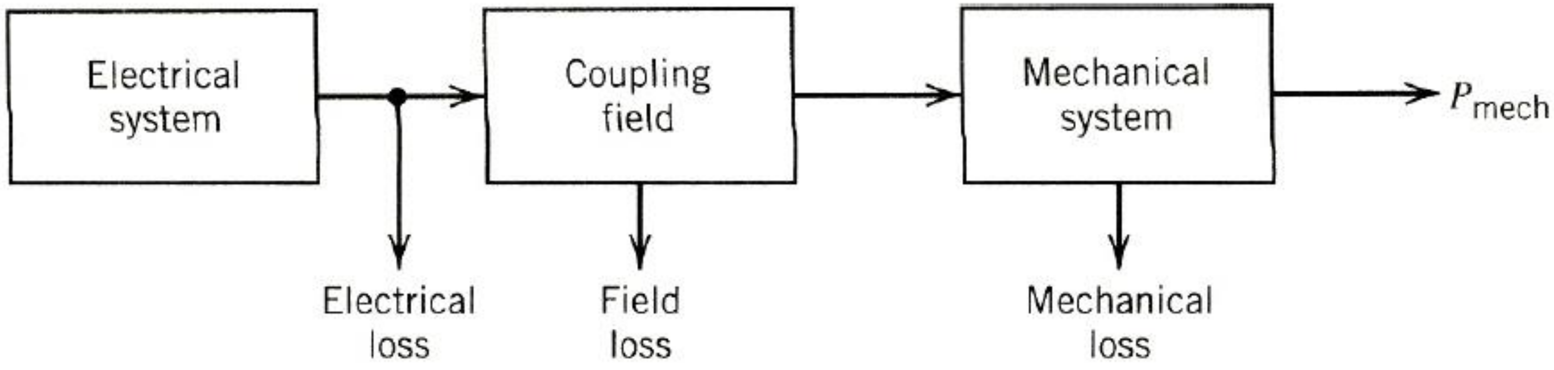
Chapter 2 Electromechanical Energy Conversions

Learning Objectives

- Analyze electromechanical energy conversion based on concepts of energy and coenergy
- Understand operating principles of rotating and cylindrical machines

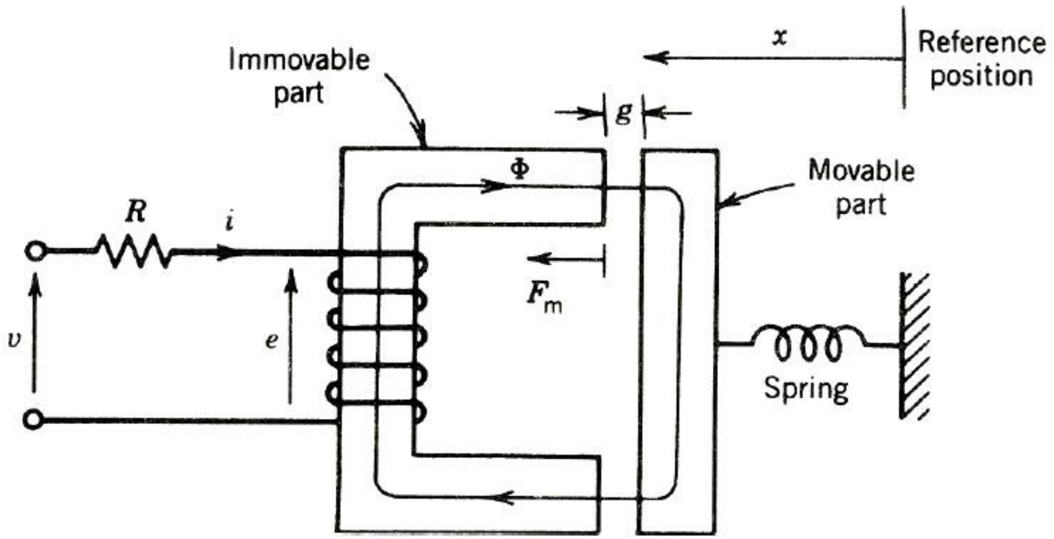
2.1 Fundamentals



- Based on the principle of conservation of energy, energy can neither be created nor destroyed but changed from one form to another
- Consider a differential time interval

