Advanced MOSFETs and Novel Devices

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6. Tutorial & Exercise

Charge Carrier Transport

6.1 Tutorial Fermi Gas

6.2 Exercise Sheet Resistance

6.3 Exercise Drift Diffusion p-n Junction

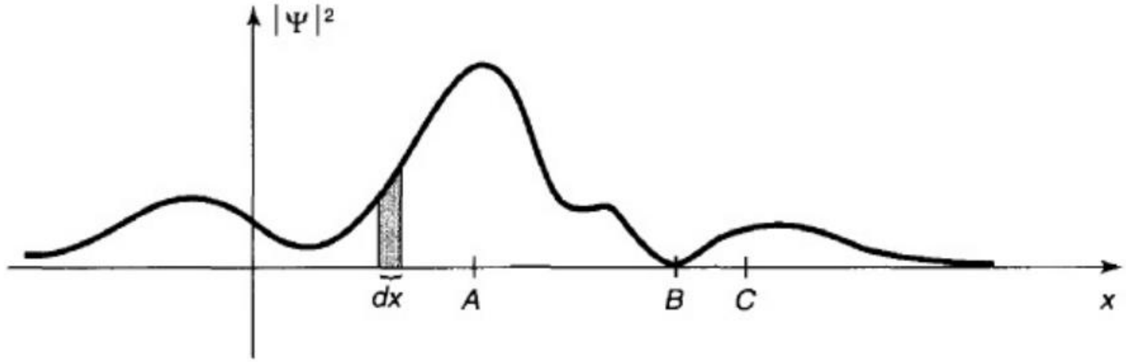
Quantum Theory:

- Louise de Broglie:
  Every particle behaves also like a wave.

- Every particle has a wave function $\Psi(x,t)$

- The probability of finding the particle between x and x + dx at time t is
  $$|\Psi(x,t)|^2\,dx$$

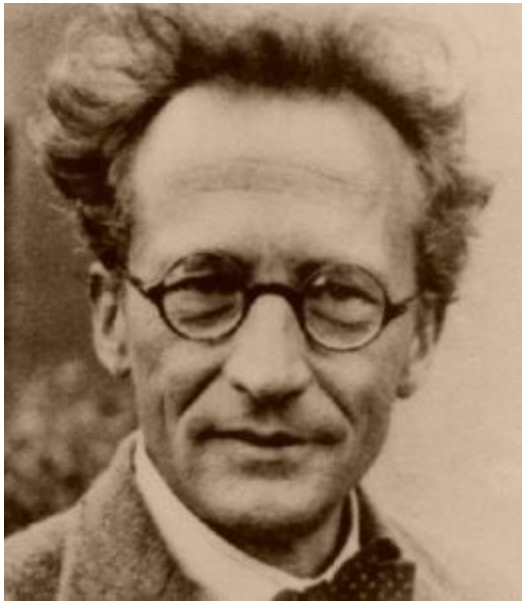
- Quantum mechanics tells us with which probability you can get a possible result

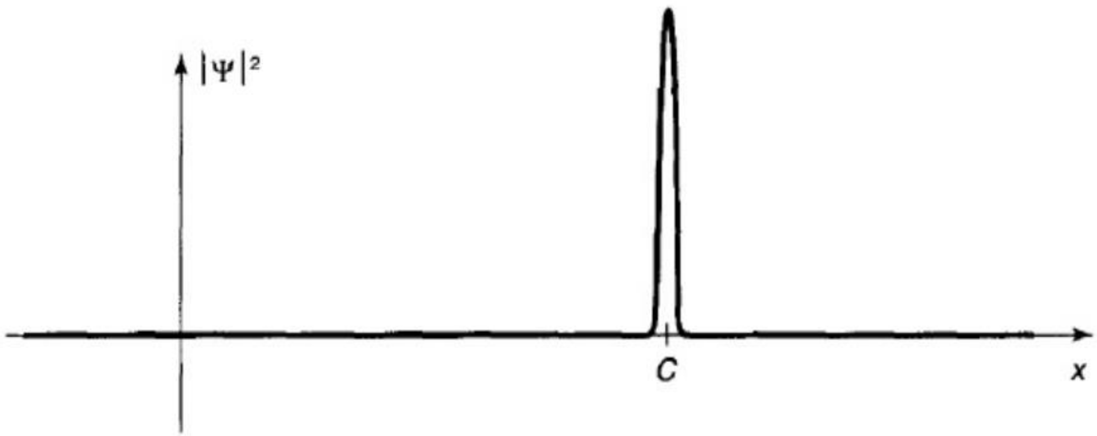


There is a high probability finding the particle at point A and nearly no chance to find it at point B.

