Modular Supervisory Control

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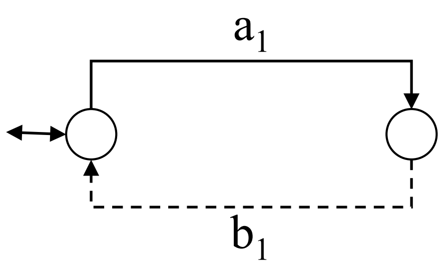
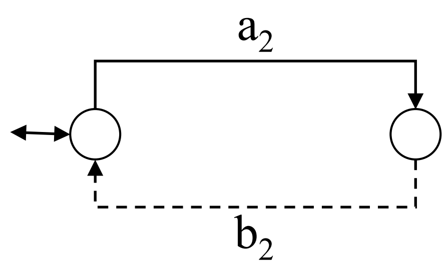
Outline

- Motivation
- Ramadge-Wonham Modular Supervisory Control
- Queiroz-Cury Extension
- Coordinated Modular Supervisory Control
- Example
- Interface-based Approach
- Conclusions

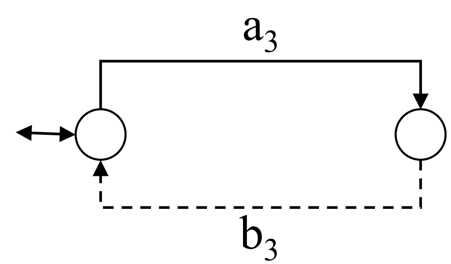
Divide & Conquer



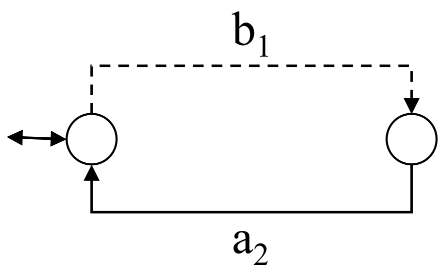
Machine G_1



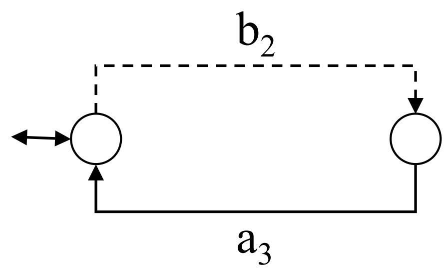
Machine G_2



Machine G_3



Specification R_1



Specification R_2

Construct Local Supervisors (by TCT)

- $G = G_1 \times G_2 \times G_3$ $(G = \text{Sync}(\text{Sync}(G_1 , G_2) , G_3)$ $(8 ; 24))$
- $\text{SPEC}_1 = \text{Selfloop}(R_1 , \{a_1,a_3,b_2,b_3\})$ $(2 ; 10)$
- $\text{SPEC}_2 = \text{Selfloop}(R_2 , \{a_1,a_2,b_1,b_3\})$ $(2 ; 10)$
- $\text{SUPER}_1 = \text{Supcon}(G , R_1)$ $(12 ; 28)$
- $\text{SUPER}_2 = \text{Supcon}(G , R_2)$ $(12 ; 28)$
- $\text{Nonconflict}(\text{SUPER}_1 , \text{SUPER}_2) = \text{true}$
- $R = R_1 \times R_2$ $(R = \text{Sync}(R_1 , R_2)$ $(4 ; 16))$
- $\text{SUPER} = \text{Supcon}(G , R)$ $(18 ; 32)$
- $\text{Isomorph}(\text{SUPER} , \text{Sync}(\text{SUPER}_1 , \text{SUPER}_2)) = \text{true}$

What to Gain ?

- $\text{Minsuper} = \text{Supreduce}(G, \text{SUPER}, \text{SUPER})$ (4 ; 13)
- $\text{Minsuper}_1 = \text{Supreduce}(G, \text{SUPER}_1, \text{SUPER}_1)$ (2 ; 2)
- $\text{Minsuper}_2 = \text{Supreduce}(G, \text{SUPER}_2, \text{SUPER}_2)$ (2 ; 2)
- $|A| :=$ the total number of states and transitions of A

$$|\text{SUPER}_1| + |\text{SUPER}_2| < |\text{SUPER}|$$

Motivation of Modular Control

Reduce complexity by allocating control tasks to local supervisors!

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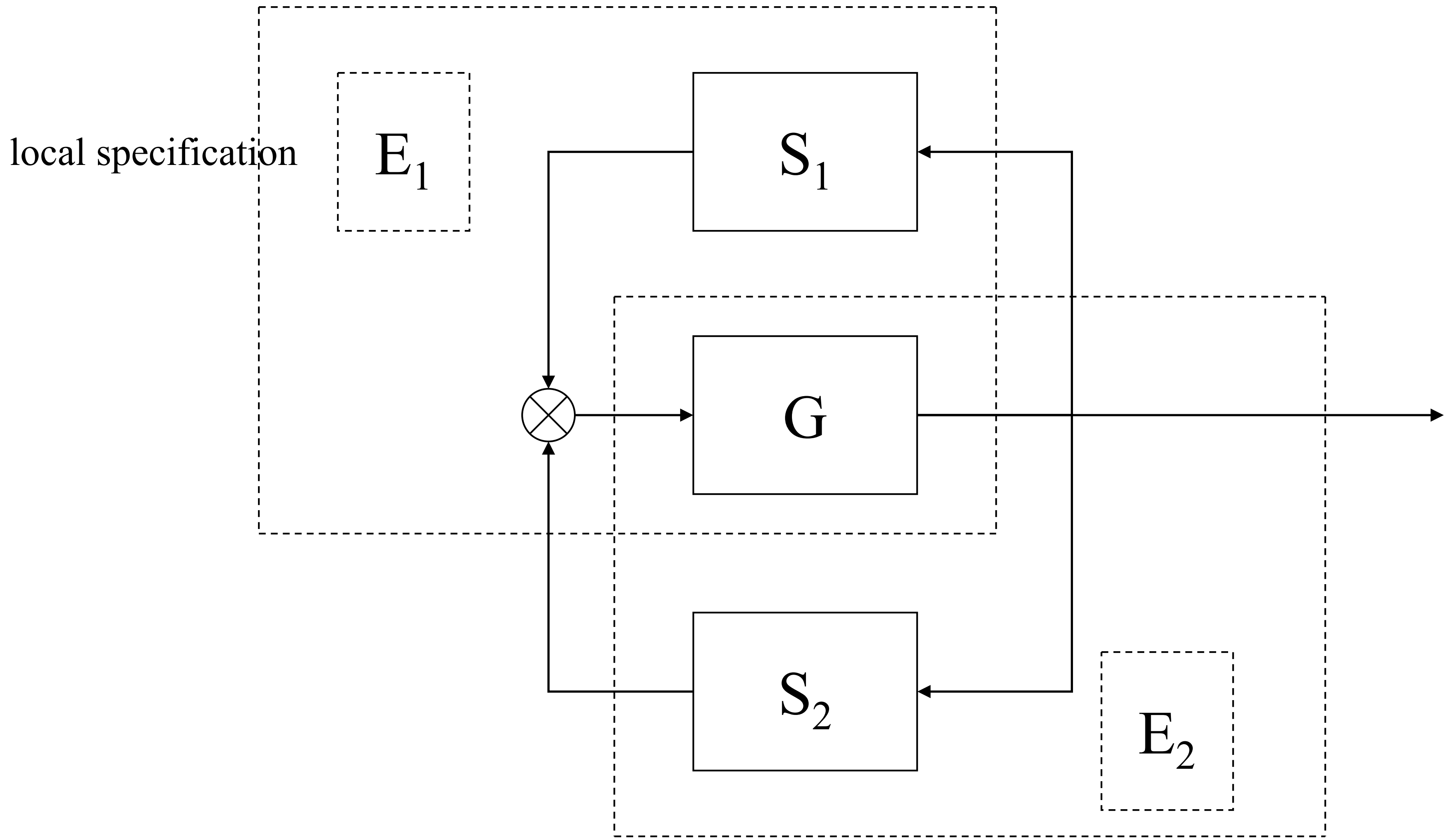
Architecture of Modular Supervisory Control



Composition of Local Supervisors (1)

- Recall that S is a *proper supervisor* of G if
  - $L_m(S) \cap L_m(G)$ is controllable with respect to G and Σ_{uc},
  - $\overline{L_m(S) \cap L_m(G)} = L(S) \cap L(G)$,
  - S is nonblocking, i.e. $\overline{L_m(S)} = L(S)$.
- Let S/G denote the supervision of S over G
  - $L_m(S/G) := L_m(S) \cap L_m(G)$,
  - $L(S/G) := L(S) \cap L(G)$.
- Given S_1 and S_2, let $S_1 \wedge S_2 :=$ reachable$(S_1 \times S_2)$

Composition of Local Supervisors (2)

- Theorem 1 (Ramadge-Wonham)
  - Given two proper supervisors S_1 and S_2 of G, we have

$$L_m((S_1 \wedge S_2)/G) = L_m(S_1/G) \cap L_m(S_2/G)$$
$$L((S_1 \wedge S_2)/G) = L(S_1/G) \cap L_m(S_2/G)$$

  - Furthermore, $S_1 \wedge S_2$ is a proper supervisor of G if and only if
    - $S_1 \wedge S_2$ is nonblocking
    - $L_m(S_1/G)$ and $L_m(S_2/G)$ are nonconflicting, i.e.

