



## Modelling and solving the milk collection problem with realistic constraints

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### ARTICLE INFO

**Keywords:**

Milk collection problem  
Vehicle routing problem  
Optimization  
Variable neighborhood search

### ABSTRACT

The milk collection problem (MCP) is concerned with the collection of raw milk of varying quality at dairy factories via tankers under problem-specific constraints. During this collection process, keeping milk of different levels of quality separate is at least as critically important as production quality because when the milk of different qualities is mixed together, the worst quality determines the final milk quality. In MCP, decisions such as which farms/milk collection centers, quality of milk, types of tankers, storage tanks, and visiting sequences will be used are made. In this study, an integrated mathematical model is proposed for the first time that aims to minimize the total distance and total network costs for tanker assignments and routing problems by simultaneously considering realistic routing, incompatibility, and loading constraints. The problem was formulated as a mixed-integer linear program and the small instances were solved using CPLEX. To solve the larger-scale real-life problems, a variable neighborhood search (VNS) metaheuristic optimization framework is developed. The proposed mathematical model and the VNS framework were evaluated on scenarios based on real-life data from a dairy company. Computational results show that the proposed VNS framework solves the realistic MCP problem efficiently.

### 1. Introduction

The vast majority of milk produced around the world is supplied by small producers, who are often dispersed in rural areas and far from processing facilities. To process the raw milk obtained from different producers, it has to be collected from milk collection centers/farms and delivered to dairy companies by tankers that have special cold storage equipment. This kind of tanker has one or more (usually three to five) insulated tanks. Since the shelf life of raw milk is very short, it needs to be processed quickly after collection (Polat et al., 2021). Therefore, the service network, including a collection of the raw milk from the farms/collection centers and delivery of them to the dairy companies, should be designed effectively. When determining tanker routes, the main decisions are related to the sequence of visits by the tankers and which milk types will be loaded. These decisions are collectively known as the Milk Collection Problem (MCP) in the Operational Research (OR) literature. Due to raw milk being a highly perishable product, MCP is a crucial logistics planning problem for the dairy industry. An example milk collection network is given in Fig. 1 to better explain the structure of MCP. In this example, the network consists of five different routes for one dairy company, one farm, 14 milk collection centers, four different types of milk, and five tankers.

The Agricultural Outlook 2021–2030, published by the Food and Agriculture Organization (FAO), states that world milk production is increasing faster than that of most other agricultural commodities and will reach 1020 Mt by 2030 (OECD/FAO, 2021). According to the (OECD/FAO, 2021), world milk production is expected to grow at an annual rate of 1.7% over the next decade. It is projected that most of this growth is provided by the countries with a majority of small-scale producers. These producers are often dispersed in rural areas and different conditions could happen due to the nature of the problem (OECD/FAO, 2021).

In this study, the constraints in the structure of the milk collection problem are considered in three main groups: routing, incompatibility, and loading. Table 1 presents the constraint groups and their details.

Routing constraints refer to the constraints addressed within the scope of the main VRP and MCP. Among these are a heterogeneous vehicle fleet constraint (R1), due to the different capacities and number of tanks on the tankers, and a fixed fleet constraint (R2), due to the special requirements of the vehicles. In the milking operation, collection from the producers must occur within a specified time interval, requiring a time window constraint (R3). As the routes must be completed within a certain time, there is a time limit constraint (R4) for each route's duration. A divisible demand constraint (R5) makes it

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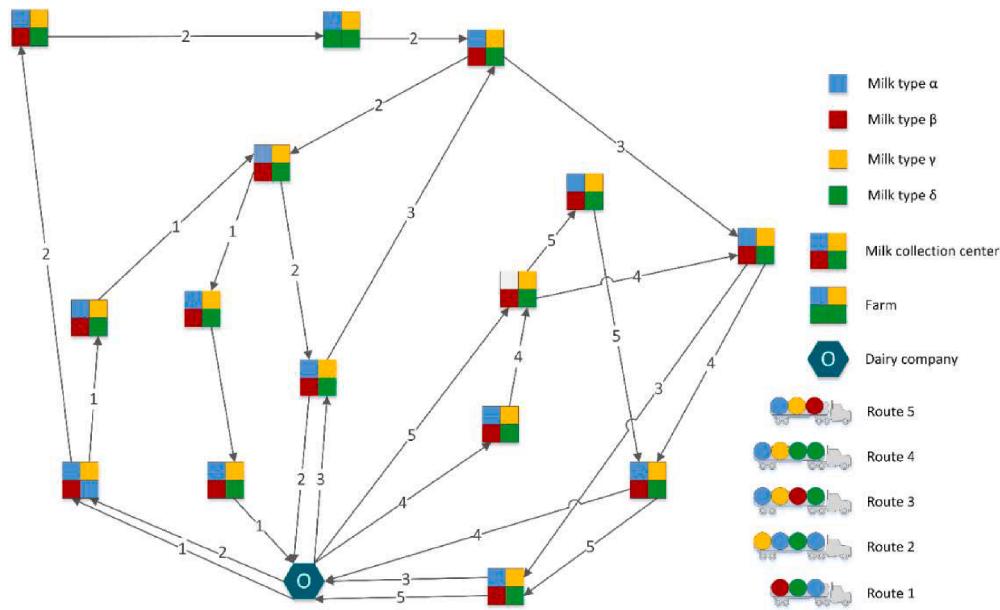


Fig. 1. Example milk collection network.

**Table 1**  
MCP constraints.

Routing Constraints	R1	Heterogenous Fleet
	R2	Fixed Fleet
	R3	Time Window
	R4	Time Limit
	R5	Divisible Demand
Incompatibility Constraints	I1	Location Requirement
	I2	Multi-Tank (Multi-compartment)
	I3	Multi Product
Loading Constraints	L1	Tank Loading Ratio
	L2	Fuel Consumption Rate

possible to load the milk onto different vehicles. Thus, a producer can be visited by more than one vehicle.

