

---

EECS 16A      Designing Information Devices and Systems I  
Fall 2019                      Midterm 2 - VERSION A

---

Exam Location: Latimer 120 / VERSION A

PRINT your student ID: \_\_\_\_\_

PRINT AND SIGN your name: \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_  
(last name) (first name) (signature)

PRINT time of your Monday section and the GSI's name: \_\_\_\_\_

PRINT time of your Wednesday section and the GSI's name: \_\_\_\_\_

Name and SID of the person to your left: \_\_\_\_\_

Name and SID of the person to your right: \_\_\_\_\_

Name and SID of the person in front of you: \_\_\_\_\_

Name and SID of the person behind you: \_\_\_\_\_

1. Tell us about something that makes you happy. (1 Point)

Netflix

2. What courses are you thinking of taking next semester? (1 Point)

Math 104, Math 185 and a CS class.

Do not turn this page until the proctor tells you to do so. You may work on the questions above.

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
Work on this page will NOT be graded.

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
**Work on this page will NOT be graded.**

PRINT your student ID: \_\_\_\_\_

### 3. Power (15 points)

In the circuit shown in Figure 3.1 calculate the total power  $P_d$  delivered by source  $I_1$  to the rest of the circuit.

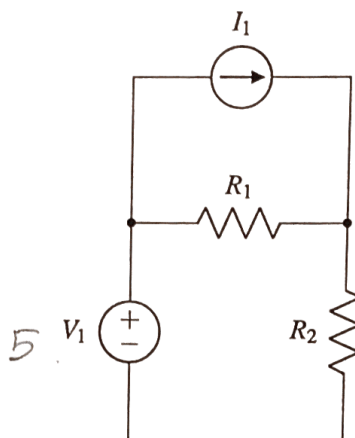


Figure 3.1: Circuit for power calculation

Use the following component values:  $R_1 = 2\Omega$ ,  $R_2 = 2\Omega$ ,  $V_1 = 5V$ ,  $I_1 = -0.5A$ .

$$\begin{aligned}
 R_1 + R_2 &= 2 + 2 = 4\Omega. & V &= IR. \\
 \therefore I_{Th} &= \frac{V}{R} = \frac{5}{4} = \underline{\underline{1.25A}}. \\
 I_1 &= -0.5 \\
 R_1 &= 2. \\
 V &= IR = -1. \\
 \therefore P_{I_1} &= VI = (-0.5)(-1) \\
 &= \underline{\underline{0.5W}}.
 \end{aligned}$$

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
Work on this page will NOT be graded.

PRINT your student ID: \_\_\_\_\_

#### 4. Equivalent Circuit (15 points)

Your friend has characterized the circuit shown in Figure 4.1(a) in the lab by first connecting a voltmeter (represented in Figure 4.1(b) by the letter V in a circle) to terminals A – B to measure  $V_m$ , then disconnecting the voltmeter and connecting an ammeter (current meter, represented in Figure 4.1(b) by the letter A in a circle) to measure  $I_m$ .

Now they ask for your help designing an equivalent circuit model (looking from terminals A and B) consisting only of a current source  $I_s$  and resistor  $R_s$ .

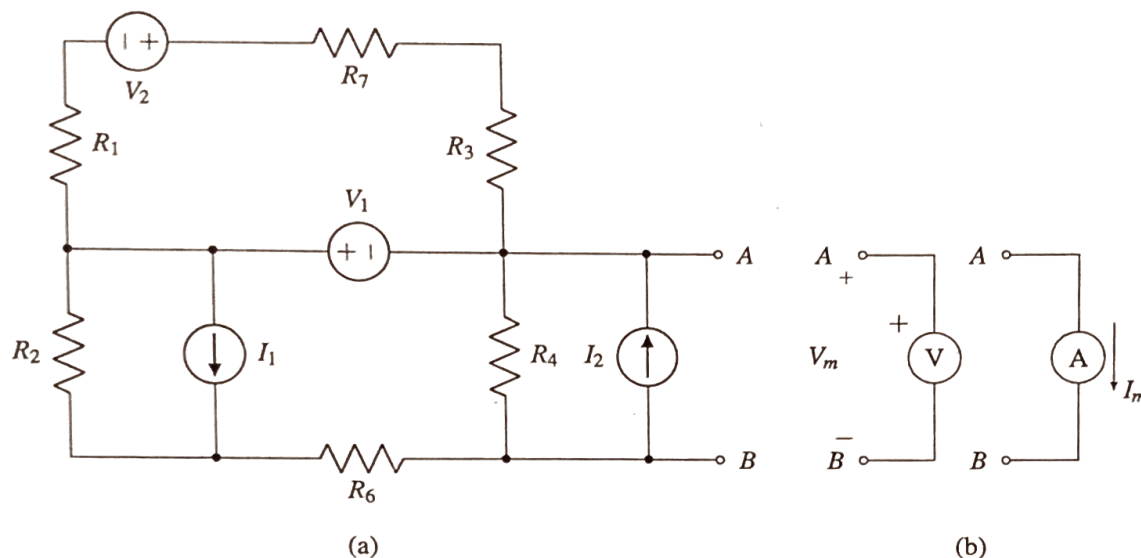
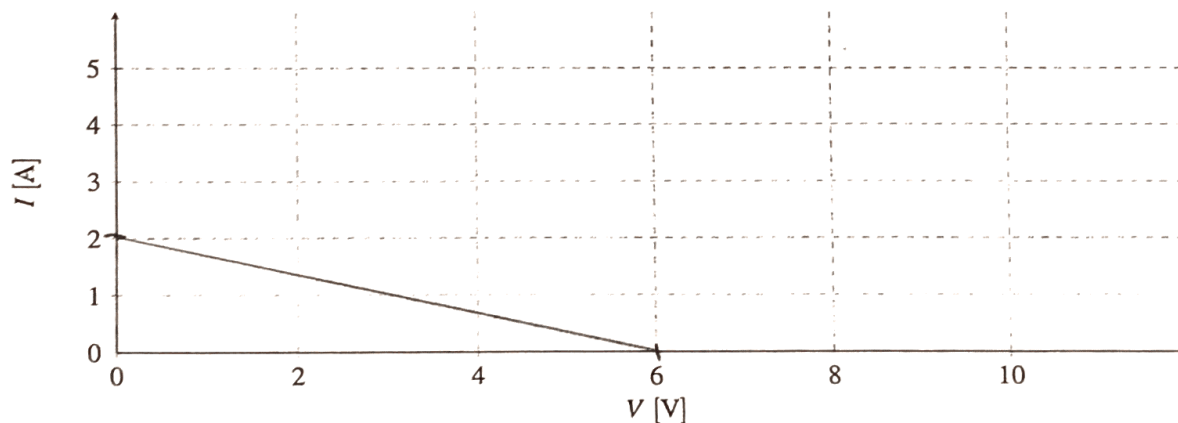


