PART A

POWER SYSTEM NETWORK COMPONENTS

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EE6511 Course Structure

Network Components

Synchronous machine (6 hours)
Elementary models, development of the general machine equations, general power equations, Blondel Transformation, steady-state machine model, equivalent circuit representation. (Chapter 4, Elgerd), machine under sudden 3-phase short-circuit (Section 10.4, Elgerd).

Power transformer (1.5 hr)
Equivalent circuits (Section 5.2 Elgerd), 3-winding transformer (Section 5.3, Elgerd).

Transmission Line (1.5 hr)
Equivalent circuits (Section 6.4 Elgerd)

System transient stability models (4 hrs)
Transient System Models (Section 12.2, 12.3, Elgerd), Speed-control (Sections 4.1-4.6, Weedy; Chapter 11, Kundur), Excitation System (Section 9.3.2, Elgerd)

Steady-state Power System Networks

Admittance matrix formulation, Power Flow (Chapter 7, Elgerd), (3hr)

Voltage and Reactive Power Control (Chapter 5 Weedy) (2 hr)

Sequence of lectures:
Synchronous machine, power transformer, transmission lines, admittance matrix formulation, power flow, voltage and reactive power control, system transient stability models.

Textbooks:

[1] OI Elgerd, "Electric Energy Systems Theory: An introduction", McGraw Hill, 1971.
[2] MB Weedy and BJ Cory, "Electric Power Systems", 4th Edition, J Wiley, 1998.
[3] P Kundur, "Power System Stability and Control", McGraw Hill, NY, 1994

1. SYNCHRONOUS MACHINE – SYSTEM MODEL REPRESENTATION

Materials of this chapter are taken from Chapter 4, Elgerd.

Intention is to focus on the generator characteristics which are important from a system point of view. Excellent text books include that by Elgerd, Kundur.

1.1 Elementary Models

Refer to Fig 4.1. Assume the machine is operating under balanced steady-state. Rotor is driven by a prime mover, while the magnetic field intensity is controlled via a dc source in the rotor (field) winding.

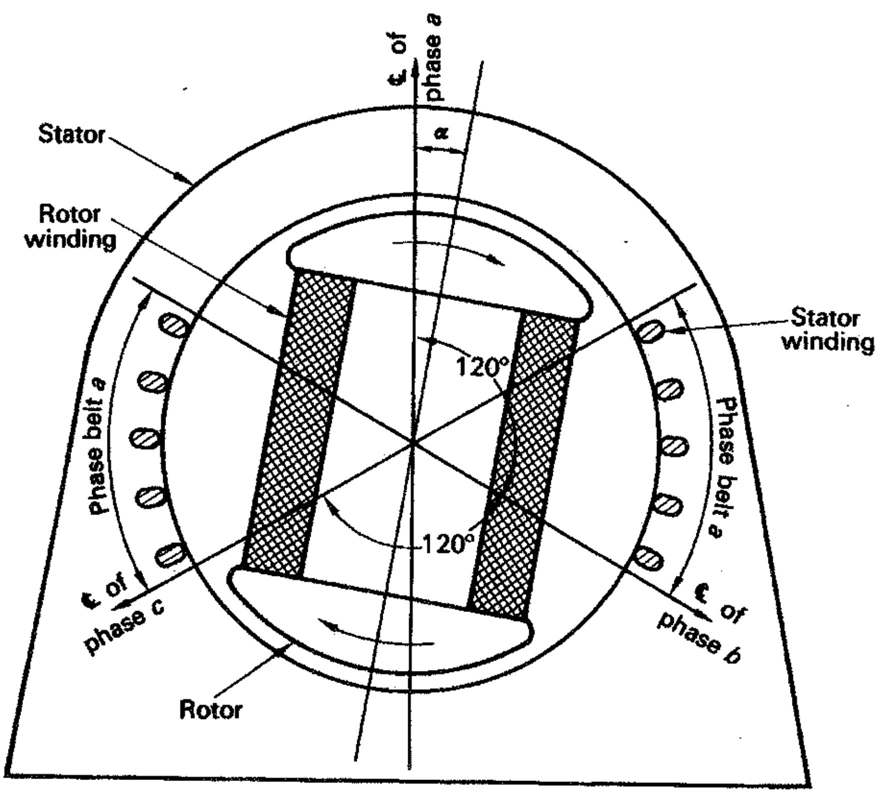


Fig. 4-1 Schematic design of 3-phase synchronous machine.

Often a synchronous generator will operate in parallel with other generators, as shown in Fig 4.2 The vth generator supplies $S_{Gv} = P_{Gv} + jQ_{Gv}$ to the system. Its bus voltage is kept at magnitude V_v and the system is running synchronously at the frequency f.

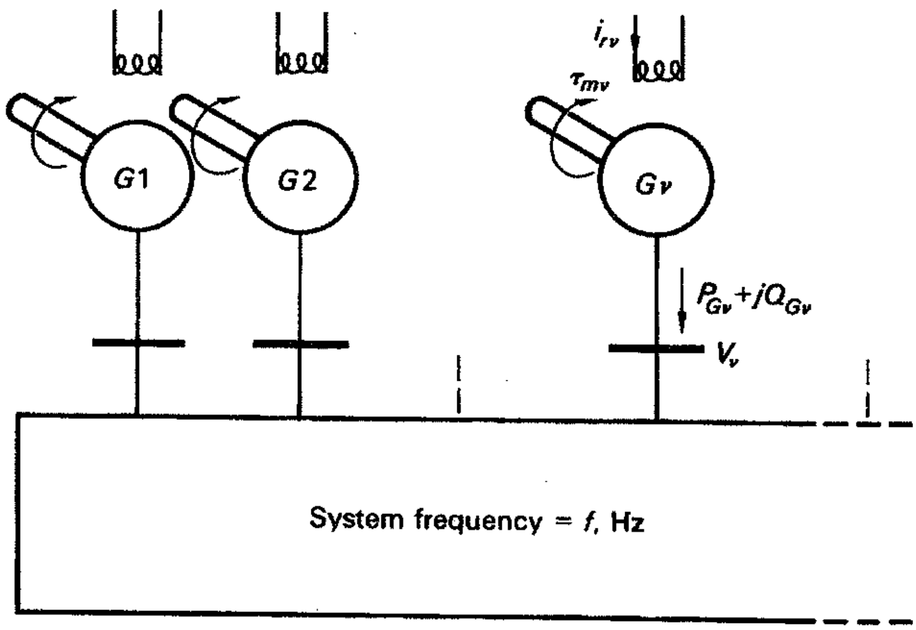


Fig. 4-2 Typical parallel operation of generators.

1.

1.1 Control of Synchronous Machines

There are two controls on each machine: field (rotor) current i_{rv} and the mechanical shaft torque t_{mv}. Outputs P_{Gv}, Q_{Gv}, $|Vv|$ and f will generally change once i_{rv} and/or t_{mv} change. An input/output model of the machine is Fig 4.3

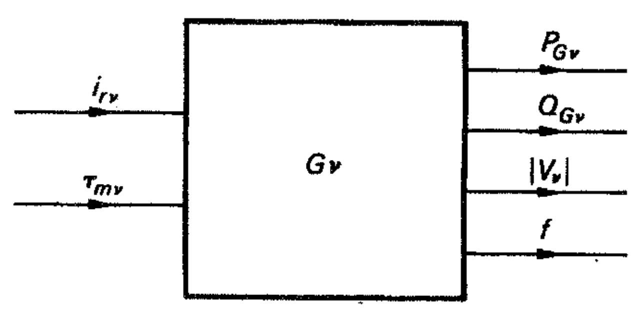


Fig. 4-3 Input-output representation of individual synchronous generator.

Note the cross coupling between the two inputs and the 4 outputs. The degree of coupling depends on the system size and structure. If the system has a very large capacity compared to that of one machine, the manipulation of t_{mv} or i_{rv} of one machine will not change f or $|V_v|$ to any significant extent. In which case, only P_{Gv} and Q_{Gv} can be manipulated. In fact, as will be shown later, i_{rv} will affect Q_{Gv} more

than on P_{Gv}, while t_{mv} will alter P_{Gv} to a greater extent than on Q_{Gv}. Hence the cross-coupling between t_{mv} and Q_{Gv} is "weak" and that between i_{rv} and P_{Gv} is also minimal. Hence, some form of non-interacting or de-coupled control has been achieved.

If there is only one machine supplying the system, then any increase in t_{mv} will cause the rotor to increase its rotational speed, assuming the load power demand is constant. Therefore f increases and the EMF will also increase. Hence, V_v rises, as well as P_{Gv} and Q_{Gv}. Thus it results in all 4 output variables changing their states.

1.2 Development of General Machine Equations

We shall use the approach of magnetically coupled circuits to derive the models. We shall consider only the 3 stator and one rotor windings, and ignore the so-called damper windings. We shall also assume the machine is magnetically linear.

1.2.1 The Basic Machine Parameters

With these assumptions and with reference to Fig 4.7, the i^{th} winding will have parameters r_i, l_{ii}, l_{ij} to represent the winding resistance, self inductance and mutual inductance respectively. Also:

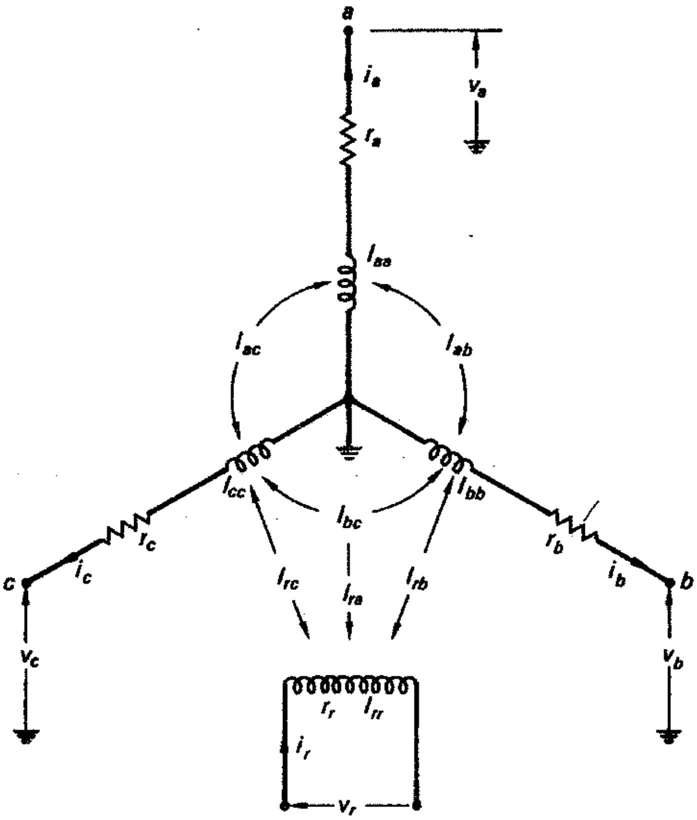


Fig. 4-7 Winding inductance and resistance parameters.

1. r_a, r_b and r_c are equal and normally small by design. Hence, set them each equal to r_s;

2. Also from Fig 4.1, for a salient-pole machine, all the inductances, except for l_{rr}, depend upon the position of the rotor and are therefore functions of the time-varying angles α;

3. If it is round-rotor machine, then all the inductance will be constant except for the mutual elements containing r in the subscripts;

4. The self inductance l_{aa} of stator winding "a" varies periodically with the position α , in a manner shown in Fig 4.8 (a), reaching peak value two times during one revolution of the rotor;

We write

$l_{aa} = L_1 + L_2 \cos 2\alpha$ (4.3)

With L_1 and L_2 are as defined in the figure. L_2 would be zero for a round-rotor machine.

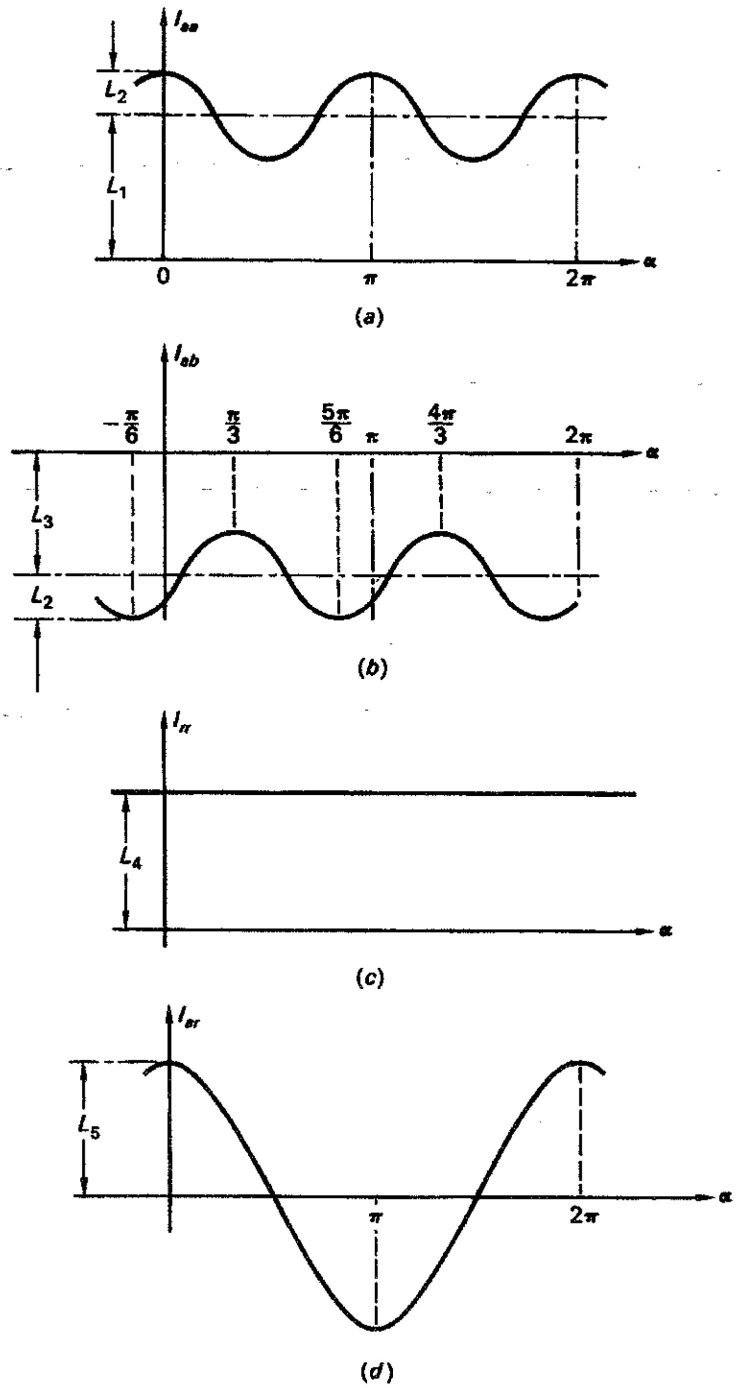


Fig. 4-8 Variation in inductance parameters with angular rotor position.

