Problem Set 2 Electromechanical Energy Conversions

1. Consider a translational motor actuator with the following condition

$$i = \lambda^{3/2} + 2.5\lambda(x-1)^2$$

For $0 < x < 1$ m, where i is the current in the coil of the actuator. Find the force acting on the moving part at $x = 0.6$ m.

2. Consider an electromagnetic system with the following condition

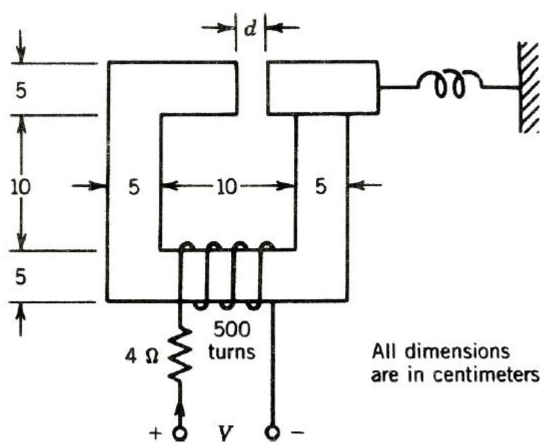
$$\lambda = \frac{1.2i^{1/2}}{g}$$

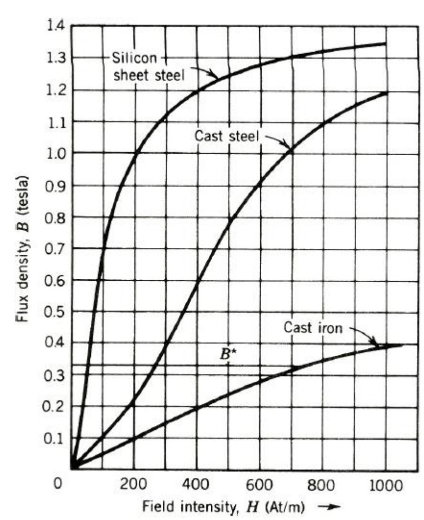
Where g is the air gap length. If i = 2 A and g = 10 cm, find the mechanical force acting on moving part

   a. By energy of the system.

   b. By coenergy of the system.

3. Consider an actuator system as shown below with all dimensions are in centimeters. The magnetic material is cast steel, with magnetization characteristics as shown below. The magnetic core and air gap both consist of a square cross-sectional area. It consists of 500 turns with resistance of 4 Ω.





   a. If gap is 1 mm, find

      i. The coil current and required DC supply voltage to produce an air gap flux density of 0.5 T.

      ii. The stored energy in the actuator system.

      iii. The attraction force on the actuator arm.

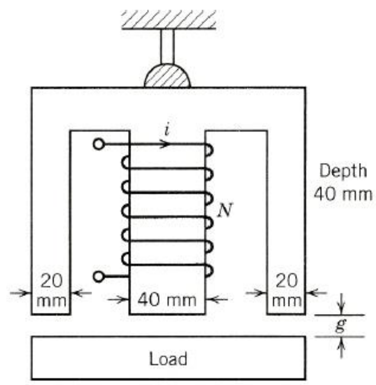
      iv. The inductance of the coil.

b. If the actuator arm is allowed to move with the air gap closed finally.

   i. For zero air gap, find the flux density in the core, force on the arm and stored energy in the actuator system.

   ii. Excluding energy loss in the coil resistance, find the energy transfer between the DC source and the actuator. Assume the arm is moved slowly. Find the direction of energy flow and how much mechanical energy generated.

4. Consider an electromagnet lift system as shown It consists of 2500 turns with flux density in air gap of 1.25 T. Assume the core material is ideal.



   a. If gap is 10 mm, find

      i. The coil current.

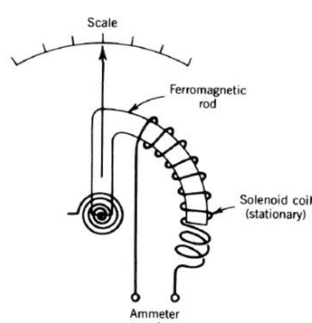
      ii. The energy stored in the system.

      iii. The force acting on the load, i.e., sheet of steel.

      iv. The mass of the load, with gravity of 9.81 m/s^2.

   b. If gap is 2 mm, find the coil current required to lift the load.

