Supervisory Control under Partial Observation

Dr Rong Su
S1-B1b-59, School of EEE
Nanyang Technological University
Tel: +65 6790-6042, Email: rsu@ntu.edu.sg

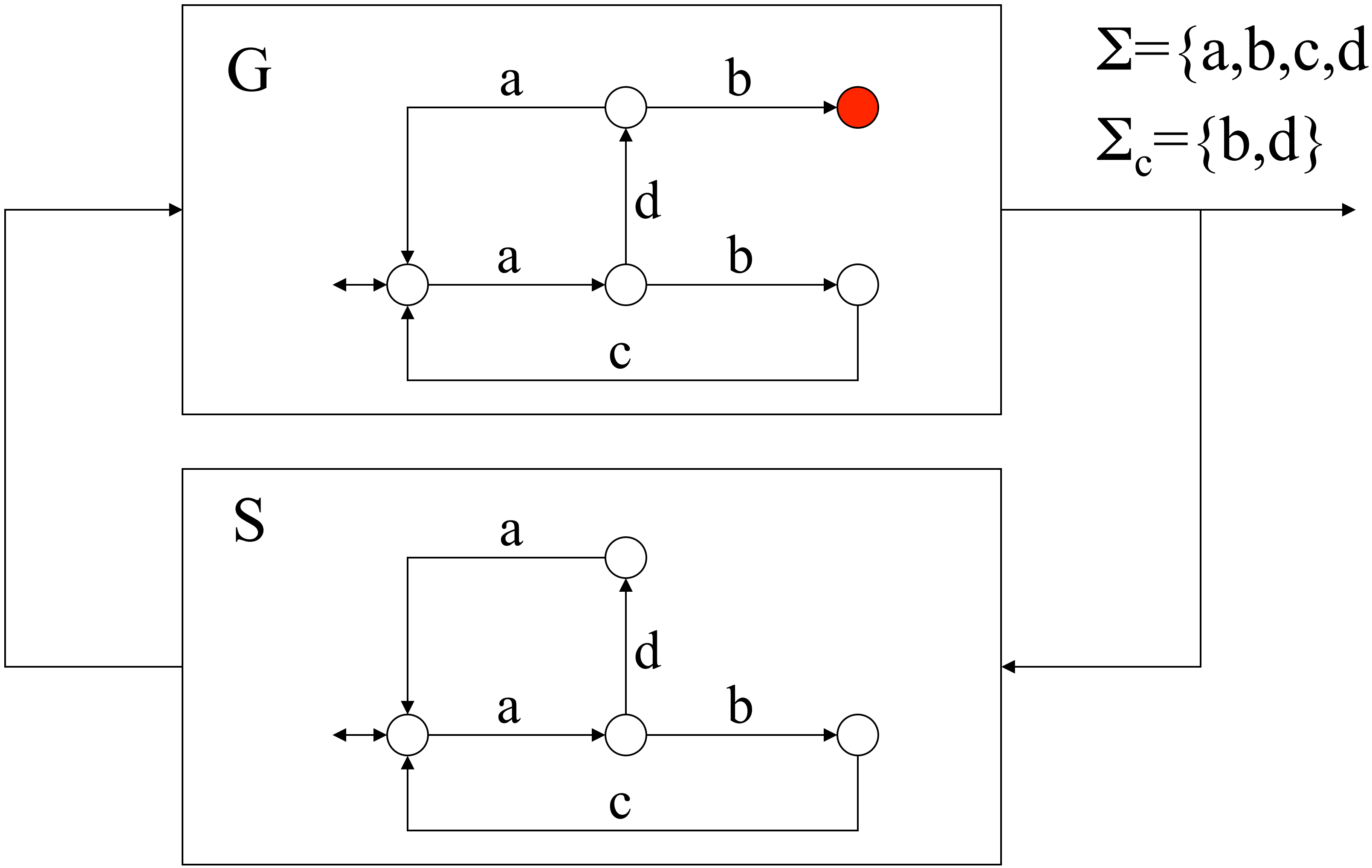
Outline

- Motivation
- The Concept of Observability
- Supervisor Synthesis under Partial Observation
- Example
- Conclusions

Three Main Concepts in Control

- Controllability
  - allows you to improve the dynamics of a system by feedback
  - e.g. controllability in the RW supervisory control theory
- Observability
  - allows you to deploy such feedback by using the system's output
- Optimality
  - gives rise to formal methods of control synthesis
  - e.g. supremality in the RW supervisory control theory