$$dW_e = dW_m + dW_f$$

2.1 Fundamentals



- Consider if the movable part is held stationary at some air gap with current is increased from zero to a value *i*

$$dW_e = dW_f$$

- If core loss is neglected, all incremental electrical energy input is stored as field energy

$$dW_e = eidt \qquad dW_f = id\lambda$$

2.1 Fundamentals

- Energy stored in the field

$$W_f = \int_0^{\lambda} i d\lambda$$

$$= \int \frac{H_c l_c + H_g l_g}{N} NAdB$$

$$= \int \left(H_c dBAl_c + \frac{B}{\mu_0} dBl_g A \right)$$

$$= w_{fc} \; x \; V_c + w_{fg} \; x \; V_g$$

$$= W_{fc} + W_{fg}$$

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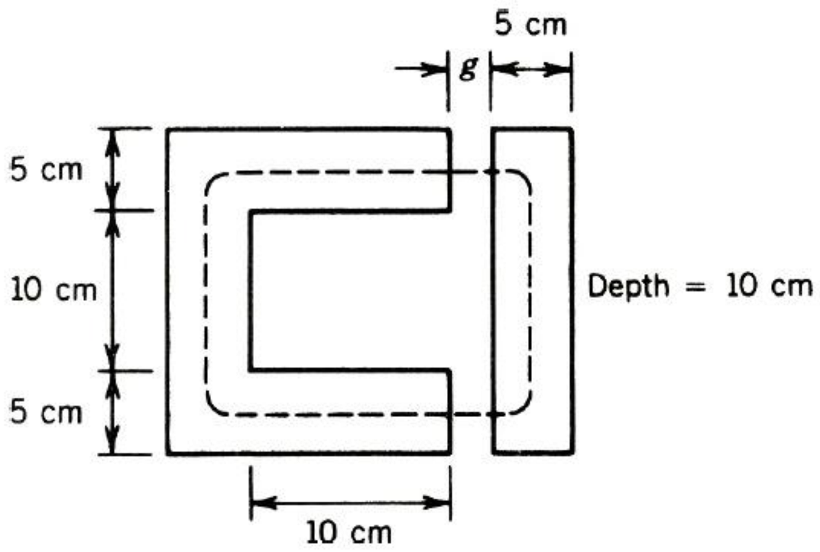
Concept Check 2.1



Consider a actuator system as shown above. It consists of coil of 250 turns and coil resistance of 5 Ω. For a fixed air gap length of 5 mm, a dc source is connected to the coil to produce a flux density of 1.0 T in the air gap. Assume the magnetic field intensity of core is a cast steel with flux density of 1.0 T is 670 At/m.

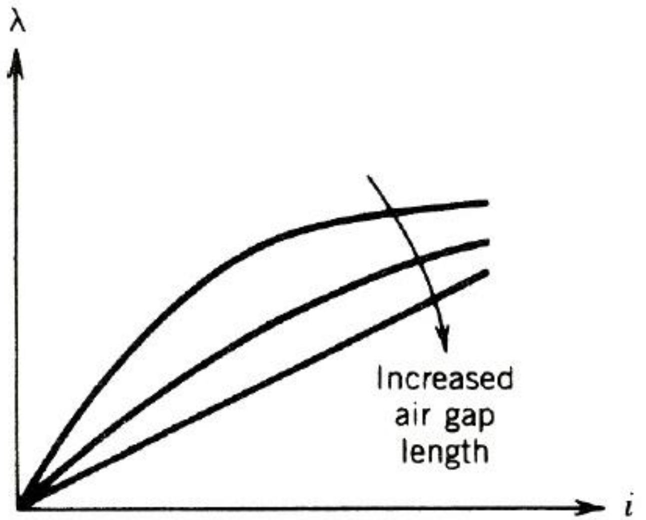
(a) Find the voltage of the dc source (Ans: 167.2 V)