5. Consider moving-iron ammeter as shown When current flows through the curved solenoid coil, a curved ferromagnetic rod is pulled into the solenoid against the torque of a restraining spring with spring constant of 0.65×10^{-3} Nm/rad. The coil consists of inductance of $L = 4.5 + 18\theta$ µH, where θ is angle of deflection in radians, and resistance of 0.015 Ω.

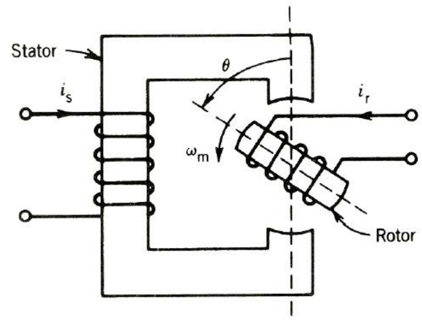


a. Show the ammeter measures the rms of the current.

b. Find the deflection in degree for a current of 10 A (rms).

c. When 10 A (rms) at 60 Hz flows through the ammeter, find the voltage drop across the terminal.

6. Consider a reluctance machine as shown below. A current of 10 A (rms) at 60 Hz flows through its stator coil. The inductance of stator winding is $L_{ss} = 0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta$ H. Find



a. The values of rotor speed where the machine can develop an average torque.

b. The maximum torque and mechanical power that can be developed by the machine at each speed.

c. The maximum torque at zero speed.

Answer

1.

$$W_f = \int i d\lambda = \int \left(\lambda^{\frac{3}{2}} + 2.5\lambda(x-1)^2\right) d\lambda = \frac{2}{5}\lambda^{\frac{5}{2}} + 2.5\frac{\lambda^2}{2}(x-1)^2$$

$$f_m = -\frac{\partial W_f(\lambda, x)}{\partial x}|_{\lambda} = -\frac{2.5}{2}\lambda^2 \cdot 2(x-1) = -2.5\lambda^2(x-1)$$

$$f_m|_{\text{x=0.6m}} = -2.5\lambda^2(0.6-1) = \lambda^2$$

2.

(a)

$$i = \left(\frac{\lambda g}{1.2}\right)^2$$

$$W_f = \int_0^{\lambda} i d\lambda = \int_0^{\lambda} \left(\frac{\lambda g}{1.2}\right)^2 d\lambda = \frac{g^2}{1.2^2} \cdot \frac{\lambda^3}{3}$$

$$f_m = -\frac{\partial W_f(\lambda, g)}{\partial g}|_{\lambda=constant} = -\frac{\lambda^3 2g}{1.2^2 \times 3}$$

For $i = 2\ A$ and $g = 10\ cm$

$$\lambda = \frac{1.2 \times 2^{\frac{1}{2}}}{10 \times 10^{-2}} = 16.97 \text{ Wb}$$

$$f_m = -\frac{16.97^3 \times 2 \times 0.1}{1.2^2 \times 3} = -226.25\ N$$

(b)

$$W_f{}' = \int_0^i \lambda di = \int_0^i \left(\frac{1.2 i^{\frac{1}{2}}}{g}\right) di = \frac{1.2}{g} \cdot \frac{2}{3} \cdot i^{\frac{3}{2}}$$

$$\begin{aligned} f_m &= \frac{\partial W_f{}'(i,g)}{\partial g}|_{i=constant} = -\frac{1.2 \times 2}{3} i^{\frac{3}{2}} \frac{1}{g^2}|_{i=constant} \\ &= -\frac{1.2 \times 2}{3} \times 2^{\frac{3}{2}} \times \frac{1}{0.1^2} \text{ N} \\ &= -226.25 \text{ N} \end{aligned}$$

3.

