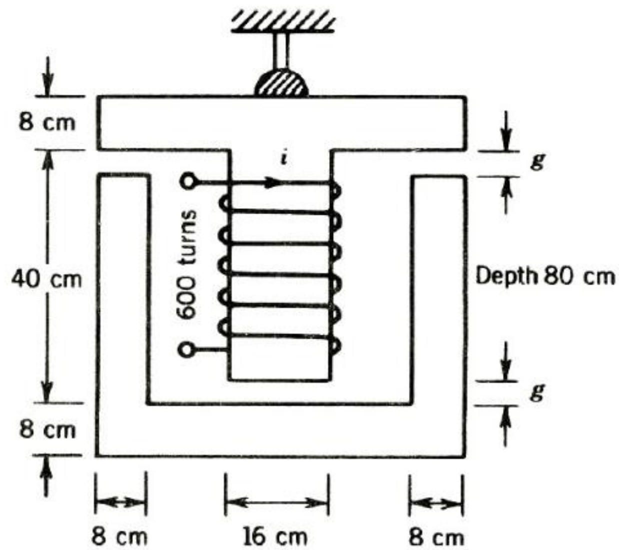
Problem Set 1 Magnetic Circuits

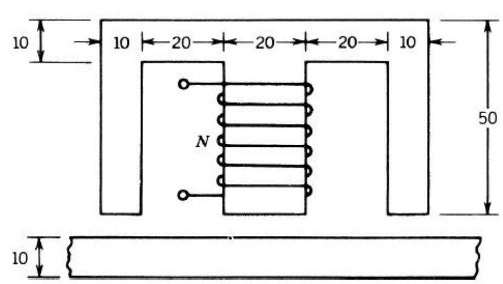
1. Consider an electromagnetic system a shown below. It is used to lift a section of steel channel and it consists of coils of 600 turns. The reluctance of the magnetic material can be neglected up to a flux density of 1.4 T.



   a. For a coil current of 15 A (DC), find the maximum air gap length at which the flux density is 1.4 T.

   b. For the air gap found in (a), find the force acting on the steel channel.

2. Consider an electromagnet that can lift a length of steel strip, as shown below. It consists of 500 turns that carry 20 A without overheating. The magnetic material consists of negligible reluctance at flux densities up to 1.4 T. Find the maximum air gap such that a flux density of 1.4 T can be established with a coil current of 20 A.

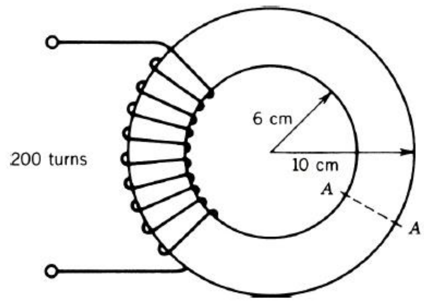


3. Consider a toroid that is made from cast steel with flux density of 1.2 T and field density of 1000 At/m as shown below.

   a. If a core flux density of 1.2 T at the mean radius of the toroid is needed, find the coil current.

   b. Assuming uniform flux density in the coil, find the core flux in Wb.

c. If a 2-mm-wide air gap is made in the toroid, find the new coil current needed to maintain a core flux density of 1.2 T.



4. Consider a toroid in Q.3. It consists of coil current of 2 A and relative permeability of the core of 2000. The core is with a square cross section.

   a. Find the maximum and minimum values of the flux densities in the core.

   b. Find the magnetic flux in the core.

   c. Find the flux density at the mean radius of the toroid and compare it with the average flux density across the core.

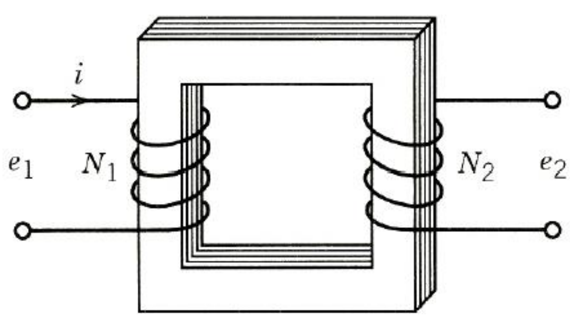
5. A coil wound on a magnetic core is excited by (i) 100 V, 50 Hz and (ii) 110 V, 60 Hz. Compare the hysteresis and eddy current losses with the two sources. For hysteresis loss, n = 2 is chosen.

6. Consider a two-winding transformer with a laminated core as shown below. It consists of primary winding of 200 turns that is able to generate a flux density in the core of B = 1.2 sin 377 t. The secondary winding consists of 400 turns that is left open-circuited. The stacking factor of the core is 0.95, i.e., the core occupies 95 % of the gross core volume. The gross cross-sectional area of the core is 25 cm^2 with relative permeability of 10,000. The core length is 90 cm.

   a. Find the rms value of the applied voltage.

   b. Find the current in the winding.

   c. Find the rms voltage induced in the secondary winding.



Answer

1.

