Waveform symmetry

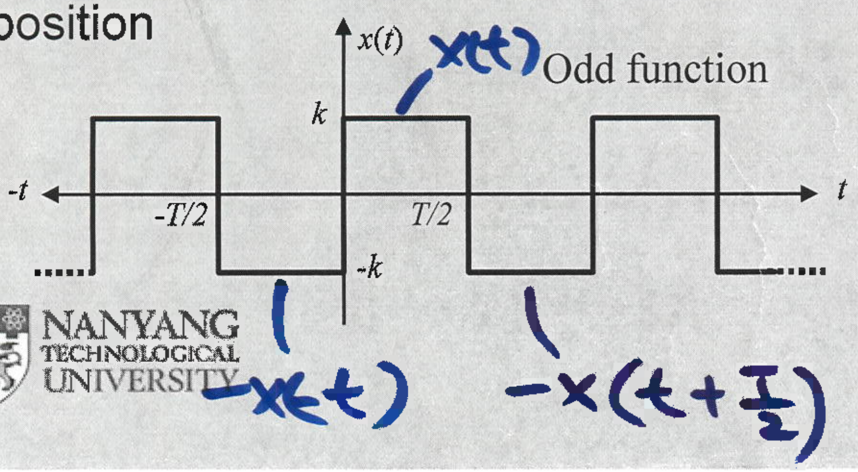
- Odd symmetry: $x(t) = -x(-t)$
  - a_n becomes zero for all n
  - The Fourier series contains only sine terms with amplitude b_n

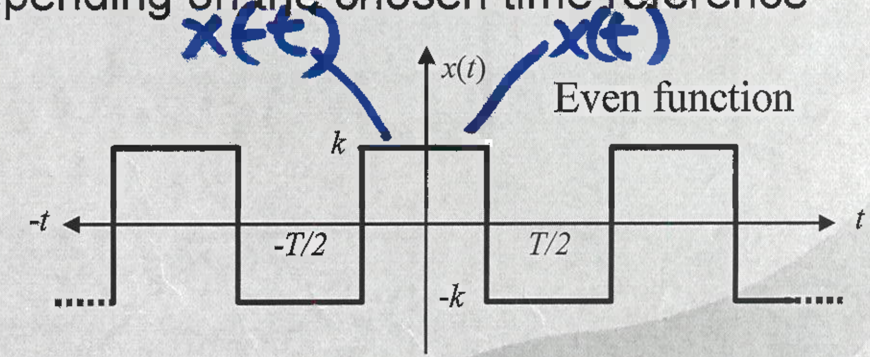
$$b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

- Even symmetry: $x(t) = x(-t)$
  - b_n becomes zero for all n
  - The Fourier series contains only cosine terms with amplitude a_n

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos\left(\frac{2\pi nt}{T}\right)dt$$

- Certain waveform can be odd or even depending on the chosen time reference position





$$\frac{2\pi n\left(t + T/2\right)}{T}$$

$$= \frac{2\pi n t}{T} + \frac{\cancel{2}\pi n}{\cancel{T}}\,\frac{\cancel{T}}{\cancel{2}}$$

$$= \underset{A}{\frac{2\pi n t}{T}} + \underset{B}{\pi n}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos\left(\frac{2\pi n t}{T} + n\pi\right)$$

$$= \cos\left(\frac{2\pi n t}{T}\right)\cos n\pi -$$

$$\sin\left(\frac{2\pi n t}{T}\right)\cancelto{0}{\sin(n\pi)}$$

Halfwave symmetry

- A function has halfwave symmetry if $x(t) = -x(t+T/2)$
  - Shape of the waveform over a period of $t+T/2$ to $t+T$ is the negative of the shape of the waveform over the period t to $t+T/2$
  - To have a common integration limits, replace (t) by $(t+T/2)$ in the interval $(-T/2,0)$

$$a_n = \frac{2}{T}\int_0^{T/2} x(t)\cos\left(\frac{2\pi nt}{T}\right)dt + \frac{2}{T}\int_{-T/2}^{0} x(t)\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$= \frac{2}{T}\int_0^{T/2} x(t)\cos\left(\frac{2\pi nt}{T}\right)dt + \frac{2}{T}\int_{-T/2+T/2}^{0+T/2} \overbrace{x(t+T/2)}^{-x(t)}\cos\left(\frac{2\pi n(t+T/2)}{T}\right)dt$$

$$= \frac{2}{T}\int_0^{T/2} x(t)\left[\cos\left(\frac{2\pi nt}{T}\right) - \cos\left(\frac{2\pi nt}{T} + n\pi\right)\right]dt$$

$$= \frac{2}{T}\int_0^{T/2} x(t)\left[\cos\left(\frac{2\pi nt}{T}\right) - \cos\left(\frac{2\pi nt}{T}\right)\underbrace{\cos(n\pi)}_{\text{1 when } n=2;\ -1 \text{ when } n=1}\right]dt$$

- a_n and b_n are zero when n is an even integer
- only when n is an odd integer, a_n and b_n are non-zero

$$a_n = \frac{4}{T}\int_0^{T/2} x(t)\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$b_n = \frac{4}{T}\int_0^{T/2} x(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