Figure 4.1: (a) Circuit tested in the lab, (b) Circuit used to measure  $V_m$  and  $I_m$

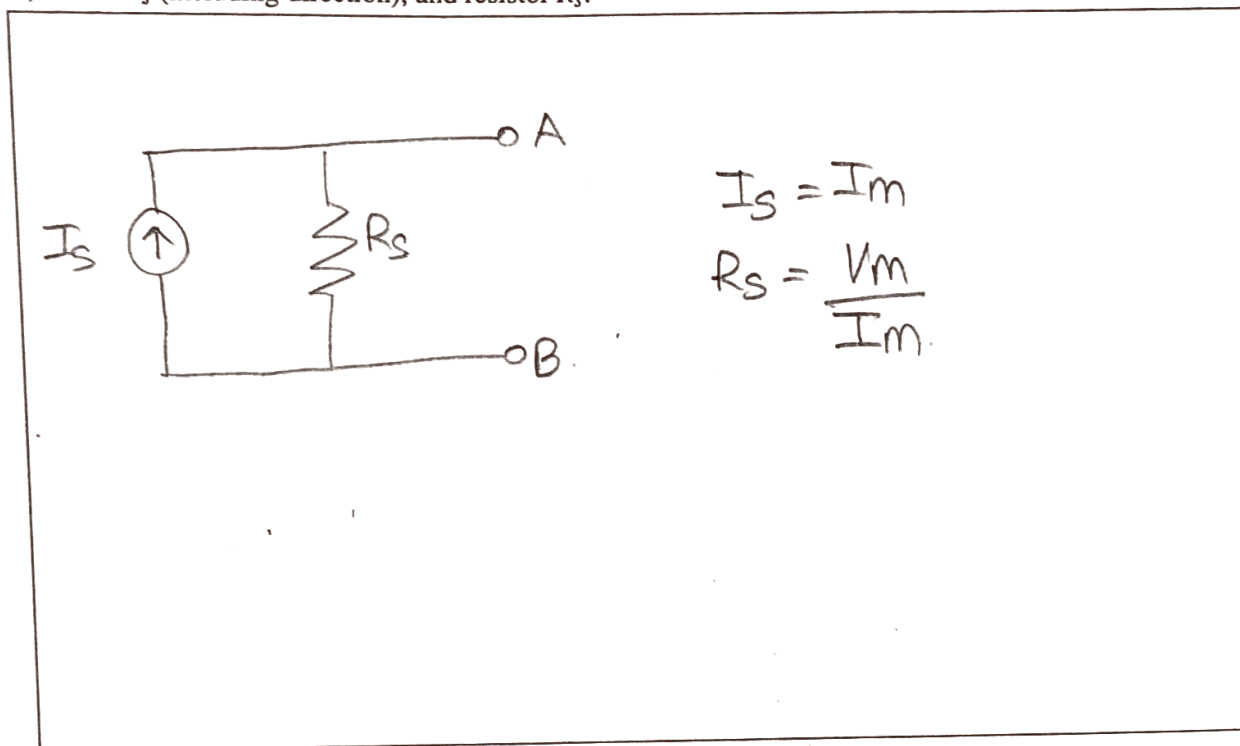
Use the following values:  $V_m = 6\text{V}$ ,  $I_m = 2\text{A}$ .

(a) Draw the  $I$ - $V$  characteristic between nodes A, B of the circuit in Figure 4.1(a).



PRINT your student ID: \_\_\_\_\_

- (b) Draw the circuit diagram of the equivalent model as seen from nodes  $A$ ,  $B$ . Clearly mark nodes  $A$  and  $B$ , current  $I_s$  (including direction), and resistor  $R_s$ .