$$\overline{L_m(S_1/G) \cap L_m(S_2/G)} = \overline{L_m(S_1/G)} \cap \overline{L_m(S_2/G)} = L(S_1/G) \cap L(S_2/G)$$

Composition of Local Supervisors (3)

- Let $\mathcal{C}(G, E) := \{K \subseteq L_m(G) \cap L_m(E) \mid \overline{K}\Sigma_{uc} \cap L(G) \subseteq \overline{K}\}$.
- Let $\sup\mathcal{C}(G, E)$ be the greatest element of $\mathcal{C}(G, E)$.
- Theorem 2 (Wonham-Ramadge)
  - Given a plant G and two specifications E_1, E_2, if $\sup\mathcal{C}(G, E_1)$ and $\sup\mathcal{C}(G, E_2)$ are nonconflicting, then

$$\sup\mathcal{C}(G, E_1 \times E_2) = \sup\mathcal{C}(G, E_1) \cap \sup\mathcal{C}(G, E_2)$$

The General Procedure for RW Modular Design

- Given G and E_1, E_2
- S_1 = Supcon(G , E_1)
- S_2 = Supcon(G , E_2)
- Nonconflict(S_1 , S_2) = true ?
  - If yes, then {S_1 and S_2} is a modular supervisor of G w.r.t. E_1, E_2
  - Otherwise, the problem is unsolvable by RW modular control theory
    - But we can compute a coordinator to solve the conflicting part of $(S_1 \wedge S_2)/G$

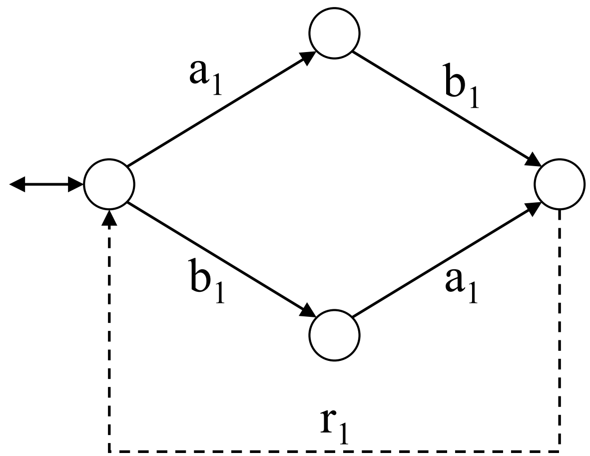
Inadequacy of RW Modular Control Theory (MCT)

- More on implementation simplicity than synthesis simplicity
  - It is computationally expensive to verify the condition

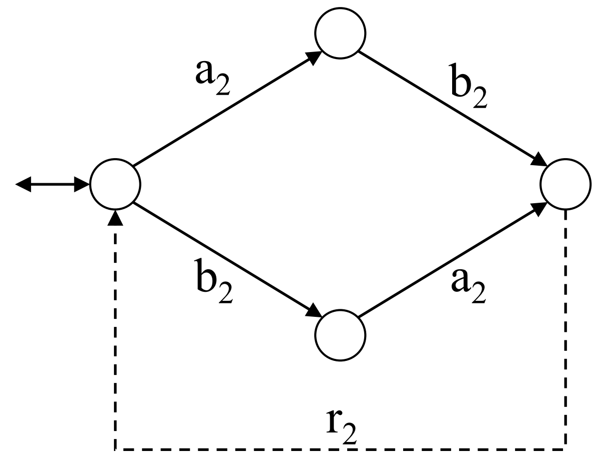
    $\sup\mathcal{C}(G, E_1)$ and $\sup\mathcal{C}(G, E_2)$ are nonconflicting

  - If the condition doesn't hold, RWMCT doesn't tell what to do next ?

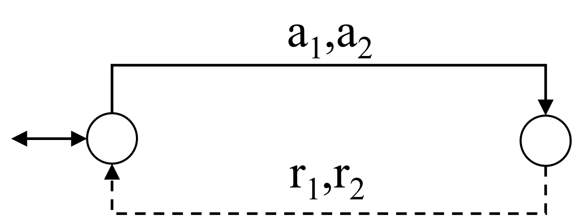
Example – Resource Competition



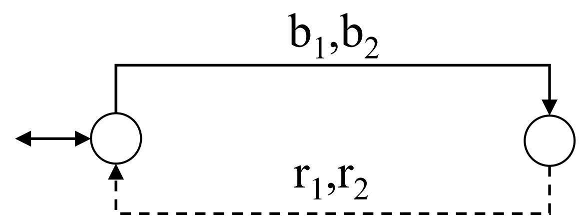
User 1: G_1



User 2: G_2



Resource A: R_A

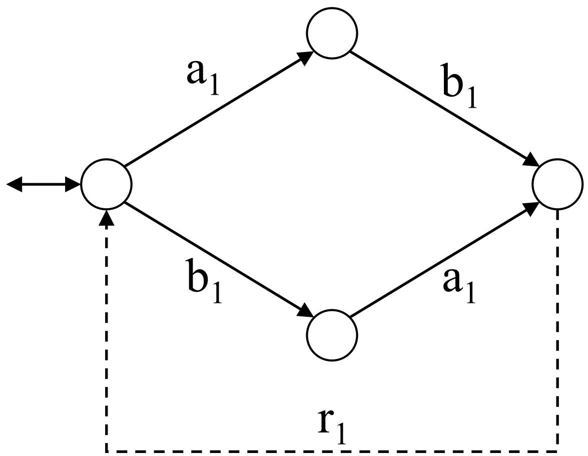


Resource B: R_B

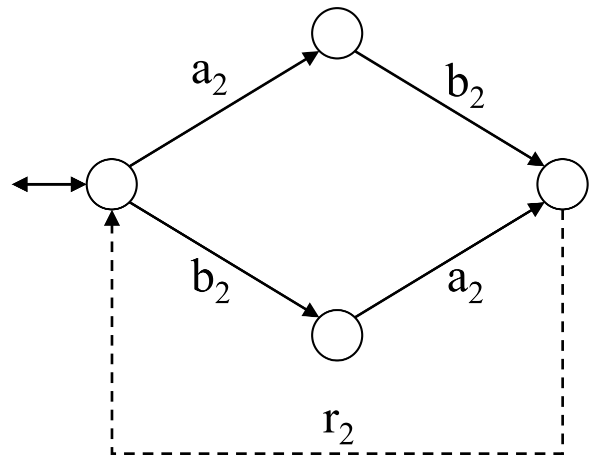
Specification

- Deadlock should not happen.

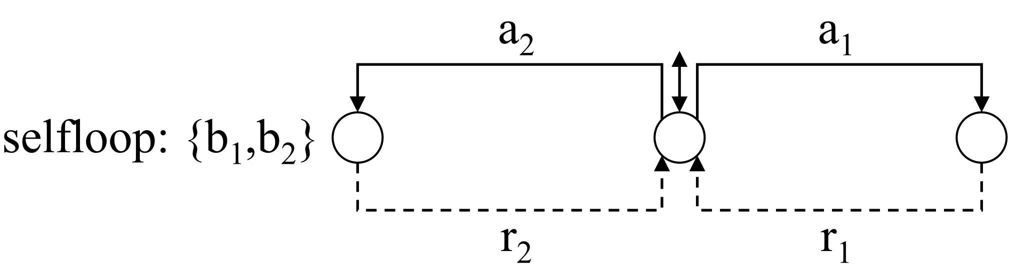
A “Naive” Modular Supervisor



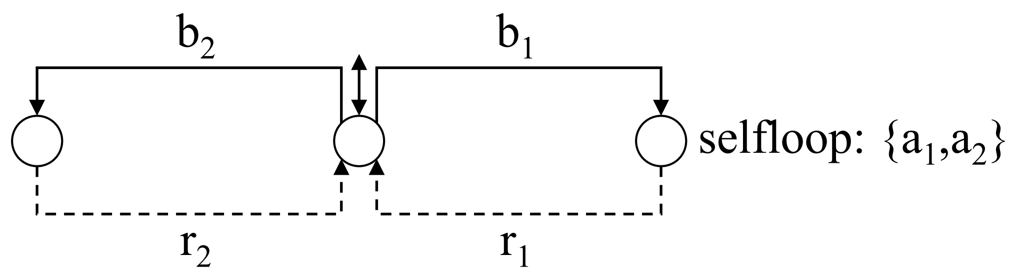
G_1



G_2



Local Supervisor: S_A



Local Supervisor: S_B

Facts

- S_A is a proper supervisor of $G_1 \times G_2 \times R_A$
- S_B is a proper supervisor of $G_1 \times G_2 \times R_B$
- Nevertheless, $L_m(S_A)$ and $L_m(S_B)$ are conflicting.
- We can check that $G_1 \times G_2 \times R_A \times R_B \times S_A \times S_B$ has deadlock!