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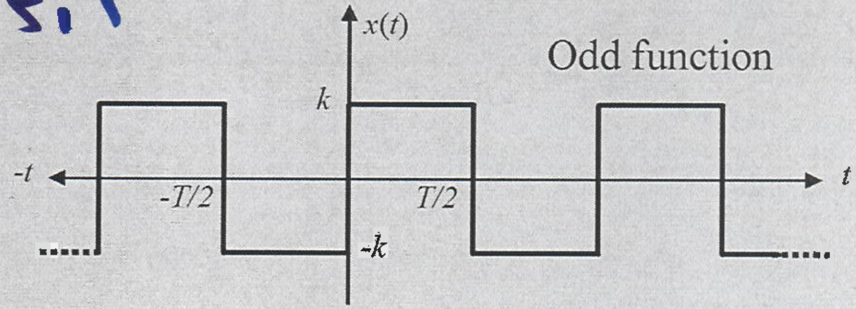
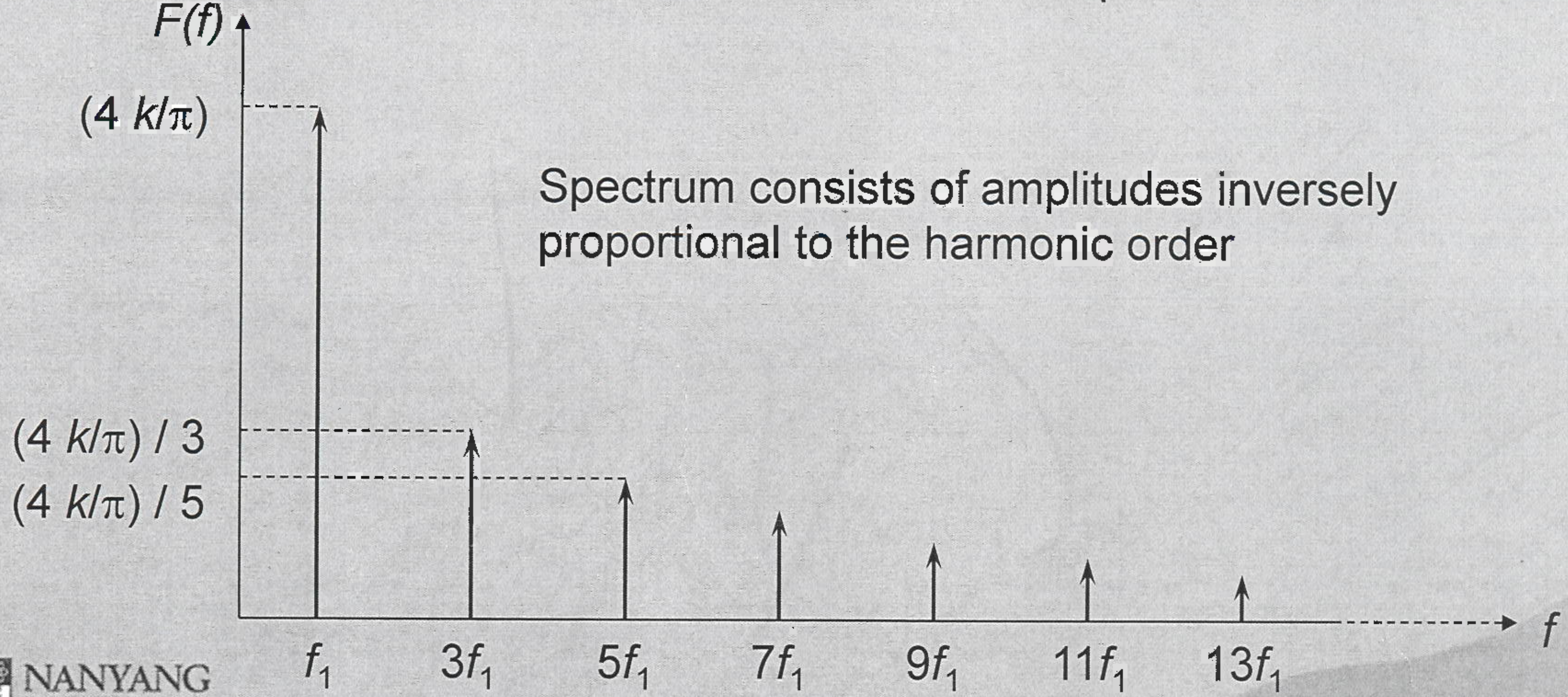
Halfwave symmetry

- The square wave is an odd function with halfwave symmetry
  - Only b_n coefficients and odd harmonics will exist

$n=1,3,5,7$

$$b_n = \frac{8}{T}\int_0^{T/4} x(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

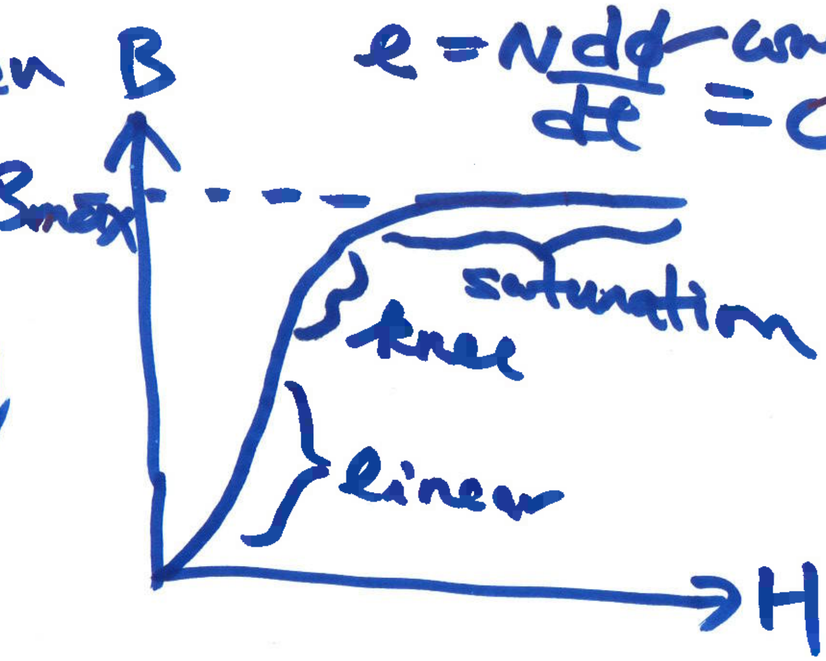




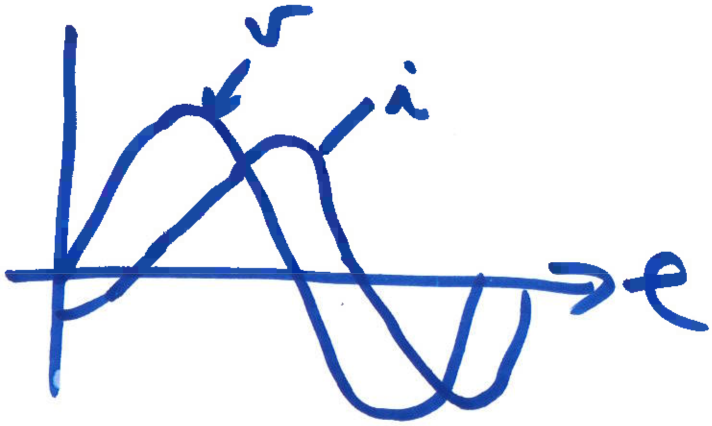
Spectrum consists of amplitudes inversely proportional to the harmonic order given

$$\uparrow B = \frac{\phi}{A \downarrow}$$

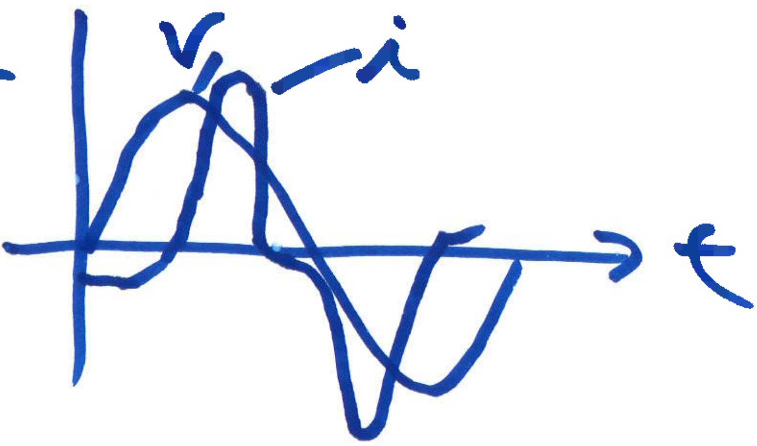
$$e = N\frac{d\phi}{dt} \quad \text{constant} \quad \frac{d\phi}{dt} = 0$$