(a)

From Figure, H_c = 350 At/m at B = 0.5 T

(i)

$$H_g = \frac{0.5}{4\pi \times 10^{-7}}$$

$$Ni = H_c l_c + H_g l_g$$

$$500i = 350 \times 60 \times 10^{-2} + \frac{0.5}{4\pi \times 10^{-7}} \times 1 \times 10^{-3}$$

$$i = 1.22 \text{ A}$$

$$V = 4 \times 1.22 = 4.88 \text{ V}$$

(ii)

$$
\begin{aligned}
W_f &= \frac{B^2}{2\mu_0}V_g + V_c\left(\int HdB\right) \\
&= \frac{0.5^2}{2\times 4\pi\times 10^{-7}}\times 5\times 5\times 10^{-4}\times 10^{-3} + 5\times 5\times 10^{-4}\times 60\times 10^{-4}\times\left(\frac{0.5\times 350}{2}\right) \\
&= 0.2487 + 0.1313 \\
&= 0.38\ \mathrm{J}
\end{aligned}
$$

(iii)

$$
f_m = \frac{B^2}{2\mu_0}\times Area = \frac{0.5^2}{2\times 4\pi\times 10^{-7}}\times 25\times 10^{-4} = 248.7\ \mathrm{N}
$$

(iv)

$$
L = \frac{\lambda}{i} = \frac{NAB}{i} = \frac{500\times 25\times 10^{-4}\times 0.5}{1.22} = 0.5123\ \mathrm{H}
$$

(b)

(i)

$$
Ni = H_c l_c
$$

$$
H_c = \frac{500\times 1.22}{60\times 10^{-2}} = 1017\ \mathrm{A}\cdot t/m
$$

From Figure, B_c = 1.2 T

$$
f_m = \frac{1.2^2}{2\times 4\pi\times 10^{-7}}\times 25\times 10^{-4} = 1432.4\ \mathrm{N}
$$

$$
W_f = V_c\int HdB = 25\times 60\times 10^{-6}\times\frac{1.2\times 1000}{2} = 0.9\ \mathrm{J}
$$

(ii) Assume moving slowly, i remains constant

$$
\begin{aligned}
dW_e &= id\lambda = iNAdB \\
&= 1.22\times 500\times 25\times 10^{-4}\times(1.2-0.5) \\
&= 1.0675\ \mathrm{J}
\end{aligned}
$$

Energy flow from source to actuator. Increase in stored energy as

$$
dW_f = 0.9 - 0.38 = 0.52\ \mathrm{J}
$$

Mechanical energy produced as

$$
dW_m = 1.0675 - 0.52 = 0.5475\ \mathrm{J}
$$

4.

(a)

$$g = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$$

(i)

$$Ni = H_g \cdot 2g = \frac{B_g}{\mu_0} \cdot 2g$$

$$i = \frac{1.25 \times 2 \times 10 \times 10^{-3}}{2500 \times 4\pi \times 10^{-7}} = 7.9577 \text{ A}$$

(ii)

$$W_f = W_{fg} = \frac{{B_g}^2}{2\mu_0} \cdot V_g = \frac{1.25^2}{2 \times 4\pi \times 10^{-7}} \times 80 \times 40 \times 10^{-6} \times 10 \times 10^{-3}$$
$$= 19.8 \text{ J}$$

(iii)

$$f_m = \frac{{B_g}^2}{2\mu_0} \cdot Area = \frac{1.25^2}{2 \times 4\pi \times 10^{-7}} \times 80 \times 40 \times 10^{-6} = 1980 \text{ N}$$

(iv)

$$Mass = \frac{1980}{9.81} = 201.64 \text{ kg}$$

(b)

$$g = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

$$f_m = \frac{{B_g}^2}{2\mu_0} \cdot Area \propto {B_g}^2 \propto \left(\frac{Ni}{2g}\right)^2 \propto \left(\frac{i}{g}\right)^2$$

Force remains same for $g = 5\ mm$ and $g = 10\ mm$.

$$\left(\frac{i_2}{g_2}\right)^2 = \left(\frac{i_1}{g_1}\right)^2 \rightarrow \left(\frac{i_2}{5}\right)^2 = \left(\frac{7.9577}{10}\right)^2$$

$$i_2 = 3.9789 \text{ A}$$

5.