- (c) Calculate the numerical values of  $I_s$  and  $R_s$ . Mark your calculated component values in the circuit diagram drawn in part (b). Clearly mark the direction of the current.

$$I_s = I_m = \underline{\underline{2A}}$$

$$R_s = \frac{V_m}{I_m} = \frac{6}{2} = \underline{\underline{3\Omega}}$$

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
**Work on this page will NOT be graded.**



PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
Work on this page will NOT be graded.

PRINT your student ID: \_\_\_\_\_

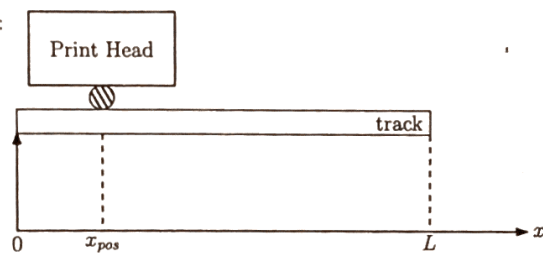
### 5. Next-Phone (15 points)

You have been hired by "Next-Phone", a promising startup that has developed a 3D printer to produce individually customized smartphones.

Only one problem remains: designing accurate position sensing for the printhead. "No problem," you tell your new boss, "I'll take care of that!"

Figure 5.1 shows your design. The printhead is supported by two rollers that move the head in the  $x$  direction. Each roller runs on two conductive tracks with resistivities  $\rho_1$  and  $\rho_2$ , respectively, length  $L$ , and cross-sectional area  $A$ . The rollers are made of metal electrically connecting the strips. The printhead is an insulator (i.e. nonconductive material) so it can be modeled as an open-circuit. You connect two voltage sources of voltage  $V_s$  as shown in Figure 5.1. You then measure voltage  $V_{AB} = V_A - V_B$  to sense position  $x$ .

Side View (cross section):



Top View:

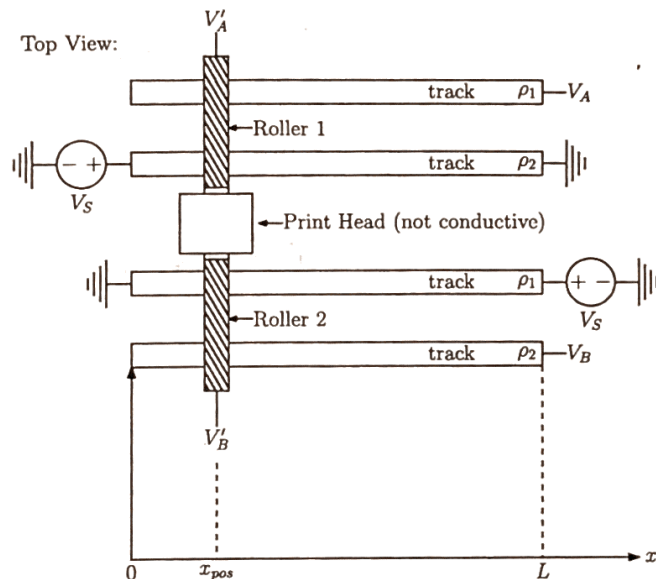
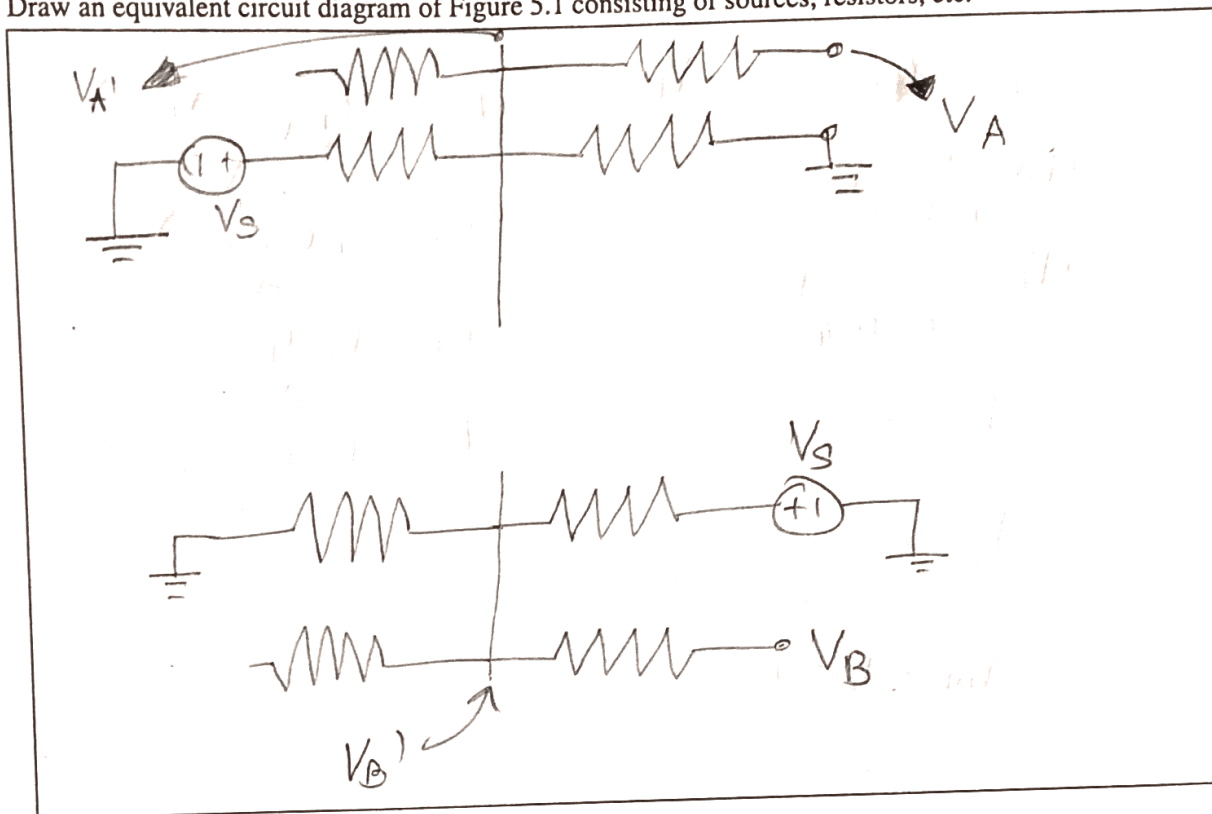