5. Since the stator coils are displaced 120° apart, then

$$l_{bb} = L_1 + L_2 \cos(2\alpha - 120^o)$$
$$l_{cc} = L_1 + L_2 \cos(2\alpha + 120^o) \qquad (4.4)$$

6. Mutual inductances also satisfy

$$l_{ij} = l_{ji} \qquad (4.5)$$

and the stator inductances l_{ab}, l_{bc}, l_{ca} are all negative.

7. The flux linked to stator winding b by a current in winding a will have its largest magnitude when α is either 30° or 150°, since the rotor in these two positions offers the least magnetic reluctance.

Hence, we have

$$l_{ab} = -L_3 - L_2 \cos 2(\alpha + \pi/6)$$

With L_3 is defined in Fig 4.8 b. L_3 is positive.

8. By similar reasoning on the 120° displacement of the stator coils, we have

$$l_{bc} = -L_3 - L_2 \cos 2(\alpha - \pi/2)$$
$$l_{ac} = -L_3 - L_2 \cos 2(\alpha - \pi/6) \qquad (4.7)$$

9. On rotor inductance elements,

(a) self-inductance l_{rr} would be constant. It is now denoted as L_4;

(b) the mutual between the rotor and stator windings vary between positive and negative maxima. So, we have

$$l_{ra} = L_5 \cos\alpha$$
$$l_{rb} = L_5 \cos(\alpha - 2\pi/3)$$
$$l_{rc} = L_5 \cos(\alpha + 2\pi/3)$$

With L_5 as defined in Fig 4.8(d)

10. The angle α represents the electrical angle, so for a p pole machine,

$\alpha_{el} = p\ \alpha_{mech}/2$

where α_{el} and α_{mech} are the electrical and mechanical angles respectively.

11. In summary, we have expressed the 16 inductances in terms of 5 positive inductance parameters, $L_1 - L_5$ and rotor angle α. We shall assume these parameters are known, either from tests or from manufacturers. In the case of round rotor machine, we need only deal with L_1, L_3, L_4 and L_5.

1.2.2. General Machine Equations

Using Kirchhoff voltage equations for all the 4 separate circuit in Fig 4.7, we have

$$
\begin{aligned}
v_a &= -i_a r_s - \frac{d}{dt}(l_{aa} i_a) - \frac{d}{dt}(l_{ab} i_b) - \frac{d}{dt}(l_{ac} i_c) + \frac{d}{dt}(l_{ar} i_r) \\
v_b &= -i_b r_s - \frac{d}{dt}(l_{ba} i_a) - \frac{d}{dt}(l_{bb} i_b) - \frac{d}{dt}(l_{bc} i_c) + \frac{d}{dt}(l_{br} i_r) \\
v_c &= -i_c r_s - \frac{d}{dt}(l_{ca} i_a) - \frac{d}{dt}(l_{cb} i_b) - \frac{d}{dt}(l_{cc} i_c) + \frac{d}{dt}(l_{cr} i_r) \\
v_r &= \ \ i_r r_r - \frac{d}{dt}(l_{ra} i_a) - \frac{d}{dt}(l_{rb} i_b) - \frac{d}{dt}(l_{rc} i_c) + \frac{d}{dt}(l_{rr} i_r)
\end{aligned}
\qquad (4\text{-}10)
$$

Note that positive sign of the stator currents means the generator action. Hence we have the mixed-sign situation in the above equation.

Define the following matrices:

$$
\mathbf{v} \triangleq \left[\begin{array}{c} v_a \\ v_b \\ v_c \\ \hdashline v_r \end{array}\right] \qquad \mathbf{i} \triangleq \left[\begin{array}{c} i_a \\ i_b \\ i_c \\ \hdashline -i_r \end{array}\right] \tag{4-11}
$$

$$
\mathbf{R} \triangleq \left[\begin{array}{ccc:c} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_s & 0 \\ \hdashline 0 & 0 & 0 & r_r \end{array}\right] \tag{4-12}
$$

$$
\begin{aligned}
\mathbf{L} \triangleq \left[\begin{array}{ccc:c} l_{aa} & l_{ab} & l_{ac} & l_{ar} \\ l_{ba} & l_{bb} & l_{bc} & l_{br} \\ l_{ca} & l_{cb} & l_{cc} & l_{cr} \\ \hdashline l_{ra} & l_{rb} & l_{rc} & l_{rr} \end{array}\right] &= \left[\begin{array}{ccc:c} L_1 + L_2\cos 2\alpha & -L_3 - L_2\cos 2\left(\alpha + \frac{\pi}{6}\right) & -L_3 - L_2\cos 2\left(\alpha - \frac{\pi}{6}\right) & L_5\cos\alpha \\ -L_3 - L_2\cos 2\left(\alpha + \frac{\pi}{6}\right) & L_1 + L_2\cos 2\left(\alpha - \frac{2\pi}{3}\right) & -L_3 - L_2\cos 2\left(\alpha - \frac{\pi}{2}\right) & L_5\cos\left(\alpha - \frac{2\pi}{3}\right) \\ -L_3 - L_2\cos 2\left(\alpha - \frac{\pi}{6}\right) & -L_3 - L_2\cos 2\left(\alpha - \frac{\pi}{2}\right) & L_1 + L_2\cos 2\left(\alpha - \frac{4\pi}{3}\right) & L_5\cos\left(\alpha - \frac{4\pi}{3}\right) \\ \hdashline L_5\cos\alpha & L_5\cos\left(\alpha - \frac{2\pi}{3}\right) & L_5\cos\left(\alpha - \frac{4\pi}{3}\right) & L_4 \end{array}\right] \\
&= \left[\begin{array}{ccc:c} L_1 + L_2\cos 2\alpha & -L_3 + L_2\cos\left(2\alpha - \frac{2\pi}{3}\right) & -L_3 + L_2\cos\left(2\alpha + \frac{2\pi}{3}\right) & L_5\cos\alpha \\ -L_3 + L_2\cos\left(2\alpha - \frac{2\pi}{3}\right) & L_1 + L_2\cos\left(2\alpha + \frac{2\pi}{3}\right) & -L_3 + L_2\cos 2\alpha & L_5\cos\left(\alpha - \frac{2\pi}{3}\right) \\ -L_3 + L_2\cos\left(2\alpha + \frac{2\pi}{3}\right) & -L_3 + L_2\cos 2\alpha & L_1 + L_2\cos\left(2\alpha - \frac{2\pi}{3}\right) & L_5\cos\left(\alpha + \frac{2\pi}{3}\right) \\ \hdashline L_5\cos\alpha & L_5\cos\left(\alpha - \frac{2\pi}{3}\right) & L_5\cos\left(\alpha + \frac{2\pi}{3}\right) & L_4 \end{array}\right]
\end{aligned} \tag{4-13}
$$

For round-rotor generator, $L_2 = 0$, then

$$
\mathbf{L} = \left[\begin{array}{ccc|c}
L_1 & -L_3 & -L_3 & L_5 \cos \alpha \\
-L_3 & L_1 & -L_3 & L_5 \cos \left(\alpha - \frac{2\pi}{3}\right) \\
-L_3 & -L_3 & L_1 & L_5 \cos \left(\alpha + \frac{2\pi}{3}\right) \\
\hline
L_5 \cos \alpha & L_5 \cos \left(\alpha - \frac{2\pi}{3}\right) & L_5 \cos \left(\alpha + \frac{2\pi}{3}\right) & L_4
\end{array}\right] \tag{4-14}
$$

with the above matrices, equation (4.10) becomes

$$
\mathbf{v} = -\mathbf{Ri} - \frac{d}{dt}(\mathbf{Li}) \tag{4-15}
$$

Note that:

1. since L depends on α, L is not a constant parameter matrix with respect to time. Hence, Laplace transform cannot be used here, directly.

2. consider a typical term in equation (4.10), say

$$
\frac{d}{dt}(l_{aa} i_a)
$$

By performing the derivation called for, we obtain terms of the types

$$
i_a \frac{d\alpha}{dt} \sin 2\alpha \qquad \text{and} \qquad \frac{di_a}{dt} \cos 2\alpha
$$

If the rotor angular velocity $d\alpha/dt$ is *constant* (which is usually the case, since the machine has large inertia), then we can write

$$\frac{d\alpha}{dt} = \omega = \text{const} \qquad (4\text{-}16)$$

and

$$\alpha = \omega t + \alpha_0$$

and the above terms will therefore reduce to

$$\omega \sin 2(\omega t + \alpha_0) i_a \qquad \text{and} \qquad \cos 2(\omega t + \alpha_0) \frac{di_a}{dt}$$

So the general differential equation is of the linear type with time-varying coefficients. However, if under changing electrodynamical torque condition, (e.g. due to a fault on the system), the speed ω is no longer constant. Then (4.15) is non-linear and general analytical solution is not possible. Numerical solution methods are needed.

1.2.3 The General Power Equation

Power delivered by stator windings is

$$p = i_a v_a + i_b v_b + v_c i_c \qquad (4.17)$$

By partitioning the stator from the rotor portion of (4.11)- (4.14), we have

$$\mathbf{v}_s \triangleq \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \qquad \mathbf{i}_s \triangleq \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{4-18}$$

$$\mathbf{R}_s \triangleq \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \tag{4-19}$$

$$\mathbf{L}_s \triangleq \begin{bmatrix} l_{aa} & l_{ab} & l_{ac} \\ l_{ba} & l_{bb} & l_{bc} \\ l_{ca} & l_{cb} & l_{cc} \end{bmatrix} \qquad \mathbf{l}_{rs} \triangleq \begin{bmatrix} l_{ar} \\ l_{br} \\ l_{cr} \end{bmatrix} \tag{4-20}$$

and

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_s \\ v_r \end{bmatrix} \qquad \mathbf{i} = \begin{bmatrix} \mathbf{i}_s \\ -i_r \end{bmatrix} \tag{4-21}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_s & \mathbf{0} \\ \mathbf{0} & r_r \end{bmatrix} \qquad \mathbf{L} = \begin{bmatrix} \mathbf{L}_s & \mathbf{l}_{rs} \\ \mathbf{l}_{rs}{}^T & l_{rr} \end{bmatrix} \tag{4-22}$$

Then

$$p = [i_a, i_b, i_c] \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \mathbf{i}_s{}^T \mathbf{v}_s \qquad \text{W} \tag{4-23}$$

and in a compact form,

$$\begin{bmatrix} \mathbf{v}_s \\ v_r \end{bmatrix} = -\begin{bmatrix} \mathbf{R}_s & \mathbf{0} \\ \mathbf{0} & r_r \end{bmatrix} \begin{bmatrix} \mathbf{i}_s \\ -i_r \end{bmatrix} - \frac{d}{dt} \left\{ \begin{bmatrix} \mathbf{L}_s & \mathbf{l}_{rs} \\ \mathbf{l}_{rs}{}^T & l_{rr} \end{bmatrix} \begin{bmatrix} \mathbf{i}_s \\ -i_r \end{bmatrix} \right\} \tag{4-24}$$

which can also be expressed as

$$\mathbf{v}_s = -\mathbf{R}_s\mathbf{i}_s - \frac{d}{dt}(\mathbf{L}_s\mathbf{i}_s - \mathbf{l}_{rs}i_r)$$
$$v_r = r_r i_r - \frac{d}{dt}(\mathbf{l}_{rs}{}^T\mathbf{i}_s - l_{rr}i_r) \tag{4-25}$$

1.2.4 The Blondel Transformation (BT)

Simplification of the above general generator equations is possible, using the Blondel Transformation. Define 3 new voltage and current variables, called direct (d), quadrature (q) and zero-sequence (0) components. The Blondel currents are defined as

$$\begin{aligned} i_d &\triangleq \frac{2}{3}\cos\alpha i_a + \frac{2}{3}\cos\left(\alpha - \frac{2\pi}{3}\right)i_b + \frac{2}{3}\cos\left(\alpha + \frac{2\pi}{3}\right)i_c \\ i_q &\triangleq -\frac{2}{3}\sin\alpha i_a - \frac{2}{3}\sin\left(\alpha - \frac{2\pi}{3}\right)i_b - \frac{2}{3}\sin\left(\alpha + \frac{2\pi}{3}\right)i_c \\ i_0 &\triangleq \tfrac{1}{3}i_a + \tfrac{1}{3}i_b + \tfrac{1}{3}i_c \end{aligned} \tag{4-26}$$