The structure of the problem is thus extended for real-life conditions by considering the incompatibility constraints. Due to geographical conditions, some customers can be visited only by certain types of vehicles; this situation is handled by the location requirement constraint (I1), which is within the incompatibility constraint group. The concepts of having more than one tank (I2) on a tanker and different types of milk (I3) are also considered in the Multi-Compartment VRP (MCVRP) literature. Unlike MCP, the MCVRP literature focuses on the determination of compartment sizes that are generally flexible for products that cannot be mixed, or where the number of products equals the number of compartments. The minimum and maximum loading ratio (L1), which is determined to ensure the driving safety of the tanker and the quality of the product, is expressed by the tank loading ratio constraint, which is within the loading constraint group. The fuel consumption rate mainly depends on the distance and the weight. The weight of the vehicle is inversely proportional to the distance traveled per unit volume of fuel used. The fuel consumption rate (L2) constraint considers the relationship between weight and a vehicle's fuel consumption.

In this study, an integrated mathematical model was developed that, for the first time, addressed routing, incompatibility, and loading restrictions that are inherent in milk collection. The remaining parts of this paper are structured as follows: Section 2 presents a literature review of the milk collection problem; section 3 describes an integrated mathematical model which includes real-life constraints; section 4 presents a metaheuristic optimization framework for MCP; section 5 presents a real-life case study to illustrate the model's and metaheuristic

approach's performance; and finally, section 6 shows the conclusions of this study and suggestions for future research.

## 2. Literature review

Fundamentally, MCP deals with the collection of raw milk of different quality levels from farms or milk collection centers located in different regions under problem-specific constraints via tankers. MCP essentially contains two sub-problems, the Tanker Assignment Problem (TAP), which includes assigning farms to tankers by considering the relevant constraints, and the Tanker Routing Problem (TRP), which determines the routes of the tankers. In the literature, MCP has been studied under different variants of VRP, such as location routing problems (LRP), rich vehicle routing problems (RVRP), and truck and trailer routing problems (TTRP).

An important part of the studies in the MCP literature addresses the problem through vehicle routing constraints without considering the incompatibility and loading constraints (Lahrichi et al., 2015; Masson et al., 2016; Rosales Vásquez and Correa Valencia, 2014). The studies included in this content used two different methods to obtain the solution. They either broke the problem down into sub-problems (assignment or routing) or else solved it sequentially (firstly assignment, then routing). While the solution time can be very fast with these methods, the quality of the solution is very low compared to simultaneous solutions. In addition to this, neglecting the problem-specific incompatibility and loading constraints makes the solution inapplicable in practice.

In the literature, several studies simultaneously solved the problem, but they did not take into account incompatibility and loading constraints. Although these studies produced better results, the results obtained are impractical due to problem-specific constraints not being considered (Dayarian et al., 2015; Expósito et al., 2018; Firouzi et al., 2018; Huang et al., 2019; Montero et al., 2019; Tarantilis and Kiranoudis, 2007).

In some of the studies in the MCP literature, in addition to some vehicle routing constraints, incompatibility constraints were also considered Hoff and Løkketangen (2007), Caramia and Guerriero (2010), Pasha et al. (2014), and Amiama et al. (2015). Hoff and Løkketangen (2007) addressed the problem as a truck trailer routing problem and developed a Tabu search algorithm to solve it. Pasha et al. (2014) solved MCP by using the Merging and Splitting Technique (MST) in the Tabu search algorithm. In these studies, the authors used a

**Table 2**  
Literature comparison.

Authors	Year	Objective	R1	R2	R3	R4	R5	I1	I2	I3	L1	L2
Hoff and Løkketangen	2007	Min Total Distance	✓	✓					✓			
Caramia and Guerrero	2010	Min Total Distance	✓		✓		✓	✓	✓	✓		
Amiama et al.	2015	Min Cost	✓	✓				✓			✓	
Sethanan and Pitakaso	2016	Min Cost	✓			✓				✓		
Paredes-Belmar et al.	2016	Max Profit	✓								✓	
Paredes-Belmar et al.	2017	Max Profit	✓						✓		✓	
Caria et al.	2018	Min Energy Consumption and CO <sub>2</sub>	✓	✓			✓				✓	✓
Rothenbächer et al.	2018	Min Cost				✓				✓		
Chokanat et al.	2019	Min Cost			✓					✓		
Polat and Topaloğlu	2019	Min Cost	✓	✓			✓	✓		✓		
Polat et al.	2019	Min Total Distance	✓	✓			✓	✓		✓		
Villagrán et al.	2020	Max Profit	✓							✓		
This Paper		Min Cost	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

heterogeneous vehicle fleet and considered location requirement constraints, but neglected different milk types and multi-compartment constraints. Caramia and Guerrero (2010) presented a study in which different types of milk were collected without mixing. However, they solved the MCP problem sequentially in two stages, therefore the quality of the solution was decreased. Also, they did not consider the time limit and loading constraint. Amiama et al. (2015) developed a spatial decision support system for the operator to see the results of different scenarios. Firstly, the VRP based on a heuristic find an initial solution, then a graphic system improved the solution by allowing the changes in the route. Rothenbächer et al. (2018) developed a branch-and-price-and-cut algorithm to simultaneously solve MCP as a TTRP. Although the authors applied location requirement constraints, they neglected the incompatibility and loading constraints that play an important role in MCP. Polat and Topaloğlu (2019) solved small-scale sized problem instances considering fuzzy demand and service time parameters. In their study, they used a heterogeneous vehicle fleet with multiple compartments, but without considering different milk types and loading constraints. Later, Polat and Topaloğlu (2021) solved the same problem by using the enhanced iterative local search approach.