Figure 5.1: 3D printhead position sensor (top and side views)

PRINT your student ID: \_\_\_\_\_

- (a) Draw an equivalent circuit diagram of Figure 5.1 consisting of sources, resistors, etc.



- (b) Derive algebraic expressions for  $V_{AA'} = V_A - V_{A'}$  and  $V_{BB'} = V_B - V_{B'}$  as a function of  $x_{pos}$ .

No current flows to R connected to  $V_A$  since not connected to anything. Same for  $V_{A'}$ . This is an open circuit  
 $\therefore V_{AA'} = V_A - V_{A'} = \underline{\underline{0}}$        $V_{BB'} = V_B - V_{B'} = \underline{\underline{0}}$

PRINT your student ID: \_\_\_\_\_

- (c) Find the value of voltage  $V_{AB}(x_{pos}) = V_A - V_B$  for  $V_s = 10V$ ,  $\rho_1 = 1 \Omega m$ ,  $\rho_2 = 2 \Omega m$ ,  $L = 200mm$ ,  $A = 1cm^2$ , and  $x_{pos} = 50mm$ .

$$V_{AB}(x_{pos}) = V_A - V_B$$

$$\bullet V_A = \frac{R_1}{R_1 + R_2} V_s$$

$$\bullet V_B = \frac{R_3}{R_3 + R_4} V_s$$

$$\therefore V_A = \frac{L - x_{pos}}{L} V_s$$

$$\therefore V_B = \frac{x_{pos}}{L} V_s$$

$$\therefore V_{AB} = V_A - V_B$$

$$= \frac{L - 2x_{pos}}{L} V_s$$

$$= \frac{(200 \times 10^{-3}) - 2(50 \times 10^{-3})}{(1 \times 10^{-2})} = \boxed{5V}$$

$$\bullet R_3 = \frac{\rho_1 (L - x_{pos})}{A}$$

$$\bullet R_4 = \frac{\rho_1 (x_{pos})}{A}$$

$$\bullet R_1 = \frac{\rho_2 x_{pos}}{A}$$

$$\bullet R_2 = \frac{\rho_2 (L - x_{pos})}{A}$$

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
Work on this page will NOT be graded.