Or in matrix form,

$$\mathbf{i}_B \triangleq \mathbf{B}\mathbf{i}_s \tag{4-27}$$

where

$$\mathbf{i}_B \triangleq \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \tag{4-28}$$

$$\mathbf{B} \triangleq \tfrac{2}{3}\begin{bmatrix} \cos\alpha & \cos\left(\alpha - \frac{2\pi}{3}\right) & \cos\left(\alpha + \frac{2\pi}{3}\right) \\ -\sin\alpha & -\sin\left(\alpha - \frac{2\pi}{3}\right) & -\sin\left(\alpha + \frac{2\pi}{3}\right) \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \tag{4-29}$$

Equation (4.27) shows the direct BT, and the inverse BT is therefore

$$\mathbf{i}_s = \mathbf{B}^{-1}\mathbf{i}_B \tag{4-30}$$

in which it can be shown that

$$\mathbf{B}^{-1} = \begin{bmatrix} \cos\alpha & -\sin\alpha & 1 \\ \cos\left(\alpha - \frac{2\pi}{3}\right) & -\sin\left(\alpha - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\alpha + \frac{2\pi}{3}\right) & -\sin\left(\alpha + \frac{2\pi}{3}\right) & 1 \end{bmatrix} \tag{4-31}$$

We can carry out BT and inverse BT for the stator voltages as

$$\mathbf{v}_B \triangleq \mathbf{B}\mathbf{v}_s \tag{4-32}$$

$$\mathbf{v}_s = \mathbf{B}^{-1}\mathbf{v}_B \tag{4-33}$$

Upon substituting the Blondel voltages and currents into (4.25), we have

$$\begin{aligned} \mathbf{B}^{-1}\mathbf{v}_B &= -\mathbf{R}_s\mathbf{B}^{-1}\mathbf{i}_B - \frac{d}{dt}\left(\mathbf{L}_s\mathbf{B}^{-1}\mathbf{i}_B - \mathbf{l}_{rs}i_r\right) \\ v_r &= r_r i_r - \frac{d}{dt}\left(\mathbf{l}_{rs}{}^T\mathbf{B}^{-1}\mathbf{i}_B - l_{rr}i_r\right) \end{aligned} \tag{4-34}$$

and pre-multiply the LHS of (4.34) with B,

$$\begin{aligned} \mathbf{v}_B &= -\mathbf{B}\mathbf{R}_s\mathbf{B}^{-1}\mathbf{i}_B - \mathbf{B}\frac{d}{dt}\left(\mathbf{L}_s\mathbf{B}^{-1}\mathbf{i}_B - \mathbf{l}_{rs}i_r\right) \\ v_r &= r_r i_r - \frac{d}{dt}\left(\mathbf{l}_{rs}{}^T\mathbf{B}^{-1}\mathbf{i}_B - l_{rr}i_r\right) \end{aligned} \tag{4-35}$$

The last equation, when expanded, will produce the following component form:

$$v_d = -r_s i_d - L_d \frac{di_d}{dt} + L_5 \frac{di_r}{dt} + L_q i_q \frac{d\alpha}{dt}$$

$$v_q = -r_s i_q - L_q \frac{di_q}{dt} - (L_d i_d - L_5 i_r) \frac{d\alpha}{dt}$$

$$v_0 = -r_s i_0 - L_0 \frac{di_0}{dt} \tag{4-36}$$

$$v_r = r_r i_r + L_4 \frac{di_r}{dt} - \tfrac{3}{2} L_5 \frac{di_d}{dt}$$

where the following new parameters have been introduced:

$$\begin{aligned} L_d &\triangleq L_1 + L_3 + \tfrac{3}{2}L_2 \\ L_q &\triangleq L_1 + L_3 - \tfrac{3}{2}L_2 \\ L_0 &\triangleq L_1 - 2L_3 \end{aligned} \tag{4-37}$$

These new inductances are referred to as the direct-axis synchronous inductance, the quadrature-axis synchronous inductance and the zero-sequence inductance, respectively.

In the event of the rotor speed is constant, or almost constant, then

$$\frac{d\alpha}{dt} = \omega = \text{const} \tag{4-38}$$

and from (4.36), one can obtain

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \\ v_r \end{bmatrix} = -\begin{bmatrix} r_s & -\omega L_q & 0 & 0 \\ \omega L_d & r_s & 0 & \omega L_5 \\ 0 & 0 & r_s & 0 \\ 0 & 0 & 0 & r_r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ -i_r \end{bmatrix} - \begin{bmatrix} L_d & 0 & 0 & L_5 \\ 0 & L_q & 0 & 0 \\ 0 & 0 & L_0 & 0 \\ \tfrac{3}{2}L_5 & 0 & 0 & L_4 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ i_0 \\ -i_r \end{bmatrix} \tag{4-39}$$

The BT has simplified the complexity of (4.10) because

1. it has transformed a set of differential equations with time-varying coefficients to one with constant parameters, so that Laplace transform could be used;
2. the new parameter matrices contain many zeros, i.e. the matrices are sparse. Whereas the physical stator currents $\mathbf{i}_s$ are strongly coupled to each other, the Blondel currents $\mathbf{i}_B$ are only weakly coupled.

Examine the power equation after BT, we have

$$\mathbf{i}_s = \mathbf{B}^{-1}\mathbf{i}_B$$

Using equations (4.23), (4.31) and (4.33), it can be shown that

$$p = \tfrac{3}{2}(v_d i_d + v_q i_q + 2v_0 i_0) \qquad (4\text{-}43)$$

1.3 Steady-state Machine Models

From the generalized machine equations shown above, one can use them to study operation of machine at any operating conditions. We start with the steady-state.

1.3.1 No Load

With the machine on no load, stator currents are zero, power and torque are also zero, we have $\mathbf{i}_s = 0$, $\mathbf{i}_r = \mathbf{i}_r^0$ = constant, then from equation (4.25),

$$\begin{aligned} \mathbf{v}_s &= \frac{d}{dt}(\mathbf{l}_{rs} i_r^0) = \frac{d}{dt}(\mathbf{l}_{rs}) i_r^0 \\ v_r^0 &= r_r i_r^0 = \text{const} \end{aligned} \qquad (4\text{-}44)$$

Since,

$$\mathbf{l}_{rs} = \begin{bmatrix} L_5 \cos\alpha \\ L_5 \cos\left(\alpha - \dfrac{2\pi}{3}\right) \\ L_5 \cos\left(\alpha + \dfrac{2\pi}{3}\right) \end{bmatrix}$$

then as $\alpha = \omega t$, speed ω is constant, equation (4.44) becomes

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = -\omega L_5 i_r{}^0 \begin{bmatrix} \sin \omega t \\ \sin\left(\omega t - \dfrac{2\pi}{3}\right) \\ \sin\left(\omega t + \dfrac{2\pi}{3}\right) \end{bmatrix} \triangleq \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} \quad (4\text{-}45)$$

It shows the stator voltages are sinusoidal and in three-phase symmetry. These voltages are referred to as stator emf, e_a, e_b and e_c. The emf in phase a is

$$e_a = -\omega L_5 i_r{}^0 \sin \omega t = \sqrt{2}\, \mathrm{Im}\, \{E_a e^{j\omega t}\}$$

where the emf phasor E_a is defined as

$$E_a \triangleq -\frac{\omega L_5 i_r{}^0}{\sqrt{2}} \quad (4\text{-}46)$$

The magnitude of E_a is proportional to the field current magnitude $i_r{}^0$ and rotor speed ω. A 3-phase stator emf phasor representation is shown in Fig 4.9.

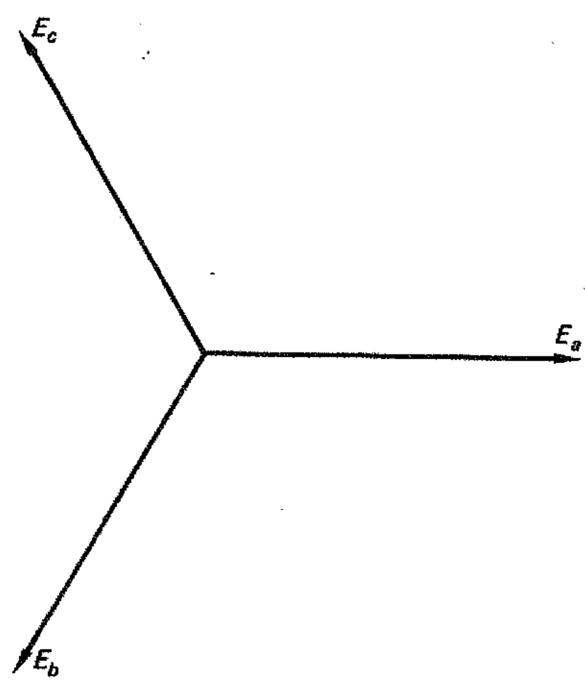


Fig. 4-9 The three stator emfs represented in phasor form. Due to the minus sign in Eq. 4-46, E_a (and also E_b and E_c) should strictly have opposite directions, but we prefer to rotate the phasors thru 180° in order to line up E_a with our normal reference direction.

1.3.2. Under Symmetrical Loading

Derivation of voltage and current relationships:

Assume the field current is constant, and therefore guarantee a set of machine emf, e_a, e_b and e_c. The emf phasor E and machine stator voltage phasor V are thus separated by a constant angle α, as shown in Fig 4.10.

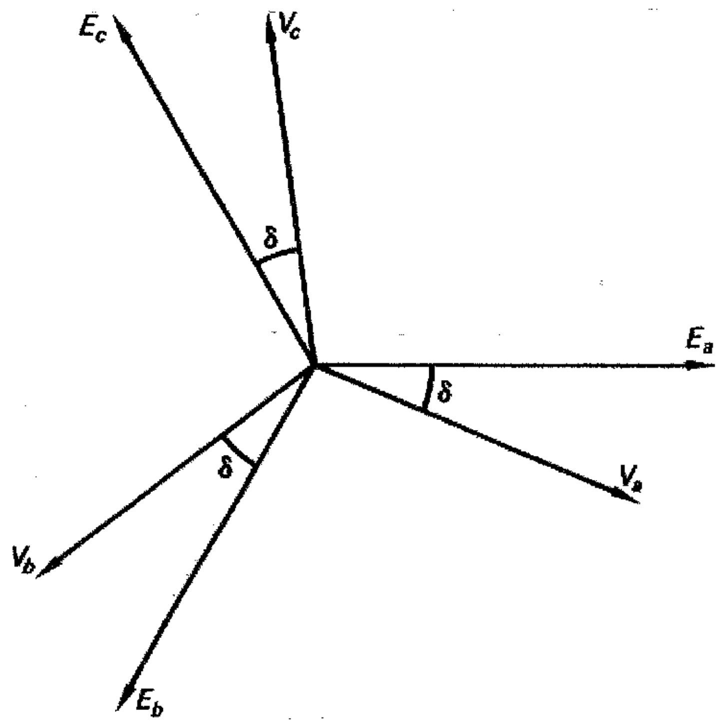


Fig. 4-10 Machine emfs and terminal voltages represented in phasor form.

Since both the emf and bus voltage are symmetrical, the same applies to the stator currents and these are

$$\begin{aligned} i_a &= \sqrt{2}\,|I|\cos(\omega t + \Psi) \\ i_b &= \sqrt{2}\,|I|\cos\left(\omega t - \frac{2\pi}{3} + \Psi\right) \\ i_c &= \sqrt{2}\,|I|\cos\left(\omega t + \frac{2\pi}{3} + \Psi\right) \end{aligned} \tag{4-47}$$

where $|I|$ is the rms value and ψ are yet to be determined. Substitute equations (4.47) into (4.26), we obtain

$$\begin{aligned} i_d &= \sqrt{2}\,|I|\cos\Psi \\ i_q &= \sqrt{2}\,|I|\sin\Psi \\ i_0 &= 0 \end{aligned} \tag{4-48}$$

The d and q axis currents are constant, while zero sequence current is zero. The machine equation (4.36) becomes

$$\begin{aligned} v_d &= -r_s i_d + \omega L_q i_q \\ v_q &= -r_s i_q + \omega L_5 i_r^0 - \omega L_d i_d \\ v_0 &= 0 \\ v_r &= r_r i_r^0 \end{aligned} \tag{4-49}$$

We can now define the direct-axis and quadrature-axis synchronous reactances as

$$\begin{aligned} X_d &\triangleq \omega L_d \\ X_q &\triangleq \omega L_q \end{aligned} \tag{4-50}$$

The first two equations of (4.49) become

$$\begin{aligned} v_d &= -r_s i_d + X_q i_q \\ v_q &= -r_s i_q - X_d i_d + \omega L_5 i_r^0 \end{aligned} \tag{4-51}$$

Hence we observe that v_d and v_q, just like i_d and i_q are constant. From the general power equation, therefore we can also conclude that power is constant.