Quantum Theory:

- The wave function is a superposition of all measurable states of the particle
- So when we do a measurement then we will get one state of the particle
- Quantum mechanics is used to calculate the expectation value which is the most, the most probable value





Fermi Gas - Introduction

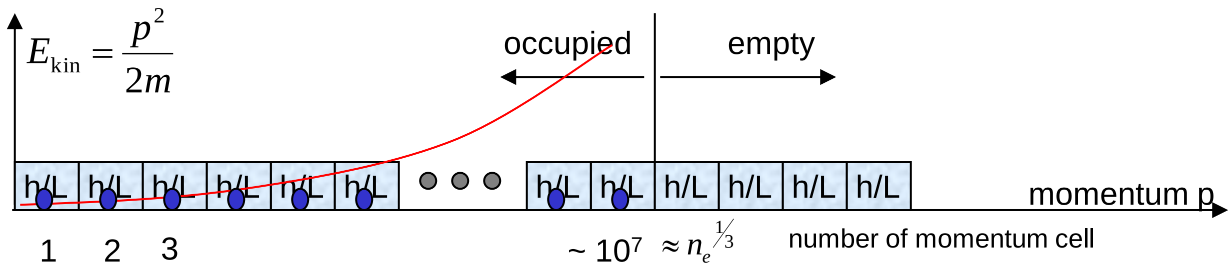
1. Due to mechanics the equation of motion for a free electron is $\vec{F} = \frac{d\vec{p}}{dt}$ with energy $E_{kin} = \frac{p^2}{2m}$.

2. Classical particles never can be on the same location, but may have identical energies (momentum) -> see Maxwell distribution.
   Quantum-mechanical particles (as superposition of planar waves) are in principle on every location present, but can never have the same energy (momentum).

   If electrons are localized in a crystal with dimension Δx=L, then every single electron must have:
   - a different value of energy (momentum) p
   - and a uncertainty of momentum $\Delta p \geq \frac{h}{l}$ which is the same for all electrons.

3. One – Dimensional momentum space:

   Without any external energy (temperature, electric fields) the electrons must occupy the states of minimum energy.



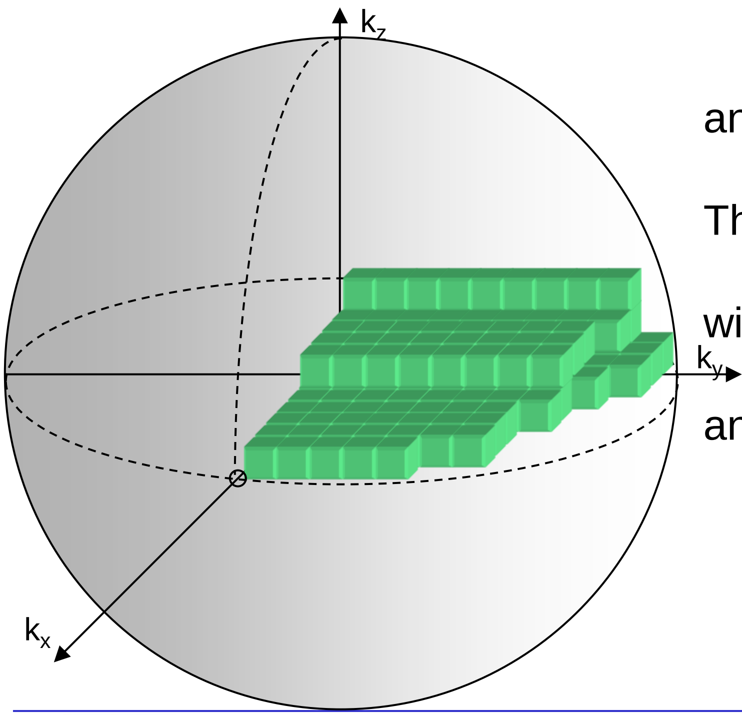
   But because every electron must have a different momentum, the momentum cells are filled from zero to the last electron.

4. The <u>highest</u> occupied momentum value is called **Fermi-momentum** p_F or k_F, the corresponding energy is called **Fermi-energy** E_F.