Paredes-Belmar et al. (2016), addressed MCP from another perspective, blending different types of milk according to cost. They solved the problem with a branch-and-cut algorithm and a three-stage heuristic. In their follow-up study, Paredes-Belmar et al. (2017), considered MCP as a location routing problem, and they included the location routing constraint and milk collection centers in the problem structure. In Villagrán et al. (2020), to which the same authors contributed, MCP, with a similar focus on multi-product vehicle routing problems with blending, was solved with the Iterated Local Search algorithm. In these studies, the time (time window and time limit), multi-tank, and loading constraints, which are important in the practical application of the problem, were not taken into account.

Sethanan and Pitakaso (2016) improved five heuristic algorithms that depend on the differential evolution algorithm to solve the MCP problem. The authors considered that raw milk collected from different nodes cannot be loaded into the same tank due to cleaning issues. However, the application of this constraint would have caused an increase in the number of tankers required. In their recent study, the authors have allowed the collection of raw milk from two different nodes into a tank (Chokanat et al., 2019). Neither study considered different milk types, location requirements, and loading constraints.

Caria et al. (2018), developed a decision support system based on Ant Colony Optimization that was used to improve the existing MCP routes on a vehicle basis in the Italian Sardinia region. The authors concluded that the decision support system improved the performance of existing routes by 6–8%. They took into account the time limit and tanker weight fuel relationship constraints, but not other constraints inherent in MCP. Polat et al. (2019) developed a mathematical model that simultaneously solved the problem as a TTRPG. Although the authors considered

heterogeneous fleet vehicles and divisible demand constraints, they did not consider time window and loading constraints.

Most of the studies presented within the scope of the milk collection problem have addressed the problem in terms of vehicle routing constraints. However, to obtain applicable solutions under real-life conditions, incompatibility and loading constraints should also be considered. In this context, studies that partially include the related constraints in the literature are presented in Table 2.

The contributions of this study to the related literature are twofold: First, a mathematical model is proposed that addresses the routing, incompatibility, and loading constraints that are inherent in MCP. Second, due to the difficulty of the problem, a VNS framework is proposed to solve MCP efficiently.

### 3. Mathematical model

In this section, an integrated mathematical model that solves MCP simultaneously is presented. The problem can be formulated on an undirected graph  $G = (N, A)$ , where  $N$  is the node-set  $N = \{0, \dots, n\}$ , with node 0 considered as the only depot, and  $A$  is the arc set  $A = \{(i, j) : i \in N, j \in N \setminus \{0\}, i \neq j\}$ . The proposed formulation is based on the following assumptions:

- Based on supplier agreement, all different types of milk produced by each farm/milk collection center should be collected.
- The amount of raw milk to be collected is definite and fixed.
- Raw milk types are categorized by an expert and are ready for collection under ideal temperature conditions.
- Farms /collection centers may have each type of milk.
- Each type of milk must be transported separately in different tanks due to quality factors.
- Mixing of the same type of raw milk collected from different farms /collection centers is allowed.
- The total amount of milk collected in tankers and tanks should not exceed the respective minimum and maximum loading ratios.
- Each tanker must start and end its route at the dairy company.
- Each tanker can only visit each farm/collection center once. However, each farm/collection center can be visited by each of the tankers.
- The milk collection processes must be completed within the specified time limit.
- Each farm/collection center service time is fixed regardless of the divisibility of the demand.
- Each farm/collection center can be visited in the allowed time window.
- Each tanker can only serve farms/collection centers according to size compatibility.

The sets, indices, variables, and parameters used for formulating the

mathematical model are described below:

Indices	
$i, j \in N$	set of nodes (0 : dairy factory, 1, ..., $N$ : farms/collection centers)
<hr/>	
$k \in K$	set of tankers
$l \in L$	set of tanks on a tanker
$m \in M$	raw milk types in the collection area
Parameters	
$a_{ij}$	distance between nodes $i$ and $j$
$Q_{kl}$	the capacity of tank $l$ on tanker $k$
$d_{im}$	amount of type $m$ raw milk to be collected from farm/collection center $i$
$s_i$	service time at farm/collection center $i$
$T$	maximum route duration for delivering collected raw milk to the dairy factory
$V$	the average speed of tankers
$P$	a sufficiently large number
$E_i$	earliest service/operation start time at node $i$
$G_i$	latest service/operation finish time at node $i$
$TB_k$	size of tanker $k$
$UB_i$	maximum tanker size allowed at farm/collection center $i$
$T_{kl}^{\min}$	minimum tank loading ratio
$T_{kl}^{\max}$	maximum tank loading ratio
$C_k$	usage cost of tanker $k$ per voyage
$\gamma_k$	fuel usage cost of tanker $k$ per km
$\alpha_k$	fuel consumption factor
$\eta_k^d$	full-load fuel consumption rate
$\eta_k^b$	no-load fuel consumption rate
Decision Variables	
$x_{ijk}$	1 if the arc between nodes $i$ and $j$ is served by tanker $k$ ; 0 otherwise
$w_{klm}$	1 if tank $l$ on tanker $k$ is assigned to milk type $m$ ; 0 otherwise
$y_{ik}$	1 if tanker $k$ visits node $i$ ; 0 otherwise
$z_{iklm}$	1 if raw milk type $m$ , which is produced at node $i$ , is loaded into tank $l$ on tanker $k$ ; 0 otherwise
$f_{ikm}$	fulfillment ratio for tank $l$ on tanker $k$ which holds raw milk type $m$ stored at node $i$
$b_{im}$	1 if raw milk type $m$ exists at node $i$ ; 0 otherwise
$Y_{mkl}$	amount of milk loaded from farm/collection center $i$ to tank $l$ on tanker $k$
$T_{mkl}$	the total amount of milk carried by tank $l$ on tanker $k$ that visited farm/collection center $i$
$TYM_{ijk}$	the total amount of milk loaded on tanker $k$ between nodes $i$ and $j$
$h_{ik}$	service/operation start time for farm/collection center $i$ by tanker $k$ .

