# Solutions to Week 2 Lecture Notes
# Exercise 1: 2D-DCT Transform

(a)

$$S_{uv} = \alpha(u)\alpha(v)\sum_{i=0}^{N-1}\sum_{j=0}^{N-1} S_{ij}\cos\frac{(2i+1)u\pi}{2N}\cos\frac{(2j+1)v\pi}{2N}$$

$$= \alpha(u)\sum_{i=0}^{N-1}\cos\frac{(2i+1)u\pi}{2N}\underbrace{\left\{\alpha(v)\sum_{j=0}^{N-1} S_{ij}\cos\frac{(2j+1)v\pi}{2N}\right\}}_{F_{iv}}$$

$$= \alpha(u)\sum_{i=0}^{N-1} F_{iv}\cos\frac{(2i+1)u\pi}{2N}$$

Hence, 2D-DCT ( $S_{uv}$ ) can be computed using a two-stage 1D-DCT:

First Stage: $$F_{iv} = \alpha(v)\sum_{j=0}^{N-1} S_{ij}\cos\frac{(2j+1)v\pi}{2N} \qquad i,v = 0,1,\cdots,N-1$$

Second Stage: $$S_{uv} = \alpha(u)\sum_{i=0}^{N-1} F_{iv}\cos\frac{(2i+1)u\pi}{2N} \qquad u,v = 0,1,\cdots,N-1$$

For 4×4 2D-DCT:

First Stage: $$F_{iv} = \alpha(v)\sum_{j=0}^{3} S_{ij}\cos\frac{(2j+1)v\pi}{8} \qquad i,v = 0,1,2,3$$

Second Stage: $$S_{uv} = \alpha(u)\sum_{i=0}^{3} F_{iv}\cos\frac{(2i+1)u\pi}{8} \qquad u,v = 0,1,2,3$$

Given image:

$$S = \begin{bmatrix} 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 10 & 10 & 10 & 10 \\ 20 & 20 & 10 & 10 \end{bmatrix}$$

First stage:

$$F_{iv} = \alpha(v)\sum_{j=0}^{3} S_{ij}\cos\frac{(2j+1)v\pi}{8}$$

$$= \alpha(v)\left\{S_{i0}\cos\frac{v\pi}{8} + S_{i1}\cos\frac{3v\pi}{8} + S_{i2}\cos\frac{5v\pi}{8} + S_{i3}\cos\frac{7v\pi}{8}\right\}$$

$$= \alpha(v)\left\{\left[S_{i0} + (-1)^v S_{i3}\right]\cos\frac{v\pi}{8} + \left[S_{i1} + (-1)^v S_{i2}\right]\cos\frac{3v\pi}{8}\right\}$$

Side note: $\cos(v\pi - \theta) = \cos v\pi\cos\theta + \sin v\pi\sin\theta = (-1)^v\cos\theta$

## apply row transform

- 1st row $S=[10 \quad 10 \quad 10 \quad 10]$

  dc coefficients: $F_{00}=\frac{1}{2}\{20+20\}=20$

  ac coefficients: $F_{01}=F_{02}=F_{03}=0$ (as the data $S$ are constant)

  $F_{0v}=[20 \quad 0 \quad 0 \quad 0]$

- 2nd and 3rd rows are similar to the 1st row

- 4th row

  $S=[20 \quad 20 \quad 10 \quad 10]$

  $$F_{3v}=\alpha(v)\left\{\left[20+(-1)^v 10\right]\cos\frac{v\pi}{8}+\left[20+(-1)^v 10\right]\cos\frac{3v\pi}{8}\right\}$$

  $$F_{30}=\frac{1}{2}\{30+30\}=30$$

  $$F_{31}=\frac{1}{\sqrt{2}}\left\{10\cos\frac{\pi}{8}+10\cos\frac{3\pi}{8}\right\}=\frac{1}{\sqrt{2}}(9.239+3.827)=9.24$$

  $$F_{32}=\frac{1}{\sqrt{2}}\left\{30\cos\frac{\pi}{4}+30\cos\frac{3\pi}{4}\right\}=0$$

  $$F_{33}=\frac{1}{\sqrt{2}}\left\{10\cos\frac{3\pi}{8}+10\cos\frac{9\pi}{8}\right\}=\frac{1}{\sqrt{2}}(3.827-9.239)=-3.83$$

