) = \frac{\Delta P_m(s)}{\Delta P_v(s)} = \frac{1}{1+\tau_T s} \tag{2.5}$$

where τ_T is the time constant, which is the range of 0.2 to 2.0 seconds.

Figure 2.12 shows that block diagram.

$$\Delta P_V(s) \longrightarrow \boxed{\frac{1}{1+\tau_T s}} \longrightarrow \Delta P_m(s)$$

Figure 2.12 Nonreheat steam turbine block diagram

GOVERNOR MODEL:

Figure 2.13 shows a speed governing system.

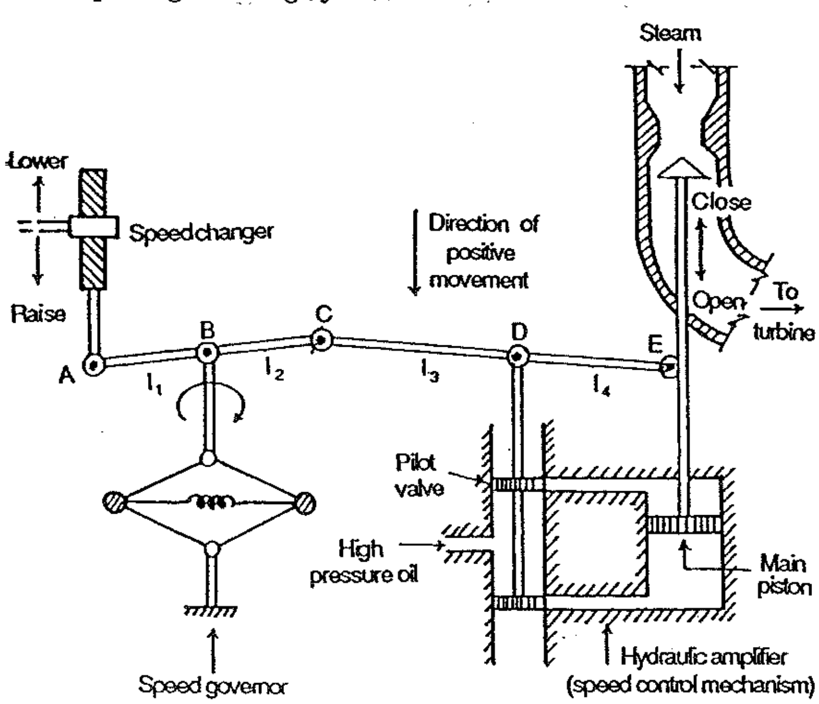


Figure 2.13 Speed governing system

When the generator electrical load is suddenly decreased, the electrical power is then less than the mechanical power input. This power difference causes the turbine speed change and, consequently, the generator frequency to increase. The change in speed is sensed by the turbine governor which acts to adjust the turbine input valve to change the mechanical power output to bring the speed to a new steady-state. Figure 2.14 shows the graphs of turbine torque, electrical torque and speed when the load on a generator suddenly falls.

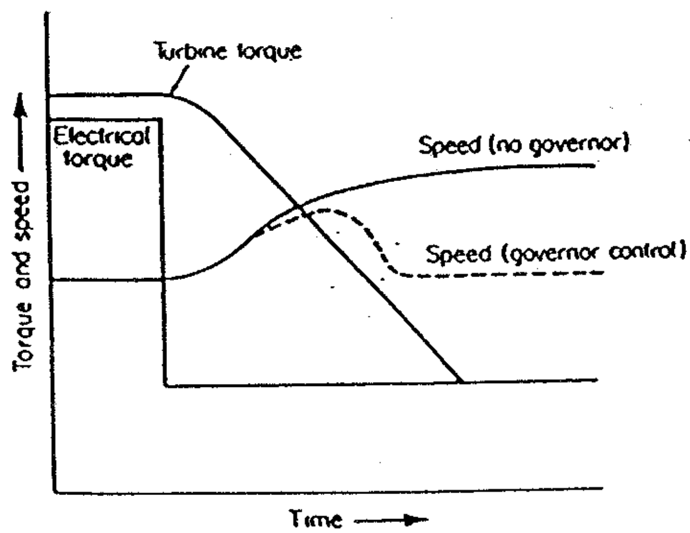


Figure 2.14 Graphs of turbine torque, electrical torque and speed when the load on a generator suddenly falls

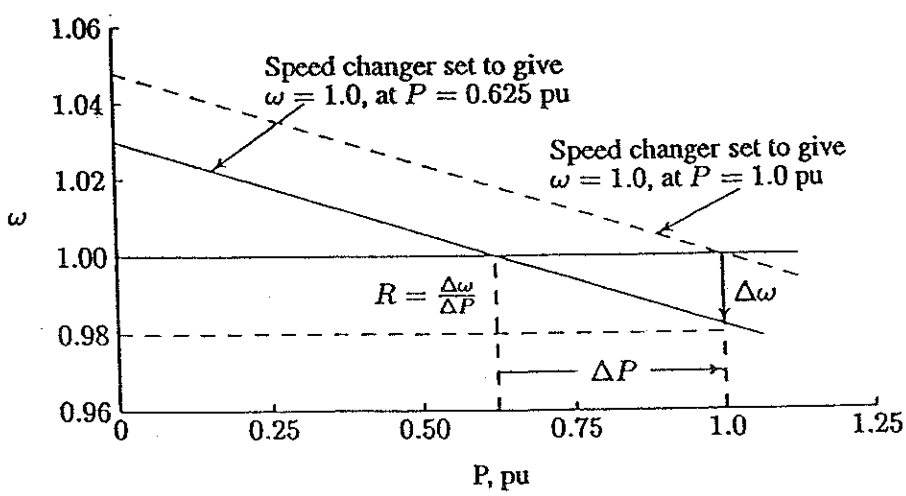


Figure 2.15 Governor steady-state speed characteristics

For a stable operation, the governors are designed to permit the speed to drop as the load is increased. The steady-state characteristics of such a governor is shown in Figure 2.15.

The slope of the curve represents the speed regulation R. Governors typically have a speed regulation of 5 -6 percent from zero to full load. The speed governor output ΔP_g is the difference between the reference set power ΔP_{ref} and the power $\frac{1}{R}\Delta\omega$ and can be described by the equation

$$\Delta P_g = \Delta P_{ref} - \frac{1}{R}\Delta\omega \tag{2.6}$$

or in s-domain

$$\Delta P_g(s) = \Delta P_{ref}(s) - \frac{1}{R}\Delta\Omega(s) \tag{2.7}$$

The s domain relationship between the steam valve position ΔP_v and the speed governor output ΔP_g is

$$\Delta P_v(s) = \frac{1}{1+\tau_g s}\Delta P_g(s) \tag{2.8}$$

where τ_g is the speed governor time constant.

Figure 2.16 shows the block diagram of the speed governor system for steam turbine.

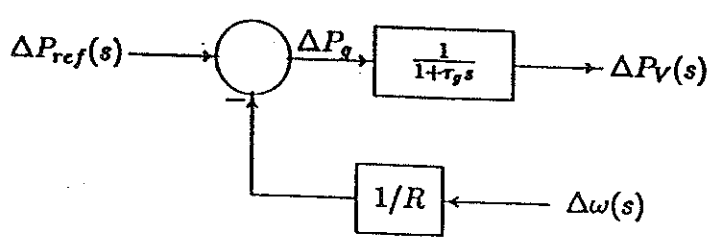


Figure 2.16 Block diagram of speed governing system for steam turbine

Figure 2.17 shows the block diagram for a single area system LFC.

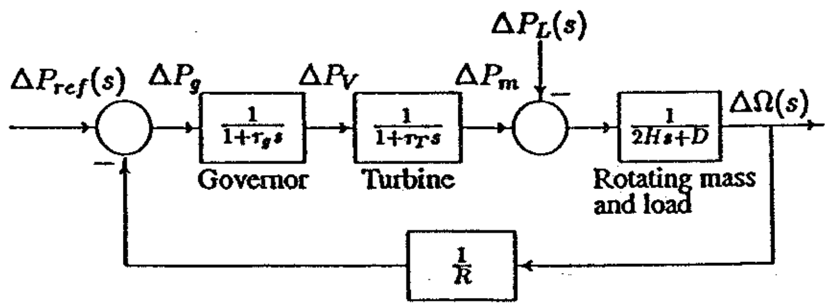


Figure 2.17 Block diagram for a single area system LFC

The transfer function of the system is

$$\frac{\Delta\Omega(s)}{\Delta P_{ref}(s)} = \frac{1}{(2Hs + D)(1 + \tau_g s)(1 + \tau_T s) + 1/R} \tag{2.9}$$

The transfer function relating the load change ΔP_L to the frequency deviation $\Delta\Omega$ is

$$\frac{\Delta\Omega(s)}{\Delta P_L(s)} = -\frac{(1 + \tau_g s)(1 + \tau_T s)}{(2Hs + D)(1 + \tau_g s)(1 + \tau_T s) + 1/R} \tag{2.10}$$

The load change is a step input, i.e., $\Delta P_L(s) = \frac{\Delta P_L}{s}$. Utilizing the final value theorem, the steady state value of $\Delta\omega$ is

$$\Delta\omega_{ss} = \lim_{s\to 0} s\,\Delta\,\Omega(s) = (-\,\Delta\,P_L)\frac{1}{D + 1/R} \tag{2.11}$$

It is clear that for the case with no frequency-sensitive load (i.e. with $D = 0$), the steady-state deviation in frequency is determined by the governor speed regulation constant R and is

$$\Delta\omega_{ss} = (-\,\Delta\,P_L)R$$

When several generators with governor speed regulations R_1, R_2 ... R_n are connected to the system, the steady-state deviation in frequency is given by

$$\Delta\omega_{ss} = (-\Delta P_L)\frac{1}{D + 1/R_1 + 1/R_2 + ... + 1/R_n} \tag{2.12}$$

where

$$\beta = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}$$

is called the area frequency response characteristic.

Example 2.1:

An single area power system, as shown in Figure 2.17, has the following parameters:
$\tau_T = 0.5$ sec.; $\tau_g = 0.2$ sec and $H = 5$ sec..
The load varies by 0.72 percent for a 0.9 percent change in frequency. Find
(a) What is the value of D?
(b) Use the Routh-Hurwitz array to find the range of R for system stable; and
(c)If $R = 0.05$ per unit and the turbine rated output is 250 MW at nominal frequency of 50 Hz, find the system steady-state frequency deviation in Hz when a sudden load change of 50 MW ($\Delta P_L = 0.2$ per unit) occurs.