PRINT your student ID: \_\_\_\_\_

### 6. Current Sensor (19 points)

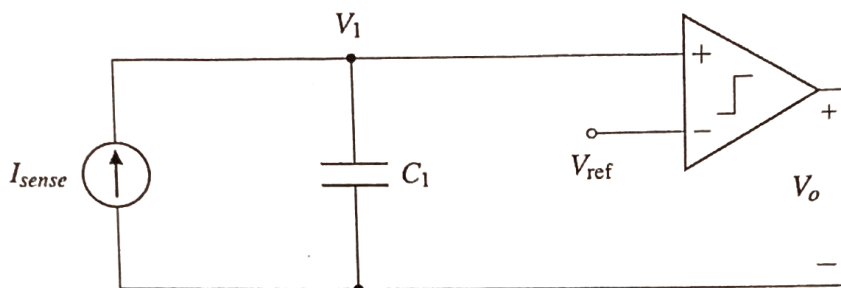


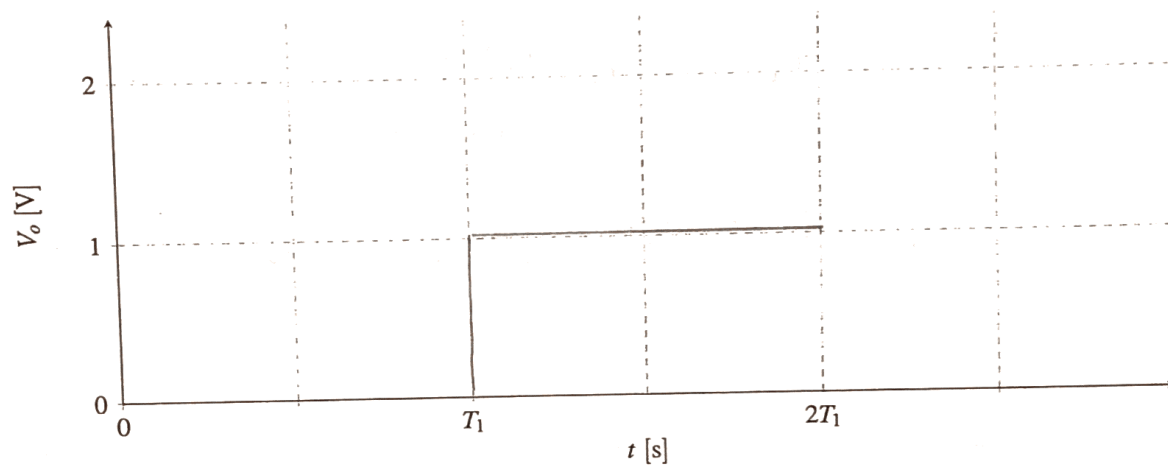
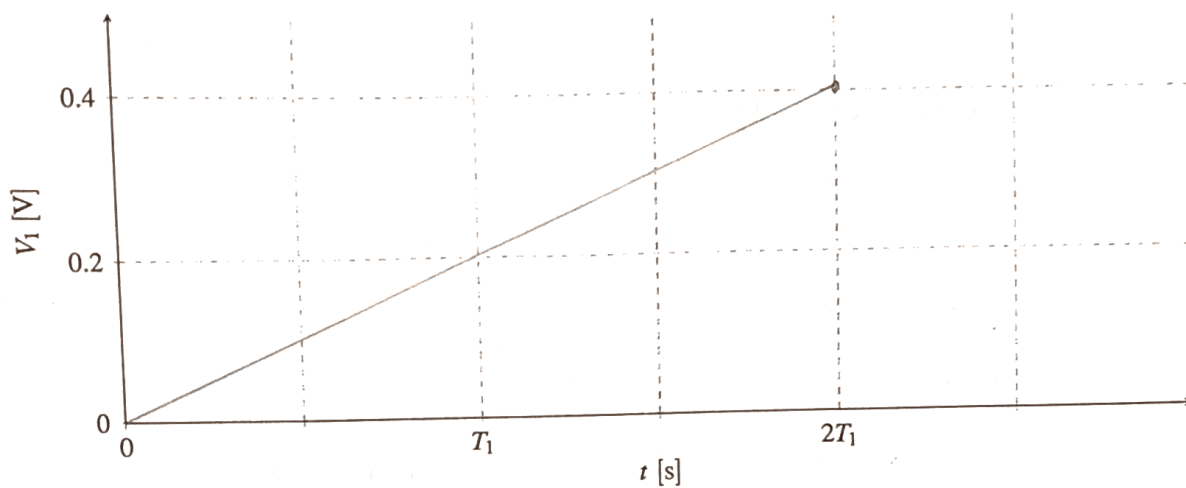
Figure 6.1: First version of the current sensor circuit

- (a) You've built a light sensor that outputs a current  $I_{sense}$  that is proportional to light intensity. Now, you need a circuit that measures this current. Figure 6.1 shows the design of the first version of the current sensor circuit. At time  $t = 0$  capacitor  $C_1$  is discharged and a timer (stop watch) is started. When voltage  $V_1$  reaches  $V_{ref}$  the comparator "trips" and stops the timer at  $T_1$ .

In the grids below, plot  $V_1$  and  $V_o$  as a function of time  $t$ , from  $t = 0$  to  $t = 2T_1$ .

Use  $I_{sense} = 0.2\text{A}$ ,  $C_1 = 1\text{F}$ , and  $V_{ref} = 0.2\text{V}$ .

PRINT your student ID: \_\_\_\_\_



PRINT your student ID: \_\_\_\_\_

- (b) Derive an expression for the time  $T_1$  measured by the timer as a function of  $I_{sense}$ ,  $C_1$ , and  $V_{ref}$ . Then, calculate its numerical value using the same component values as given in part (a).

time:  $(0, T_1)$

$$I_{sense} = \frac{C \, dV_c}{dt}$$

$$\int \frac{I_{sense}}{C} \cdot dt = \int_0^{V_c(T_1)} dV_c$$

$$\frac{I_{sense} \cdot T_1}{C} = V_c(T_1) - 0$$

$$T_1 = \frac{V_c(T_1) \cdot C}{I_{sense}}$$

$$\therefore T_1 = \frac{V_{ref} \cdot C}{I_{sense}} = \boxed{1}$$



PRINT your student ID: \_\_\_\_\_

(c) Figure 6.2 shows an improved current sensor that does not depend on the value of capacitor  $C_1$ .