Linear region

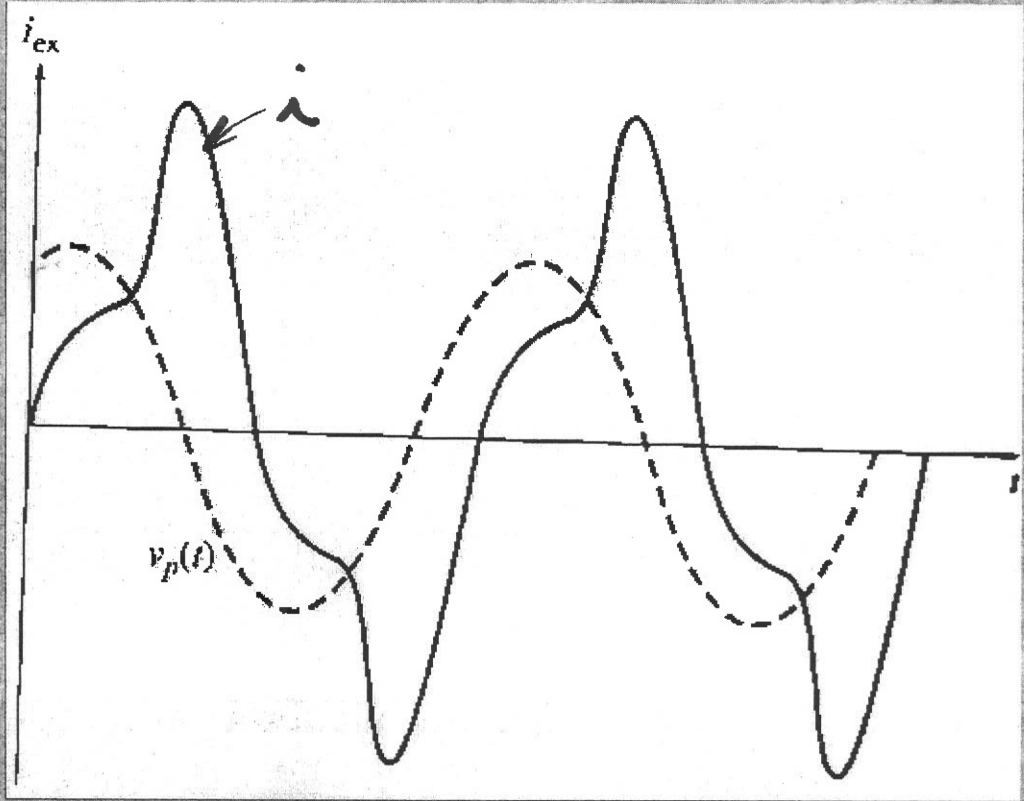


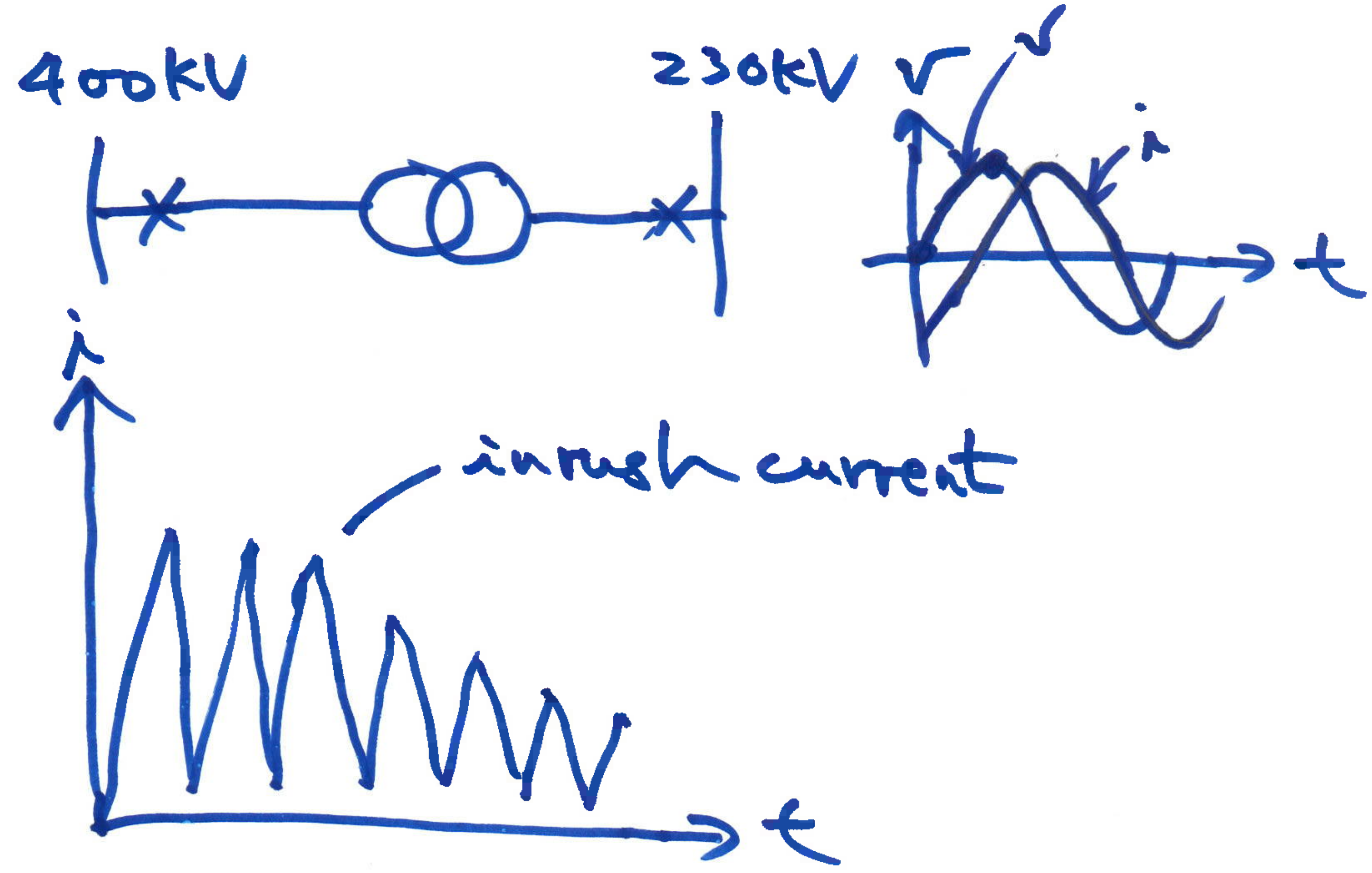
Non-linear region (Saturation)

