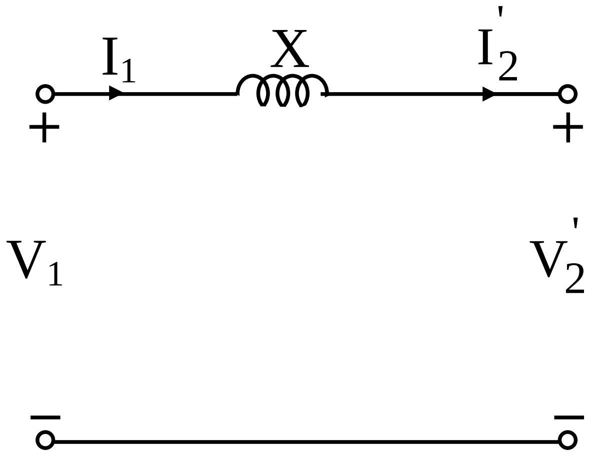

In other words, the transformer can be replaced by an equivalent impedance, or just an equivalent reactance.

3. <u>PER-UNIT (P.U.) SYSTEM</u>

- Power transmission lines are operated at very high voltage levels (kilovolts). Due to the large amount of power transmitted, megawatts & megavoltamps are commonly-used terms!

- It therefore would be more meaningful to scale down all physical values of Ω, A, kV, MVA, MW using scaling factors called <u>based values</u>. For example, if a base voltage of 100 kV is selected, then physical system voltages of 80 kV, 110 kV & 100 kV become $\frac{80}{100}$, $\frac{110}{100}$ & $\frac{100}{100}$ <u>per unit</u> respectively!

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- The per-unit value of any quantity is defined as the ratio of the actual quantity to an "arbitrarily" chosen value (base or reference) of the same dimensions.

$$\therefore \text{ Quantity in per unit} = \frac{\text{physical quanity}}{\text{base value}}$$

$$\text{\& Quantity in percent} = \frac{\text{physical quantity}}{\text{base value}} \times 100$$

- Percent system should be used with caution (due to mult. factor of 100)
- Per-unit system is preferred in power system calculations as it offers the following advantages :

Advantages of Per-unit System

- Analysis is greatly simplified, e.g. all impedances of a given equivalent CKT can be directly added without considering system voltages.
- Use of "$\sqrt{3}$" is eliminated! The base values account for these easily.
- Manufacturers of electrical equipment usually specify the impedance in per unit or percent of nameplate ratings.

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- Electrical machines & transformers have widely varying internal impedances with size & rating. However, it turns out that in the p.u. system, these impedances fall within a fairly narrow range. Hence, if the actual impedance of a machine is not known, its per unit value can be easily assigned!

- Circuit analyst is relieved of the worry of referring quantities to one side or other side of transformer, especially in large networks containing many transformers of different turns ratios.

$\Rightarrow$Eliminates the possible cause of making serious calculation mistakes!

- P.U. values are more convenient in simulating machine systems on digital computers.

Significant advantage : Per unit impedance of transformer is the same on both sides of the transformer!

Per Unit (p.u.) Quantities in 3-phase Power System

- There are 4 base values

  1. Power base S_b, usually in MVA.
  2. Impedance base Z_b, usually in OHMS.
  3. Current base I_b, usually in A.
  4. Voltage base V_b, usually in kV.

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- In 3-phase systems, usually
  - → S, P, Q are three-phase powers in MVA, MW & MVAr respectively
  - → Voltages considered are line-to-line values
  - → Currents considered are line values
  - → Impedances considered are phase values of equivalent star (Y) configuration (This means that all impedances in Ω must be converted to equivalent Y value in Ω, before converting to its p.u. value)

- The 4 base values (S_b, Z_b, I_b & V_b) are related as follows :

$$S_b = \sqrt{3}\frac{V_b I_b}{1000} \text{ MVA} \qquad (1)$$

where V_b is line-to-line base voltage in kV & I_b is the line current in AMPS.

$$\Rightarrow \quad I_b = \frac{S_b \times 1000}{\sqrt{3} \times V_b} \qquad (2)$$

$$\text{Next, } Z_b = \frac{(V_b / \sqrt{3})}{I_b} \times 1000 \quad \Omega \qquad (3)$$

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<u>Note</u> : $\dfrac{V_b}{\sqrt{3}}$ is the phase voltage of the equivalent Y system.