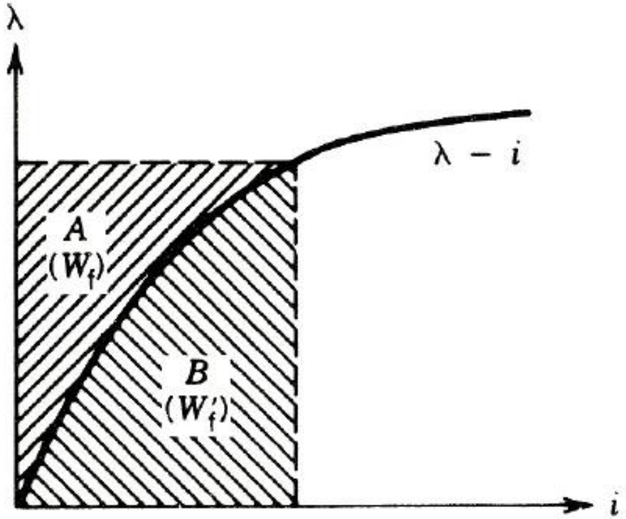
(b) Find the stored field energy (Ans: 20.9 J)

Concept Check 2.1

Concept Check 2.1

2.2 Mechanical Force





- For larger air gap length, the $\lambda - i$ characteristic will become more linear
- The area *B* is known as *coenergy*

$$W_f' = \int_0^{\lambda} i d\lambda$$

- If it is linear

$$W_f' = W_f = \lambda i$$

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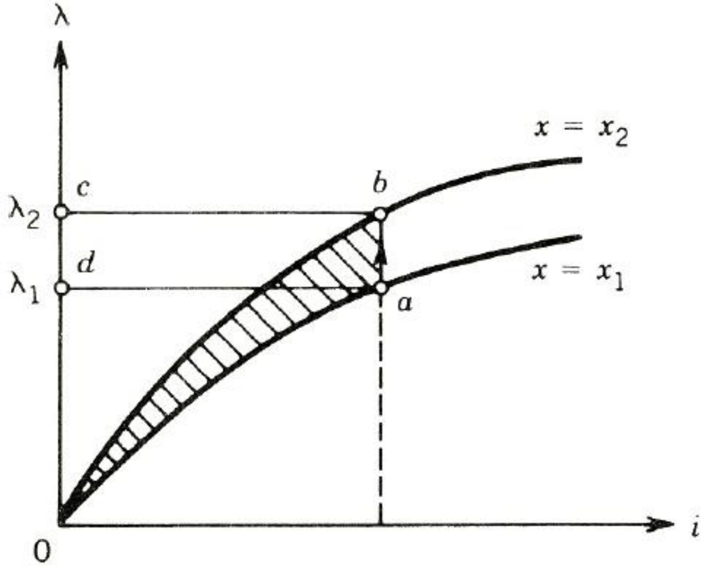
2.2 Mechanical Force



- If the movable part has moved slowly with current remained constant, the operating point will move upward from point *a* to *b*

$$dW_e = \int eidt = \int_{\lambda_1}^{\lambda_2} id\lambda = area\ abcd$$

$$dW_f = area\ 0bc - 0ad$$

$$dW_m = dW_c - dW_f = area\ 0ab$$

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2.2 Mechanical Force

- If the motion is under constant-current conditions, the mechanical work done is the incremental part in the coenergy and the mechanical force is