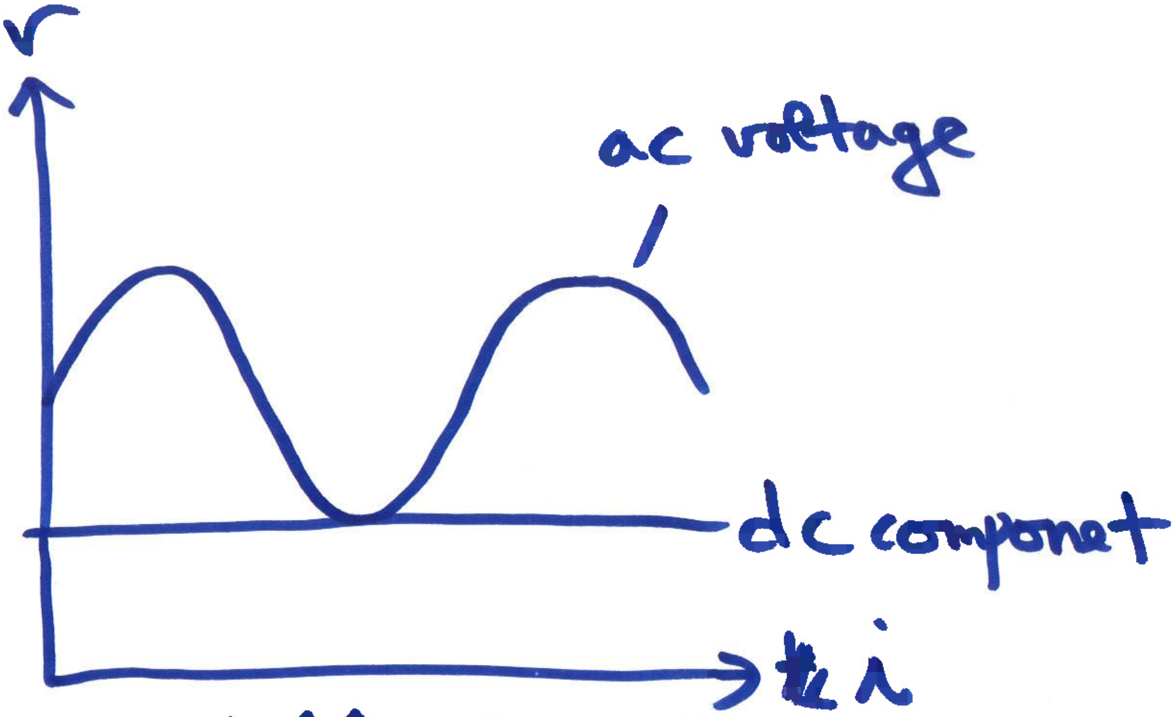


No load Current

Non linear nature of B-H curves make the no load current non-sinusoidal in nature.