   The relation of momentum and the wave vector k is: $\vec{p} = \hbar\vec{k}$ with $\hbar = \frac{h}{2\pi}$

5. For 3 dimensions the states with same energy form a sphere with radius

$$|k| = \sqrt{k_1^2 + k_2^2 + k_3^2}$$



and volume

$$V_k = \frac{N}{2} \cdot \frac{h^3}{L^3}$$

The ground state forms the so called Fermi Sphere

with a volume

$$V_k = \frac{4}{3}\pi k_F^3$$

and wave vector

$$k_F = (3\pi^2 \cdot n)^{1/3}$$

$$k_F[cm] \approx 3.1 \cdot n^{\frac{1}{3}} cm^{-3}$$

N/2 states are occupied with N electrons (Spin).

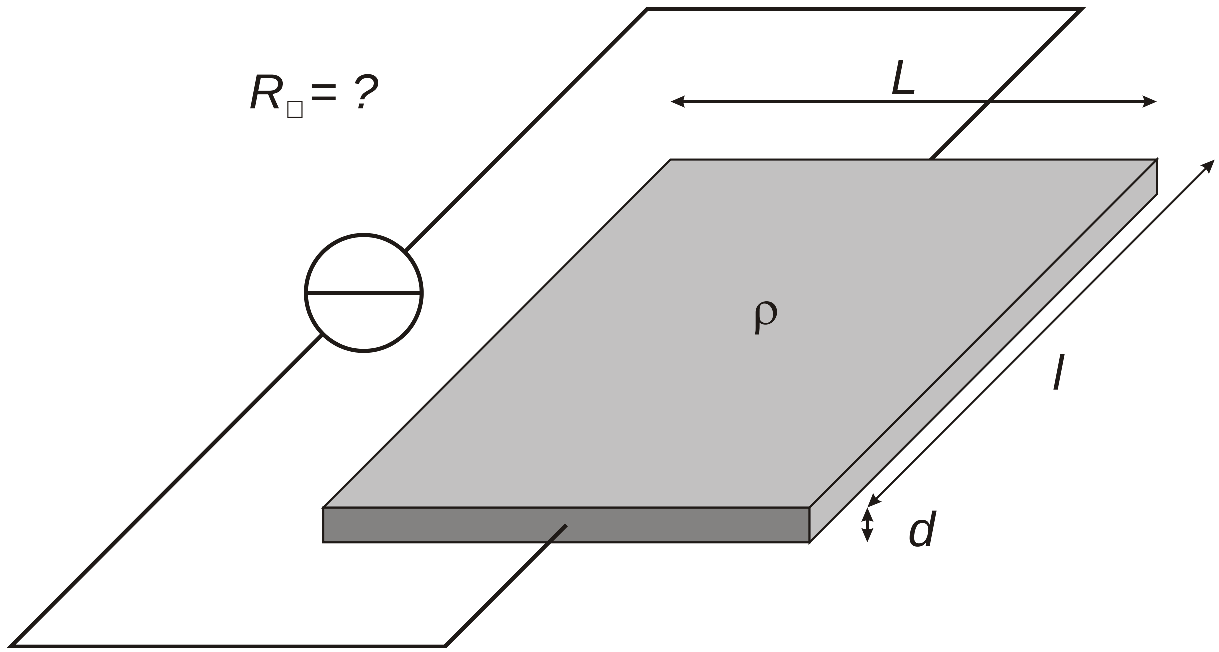
Using the energy of free electrons $E_F = \frac{{p_F}^2}{2m} = \frac{\hbar^2 {k_F}^2}{2m}$

the Fermi Energy is $E_F = \frac{\hbar}{2m}(3\pi^2 \cdot n)^{2/3}$, $E_F[eV] \approx 3.65 \cdot 10^{-15} \cdot n^{2/3} eV$.

The velocity of an electron with Fermi Momentum is $v_F = \frac{\hbar k_F}{m}$.