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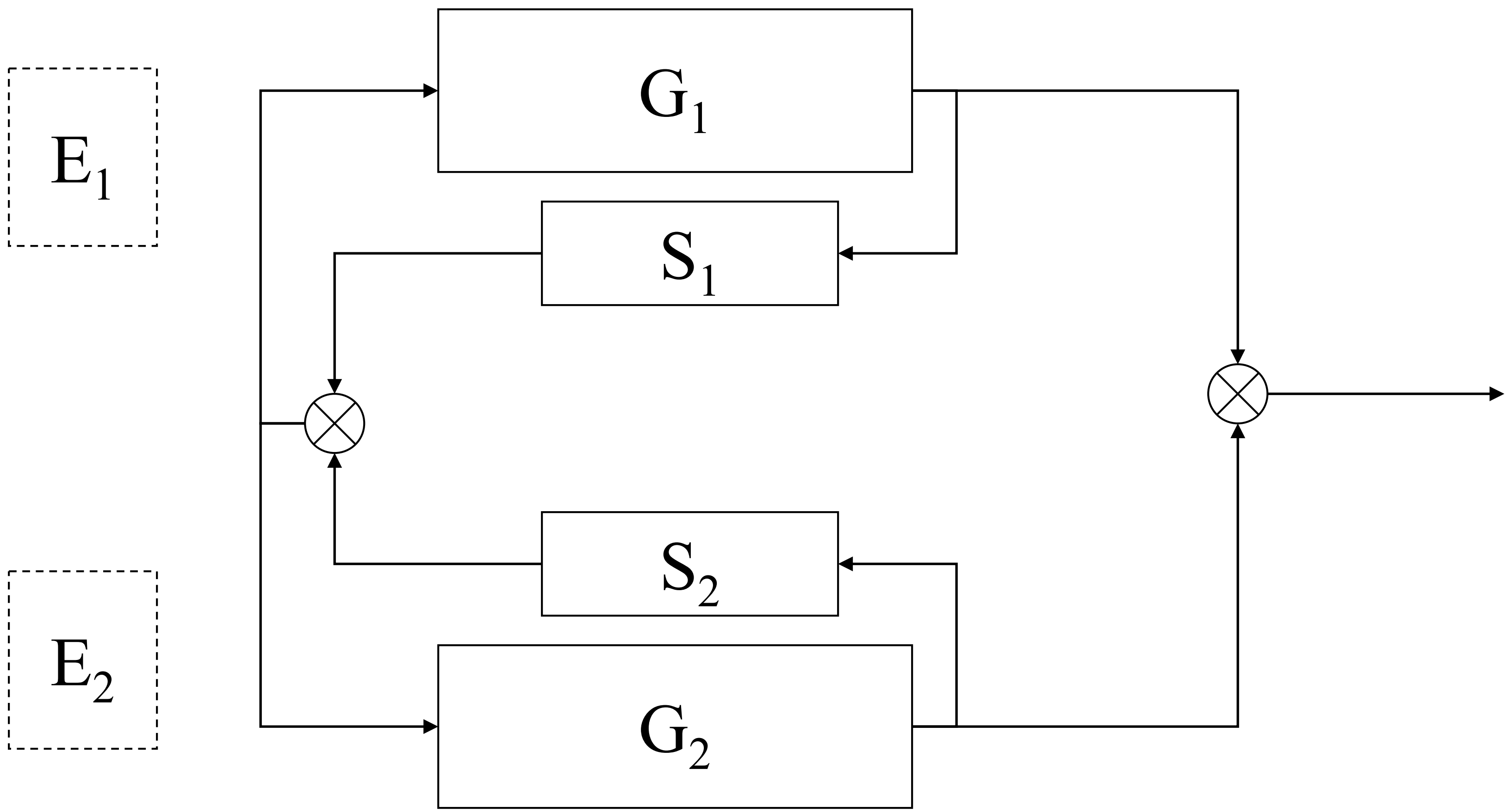
An Extended Architecture



Main Result

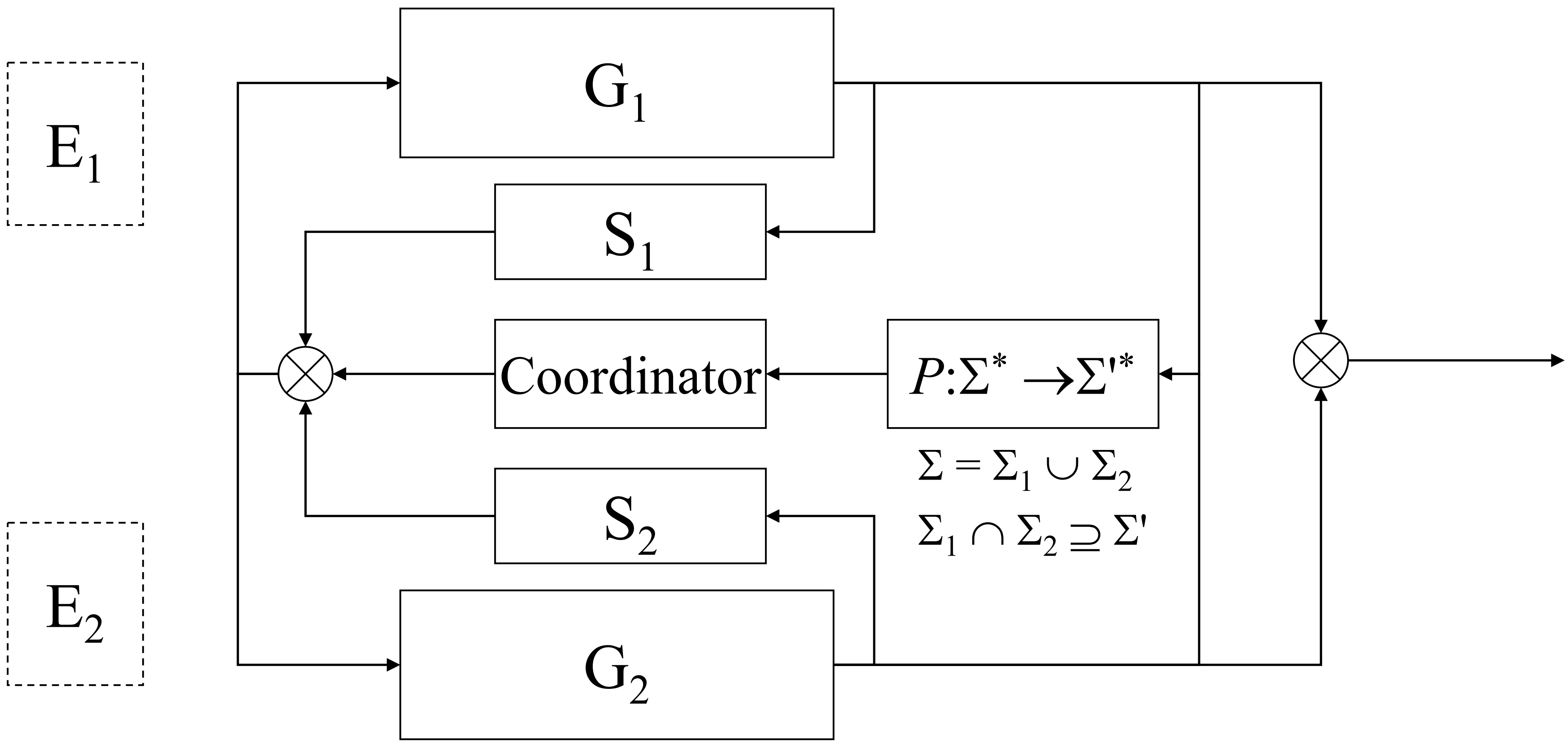
- (Product) Plant: $\{G_i \in \phi(\Sigma_i) \mid i \in I \wedge (\forall j \in I)\ j \neq i \Rightarrow \Sigma_i \cap \Sigma_j = \emptyset\}$
- Specifications: $\{E_i \in \phi(\Sigma_i) | i \in I\}$
- Let $G = \times_{i \in I} G_i$ and $E = \times_{i \in I} E_i$
- Let S_i be a proper supervisor of G_i with respect to E_i
- Theorem 3 (Queiroz-Cury)
  - $\wedge_{i \in I} S_i$ is a proper supervisor of G with respect to E if $\wedge_{i \in I} S_i$ is nonblocking and $\{L_m(S_i/G_i) | i \in I\}$ is (synchronously) nonconflicting.
  - Furthermore, if $\{\sup\mathcal{C}(G_i,E_i) \mid i \in I\}$ is (synchronously) nonconflicting then

$$\sup\mathcal{C}(G,E) = \|_{i \in I} \sup\mathcal{C}(G_i,E_i)$$

The inadeqracy of RW modular control theory still exists!

