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Numerical Modeling Project
One-Dimensional Lake Model
5-9-12

Background & Motivation

The Kauhako Crater on the Kalaupapa Peninsula on the island of Molokai is a very interesting lake. It has a very small surface area relative to its depth. At its widest point it is approximately 85 meters across, but its depth exceeds 250 meters! The lake is highly stratified with fresher water layer at the surface of around 18 psu while the waters below the stratified layer (about 8 meters depth) remain around 32 psu. The water at depth also contains high quantities of dissolved sulfur which do not reach the surface waters and is thus prevented from being released into the atmosphere. However, in 2011 large amounts of sulfur were able to escape the lake. The gaseous sulfur is heavier than air and filled the crater. Once full, the gas flowed over the crater walls and enveloped the nearby settlement in an unpleasant stench. The cause of this sulfur release is not know, but an overturning event has been hypothesized as to the cause. The lake model presented uses data from CTD's taken in the lake as well as nearby weather station data in an attempt to simulate the lake and determine what conditions could cause an overturning event.

Model Equations

The model uses the one-dimensional (vertical) diffusion equation with a flux term at the surface as its governing equation. The variables being diffused are temperature and salinity, while density is calculated as a function of temperature and salinity using a linearized version of the equation of state (Gill, 1982),

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial z}\kappa_T\frac{\partial T}{\partial z}+QT \qquad \frac{\partial S}{\partial t}=\frac{\partial}{\partial z}\kappa_S\frac{\partial S}{\partial z}+QS \qquad \rho=\rho_0(1+\alpha_S(S-S_0)-\alpha_T(T-T_0))$$

where T is the temperature, κ_T is the diffusivity of heat, QT is the heat flux term, S is salinity, κ_S is the diffusivity of salt, QS is the salt flux term, ρ is the density, α_T is the coefficient of thermal expansion, α_S is the coefficient of haline contraction, and ρ_0, T_0, S_0 are reference values of density, temperature, and salinity. Levels below the surface layer do not include the flux terms. Heat diffusivity is a function of depth and is parameterized as:

$$\kappa_T(z)=\kappa_{0\mathrm{T}}(1+\beta_T\tanh(\delta_T\frac{\partial\rho}{\partial z}))$$

where κ_0 is a reference value determined from a review of the limnological literature, and β_T and δ_T are calibrated parameters. This parameterization ensures that high values of diffusivity are found when stratification is weak and low values found when stratification is strong. The diffusivity of salt is parameterized in the same manner, with lower values where stratification is stronger (Figure 1):

$$\kappa_S(z)=\kappa_{0\mathrm{S}}(1+\beta_S\tanh(\delta_S\frac{\partial\rho}{\partial z}))$$

The heat flux term QT uses the Haney relaxation method (Haney, 1972) and is a function of the air-water temperature difference

$$QT=\gamma(Tsurface-Tair)$$

where the coupling variable γ acts as a damping time scale with units of inverse time and determines the strength of the coupling between the air and the surface of the lake. Large values indicate weak coupling while lower values indicate strong coupling. Average daily temperature data between 2008 to 2011 from a nearby weather station was used for input into the heat flux equation. The initial surface temperature is given from CTD casts taken at the lake and the remaining surface temperature values are calculated from the prior time step and the associated heat flux.

The salinity flux term QS is a function of the surface salinity, evaporation, precipitation, and the surface layer depth in which the salinity flux acts:

$$QS=S\frac{(E-P)}{Surface\,Layer\,Depth}$$

Evaporation is calculated via an energy balance method (Vallet-Coulomb et al., 2001):

$$E=\frac{R_{net}}{\lambda(\beta+1)}$$

where R_{net} is the net radiation on the surface of the lake, λ is the latent heat of vaporization of water, and β is the Bowen ratio of sensible heat flux to latent heat flux. Net radiation values were determined from nearby weather station data and the Bowen ratio was chosen from a review of the limnological literature. This method gives evaporation of approximately 3000mm per year. Precipitation values were also taken from local weather station data. The surface layer depth was taken as 2 meters. The salinity value used in the flux term is from the prior time step, with the initial surface salinity value given from CTD casts at the lake. Since real data is being used for precipitation and the true evaporation rate is unknown, the mean has been taken out of the salt flux values so that trends toward saline or fresh water can be prevented.