Substituting (2) in (3), we get :

$$Z_b = \frac{(V_b/\sqrt{3})}{\left(\dfrac{S_b \times 1000}{\sqrt{3} \times V_b}\right)} \times 1000$$

$$\Rightarrow \quad Z_b = \frac{{V_b}^2}{S_b}\ \Omega \qquad (4)$$

- From the above, it is clear that we <u>need to select only 2 base values</u>, instead of all 4. For example, if S_b & V_b are selected, then I_b & Z_b can be calculated using equations (2) & (4) respectively!
- V_b (and consequently Z_b & I_b too) changes from <u>transformer</u> primary to secondary as follows :

$$\frac{V_b\ \text{(primary)}}{V_b\ \text{(secondary)}} = \frac{\text{No. of turns (primary)}}{\text{No. of turns (secondary)}}$$

$$= \frac{N_1}{N_2}\text{(line} - \text{to} - \text{line turns ratio, a)}$$

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- Per unit values of Z, R, X & I are the same on either side of the transformer, but the actual values are not!

⇒ Per unit quantities can now be evaluated as :

$$Z_{p.u.} = \frac{Z(\Omega)}{Z_b(\Omega)}; \; I_{p.u.} = \frac{I(AMPS)}{I_b(AMPS)};$$

$$V_{p.u.} = \frac{V(kV)}{V_b(kV)}; \; P_{p.u.} = \frac{P(MW)}{S_b(MVA)}$$

⇒ If necessary, actual voltages/currents/ohms etc. can also be obtained as :

Actual value = Base value × p.u. value

General Guidelines for Obtaining P.U. Values

Objective : To reduce the number of computations by selecting suitable values for V_b & S_b.

- Base MVA (S_b) is the same for all parts of the system. Normally S_b in MVA is used.
- Base kV (V_b) is selected in one part of the system; for other parts base kV is obtained according to the line-to-line voltage ratios of transformers.

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- Base impedances will be different in different parts of the system.
- In general, the p.u. (or %) impedances of electrical equipment are specified in terms of their own MVA & kV ratings. These values need to be converted to the <u>system bases</u> selected in (1) & (2) above. This conversion is done using the following formula :

$$Z_{\text{p.u. NEW}} = \frac{Z_{\text{ACTUAL}}}{Z_{\text{b NEW}}}$$

$$Z_{\text{p.u. OLD}} = \frac{Z_{\text{ACTUAL}}}{\underset{\displaystyle\uparrow}{Z_{\text{b OLD}}}}$$

Manufacturer's base (Equipment base)

where : $Z_{\text{b NEW}}$ = Base impedance selected in that part of the system where this component is placed

$$= \frac{(V_{\text{b NEW}})^2}{S_{\text{b NEW}}} \Leftarrow \text{these are base values in that section}$$

$$\& \ Z_{\text{b OLD}} = \frac{(V_{\text{b OLD}})^2}{S_{\text{b OLD}}} \Leftarrow \text{these are component bases/ratings}$$

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$$\therefore \frac{Z_{\text{p.u. NEW}}}{Z_{\text{p.u. OLD}}} = \frac{Z_{\text{b OLD}}}{Z_{\text{b NEW}}} = \frac{(V_{\text{b OLD}})^2 / S_{\text{b OLD}}}{(V_{\text{b NEW}})^2 / S_{\text{b NEW}}}$$

$$\Rightarrow Z_{\text{p.u. NEW}} = Z_{\text{p.u. OLD}} \times \left[\frac{V_{\text{b OLD}}}{V_{\text{b NEW}}}\right]^2 \times \left[\frac{S_{\text{b NEW}}}{S_{\text{b OLD}}}\right] \qquad (*)$$

Example 1 : A component rated for 13.2 kV, 30 MVA & with Z = 0.2 p.u. (on its own ratings) is placed in a power system portion where V_b = 13.8 kV & S_b = 50 MVA. What is the new p.u. Z of the component?

Here,

$S_{\text{b NEW}} = 50$	$S_{\text{b OLD}} = 30$
$V_{\text{b NEW}} = 13.8$	$V_{\text{b OLD}} = 13.2$
$Z_{\text{p.u. OLD}} = 0.2$	$Z_{\text{p.u. NEW}} = ?$

$$\therefore Z_{\text{p.u. NEW}} = 0.2 \times \left[\frac{13.2}{13.8}\right]^2 \times \left[\frac{50}{30}\right] = 0.306 \text{ p.u.}$$

- If a transformer impedance is given in p.u., then this value is based on the MVA rating of the transformer.