But we can do something about it …

One Solution to The Inadequacy



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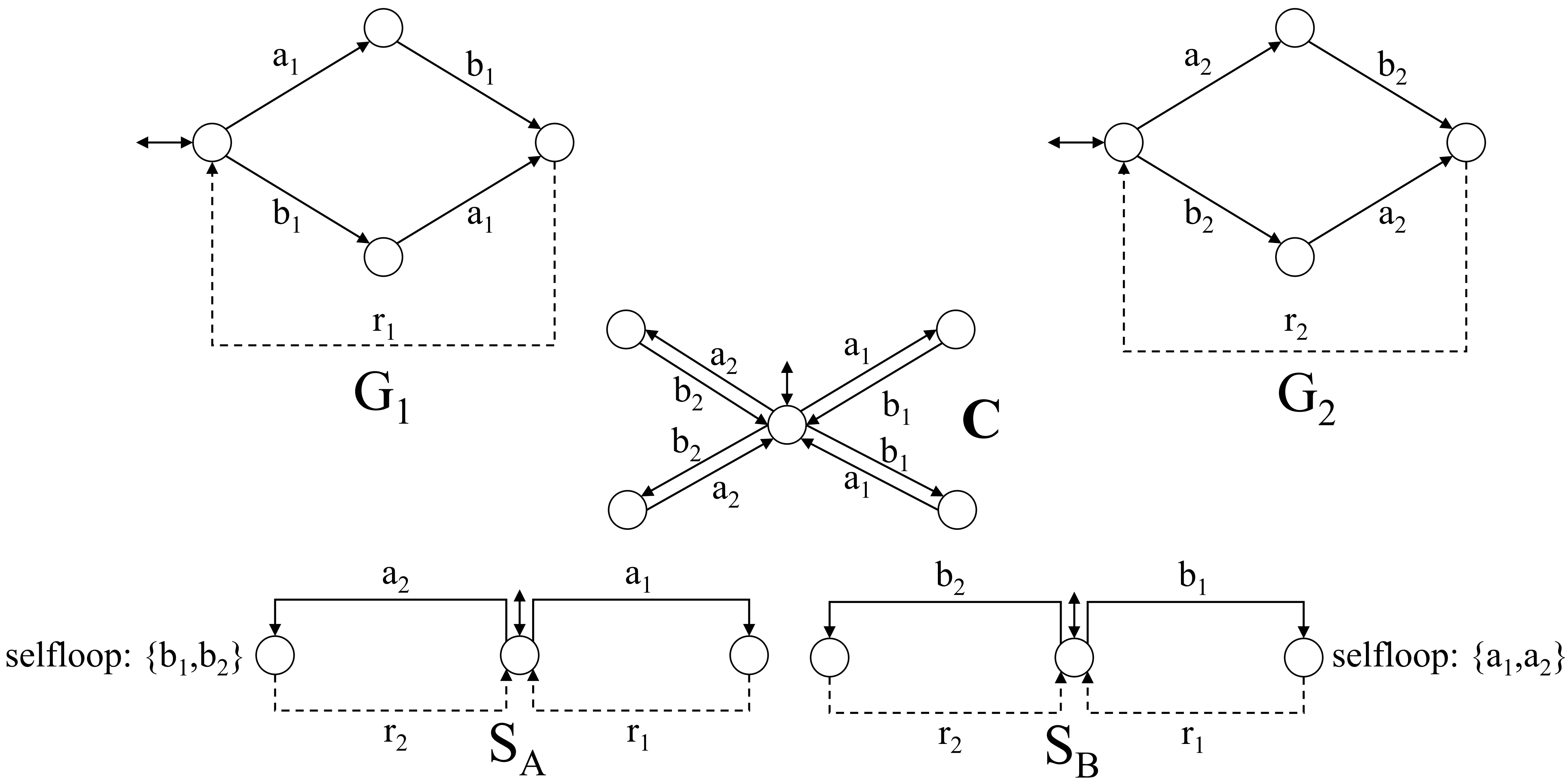
Example – Resource Competition Revisit

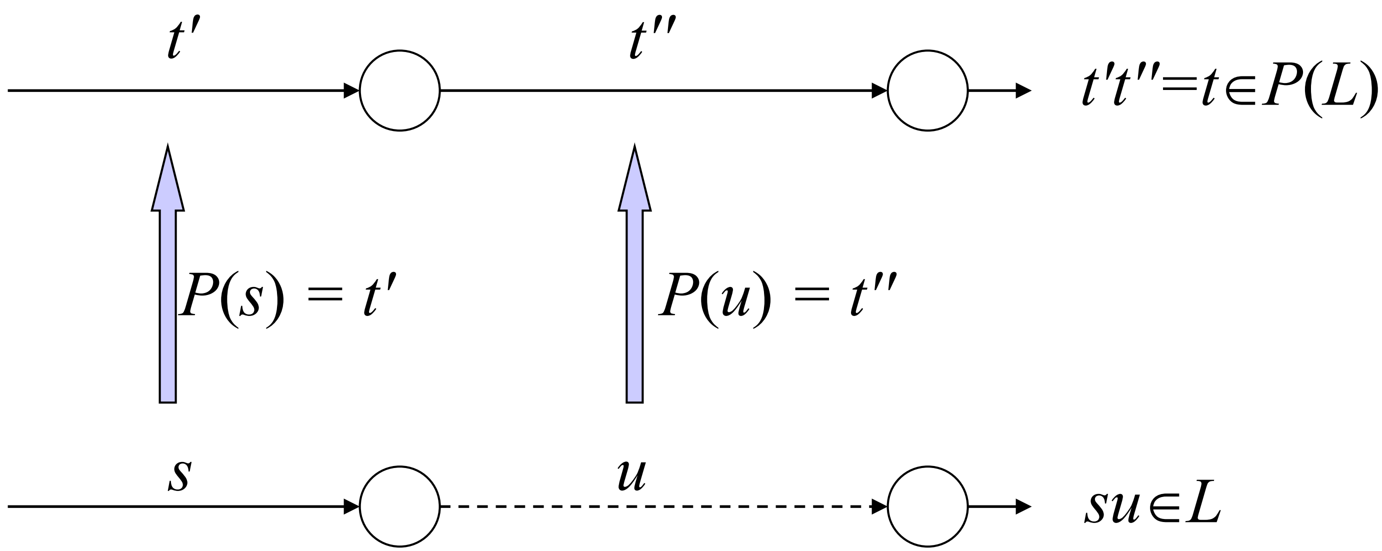


$S_A \wedge S_B \wedge C$ is a proper supervisor of $G_1 \times G_2 \times R_A \times R_B$

The Concept of L-observer

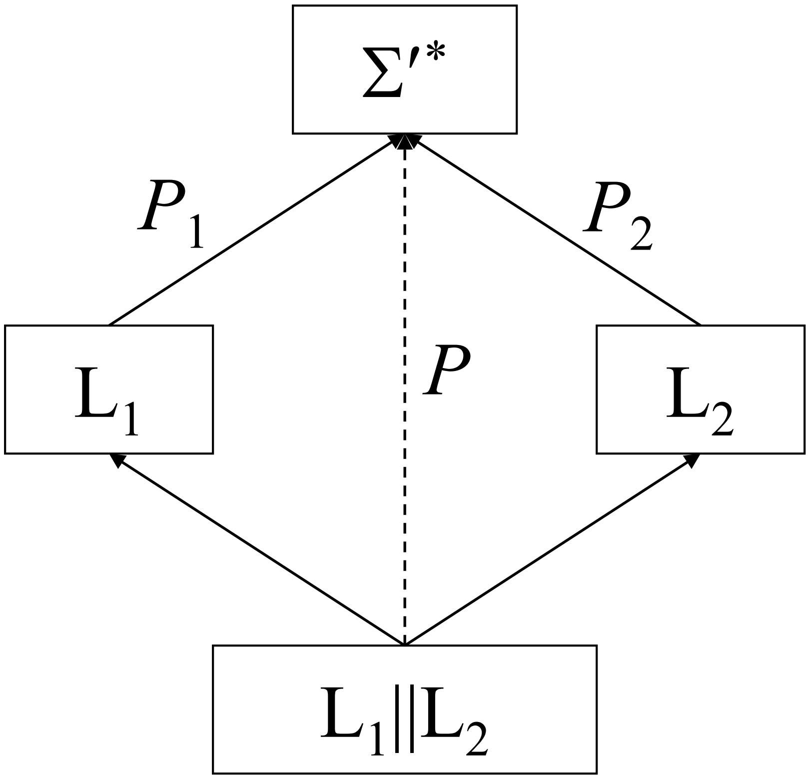
- Given $L \subseteq \Sigma^*$ and $\Sigma' \subseteq \Sigma$, let $P: \Sigma^* \rightarrow \Sigma'^*$ be the natural projection
- P is called an L-*observer* if

$$(\forall t \in P(L))(\forall s \in \overline{L})\ P(s) \leq t \Rightarrow (\exists u \in \Sigma^*)\ su \in L \wedge P(su)=t$$



The Main Property of L-observer (MPLO)

- $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma' \subseteq \Sigma_1 \cup \Sigma_2$
- If
  - $P_1 : \Sigma_1^* \rightarrow (\Sigma_1 \cap \Sigma')^*$ is L_1-observer
  - $P_2 : \Sigma_2^* \rightarrow (\Sigma_2 \cap \Sigma')^*$ is L_2-observer
- then
  - $P : (\Sigma_1 \cup \Sigma_2)^* \rightarrow \Sigma'^*$ is $L_1 \| L_2$-observer



Application of MPLO

- Given $L \subseteq \Sigma^*$ and $\Sigma' \subseteq \Sigma$, let $P:\Sigma^* \to \Sigma'^*$ be the L-observer.
- Let $\Sigma'' \subseteq \Sigma'$ and $L'' \subseteq \Sigma''^*$, then

$$P(L) \text{ and } L'' \text{ is nonconflicting} \Leftrightarrow L \text{ and } L'' \text{ is nonconflicting}$$

$$\overbrace{L \qquad\qquad L''}^{\text{nonconflicting}} \qquad \overline{L \,\|\, L''} = \overline{L} \,\|\, \overline{L''}$$

$$\downarrow P \qquad\qquad\qquad\qquad \Uparrow$$

$$\underbrace{P(L) \qquad\qquad L''}_{\text{nonconflicting}} \qquad \overline{P(L) \,\|\, L''} = \overline{P(L)} \,\|\, \overline{L''}$$