In fact, one can make use of the inverse BT, we can obtain the stator voltages as

$$
\begin{aligned}
v_a &= v_d \cos \omega t - v_q \sin \omega t \\
&= -(r_s i_d - X_q i_q) \cos \omega t + (r_s i_q + X_d i_d - \omega L_5 i_r^0) \sin \omega t \\
v_b &= -(r_s i_d - X_q i_q) \cos (\omega t - 120^\circ) \\
&\quad + (r_s i_q + X_d i_d - \omega L_5 i_r^0) \sin (\omega t - 120^\circ) \\
v_c &= -(r_s i_d - X_q i_q) \cos (\omega t + 120^\circ) + (r_s i_q + X_d i_d \\
&\quad - \omega L_5 i_r^0) \sin (\omega t + 120^\circ)
\end{aligned} \tag{4-52}
$$

Since in generators the winding resistances would only introduce relatively small voltage drops, these can be neglected from (4.52) and the phase "a" voltage and current equations become

$$v_a \approx X_q i_q \cos \omega t + (X_d i_d - \omega L_5 i_r^0) \sin \omega t \tag{4-53}$$

$$i_a = i_d \cos \omega t - i_q \sin \omega t \tag{4-54}$$

Similar equations, with the appropriate phase shift describe phases b and c voltages and currents.

***Phasor Diagrams*:**

From the last two equations, we can write them in the complex form:

$$
\begin{aligned}
v_a &= \operatorname{Im} \{(X_d i_d - \omega L_5 i_r^0) e^{j\omega t} + j X_q i_q e^{j\omega t}\} \\
i_a &= \operatorname{Im} \{j i_d e^{j\omega t} - i_q e^{j\omega t}\}
\end{aligned} \tag{4-55}
$$

where

$$
\begin{aligned}
\sin \omega t &= \operatorname{Im} \{e^{j\omega t}\} \\
\cos \omega t &= \operatorname{Im} \{j e^{j\omega t}\}
\end{aligned}
$$

Introduce the phasors

$$
\begin{aligned}
I_q &\triangleq -\frac{i_q}{\sqrt{2}} \\
I_d &\triangleq j \frac{i_d}{\sqrt{2}}
\end{aligned} \tag{4-56}
$$

From equation (4.55), we have

$$v_a = \sqrt{2}\,\mathrm{Im}\left\{\left(-jX_dI_d - \frac{\omega L_5 i_r^{\,0}}{\sqrt{2}} - jX_qI_q\right)e^{j\omega t}\right\} \qquad (4\text{-}57)$$
$$i_a = \sqrt{2}\,\mathrm{Im}\,\{(I_d + I_q)e^{j\omega t}\}$$

which means that we can represent v_a and i_a by the phasors

$$V_a \triangleq E_a - jX_dI_d - jX_qI_q \qquad (4\text{-}58)$$
$$I_a \triangleq I_d + I_q$$

where we have made use of (4.46) to identify the emf E_a.

The phasor diagram (Fig 4.11) shows the relationship of the voltage and current of phase "a" of a symmetrically loaded generator.

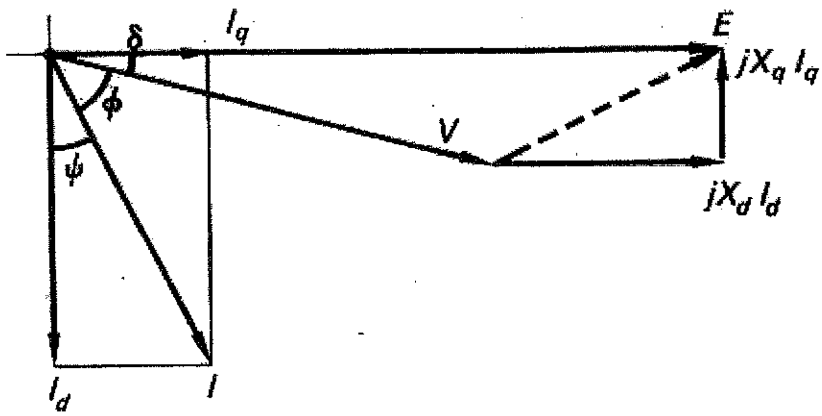


Fig. 4-11 Phasor diagram of symmetrically operated machine. Only phase a is shown. (Compare note in Fig. 4-9 concerning phasor directions.)

Note:

1. The power angle δ has been identified. It shall be defined as positive when E leads V,

2. the phase angle ψ is shown. There is another angle Φ between the stator voltage and current. Φ is defined positive when V leads I. It is called the power factor angle.

Power relations:

We shall try to derive the power equations from the phasor diagram Fig 4.11. We have

$$S_G = P_G + jQ_G = |V|\,|I| \cos\phi + j\,|V|\,|I| \sin\phi$$

where

$$\begin{aligned} P_G &= |V|\,|I| \cos\phi \quad \text{MW/phase} \\ Q_G &= |V|\,|I| \sin\phi \quad \text{Mvar/phase} \end{aligned} \tag{4-59}$$

Positive P_G and Q_G are for generator action. Also, from Fig 4.11, project along and perpendicular to E, we get

$$\begin{aligned} |E| - |I_d|\,X_d &= |V| \cos\delta \\ |I_q|\,X_q &= |V| \sin\delta \end{aligned} \tag{4-60}$$

Since

$$\begin{aligned} |I_q| &= |I| \sin\Psi \\ |I_d| &= |I| \cos\Psi \end{aligned} \tag{4-61}$$

and

$$\phi + \delta + \Psi = 90^\circ \tag{4-62}$$

from which one can obtain

$$|I| \cos\phi = |I| \sin\Psi \cos\delta + |I| \cos\Psi \sin\delta \tag{4-63}$$

and with equation (4.61), the last equation becomes

$$|I| \cos\phi = |I_q| \cos\delta + |I_d| \sin\delta \tag{4-64}$$

Then substitute equations (4.64) and (4.60) into (4.59), we obtain

$$P_G = \frac{|V|\,|E|}{X_d} \sin\delta + \frac{|V|^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta \tag{4-65}$$

In practice, if one deals with large system so that V is fairly constant and by keeping the excitation constant, $|E|$ is also constant. P_G is a function of the power angle δ.

For cylindrical pole (round-rotor) machine, $X_d = X_q$ and

$$P_G = \frac{|V|\,|E|}{X_d} \sin \delta \tag{4-66}$$

The corresponding reactive power equation can be derived similarly and is

$$Q_G = \frac{|E|\,|V|}{X_d} \cos \delta - \frac{|V|^2}{X_d} \tag{4-67}$$

Generator Real Power P_G:

Assuming round-rotor machine, and if the machine is connected to a relatively large system, $|V|$ is constant. By keeping the excitation constant, $|E|$ is also constant. Then

$$P_G = P_{\max} \sin \delta \tag{4-68}$$

where

$$P_{\max} \triangleq \frac{|E|\,|V|}{X_d} \tag{4-69}$$

Fig 4.12 shows the relationship between P_G and δ. Pullout power is the power level when the machine steps out of synchronization, i.e.

$$P_{po} = \pm P_{max} = \pm \frac{|E|\,|V|}{X_d} \qquad \text{MW} \qquad (4\text{-}70)$$

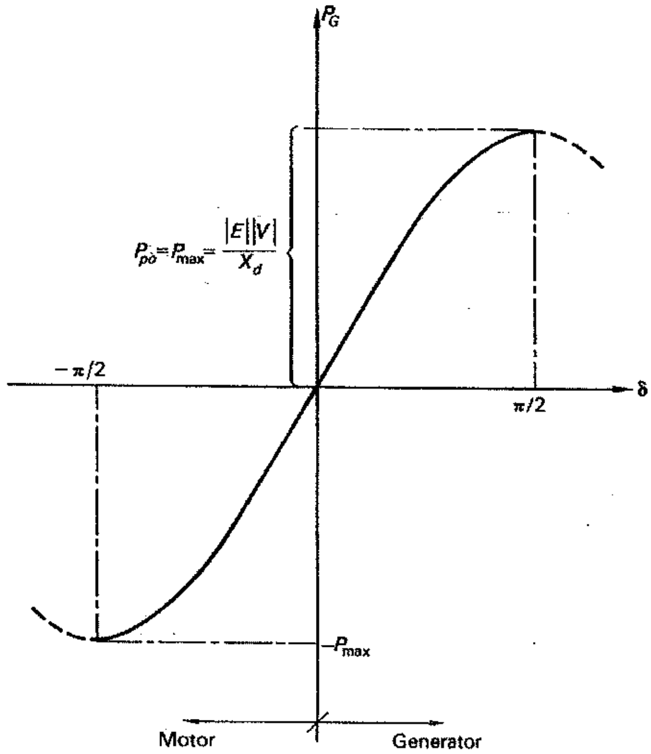


Fig. 4-12 Real generated power versus power angle for nonsalient machine.

The stiffness of the machine is defined as

$$\frac{dP_G}{d\delta} = \frac{|E|\,|V|}{X_d} \cos\delta \qquad \text{MW/rad} \qquad (4\text{-}71)$$

One can increase the stiffness by increasing the excitation or by making X_d smaller.

Under steady-state condition, the speed is constant. The mechanical torque emanating from the prime mover τ_m must equal the electro-dynamic torque τ_e. Assume no loss in the machine, then the output power P_G must be related to the torques as

$$\tau_e = \tau_m = \frac{P_G}{\omega_{mech}} \qquad \text{MN} \cdot \text{m} \qquad (4\text{-}72)$$

So by adjusting τ_m, the rotor will accelerate/retard so that E becomes more leading/lagging with respect to V. Hence the power angle will increase/decrease, until a new steady-state is re-established, to exactly produce the new P_G.

If the torque is increased beyond P_{max}/ω_{mech}, a further increase in τ_m will not produce a corresponding increase in P_G. In fact, P_G will decrease, which makes δ even larger, and in the process, it leads to the machine speed accelerating and no chance for it to return to the synchronous speed. This is called the loss of synchronism or skip pole. This is most unacceptable.

Negative δ means motoring action, and too large a negative motor-torque can also lead to de-synchronization.

The reactive power Q_G:

Again assume a round-rotor machine, from equation (4.67), if

$$|E| \cos \delta > |V| \tag{4-73}$$

then Q_G is positive and the generator produces reactive power. That is, the machine acts like a capacitor from the network point of view. Generally speaking, high magnitude E is needed to satisfy the inequality of equation (4.73). We refer to this case as an over-excited generator.

Conversely, an under-excited machine absorbs or consumes reactive power. When this happens,

$$|E| \cos \delta < |V| \tag{4-74}$$

Thus by adjusting the excitation level, one can continuously modify Q_G.

Equivalent diagram representation of alternator:

Again for a round-rotor machine, Fig 4.11 becomes Fig 4.13, in which one can consider the voltage drop across the reactance X_d caused by the current I in accordance of the phasor equation

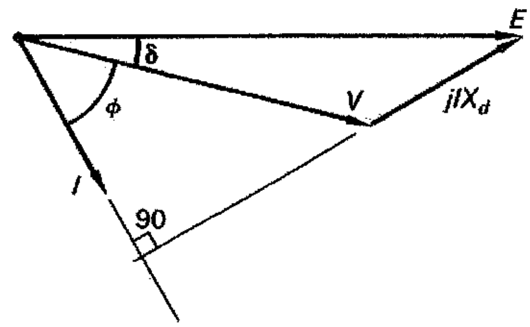


Fig. 4-13 Phasor diagram for nonsalient synchronous machine. Resistive voltage drop neglected.

$$V = E - jIX_d \qquad (4\text{-}76)$$

The equivalent circuit diagram is Fig 4.14.

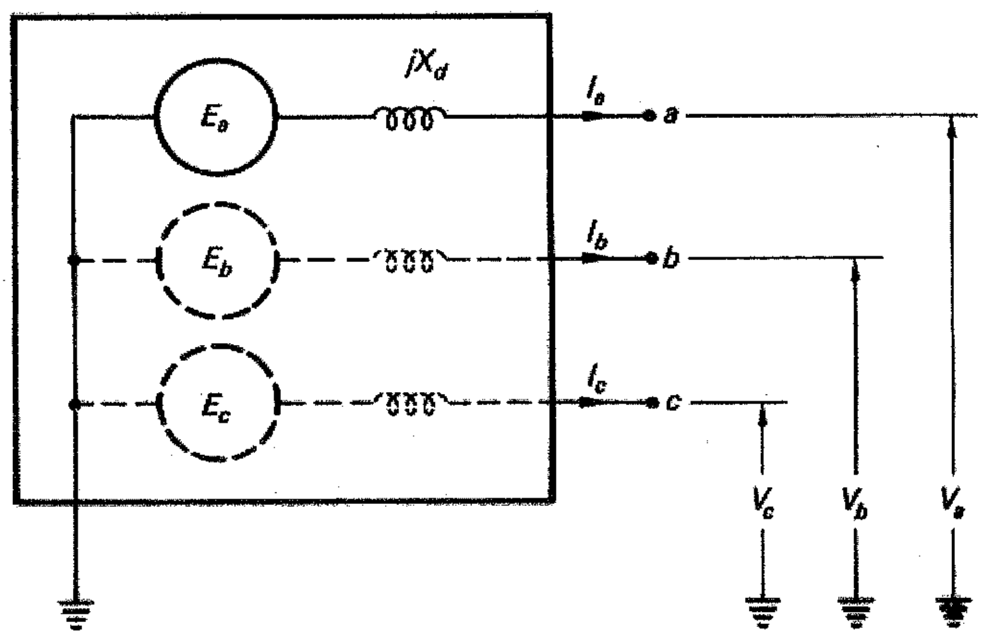


Fig. 4-14 Equivalent network representation of symmetrically operated machine. Nonsaliency assumed.