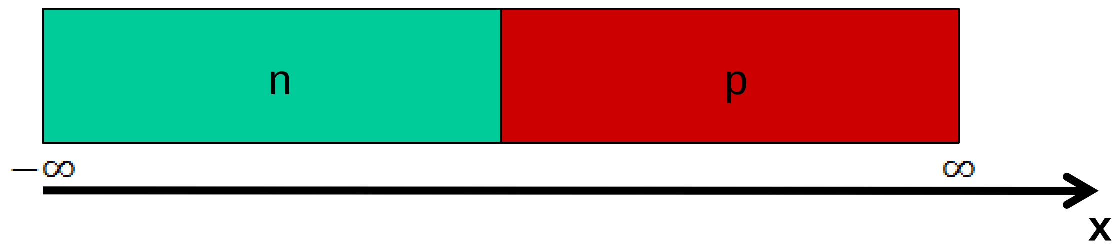
The Fermi Temperature is $E_F = k_B T_F$, $T[eV] = \frac{T[K]}{11600}$.

Determine the maximum sheet resistance $R_{\square}$ of a metallic **quadratic** monolayer. The free mean path time τ is caused by surface scattering so that $\tau = d / v_F$, where d is the thickness of the film and v_F is the Fermi velocity.



Derive the built in voltage V_{bi} of a *pn*-diode starting from the drift-diffusion equation:

$$\vec{J}_{Total} = \vec{J}_{drift} + \vec{J}_{diff} = e\mu\left(n(x)\vec{E}(x) + \frac{k_B T}{e}\frac{\partial n(x)}{\partial x}\right)$$



Sheet Resistance - Solution

Resistance:
with l = L

$$R = \frac{\rho \cdot l}{A} = \frac{\rho \cdot L}{L \cdot d} = \frac{\rho}{d} \qquad \Rightarrow R_{\blacksquare} = \frac{\rho}{d}$$

Ohm's law:

$$\vec{j} = \rho^{-1} \cdot \vec{E}$$

and

$$\vec{v}_{drift} = \mu\vec{E} = \frac{e \cdot \tau}{m}\vec{E}$$

Specific resistance:

$$\rho = \frac{m}{n \cdot e^2 \cdot \tau}$$

Sheet resistance:

$$R_{\blacksquare} = \frac{m}{n \cdot e^2 \cdot \tau \cdot d}$$

with

$$\tau \approx \frac{d}{v_F} \qquad and \quad v_F = \frac{\hbar \cdot k_F}{m}$$

$$R_{\blacksquare} = \frac{\hbar \cdot k_F}{n \cdot e^2 \cdot d^2}$$

Monolayer: $n = \frac{a}{d^3}$ with $a = \frac{electrons}{atom}$

Sheet resistance: $R_{\blacksquare} = \frac{\hbar}{e^2} \cdot \frac{k_F \cdot d}{a}$

L >> d k_F can be calculated one dimensional: $\Delta p = \frac{h}{d}$

$\vec{p} = \hbar \vec{k}$ and $p_F = \frac{N}{2} \cdot \Delta p = \frac{N}{2} \cdot \frac{h}{d}$

$$k_F = \frac{p_F}{\hbar} = \frac{a}{2} \cdot \frac{h}{d} \cdot \frac{2 \cdot \pi}{h} = \frac{\pi \cdot a}{d}$$

Sheet resistance: $R_{\blacksquare} = \frac{\hbar}{e^2} \cdot \frac{d}{a} \cdot \frac{\pi \cdot a}{d} = \frac{\pi \cdot \hbar}{e^2}$

$$R_{\blacksquare} = \pi \cdot \frac{1.05 \cdot 10^{-34} Js}{(1.6 \cdot 10^{-19} As)^2} = 12.9 k\Omega$$

Equilibrium: $\vec{J}_{Total} = e\mu\left(n(x)\vec{E}(x) + \frac{k_B T}{e}\frac{\partial n(x)}{\partial x}\right) = \sigma$

1st order differential Equation (Bronstein): $n(x) = C \cdot exp\left[-\int_{-\infty}^{x}\frac{e}{k_b T}E(x')dx'\right]$

Solution: $n(x) = N_D \cdot exp\left[-\frac{e}{k_b T}\left(V(x) - V(-\infty)\right)\right]$

with $C = n(-\infty) = N_D$

Built in potential: $V_{bi} = V(\infty) - V(-\infty) = \frac{k_B T}{e} ln\left(\frac{N_D}{n(\infty)}\right) = \frac{k_B T}{e} ln\left(\frac{N_A \cdot N_D}{n_i^2}\right)$

$$n(x) \cdot p(x) = n_i^2 \Rightarrow n(\infty) = \frac{n_i^2}{N_A}$$