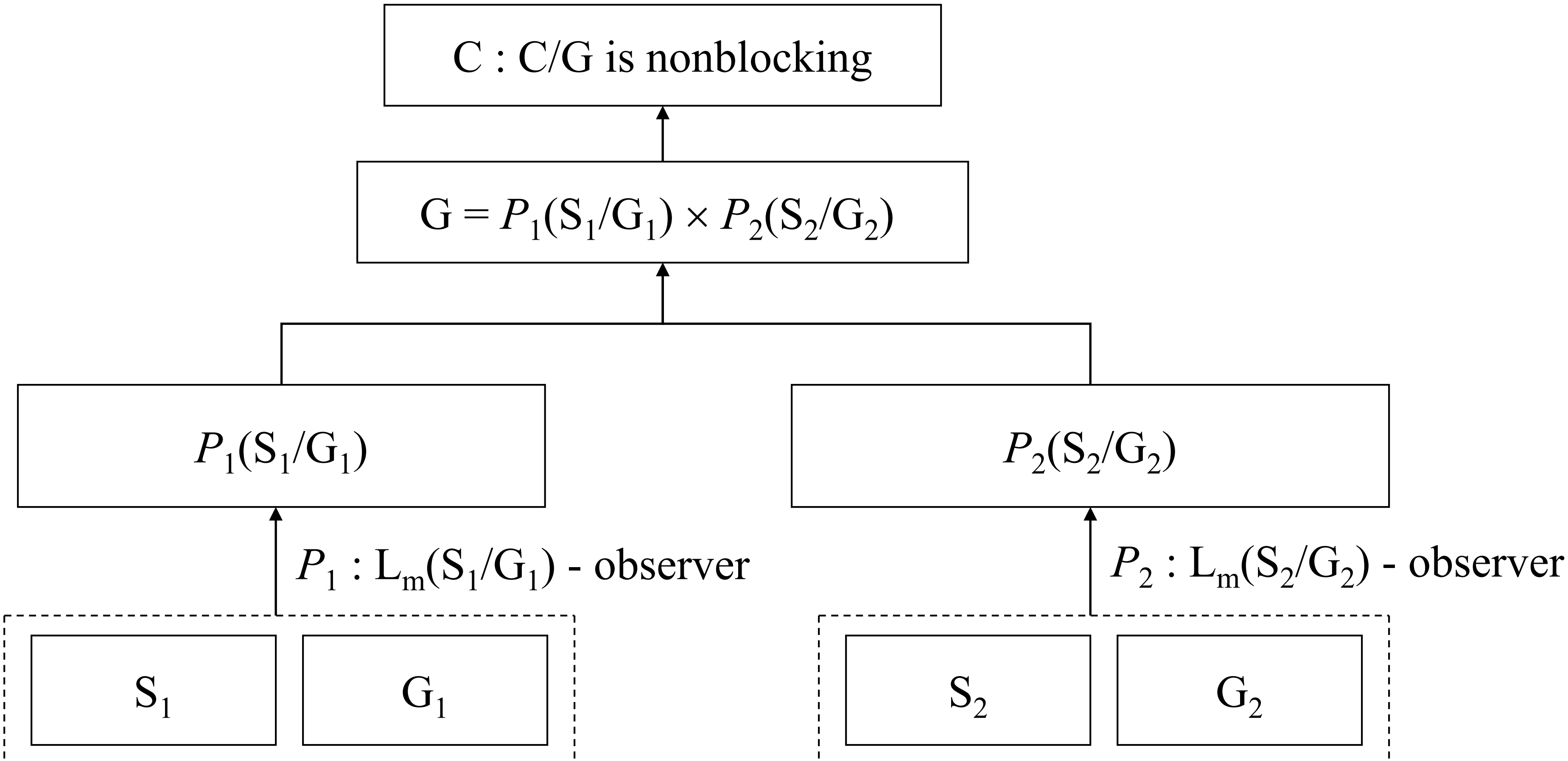
Coordinated Modular Supervisory Control

- Given Σ, let $\phi(\Sigma)$ denote the set of all FSAs over Σ.
- Given two alphabets Σ_1 and Σ_2, let $G_1 \in \phi(\Sigma_1)$ and $G_2 \in \phi(\Sigma_2)$.
- Let S_i be a proper supervisor of G_i (i=1,2).
- Let $\Sigma' \subseteq \Sigma_1 \cup \Sigma_2$ with $\Sigma_1 \cap \Sigma_2 \subseteq \Sigma'$.
- Suppose $P_i : \Sigma_i^* \to (\Sigma_i \cap \Sigma')^*$ be an $L_m(S_i/G_i)$-observer, where i=1,2.
- Let $P_1(S_1/G_1)$ denote an automaton, where
  - $L(P_1(S_1/G_1)) = P_1(L(S_1/G_1))$ and $L_m(P_1(S_1/G_1)) = P_1(L_m(S_1/G_1))$
- Let $G := P_1(S_1/G_1) \times P_2(S_2/G_2)$
- Compute a coordinator $C \in \phi(\Sigma')$ such that C/G is nonblocking

Theorem 4

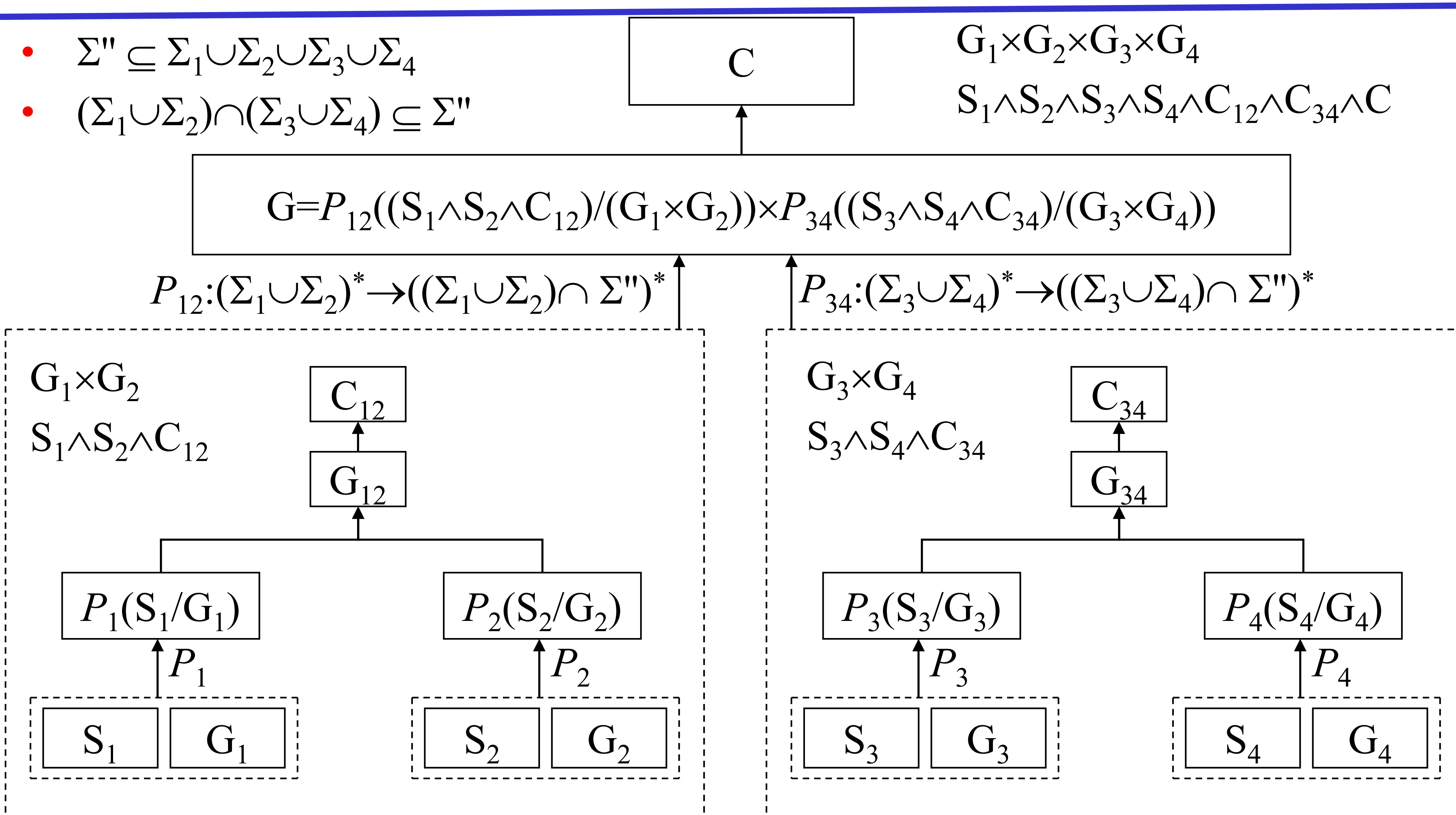
- Given the above setup, $S_1 \wedge S_2 \wedge C$ is a proper supervisor of $G_1 \times G_2$.