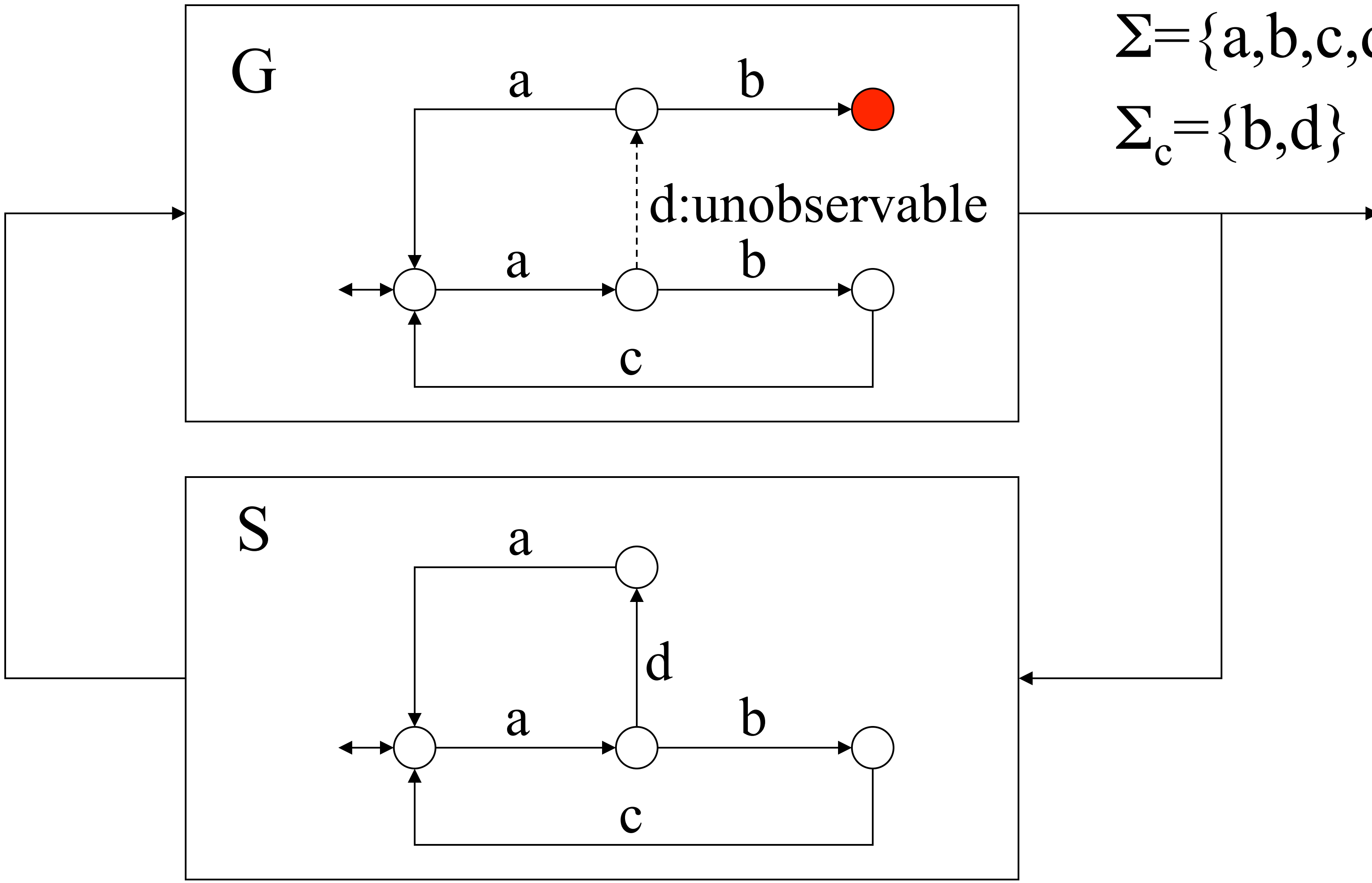
Example



$\Sigma=\{a,b,c,d\}$

$\Sigma_c=\{b,d\}$

Example (cont.)



$\Sigma=\{a,b,c,d\}$

$\Sigma_c=\{b,d\}$

Some Intuitions

- Supervisor can only act upon receiving observable events
- Partial observation forces a supervisor to be conservative
- We can enable or disable an unobservable event

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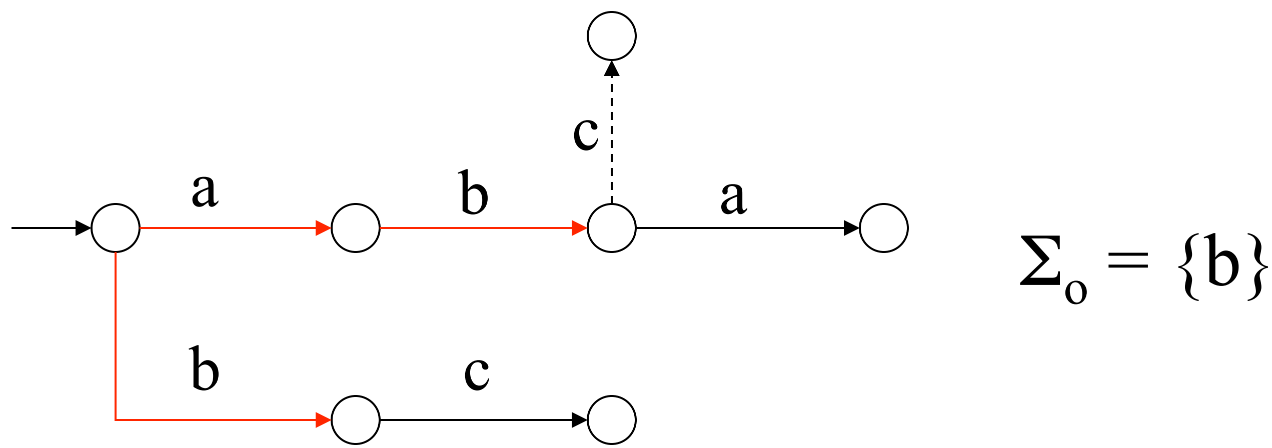
Observability

- Given $G \in \phi(\Sigma)$, let $\Sigma_o \subseteq \Sigma$ and $P: \Sigma^* \rightarrow \Sigma_o^*$ be the natural projection.
- A language $K \subseteq L(G)$ is (G,P)-*observable,* if

$$(\forall s \in \overline{K})(\forall \sigma \in \Sigma)\ s\sigma \in L(G) - \overline{K} \Rightarrow P^{-1}P(s)\sigma \cap \overline{K} = \varnothing$$



Or equivalently …

- $K \subseteq L(G)$ is (G,P)-*observable,* if for all $s \in K$, $s' \in \Sigma^*$ and $\sigma \in \Sigma$,

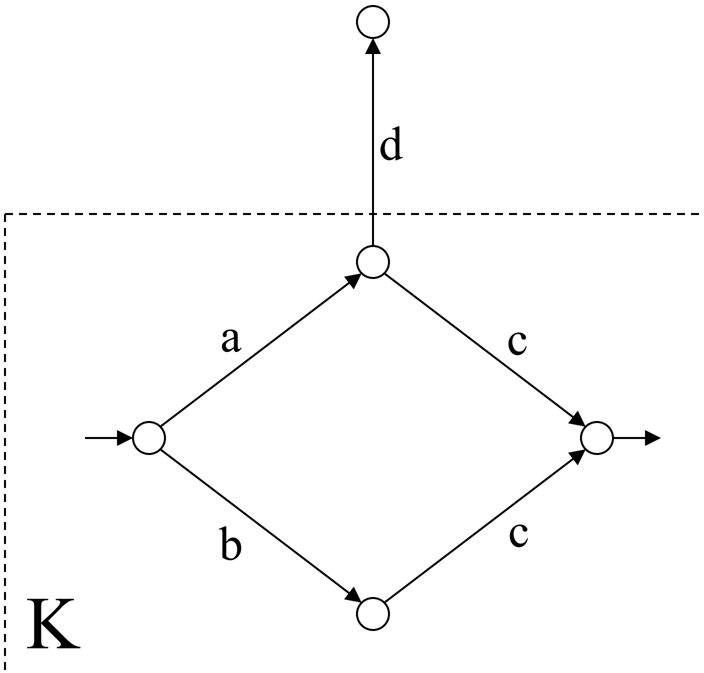
$$s\sigma \in L(G) - \overline{K} \wedge s'\sigma \in L(G) \wedge P(s)=P(s') \Rightarrow s'\sigma \in L(G) - \overline{K}$$

or equivalently,

$$s\sigma \in \overline{K} \wedge s'\sigma \in L(G) \wedge P(s)=P(s') \Rightarrow s'\sigma \in \overline{K}$$

(Think about why they are equivalent)