In vector form, the 3-phase voltage and current become

$$\mathbf{V} = \mathbf{E} - \mathbf{Z}_i\mathbf{I} \qquad (4\text{-}77)$$

where

$$\mathbf{Z}_i \triangleq \begin{bmatrix} jX_d & 0 & 0 \\ 0 & jX_d & 0 \\ 0 & 0 & jX_d \end{bmatrix} \tag{4-78}$$

and the vectors are

$$\mathbf{V} \triangleq \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad \mathbf{E} \triangleq \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} \qquad \mathbf{I} \triangleq \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

For the balanced case, therefore, we note that $\mathbf{Z}_i$ is diagonal and there is zero cross-coupling between phases.

Fig 4.15 shows the 4 possible operational cases of a synchronous machine.

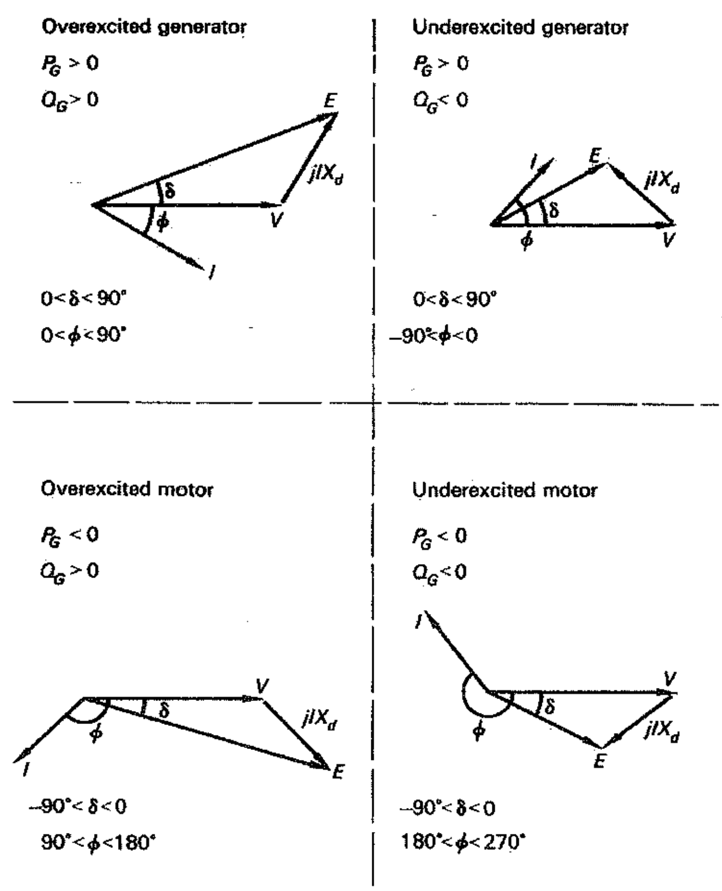


Fig. 4-15 The four possible operating cases of a synchronous machine.

WORKED EXAMPLES

4-2. A generator having a synchronous reactance of 0.90 pu based on its own rating is connected via a transmission line to a remote bus, the voltage of which we may assume is kept constant at a value of 100 percent. The line has an impedance of $j0.15$ pu per phase, based upon the machine megavoltamperes. The internal emf E of the machine is kept at a constant magnitude of 135 percent.

(*a*) Compute the pullout power of the machine if operated as a generator.

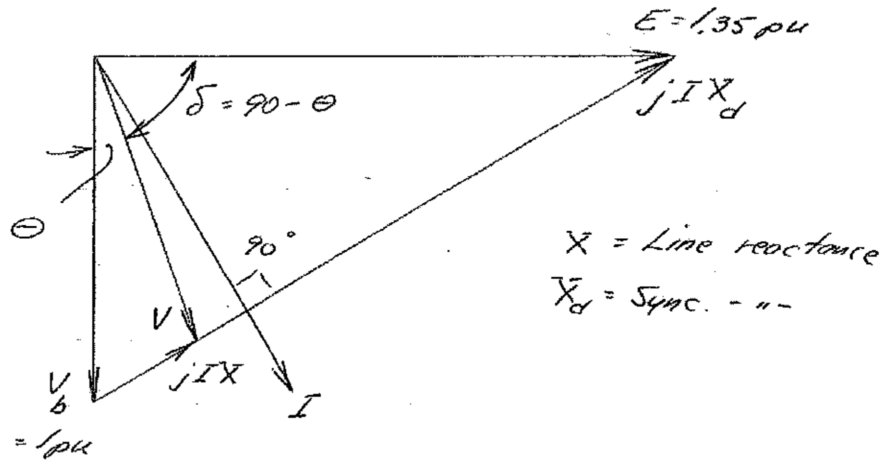
(*b*) At the moment of pullout, determine magnitude and direction of the reactive power at both the generator terminal and the remote bus.

(*c*) At the moment of pullout, what is the terminal voltage?

(*d*) Is the machine current overloaded at pullout?

Solution:

If V_b us the bus voltage, V the generator terminal voltage and I the current we have the following phasor diagram at the point of pull-out.



Since the line is loss-free the generator power will equal the power delivered to the bus, i.e.

$$P_G = P_b$$

$$\therefore \quad \frac{|E||V|}{X_d} \sin (90 - \Theta) = \frac{|V||V_b|}{X} \sin \Theta$$

Since $\sin (90 - \Theta) = \cos \Theta$ we thus get:

$$\tan \Theta = \frac{|E|}{|V_b|} \cdot \frac{X}{X_d} = \frac{1.35}{1.00} \cdot \frac{0.15}{0.90} = 0..225$$

$\therefore \quad \Theta = 12.68°$ $\qquad\qquad 90 - \Theta = 77.32°$

We also have for the current:

$$1.05\ |I| = \sqrt{1.00^2 + 1.35^2} \qquad \text{(see fig.)}$$

$\therefore \quad |I| = 1.60$ pu

For the generator voltage we have:

$$(|I| \cdot 0.90)^2 = 1.35^2 + |V|^2 - 2 \cdot 1.35 \cdot |V| \cos 77.32$$

$\therefore \quad |V| = 0.879$ pu

We now summarize:

a) Pullout power:

$$P_{po} = \frac{1.35 \times 1.00}{1.05} = 1.287 \text{ pu}$$

b) $Q_G = \dfrac{|E|\,|V|}{X_d} \cos \delta - \dfrac{|V|^2}{X_d} = -0.57$ pu

(Generator behaves like shunt reactor)

Reactive power at bus (defined positive <u>from</u> bus)

$$Q_b = \frac{|V_b|^2}{0.15} - \frac{|V|\,|V_b|}{0.15} \cos \Theta = 0.95 \text{ pu}$$

c) Generator voltage = 0.879 pu

d) $|I| = 1.60$ pu (overloaded!)

4-4. A synchronous machine is running overexcited with $|E| = 150$ percent. The synchronous reactance has the value 120 percent, and the machine delivers a real power of 0.4 pu. Terminal voltage is 1 pu.

If the prime mover torque is increased by 1 percent, with how many percents will P_G and Q_G change?

Solution:

For the real power we have

$$0.4 = \frac{1.5 \times 1.0}{1.20} \sin \delta$$

$$\sin \delta = 0.320$$

$$\delta = 18.7^{\circ}$$

a) Since torque ~ power,

1% change in torque will correspond to
1% change (= 0.004 pu) in power

b) $$Q_G = \frac{|E||V|}{X_d} \cos \delta - \frac{|V|^2}{X_d} \tag{1}$$

$$\therefore \frac{dQ_G}{d\delta} = - \frac{|E||V|}{X_d} \sin \delta \tag{2}$$

We also have

$$\frac{dP_G}{d\delta} = \frac{|E||V|}{X_d} \cos \delta \tag{3}$$

By dividing (2) by (3) we get

$$\frac{dQ_G}{dP_G} = - \tan \delta$$

$$\tan \delta = \tan 18.7^{\circ} = 0.339$$

$$dP_G = 1\%$$

$$\therefore dQ_G = - 0.339 \times 1\% = -0.339\%$$

4-6. A synchronous machine has been synchronized onto a big network. All three conditions for a smooth "lock-in" were satisfied. Without changing the field current, we now make the machine generate real power. Will it, at the same time, generate or consume reactive power? Explain.

Solution:

For perfect synchronization we have

$$|E| = |V|$$

We therefore have

$$Q_G = \frac{|E||V|}{X_d} \cos \delta - \frac{|V|^2}{X_d} = \frac{|V|^2}{X_d} \cos \delta - \frac{|V|^2}{X_d} = \frac{|V|^2}{X_d} (\cos \delta - 1)$$

If we now make $P_G \neq 0$,

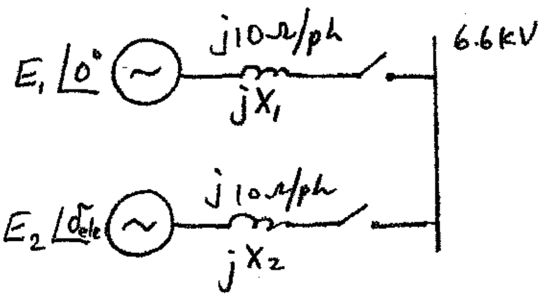
then $\delta \neq 0$

and $\cos \delta < 1$

Therefore $Q_G < 0$, and the generator <u>absorbs</u> reactive power.

3.1 When two four pole, 50 Hz synchronous generators are paralleled their phase displacement is 2° mechanical. The synchronous reactance of each machine is 10 Ω/phase and the common busbar voltage is 6.6 kV. Calculate the synchronizing torque.

Solution:



We assume that prior to the paralleling, the generator emf are adjusted such that they have the same voltage level as the external bus, i.e. $|E_1| = |E_2| = 6.6\ \text{kV}/\sqrt{3} = 3810$ V. The angle of E_1 is 0° while that of E_2 is therefore 2° mechanical ahead of E_1.

Since $\delta_{ele} = p\delta_{mech}/2 = 2 \times 2^\circ = 4^\circ$ where p = 4 poles.

Once the switches are closed suddenly, there would be an instantaneous circulating power P (3 phase) from generator 2 to generator 1, as E_2 leads E_1 by 4°. We assume negligible amount of the power is "leaked" out to the external system, because the external impedance could be much larger.

$P = 3 \times (E_1 \times E_2) \sin \delta_{ele} / (X_1 + X_2) = 3\text{x}(3810)^2 \sin 4^\circ / (20) = 151929$ W

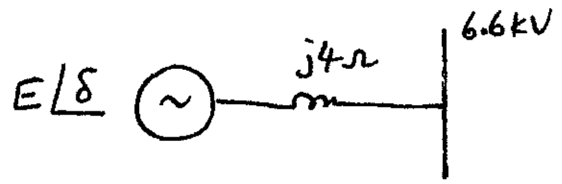
But we have

$f = np/120$, so n = 1500 rpm, or $\omega = 50\pi$ rad/sec.

Therefore, synchronizing torque = P/ω = 967 Nm.

3.3 A 6.6 kV synchronous generator has negligible resistance and synchronous reactance of 4 Ω/phase. It is connected to an infinite busbar and gives 2000 A at unity power factor. If the excitation is increased by 25 per cent find the maximum power output and the corresponding power factor. State any assumptions made.

Solution:



At unity power factor,

$I = 2000 \angle 0°$ where we use the terminal voltage as reference, i.e. $V = (6.6 \text{ kV} / \sqrt{3}) \angle 0°$

or $3810 \angle 0°$ V (terminal voltage phasor).

Therefore, before the excitation is increased,

$|E| \angle \delta = 3810 \angle 0° + j4 \times 2000 \angle 0° = 8861 \angle 64.5°$ Volts

With 25% increase in excitation, we have a new excitation E' where

$E' = 1.25 |E| \angle \delta' = 11076 \angle \delta'$ volts

Note the angle of the emf can change.

Maximum power output occurs when $\delta' = 90°$, i.e.

$P_{max} = 3\, E' \times V / X_d = 3 \times 3810 \times 11076/4 = 31.6$ MW (3 phase)

And the corresponding stator current

$I' = (E' - V)/ X_d = 11076 \angle 90° - 3810 \angle 0° /(j4) = 2928 \angle 19°$.

Hence power factor at generator terminal is $\cos 19° = 0.95$ (lead).

3.5 A salient-pole, 75 MVA, 11 kV synchronous generator is connected to an infinite busbar through a link of reactance 0.3 p.u. and has $X_d = 1.5$ p.u. and $X_q = 1$ p.u., and negligible resistance. Determine the power output for a load angle 30° if the excitation e.m.f. is 1.4 times the rated terminal voltage. Calculate the synchronizing coefficient in this operating condition. All p.u. values are on a 75 MVA base.

Solution:

We have $X_d = 1.5$ p.u., $X_q = 1$ p.u., and the link reactance $X_l = 0.3$ p.u.

From lecture note, the power equation of a salient pole machine is

$$P_G = \frac{|V||E|}{\tilde{X}_d} \sin\delta + \frac{|V|^2}{2}\left(\frac{1}{\tilde{X}_q} - \frac{1}{\tilde{X}_d}\right) \sin 2\delta \qquad (4\text{-}65)$$

Using the generator rated voltage (11kV) and 75 MVA as base values, therefore, we have $|E| = 1.4$ p.u., $|V| = 1$ p.u., and $\delta = 30°$. Substituting the numerical values into (4.65), thus the output power is

$P_G = 0.7777 \sin\delta + 0.1068 \sin 2\delta = 0.48$ p.u.

And the synchronizing coefficient is

$dP/d\delta = 0.777 \cos\delta + 0.2136 \cos 2\delta = 0.78$ p.u./ °.

$\tilde{X}_d = X_d + X_l$

$\tilde{X}_q = X_q + X_l$

2. Synchronous Machine – under a Balanced Short Circuit

2.1. Analysis – no winding resistances

Materials of this Section are taken from Elgerd Section 10.4. One could also refer to Section 3.3 of Weedy.