Considering the nature of the problem, finding the shortest route is not an effective solution due to operating costs, therefore the mathematical model aims to minimize the total cost. One of the most important components of the total cost is the fuel cost. To minimize this, the tanker weight-fuel relationship factor should be included in the model. For this reason, the FCR formula developed by Xiao et al. (2012) to calculate the fuel consumption coefficients of tankers is applied to the objective function of the model. The formula is given in Equation (1):

$$\alpha_k = \frac{\eta_k^d - \eta_k^b}{\sum_{l \in L} Q_{kl}} \forall k \in K \quad (1)$$

To calculate the total fuel cost, Equation (2) is added to the objective function

$$\sum_{i \in N} \sum_{j \in N} \sum_{k \in K} \gamma_k a_{ij} (\eta_k^b x_{ijk} + \alpha_k TYM_{ijk}) \quad (2)$$

The proposed mixed-integer linear mathematical model is given as follows:

$$\text{minimize } Z = \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} \gamma_k a_{ij} (\eta_k^b x_{ijk} + \alpha_k TYM_{ijk}) + \sum_{k \in K} C_k Y_{0k} \quad (3)$$

s.t.

$$\sum_{j \in N \setminus \{0\}} x_{ijk} \leq y_{ik} \forall i \in N \setminus \{0\}, k \in K \quad (4)$$

$$\sum_{i \in N} x_{ijk} - \sum_{j \in N} x_{jik} = 0 \forall j \in N, k \in K \quad (5)$$

$$x_{i0k} \leq 1 \forall i \in N \setminus \{0\}, k \in K \quad (6)$$

$$x_{0ik} \leq 1 \forall i \in N \setminus \{0\}, k \in K \quad (7)$$

$$\sum_{i \in N} \sum_{k \in K} x_{i0k} = 0 \quad (8)$$

$$\sum_{j \in N} x_{ijk} + \sum_{j \in N} x_{jik} = 2y_{ik} \forall i \in N, k \in K \quad (9)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \leq |S| - 1 \quad \forall S \subset N : S \neq \emptyset, k \in K \quad (10)$$

$$z_{iklm} \leq y_{ik} \forall i \in N \setminus \{0\}, k \in K \quad (11)$$

$$z_{iklm} \leq w_{klm} \forall i \in N \setminus \{0\}, k \in K, l \in L, m \in M \quad (12)$$

$$\sum_{k \in K} \sum_{l \in L} z_{iklm} \leq 0 \forall i \in N \setminus \{0\}, m \in M : D_{im} = 0 \quad (13)$$

$$w_{klm} \leq \sum_{i \in N \setminus \{0\}} z_{iklm} \forall k \in K, l \in L, m \in M \quad (14)$$

$$\sum_{m \in M} w_{klm} \leq 1 \forall k \in K, l \in L \quad (15)$$

$$\sum_{i \in N} \sum_{j \in N, i \neq j} (a_{ij} x_{ijk}) / V + \sum_{i \in N} s_i y_{ik} \leq Tk \forall k \in K \quad (16)$$

$$h_{ik} + a_{ij} - P(1 - x_{ijk}) + s_i \leq h_{jk} \forall i, j \in N, k \in K \quad (17)$$

$$h_{ik} \geq E_i * y_{ik} \forall i \in N \setminus \{0\}, k \in K \quad (18)$$

$$h_{ik} \leq G_i * y_{ik} \forall i \in N \setminus \{0\}, k \in K \quad (19)$$

The objective function (3) aims to minimize the total cost by reducing the total distance and tanker usage cost. Constraint (4) states that each farm/collection center must be visited by at least one tanker. Constraint (5) is the typical flow conservation equation that ensures the same vehicle arrives at and departs from each node. Constraint (6) and (7) ensure that, if tanker  $k$  is used, the start and endpoints of its route are the depot. Constraint (8) aims to improve model performance. Constraint (9) specifies the degree of each node, while constraint (10) prohibits the formation of illegal sub-tours (Dantzig et al., 1954). If raw milk type  $m$ , which is produced at node  $i$ , is loaded into tank  $l$  on tanker  $k$ , then tanker  $k$  must visit node  $i$ , and milk type  $m$  must be loaded into tank  $l$  on tanker  $k$ , as stipulated by constraints (11) and (12). Constraint (13) provides that for tank  $l$  on tanker  $k$  to be assigned to node  $i$ , that node must have at least one milk type  $m$ . Constraint (14) ensures that if tank  $l$  on tanker  $k$  is assigned to milk type  $m$ , then milk type  $m$ , produced by at least one node, is assigned to this tanker. Constraint (15) prevents the mixing of different types of milk. Constraint (16) ensures that the time spent by a tanker in the collection of milk does not exceed the maximum route duration limit. Constraints (17)-(19) are related to the time window constraints. Constraint (17) indicates that the beginning of service to farm/collection center  $j$  will be after the visit to farm/collection center  $i$ . Constraints (18) and (19) ensure that the visit to farm/collection center  $i$  is within the specified time window interval.

### 3.1. Logical inequalities constraints

To strengthen the logical relationships of routing, visiting, and assignment variables in the formulation, logical inequalities constraints (20)-(23), proposed by (Coelho and Laporte, 2015), were added to the model.

$$x_{i0k} \leq 2y_{ik} \forall i \in N \setminus \{0\}, k \in K \quad (20)$$

$$y_{ik} \leq y_{0k} \forall i \in N \setminus \{0\}, k \in K \quad (21)$$

$$y_{ik} \leq \sum_{l \in L} \sum_{m \in M} z_{iklm} \forall i \in N \setminus \{0\}, k \in K \quad (22)$$

$$y_{ik} \leq \sum_{l \in L} \sum_{m \in M} w_{klm} \forall i \in N \setminus \{0\}, k \in K \quad (23)$$

Constraint (20) ensures that if farm/collection center  $i$  is on tanker  $k$ 's route, then farm/collection center  $i$  is visited by tanker  $k$ . Constraint (21) provides that each tanker returns to the dairy factory after completing its route. Constraints (22) and (23) ensure that if tanker  $k$  visits node  $i$ , then at least one of the raw milk types  $m$  which is produced at node  $i$  is assigned to and loaded into tank  $l$  on tanker  $k$ .