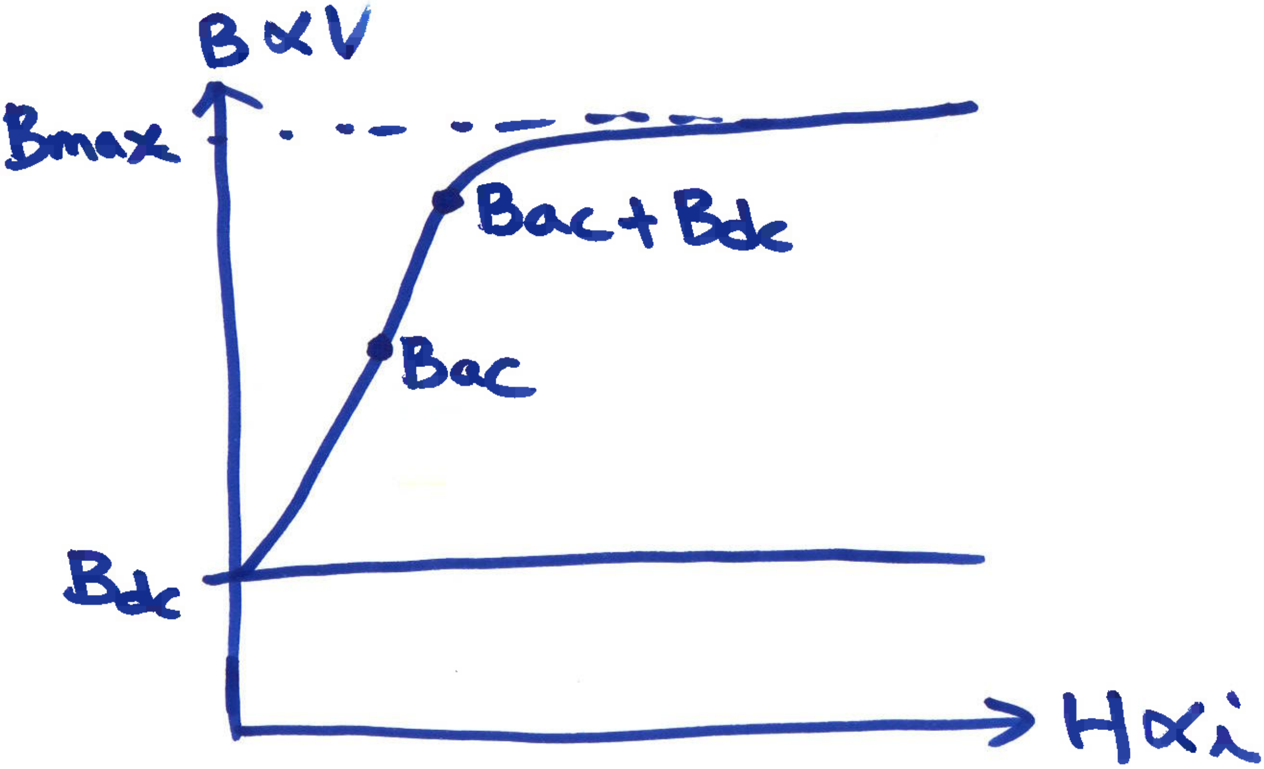



400kV
230kV
v
v
i
t
i
inrush current
t

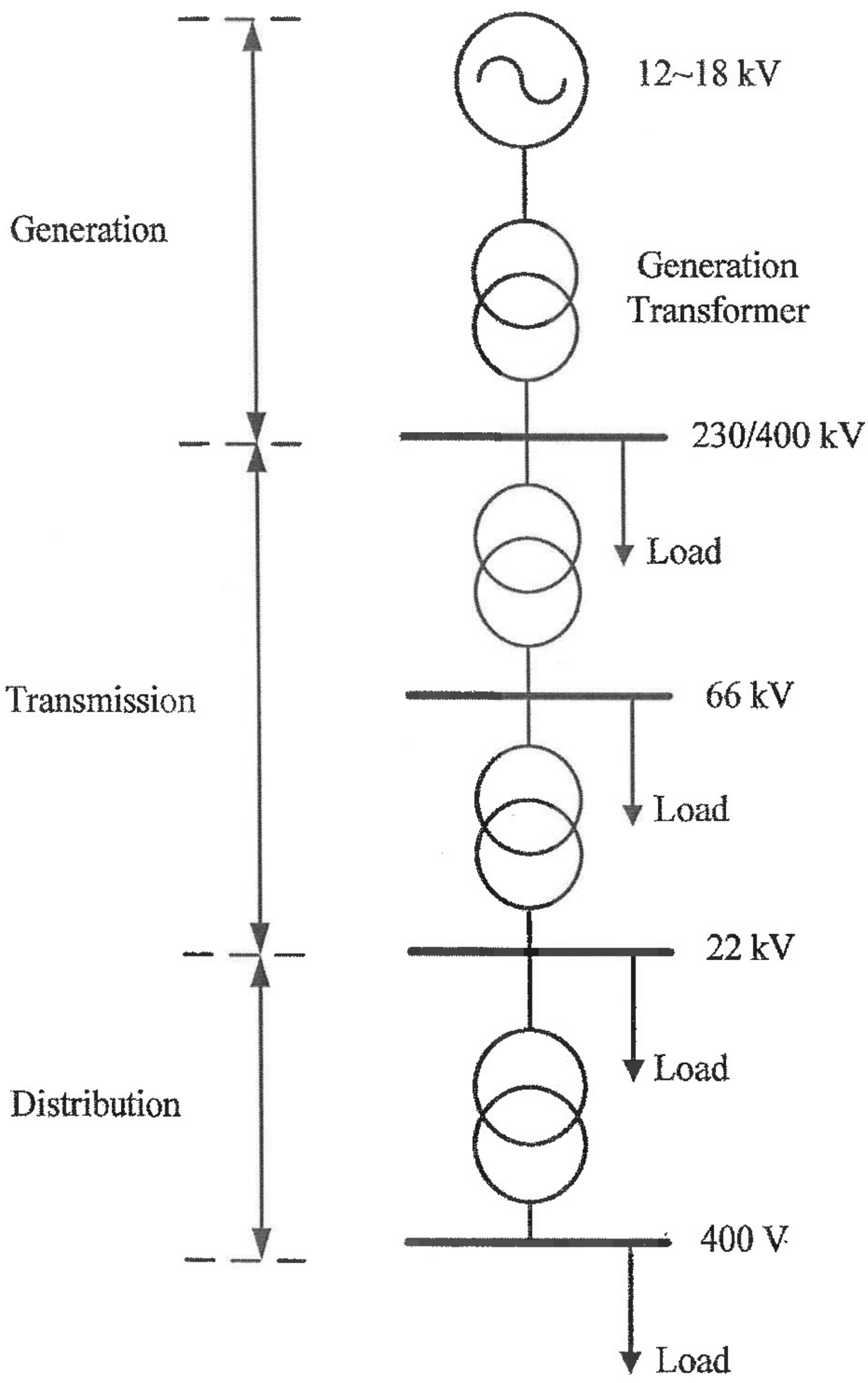


$$V = 4.44\,N B_m A f$$

$$\uparrow V \propto \uparrow B_m f \text{ — } 50\,Hz$$



Singapore Power System



Standard ratings of 22/0.4 kV power transformers: 1 MVA, 1.5 MVA, 2 MVA, 2.5 MVA and 3 MVA.

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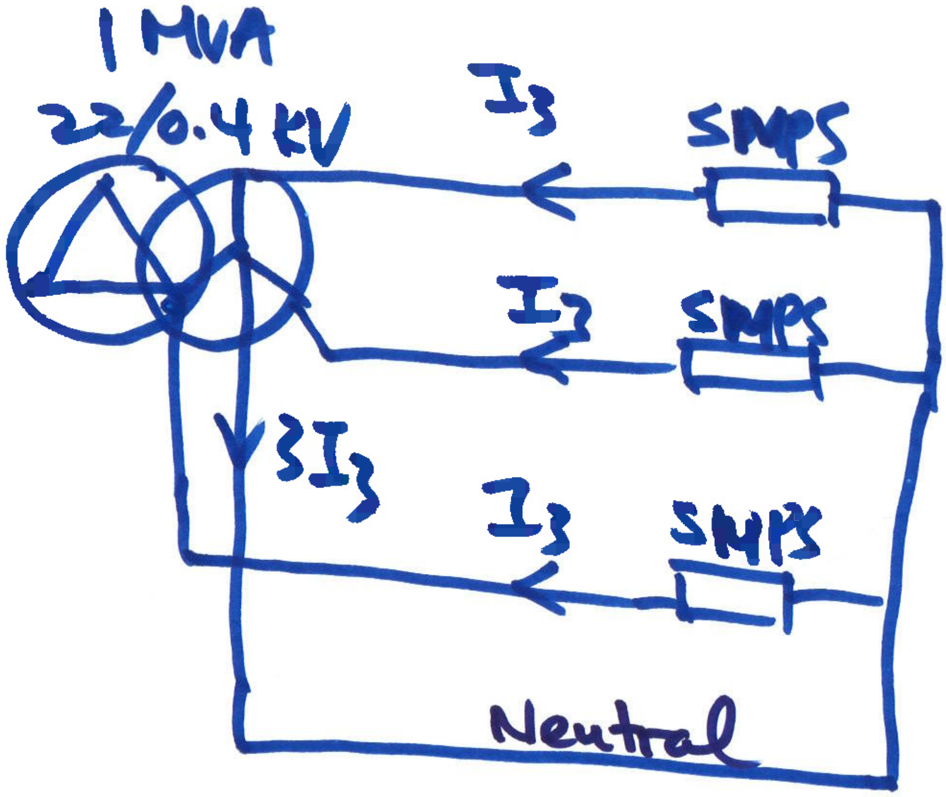
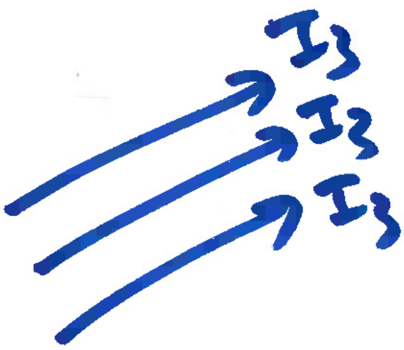

1 MVA
22/0.4 kV
I3
SMPS
I3
SMPS
3I3
I3
SMPS
Neutral


I3
I3
I3

$$A_0 = \frac{1}{2\pi}\int_{-W/2}^{W/2} d(\omega t) = \frac{W}{2\pi} = \frac{1}{2\pi}\,\frac{2\pi}{p} = \frac{1}{p}$$

dc component

$$A_0 = \frac{2}{\pi}\,\frac{W}{4} = \frac{2}{\pi}\,\frac{2\pi}{4p} = \frac{1}{p}$$

$$A_n = \frac{2}{n\pi}\sin\left(\frac{n\pi}{p}\right)$$

Fundamental Component

$$A_1 = \frac{2}{\cancelto{1}{n}\pi}\sin\left(\frac{\cancelto{1}{n}\pi}{p}\right)$$

$$\frac{\pi}{p} = \frac{2\pi}{2p} = \frac{W}{2}$$

$$= \frac{2}{\pi}\sin\left(\frac{W}{2}\right)$$

Second Harmonic

$$A_2 = \frac{2}{\cancelto{2}{n}\pi}\sin\left(\frac{\cancelto{2}{n}\pi}{p}\right)$$