Solution:

(a)

$$D = \frac{0.72}{0.9} = 0.8$$

(b) Using the equation (2.9) gives that the characteristic equation is

$$(2Hs + D)(1 + \tau_g s)(1 + \tau_T s) + 1/R = 0$$

setting $K = 1/R$ and substituting the parameters into the equation gives that

$$s^3 + 7.08s^2 + 10.56s + 0.8 + K = 0$$

$a_4 s^3 + a_3 s^2 + a_2 s + a_1 = 0$

The Routh-Hurwitz array for this polynomial is then

$$\begin{array}{c|cc} s^3 & 1 & 10.56 \\ s^2 & 7.08 & 0.8+K \\ s^1 & \frac{73.965-K}{7.08} & 0 \\ s^0 & 0.8+K & 0 \end{array}$$

$$\begin{array}{c|cc} s^3 & a_4 & a_2 \\ s^2 & a_3 & a_1 \\ s^1 & \frac{a_2 a_3 - a_4 a_1}{a_3} & 0 \\ s^0 & a_1 & \end{array}$$

From the s^1 row, we can see that for the system stability, K must be less than 73.965. Also form s^0 row, K must be greater than -0.8. Thus, with positive values of K,

$$K < 73.965$$

Since $R = 1/K$, for the system stability, the governor speed regulation must be

$$R > \frac{1}{73.965} = 0.0135$$

(c) Using (2.11) gives that

$$\Delta\omega_{ss} = \lim_{s\to 0} s \bigtriangleup \Omega(s) = (-\bigtriangleup P_L)\frac{1}{D + 1/R}$$

$$= (-0.2)\frac{1}{0.8 + 1/0.05} = (-0.2)\frac{1}{20.8} = -0.0096 \ \ per\ unit$$

Thus, the steady-state frequency deviation in hertz due to the sudden application of a 50-MW load is

$$\Delta f = (-0.0096)(50) = -0.48 \ \ Hz$$

Example 2.2:

One area of an interconnected 50 Hz power system has three turbine generator units rated 1000, 750 and 500 MVA, respectively. The regulation constant of each unit is $R = 0.05$ per unit based on its own rating. Each unit is initially operating at one-half of its own rating, when the system load suddenly increases by 200 MVW. Determine:

(a) The per unit area frequency response β on a 1000 MVA system base;

(b) The steady state drop in area frequency; and

(c) the increase in turbine mechanical power output of each unit.

Assume that the reference power setting of each turbine-generator remains constant. Neglect losses and the dependence of load on frequency.

Solution:

(a) The regulation constant are converted to per-unit on the system base using

$$R_{new} = R_{old}\frac{S_{base(new)}}{S_{base(old)}}$$

It gives that

$$R_{1\ new} = R_{1\ old} = 0.05\ per\ unit$$

$$R_{2\ new} = (0.05)\frac{1000}{750} = 0.06667\ per\ unit$$

$$R_{3\ new} = (0.05)\frac{1000}{500} = 0.10\ per\ unit$$

$$\beta = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{0.05} + \frac{1}{0.6667} + \frac{1}{0.10} = 45.0\ per\ unit$$

(b) Neglecting losses and the dependence of load on frequency, the steady state increase in total turbine mechanical power equals the load increase 200 MW or 0.20 per unit. With $\Delta P_{ref} = 0$

$$\Delta f = \frac{-1}{\beta}\,\Delta\, P_m = \frac{-1}{45}(0.20) = -4.444 \times 10^{-3}\ per\ unit$$

$$= (-4.444 \times 10^{-3})(50) = 0.2222\ Hz$$

The steady-state frequency drop is 0.2222 Hz.

(c) Using $\Delta f = -4.444 \times 10^{-3}$ per unit

$$\Delta P_{m1} = \frac{-1}{R_1}\,\Delta\, f = \frac{-1}{0.05}(-4.444 \times 10^{-3}) = 0.08888\ per\ unit = 88.88\ MW$$

$$\Delta P_{m2} = \frac{-1}{R_2}\,\Delta\, f = \frac{-1}{0.06667}(-4.444 \times 10^{-3}) = 0.06667\ per\ unit = 66.67\ MW$$

$$\Delta P_{m3} = \frac{-1}{R_3}\,\Delta\, f = \frac{-1}{0.10}(-4.444 \times 10^{-3}) = 0.04444\ per\ unit = 44.44\ MW$$

Note that Unit 1, whose MVA rating is 33.33 % larger than that of Unit 2 and 100 % larger than that of Unit 3, pick up 33.33 % more load than Unit 2 and 100 % more load

than unit 3. That is, each unit shares the total load change in proportion to its own rating.

2.2.3 Area Frequency Control

The main objectives of LFC are to follow a load change and to maintain the steady-state frequency error $\triangle f$ to be zero. In order to eliminate the steady state frequency error, we introduce the concept of *Area Control Error* (ACE), which is defined as

$$ACE = (P_{tie} - P_{tie\ sched}) + B_f \triangle f = \triangle P_{tie} + B \triangle f \tag{2.13}$$

where $\triangle P_{tie}$ is the deviation in net tie-line power flow out of the area from its scheduled value $P_{tie\ sched}$; $\triangle f$ is the deviation of the area frequency from 50 Hz and B is the frequency bias constant, which determines the amount of interaction during a disturbance in the neighboring areas.

The change in each reference power setting $\triangle P_{i\ ref}$ of each turbine generator operating under LFC is

$$\triangle P_{i\ ref} = -K_{Ii} \int ACEdt \tag{2.14}$$

The negative sign indicates that if the ACE is negative (either the net tie-line power flow out of the area or the area frequency is low), then the area should increase its generation (raise request).

LFC system block diagrams for single area and two-area systems are shown in Figures 2.18 and 2.19, respectively.

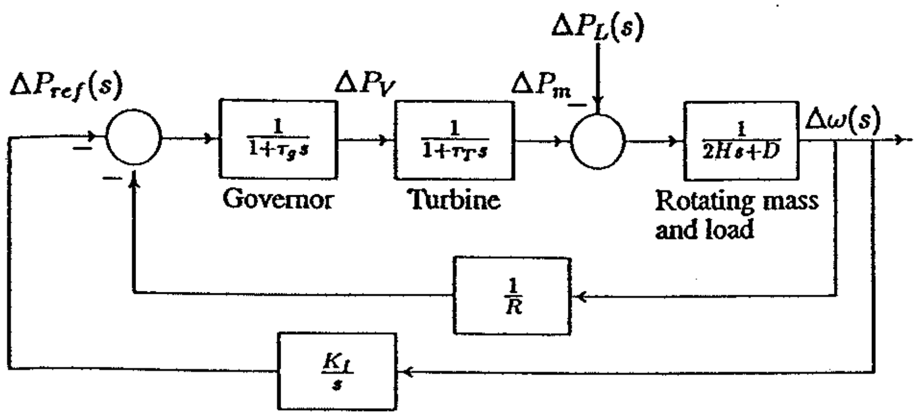


Figure 2.18 LFC system block diagram for a single area system

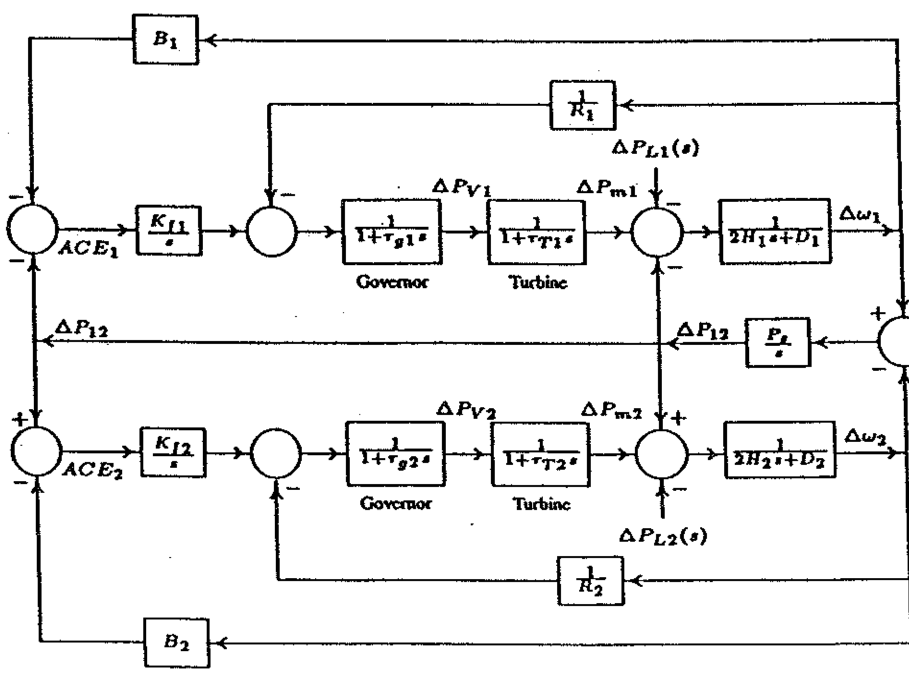


Figure 2.19 LFC system block diagram for a two-area system

Consider the LFC single area system, as shown in Figure 2.18. We have

$$\frac{\Delta\Omega(s)}{\Delta P_L(s)} = -\frac{s(1+\tau_g s)(1+\tau_T s)}{s(2Hs+D)(1+\tau_g s)(1+\tau_T s)+K_I+s/R} \tag{2.15}$$

If the load change is a step input, i.e., $\Delta P_L(s) = \frac{\Delta P_L}{s}$, we can obtain that by utilizing the final value theorem, the steady state value of $\Delta\omega$ is

$$\Delta\omega_{ss} = \lim_{s\to 0} s \,\Delta\, \Omega(s) = (-\frac{\Delta P_L}{s})s\frac{s(1+\tau_g s)(1+\tau_T s)}{s(2Hs+D)(1+\tau_g s)(1+\tau_T s)+K_I+s/R} = 0 \quad (2.16)$$

Introducing the ACE into LFC can maintain the steady-state frequency error Δf to be zero. The ACE is zero in every area only when both ΔP_{tie} and Δf are zero.