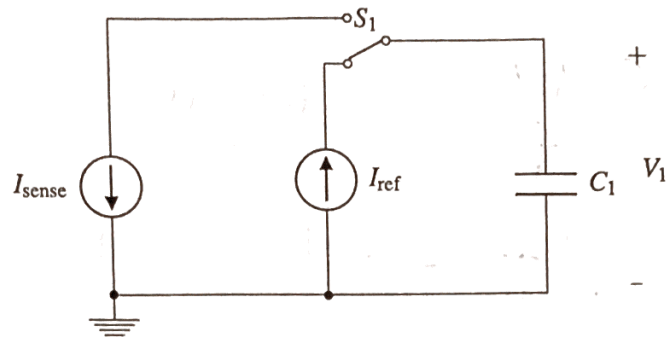
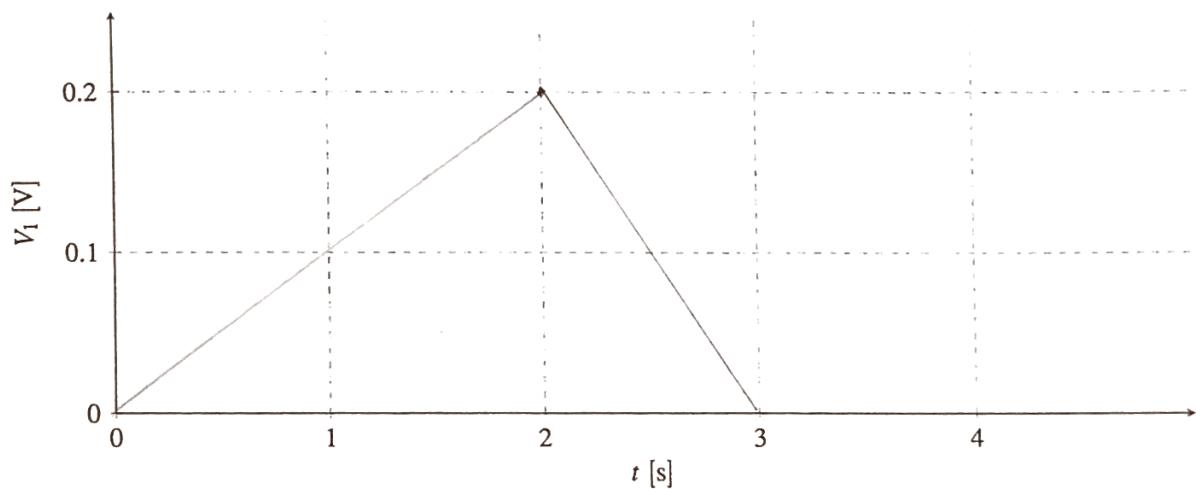


Figure 6.2: Improved Current Sensor

Capacitor  $C_1$  is discharged at time  $t = 0$  and switch  $S_1$  is connected to  $I_{\text{ref}}$ . When  $V_1$  reaches  $V_{\text{ref}}$  at time  $T_1$ , switch  $S_1$  is disconnected from  $I_{\text{ref}}$  and connected to  $I_{\text{sense}}$  instead.  $T_2$  is defined as the time from connecting  $S_1$  to  $I_{\text{sense}}$  to the moment  $V_1$  reaches 0V.

Plot  $V_1$  versus time from  $t = 0$  to  $t = T_1 + T_2$ .

Use  $I_{\text{sense}} = 0.2\text{A}$ ,  $C_1 = 1\text{F}$ ,  $V_{\text{ref}} = 0.2\text{V}$  and  $I_{\text{ref}} = 0.1\text{A}$ .



PRINT your student ID: \_\_\_\_\_

- (d) Derive an expression for the ratio  $\frac{T_1}{T_2}$  (the times calculated in part (c)) as a function of  $I_{sense}$ ,  $C_1$ , and  $V_{ref}$ . Then, calculate its numerical value using the same component values as given in part (c).

$$T_1 = \frac{C_1 V_{ref}}{I_{ref}} \leftarrow \text{charging}$$

$$T_2 = \frac{C_1 V_{ref}}{I_{sense}} \leftarrow \text{Discharging}$$

$$\frac{T_1}{T_2} = \boxed{\frac{I_{sense}}{I_{ref}}}$$

$$= \boxed{2}$$

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
Work on this page will NOT be graded.

PRINT your student ID: \_\_\_\_\_

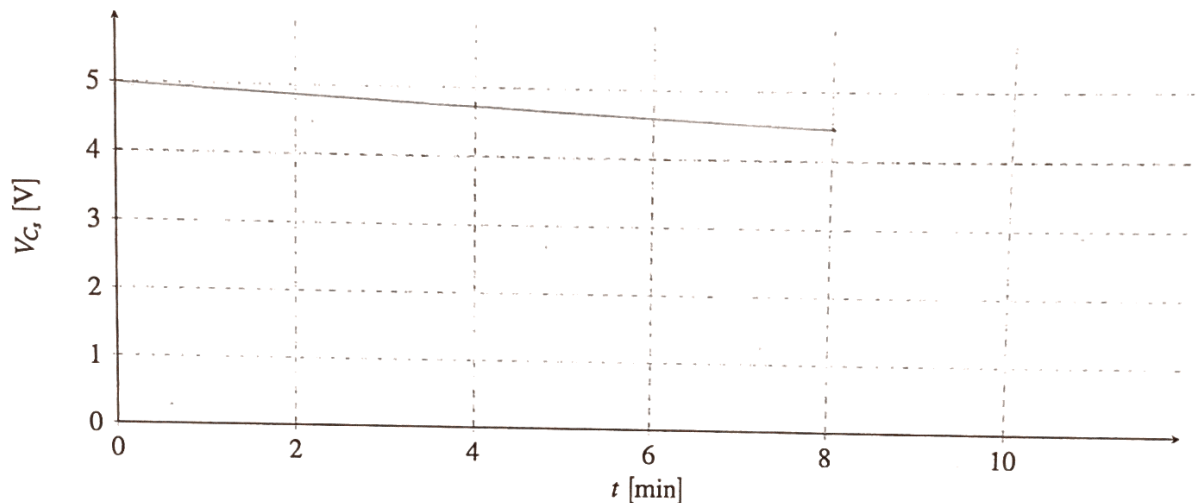
### 7. Capacitor Powered Quadcopter (15 points)