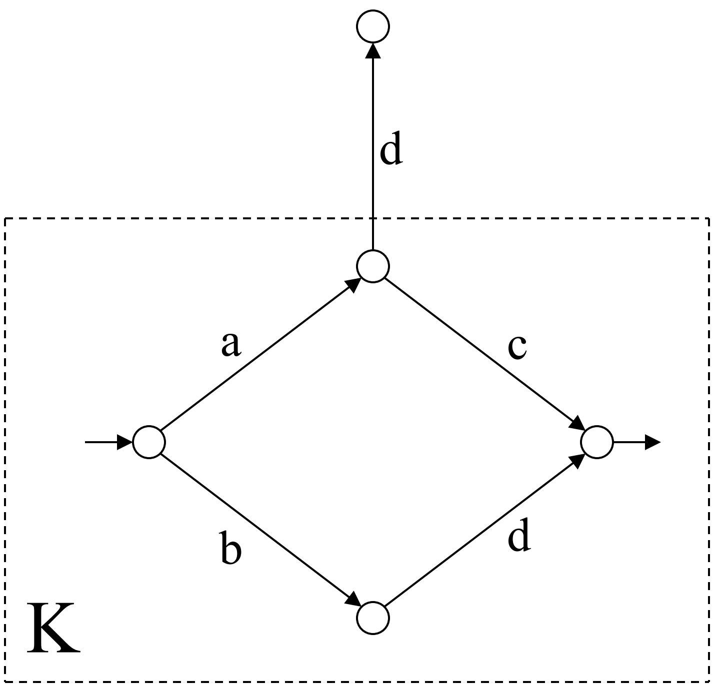
Example 1



- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$

Question: is K (G,*P*)-observable? yes

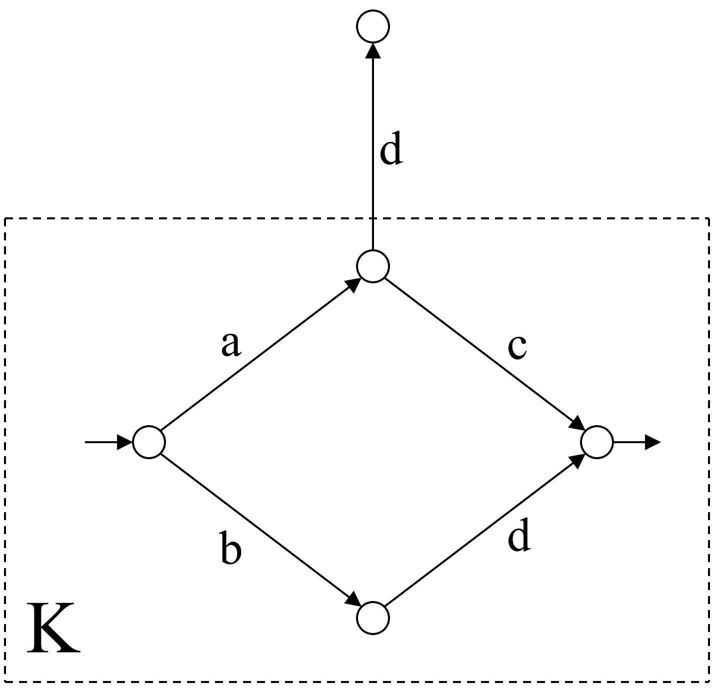
Example 2



- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$

Question: is K (G,*P*)-observable? no

Example 3



- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{a,c\}$
- $K = \{ac, bc\}$

Question: is K (G,P)-observable? yes

(G,P)-observability is *decidable*. But how?

Procedure of Checking Observability : Step 1

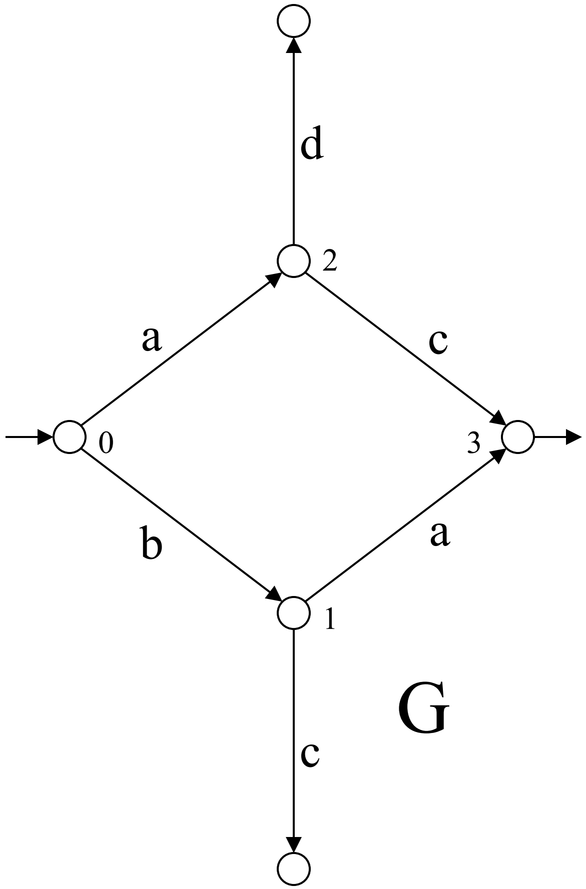
- Let $G = (X,\Sigma,\xi,x_0,X_m)$
- Suppose K is recognized by $A = (Y,\Sigma,\eta,y_0,Y_m)$, i.e. $K=L_m(A)$
- Let $A' = G\times A = (X\times Y,\Sigma,\xi\times\eta,(x_0,y_0),X_m\times Y_m)$
  - Since $\overline{K}=L(A)\subseteq L(G)$, we have $L(G\times A)=L(A)$
- A state $(x,y)\in X\times Y$ is a *boundary state* of A' w.r.t. G, if
  - $(\exists s\in L(A'))\ \xi\times\eta((x_0,y_0),s)=(x,y)$, i.e. (x,y) is reachable from (x_0,y_0)
  - $(\exists\sigma\in\Sigma)\ \xi(x,\sigma)!\wedge\neg\eta(y,\sigma)!$, where "!" denotes "is defined"
- Let B be the collection of all boundary states of A' w.r.t. G
  - B is a finite set. (Why?)