### 3.2. Demand split constraints

During the milk collection process, a product can be loaded into more than one tanker. To represent this situation, demand split constraints (24) - (29) were attached to the model.

$$\sum_{k \in K} \sum_{l \in L} f_{iklm} = b_{im} \forall i \in N \setminus \{0\}, m \in M \quad (24)$$

$$b_{im} P \geq d_{im} \forall i \in N, m \in M \quad (25)$$

$$b_{im} \leq d_{im} \forall i \in N, m \in M \quad (26)$$

$$\sum_{i \in N \setminus \{0\}} \sum_{m \in M} d_{im} f_{iklm} \leq Q_{kl} T_{kl}^{\max} \quad k \in K, l \in L \quad (27)$$

$$z_{iklm} \geq f_{iklm} \forall i \in N, k \in K, l \in L, m \in M \quad (28)$$

$$f_{iklm} \leq 1 \forall i \in N \setminus \{0\}, k \in K, l \in L, m \in M \quad (29)$$

Constraints (24), (25), and (26) allow the milk which is ready for collection to be divided. Constraint (27) guarantees that the amount of raw milk assigned to tank  $l$  on tanker  $k$  cannot exceed that tank's capacity. Constraint (28) creates a connection between a node assignment and its demand ratio. Constraint (29) ensures that the division ratio of demand can be at most one.

### 3.3. Capacity control and loading constraints

It is very important that the loading of the tanks on the tankers is under the minimum and maximum loading ratios to maintain the quality of the milk and ensure driver safety. Constraints (30) - (37) provide that capacity control of the tanks and loading constraints.

$$Y_{m_{ikl}} = \sum_m d_{im} f_{iklm} \forall i \in N \setminus \{0\}, k \in K, l \in L \quad (30)$$

$$T_{m_{ikl}} \geq Y_{m_{ikl}} \forall i \in N \setminus \{0\}, k \in K, l \in L \quad (31)$$

$$T_{m_{ikl}} + Y_{m_{jkl}} - P(1 - x_{ijk}) \leq T_{m_{jkl}} \forall i \in N, \forall j \in N \setminus \{0\}$$

$$k \in K, l \in L \quad (32)$$

$$T_{m_{ikl}} + Y_{m_{jkl}} + P(1 - x_{ijk}) \geq T_{m_{jkl}} \forall i \in N, \forall j \in N \setminus \{0\},$$

$$k \in K, l \in L \quad (33)$$

$$T_{m_{ikl}} \geq z_{iklm} Q_{kl} T_{kl}^{\min} \forall i \in N \setminus \{0\}, k \in K, l \in L, m \in M \quad (34)$$

$$T_{m_{ikl}} = 0 \forall i \in N \setminus \{0\}, k \in K, l \in L \quad (35)$$

$$TYM_{ijk} \leq x_{ijk} P \forall i \in N, \forall j \in N, k \in K \quad (36)$$

$$\sum_{i \in N, i \neq j} TYM_{ijk} = \sum_l T_{m_{jkl}} \forall j \in N \setminus \{0\}, k \in K \quad (37)$$

Constraint (30) provides a calculation of the amount of milk loaded into tank  $l$  on tanker  $k$  from farm/collection center  $i$ . Constraint (31) ensures that the amount of milk in tank  $l$  is at least the amount of milk loaded from farm/collection center  $i$ . Constraints (32) and (33) provide the flow relationship of the amount of milk collected among the sequential nodes. Constraint (34) ensures that the amount of milk loaded into a tank  $l$  is more than the minimum loading ratio of the tank. Constraint (35) ensures that all tanks are empty at the start of the tanker's route. Constraint (36) provides a relationship between milk loading and the assignment of the nodes. Constraint (37) specifies that the total amount of milk loaded in tanker  $k$  is equal to the total amount of milk in the tanks of the tanker.

### 3.4. Location requirement constraint

In real-life conditions, because of geographical conditions and facility restrictions, some of the nodes have specific conditions, so that only certain types of vehicles may visit these nodes. Constraint (38) enforces this restriction by specifying that if the size of tanker  $k$  is acceptable to farm/collection center  $j$  then tanker  $k$  can visit node  $i$ .

$$y_{ik} * TB_k \leq UB_i \forall i \in N, k \in K$$

### 3.5. Variable nature of the integer and binary constraints

$$x_{ijk} \in \{0, 1\} \forall i, j \in N, k \in K \quad (39)$$

$$0 \leq f_{iklm} \forall i \in N \setminus \{0\}, k \in K,$$

$$l \in L, m \in M \quad (40)$$

$$Y_{m_{ikl}} \geq 0 \forall i, j \in N, k \in K, l \in L \quad (41)$$

$$TYM_{ijk} \geq 0 \forall i, j \in N, k \in K \quad (42)$$

$$h_{ik} \geq 0 \forall i \in N, k \in K \quad (43)$$

$$w_{klm}, z_{iklm}, y_{ik}, b_{im} \in \{0, 1\} \forall i, j \in N, k \in K, l \in L, m \in M \quad (44)$$

Constraints (39), (40), (41), (42), (43) and (44) are variable domains.

