EE6403 Distributed Multimedia Systems Assignment 1

Important:
Please submit ONLY the **hardcopy** of your completed assignment to A/P Yap Kim Hui at office S2-B2b-53 by **13 March 2020**. You may slot your assignment under the office's door if the instructor in not in the office.

The objective of this assignment is to explore image compression techniques introduced in the lecture. The completed assignment should include printouts of the source codes, illustrative figures (if any), and all the necessary computations to answer the questions. You are encouraged to use MATLAB to solve the problems, unless stated otherwise in the questions.

1. Karhunen-Loeve Transform (KLT) is used in an image compression scheme. A 6×6 image is given as follows:

$$\mathbf{U} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 1 & 2 \\ 1 & 1 & 2 & 4 & 4 & 4 \\ 1 & 2 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 3 & 4 & 4 \end{bmatrix}$$

   (a) Partition the image into 9 2×2 blocks (subimages), and order them lexicographically to form 9 4×1 column vectors (**x**). Calculate the mean vector, and show that its sample covariance matrix is given by:

$$\mathbf{C} = \begin{bmatrix} 1.6944 & 1.7361 & 1.4028 & 1.4167 \\ 1.7361 & 2.0278 & 1.6944 & 1.6667 \\ 1.4028 & 1.6944 & 2.1111 & 1.9167 \\ 1.4167 & 1.6667 & 1.9167 & 2.2500 \end{bmatrix}$$

   (b) Determine the corresponding eigenvectors and eigenvalues for the covariance matrix **C** using MATLAB.

   (c) A 2×2 block after zero-mean centering is given by:

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

   Calculate the total squared error of the reconstructed image if the image block is approximated using two dominant principal components.

2. The singular value decomposition (SVD) of a matrix **A** is given by:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & 5 \end{bmatrix} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$$

(a) Find the singular values in the SVD of **A manually**.

(b) Explain how SVD can be used to perform image compression.

(c) Write a MATLAB program to perform color image compression using singular value decomposition (SVD). Choose a suitable color image and demonstrate how SVD can achieve image reconstruction using low-rank approximation with different ranks. Your answers should contain the program listing and various figures.

(d) Discuss the limitations of using SVD to perform image compression.