The lower boundary conditions for temperature, salinity, and density are that they are both constant. The CTD casts indicate that these variables are nearly constant throughout the water column below 10 meters indicating that constant values at the bottom of the lake would be applicable and realistic.

Model Parameters

The model has 250 vertical grid points coinciding with 1 meter grid spacing. A convenient time step of one hour was chosen such that the stability criteria for diffusion models was satisfied, i.e.

$$\frac{\kappa\Delta t}{\Delta z^2}\leq\frac{1}{2}$$

Diffusivity values remain sufficiently small so that a stable time step can be ensured. A variable time step was considered but not used in this model. The run times for the model presented here were 4 years and 12 years. Discretization of the model equations was implemented through a forward in time, centered in space discretization scheme.

Results

After numerous model runs using various values for the calibration variables, precipitation, evaporation, run time, and other inputs to the model, some interesting effects can be seen. Two model runs will be presented here. The seasonal cycle of the surface and its diffusion into subsurface layers can clearly be seen in Hoffmuller diagrams of both model runs. The effects of precipitation on the surface salinity are also clear as a freshening of the surface waters occurs during times of heavy rainfall. Density is seen to be a stronger function of salinity than density and fluctuates much more with the salinity fluctuations rather than the fluctuations in temperature. The diffusion of salt is much less apparent than heat which is expected given its much lower value. Hoffmuller diagrams of heat diffusivity and bouyancy frequency show that when stratification is high, as is the case near the surface, the diffusivity drops. The effects combined yield average vertical profiles that closely resemble the CTD casts collected at the lake which somewhat validates that the model can produce realistic results (Figure 2, model values in red, collected data in blue).

The four year model run (Figure 3) used real data from 2008 to 2011 in an attempt to get as close as possible to the environment experienced at the lake in recent years. Hopefully, signs of an overturning event, such as a drastic decrease in density stratification, would emerge, however the model needed some adjustment to produce those signs. Precipitation values at the end of the time period dropped and the model does show some decrease in density in the surface layers during that time, but only slightly. To increase the effect of the salt flux to see how this may affect the stratification, the amplitude of the salt flux was increased 70%. This value was chosen so that it was high enough to produce a noticeable effect without bringing salinity values below zero. The increased amplitude does show that an extreme event of precipitation or drought can have a drastic effect on the stratification (Figure 4). To test the effects of drought another way, the model was run for 12 years (Figure 5), but with the last 8 years without precipitation. This had the effect of making the stratification much weaker at the surface as can be seen in Hoffmuller diagrams of the bouyancy frequency (Figure 6). Although this configuration is extreme, there are many aspects of this model that are not know to be accurate, such as diffusivity. Hopefully this model shows that an overturning event may be possible at Kauhako Crater Lake given a set of extreme circumstances.