Example 2.3:

A two-area system connected by a tie line has the following parameters on a 1000 MVA common base

Area	1	2
R	$R_1 = 0.05$	$R_1 = 0.0625$
D	$D_1 = 0.6$	$D_2 = 0.9$
H	$H_1 = 5$	$H_2 = 4$
Base Power	1000 MVA	1000 MVA

The units are operating in parallel at the nominal frequency of 50 Hz. A load change of 187.5 MW occurs in area 1. Determine the new steady state frequency and the change in tie line flow.

Solution:

The per unit load change in area 1 is

$$\Delta P_{L1} = \frac{187.5}{1000} = 0.1875 \; per \; unit$$

The per unit steady state frequency deviation is

$$\Delta\omega_{ss} = \frac{-\Delta P_{L1}}{(D_1+\frac{1}{R_1})+(D_2+\frac{1}{R_2})} = \frac{-0.1875}{(20+0.6)+(16+0.9)} = -0.005 \; per \; unit$$

Thus, the steady state frequency deviation in Hz is

$$\Delta f = (-0.005)(50) = -0.25 \; Hz$$

and the new frequency is

$$f = f_0 - \Delta f = 50 - 0.25 = 49.75\ Hz$$

The change in mechanical power in each area is

$$\Delta P_{m1} = -\frac{\Delta\omega}{R_1} = -\frac{-0.005}{0.05} = 0.10\ per\ unit = 100\ MW$$

$$\Delta P_{m2} = -\frac{\Delta\omega}{R_2} = -\frac{-0.005}{0.0625} = 0.08\ per\ unit = 80\ MW$$

Thus, area 1 increases the generation by 100 MW and area 2 by 80 MW at the new operating frequency 49.25 Hz. The total change in generation is 180 MW, which is 7.5 MW less than the load change because of the change in the area loads due to the frequency drops.

The change in the area 1 load is

$$\Delta\omega D_1 = (-0.005)(0.6) = -0.003\ per\ unit = -3.0\ MW$$

and the change in area 2 is

$$\Delta\omega D_2 = (-0.005)(0.9) = -0.0045\ per\ unit = -4.5\ MW$$

Thus, the change in the total area load is -7.5 MW.

The tie line power flow is

$$\Delta P_{12} = \Delta\omega(\frac{1}{R_2} + D_2) = (-0.005)(16.9) = -0.0845\ per\ unit = -84.5\ MW$$

That is, 84.5 MW flows from area 2 to area 1. 80 MW comes from the increased generation in area 2, and 4.5 MW comes from the reduction in area 2 load due to frequency drop.

load relief concept

2.2.4 Under-frequency Load Shedding (UFLS)

In situation when disturbances have resulted in excessively low frequency operation for extended period, such is not acceptable because it would damage thermal generating units. The problems are related to mechanical stress on the turbine blades and the reduced outputs of feed-pumps and fans associated with the generators. In general, the ability of thermal units to restore frequency is rather limited if the frequency excursion is large. One practical way, and only under such an emergency condition, to solve this problem is the automatic shedding of loads. For example, a typical UFLS scheme would entail

- **10% load shed when frequency drops to 59.2 Hz (nominal is 60Hz)**
- **15% additional load is shed when frequency drops to 58.8 Hz**
- **20% additional load is shed when frequency drops to 58 Hz.**

Typical operating time with solid-state UFLS relays is in the range 0.1-0.2 s.

Others schemes include the use of frequency drop and the rate of frequency change, so that it provides increased selectivity and prevent unnecessary tripping of loads (over-shed).

Selection of the shedding schemes must be carried out after a detailed system study, involving dynamic simulation of the power system. Interested readers may refer to the reference by Kundur for further details.

A typical frequency decay plot, under different load damping constant (D) and generator inertia constant (M) conditions is shown in the figure, below. ΔL represents the amount of generation deficiency.

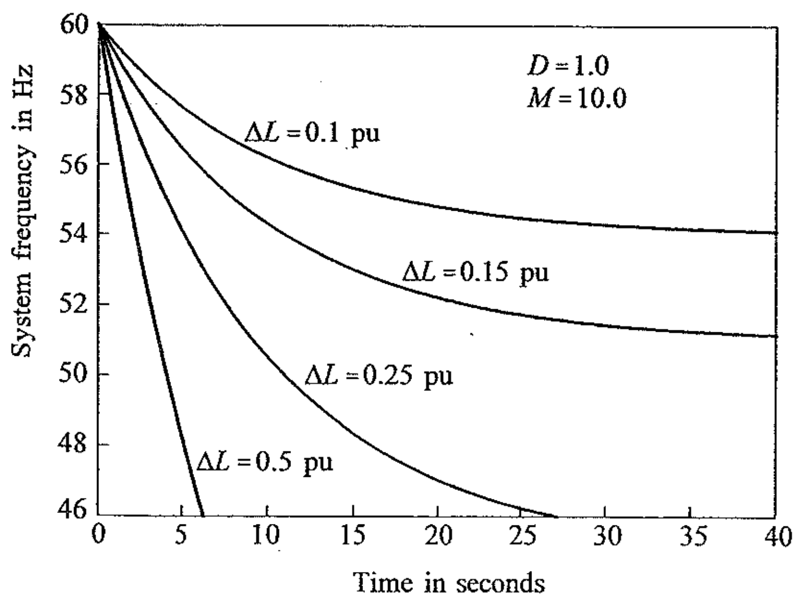


Figure 11.30 Frequency decay due to generation deficiency

WORKED EXAMPLES:

(1) A small isolated system is supplied by two diesel generators rated 10 MVA and 5 MVA respectively. The diesel sets inertia constants in their individual rating are H = 2 for the 10MW unit and H = 3 for the 5 MVA unit.
The load consists of 8 MW of pumping load, which varies approximately as the cube of speed and 2 MW of resistive load.

(a) Derive a linearized model of speed output/mechanical power input of this power system for small changes in the prime mover input power. Express the model on 10 MW base.
(b) If the initial loading on the 5MW unit is 1 MW. Obtain an estimated resulting speed deviation on the remaining unit following the tripping of the 5 MW unit. Assume there is no change in the prime mover power on the 10 MW unit.
(c) Repeat (b) if the prime mover power of the 10MW unit varies directly proportional to the speed.

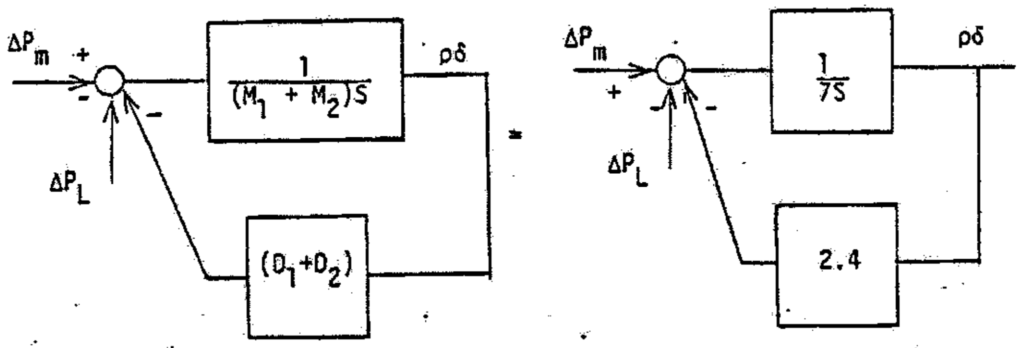
Ans: (a) Linearized model $\Delta\omega/\Delta Pm$ has the total M = 7, D = 2.4 on 10 MVA base. (b) Tripping of the 5 MW unit results in an approximate speed deviation $\Delta\omega = -0.0416\ (1 - e^{-t/1.67})$ p.u. (c) $\Delta\omega = -0.0667\ (1 - e^{-t/2.66})$ p.u.

Refer to additional notes

Solution:

(a)

Linearized model of system



$M_1 = 2H_1 = 4$ on 10,000 KVA base

$M_2 = 2H_2 = 6$ on 5,000 KVA base $= 6 \times \frac{5,000}{10,000} = 3$ on 10,000 KVA base

$\underline{M_1 + M_2 = 7}$

Pumping load $L_1 = L_o\ (1 + \rho\delta)^3 = L_o\ (1 + 3\rho\delta + \ldots)$

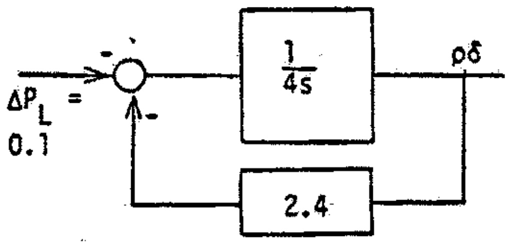
$D_1 = \frac{\Delta L_1}{\rho\delta} = 3\ L_o = 3 \times 8,000$ KW/p.u. $\rho\delta = \frac{3 \times 8,000}{10,000}\ \frac{\text{p.u.}\Delta P}{\text{p.u.}\rho\delta} = 2.4$

D_2 for resistive load $= 0$

$\therefore \underline{D_1 + D_2 = 2.4}$

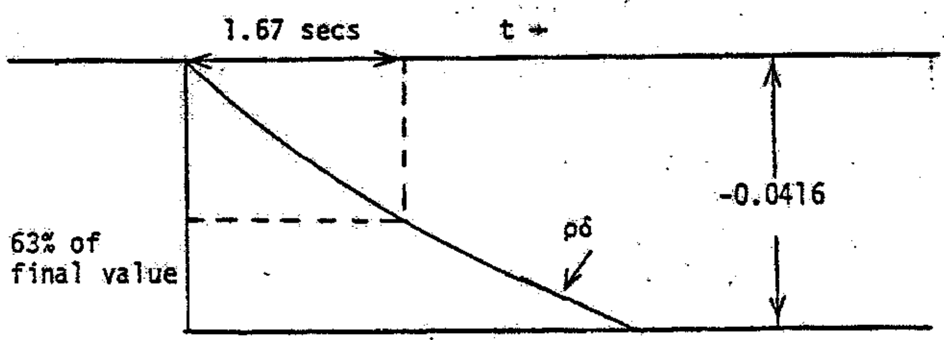
(b)