  $F_{3v}=[30 \quad 9.24 \quad 0 \quad -3.83]$

After the first stage 1D-DCT, we obtain:

$$F=\begin{bmatrix}20 & 0 & 0 & 0\\ 20 & 0 & 0 & 0\\ 20 & 0 & 0 & 0\\ 30 & 9.24 & 0 & -3.83\end{bmatrix}$$

Second stage:

$$S_{uv}=\alpha(u)\left\{F_{0v}\cos\frac{u\pi}{8}+F_{1v}\cos\frac{3u\pi}{8}+F_{2v}\cos\frac{5u\pi}{8}+F_{3v}\cos\frac{7u\pi}{8}\right\}$$

$$=\alpha(u)\left\{\left[F_{0v}+(-1)^u F_{3v}\right]\cos\frac{u\pi}{8}+\left[F_{1v}+(-1)^u F_{2v}\right]\cos\frac{3u\pi}{8}\right\}$$

v=0时
F=1/2(F00+F01+F02+F03)=1/2(10+10+10+10)=20

v=1时
F=1/根号二*10(cos π /8+cos3 π /8+cos5 π /8 +7 π /8)=0

v=2 F=0

v=3 F=0

v=0 F=1/2(20+20+10+10)=30

v=1 F=1/根号二 (20cos π /8+20cos3 π /8+10cos5 π /8+10cos π 7/8)=

1st column:

$$S_{u0} = \alpha(u)\left\{\left[20+(-1)^u 30\right]\cos\frac{u\pi}{8}+\left[20+(-1)^u 20\right]\cos\frac{3u\pi}{8}\right\}$$

$$S_{00} = \frac{1}{2}\{50+40\} = 45$$

$$S_{10} = \frac{1}{\sqrt{2}}\left\{-10\cos\frac{\pi}{8}\right\} = -6.53$$

$$S_{20} = \frac{1}{\sqrt{2}}\left\{50\cos\frac{\pi}{4}+40\cos\frac{3\pi}{4}\right\} = 5$$

=20/根号二（cos2π/8+cos6π/8
+cos10π/8）+30/根号二*cos14π/8

$$S_{30} = \frac{1}{\sqrt{2}}\left\{-10\cos\frac{3\pi}{8}\right\} = -2.706$$

$$F = \begin{bmatrix}20\\20\\20\\30\end{bmatrix},$$

$$\therefore S_{u0} = \begin{bmatrix}45\\-6.53\\5\\-2.706\end{bmatrix}$$

2nd column:

$$S_{u1} = \alpha(u)\times(-1)^u \times 9.24 \times \cos\frac{u\pi}{8}$$

$$S_{01} = \frac{1}{2}\times 9.24 = 4.62$$

$$S_{11} = \frac{1}{\sqrt{2}}\times(-9.24)\times\cos\frac{\pi}{8} = -6.04$$

$$S_{21} = \frac{1}{\sqrt{2}}\times 9.24\times\cos\frac{\pi}{4} = 4.62$$

$$S_{31} = \frac{1}{\sqrt{2}}\times(-9.24)\times\cos\frac{3\pi}{8} = -2.5$$

$$F_{i1} = \begin{bmatrix}0\\0\\0\\9.24\end{bmatrix},$$

$$\therefore S_{u1} = \begin{bmatrix}4.62\\-6.04\\4.62\\-2.5\end{bmatrix}$$

3rd column:

$$S_{u2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

4th column

$$F_{i3} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.83 \end{bmatrix},$$

$$S_{u3} = \alpha(u) \times (-1)^u \times (-3.83) \times \cos\frac{u\pi}{8}$$

$$S_{03} = \frac{1}{2} \times (-3.83) = -1.915$$

$$S_{13} = \frac{1}{\sqrt{2}} \times (-1) \times (-3.83) \times \cos\frac{\pi}{8} = 2.5$$

$$S_{23} = \frac{1}{\sqrt{2}} \times (-3.83) \times \cos\frac{\pi}{4} = -1.915$$

$$S_{33} = \frac{1}{\sqrt{2}} \times (-1) \times (-3.83) \times \cos\frac{3\pi}{8} = 1.036$$

$$\therefore S_{u3} = \begin{bmatrix} -1.915 \\ 2.5 \\ -1.915 \\ 1.036 \end{bmatrix}$$

**2D-DCT:**

$$S_{uv} = \begin{bmatrix} 45 & 4.62 & 0 & -1.915 \\ -6.53 & -6.04 & 0 & 2.5 \\ 5 & 4.62 & 0 & -1.915 \\ -2.71 & -2.5 & 0 & 1.036 \end{bmatrix}$$