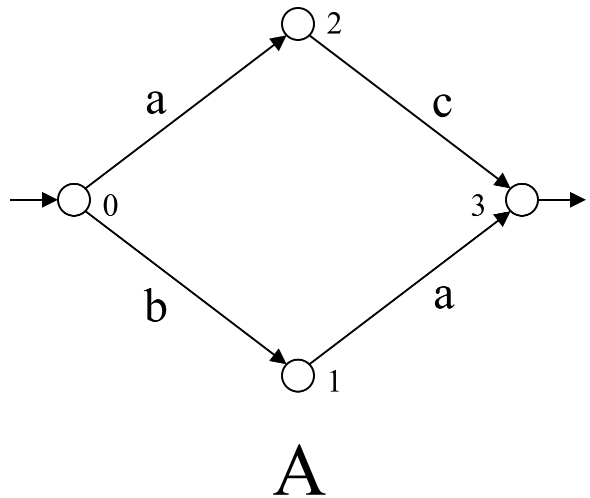
Procedure of Checking Observability : Step 2

- For each boundary state $(x,y) \in B$, we define two sets
  - $T(x,y) := \{s \in L(A') | \xi \times \eta((x_0,y_0),s) = (x,y)\}$ (T(x,y) is regular, why?)
  - $\Sigma(x,y) := \{\sigma \in \Sigma | \xi(x,\sigma)! \wedge \neg \eta(y,\sigma)!\}$
- Theorem
  - K is observable w.r.t. G and *P*, iff for any boundary state $(x,y) \in B$,

$$P^{-1}P(T(x,y))\Sigma(x,y) \cap \overline{K} = \varnothing$$

Example





- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{c\}$
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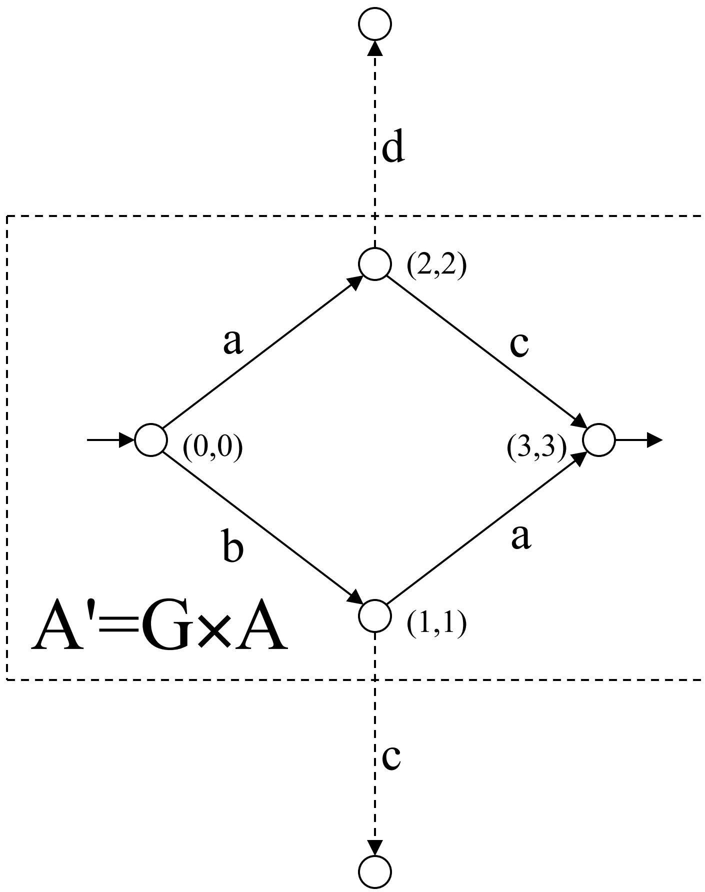
Example – Step 1

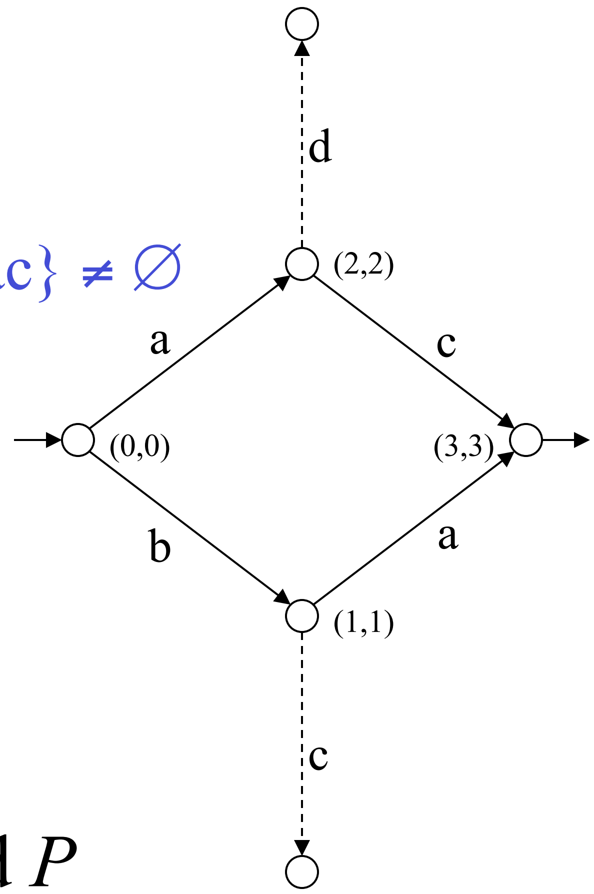
- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$



- $B = \{(1,1)\ ,\ (2,2)\}$

Example – Step 2

- For the boundary state (1,1) we have
  - $T(1,1) = \{b\}$
  - $\Sigma(1,1) = \{c\}$
  - $P^{-1}P(T(1,1))\Sigma(1,1) \cap \overline{K} = \{bc,ac\} \cap \{ac,ba\} = \{ac\} \neq \emptyset$
- For the boundary state (2,2) we have
  - $T(2,2) = \{a\}$
  - $\Sigma(2,2) = \{d\}$
  - $P^{-1}P(T(2,2))\Sigma(2,2) \cap \overline{K} = \{ad\} \cap \{ac,ba\} = \emptyset$



K is not observable w.r.t. G and *P*

Properties of Observable Languages

- Suppose K_1 and K_2 are closed, observable w.r.t. G and *P*. Then
  - $K_1 \cap K_2$ is observable w.r.t. G and *P*
  - $K_1 \cup K_2$ may not be observable w.r.t. G and *P*
- Given a plant G, let

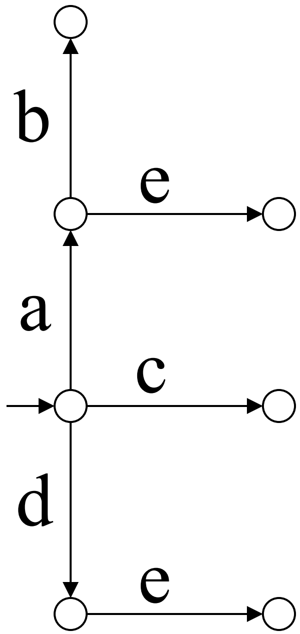
$$\mathcal{O}(G) := \{K \subseteq L(G) \mid K \text{ is closed and observable w.r.t. } G \text{ and } P\}$$