Following the tripout of the 5,000 KVA unit the conditions are described by:



$$\rho\delta\ (s) = \frac{-0.1}{s4(s + \frac{2.4}{4})}$$

$$\therefore\ \rho\delta(t) = -0.0416\ (1 - \varepsilon^{-t/1.67})$$



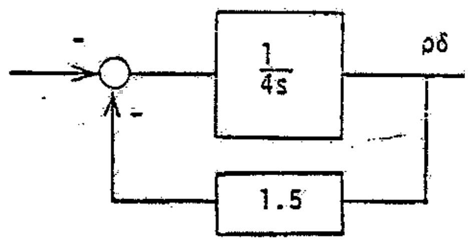
(c)

$$P_{mo} + \Delta P_m = P_{mo}(1 + \rho\delta)$$

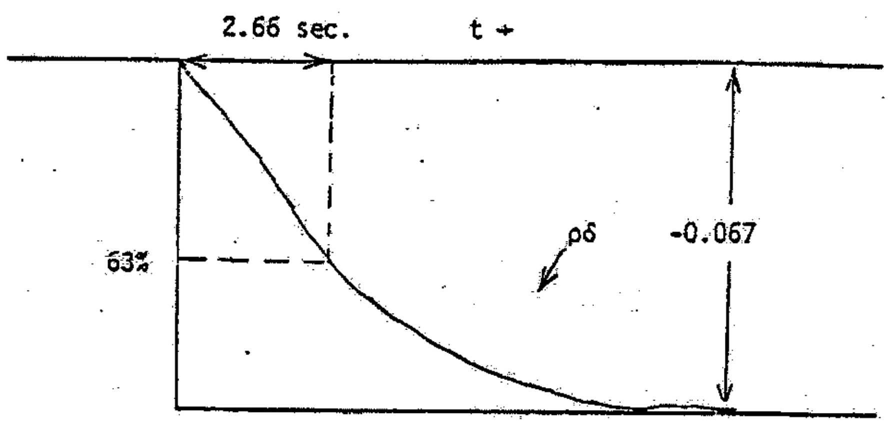
$$\Delta P_m = P_{mo}\rho\delta$$

$$D = (2.4 - P_{mo})$$

$$= (2.4 - 0.9) = 1.5$$



$$\rho\delta(t) = \frac{-0.1}{1.5}(1 - \varepsilon^{-t/2.66})$$

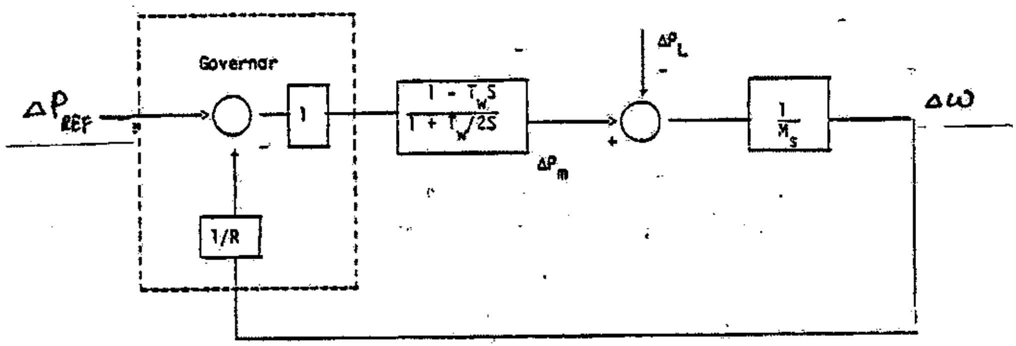


68.6

(2) Shown below is a simplified block diagram of a hydro turbine and generator supplying an isolated load. The speed-governor is shown as a pure gain $1/R$ with no time delay.

(a) Express in terms of T_w and M, what is the lowest value of regulation R for which stable operation can be expected?
(b) For $T_w = 4$ sec, and $M = 10$s, what is the value of R for which the speed governing system is critically damped?
(c) For a load increase of 0.1 p.u., derive the speed deviation for the value of the R determined in (b).

Ans: (a) $R > T_w/M$, (b) $R = 1.49$ for critical damping; (c) $\Delta\omega = -0.149 + (0.0173t + 0.149)\, e^{-0.183\,t}$.



Solution:

(1) and (2) $$\frac{pS(s)}{\Delta P_L(s)} = \frac{-\frac{1}{Ms}}{1 + \frac{1}{RMs}\left(\frac{1 - T_w s}{1 + \frac{T_w}{2} s}\right)} = \frac{R(1 + \frac{T_w}{2} s)}{RMs(1 + \frac{T_w}{2} s) + (1 - T_w s)}$$

Characteristic equation $s^2 RM \frac{T_w}{2} + S(RM - T_w) + 1$

Critical damping when $b^2 = 4ac$

i.e., $R^2M^2 - 2RM\,T_w + T_w^2 = 2RM\,T_w$

or $R^2M^2 - 4RMT_w + T_w^2 = 0$

Solving for R

$$R = \frac{4MT_w \pm \sqrt{16M^2T_w^2 - 4M^2T_w^2}}{2M^2}$$

$$R = \frac{T_w}{M}[2 \pm \sqrt{3}\,]$$

Examine relevance of roots noting that for stable solution $b > 0$. I.E., $RM > T_w$ is satisfied only by root

$R = \frac{T_w}{M}[2 + \sqrt{3}\,]$ $\therefore$ $R = \frac{4 \times 3.73}{10} = 1.49$ p.u. ~~or~~ $R = \frac{T_w}{M}(2 - 1.7321) = 0.107$

Lowest value of R for stability is $\frac{T_w}{M}$ or 40 %

Not satisf ~~for~~ because $R > \frac{T_w}{M}$ for $b > 0$.

$R > \frac{T_w}{M} = 0.$

$\therefore R = 40$

(3) $$pS(s) = \frac{-R\,(1 + \frac{T_w}{2} s)}{RM\,\frac{T_w}{2}\,[S^2 + S\,\frac{(RM - T_w)}{RMT_w/2} + \frac{1}{RMT_w/2}]} \times \frac{0.1}{s}$$

Quadratic in the form $S^2 + 2\zeta W_0 S + W_0^2$ has roots $S = [-\zeta \pm \sqrt{1 - \zeta^2}\,]\,W_0$, ζ = dampin

For critical damping, $\zeta = 1$ and $S_1 = S_2 = -\zeta W_0$

$$S_1 = S_2 = \frac{(RM - T_w)}{RMT_w} = \frac{14.9 - 4}{14.9 \times 4} = 0.183$$

$$pS(s) = \frac{-1.49(1 + 2s) \times 0.1}{S\ 29.8(s + 0.183)^2}$$

$$\therefore \quad pS(s) = \frac{K_o}{S} + \frac{K_{11}}{(S + 0.183)^2} + \frac{K_{12}}{(S + 0.183)}$$ See Appendix 8, page 11

$$K_o = \frac{-1.49 \times 0.1}{29.8 \times 0.183^2} = -0.149$$

$$K_{11} = \frac{-1.49(1 - 2 \times 0.183) \times 0.1}{-29.8 \times 0.183} = 0.0173$$

$$K_{12} = \frac{d}{ds} \left. \frac{-0.149(1 + 2s)}{29.8s} \right|_{s= -0.183} = \frac{-0.149}{29.8} \left[\frac{2s - (1 + 2s)}{s^2} \right]_{s = -.183}$$

$$= +0.149$$

(3) $\therefore$ $pS(t) = -0.149 + (0.0173t + 0.149)\varepsilon^{-0.183t}$

3. SYNCHRONOUS MACHINE – EXCITATION SYSTEM

Materials of this chapter are taken from part of Elgerd Chapter 9.
Excitation system comes in many varieties, as is for turbine control. Fig 9.23 depicts a typical type, in which the generator field is fed from a dc source (exciter), which is usually driven by the same turbine shaft as the main generator itself. The exciter field is controlled via an amplifier, and due to the exciter size, there could be several stages in the amplification. Fig 9.23 shows only one stage.

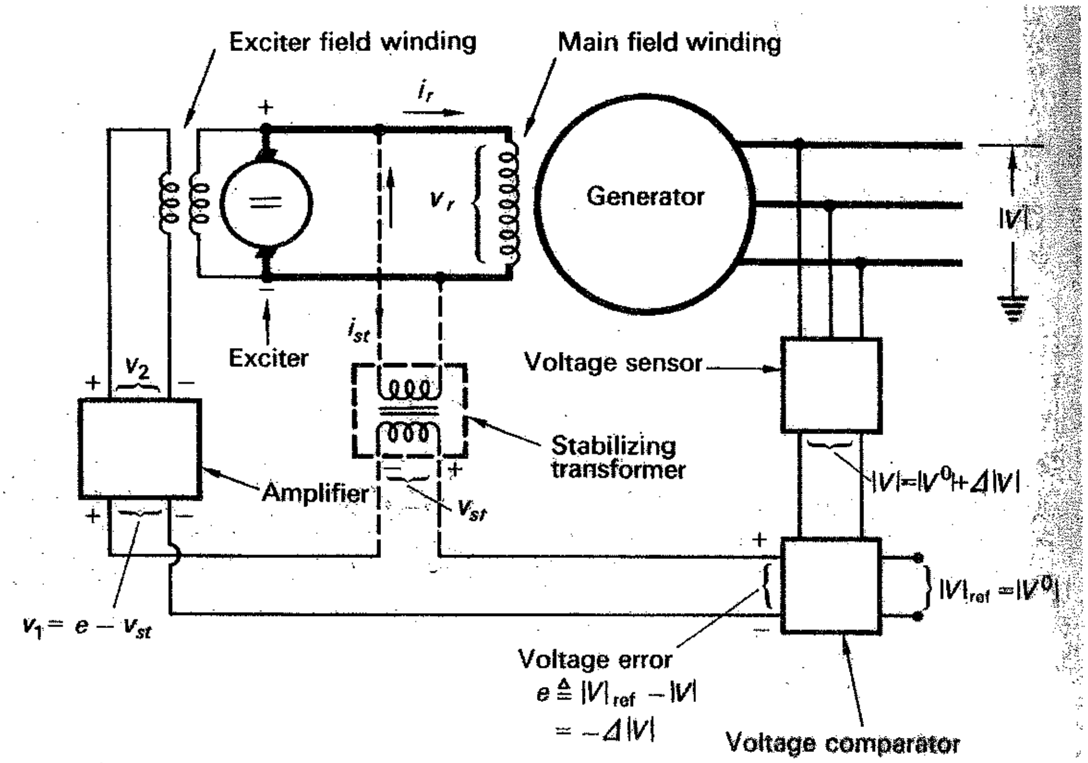


Fig. 9-23 Typical voltage regulator.