$$f_m dx = dW_m = W_f'$$

$$f_m = \left.\frac{\partial W_f'(i,x)}{\partial x}\right|_{i=constant}$$

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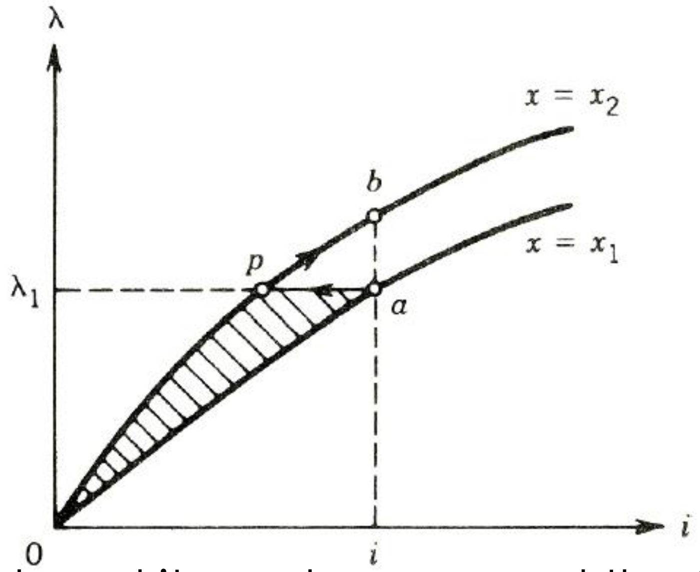
2.2 Mechanical Force



- If the movement is very quick and it can be assumed the flux linkage is constant
- The mechanical work done is shown in area *0ap*, which is decreased part in field energy

$$f_m dx = dW_m = W_f$$

$$f_m = \left. \frac{\partial W_f(\lambda, x)}{\partial x} \right|_{\lambda = constant}$$

- If the displacement is small, areas *0ab* of both cases are equal so as the force

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Concept Check 2.2

The $\lambda - i$ relationship of an electromagnetic system is given as

$$i = \left(\frac{\lambda g}{0.09}\right)^2$$

Given that it is valid for the limits $0 < i < 4$ A and $3 < g < 10$ cm. For current $i = 3$ A and air gap length $g = 5$ cm. Find the mechanical force on the moving part, by using energy and coenergy of the field (Ans: - 124.7 Nm; - 124.7 Nm)

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Concept Check 2.2

Concept Check 2.2

2.2 Mechanical Force

- In the reluctance of the magnetic core path is negligible as compared to that of the air gap path, the $\lambda - i$ relation becomes linear

$$\lambda = L(x)i$$

$$W_f = \int_0^{\lambda} \frac{\lambda}{L(x)} d\lambda$$

$$= \frac{1}{2} L(x) i^2$$

$$f_m = -\frac{\partial}{\partial x}\left(\frac{\lambda^2}{2L(x)}\right)\Bigg|_{\lambda = constant}$$

$$= \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

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2.2 Mechanical Force

- For linear system

$$W_f = W_f' = \frac{1}{2}L(x)i^2$$

$$f_m = \frac{ð}{\partial x}\left(\frac{1}{2}L(x)i^2\right)\Bigg|_{i=constant}$$

$$= \frac{1}{2}i^2\frac{dL(x)}{dx}$$

- If the reluctance of the magnetic core path is negligible

$$Ni = H_g 2g = \frac{B_g}{\mu_0}2g$$

$$f_m = \frac{B_g^2}{2\mu_0}(2A_g)$$

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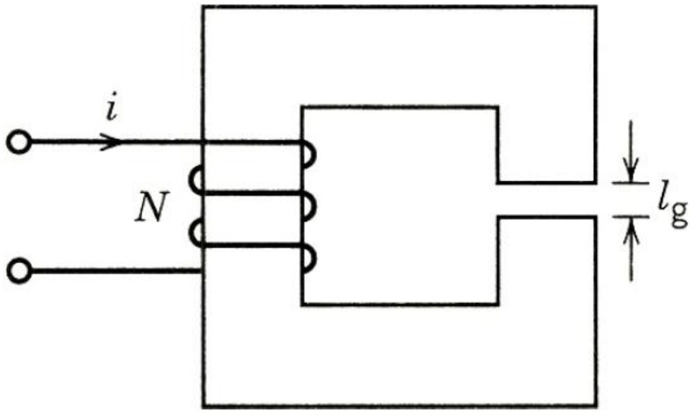
Concept Check 2.3



Consider a magnetic system as shown above. It consists of coil of 500 turns, current of 2 A, width of air gap of 2.0 cm, depth of air gap of 2.0 cm and length of air gap of 1 mm. Neglect the reluctance of the core, leakage flux and fringing flux.