(a)

$$L = 4.5 + 18\theta\ \mu\text{H},\ K_s = 0.65 \times 10^{-3} \text{ Nm/rad}$$

$$T = \frac{1}{2} i^2 \frac{dL}{d\theta} = \frac{1}{2} i^2 \cdot 18 \cdot 10^{-6} = 9 \cdot 10^{-6} i^2$$

$$T|_{avg} = 9 \cdot 10^{-6} |i^2|_{avg} = 9 \cdot 10^{-6} {I_{rms}}^2 \text{ Nm}$$

In the equilibrium position

$$0.65 \times 10^{-3}\theta = 9 \times 10^{-6} I_{rms}$$

$$\theta = 13.85 \times 10^{-3} {I_{rms}}^2$$

Deflection is proportional to the square of rms current, scale on the ammeter will be nonlinear

(b)

$$\begin{aligned} \theta &= 13.85 \times 10^{-3} \times 10^2 = 1.385 \text{ radian} \\ &= \frac{180}{\pi} \times 1.385 = 79.35^\circ \end{aligned}$$

(c)

$$L = 4.5 + 18 \times 1.385\ \mu\text{H} = 29.43\ \mu\text{H}$$

$$Z = 0.015 + j377 \times 29.43 \times 10^{-6} = 0.015 + j0.011$$

$$|Z| = 0.0186\ \Omega$$

$$|V| = 0.0186 \times 10 = 0.186\ \text{V}$$

6.

(a)

$$T = \frac{1}{2} i^2 \frac{dL}{d\theta} = \frac{1}{2} i^2 (0.6 \sin 2\theta + 0.8 \sin 4\theta)$$

Let $i = 10\sqrt{2} \sin \omega t$, $\theta = \omega_m t - \delta$.

$$\begin{aligned} T &= 100 \sin^2 \omega t \left[0.6 \sin(2\omega_m t - 2\delta) + 0.8 \sin(4\omega_m t - 4\delta)\right] \\ &= 30 \sin(2\omega_m t - 2\delta) - 30 \cos(2\omega t) \sin(2\omega_m t - 2\delta) \\ &+ 40 \sin(4\omega_m t - 4\delta) - 40 \cos(2\omega t) \sin(4\omega_m t - 4\delta) \end{aligned}$$

$$\begin{aligned} T &= 30 \sin(2\omega_m t - 2\delta) + 40 \sin(4\omega_m t - 4\delta) \\ &- 15 \left[\sin(2\omega_m t - 2\delta + 2\omega t) + \sin(2\omega_m t - 2\delta - 2\omega t)\right] \\ &- 20 \left[\sin(4\omega_m t - 4\delta + 2\omega t) + \sin(4\omega_m t - 4\delta - 2\omega t)\right] \end{aligned}$$

For $\omega_m \neq 0$, average torque are produced where,

$$\omega_m = \pm\omega = \pm 377 \text{ rad/s}$$

$$\text{or} \quad \omega_m = \pm\frac{\omega}{2} = \pm 188.5 \text{ rad/s}$$

(b)
For $\omega_m = \pm 377$ rad/s,

$$T_{avg} = 15 \sin 2\delta$$

$$T_{\max} = 15 \text{ Nm}$$

$$P_{\max} = 15 \times 377 = 5655 \text{ W}$$

For $\omega_m = \pm 188.5$ rad/s,

$$T_{avg} = 20 \sin 4\delta$$

$$T_{\max} = 20 \text{ Nm}$$

$$P_{\max} = 20 \times 188.5 = 3770 \text{ W}$$

For $\omega_m = 0$,

$$T_{avg} = 30 \sin 2\delta + 40 \sin 4\delta$$

For maximum torque, $\frac{dT_{avg}}{d\delta} = 0 \; \rightarrow$

$$0 = 60 \cos 2\delta + 160 \cos 4\delta \rightarrow \delta = 25.86^{\circ}$$

$$T_{\max} = 30 \sin 51.72 + 40 \sin 103.44 = 62.45 \text{ Nm}$$