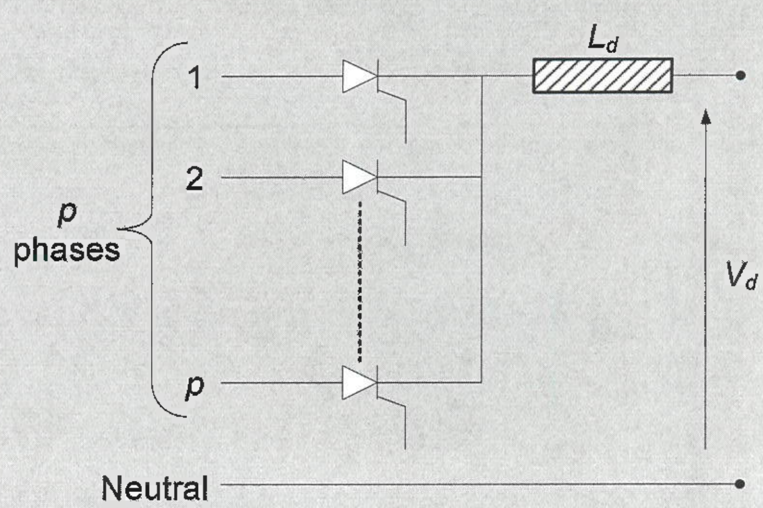
$$\frac{2\pi}{p} = W = \frac{2W}{2}$$

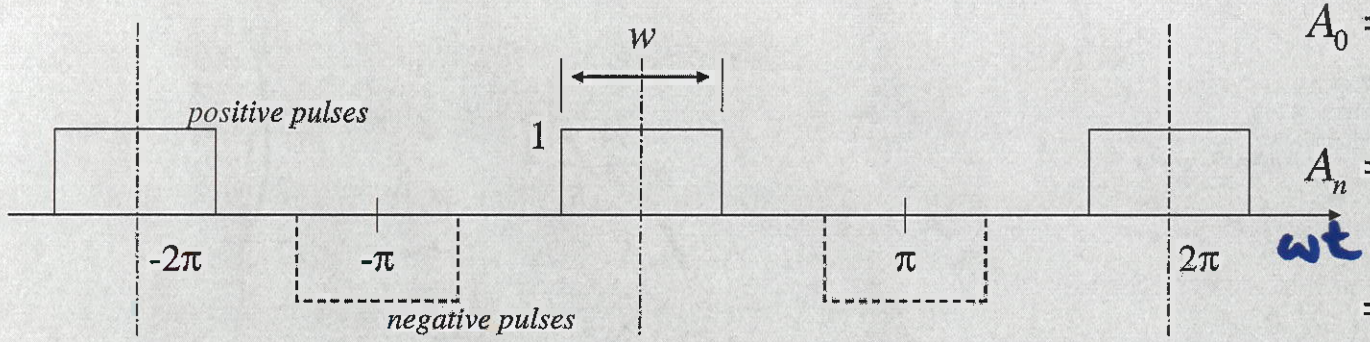
$$= \frac{2}{\pi}\left(\frac{1}{2}\sin\left(\frac{2W}{2}\right)\right)$$

$\vdots$

Current source conversion harmonics

- Ideal *p*-phase one-way converter
  - Zero ac system impedance
  - Infinite dc side smoothing inductance
  - ac phase current consists of periodic positive rectangular pulses of width $w = 2\pi/p$, repeating at supply frequency
    - Even function ➔ only cosine terms
    - Fourier series w.r.t. 1 p.u. dc current





$$A_0 = \frac{1}{2\pi}\int_{-w/2}^{w/2} d(\omega t) = \frac{w}{2\pi} = \frac{1}{p}$$

$$A_n = \frac{1}{\pi}\int_{-w/2}^{w/2} \cos(n\omega t)\, d(\omega t)$$

$$= \frac{2}{n\pi}\sin\left(\frac{nw}{2}\right) = \frac{2}{n\pi}\sin\left(\frac{n\pi}{p}\right)$$

- Fourier series of positive current pulses is

$$F_p = \frac{2}{\pi}\left(\overset{A_0}{\frac{w}{4}} + \overset{A_1}{\sin\left(\frac{w}{2}\right)}\cos(\omega t) + \overset{A_2}{\frac{1}{2}\sin\left(\frac{2w}{2}\right)}\cos(2\omega t) + \overset{A_3}{\frac{1}{3}\sin\left(\frac{3w}{2}\right)}\cos(3\omega t)\right.$$

$$\left. + \overset{A_4}{\frac{1}{4}\sin\left(\frac{4w}{2}\right)}\cos(4\omega t) + \cdots\right)$$

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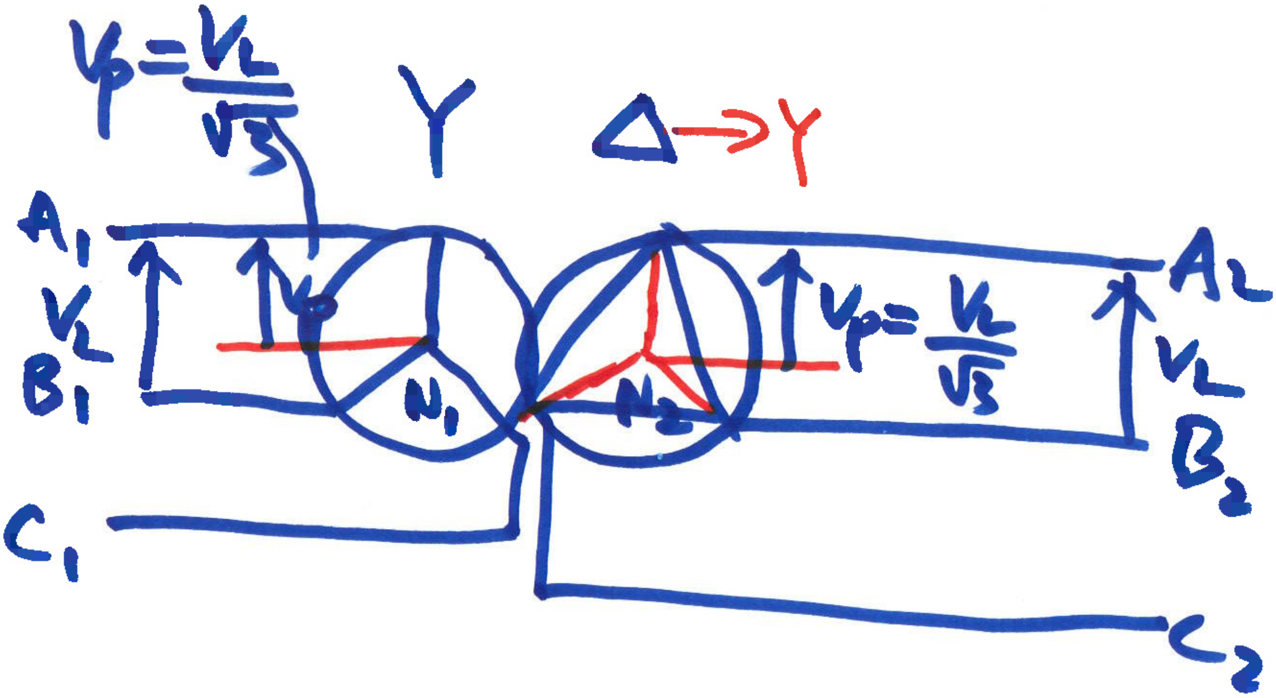

$V_\phi = \frac{V_L}{\sqrt{3}}$
Y
$\Delta \to Y$
A_1
V_L
B_1
V_ϕ
N_1
N_2
C_1
A_2
$V_\phi = \frac{V_L}{\sqrt{3}}$
V_L
B_2
C_2