(a) Find the force of attraction between both sides of the air gap (Ans: 251.3 N)

(b) Find the energy stored in the air gap (Ans: 0.251 J)

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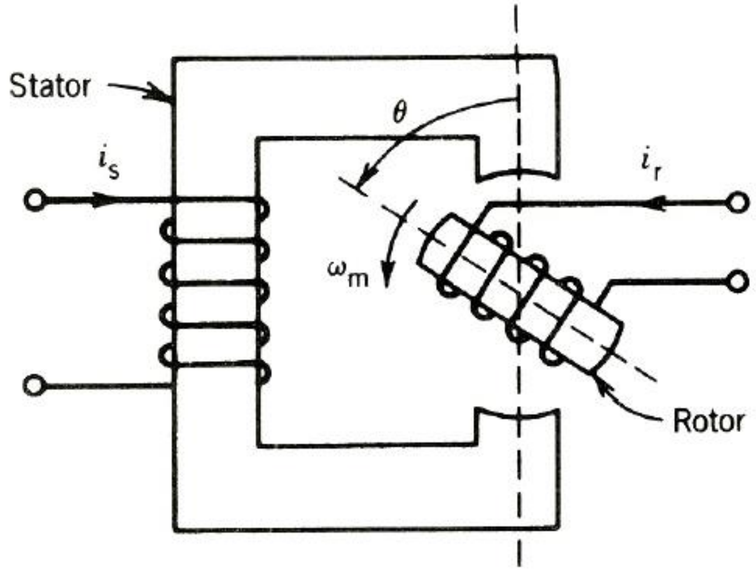
Concept Check 2.3

Concept Check 2.3

2.3 Rotating Machine



- Consider a rotating electromagnetic system that consists of both stator and rotor with winding carrying currents
- Assume the stored field energy is kept as static, i.e., no mechanical output

$$dW_f = e_s i_s dt + e_r i_r dt$$

$$= i_s d\lambda_s + i_r d\lambda_r$$

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2.3 Rotating Machine

- For a linear magnetic system

$$\lambda_s = L_{ss} i_s + L_{sr} i_r$$

$$\lambda_r = L_{rs} i_s + L_{rr} i_r$$

- As a matrix form

$$\begin{vmatrix} \lambda_s \\ \lambda_r \end{vmatrix} = \begin{vmatrix} L_{ss} & L_{sr} \\ L_{sr} & L_{rr} \end{vmatrix} \quad \begin{vmatrix} i_s \\ i_r \end{vmatrix}$$

- Hence

$$dW_r = i_s d(L_{ss} i_s + L_{sr} i_r) + i_r d(L_{sr} i_s + L_{rr} i_r)$$

$$= L_{ss} i_s d i_s + L_{rr} i_r d i_r + L_{sr} d(i_s i_r)$$

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2.3 Rotating Machine

- The field energy

$$W_f = L_{ss}\int_0^{i_s} i_s\,di_s + L_{rr}\int_0^{i_r} i_r\,di_r + L_{sr}\int_0^{i_s,i_r} d\,(i_s i_r)$$

$$= \frac{1}{2}L_{ss}i_s^2 + \frac{1}{2}L_{rr}i_r^2 + L_{sr}i_s i_r$$

- In a linear magnetic system, energy and coenergy are the same

$$T = \left.\frac{\partial W_f'(i,\theta)}{ð\theta}\right|_{i=constant}$$

$$= \frac{1}{2}i_s^2\frac{dL_{ss}}{d\theta} + \frac{1}{2}i_r^2\frac{dL_{rr}}{d\theta} + i_s i_r\frac{dL_{sr}}{d\theta}$$