Several IEEE Committee papers have been published which provide details of the various excitation systems and their models. Interested readers may wish to refer to them.

3.1. Transfer function model:

The transfer function model of the excitation system shown in Fig 9.23 is shown in Fig 9.24 where:

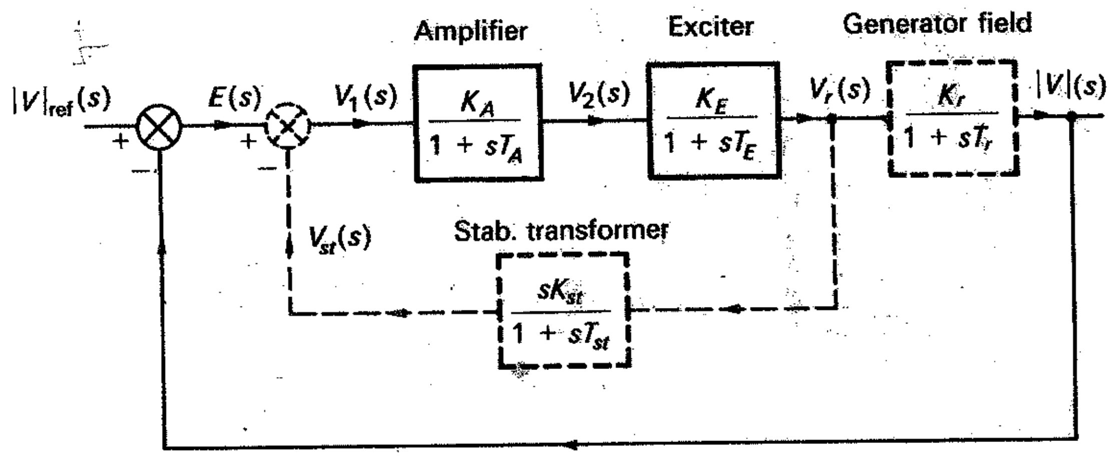


Fig. 9-24 Block-diagram representation of voltage regulator in Fig. 9-23.

- **The voltage comparator is used to compare the machine terminal voltage with a set nominal value. The error is**

$$e \triangleq |V|_{ref} - |V| = |V^0| - (|V^0| + \Delta|V|) = -\Delta|V| \tag{9-74}$$

- **Amplifier is characterized by a gain K_A and a time constant T_A. Typically TA is less than 100 ms.**

- **Exciter is basically also an amplifier and has the same form of transfer function. T_E is the time constant of the field winding, typically it is about 1 sec or so.**

Generator representation:

The above only deals up to the terminals of the generator field. In order to see how the excitation system would affect generator operations, we need to close the loop.

The transfer function which relates the field voltage V_r and the generator terminal voltage depends on the operating conditions of the machine. Under the case of open-circuited or very lightly loaded condition, the stator current is small and the direct axis current i_d will be negligible. From the last of equation (4.36), therefore,

$$v_r = r_r i_r + L_4 \frac{di_r}{dt} \tag{9-77}$$

Under no load condition, the terminal voltage is equal to the generator emf, and from equation (4.46), we have

$$|V| = |E| = \frac{\omega L_5 i_r}{\sqrt{2}} \tag{9-78}$$

From (9-77) and (9-78), therefore,

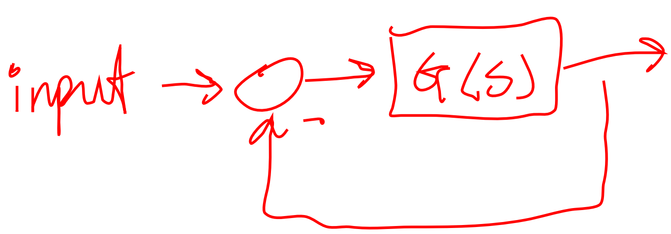
$$\frac{|V|(s)}{V_r(s)} = \frac{\omega L_5}{\sqrt{2}(r_r + sL_4)} = \frac{\omega L_5}{\sqrt{2} r_r} \frac{1}{1 + s(L_4/r_r)} \triangleq \frac{K_r}{1 + sT_r} \tag{9-79}$$

With this transfer function, we can close the loop. The time constant $T_r = L_4/r_r$ and typically has a value of several sec.

When the machine is on load, we cannot neglect the term di_d/dt in equation (4.36) and the resulting magnetic coupling between the stator and rotor introduces considerable complexity to the model. This will be taken up later.

Stabilizing Circuit:

From the root-locus plot Fig 9.25, one notices that the stability of the excitation system would be impaired by high loop gains. Since T_r is very large, we need very high loop gain in order to reduce the response time of the loop. However, this will inevitably render the system unstable.



$$\frac{o/p}{i/p} = \frac{G(s)}{1+G(s)}$$

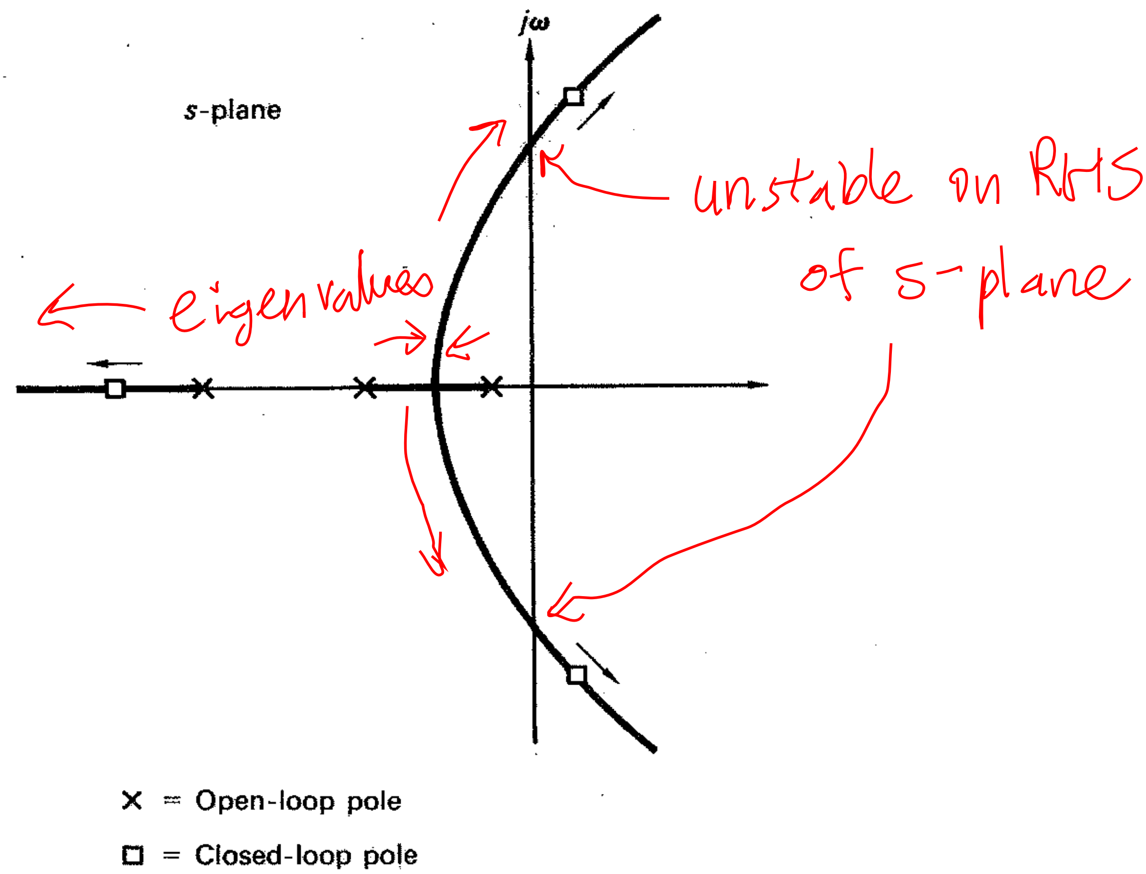


Fig. 9-25 Root loci for unstabilized regulator. Instability indicated for high loop gain.

There are several methods to stabilize the system. One arrangement is shown in Fig 9.23, in which a stabilizing transformer has been added. The transformer provides derivative feedback to cure the instability problem. It provides a voltage v_{st} which is proportional to the derivative of i_{st} and thus essentially, also to the derivative of v_r. This derivative is added before v_r has changed, and subtracted from the error signal, so that it "soften" the amplifier action. Mathematically,

$$v_r = Ri_{st} + L\frac{di_{st}}{dt}$$

Thus

$$\frac{V_{st}(s)}{V_r(s)} = \frac{sM}{R+sL} \triangleq \frac{sK_{st}}{1+sT_{st}} \approx sK_{st} \qquad (9\text{-}80)$$

The last approximation is valid if we intentionally make the transformer *R*>>*L*. By so doing, the root-locus of the resulting closed-loop system becomes Fig 9.26.