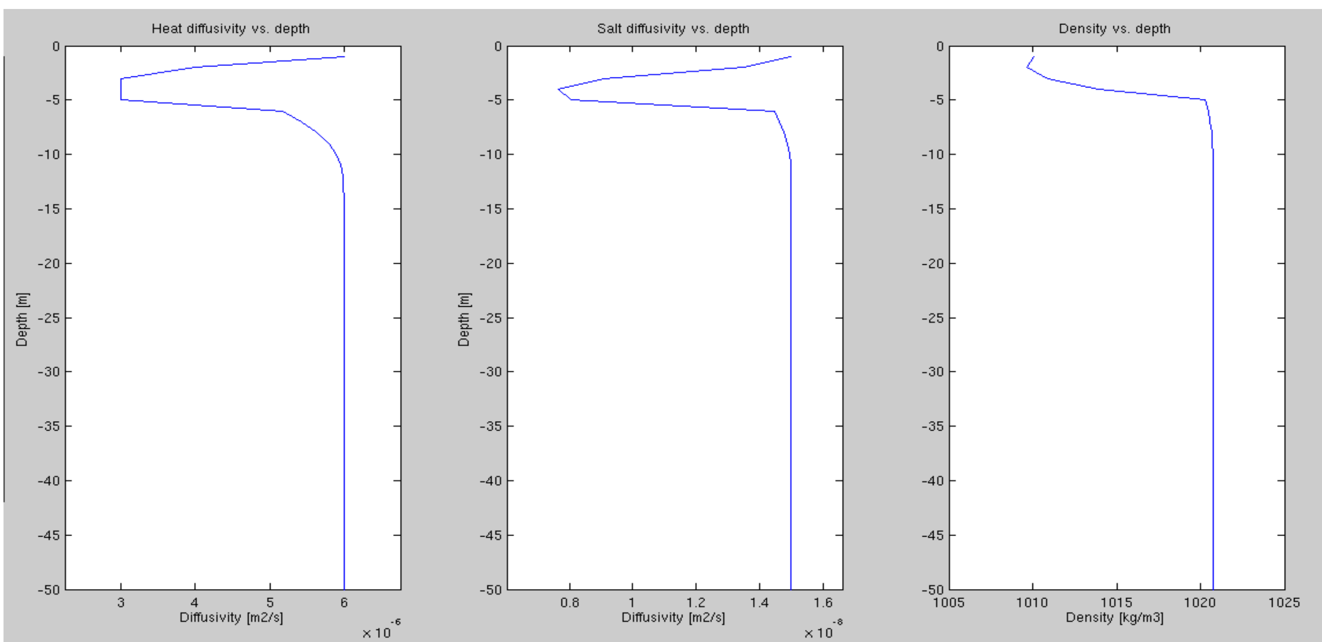

Figure 1

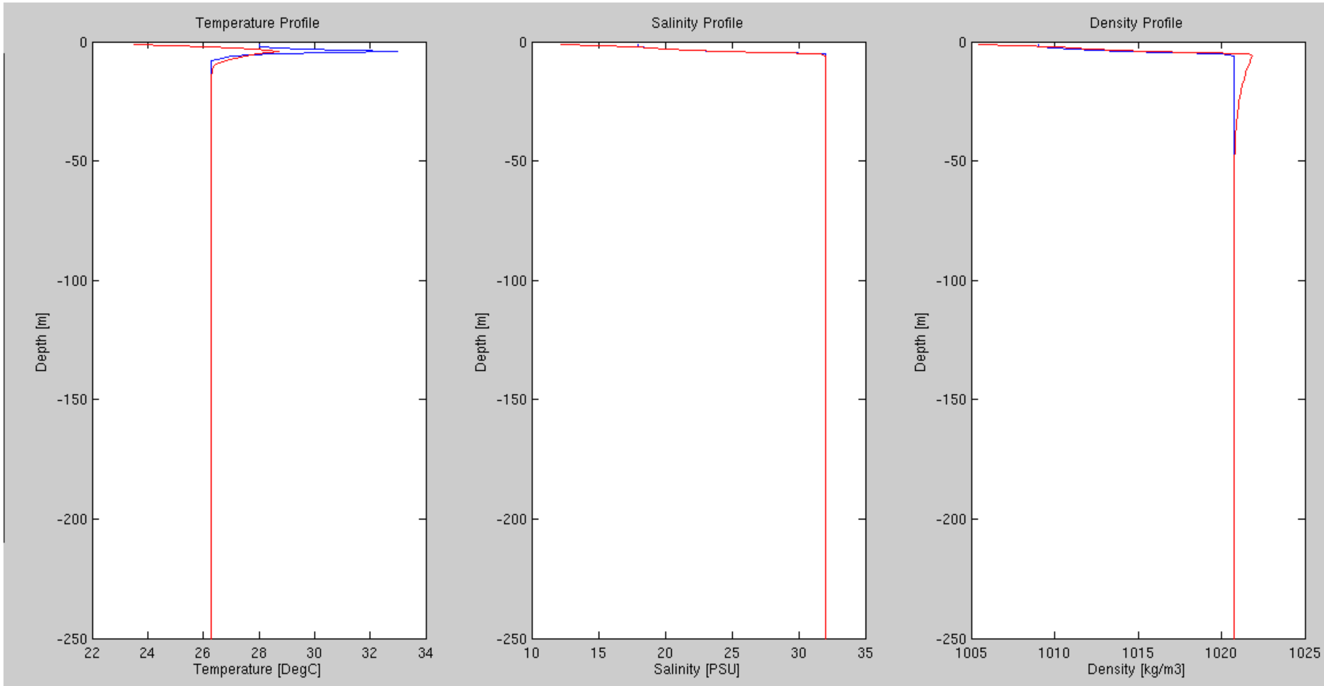

Figure 2

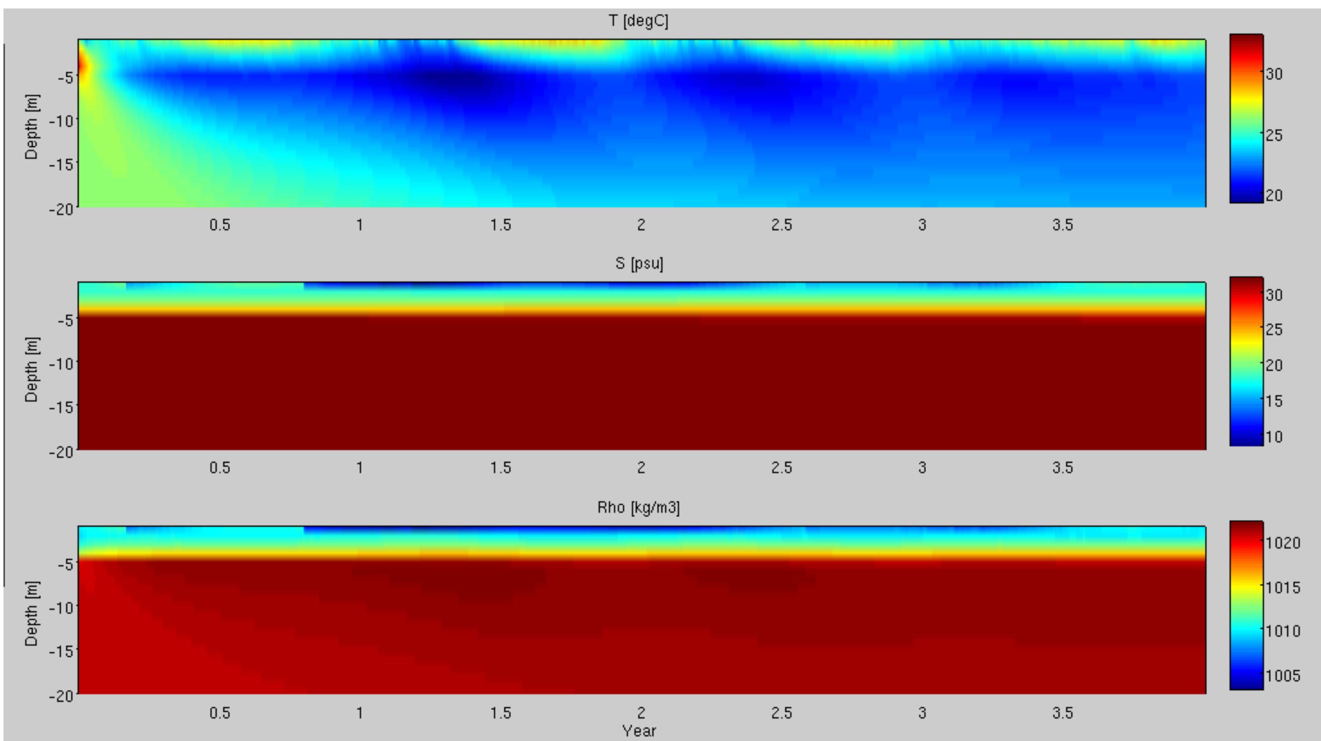

Figure 3