Consider firstly the machine is on open-circuit, at constant speed and excited with constant excitation. A symmetrical 3-phase short circuit occurs at the machine terminals at t = 0. If winding resistances are ignored, (to simplify analysis), the terminal voltage is equal to the induced emf of the machine, i.e. from equation (4.46),

$$|V_a| = |V_b| = |V_c| = |E| = \frac{\omega L_5 i_r^0}{\sqrt{2}} \qquad (10\text{-}15)$$

With the short circuit, for t>0, the stator voltage collapses to zero and therefore,

$$v_a = v_b = v_c = 0 \qquad (10\text{-}16)$$

Suppose we also short-circuit the rotor circuit, i.e. set $v_r = 0$ for $t > 0$. The reason is because we assume the rotor resistance is zero, so without making $v_r = 0$, we will have an infinite rotor current. So with these assumptions, equation (4.36) from earlier chapter on synchronous machines becomes

$$\begin{aligned}
0 &= -L_d \frac{di_d}{dt} + L_5 \frac{di_r}{dt} + L_q \omega i_q \\
0 &= -L_q \frac{di_q}{dt} + (L_5 i_r - L_d i_d)\omega \\
0 &= -L_0 \frac{di_0}{dt} \\
0 &= L_4 \frac{di_r}{dt} - \tfrac{3}{2} L_5 \frac{di_d}{dt}
\end{aligned} \qquad (10\text{-}18)$$

The equations are now sufficient for us to find the solution of the unknown currents i_d, i_q, i_0 and i_r. The initial conditions of the currents are

$$i_d(0) = i_q(0) = i_0(0) = 0 \tag{10-19}$$

The initial rotor current is i_r^0. From the 3rd equation of (10.18), upon integration, we have

$$i_0(t) = \text{const} = i_0(0) = 0 \tag{10-20}$$

***Transient inductance and reactance*:**

Laplace transform the remaining (10.18), use capital letters to represent the Laplace-transformed currents,

$$\begin{aligned} 0 &= -L_d sI_d + L_5(sI_r - i_r^0) + \omega L_q I_q \\ 0 &= -L_q sI_q + (L_5 I_r - L_d I_d)\omega \\ 0 &= L_4(sI_r - i_r^0) - \tfrac{3}{2}L_5 sI_d \end{aligned} \tag{10-21}$$

Solving for I_d, we have

$$I_d = \frac{\omega^2 L_5 i_r^0}{L_d'} \frac{1}{s(s^2 + \omega^2)} \tag{10-22}$$

where we have, for brevity, introduced a new inductance

$$L_d' \triangleq L_d - \frac{3}{2}\frac{L_5^2}{L_4} \tag{10-23}$$

***L_d'* is referred to as the d-axis transient inductance. Laplace transform (10.22) produces**

$$i_d(t) = \frac{L_5 i_r^0}{L_d'}(1 - \cos \omega t) \tag{10-24}$$

or in view of equation (10.15),

$$i_d(t) = \sqrt{2}\frac{|E|}{\omega L_d'}(1 - \cos \omega t) = \sqrt{2}\frac{|E|}{X_d'}(1 - \cos \omega t) \tag{10-25}$$

The new parameter

$$X_d' \triangleq \omega L_d' \tag{10-26}$$

is referred to as the d-axis transient reactance.

Similarly, one can solve for i_q and i_r to give

$$i_q(t) = \sqrt{2}\,\frac{|E|}{X_q}\sin\omega t \tag{10-27}$$

$$i_r(t) = i_r^{\,0} + \frac{3}{2}\frac{L_5}{L_4}\frac{|E|}{X'_d}(1 - \cos\omega t) \tag{10-28}$$

General stator current expressions

Actual stator currents are obtained directly using the inversed transform equation (4.30), i.e. for phase a,

$$i_a = i_d\cos\alpha - i_q\sin\alpha \tag{10-29}$$

and the rotor angle is

$$\alpha = \omega t + \alpha_0 \tag{10-30}$$

where α_0 is the initial rotor angle at time of the fault initiation. Upon substituting equations (10.25), (10.27) and (10.30) into (10.29), we obtain

$$i_a = \sqrt{2}\,\frac{|E|}{X'_d}\cos(\omega t + \alpha_0) - \frac{|E|}{\sqrt{2}}\left(\frac{1}{X'_d} + \frac{1}{X_q}\right)\cos\alpha_0 - \frac{|E|}{\sqrt{2}}\left(\frac{1}{X'_d} - \frac{1}{X_q}\right)\cos(2\omega t + \alpha_0) \tag{10-31}$$

Similar expressions can be obtained for the other two phases.

Note:

1. stator current has 3 components: a fundamental frequency component, a dc component and a double-frequency component;

2. magnitude of the dc component varies with α_0. It is as large as the fundamental component, if the short-circuit is initiated at $\alpha_0 = 0$;

3. double-frequency component is usually quite small. Its presence can often be neglected;

4. since resistances are ignored, none of the 3 components contains any decaying factors;

5. the rms value of the fundamental-frequency component equals $|E|/X_d'$. In steady-state, the short-circuit current equals to the rms value of $|E|/X_d$. Since $X_d > X_d'$, therefore, the transient current component at frequency ω is of higher magnitude than the steady-state value. Often it is of 3-10 times higher.
6. Field current will contain a large fundamental frequency component post-fault. With resistance in the circuit, this will decay and eventually die out. It is because of the presence of the induced rotor current that results in the temporary reductions of the inductance as felt on the stator side.

2.2. Effects of Winding Resistances and Damper Windings

Adding the winding resistances introduces certain mathematical complications. Instead of devoting time to analyst this effect, it suffices to point out what is typically happening in a synchronous machine with stator resistances are included:
1. the magnitude of the 3 stator currents as measured immediately following the short circuit will be approximately equal to that given by equation (10.31);
2. the fundamental frequency component will decay exponentially from its initial amplitude of rms = $|E|/X_d'$ to its steady-state rms $|E|/X_d$, with the time constant

$$T_d' \approx \frac{X_d'}{X_d} T_r \quad \text{s} \qquad (10\text{-}32)$$

where T_r is the time constant of the rotor winding, i.e.

$$T_r = \frac{L_4}{r_r} \qquad (10\text{-}33)$$

The transient time constant T_d' is typically 1-2 s for large generator. Hence steady-state value is reached in some 5-10 s.
3. The dc component dies out in 8-10 cycles;
4. With damper windings, the induced currents in the damper bars will also reduce the inductance seen from the stator side, just like the rotor winding transient current effects. This significant effect is seen only over a short interval, typically some 2 cycles of the fundamental frequency. It is customary to define a sub-

transient reactance value X_d'', which can then be used to describe the behavior of the rms value of the stator current in the initial 2 cycles or so. This is computed as $|E|/X_d$''.

Fig 10.11 shows a typical oscillogram of the stator short-circuit current in one phase of a synchronous generator.

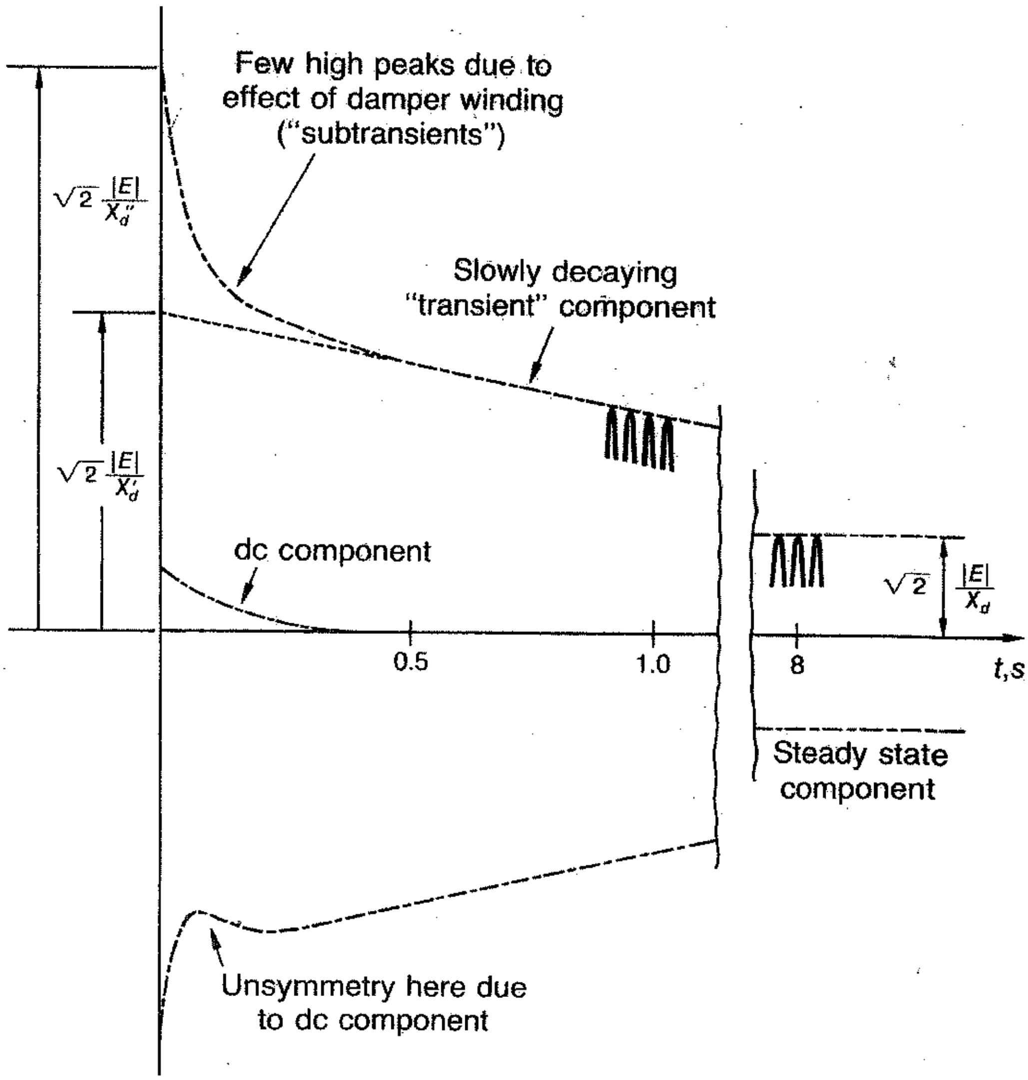


Fig. 10-11 Transient short-circuit current of synchronous machine. Only the envelope of one phase shown.

3. POWER TRANSFORMERS

Materials of this chapter are taken from Chapter 5 of Elgerd.

3.1. Equivalent Circuits of YY-connected Two-winding Transformers

Consider Fig 5.1, single phase of the transformer. We have the voltage equations

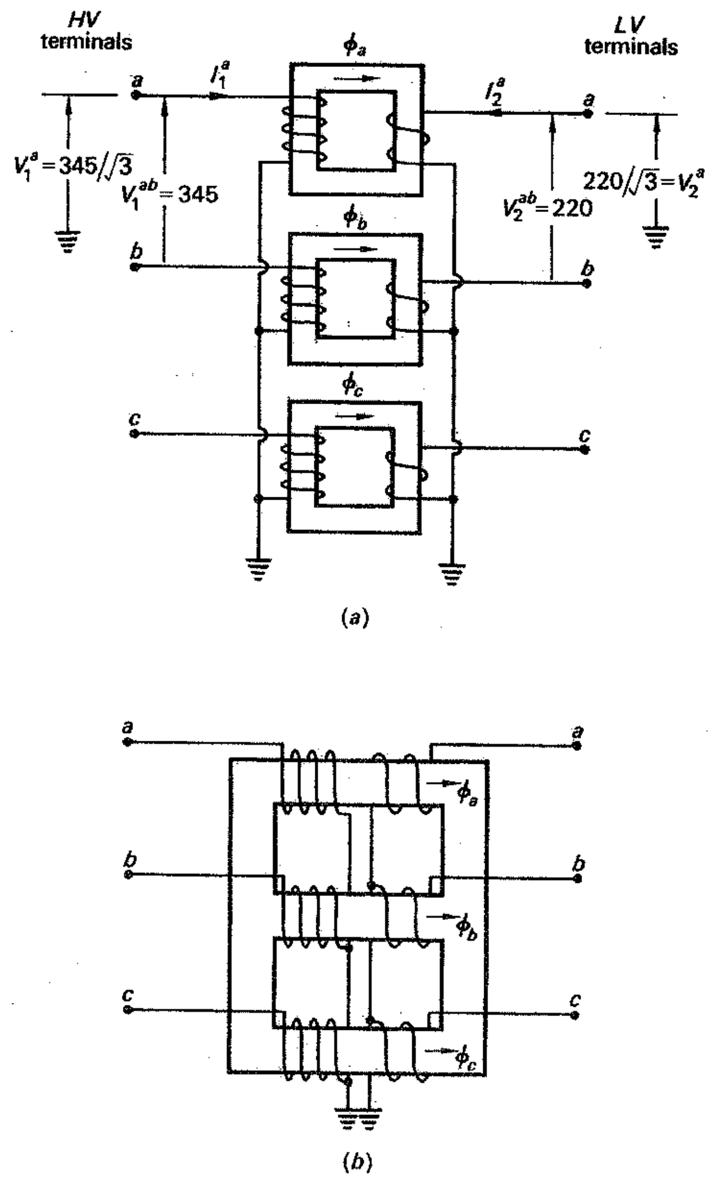


Fig. 5-1 Three-phase transformation. (*a*) Bank arrangement;
(5.1): (*b*) core arrangement.

$$V_1 = I_1R_1 + I_1j\omega L_1 + I_2j\omega M_{12}$$
$$V_2 = I_2R_2 + I_2j\omega L_2 + I_1j\omega M_{12} \quad (5\text{-}1)$$

where R_i is the respective winding resistance, L_i is the self inductance and M_{ij} is the mutual inductance between winding i and j.