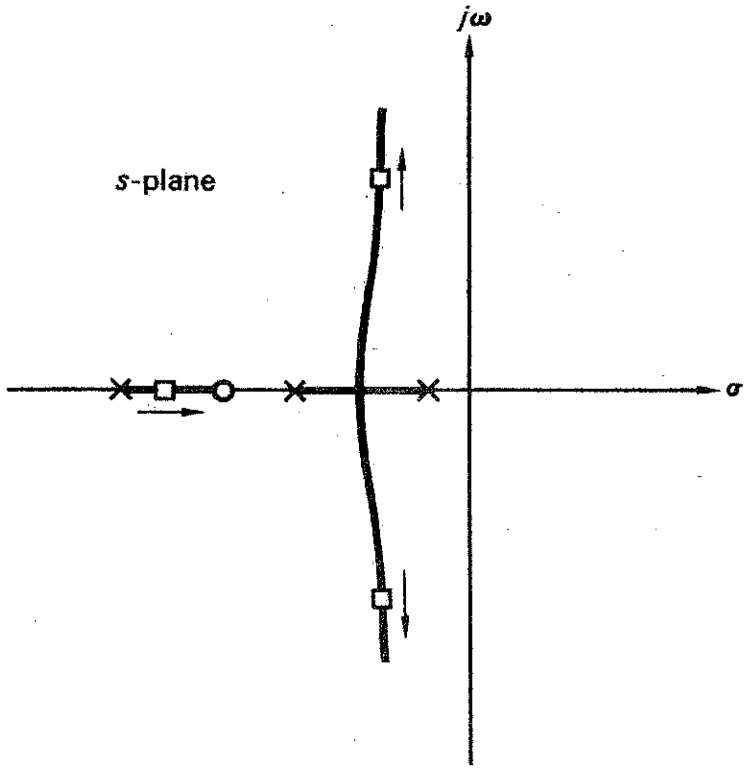


Fig. 9-26 Root loci for stabilized regulator. Loop stable for high gains.

3.2. Effects of voltage control loop

Contents of this Section are taken from Elgerd Section 12.5.2.

The emf E', described in the chapter on the transient model of synchronous generator, changes due to the action of the excitation system. From Fig 9.24, one obtains the following equations:

$$\frac{V_r(s)}{V_2(s)} = \frac{K_E}{1+sT_E}$$
$$\frac{V_2(s)}{V_1(s)} = \frac{K_A}{1+sT_A}$$
$$\frac{V_{st}(s)}{V_r(s)} = \frac{sK_{st}}{1+sT_{st}} \tag{12-69}$$
$$V_1(s) = |V|_{\mathrm{ref}}(s) - |V|(s) - V_{st}(s)$$

Inverse Laplace and we obtain

$$\dot{v}_r = \frac{1}{T_E}(K_E v_2 - v_r)$$
$$\dot{v}_2 = \frac{1}{T_A}[K_A(|V|_{\mathrm{ref}} - |V| - v_{st}) - v_2] \tag{12-70}$$
$$\dot{v}_{st} = \frac{1}{T_E T_{st}}(K_{st}K_E v_2 - K_{st}v_r - T_E v_{st})$$

In computer model, one uses state variable representation and is of the form

$$\dot{x}_6 = \frac{1}{T_E}(K_E x_7 - x_6)$$
$$\dot{x}_7 = \frac{1}{T_A}[K_A(|V|_{\mathrm{ref}} - |V| - x_8) - x_7] \tag{12-71}$$
$$\dot{x}_8 = \frac{1}{T_E T_{st}}(K_{st}K_E x_7 - K_{st}x_6 - T_E x_8)$$

where we have introduced the 3 new state variables

$$x_6 \triangleq v_r$$
$$x_7 \triangleq v_2 \tag{12-72}$$
$$x_8 \triangleq v_{st}$$

Under steady-state condition, v_{sr} is zero and the initial values of the state vector is

$$\begin{bmatrix} x_6{}^0 \\ x_7{}^0 \\ x_8{}^0 \end{bmatrix} = \begin{bmatrix} v_r{}^0 \\ v_2{}^0 \\ 0 \end{bmatrix} \tag{12-73}$$

The effect of finite field time constant T_r:

There is still an additional important link missing in equation (12.70). This is because we must establish the relationship between E' and $v_r = x_6$.

From equation (4.36) and following the derivation given in Section 12.5.2 of Elgerd, one can obtain the following expressions

$$\frac{1}{T_r}(|E_v| - |E_i|) = \frac{d}{dt}|E'| \qquad (12\text{-}78)$$

where

$$T_r \triangleq \frac{L_4}{r_r} \qquad \text{field time constant}$$

$$|E_v| \triangleq \frac{\omega L_5}{\sqrt{2}}\frac{v_r}{r_r} \triangleq K_v v_r \qquad (12\text{-}79)$$

$$|E_i| \triangleq \frac{\omega L_5}{\sqrt{2}} i_r \triangleq K_i i_r$$

In arriving equations (12.78) and (12.79), one can interpret E_v as the steady-state stator emf we would get if the field voltage is held at the constant value of v_r, and E_i is that when the field current is held constant at i_r.

Again in terms of state variable form, we have

$$x_9 \triangleq |E'| \qquad (12\text{-}80)$$

and from equation (12.78)

$$\dot{x}_9 = \frac{1}{T_r}(K_v x_6 - |E_i|) \qquad (12\text{-}81)$$

The initial value of x_9 is

$$x_9{}^0 = |E'|^0 \qquad (12\text{-}82)$$

Fig 12.27 provides the phasor diagram to show how this initial value of x_9 can be determined. From this diagram,

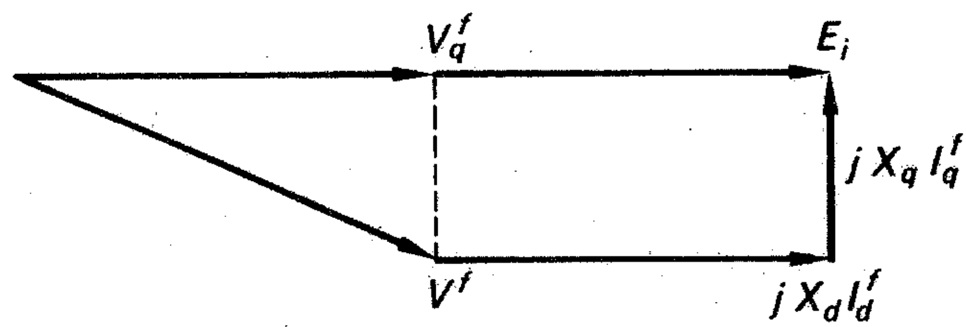


Fig. 12-27 Phasor diagram for finding E_i.

$$|E_i| = |V_q| + X_d\,|I_d| \tag{12-84}$$

If there is no saliency,

$$|E_i| = |V + jX_d\,I_G| \tag{12-85}$$

WORKED EXAMPLES

Modern exciter tends to be either brushless type or static design. A typical scheme is shown in Fig 9.2. Its ac armature voltage is rectified in diodes mounted on the rotating shaft of the main generator, and then fed directly to the generator field. It eliminates the use of slip-rings and brushes and hence reduces wear and tear.
Fig 9.4 shows the transfer function model of the excitation system, where again, the generator is assumed to be under no load condition. Fig 9.5 shows the root-loci of the closed-loop system as the amplifier gain K_A (equivalently open-loop gain K) varies for a typical set of excitation system values.

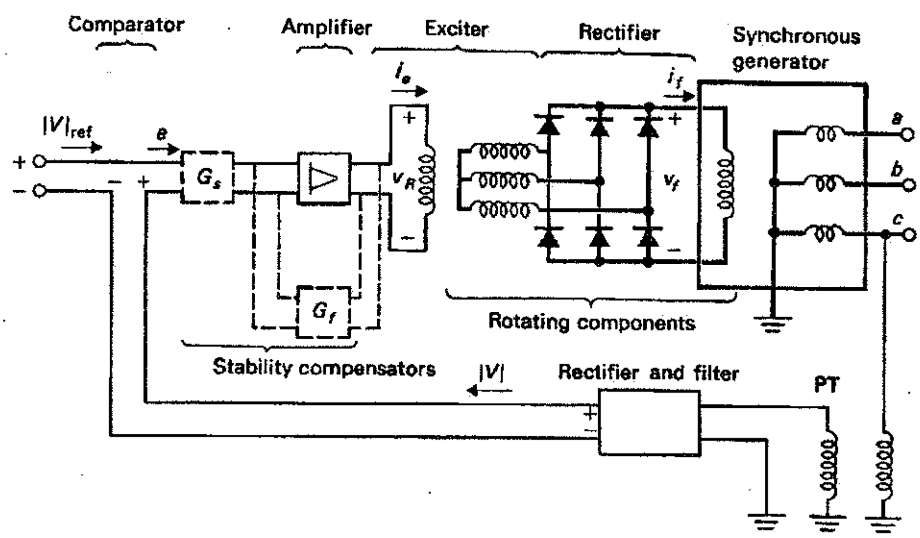


Figure 9-2 Brushless AVR loop

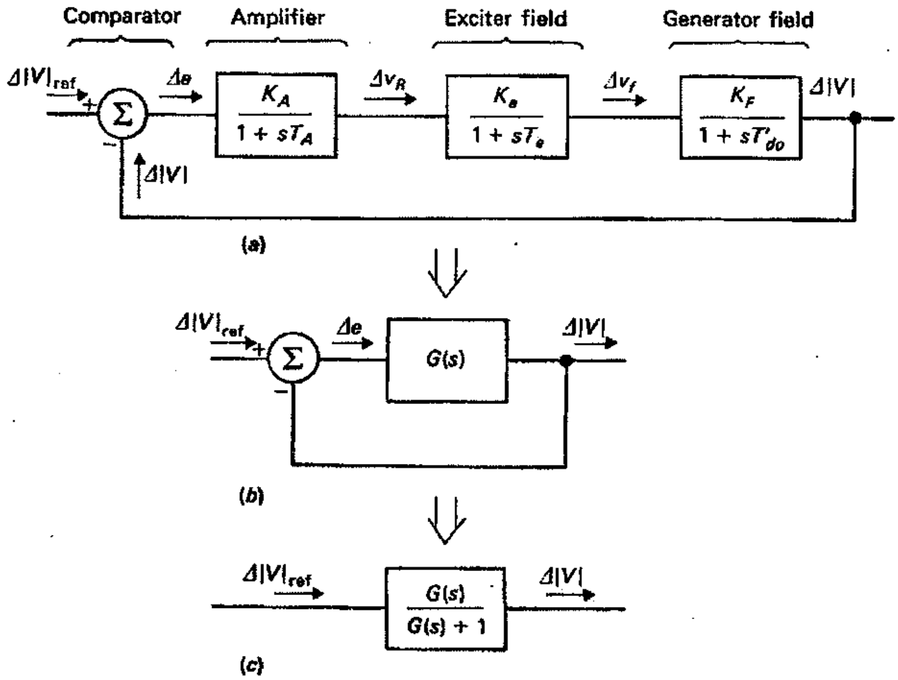


Figure 9-4 Closing the AVR loop; (*a*) individual block representation; (*b*) condensed model; (*c*) closed loop model.