Illustration of Coordinator Synthesis



Multi-Level Coordinators

- $\Sigma'' \subseteq \Sigma_1 \cup \Sigma_2 \cup \Sigma_3 \cup \Sigma_4$
- $(\Sigma_1 \cup \Sigma_2) \cap (\Sigma_3 \cup \Sigma_4) \subseteq \Sigma''$



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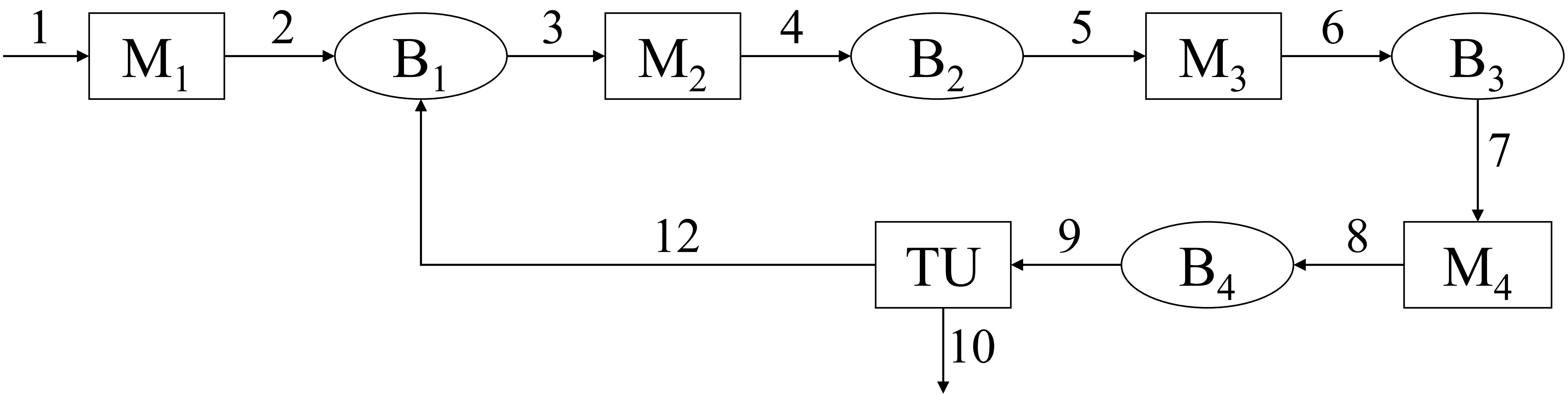
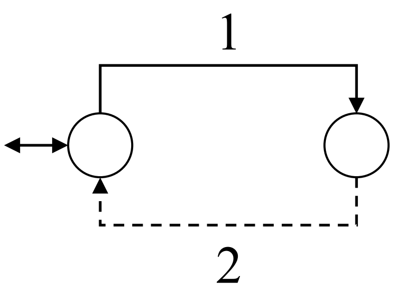
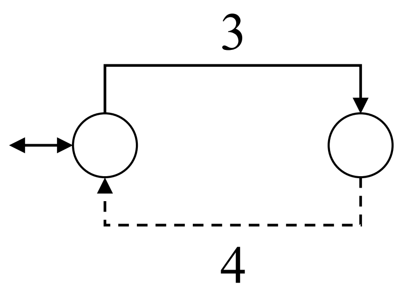
Simple Transfer Line (STL)



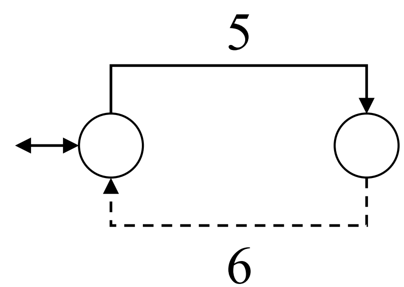
Component Models



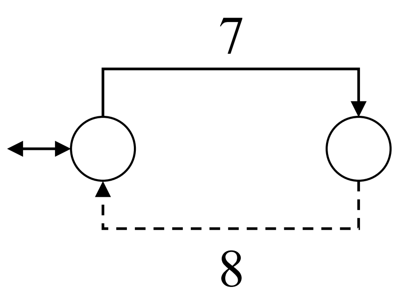
M_1



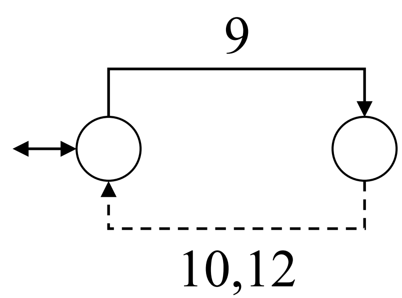
M_2



M_3



M_4



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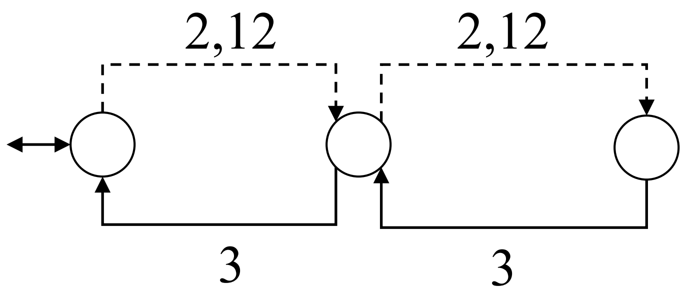
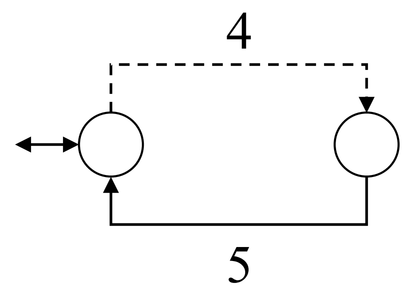
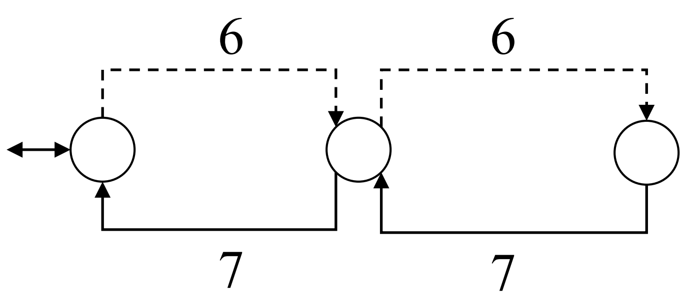
Buffer Specifications