- The partially ordered set (poset) $(\mathcal{O}(G), \subseteq)$ is a meet-semi-lattice
  - The greatest element may not exist (i.e. no supremal observable sublanguage)

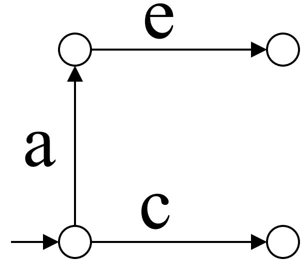
Example

$\Sigma=\{a,b,c,d,e\}$

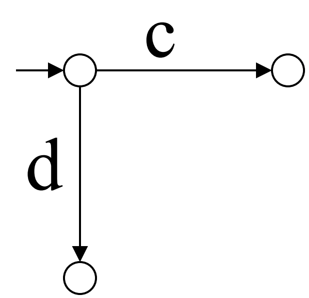
$\Sigma_o=\{c\}$



G



K_1



K_2

- $K_1 \cap K_2$ is observable, but $K_1 \cup K_2$ is not. (Why?)

Main Existence Result

- Theorem 1
  - Let $K \subseteq L_m(G)$ and $K \neq \emptyset$. There exists a proper supervisor iff
    - K is controllable with respect to G
    - K is observable with respect to G and *P*
    - K is $L_m(G)$-closed, i.e. $K = \overline{K} \cap L_m(G)$

Supervision under Partial Observation

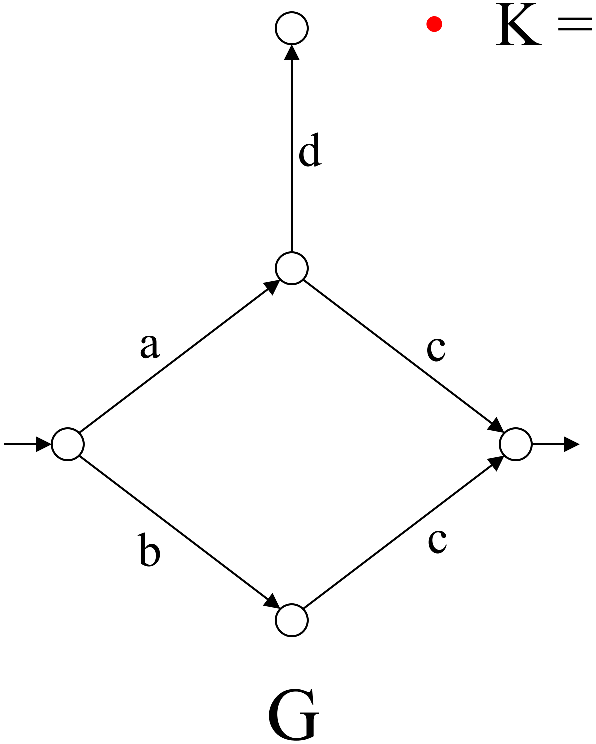
- Suppose K is controllable, observable and $L_m(G)$-closed.
- Let $A=(Y,\Sigma_o,\eta,y_0,Y_m)$ be the canonical recognizer of $P(K)$.
- We construct a new automaton $S=(Y,\Sigma,\lambda,y_0,Y_m)$ as follow:
  – For any $y\in Y$, an event $\sigma\in\Sigma-\Sigma_o$ is *control-relevant* w.r.t. y and K, if

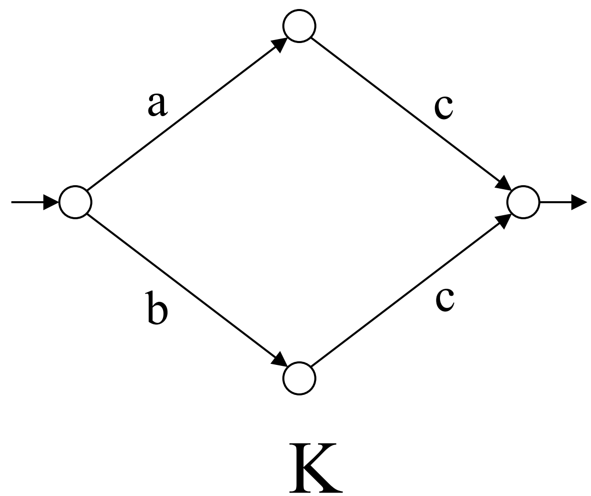
$$(\exists s\in\overline{K})\ \eta(y_0,P(s))=y \wedge s\sigma\in\overline{K}$$

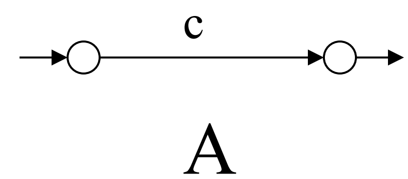
  – Let $\Sigma(y)$ be the collection of all events in $\Sigma-\Sigma_o$ control-relevant w.r.t. y, K
  – We define the transition map $\lambda:Y\times\Sigma\rightarrow Y$ as follows:
    - λ is the same as η over $Y\times\Sigma_o$
    - For any $y\in Y$ and $\sigma\in\Sigma(y)$, define $\lambda(y,\sigma):=y$ (i.e. selfloop all events of $\Sigma(y)$ at y)
    - For all other (y,σ) pairs, $\lambda(y,\sigma)$ is undefined
- S is a *proper supervisor of* G *under PO* such that $L_m(S/G)=K$

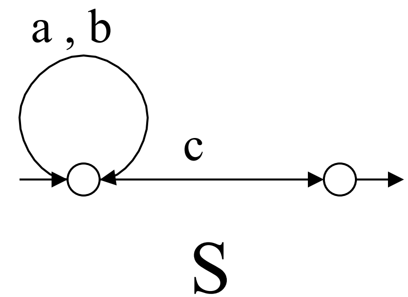
Example

- $\Sigma = \{a,b,c,d\}$
- $\Sigma_o = \{c\}$
- $K = \{ac, bc\}$









$L_m(S/G)=K$?

Difficulty of Synthesis

- Given a plant G and a specification SPEC, let

$\mathcal{O}(G,SPEC) := \{K \subseteq L_m(G) \cap L_m(SPEC) \mid K \text{ is controllable and observable}\}$

- Unfortunately, there is no supremal element in $\mathcal{O}(G,SPEC)$.