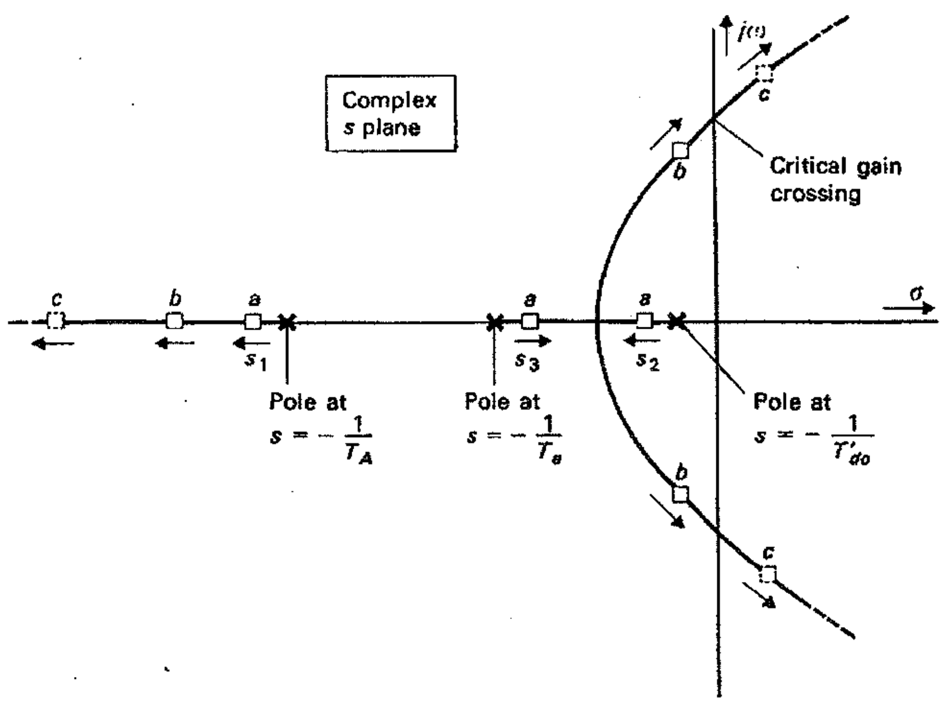


Figure 9-5 Closed-loop root loci for AVR control loop in Fig. 9-4.

*9-2 Consider the AVR loop discussed in the text having the block representation shown in Fig. 9-4. The time constants have the following numerical values:

$$T_A = 0.050 \text{ s}$$

$$T_e = 0.50 \text{ s}$$

$$T_{d0}' = 3.0 \text{ s}$$

Assume the loop uncompensated. We would like the static "error" not to exceed one percent which means, according to Eq. (9-13), that the open-loop gain K must exceed 99.

(*a*) Construct a carefully drawn root-locus plot (Fig. 9-5) and prove that the static accuracy requirement conflicts with the requirement for stability.

(*b*) Specifically, compute the closed-loop poles (eigenvalues) if K is set at 99.

(*c*) Reduce K by 50 percent and repeat part *b*.

Solution:

Substitute the numerical values into G(s), one obtains

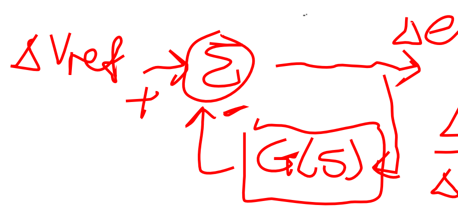
$G(s) = K/ (1 + 0.05s)(1 + 0.5s)(1 + 3s)$.

$K = K_A K_e K_F$

If the loop is uncompensated, the static gain is obtained by setting s = 0 into G(s)

$G(0) = K$

The static error between the output voltage change ΔV and the reference voltage change ΔVref therefore becomes



$$\frac{\Delta e}{\Delta V_{ref}} = \frac{1}{1+G(s)}$$

$\Delta e_0 = 1/(1 + G(0))\ \Delta V_{ref}$

If we want Δe_0 to be less than 1%, then,

$1/(1+ K) < 0.01$, or $K > 99$.

(a) With G(s) given above, let's examine the closed-loop transfer function $\Delta V/\Delta V_{ref}$ and it is given by

$G(s) /(1 + G(s))$

(FVT) $\Delta e_{ss} = \lim_{s \to 0} s \times \frac{\Delta V_{ref}}{1+G(s)}$

The denominator after expanding it out becomes

$0.075s^3 + 1.675\ s^2 + 3.55\ s + (1+K)$

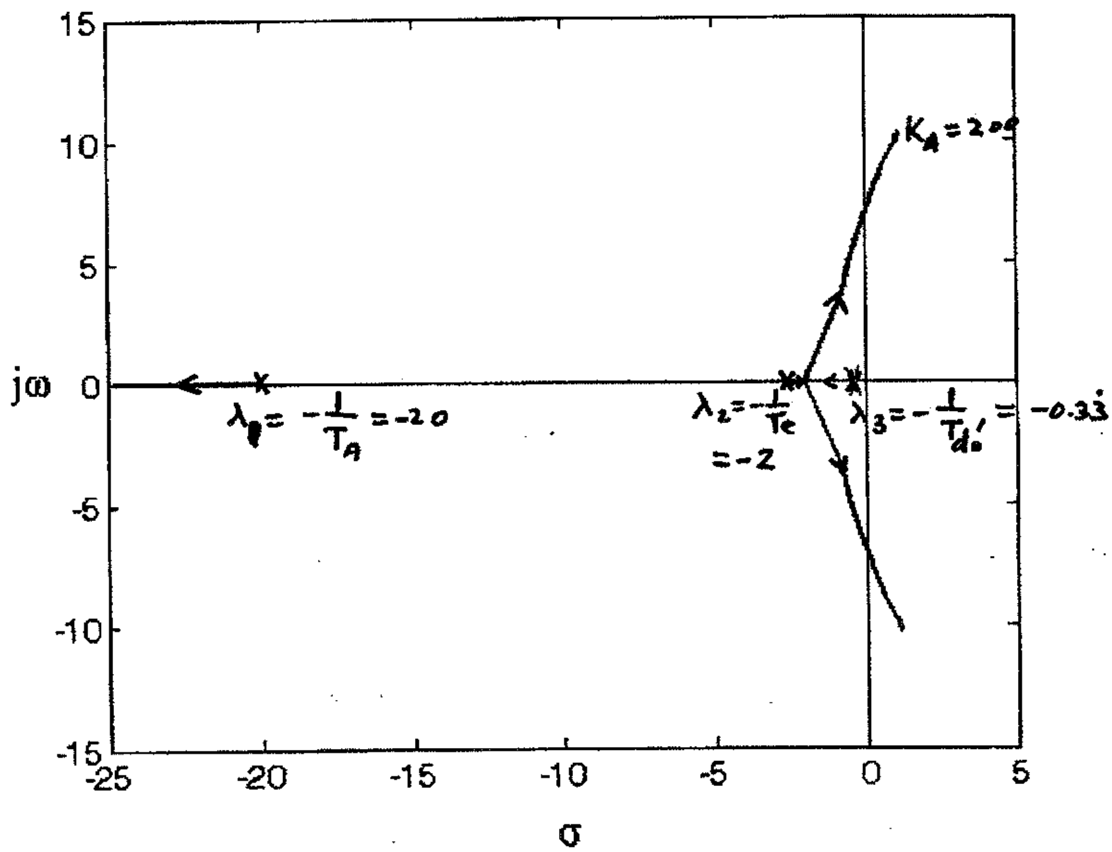
This is the so-called characteristic equation, and the roots of the equation are the poles (eigenvalues) of the closed-loop system. It is a third order equation, so a numerical method to solve for the roots ($\lambda_1, \lambda_2, \lambda_3$) is the most convenient way. With the help of Matlab software*, the following results were obtained.

$G(s)= 0.075s^3+1.675s^2+3.55s+(1+K)$

K	λ_1	λ_2	λ_3
0	-20.0000	-2.0000	-0.3333
20	-20.7002	-0.8166 + 3.5860i	-0.8166 - 3.5860i
40	-21.3159	-0.5087 + 5.0386i	-0.5087 - 5.0386i

	60	-21.8695	-0.2319 + 6.0940i	-0.2319 - 6.0940i
unstable →	80	-22.3751	0.0209 + 6.9475i	0.0209 - 6.9475i
	100	-22.8421	0.2544 + 7.6740i	0.2544 - 7.6740i
	120	-23.2774	0.4720 + 8.3118i	0.4720 - 8.3118i
	140	-23.6860	0.6763 + 8.8834i	0.6763 - 8.8834i
	160	-24.0717	0.8692 + 9.4033i	0.8692 - 9.4033i
	180	-24.4375	1.0521 + 9.8817i	1.0521 - 9.8817i
	200	-24.7860	1.2263 +10.3258i	1.2263 -10.3258i

The corresponding root-loci plot is shown below.



From the results, one note that as K (i.e. K_A) increases, λ_2 and λ_3 move toward each other, and then form a complex conjugate pair. At some K value, the poles move into the RHS and this signifies an unstable system. One can use the table above to estimate when this occurs.

[Or one could use the Hurwitz criteria to find this limiting value of K.

Application of Hurwitz criteria gives the following table

0.075   1.675

3.55   (1+ K)

for which in this case, the condition for stability is

3.55 x 1.675 – 0.075 x (1 + K) > 0

or

K < 78.]

From the above, therefore one concludes that the steady-state accuracy requirement conflict with the stability consideration.

(b) Again from the Matlab program, one obtains the closed-loop eigenvalues as

K = 99	**λ_1= -22.8196**	**λ_2 = 0.2431 + 7.6400i**	**λ_3 = 0.2431 - 7.6400i**

Again confirming it is an unstable amplifier gain setting.

(c) When K = 49.5, the poles are

K = 49.5	**-21.5856**	**-0.3739 + 5.5726i**	**-0.3739 - 5.5726i**

However, the static error is 1/(1+ 49.5) or 0.0195 or almost 2%.

(* Many thanks to Ms Q Wang for obtaining the numerical solutions to the problem).