  $\Rightarrow$ Use equation in (*) above to convert this given p.u. to the new p.u. on the system base kVA, Sb.

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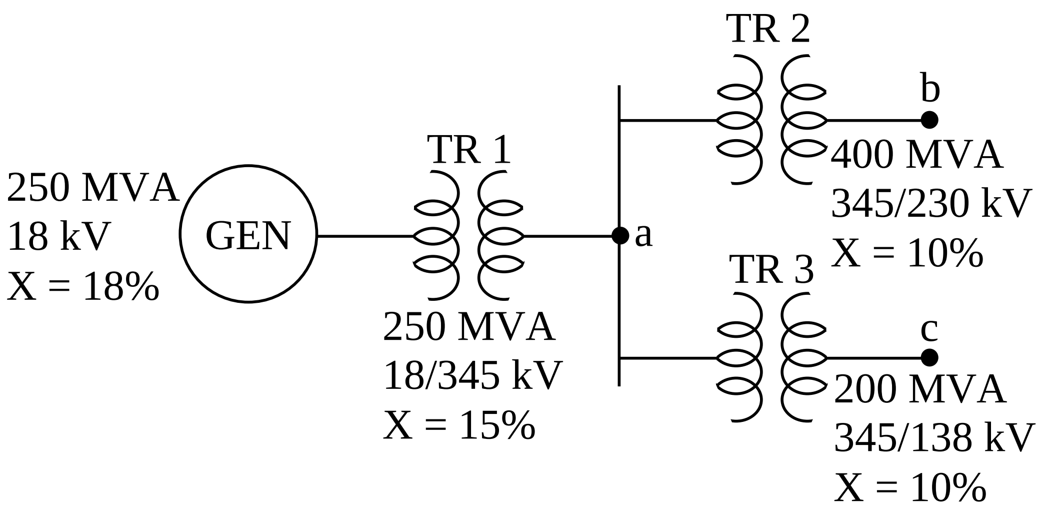
- If the Sb (base MVA) is not specified (and that is usually the case), the system component that has the largest MVA rating is chosen to give us the base MVA. Occasionally, a nice round number such as 100 MVA is selected as the base MVA!

Example 2 : For the power system shown below,

(i) Find appropriate voltage bases by selecting V_b = 18 kV at the generator terminals.

(ii) Find all impedances in p.u. Use S_b = 100 MVA.



Solution : Given $V_{b,\,GEN}$ = 18 kV (generator circuit)

Base voltage at point a = $V_{b,\,a}$

$$V_{b,\,GEN} \times \frac{345}{18} = 345 \text{ kV}$$

Base voltage at point b = $V_{b,\,b} = V_{b,\,a} \times \frac{230}{345} = 230\text{ kV}$

Finally, base voltage at point c = $V_{b,\,c} = V_{b,\,a} \times \frac{138}{345} = 138\text{ kV}$

Next, assuming a power base of S_b = 100 MVA, let us find all the impedances in p.u.

$$Z_{\text{GEN,NEW p.u.}} = Z_{\text{GEN,OLD p.u.}} \times \left[\frac{V_{\text{b OLD,GEN}}}{V_{\text{b,GEN}}}\right]^2 \times \frac{S_{\text{b NEW}}}{S_{\text{b OLD,GI}}}$$

$$= 0.18 \times \left(\frac{18}{18}\right)^2 \times \left(\frac{100}{250}\right) = 0.072\text{ p.u.}$$

$$Z_{\text{TR1,NEW p.u.}} = Z_{\text{TR1,OLD p.u.}} \times \left[\frac{V_{\text{b OLD,TR1}}}{V_{\text{b,GEN}}}\right]^2 \times \left[\frac{S_{\text{b NEW}}}{S_{\text{b OLD,TR1}}}\right]$$

↑

Calculated on the primary side

$$= 0.15 \times \left[\frac{18}{18}\right]^2 \times \left[\frac{100}{250}\right]$$

$$= 0.06\text{ p.u.}$$

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$$Z_{TR\,2,NEW\ p.u.} = Z_{TR2,OLD\ p.u.} \times \left[\frac{V_{b\ OLD,TR2}}{V_{b,a}}\right]^2 \times \left[\frac{S_{b\ NEW}}{S_{b\ OLD,TR2}}\right]$$