## 4. Solution framework

In a broader perspective, VRP can be considered as a combination of the bin packing problem (BPP) and the traveling salesman problem (TSP). Both BPP and TSP are NP-hard problems (Falkenauer, 1996; Wolsey, 2020); thus, VRP and its variant MCP are also NP-hard. Since the real-life problem instance cannot be solved in polynomial time with exact solution methods in the MCP, the metaheuristics approaches are developed to obtain effective solutions within a reasonable time in the literature (i.e., Hoff and Løkketangen (2007), Pasha et al. (2014), Caria et al. (2018)). Among the metaheuristics that exist in the literature (Elshaer and Awad, 2020), the variable neighborhood search (VNS) stands out with its success in VRP applications (i.e., Salhi et al. (2014), Polat et al. (2015), Kalayci and Kaya (2016), Polat (2017), Karakostas et al. (2019), Karakostas et al. (2020a), Karakostas et al. (2020b), Pitakaso et al. (2020), Chen et al. (2020) and Hesam Sadati et al. (2021)). Recent very effective applications of the VNS framework to various VRP variants in different sectors led us to an adaptation of this framework for solution of the MCP.

The VNS metaheuristic is an optimization framework that uses different neighborhood structures simultaneously (Mladenović and Hansen, 1997). Since its introduction to the literature, the VNS metaheuristic, which has been continuously improved and has applications in numerous fields. The VNS provides a flexible metaheuristic optimization framework for approximately solving combinatorial optimization problems. It is based on the systematic change of neighborhood

**Table 3**

Notation used in proposed VNS framework.

$k_{max}$	the number of shaking structures of VNS
$m_{max}$	the number of local search structures of VNS
$s_{max}$	the termination number of VNS
$p_{max}$	the number of perturbations namely PM
$N_k$	the set of shaking neighborhood structures
$N_m$	the set of local search neighborhood structures
$N_r$	the set of perturbation neighborhood structures
$\pi^0$	the best solution found so far
$\pi^1$	the incumbent best solution in each iteration
$\pi^2$	shaking solution
$\pi^3$	local search best solution
$\pi_k^1$	a random solution in the $k^{th}$ neighborhood of $\pi^1$
$\pi_m^2$	the best solution in the $m^{th}$ neighborhood of $\pi^2$
$\pi_0^r$	the temporary best solution; a random solution in the random $r^{th}$ neighborhood of $\pi^0$

structures during the search for an optimal or near-optimal solution (Hansen et al., 2017).

While reaching a solution, the VNS framework uses three main phases following the specified procedure (Hansen and Mladenović, 2001). These are: the shaking phase, which allows temporary jumps in the solution space; the local search phase, which allows a detailed search of the temporary points in the solution space; and the move or stop phase, which either classifies the defined temporary solution as the main solution or moves to a new possible solution. The structure of the VNS framework is shown in Algorithm 1.

#### Algorithm 1: Structure of the VNS framework

##### Initialization

- (1)  $N_k$  Select the set of neighborhood structures  $k = 1, \dots, k_{max}$
- (2)  $\pi$  Find an initial solution
- (3) Choose a stopping condition

##### Solution procedure

- Repeat the following sequence until the stopping condition is met:
- (1) Set  $k \leftarrow 1$
  - (2) Repeat the following steps until  $k = k_{max}$ 
    - (a) Shaking: Generate a point  $\pi'$  at random from the  $k^{th}$  neighborhood  $\pi, \pi' \in N_k(\pi)$ .
    - (b) Local search: Apply a local search method to solution  $\pi', \pi' \in N_m(\pi')$ .
  - Move or not: If the solution  $\pi'$  is better than  $\pi$ , set  $\pi \leftarrow \pi'$  and keep searching; otherwise, set  $k \leftarrow k + 1$

In the literature, various VNS variants have been deduced from the basic VNS framework for solving NP-hard problems. Among them, the perturbation-based VNS (Kalayci and Kaya, 2016; Polat et al., 2015) is a powerful alternative that aims the increase search space diversity by allowing the improvement of infeasible solutions. The notation used in the proposed VNS framework in Algorithm 2 for solving the MCP is

**Table 4**

Producer nodes and the supply amounts.

Producer Node	Milk Type (L)		
	1	2	3
1	50	100	–
2	80	100	50
3	60	–	–
4	–	70	–
5	–	–	200
6	–	100	160
7	30	40	50

given in Table 3.

#### Algorithm 2: Proposed Pseudocode for proposed VNS framework

```

1  input: parameters and  $k_{max}, m_{max}, s_{max}, p_{max}$ 
2  output:  $\pi^0$ 
3  start
4    construct  $\pi^0$ 
5     $\pi^1 \leftarrow \pi^0, p \leftarrow 0, s \leftarrow 1;$ 
6    repeat
7      repeat
8         $k \leftarrow 1$ 
9        repeat
10        $\pi^2 \leftarrow \pi_k^1 \in N_k(\pi^1)$ 
11       if  $f(\pi^2) < f(\pi^1)$ 
12          $\pi^1 \leftarrow \pi^2, k \leftarrow 1;$ 
13       else if
14          $m \leftarrow 1; i \leftarrow false;$ 
15         repeat
16            $\pi^3 \leftarrow argmin_{\{\pi_m^2 \in N_m(\pi^2)\}} f(\pi_m^2)$ 
17           if  $f(\pi^3) < f(\pi^1)$ 
18              $\pi^1 \leftarrow \pi^3$ 
19              $i \leftarrow true$ 
20           else
21              $m \leftarrow m + 1$ 
22           end if
23         until  $m = m_{max}$ 
24         If  $i = true$ 
25            $k \leftarrow 1$ 
26         else
27            $k \leftarrow k + 1$ 
28         end if
29       end if
30     until  $k = k_{max}$ 
31     if  $f(\pi^1) < f(\pi^0)$ 
32        $\pi^0 \leftarrow \pi^1;$ 
33        $p \leftarrow 0, s \leftarrow 1;$ 
34     else if
35        $s \leftarrow s + 1; p \leftarrow p + 1;$ 
36     end if
37   until  $p = p_{max}$ 
38   if  $p > p_{max}$ 
39      $\pi^1 \leftarrow \pi_0^r \in N_r(\pi^0)$ 
40      $p \leftarrow 0;$ 
41   end if
42   until  $s = s_{max}$ 
43 end

```

#### 4.1. Solution representation

In this study, it is taken into consideration that each farm/collection center may supply different types and amounts of milk, the dairy company can use a heterogeneous vehicle fleet, and the capacities of the tankers and the number of tanks on the tankers may differ from each other.