The circuit diagram on Fig 5.4a shows the ideal transformer in which V_1 applied to the input will produce an output voltage of V_1/a, input current I_1 produces an output current aI_1.

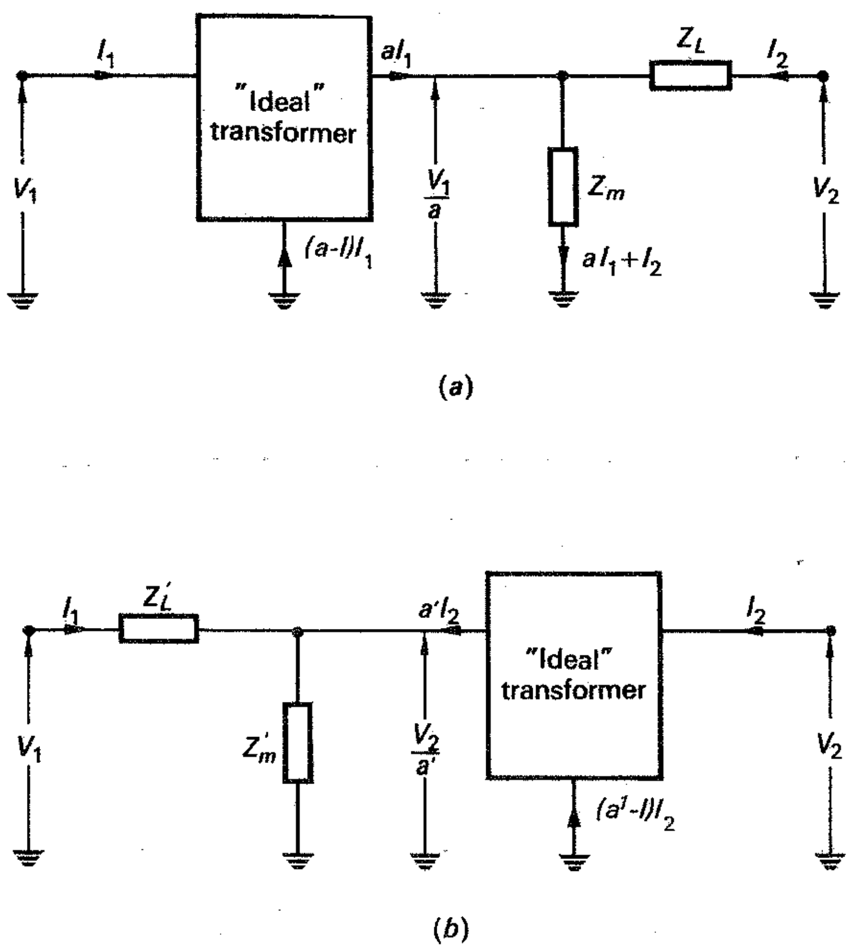


Fig. 5-4 Equivalent transformer circuit. (*a*) Leakage and magnetizing impedances referred to secondary side. (*b*) Impedances now referred to primary side.

Impedances Z_m and Z_L are referred to as the magnetizing and leakage impedances respectively. We can write

$$\frac{V_1}{a} = Z_m(aI_1 + I_2)$$
$$V_2 = Z_m(aI_1 + I_2) + Z_L I_2 \qquad (5\text{-}2)$$

Comparing (5.1) and (5.2), the two equations are identical if

$$a \triangleq \frac{R_1 + j\omega L_1}{j\omega M_{12}}$$
$$Z_m = -\frac{\omega^2 M_{12}^2}{R_1 + j\omega L_1} \qquad (5\text{-}3)$$
$$Z_L = R_2 + j\omega L_2 + \frac{\omega^2 M_{12}^2}{R_1 + j\omega L_1}$$

3.2. Approximate Equivalent Circuits for YY-connected Transformers

From magnetic circuit analysis, we can show that

$$L_1 = b\mu N_1^2$$
$$L_2 = b\mu N_2^2 \qquad (5\text{-}4)$$
$$M_{12} = k\sqrt{L_1 L_2} = k\mu b N_1 N_2$$

where

N_1 = primary-winding turns
N_2 = secondary-winding turns
μ = permeability for the core
b = constant, depending upon core geometry
k = leakage coefficient

The permeability is very high for iron core, all inductances are very large and since by design resistances are small, we have

$$\omega L_1 \gg R_1$$
$$\omega L_2 \gg R_2 \qquad (5\text{-}5)$$

With these inequalities, from equation (5.3), we obtain

$$a \approx \frac{L_1}{M_{12}} = \frac{N_1}{kN_2}$$

$$Z_m \approx j\omega \frac{{M_{12}}^2}{L_1} = j\omega k^2 L_2 \triangleq jX_m \qquad (5\text{-}6)$$

$$Z_L = \frac{(R_1 + j\omega L_1)(R_2 + j\omega L_2) + \omega^2 {M_{12}}^2}{R_1 + j\omega L_1} \approx R_2 + \frac{R_1}{a^2} + j\omega L_2 (1\text{-}k^2) \triangleq R_L + jX_L$$

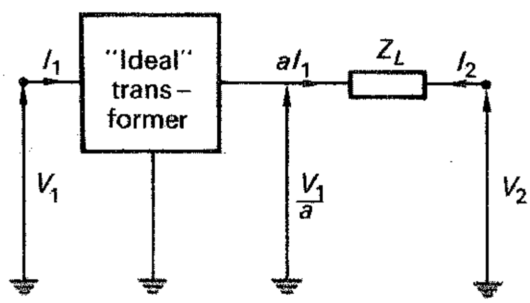
For normal transformers the leakage reactance is essentially reactive. Since *k* is close to unity, we have

$$R_L \ll X_L \lll |X_m| \qquad (5\text{-}7)$$

and

$$a \approx \frac{N_1}{N_2} = \text{phase voltage ratio} \qquad (5\text{-}8)$$

In system studies, in view of equation (5.7), we can set $X_m = \infty$ and the equivalent circuit Fig 5.5 is obtained. From it, we conclude that



From Fig 5.5,
$I_2 \approx -a I_1$
$= -\frac{N_1}{N_2} I_1$
$\Rightarrow N_2 I_2 = -I_1 N_1$

Fig. 5-5 Approximate equivalent transformer circuit.

$$N_1 I_1 + N_2 I_2 \approx 0 \qquad (5\text{-}9)$$

And in the event of the transformer at no load, I_1=I_2 =0, there is no voltage drop across Z_L, we then have

$$\frac{V_1}{V_2} = a \approx \frac{N_1}{N_2} \qquad (5\text{-}10)$$

The parameter *a* can therefore be obtained when the transformer is under no load condition. On per unit of rated voltage bases, we arrive at Fig 5.6 in which the ideal transformer no longer needs to be included.

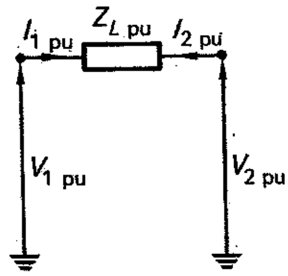


Fig. 5-6 Equivalent circuit for YY-connected transformer. Voltages and impedance expressed in per units.

3.3. Equivalent Circuit for Δ-connected Transformers

If one of the windings is delta-connected, one must take into account the ±30° phase shift. This can be accomplished most conveniently by letting the parameter *a* in the ideal transformer be represented with a complex value

$$a \triangleq |a|\ e^{\pm j30^\circ} \tag{5-11}$$

as shown in Fig 5.7.

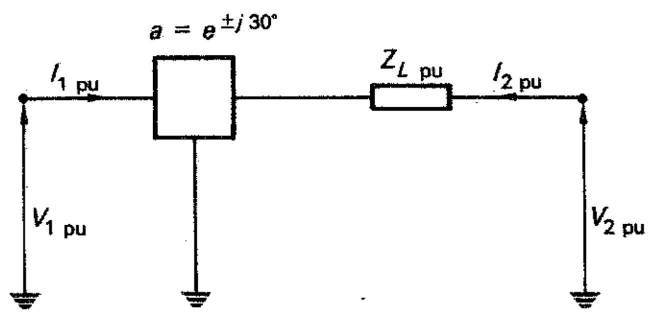


Fig. 5-7 Equivalent circuit for YΔ-connected transformer. All variables and parameters in pu.

3.4. π -Equivalent

From Fig 5.9,

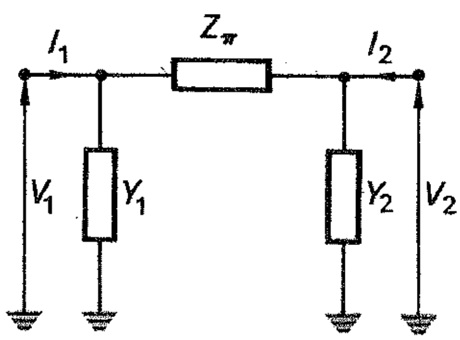


Fig. 5-9 π equivalent of transformer.

we have:

$$I_1 = \frac{V_1 - V_2}{Z_\pi} + V_1 Y_1 = \left(\frac{1}{Z_\pi} + Y_1\right) V_1 - \frac{1}{Z_\pi} V_2$$
$$I_2 = -\frac{V_1 - V_2}{Z_\pi} + V_2 Y_2 = -\frac{1}{Z_\pi} V_1 + \left(\frac{1}{Z_\pi} + Y_2\right) V_2 \tag{5-17}$$

From Fig 5.7,

$$aI_1 = \frac{V_1/a - V_2}{Z_L}$$
$$I_2 = -aI_1 = -\frac{V_1/a - V_2}{Z_L} \tag{5-18}$$

The latter equation can be written as

$$I_1 = \frac{1}{a^2 Z_L} V_1 - \frac{1}{a Z_L} V_2$$
$$I_2 = -\frac{1}{a Z_L} V_1 + \frac{1}{Z_L} V_2 \tag{5-19}$$

Clearly, equations (5.17) and (5.19) are identical if we set

$$\frac{1}{Z_\pi} + Y_1 = \frac{1}{a^2 Z_L}$$
$$\frac{1}{Z_\pi} = \frac{1}{a Z_L} \tag{5-20}$$
$$\frac{1}{Z_\pi} + Y_2 = \frac{1}{Z_L}$$

So for the π equivalent circuit, we have

$$Z_\pi = a Z_L$$
$$Y_1 = \frac{1}{Z_L} \frac{1 - a}{a^2} \tag{5-21}$$
$$Y_2 = \frac{1}{Z_L} \frac{a - 1}{a} = -a Y_1$$

In the foregoing equations, the turns-ratio "*a*" could be complex, as would be the case when one winding is delta-connected.

3.5. Equivalent Circuits for Three-winding Transformers

This is best illustrated through an example. Consider Fig 5.13 for a YYD transformer. The 3 windings are represented by the 3 leakage reactances as shown in Fig 5.13b. Then through a delta-star transformation, one obtains the star equivalent of Fig 5.13c. Note the appearance of the 30° phase shift. The last equivalent circuit is what we could use as the transformer model in power flows and system studies.

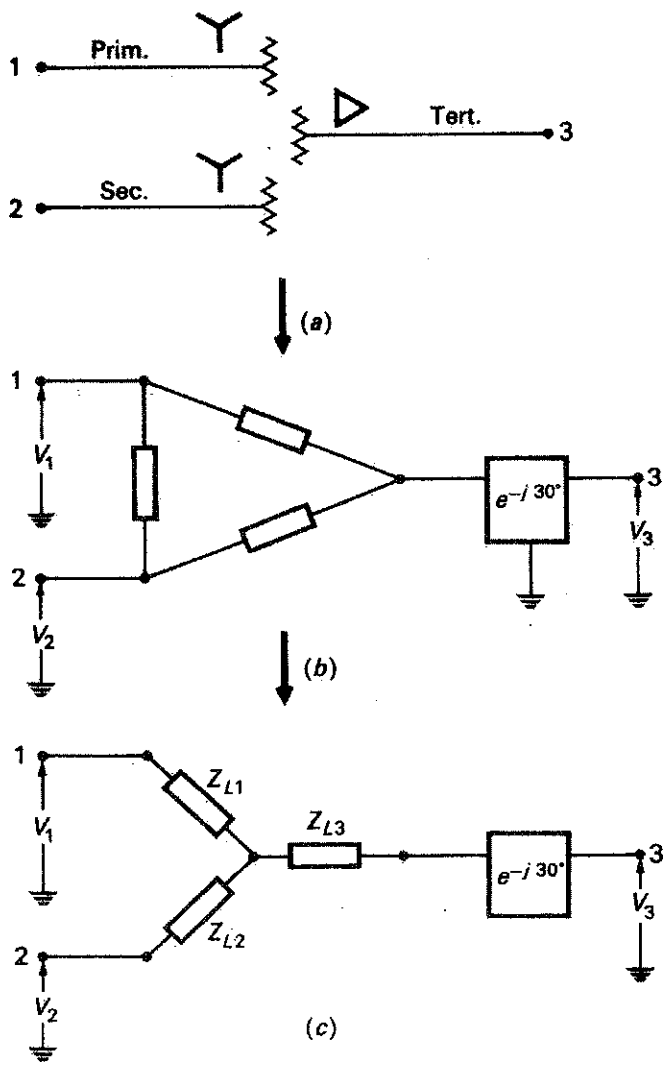


Fig. 5-13 Three-winding transformer and its circuit equivalent.