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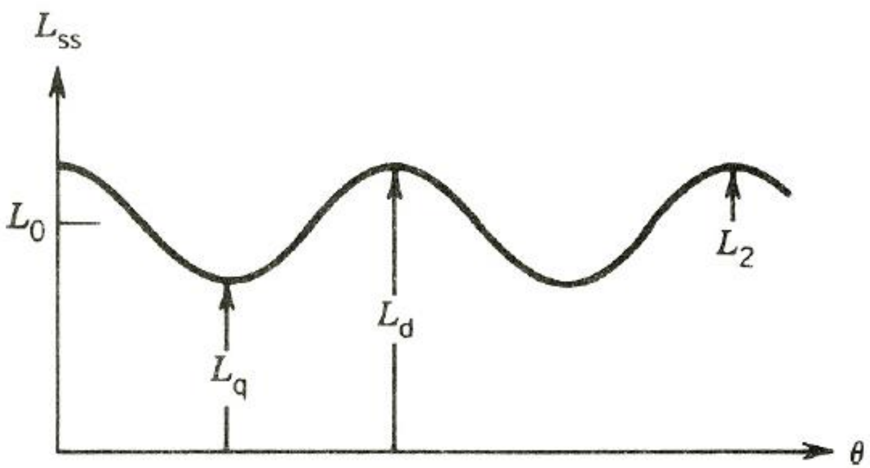
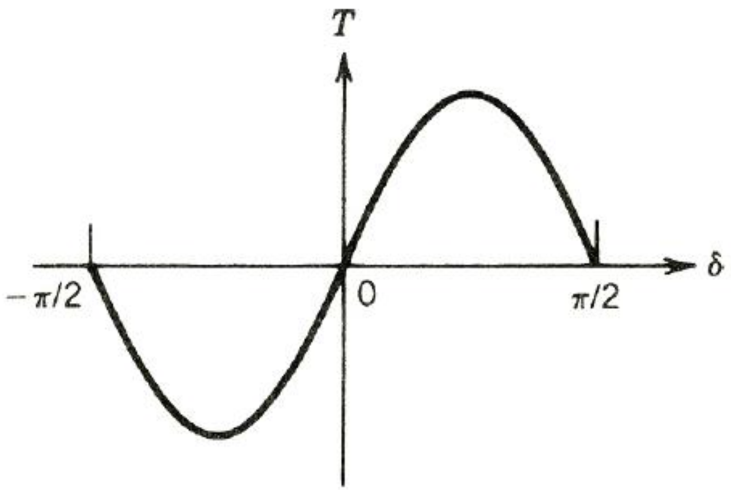
Concept Check 2.4





Consider an electromagnetic system whose rotor consists of no winding, i.e., it is a reluctance motor. Its stator current is $i_s = I_{sm} \sin\omega t$ with stator inductance is as a function of the rotor position as $L_{ss} = L_0 + L_2 \cos 2\theta$, as shown above.

(a) Find an expression for the torque acting on the rotor (Ans: $T = -I_{sm}^2 L_2 \sin 2\theta \sin^2 \omega t$)

(b) Let $\theta = \omega_m t + \delta$, find the condition for nonzero average torque and also an expression for the torque (Ans: 0; $T_{avg} = -1/4\ I_{sm}^2 (L_d - L_q) \sin 2\delta$; $\pm\omega$; $T_{avg} = 1/8\ I_{sm}^2 (L_d - L_q) \sin 2\delta$)