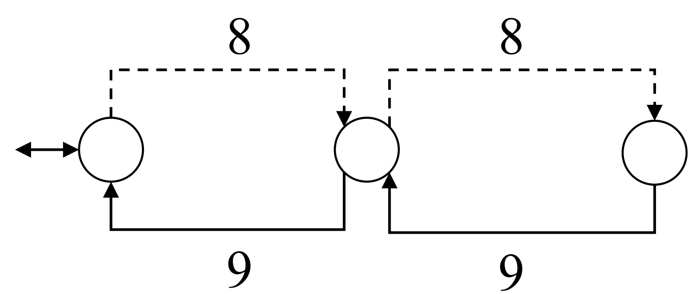
B_1



B_2

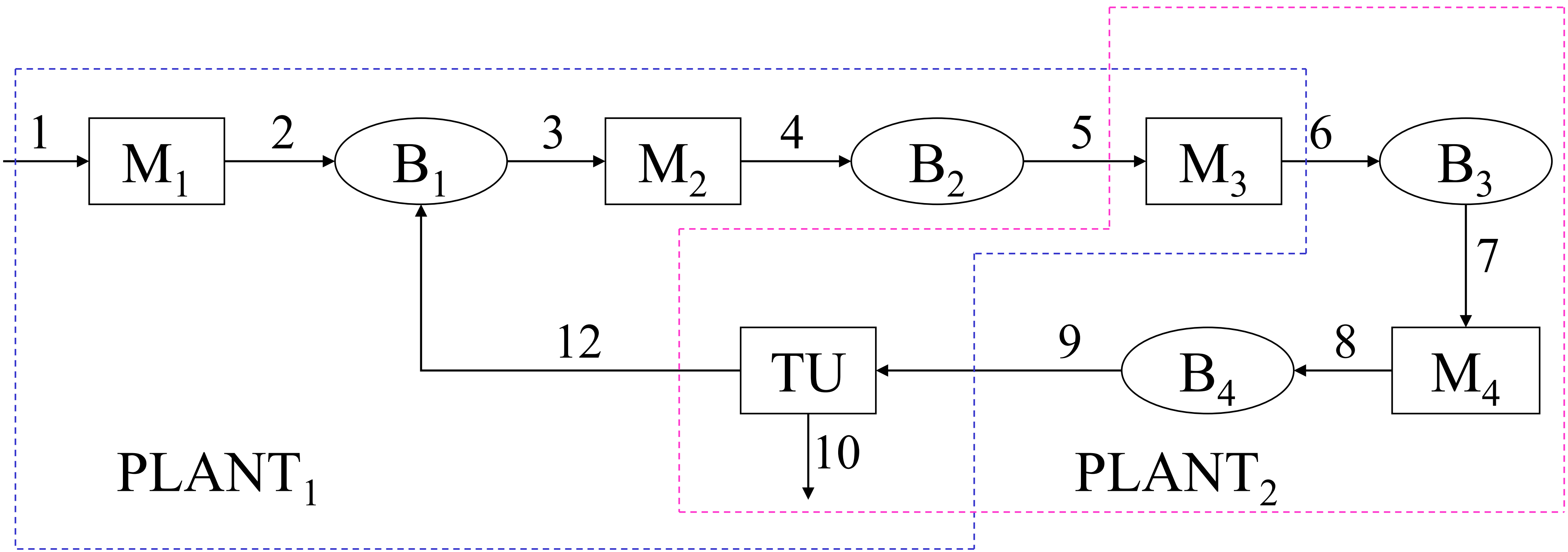


B_3



B_4

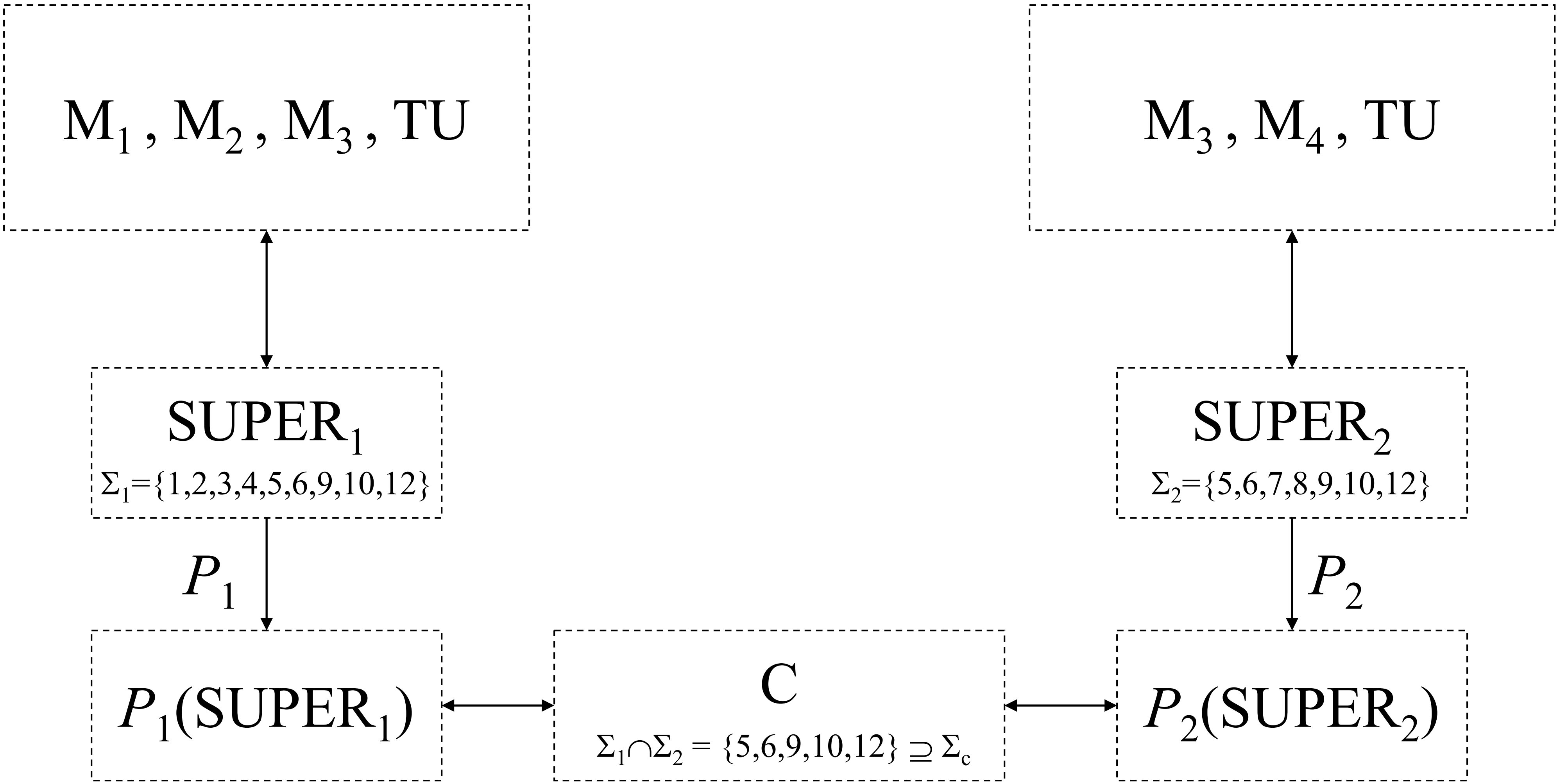
Partition of STL



Local Synthesis with TCT

- $\mathrm{PlANT}_1 = M_1 \times M_2 \times M_3 \times TU$ (use Sync) (16 , 72)
- $\mathrm{PlANT}_2 = M_3 \times M_4 \times TU$ (8 , 28)
- $\mathrm{SPEC}_1 = \mathrm{Selfloop}(\mathrm{Sync}(B_1,B_2),\{1,6,9,10\})$ (4 , 42)
- $\mathrm{SPEC}_2 = \mathrm{Selfloop}(\mathrm{Sync}(B_3,B_4),\{5,10,12\})$ (9 , 51)
- $\mathrm{SUPER}_1 = \mathrm{Supcon}(\mathrm{PLANT}_1,\mathrm{SPEC}_1)$ (48 , 146)
- $\mathrm{SUPER}_2 = \mathrm{Supcon}(\mathrm{PLANT}_2,\mathrm{SPEC}_2)$ (50 , 137)
- SUPER_1 and SUPER_2 are conflicting

Create an Coordinator



Coordinator Synthesis

- Preparation
  - Set the coordinator's alphabet as $\Sigma_c = \{1,5,6,9,10,12\}$
  - We can check that both P_1 and P_2 are observers.
- Create local abstractions
  - $PPLANT_1$=Project($SUPER_1$,{1,5,6,9,10,12}) (14 , 40)
  - $PPLANT_2$=Project($SUPER_2$,{5,6,9,10,12}) (18 , 41)
- Create a specification SPEC, recognizing Σ_c^*.
- Synthesis
  - PPLANT=Sync($PPLANT_1$,$PPLANT_2$) (63 , 168)
  - C = Supcon(PPLANT,SPEC) (59 , 158)

Verification

- C, SUPER_1 and SUPER_2 are nonconflicting

Monolithic Supervisor Synthesis

- PLANT = Sync($PLANT_1$,$PLANT_2$) (32 , 176)
- SPEC = Selfloop(Sync(Sync(Sync(B_1,B_2),B_3),B_4), {1,10}) (54 , 414)
- SUPER = Supcon(PLANT,SPEC) (568 , 1927)

Isomorphic(Sync(C,Sync($SUPER_1$,$SUPER_2$)),SUPER)=true

Comparison

- Monolithic Approach
  - Plant : (32 , 176)
  - Supervisor : (568 , 1927)
  - The largest intermediate computational result : (568 , 1927)

- Coordinated Modular Approach
  - Local Plants : (16 , 72) , (8 , 28)
  - Local Supervisors : (48 , 146) , (50 , 137)
  - Coordinator : (59 , 158)
  - The largest intermediate computational result : (63 , 168)