Solution 1: A New Supervisory Control Problem

- Given G, suppose we have $A \subseteq E \subseteq L(G)$ and $\Sigma=\Sigma_o \cup \Sigma_c$.
- To synthesize a supervisor S under partial observation such that

$$A \subseteq L(S/G) \subseteq E \qquad (*)$$

- Let $\mathcal{O}(A) := \{K \subseteq A \mid K$ is closed and observable w.r.t. G and $P\}$
- Let $\mathcal{C}(E) := \{K \subseteq E \mid K$ is closed and controllable w.r.t. G$\}$
- Theorem (Feng Lin)
  - Assume $A \neq \emptyset$. The (*) problem has a solution S iff $\inf \mathcal{O}(A) \subseteq \sup \mathcal{C}(E)$

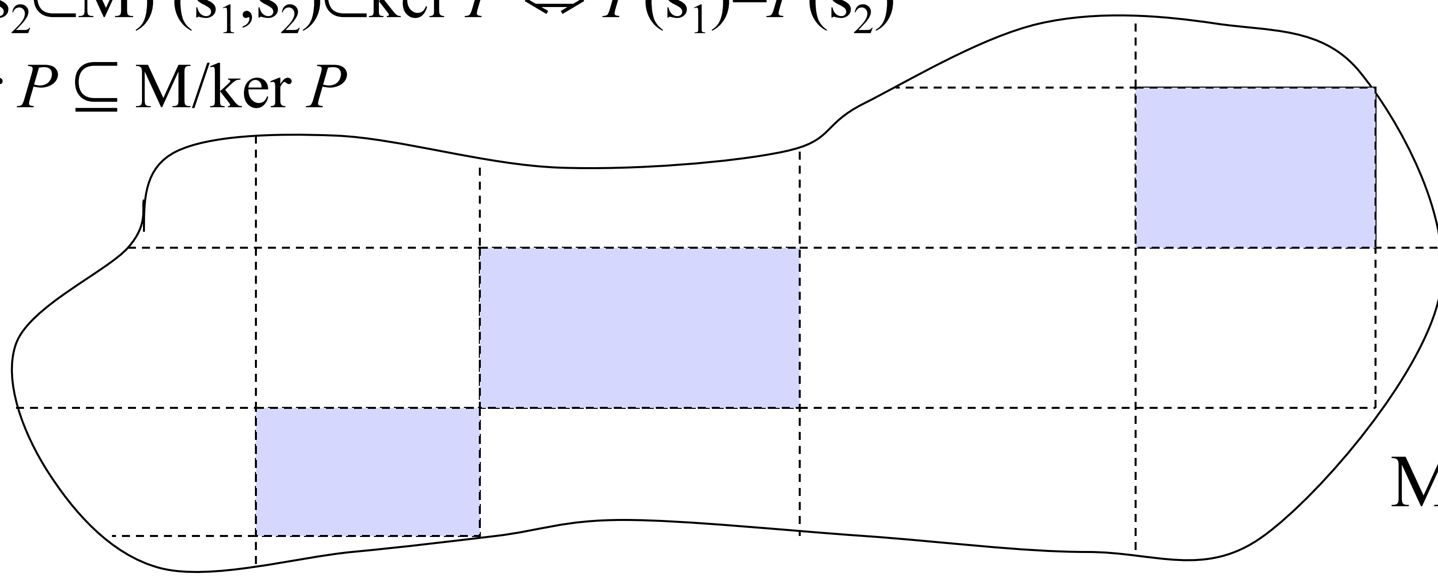
Solution 2 : The Concept of Normality

- Given $N \subseteq M \subseteq \Sigma^*$, we say N is (M,$P$)-*normal* if

$$N = M \cap P^{-1}P(N)$$

  - In particular, take $N=M\cap P^{-1}(K)$ for any $K\subseteq\Sigma_o{}^*$. Then N is (M,P)-normal.

- $(\forall s_1,s_2\in M)\ (s_1,s_2)\in\ker P \Leftrightarrow P(s_1)=P(s_2)$
- $N/\ker P \subseteq M/\ker P$



Properties of Normality

- Let $\mathcal{N}(E\ ;\ M) := \{N \subseteq E \mid N \text{ is } (M,P)\text{-normal}\}$ for some $E \subseteq \Sigma^*$
  - The poset $(\mathcal{N}(E\ ;\ M),\subseteq)$ is a complete lattice
    - The union of (M,P)-normal sublanguages is normal (intuitive explanation ?)
    - The intersection of (M,P)-normal sublanguages is normal (intuitive explanation ?)
  - Lin-Brandt formula : $\sup \mathcal{N}(E\ ;\ M) = E - P^{-1}P(M - E)$
    - In TCT : N = Supnorm(E,M,Null/Image)
- Let $E \subseteq L_m(G)$, and $\overline{\mathcal{N}}(E\ ;\ L(G)) := \{N \subseteq E \mid \overline{N} \text{ is } (L(G),P)\text{-normal}\}$
  - $\overline{\mathcal{N}}(E\ ;\ M)$ is closed under arbitrary unions, but not under intersections

Relationship between Normality and Observability

- Let $K \subseteq L_m(G)$. Then

$$\overline{K} \text{ is } (L(G), P)\text{-normal} \Rightarrow K \text{ is observable w.r.t. } G \text{ and } P$$

- Let $\Sigma(K) := \{\sigma \in \Sigma \mid (\exists s \in \overline{K})\ s\sigma \in L(G) - \overline{K}\}$
  - $\Sigma(K)$ is the collection of all boundary events of K w.r.t. G

$$K \text{ is observable w.r.t. } G, P \wedge \Sigma(K) \subseteq \Sigma_o \Rightarrow \overline{K} \text{ is } (L(G),P)\text{-normal}$$

Supervisory Control under Normality

- Given a plant G and a specification E, let
  - $\mathcal{C}(G,E) := \{K \subseteq L_m(G) \cap L_m(E) \mid K \text{ is controllable w.r.t. } G\}$
- We define a new set

$$\mathcal{S}(G,E) := \{K \subseteq \Sigma^* \mid K \in \mathcal{C}(G,E) \wedge \overline{\mathcal{N}}(L_m(E), L(G)) \wedge L_m(G)\text{-closed}\}$$