You've made a fun little quadcopter that juggles 10 colorful balls - all while flying in circles above the heads of its stunned audience that cannot get enough of the spectacle.

Unfortunately it takes quite a bit of time to recharge the battery after each demonstration. To shorten the time you decide to replace the battery with a capacitor  $C_s$  which can be charged virtually instantaneously.

The drone consumes a constant current  $I_d = 0.5\text{A}$ . The nominal supply voltage of the drone is  $V_{nom} = 5\text{V}$ , but it works with voltages as low as  $V_{min} = 4\text{V}$  (i.e. when the capacitor voltage drops below  $4\text{V}$  the drone crashes).

- (a) Plot the voltage across the capacitor  $V_{C_s}$  as a function of time from 0 to 10 minutes in the graph space provided to you below. Use  $C_s = 600\text{F}$ .



PRINT your student ID: \_\_\_\_\_

- (b) Calculate the minimum value of the capacitor  $C_s$  required to support 10 minutes of flying time.

$$I_d = C_s \frac{dV_o}{dt} = C_s \frac{\Delta V}{\Delta t}$$

$$\begin{aligned} \therefore \Delta t &= C_s \frac{\Delta V}{I_d} \\ &= \frac{C_s (V_{nom} - V_{min})}{I_d} \end{aligned}$$

$$\therefore 600 = \frac{(C_s) (5 - 4)}{0.5}$$

$$\therefore C_s = \underline{\underline{800F}}$$

- (c) Regardless of your answer in (b), assume  $C_s = 5F$ . Calculate the ratio of the energy  $E_2$  remaining in the capacitor at the end of the flight divided by the energy  $E_1$  initially stored in the capacitor  $C_s$  when it is fully charged.

$$E = \frac{1}{2} C V^2$$

$$\frac{E_1}{E_2} = \frac{\frac{1}{2} C_s V_{min}^2}{\frac{1}{2} C_s V_{nom}^2} = \frac{16}{25} = \underline{\underline{0.64}}$$

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
**Work on this page will NOT be graded.**

PRINT your student ID: \_\_\_\_\_

**8. Fun with charge sharing (19 points)**

- (a) In Figure 8.1, capacitors  $C_1$  and  $C_2$  are charged to  $V_1$  and  $V_2$  and switch  $S_1$  is open for time  $t < 0$ . At time  $t = 0$ , switch  $S_1$  is closed. Calculate  $V_1$  at time  $t > 0$ .

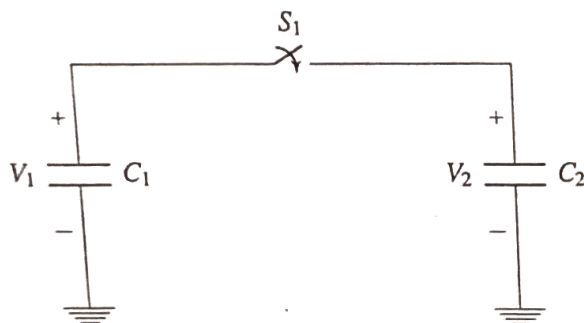


Figure 8.1: Capacitor Charge Sharing

Use the following values:  $C_1 = 1\text{F}$ ,  $C_2 = 4\text{F}$ ,  $V_1 = 6\text{V}$ ,  $V_2 = 1\text{V}$ .

Phase Initial  $\rightarrow Q_1^i = C_1 V_1$   
 $Q_2^i = C_2 V_2$

Phase final  $\rightarrow Q_{\text{total, final}} = Q_1^f + Q_2^f$

$$\therefore Q_1^f + Q_2^f = C_1 V_1 + C_2 V_2$$

$$C_1 V_1^f + C_2 V_2^f = C_1 V_1 + C_2 V_2$$

$$V_1^f = V_2^f$$

$$\therefore C_1 V_1^f + C_2 V_2^f = C_1 V_1 + C_2 V_2$$

$$\therefore V_1^f = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} = \frac{6 + 4}{5} = \underline{\underline{2\text{V}}}$$

PRINT your student ID: \_\_\_\_\_

- (b) The circuit shown in Figure 8.2 operates in two phases. During phase 1, switches labeled  $S_1$  are closed and switches  $S_2$  are open. During phase 2, switches  $S_1$  are open and switches  $S_2$  are closed, as illustrated in the timing diagram shown in Figure 8.3.