$\uparrow$

Calculated on the primary side

$$= 0.10 \times \left[\frac{345}{345}\right]^2 \times \left[\frac{100}{400}\right]$$

$$= 0.025 \text{ p.u.}$$

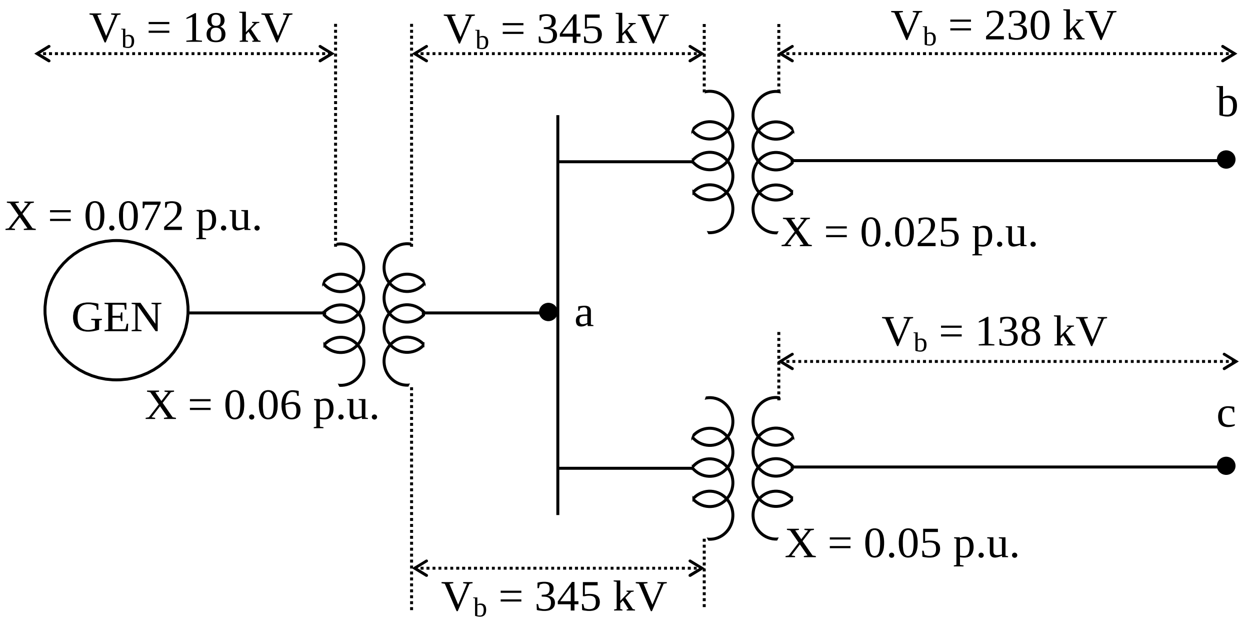
$$Z_{TR\,3,NEW\ p.u.} = Z_{TR3,OLD\ p.u.} \times \left[\frac{V_{b\ OLD,TR3}}{V_{b,a}}\right]^2 \times \left[\frac{100}{200}\right]$$

$\uparrow$

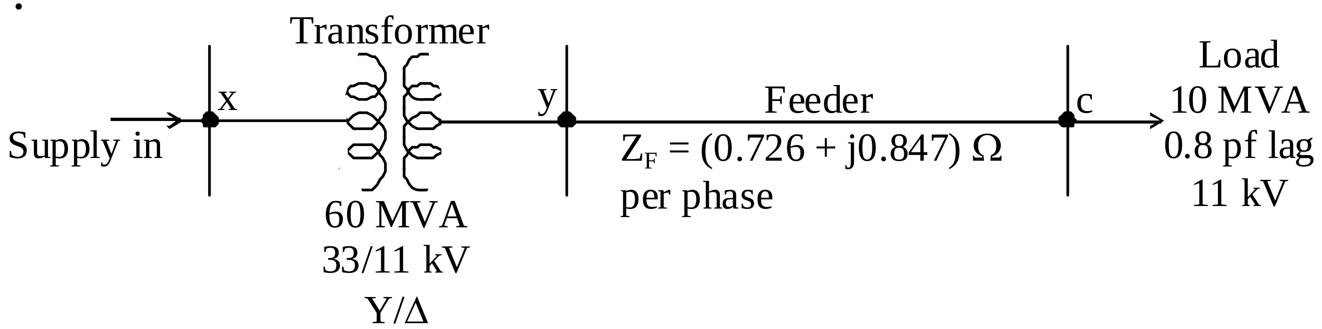
Calculated on the primary side

$$= 0.10 \times \left[\frac{345}{345}\right]^2 \times \left[\frac{1}{2}\right]$$

$$= 0.05 \text{ p.u.}$$



Example 3 :



Z_T = (0.2723 + j0.726) Ω per phase (Referred to H.V. side)

Calculate the input & output line voltages of the 3-ϕ transformer i.e. voltages at x and y.