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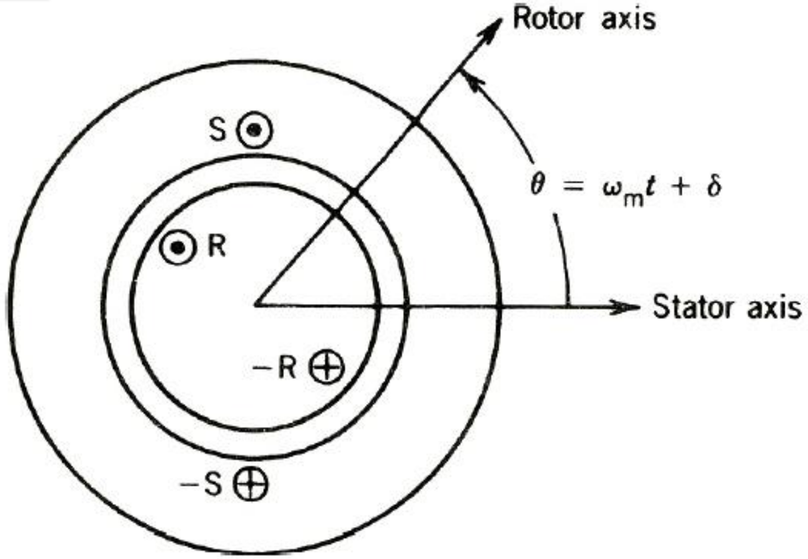
Concept Check 2.4

Concept Check 2.4

2.4 Cylindrical Machine



- If the effects of the slots are neglected, the reluctance of the magnetic path is independent of the position of the rotor
- Self-inductances are constant and no reluctance torque is produced
- The mutual inductance varies with position, and the produced torque becomes

$$T = i_s i_r \frac{dL_{sr}}{d\theta}$$

2.4 Cylindrical Machine

- Assume

$$L_{sr} = Mcos\theta$$

$$i_r = I_{rm}cos(\omega_r t + \alpha)$$

$$i_s = I_{sm}cos\omega_s t$$

$$\theta = \omega_m t + \delta$$

- Torque becomes

$$T = -I_{sm}I_{rm}\cos(\omega_s t)\,cos(\omega_r t + \alpha)\sin(\omega_m t + \delta)$$

$$= -\frac{I_{sm}I_{rm}M}{4}\left[sin\{(\omega_m + (\omega_s + \omega_r))t + \alpha + \delta\}\right]$$
$$+sin\{(\omega_m - (\omega_s + \omega_r))t - \alpha + \delta\}$$
$$+\,sin\{(\omega_m + (\omega_s - \omega_r))t - \alpha + \delta\}$$
$$+\,sin\{(\omega_m - (\omega_s - \omega_r))t + \alpha + \delta\}$$

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2.4 Cylindrical Machine

- Average torque becomes nonzero when

$$\omega_m = \pm(\omega_s \pm \omega_r)$$

- Consider when $\omega_r = 0$, $\alpha = 0$, $\omega_m = \omega_s$

$$T = -\frac{I_{sm} I_R M}{2} \{\sin(2\omega_s t + \delta) + sin\delta\}$$

$$T_{avg} = -\frac{I_{sm} I_R M}{2} sin\delta$$

- When the machine is operated under synchronous speed, it can develop an average unidirectional torque
- Even it can develop an average torque, it produce a pulsating value

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2.4 Cylindrical Machine

- Consider when $\omega_m = \omega_s - \omega_r$, the machine is operated at an asynchronous speed

$$\mathrm{T} = -\frac{I_{sm} I_{rm} M}{4} \sin(2\omega_s t + \alpha + \delta) + sin(-2\omega_r t - \alpha + \delta)$$

$$+[\sin(2\omega_s t - 2\omega_r t - \alpha + \delta) + \sin(\alpha + \delta)]$$

$$T_{avg} = -\frac{I_{sm} I_{rm} M}{4} \sin(\alpha + \delta)$$

- This is the fundamental operation of an induction machine
- No average torque is produced at $\omega_m = 0$
- It can produce average torque when speed is brought up to $\omega_m = \omega_s - \omega_r$