  - $\mathcal{S}(G,E)$ is nonempty and closed under arbitrary unions. sup $\mathcal{S}(G,E)$ exists
- Supervisory Control and Observation Problem (SCOP)
  - to compute a proper supervisor S under partial observation such that

$$L_m(S/G) = \sup \mathcal{S}(G,E)$$

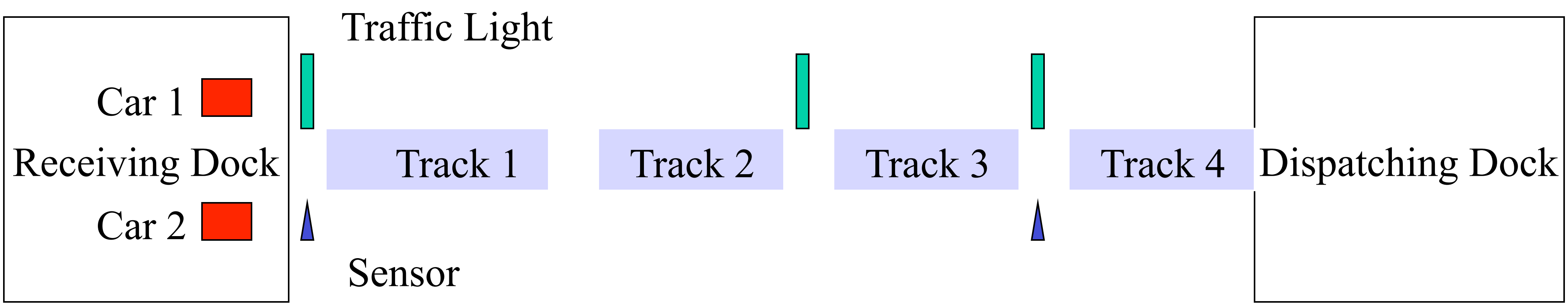
The TCT Procedure for SCOP

- Given a plant G and a specification E, let

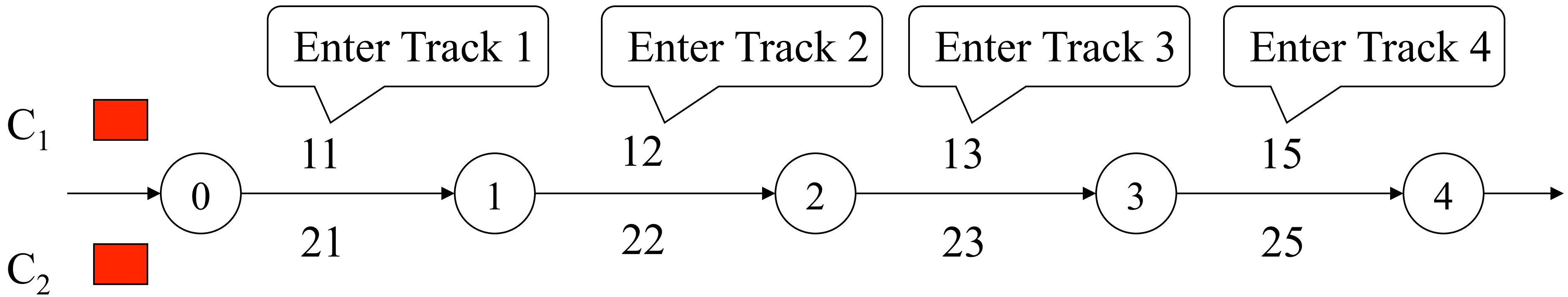
$$A = \text{Supscop}(E,G,\text{Null/Image})$$

  - $L_m(A) = \sup \mathcal{S}(G,E)$
  - Based on A, we construct a proper supervisor S under partial observation
    - Why can we do that? Because $\sup \mathcal{S}(G,E)$ is controllable and observable

Warehouse Collision Control



Plant Model



- $\Sigma_1 = \{11, 12, 13, 15\}$, $\Sigma_{1,c} = \{11, 13, 15\}$, $\Sigma_{1,o} = \{11, 15\}$
- $\Sigma_2 = \{21, 22, 23, 25\}$, $\Sigma_{2,c} = \{21, 23, 25\}$, $\Sigma_{2,o} = \{21, 25\}$

Specification

- To avoid collision, C_1 and C_2 can't reach the same state together
  - States (1,1), (2,2), (3,3) should be avoided in $C_1 \times C_2$