In case when the windings are not of the same power ratings, one needs to express the impedances to the same power base when dealing with their p.u. representation.

4. TRANSMISSION LINES

4.1. Long-line Equations

Materials of this chapter are taken from Chapter 6 of Elgerd.

Fig 6.16 depicts a differential section of one–phase of a line. The line has a series impedance of

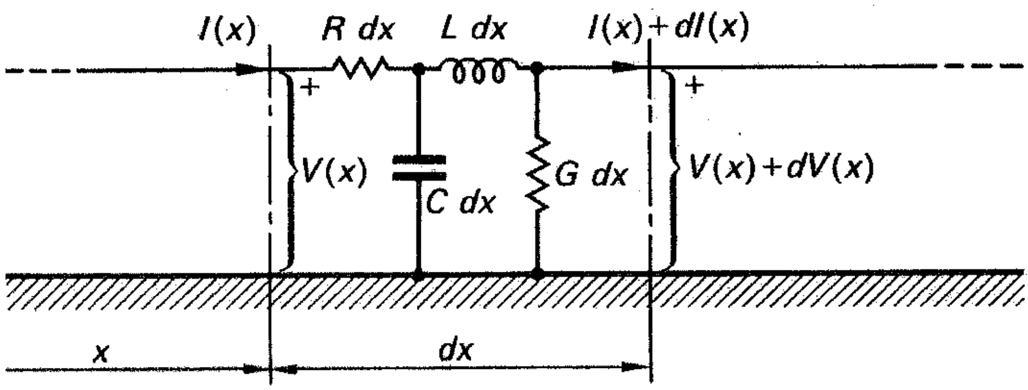


Fig. 6-16 Differential line element.

$dZ = (R + j\omega L)\,dx \qquad \Omega/\text{phase}$

and a shunt admittance of

$dY = (G + j\omega C)\,dx \qquad \mho/\text{phase}$

We consider one phase of the line and obtain the following voltage and current equations

$$\begin{aligned} V(x) - [V(x) + dV(x)] &= dZ\, I(x) \\ I(x) - [I(x) + dI(x)] &= dY\, V(x) \end{aligned} \qquad (6\text{-}69)$$

Upon substituting *dZ* and *dY* into equation (6.69), we have

$$\begin{aligned} \frac{dV(x)}{dx} &= -(R + j\omega L)I(x) \\ \frac{dI(x)}{dx} &= -(G + j\omega C)V(x) \end{aligned} \qquad (6\text{-}70)$$

By separation of the variables, we obtain the long-line differential equations for *V*(x) and *I*(x)

$$\frac{d^2V(x)}{dx^2} = (R + j\omega L)(G + j\omega C)V(x)$$

$$\frac{d^2I(x)}{dx^2} = (R + j\omega L)(G + j\omega C)I(x) \qquad (6\text{-}71)$$

Next introduce the propagation constant γ and wave (or characteristic) impedance Z_w, defined as

$$Z_w \triangleq \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \text{(dimension = ohms)}$$

$$\gamma \triangleq \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (\text{dimension} = \text{m}^{-1}) \qquad (6\text{-}72)$$

we can re-write equation (6.71) in a shorter form

$$\frac{d^2V}{dx^2} = \gamma^2 V$$

$$\frac{d^2I}{dx^2} = \gamma^2 I \qquad (6\text{-}73)$$

Solutions to the last two equations depend on the two independent initial conditions for *V(x)* and *I(x)*. Usually we choose the initial conditions at the same end-point.

Also in practice, we are interested in the power flow along the line. The power equation is

$$P(x) + jQ(x) = S(x) = V(x)\, I^*(x)$$

Equation (6.73) can be integrate, and the solutions are actually

$$V(x) = A \cosh \gamma x + B \sinh \gamma x$$

$$I(x) = C \cosh \gamma x + D \sinh \gamma x \qquad (6\text{-}75)$$

Of the integration constants A, B, C and D, one can only choose two. Suppose we choose the sending end (x = 0) voltage and current as the boundary conditions known, i.e. set x = 0, and from (6.75)

$$V(0) = A \cdot 1 + B \cdot 0$$

$$I(0) = C \cdot 1 + D \cdot 0$$

Hence

$$A = V(0)$$
$$C = I(0) \qquad (6\text{-}76)$$

For which one can determine B and D from equations (6.75) and (6.70) as

$$B = -Z_w I(0)$$
$$D = -\frac{V(0)}{Z_w} \qquad (6\text{-}77)$$

use (6-70)

$\left.\frac{dv}{dx}\right|_{x=0} = -(R+j\omega L)I(0)$

use (6-75)

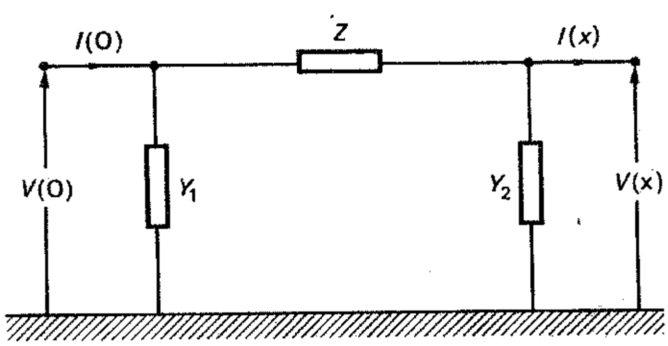
$\left.\frac{dv}{dx}\right|_{x=0} = B\gamma$

In terms of *V(0)* and *I(0)*, therefore,

$$V(x) = V(0)\cosh\gamma x - Z_w I(0)\sinh\gamma x$$
$$I(x) = I(0)\cosh\gamma x - \frac{V(0)}{Z_w}\sinh\gamma x \qquad (6\text{-}78)$$

Thus if we assume $V(0)$ and $I(0)$ are known, one can use (6.78) to calculate $V(x)$ and $I(x)$ and then the powers along the line.

1.1 Equivalent Network of Long-line



$$Z = Z_w \sinh\gamma x$$
$$Y_1 = Y_2 = \frac{1}{Z_w}\tanh\frac{\gamma x}{2}$$

Fig. 6-17 Equivalent circuit of long line.

Consider the π network in Fig 6.17. We can write the circuit voltage equation as

$$V(x) = V(0) - Z[I(0) - Y_1 V(0)] \qquad (6\text{-}89)$$

and for the current

$$I(x) = I(0) - Y_1 V(0) - Y_2 V(x) \qquad (6\text{-}90)$$

Using these two equations, we obtain

$$V(x) = V(0)(1+ZY_1) - Z\,I(0) \qquad (6\text{-}91)$$

$$I(x) = I(0)(1 + ZY_2) - V(0)(Y_1 + Y_2 + ZY_1Y_2) \tag{6-92}$$

If we compare this equation with equation (6.78), we note that they are identical if by setting

Also, compare first equation of (6-78) with (6-89)

$$\Rightarrow \left.\begin{aligned} 1 + Y_1 Z &= \cosh \gamma x \\ Z_w \sinh \gamma x &= Z \end{aligned}\right\}$$

$$1 + ZY_1 = 1 + ZY_2 = \cosh \gamma x$$

$$Z = Z_w \sinh \gamma x \tag{6-93}$$

$$Y_1 + Y_2 + ZY_1Y_2 = \frac{1}{Z_w} \sinh \gamma x$$

Compare 2nd equation of (6-78) with (6-92)

$$\left\{\begin{aligned} 1 + Y_2 Z &= \cosh \gamma x \\ Y_1 + Y_2 + ZY_1Y_2 &= \frac{\sinh \gamma}{Z_w} \end{aligned}\right.$$

that is (6-93)

Hence we can readily confirm that these equations are identical if we set

$$Z = Z_w \sinh \gamma x$$

$$Y_1 = Y_2 = \frac{1}{Z_w} \frac{\cosh \gamma x - 1}{\sinh \gamma x} = \frac{1}{Z_w} \tanh \frac{\gamma x}{2} \tag{6-94}$$

With such a substitution, the parameter values therefore constitute the branch elements of the equivalent network.

Since $Y_1 = Y_2$

$\therefore$ From 1st equation of (6-93), we have

$$1 + ZY_1 = \cosh \gamma x$$

$$Y_1 = \frac{\cosh \gamma x - 1}{Z}$$

Substitution from 2nd equation (6-93),

$$= \frac{\cosh \gamma x - 1}{Z_w \sinh \gamma x}$$

1.2 The lossless line

Usually line series *R* is small compared to *ωL* and shunt *G* is negligible compared to ωC. In which case,

$$Z_w = R_w = \sqrt{\frac{L}{C}} \tag{6-96}$$

$$\gamma = j\beta = j\omega\sqrt{LC}$$

But

$$\sinh \gamma x = 2 \sinh \frac{\gamma x}{2} \cosh \frac{\gamma x}{2}$$

$$\cosh \gamma x = \cosh^2 \frac{\gamma x}{2} + \sinh^2 \frac{\gamma x}{2}$$

But $\cosh^2 \frac{\gamma x}{2} - \sinh^2 \frac{\gamma x}{2} = 1$

Since *γx* is now purely imaginary, the hyperbolic function in equation (6.78) thus becomes

$$\cosh \gamma x = \cos \beta x$$
$$\sinh \gamma x = j \sin \beta x$$

$$\therefore \cosh \gamma x = 2\cosh^2 \frac{\gamma x}{2} - 1 = 1 + 2\sinh^2 \frac{\gamma x}{2}$$

and equation (6.78) becomes

$$\therefore Y_1 = \frac{\cosh \gamma x - 1}{Z_w \sinh \gamma x}$$

$$V(x) = V(0) \cos \beta x - jR_w I(0) \sin \beta x$$

$$I(x) = I(0) \cos \beta x - j\frac{V(0)}{R_w} \sin \beta x \tag{6-97}$$

$$= \frac{1 + 2\sinh^2 \frac{\gamma x}{2} - 1}{2 \cdot Z_w \sinh \frac{\gamma x}{2} \cosh \frac{\gamma x}{2}} = \frac{\sinh \frac{\gamma x}{2}}{Z_w \cosh \frac{\gamma x}{2}} = \frac{\tanh \frac{\gamma x}{2}}{Z_w}$$

Concept of wavelength

Equation (6.97) indicates that the voltage and current vary sinusoidally along the line with respect to space coordinate *x*. A full voltage and current cycle corresponds to a change in the angular argument, βx of 2π radians. The wavelength evidently is obtained from the equality

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f\sqrt{LC}} \tag{6-98}$$

Hence, a typical line operating in a 60Hz system will have a wavelength λ of approximately 4890 km or about 3000 miles.

Equivalent network for lossless line

From Fig 6.17, under lossless condition, all the branch elements are pure reactive.

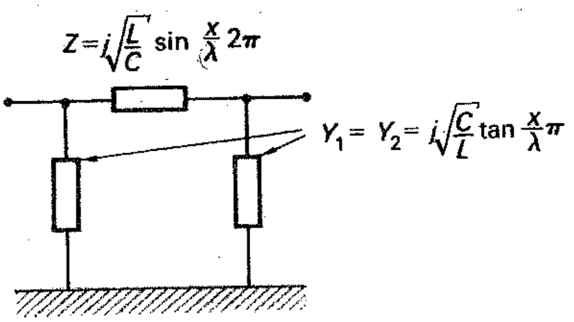


Fig. 6-19 Equivalent circuit for lossless long line.

Hence, in Fig 6.19, all the βx value is replaced by the expression

$$\beta x = \frac{x}{\lambda} 2\pi \tag{6-99}$$

The shunt admittance becomes

$$Y = jB$$

where *B* is positive for tan $(x\pi/\lambda) > 0$. This means that the admittance is capacitive for $x < \lambda/2$.

The series reactance is of the form

$Z = jX$

And is obviously inductive for

$$\sin\frac{x2\pi}{\lambda} > 0 \qquad \text{i.e., for } x < \frac{\lambda}{2}$$

When line length exceeds half a wavelength, the series reactance turns capacitive, and the shunt admittance turns inductive. In practice, all AC transmission lines at present are actually less than a quarter wavelength (about 750 miles at 60 Hz).

// Again this case would be unlikely.

Concept of electrically short line

If the line is short, the factor x/λ is so small that we can use

$$\sin\frac{x}{\lambda}2\pi \approx \frac{x}{\lambda}2\pi$$

$$\tan\frac{x}{\lambda}\pi \approx \frac{x}{\lambda}\pi$$

Then the impedance and admittance elements of Fig 6.19 become

From (6-98)

$$\lambda = \frac{1}{f\sqrt{LC}}$$

$$Z = j\sqrt{\frac{L}{C}}\sin\frac{x}{\lambda}2\pi \approx j\sqrt{\frac{L}{C}}\frac{x}{\lambda}2\pi = j\omega Lx$$

$$Y_1 = Y_2 = j\sqrt{\frac{C}{L}}\tan\frac{x}{\lambda}\pi \approx j\sqrt{\frac{C}{L}}\frac{x}{\lambda}\pi = j\frac{\omega Cx}{2} \qquad (6\text{-}100)$$

$$\therefore j\sqrt{\frac{L}{C}}\frac{x}{\lambda}2\pi = j\sqrt{\frac{L}{C}}\,x\,2\pi\cdot f\sqrt{LC} = j\,2\pi f L x = j\omega L x$$

The line series impedance is simply the line reactance per meter, multiply by the line length in m. Similarly the shunt admittance can be obtained in a similar manner.