The constraints on the nature of MCP are very important, and it complicates the related problem structure. Therefore, matrix representation is used as an encoding mechanism in the developed VNS framework to easily express the structure and constraints of the problem. A new solution representation procedure is designed to include node number, milk type, and the loaded amount of milk from this node and type by using 10 digits. The generated solution representation is explained with an example where there are seven farms/collection centers (producers) and three different types of milk (Table 4).

In the given example, considering an undivided supply, the load of producer 1 from milk type 1 can be represented as "0010100050". In this representation, the first three digits specify the producer (at most 999), the next two digits the product type (up to 99), and the final five digits the number of liters of milk (at most 99,999). This representation helps to signify when the supply of producers from one type or more types is divided (split) into different tanks or tankers.

**Table 5**

The random sequence.

Insertion order	1	2	3	4	5	6	7	8	9	10	11	12	13
Supply code	9	3	4	1	8	12	13	6	2	10	5	11	7
Producer node	6	2	2	1	5	7	7	3	1	6	2	7	4

**Table 6**

Tanker-Tank capacities.

Tanker	Tank 1 (L)	Tank 2 (L)	Tank 3 (L)
1	100	100	
2	100	100	100
3	150	150	150
4	100	100	

#### 4.2. Initial solution

In this study, the initial solution is generated randomly by following the procedure with three steps. First, a random sequence is generated according to the total number of supplies from different producers and milk types. For the example given in Table 4, a random sequence is created for thirteen supplies in Table 5. According to this example, supply with code 9 (6th producers' type 2 milk) will be first assigned to a tanker in the next step.

For the same example, the dairy company has four tankers, and the tank capacities of these tankers are given in Table 6.

In the second step, supplies are assigned to the randomly preordered tankers according to the randomly generated sequence by controlling tank capacities and milk type of the first assigned supply to the related tank. If a supply could not be assigned to any tanks of the current tanker, the next tanker is used and the current tanker is passed. At this stage, if all tankers are used and there are still unassigned supplies, then dummy tankers are used by repeating the same order of tankers. When this procedure is applied to the current example, five routes are created by using one dummy tanker. The top part (representation matrix) of Table 7 shows this assignment by using supply representations.

In the last step, the representation matrix can be easily converted to the network matrix to achieve routes of the tankers by gathering the node numbers from representations. Note that if the related node is already on the route, it is not repeated in the same route.

These two matrices are transferred throughout generations during the run of the VNS framework. All the necessary information for the solution is kept through the iterations by using them. The neighborhood structures in the next subsection are utilized with the help of these matrices.

#### 4.3. Neighborhood structures

To explore the solution space of MCP, several intra- and inter-route neighborhood structures are employed in the shaking, local search, and perturbation steps.

In this VNS framework, the designed operators are categorized into four groups based on their usage area. Group 1 is made up of the *intra-route* operators that aim to improve their route by making in-route changes. Group 2 consists of *inter-route* operators that are used in the development of routes by performing inter-route exchange operations. Group 3 contains the *split inter-route* operators, which enable producers' products to be divisible both by type and by quantity. Group 1, 2, and 3 operators work on the sequence matrix. The modifications related to these operators are applied to all positions of affected customers in that route. On the other hand, Group 4 operators, *inter-route representation change*, aim to improve routes by ensuring the exchange of positions between routes.

The operators in Groups 1, 2, 3, and 4 are used in both Shaking and Local Search phases. The probability of selecting each operator group and each operator in this group is the same during the shaking and local search phases. While the routes are randomly selected in the shaking phase, a route selection rule is used in the local search phase. According to this rule, at least one route must be selected from the routes that have been modified during the shaking phase. Thus, it is aimed at reaching diversified solutions.

##### 4.3.1. Group 1 (Intra-route operators)

*Best insert:* In this operator, one customer on a randomly selected route is placed sequentially in all other positions on that route. This operation is repeated for all customers on the route. The output of the operator is the best route among the alternatives.

*Best swap:* In this operator, two customers of a randomly selected route are swapped to each other's positions. This operation is repeated for all customer pairs in the route. The output of the operator is the best route among the alternatives.

*2\_opt and 3\_opt:* In the 2\_opt operator, two randomly selected edges from the route are deleted, inverted, and added back to the route. In the 3\_opt operator, three randomly selected edges from the route are deleted, then reconnected to the route in the same way as the 2\_opt operator.

##### 4.3.2. Group 2 (Inter-route operators)

*Exchange:* It provides new route sequences by exchanging two randomly selected customer groups from two randomly selected routes.

**Table 7**

Representation and network matrices for the example.

Representation Matrix	Tank 1				Tank 2		Tank 3	
	Tanker 1	0060200100			0020100080	0030100020	0050300100	
Tanker 2	0020200100			0010100050	0030100040	0060300150		
Tanker 3	0050300100		0070300050		0070200040	0010200100	0060300150	
Tanker 4	0060300010		0020300050		0070100030			
Dummy Tanker 1	0040200070							
Network Matrix	Tanker 1	6	→	2	→	3		
	Tanker 2	2	→	1	→	5	→	3
	Tanker 3	5	→	7	→	1	→	6
	Tanker 4	6	→	2	→	7		
	Dummy Tanker 1	4						

**Table 8**

CPLEX and VNS framework results for different numbers of customers.