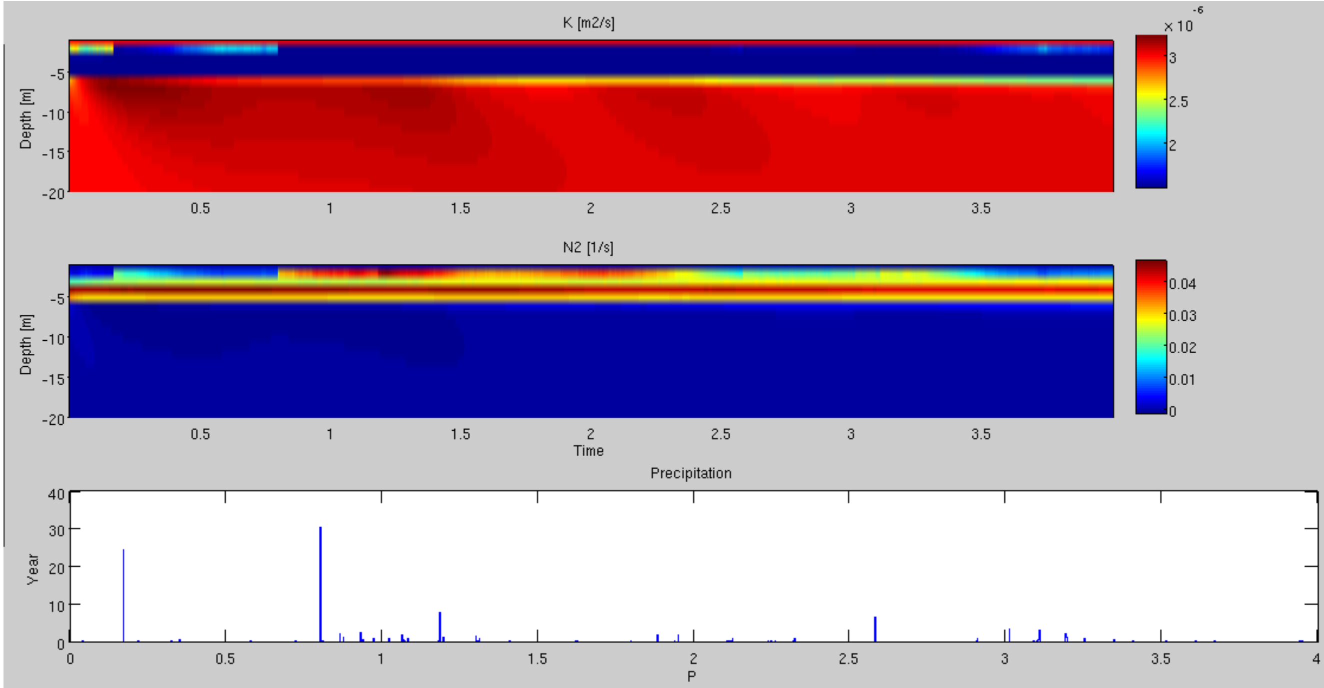

Figure 4

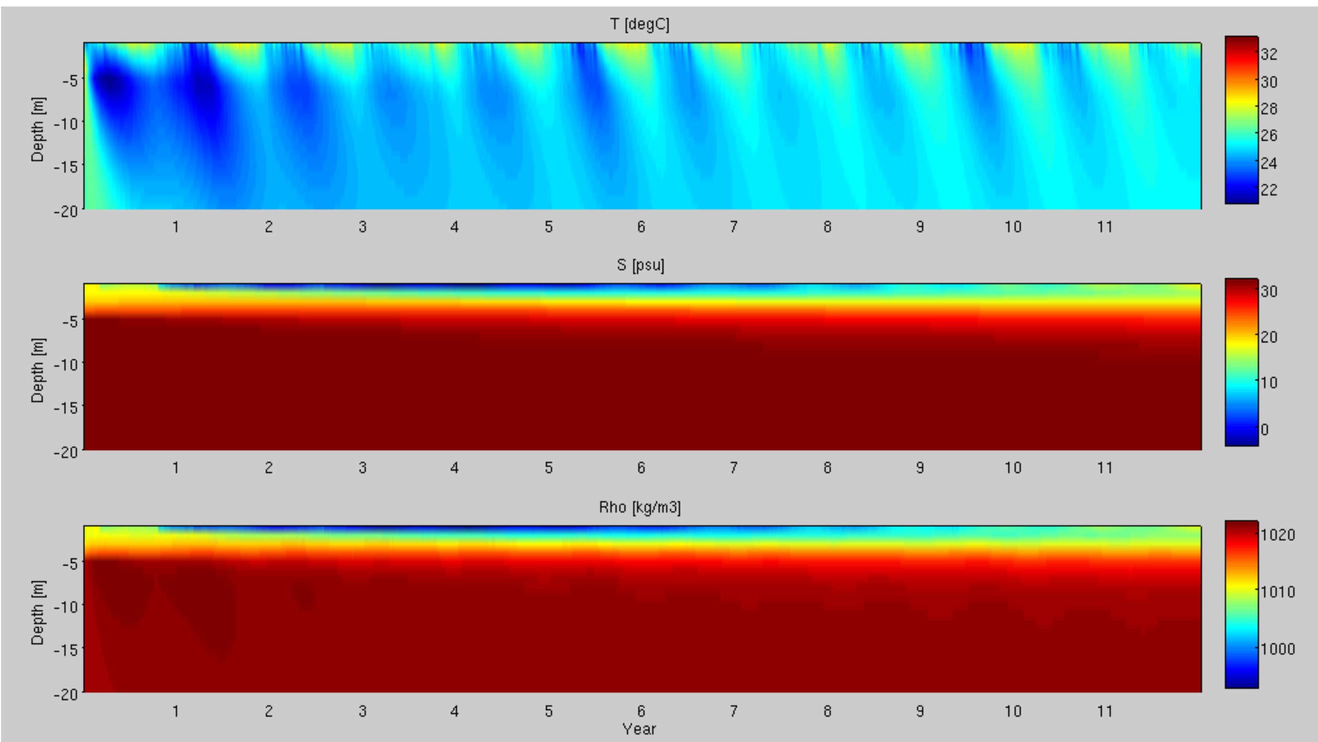

Figure 5

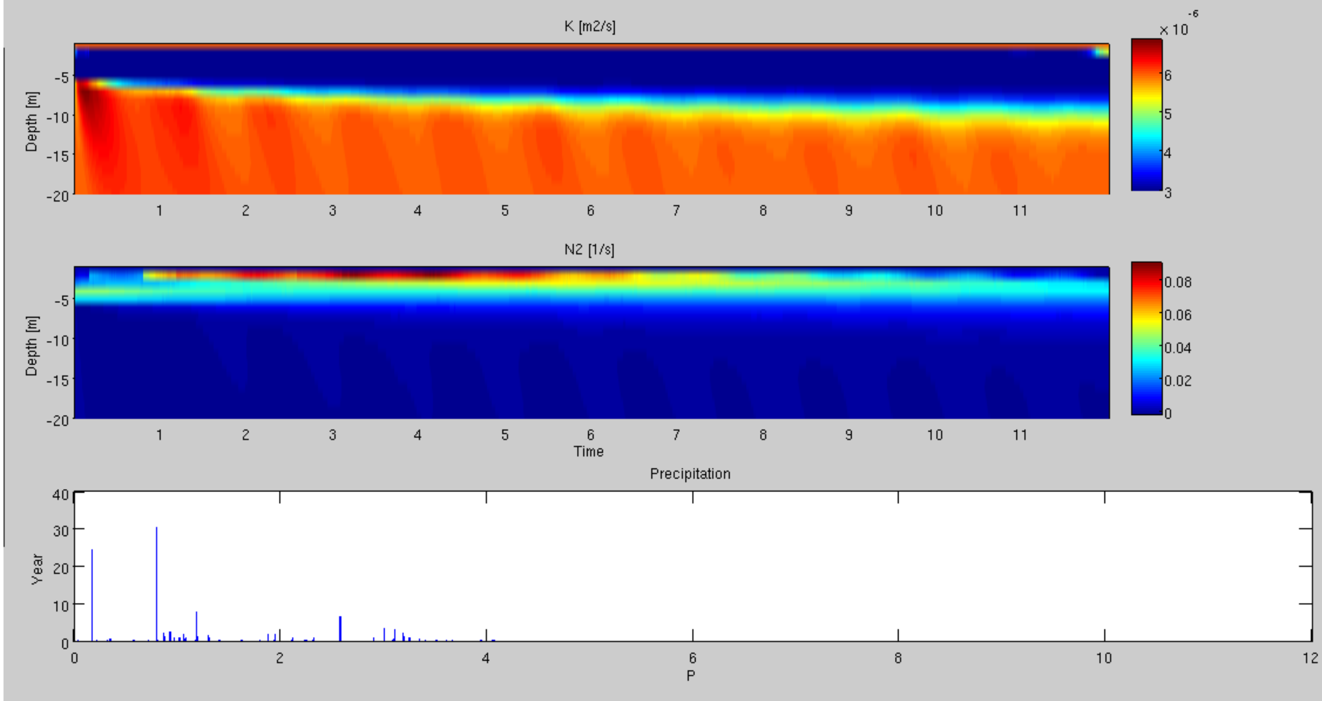

Figure 6

References

Haney, R. 1971. Surface thermal boundary condition for ocean circulation models. Journal of Physical Oceanography, 1, 241-248.

Vallet-Coulomb, C., Legesse, D., Gasse, F., Travi, Y., & Chernet, T. 2001. Lake evaporation estimates in tropical Africa (Lake Ziway, Ethiopia). Journal of Hydrology, 245, 1-18.