9-3 In working the previous problem you have now determined that the time response will contain a dominant response term of the oscillatory type

$$e^{\sigma t} \sin(\omega t + \alpha)$$

For stability we not only want σ to be negative but also *sufficiently* negative to make the response term decay fast. Specifically, we require that

$$\sigma < -0.6$$

From the root-locus plot in Prob. 9-2 determine the constraints that this will put on the open-loop gain K.

Solution:

From the table, it is seen that the real parts of λ_2 and λ_3 (i.e. σ) are -0.51 and -0.82 for K = 40 and 20 respectively. One could interpolate between these two points to obtain an estimate of the limiting K so that σ < -0.6 as

K = 40 – (0.09/0.31) x 20 = 34.

While this will give us the required decay rate, however, the steady-state error is close to 3%. Not so good.

9-4 It is most desirable that a control loop be "robust," meaning that the loop should retain its stability even if its parameters will undergo wide ranging changes. Consider the above AVR loop with the gain set at the value computed in Prob. 9-3. We assume now that the generator load increases resulting in a decrease in the effective field inductance according to Eq. (9-20). Specifically, let us assume that the field time constant decreases from 3 to 1 s. All other loop parameters (including *K*) remain unchanged.

Adjust your root-locus plot and determine whether the AVR loop remains stable.

Solution:

[Although not covered in the lecture, the effective field time constant actually decreases approximately according to the equation

$T_{d,\,load} = [(X_d' + X_{ext})/(X_d + X_{ext})] \times T_{do}'$

Where $T_{d,\,load}$ is the effective time constant, X_{ext} is the exterior reactance viewed from the generator terminals and T_{do}' is the field time constant under no load condition.]

Again thanks to the Matlab program, one can obtain the following table with the new generator operating condition when the field time constant is 1 sec. The new characteristic equation is

$0.025s^3+0.575s^2+1.55s+(1+K)$

K	λ_1	λ_2	λ_3
0	-20.0000	-2.0000	-1.0000
20	-21.9198	-0.5401 + 6.1668i	-0.5401 - 6.1668i
40	-23.3524	0.1762 + 8.3784i	0.1762 - 8.3784i
60	-24.5280	0.7640 + 9.9446i	0.7640 - 9.9446i
80	-25.5396	1.2698 +11.1915i	1.2698 -11.1915i
100	-26.4357	1.7178 +12.2423i	1.7178 -12.2423i
120	-27.2448	2.1224 +13.1584i	2.1224 -13.1584i
140	-27.9858	2.4929 +13.9756i	2.4929 -13.9756i
160	-28.6716	2.8358 +14.7164i	2.8358 -14.7164i
180	-29.3116	3.1558 +15.3962i	3.1558 -15.3962i
200	-29.9128	3.4564 +16.0261i	3.4564 -16.0261i
99	-26.3931	1.6966 +12.1933i	1.6966 -12.1933i

From the table, it would appear stability may be a problem. Use Hurwitz criteria to check:

Form the Hurwitz table

0.025	**1.55**
0.575	**(1 + K)**

so for stable operation,

K < 34.65.

K was estimated to be 34 in Problem 9.3. The gain setting is not suitable as it has resulted in the two complex poles very close to the imaginary axis, if not already in the RH plane.

The problem also demonstrates a very important point: as the generator loading increases, the stability margin decreases: the generator would be stable under load condition only if the open loop gain is reduced significantly from 78 under no load to about 34 under this load condition.

Hence, it seems to suggest that the setting of (say) the amplifier gain has to be carried out under load condition. An interesting question: what would be the "worst" case and how could one determine a setting which would guarantee that it would ensure stable operation under other load conditions? This is where the "robustness" of the control must come in.

9-5 The previous problems have demonstrated the need for loop stabilization. Figure 9-24 depicts a circuit often used for this purpose. It consists of a *stabilizing* or *damper* transformer, the primary of which is connected to the regulator voltage v_R or (as shown dashed) to the field voltage v_f. (The latter voltage is obviously not easily accessible in a brushless exciter.) Compare Fig. 9-2.

The transformer has the primary parameters R and L and a mutual inductance M. If it feeds into an amplifier with high input impedance its secondary current is zero and we can model its response with the following two equations:

$$v_R = Ri + L\frac{di}{dt} \tag{9-130}$$

$$v_0 = M\frac{di}{dt} \tag{9-131}$$

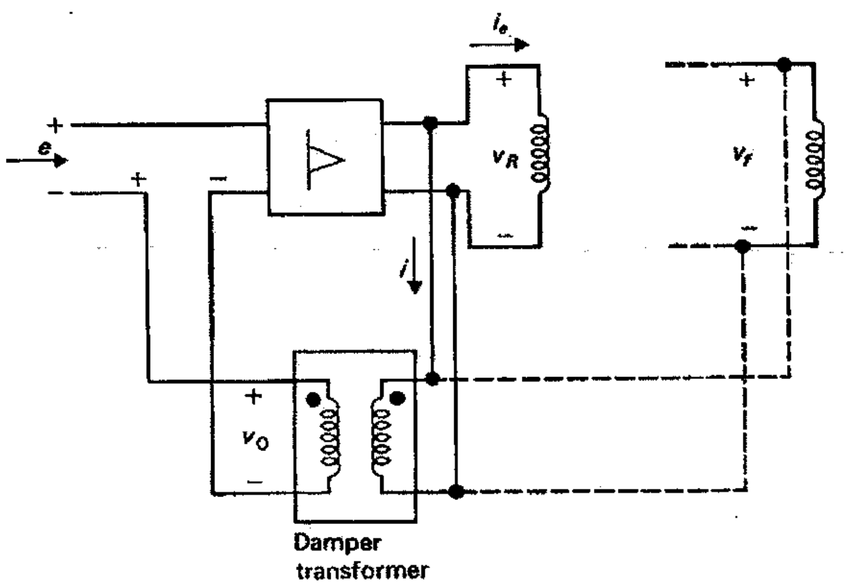


Figure 9-24 Damper transformer.

These equations yield the feed-back stabilizing transfer function

$$G_f = \frac{V_0(s)}{V_R(s)} = \frac{sM}{R + sL} \tag{9-132}$$

If the resistance is dominating we have

$$G_f \approx \frac{M}{R}s \triangleq K_f s \tag{9-133}$$

which then exposes the *derivative feed-back* character of the device.

(*a*) In view of Eq. (9-133) give a *physical* reasoning why this transformer would introduce damping.

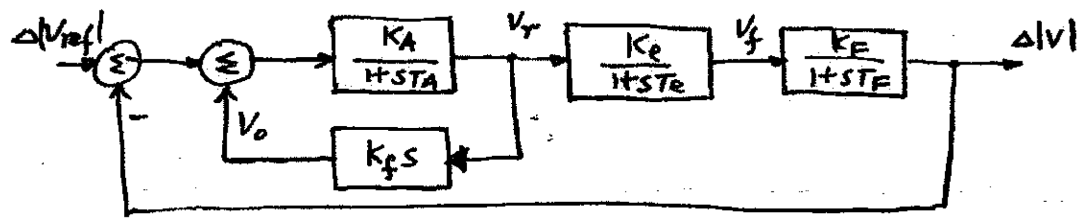
(*b*) Use the transfer function (9-133) and study how the device changes the character of the root loci in Fig. 9-5.

Solution:

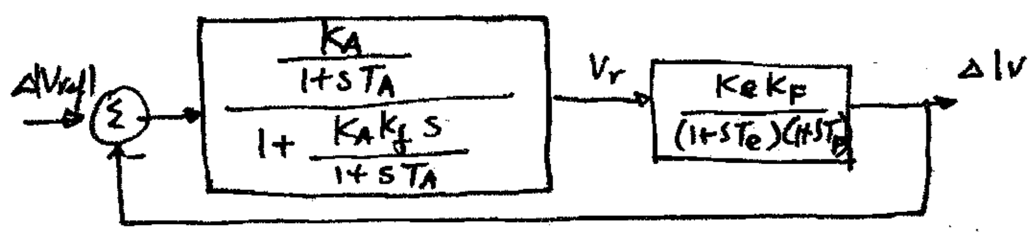
(a) In the lecture note, the explanation of the stabilizing signal has been explained.

(b) Since from Problem 9.4, we note that the loaded condition reduces the open-loop gain to about 34. So we can treat this as a strenuous condition and shall use it to determine the stabilizing system setting. In order to reduce the amount of mathematical manipulation, let's assume that the amplifier gain $K_e = K_F = 1$, and $T_{d,load} = T_F = 1$ sec, $T_e = 0.5$ s, $T_A = 0.05$ s as before. Then the only unknown is K_f, the stabilizer gain. Let $K = K_A K_e K_F$.

Suppose we still want to satisfy the static error of no more than 1%. From the transfer function block diagram shown below, we note that:



Which reduces to



Of which we have the open loop transfer function G(s)

$$G(s) = \frac{K}{[1+s(T_A + K_A K_f)](1+T_e)(1+sT_F)}$$

Steady-state error:

At s = 0,

G(0) = K

And the loop gain becomes

1/(1 + K) < 0.01

thus,

K > 99 (as before)

So we set $K = 100$, i.e. amplifier gain $K_A = 100$, since $K_e = K_F = 1$. So steady state requirement can be met.

Stability consideration:

From G(s), the characteristic equation is

$$[1 + s(0.05 + 100K_f)]\,(1+ 0.5s)\,(1 + s) + 100 = 0$$

After a few steps to collect terms, we have

$$S^3\,(0.5)(0.05 + 100K_f) + s^2\,\{0.5 +(0.05+100K_f)\text{x}1.5\} + s\,\{0.05 + 100K_f + 1.5\} + 101 = 0$$

One notices that the stabilizer setting K_f affects almost all the coefficients of the characteristic equation. To ensure stability, again one could make use of the Hurwitz criteria and determine the range of value on K_f that would be suitable.

From the characteristic equation, the Hurwitz table is

$0.025 + 50\,K_f$	$1.55 + 100\,K_f$
$0.575 + 150\,K_f$	101

Apply Hurwitz criteria, the condition is (after a few steps)

$$15000\,K_f^2 - 4760\,K_f - 1.63375 > 0$$

or

$$K_f > 0.317.$$

(The other solution is a negative value, and is not acceptable).

One could try different values of K_f to see whether the setting could ensure stability, even under other load condition.