(a) Flux density is same at all these airgaps:

$$Ni = H_g \times 2g = \frac{B_g}{\mu_0} \times 2g$$

$$g = \frac{Ni\mu_0}{2B_g} = \frac{600 \times 15 \times 4\pi \times 10^{-7}}{2 \times 1.4} = 4.04mm$$

(b)

$$f_m = \frac{{B_g}^2}{2\mu_0} \times Area = \frac{1.4^2}{2 \times 4\pi \times 10^{-7}} \times (8 \times 80 \times 10^{-4} + 16 \times 80 \times 10^{-4} + 8 \times 80 \times 10^{-4})$$

$$= \frac{1.4^2}{8\pi \times 10^{-7}} \times 0.256 = 199.64KN$$

2.

B=1.4T throughout. H_c=0

$$Ni = H_{g_1} g_1 + H_{g_2} g_2$$

$$H_{g_1} = H_{g_2} = \frac{B}{\mu_0}$$

$$g_1 = g_2 = g$$

$$Ni = 2\frac{B}{\mu_0} g$$

$$g = \frac{\mu_0 Ni}{2B} = \frac{4\pi \times 10^{-7} \times 500 \times 20}{2 \times 1.4} m = 4.5mm$$

3.

(a) Mean length of core, $l_c = 2\pi \times \frac{10+6}{2} \times 10^{-2} m = 0.503m$.

For cast steel, H_c=1000 At/m at B=1.2T.

$$i = \frac{H_c l_c}{N} = \frac{1000 \times 0.503}{200} = 2.51A$$

(b) $A_c = \pi \times (2 \times 10^{-2})^2 = 1.26 \times 10^{-3} m^2$

$$\Phi_c = A_c \times B_c = 1.26 \times 10^{-3} \times 1.2 = 1.51 \times 10^{-3} Wb$$

(c) $Ni = H_c l_c + H_g l_g = H_c l_c + \frac{B_g}{\mu_0} l_g$

$$i = \frac{1000 \times 0.503}{200} + \frac{1.2 \times 2 \times 10^{-3}}{4\pi \times 10^{-7} \times 200} = 12.06A$$

4.

(a) $B = \mu_r \mu_0 H = \mu_r \mu_0 \frac{Ni}{2\pi r}$

$$B_{\max} = 2000 \times 4\pi \times 10^{-7} \times \frac{200 \times 2.0}{2\pi \times 6 \times 10^{-2}} T = 2.667T \Big| at\ inside$$

$$B_{\min} = 2000 \times 4\pi \times 10^{-7} \times \frac{200 \times 2.0}{2\pi \times 10 \times 10^{-2}} T = 1.6T \Big| at\ outside$$

(b)

$$\begin{aligned}\Phi &= \int BdA = \int \mu H \times (10-6) \times 10^{-2} dr \\ &= \int \mu \frac{Ni}{2\pi r} \times 0.04 dr = \frac{0.04 \mu Ni}{2\pi} \ln \frac{r2}{r1} \\ &= \frac{0.04 \times 2000 \times 4\pi \times 10^{-7} \times 200 \times 2.0}{2\pi} \times \ln\left(\frac{10}{6}\right) \\ &= 3.269 \times 10^{-3} Wb\end{aligned}$$

(c) $B\big|_{centre} = 2000 \times 4\pi \times 10^{-7} \times \frac{200 \times 2.0}{2\pi \times 8 \times 10^{-2}} = 2.0T$

$$B\big|_{avg} = \overline{B} = \frac{\Phi}{A} = \frac{3.269 \times 10^{-3}}{4 \times 4 \times 10^{-4}} = 2.0432T$$

5.

$$E = 4.44N \cdot f \cdot A \cdot B_{\max}$$

$$B_{\max} = \frac{E}{4.44N \cdot f \cdot A}$$

$$P_h = K_h f \frac{E^2}{(4.44NfA)^2} = K_1 \frac{E^2}{f}$$

$$P_e = K_e f^2 \frac{E^2}{(4.44NfA)^2} = K_2 E^2$$

Hysteresis loss:

$$\frac{P_{h(60)}}{P_{h(50)}} = \left(\frac{E_{60}}{E_{50}}\right)^2 \times \left(\frac{f_{50}}{f_{60}}\right) = \left(\frac{110}{100}\right)^2 \times \left(\frac{50}{60}\right) = 1.008$$

Eddy current loss:

$$\frac{P_{e(60)}}{P_{e(50)}} = \left(\frac{E_{60}}{E_{50}}\right)^2 = \left(\frac{110}{100}\right)^2 = 1.21$$

6.

(a)

$$\Phi = BA \times 0.95 = 1.2 \times 25 \times 10^{-4} \times 0.95 \sin(377\,\mathrm{t})$$
$$= 28.5 \times 10^{-4} \sin(377\,\mathrm{t})$$
$$\mathrm{e}_1 = N_1 \frac{d\Phi}{dt} = 200 \times 28.5 \times 10^{-4} \times 377 \cos(377\,\mathrm{t})$$
$$= 214.89 \cos(377\,\mathrm{t})$$
$$E_1 = \frac{214.89}{\sqrt{2}} = 151.973V$$

(b) $B_{\max} = 1.2T,\ H_{\max} = \dfrac{1.2}{10000 \times 4\pi \times 10^{-7}} = 95.49 At/m$

$$i_{\max} = \frac{95.49 \times 90 \times 10^{-2}}{200} = 0.4297A$$
$$i_1 = 0.4297 \sin(377t)$$

(c) $E_2 = \dfrac{N_2}{N_1} E_1 = \dfrac{400}{200} \times 151.973 = 303.946V$