Effect of transformer delta connection

– For $w = \pi / 3$,

$$F_2 = \frac{4}{\pi}\left(\sin\left(\frac{\pi/3}{2}\right)\cos(\omega t) + \frac{1}{3}\sin\left(\frac{3\pi/3}{2}\right)\cos(3\omega t) + \frac{1}{5}\sin\left(\frac{5\pi/3}{2}\right)\cos(5\omega t) + \cdots \right)$$

$$= \frac{4}{\pi}\left(\frac{1}{2}\cos(\omega t) + \frac{1}{3}\cos(3\omega t) + \frac{1}{5}\cdot\frac{1}{2}\cos(5\omega t) - \frac{1}{7}\cdot\frac{1}{2}\cos(7\omega t) + \cdots \right)$$

– Incorporating the √3 factor and the dc current I_d

$$i_a = (F_1 + F_2)\times\frac{I_d}{\sqrt{3}} = \frac{4}{\pi}\left(\frac{3}{2}\cos(\omega t) + \frac{1}{5}\cdot\frac{3}{2}\cos(5\omega t) - \frac{1}{7}\cdot\frac{3}{2}\cos(7\omega t) + \cdots \right)\times\frac{I_d}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{\pi} I_d \left(\cos(\omega t) + \frac{1}{5}\cos(5\omega t) - \frac{1}{7}\cos(7\omega t) - \frac{1}{11}\cos(11\omega t) + \frac{1}{13}\cos(13\omega t) + \frac{1}{17}\cos(17\omega t) - \frac{1}{19}\cos(19\omega t) \cdots \right)$$

6k±1 when k=1

6k±1 when k=3

– Differences in the sign or sequence of the $6k\pm1$ harmonics with odd values of k from those of the star-star connection

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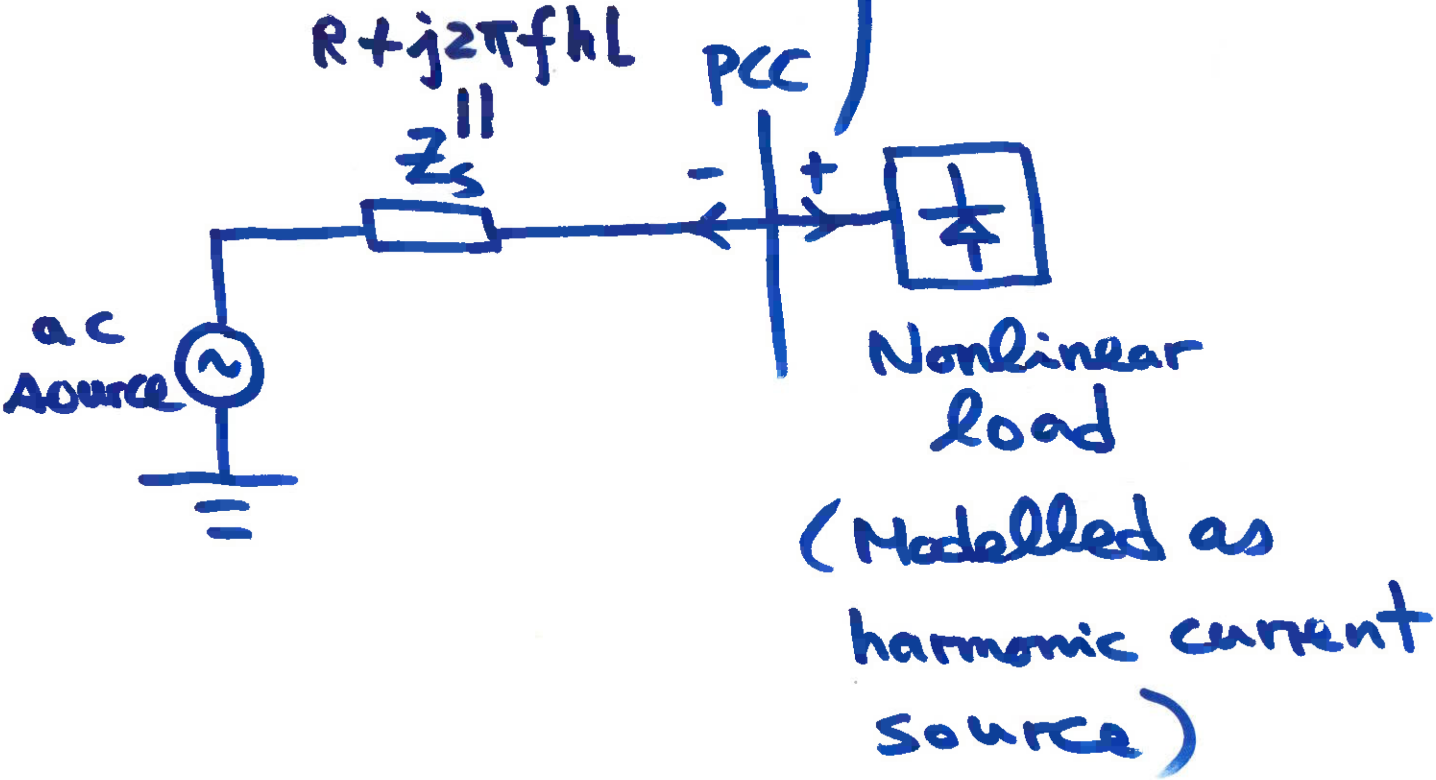
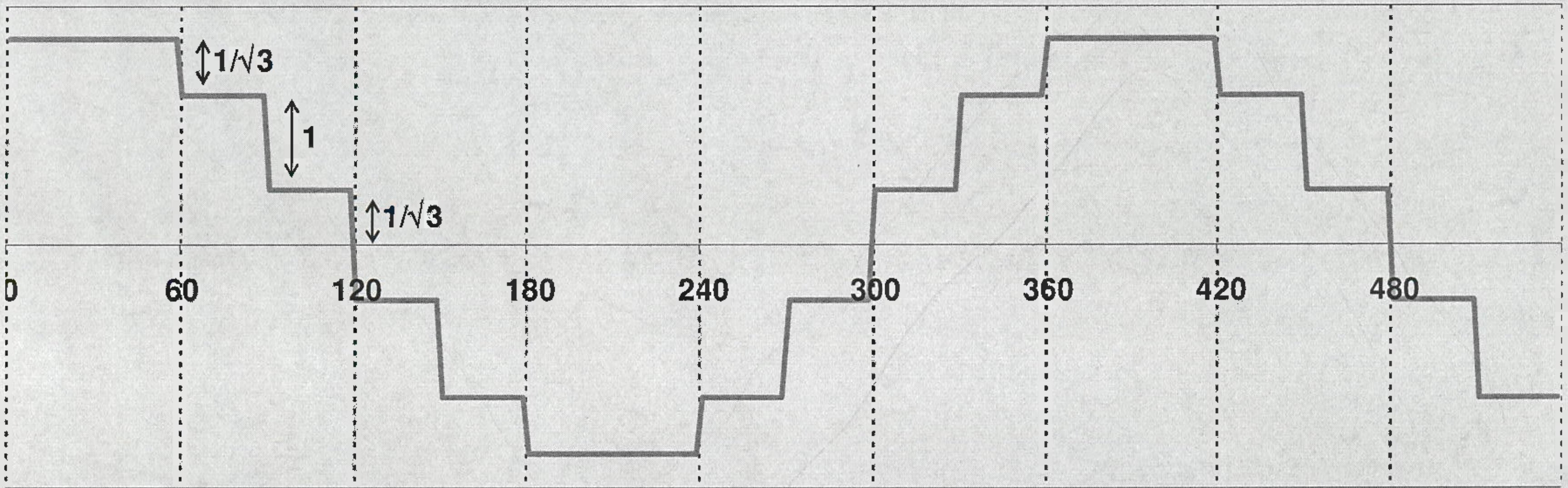
$$i_a = I_1 + I_5 - I_7 - I_{11} + I_{13} + I_{17} - I_{19} \cdots$$