Solution : Let base MVA = S_b = 60 MVA (constant throughout the system)`

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Transformer H.V. side

Base kV, $V_{b\,x}$ = 33 kV (assumed)

$$Z_{b\ \text{primary of trans.}} = Z_{b\,x} = \frac{V_{b\,x}^{\ 2}}{S_b} = \frac{33^2}{60} = 18.15\ \Omega = Z_{b\ \text{pri}}$$

$$\therefore Z_{T\ \text{p.u.}} = \frac{Z_{T\ \text{actual pri}}}{Z_{b\ \text{pri}}} = \frac{0.2723 + j0.726}{18.15} = 0.015 + j0.04 = 0.0427\angle 69.44°\ \text{p.u.}$$

Transformer L.V. side

$$\text{Base kV} = V_{b\,y} = V_{b\,x}\frac{11}{33} = 11\ \text{kV}$$

$$\therefore Z_{b\,y} = \frac{(V_{b\,y})^2}{S_b} = \frac{11^2}{60} = 2.017\ \Omega$$

$$\therefore Z_{F\ \text{p.u.}} = \frac{Z_F(\Omega)}{Z_{b\,y}} = \frac{0.726 + j0.847}{2.017} = 0.36 + j0.42 = 0.5532\ \angle 49.4°\ \text{p.u.}$$

$\therefore$ Total impedance

$Z_{TOT\ p.u.} = Z_{T\ p.u.} + Z_{F\ p.u.} = 0.5935\angle 50.81°$ p.u.

S_{Load} = 10 MVA, 0.8 pf lag = $10\angle 36.87^o$ MVA

$$\therefore S_{\text{Load p.u.}} = \frac{S_{\text{Load}}}{S_b} = \frac{10\angle 36.87^\circ}{60} = 0.1667\angle 36.87^\circ \text{ p.u.}$$

$$V_{\text{Load}} = 11 \text{ kV} \Rightarrow V_{\text{Load p.u.}} = \frac{V_{\text{Load}}}{V_{\text{b c}}} = \frac{11}{11} = 1.0\angle 0^\circ \text{ p.u.}$$

$\Rightarrow$ Load current

$$I^*_{\text{Load p.u.}} = \frac{S_{\text{Load p.u.}}}{V_{\text{Load p.u.}}} = \frac{0.1667\angle 36.87^\circ}{1\angle 0^\circ} = 0.1667\angle 36.87^\circ \text{ p.u.}$$

$$\Rightarrow I_{\text{Load p.u.}} = 0.1667 \angle\text{-}36.87^\circ \text{ p.u.}$$

$\therefore$ Voltage at the L.V. side of transformer

$$V_{\text{y p.u.}} = (I_{\text{Load p.u.}})(Z_{\text{F p.u.}}) + V_{\text{Load p.u.}}$$

$$= (0.1667\angle\text{-}36.87^\circ)(0.5532\angle 49.4^\circ) + 1\angle 0^\circ = 1.09\angle 1.05^\circ \text{ p.u.}$$

Actual voltage $|V_y| = V_{\text{y p.u.}}$ x Base voltage at y = 1.09 x 11 kV $\simeq$ $\underline{\underline{12 \text{ kV}}}$

Finally, voltage at H.V. side of transformer

$$V_{\text{x p.u.}} = (Z_{\text{T p.u.}})(I_{\text{Load p.u.}}) + V_{\text{y p.u.}}$$

$$= (0.0427\angle 69.44^\circ)(0.1667\angle\text{-}36.87^\circ) + 1.09\angle 1.05^\circ = 1.0963\angle 1.24^\circ \text{ p.u.}$$

Actual voltage, $|V_x| = V_{\text{x p.u.}}$ x Base voltage at x

= 1.0963 x 33 = $\underline{\underline{36.18 \text{ kV}}}$

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