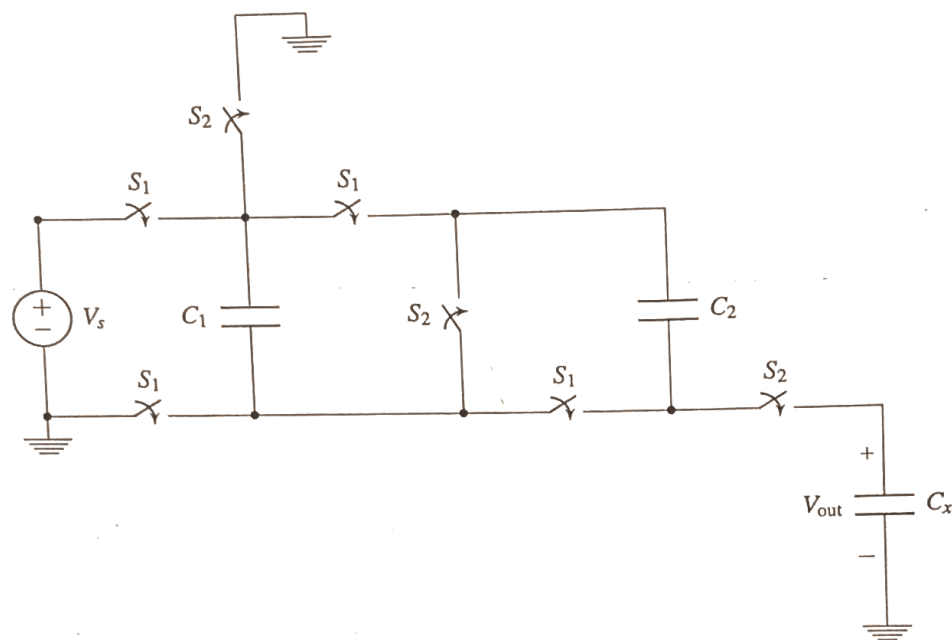


Figure 8.2: Capacitor Charge Sharing

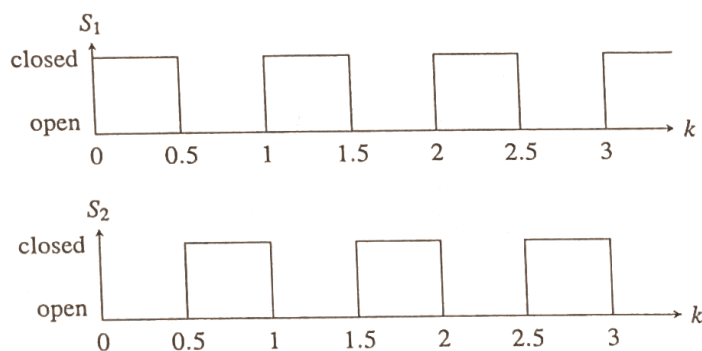


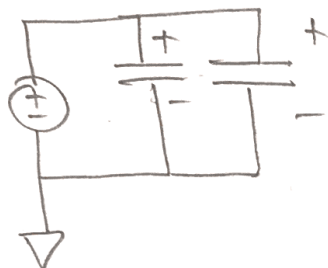
Figure 8.3: Timing diagram for switches



PRINT your student ID: \_\_\_\_\_

- i. Redraw the circuit during phase 1. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use  $C_1 = C_2 = C_0$ .

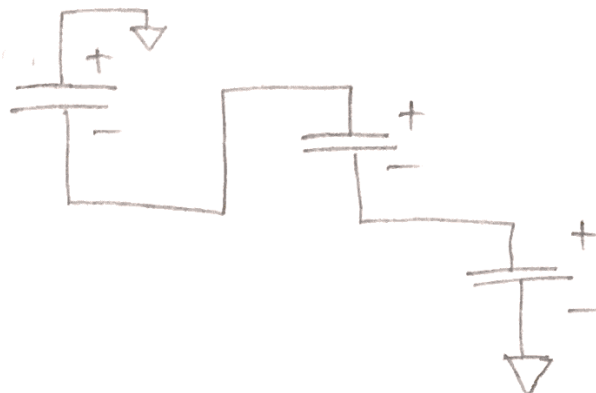
Phase 1



PRINT your student ID: \_\_\_\_\_

- ii. Redraw the circuit during phase 2. Replace closed switches with "wires" and open switches with "open circuits" (i.e. just omit them from the diagram). Use  $C_1 = C_2 = C_0$ .

Phase 2



PRINT your student ID: \_\_\_\_\_

iii. Calculate the value of the voltage  $V_{out}$  during phase 2 as a function of  $C_0$ ,  $C_x$ , and  $V_s$ .

$$Q_{C_1} = C_1 V_1$$

$$Q_{C_2} = C_2 V_s$$

$$Q_{total 1} = C_2 V_s - C_1 V_s$$

$$Q_{total 2} = -C_2 V_s$$

$$\therefore V_{out} = \frac{-2C_1 C_2 V_s}{C_1 C_x + C_1 C_2 + C_2 C_x} = \frac{-2V_s}{1 + \frac{2C_x}{C_0}}$$

PRINT your student ID: \_\_\_\_\_

Extra page for scratchwork.  
Work on this page will NOT be graded.

PRINT your student ID: \_\_\_\_\_

Doodle page!

Draw us something if you want or give us suggestions, compliments, or complaints.  
You can also use this page to report anything suspicious that you might have noticed.