12-pulse configuration - harmonics

- Resultant waveform is sum of the two waveforms of the star-star and star-delta transformers



- Fourier series is the sum of those of star-star and star-delta

$$i_a = 2\left(\frac{2\sqrt{3}}{\pi}\right)I_d\left(\cos(\omega t)-\frac{1}{11}\cos(11\omega t)+\frac{1}{13}\cos(13\omega t)-\frac{1}{23}\cos(23\omega t)+\frac{1}{25}\cos(25\omega t)\cdots\right)$$

5, 7, 17, 19

  - Harmonics of order $12k\pm1$, k are integers
  - Harmonic currents of order $6k\pm1$ with odd k, circulate between the transformers but do not penetrate the ac network

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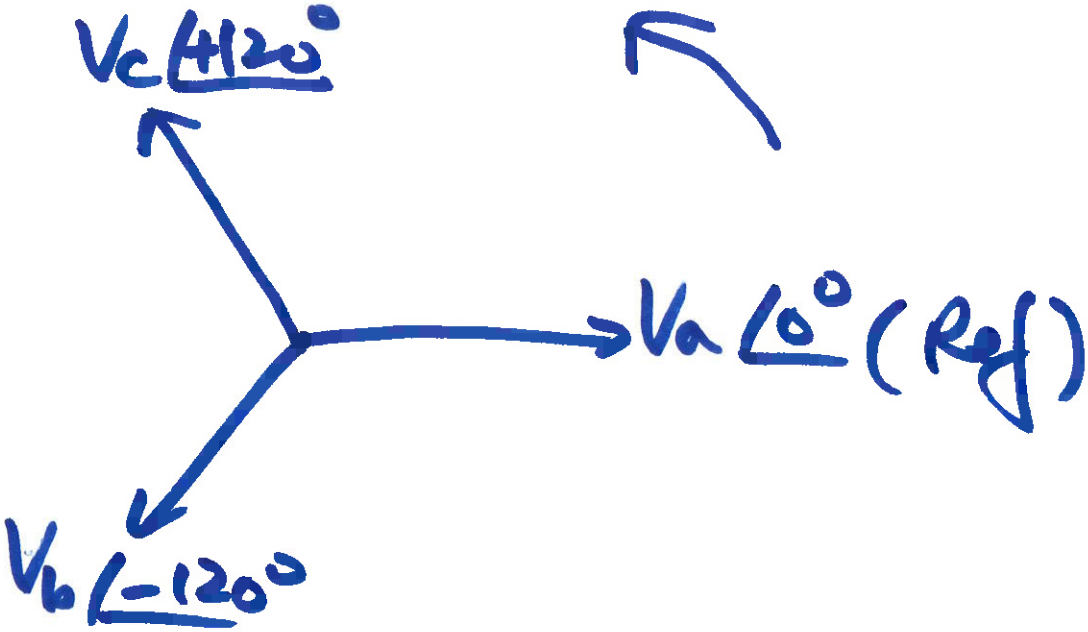
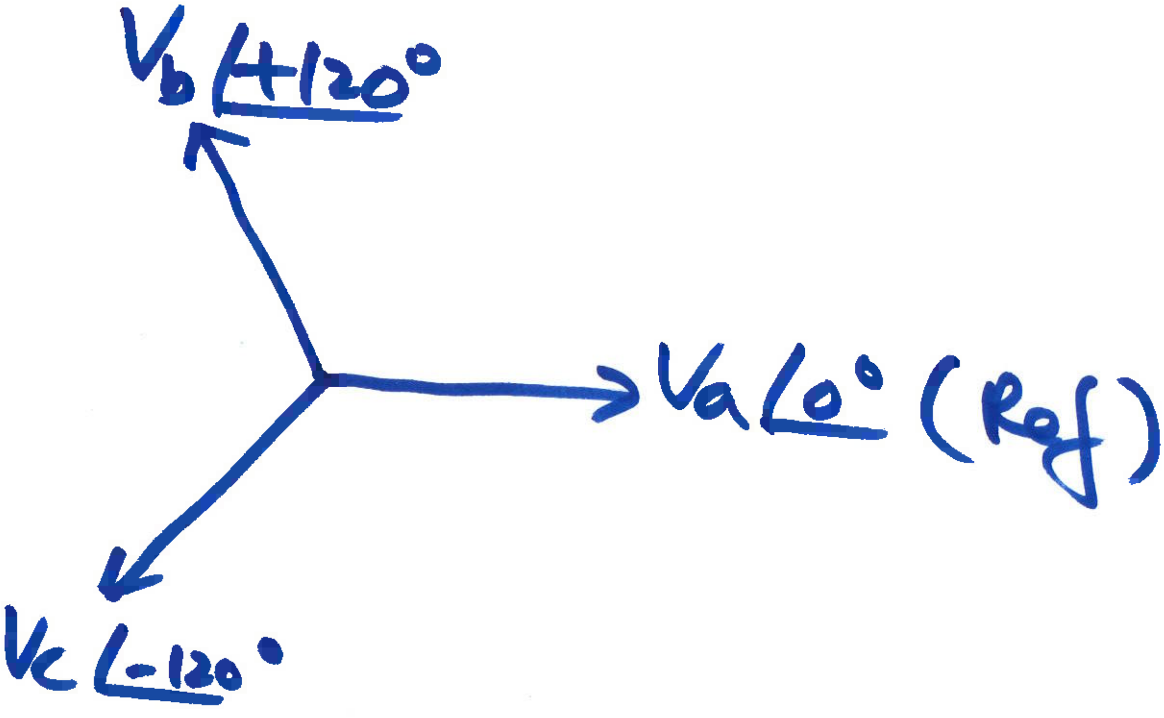
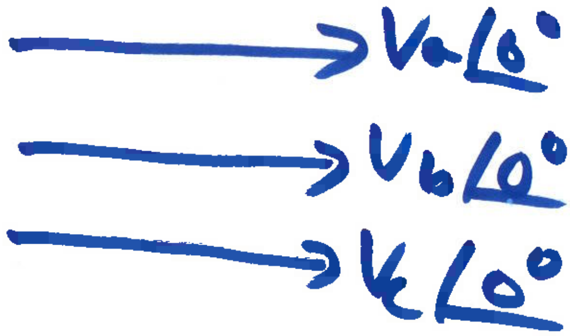


PPS - a, b, c



NPS - a, c, b



ZPS