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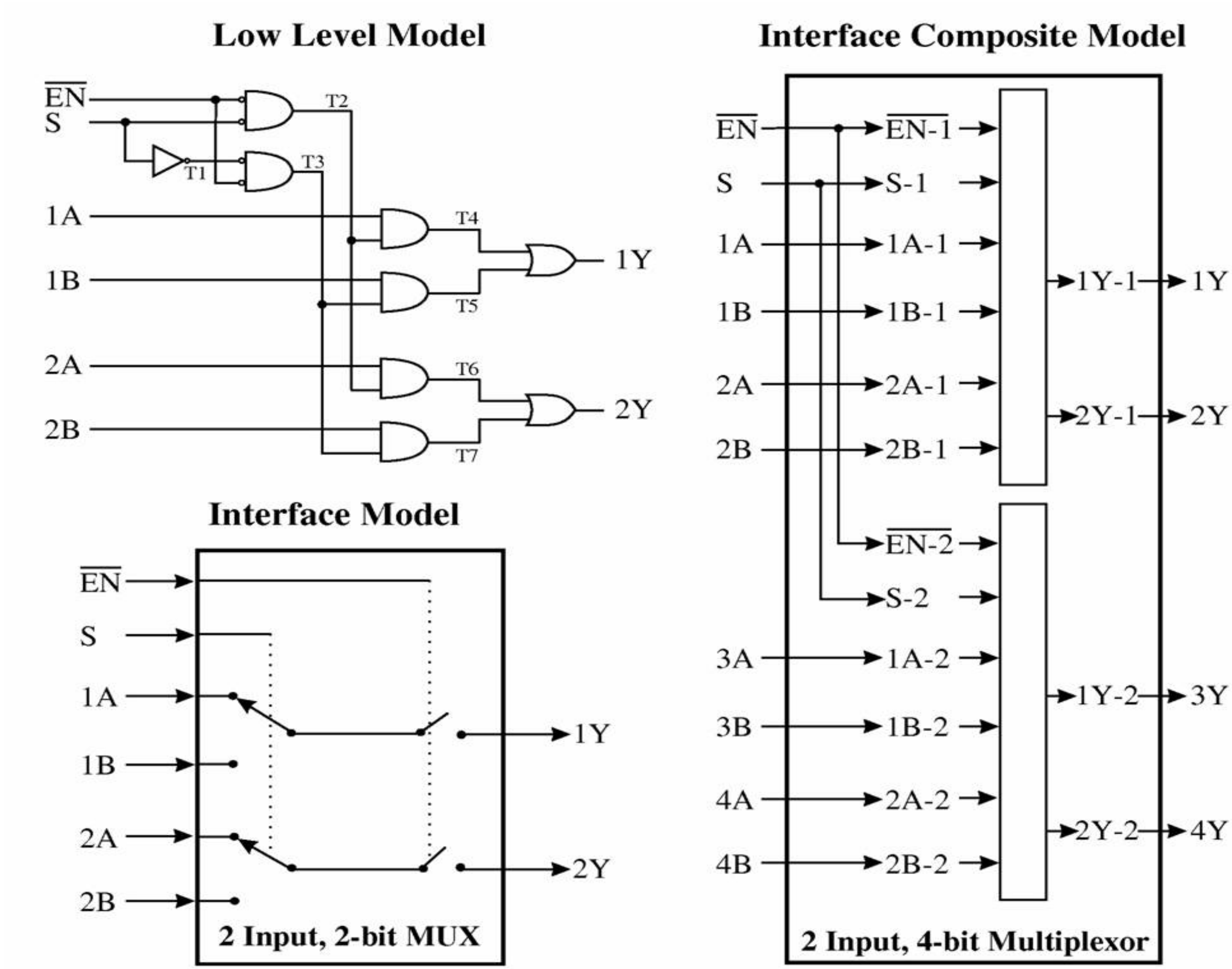
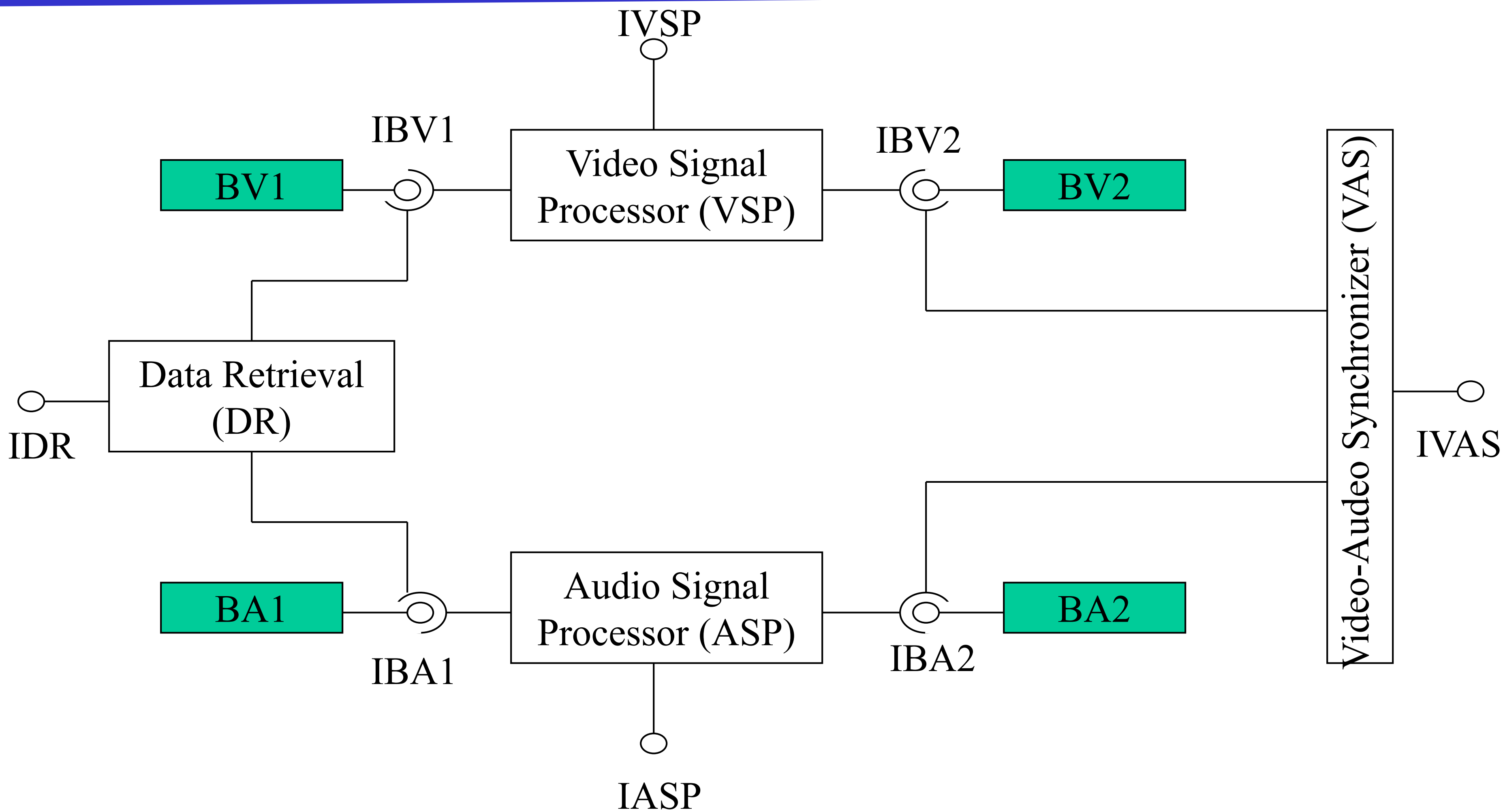
Motivations

- Each component has a fixed interface
- Each component’s internal behaviour is unseen to outsiders
- Components communicate with each other through interfaces

Example 1 – Digital Circuit



Example 2 – Component-Based Software



Our Goal

- Use interfaces to separate components, allowing local synthesis

The System Architecture

High-Level Component $G_H \in \phi(\Sigma_H)$
(where $\phi(\Sigma_H)$ contains all FSAs over Σ_H)

Interfaces

$G_{I1} \in \phi(\Sigma_{I1})$ … $G_{I1} \in \phi(\Sigma_{I1})$

$G_{L1} \in \phi(\Sigma_{L1})$ … $G_{Ln} \in \phi(\Sigma_{Ln})$

Low-Level Components

- For any $i,j \in \{H, L1, \ldots, Ln\}$
  - $\Sigma_i = \Sigma_{i,c} \cup \Sigma_{i,uc}$
  - $i \neq j \Rightarrow \Sigma_{i,c} \cap \Sigma_{j,uc} = \varnothing$
- For any $i,j \in \{L1, \ldots, Ln\}$
  - $\Sigma_H \cap \Sigma_{Li} = \Sigma_{Ii}$
  - $i \neq j \Rightarrow \Sigma_{Li} \cap \Sigma_{Lj} = \Sigma_{Ii} \cap \Sigma_{Ij}$

Separable Requirements

- At the high level: $E_H \in \phi(\Sigma_H)$
- At the low level: $\{E_{Li} \in \phi(\Sigma_{Li}) \mid i=1,\ldots,n\}$

A requirement can "touch" different components via interface events!

A Supervisor Synthesis Problem

- Compute an interface-based modular (IBM) supervisor $\{S_H, S_{L1}, \ldots, S_{Ln}\}$,
  - Requirements: $L_m(S_H/G_H) \subseteq L_m(E_H) \wedge (\forall i \in \{1,\ldots,n\})\ L_m(S_{Li}/G_{Li}) \subseteq L_m(E_{li})$
  - Nonblockingness : $\overline{L_m(S/G)} = L(S/G)$ where
    - $L_m(G) = L_m(G_H)||L_m(G_{L1})||\ldots||L_m(G_{Ln})$ and $L(G) = L(G_H)||L(G_{L1})||\ldots||L(G_{Ln})$
    - $L_m(S) = L_m(S_H)||L_m(S_{L1})||\ldots||L_m(S_{Ln})$ and $L(S) = L(S_H)||L(S_{L1})||\ldots||L(S_{Ln})$
  - Controllability: $L(S/G)\Sigma_{uc} \cap L(G) \subseteq L(S/G)$
  - Interface Invariance:

$$(\forall i \in \{1,\ldots,n\})\ P_i(L_m(S_{Li}/G_{Li})) = L_m(G_{Ii})$$

where $P_i : \Sigma_{Li}^* \rightarrow \Sigma_{Ii}^*$ is an $L_m(S_{Li}/G_{Li})$-observer

Theorem 1 :

Given $\mathcal{G} = \{G_H, G_{Li}, G_{Ii} \mid i=1,\ldots,n\}$ and $\mathcal{E} = \{E_H, E_{Li} \mid i=1,\ldots,n\}$, the largest IBM supervisor, denoted as the supremal IBM supervisor, in terms of component-wise set inclusion exists.

Local Supervisor Synthesis (1)

- At the high level
  - Plant : $G = G_H \times G_{I1} \times \ldots \times G_{In}$
  - Requirement : E_H
  - Synthesize $S_H \in \phi(\Sigma_H)$, where
    - $L_m(S_H/G) \subseteq L_m(E_H)$
    - $\overline{L_m(S_H/G)} = L(S_H/G)$
    - $L(S_H/G)\Sigma_{H,uc} \cap L(G) \subseteq L(S_H/G)$

Local Supervisor Synthesis (2)

- At the low level, for each local component G_{Li} ($i \in \{1,\ldots,n\}$)
  - Plant : G_{Li}
  - Requirement : E_{Li}
  - Synthesize $S_{Li} \in \phi(\Sigma_{Li})$, where
    1. $L_m(S_{Li}/G_{Li}) \subseteq L_m(E_{Li})$
    2. $\overline{L_m(S_{Li}/G_{Li})} = L(S_{Li}/G_{Li})$
    3. $L(S_{Li}/G_{Li})\Sigma_{Li,uc} \cap L(G_{Li}) \subseteq L(S_{Li}/G_{Li})$
    4. $P_i(L_m(S_{Li}/G_{Li})) = L_m(G_{Ii})$, where $P_i : \Sigma_{Li}^* \rightarrow \Sigma_{Ii}^*$ is an $L_m(S_{Li}/G_{Li})$-observer

Theorem 2:

The largest language $L_m(S_{Li}/G_{Li})$ satisfying conditions 1-4 exists.

(Why ?)

- The largest language $L_m(S_{Li}/G_{Li})$ in Theorem 2 is computable.

Theorem 3:

$\{S_H, S_{L1}, \dots , S_{Ln}\}$ is the supremal IBM supervisor w.r.t. $\mathcal{G}$ and $\mathcal{E}$.

Conclusions

- Advantages of Modular Supervisory Control
  - It is easy to present a system in a modular way
  - It is computationally tractable compared to the monolithic approach
  - It possesses a certain level of implementation flexibility

- Disadvantage of Modular Supervisory Control
  - Modular control is more conservative than centralized control (why?)
  - The observer property is required during model abstraction