Synthesis Procedure in TCT

- Create the plant

$$G = \mathrm{Sync}(C_1,C_2) \quad (25\,;\,40)$$

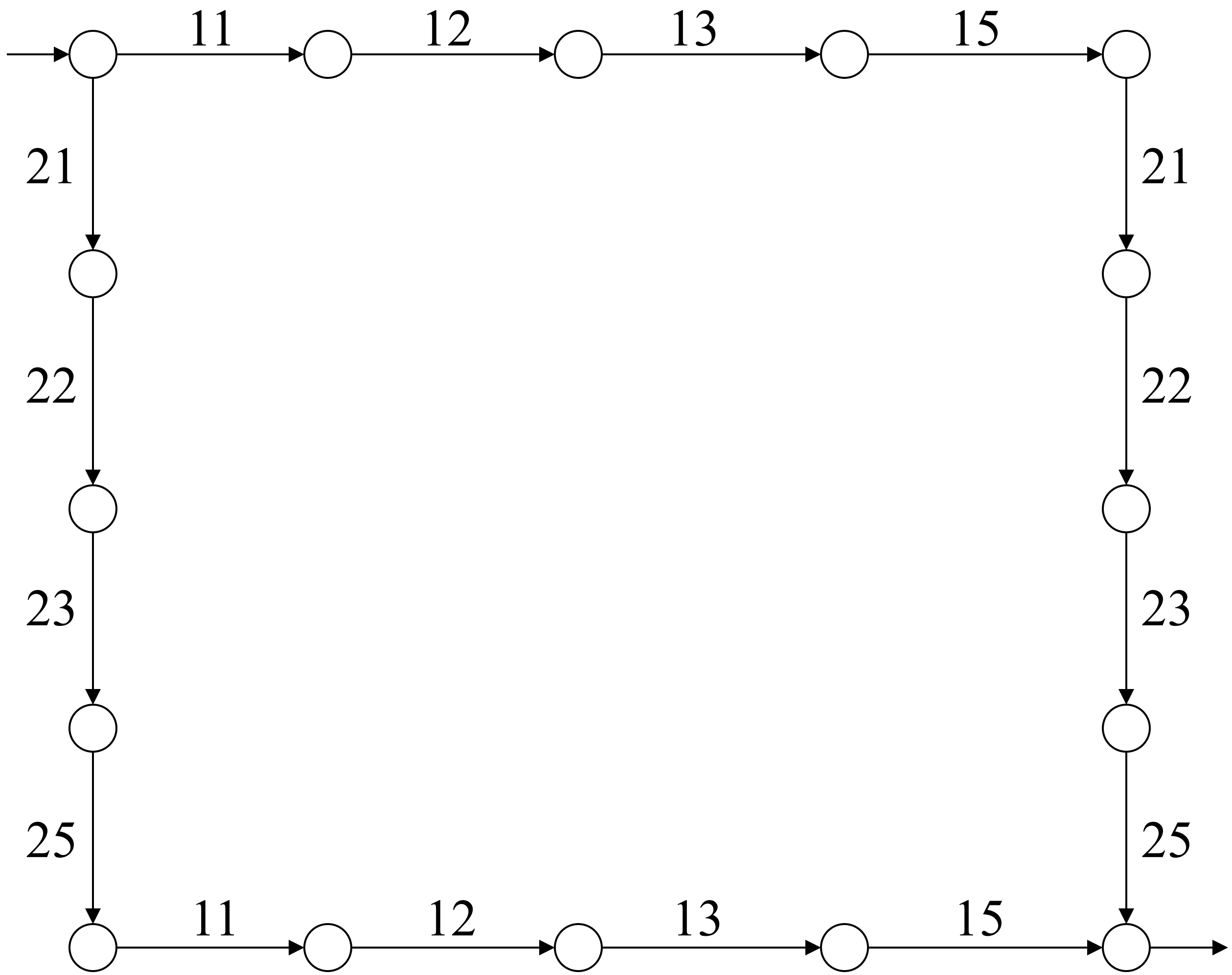
- Create the specification

$$E = \mathrm{mutex}(C_1,C_2,[(1,1),(2,2),(3,3)]) \quad (20\,;\,24)$$

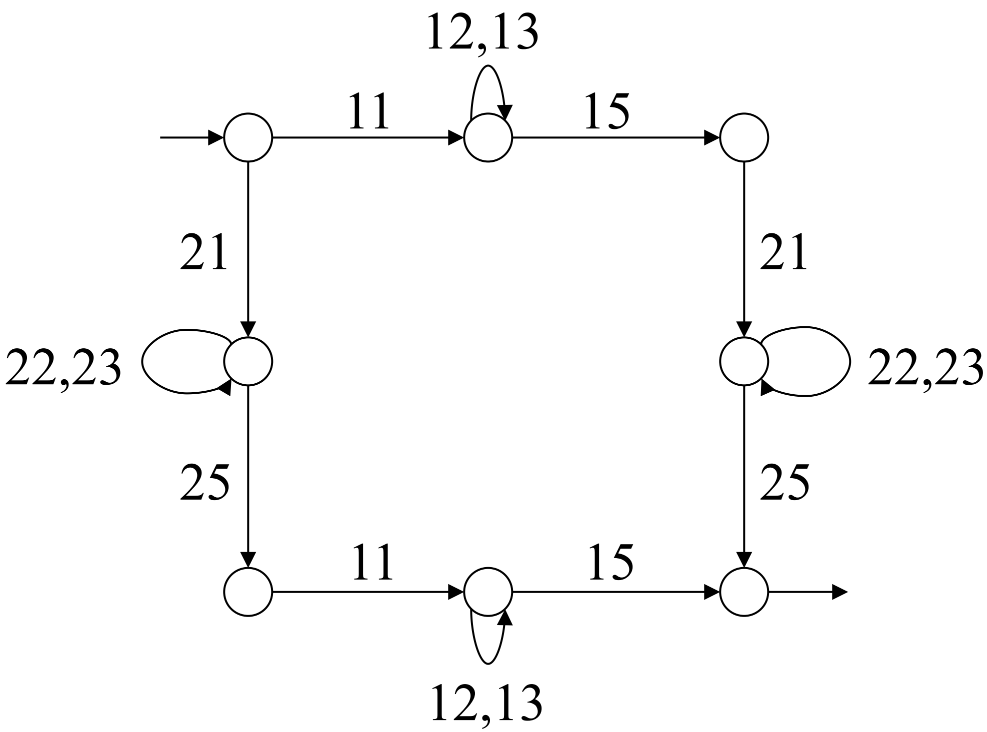
- Supervisor Synthesis

$$K = \mathrm{Supscop}(E,G,[12,13,22,23]) \quad (16\,;\,16)$$

Transition Structure of K



A Proper Supervisor S under Partial Observation



$K = L_m(S/G)$

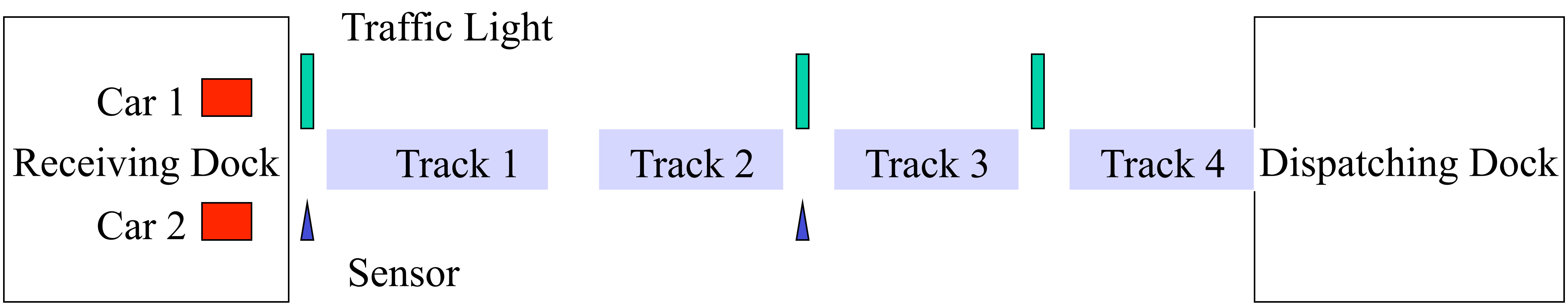
Some Fact

- Perform the following TCT operation

$$W = \mathrm{Condat}(G,K)$$

  - Only events 11 and 21 are required to be disabled.
  - Therefore, we only need one traffic light at Track 1.

A Slight Modification



- $\Sigma_{1,o} = \{11, 15\}$
- $\Sigma_{2,o} = \{21, 25\}$

→

- $\Sigma_{1,o} = \{11, 13\}$
- $\Sigma_{2,o} = \{21, 23\}$

Synthesis Result

- Create the plant

$$G = \mathrm{Sync}(C_1,C_2) \quad (25 ; 40)$$

- Create the specification

$$E = \mathrm{Mutex}(C_1,C_2,[(1,1),(2,2),(3,3)]) \quad (20 ; 24)$$

- Supervisor Synthesis

$$K = \mathrm{Supscop}(E,G,[12,15,22,25]) \quad (\text{empty})$$

  – Explain intuitively why this can happen (homework)

Conclusions

- Partial observation is important for implementation.
  - A supervisor can make a move only based on observations.
- The current observability is not closed under set union.
  - Thus, there is no supremal observable sublanguage (unfortunately).
- Normality is closed under set union.
  - Thus, the supremal normal sublanguage exists.
  - But the concept of normality is too conservative.