Number of Nodes	CPLEX					VNS			Gap (%)
	Upper Bound	Lower Bound	Relative Gap	Time (sec.)	Number of Vehicles	Best Found Solution	Time (sec.)	Number of Vehicle	
9	793.48	793.48	0.00%	273.97	1	793.48	1.25	1	0.00%
10	825.86	825.86	0.00%	732.58	1	825.86	1.40	1	0.00%
11	847.95	847.95	0.00%	1971.95	1	847.95	1.43	1	0.00%
12	1278.65	766.20	40.08%	3599.89	2	1211.11	5.61	2	36.74%
13	1319.84	658.53	50.11%	3599.69	2	1217.41	6.58	2	45.91%
14	1485.80	753.47	49.29%	3599.73	2	1242.23	10.14	2	39.35%
15	1867.81	744.76	60.13%	3596.27	2	1328.73	12.45	2	43.95%
16	3154.87	600.61	80.96%	3588.95	5	1830.80	19.98	2	67.19%
17	1686.83	625.33	62.93%	3581.42	2	1505.69	23.76	2	58.47%
18	–	–	–	3552.16	–	1830.80	34.54	3	–

This operator does not limit the size of the customer group selected.

*1-1 Exchange:* In this operator, the number of transferable customers is set at one for each of the routes.

*m-n Exchange:* The upper limit of the number of transferable customers at this operator is set at three for each route.

*Shift:* A randomly selected customer on a randomly selected route is assigned to a randomly selected position in another randomly selected route.

#### 4.3.3. Group 3 (Split inter-route operators)

*Shift-Split Type 1:* From a randomly selected route, a random milk type is selected from a random customer, and the related position is discarded from this route. This milk type is randomly distributed to two random routes which have the same milk type or empty tanks. If the capacity of the route with the highest capacity among the specified routes is more than the amount of milk to be assigned, all the milk is shifted to this route.

*Shift-Split Type 2:* From a randomly selected route, all milk types of a randomly selected customer are randomly assigned to two routes that have the same milk type or empty tanks. In this operator, the customer selected for the assignment must have more than one milk type.

#### 4.4. Group 4 (Inter-route representation change operators)

*Partial represent shift:* This operator aims to ensure that a randomly selected representation found in a randomly selected route is shifted to another suitable route. The suitable route is chosen from routes that have the same milk type or empty tanks.

*Partial represent exchange:* This operator aims to exchange randomly selected representations between two randomly selected routes.

#### 4.5. Fitness function

In the calculation of the fitness value, the total cost is included. Unit delays and the number of delayed route penalties are added to the fitness function in case the time window or time limits are exceeded. In case of violation of the specified limits of the time window, location requirement, and loading constraints, the repair and penalty structure are applied together. If the relevant violation limit can be resolved by an *intra-route* operation, the repair is applied. Otherwise, the relevant route receives a very high penalty. If there is no violation of the route, the route becomes feasible, and the fitness value is calculated from the total cost of the route. This infeasibility allowed approach increases the diversification in the shaking and intensification in the local search.

#### 4.6. Perturbation mechanism

One of the effective strategies developed to improve the solutions found by the algorithm and to prevent getting stuck at local optimum points is the perturbation strategy (Jun and Kim, 2012; Papalitsas et al.,

2019; Subramanian and Cabral, 2008). In this study, the perturbation mechanism is applied when there is no improvement in the local optimum solution after a certain number of iterations ( $p_{max}$ ). In this phase, the inter-route operators in Group 2 are used to provide the temporary best solution ( $\pi_0^r$ ), a partial modification of the best solution ( $\pi^0$ ) that does not improve within a certain time. Infeasible temporary best solutions are also accepted in the perturbation stage with a penalty value as stated in the fitness function section.

#### 4.7. Termination criteria

The VNS framework should be able to work in an acceptable length of time to produce the best result. It should not continue to work without improvement. Therefore, one of the critical steps of the VNS framework is determining the stopping condition. A maximum number of iterations between two improvements ( $p_{max}$ ) is used as the termination criteria.

### 5. Case study

To evaluate the mathematical model and the VNS framework, we used a data set obtained from a dairy company. In the real-life data set, there are 87 farm/collection centers and one dairy company. The company's fleet consists of 11 tankers with different capacities and several tanks. The routes of these tankers and the milk assignments to the tanks on the tankers are determined by the company's technicians based on experience. Each production center has a service time for the milk transfer operation and this parameter is considered independent of the divisibility of the demand to avoid increasing the complexity of the model. Since milk is a perishable food, all transportation processes must be finished under the specified time limit of 540 min. The types and amounts of milk produced by each producer, service time, location requirement, and time window information are presented in Table A1. in the appendix section. The information of the tanker fleet used in the scenarios is given in . in the appendix section. Fuel cost is set at 6.24 TL in all scenarios. Since the case dealt with in this study is from real life, actual distances between each pair of nodes were obtained from the Google Map API.

The case study has been evaluated under five scenarios. Scenario 0 was based on the situation where the milk produced by the farms is not separated by quality. There is two milk type in Scenario 1, three in Scenario 2 and four in Scenario 3. In these scenarios, producers could supply only the milk types. On the other hand, in Scenario 4, there are again two types of milk, but each producer may have each type of milk.

The first 18 nodes of the Scenario 1 data set were used to compare the results of the exact solution method and the VNS framework. The mathematical model was coded by using GAMS 23.4.3 with the CPLEX 12.1.0 solver and executed with a 3600 CPU seconds run-time limit. The proposed VNS framework was coded using MATLAB R2019a. Both of them were executed on a workstation computer with an Intel(R) Xeon (R) W-2145 CPU 3.70 GHz processor and 32 GB of RAM. The proposed VNS framework has two parameters:  $s_{max}$ , the maximum total number of

**Table 9**  
Tank assignment of Scenario 1.

Tank	Milk Assignment	Loading Rate (%)	Route											Total Usage Rate (Liter/Capacity)	
			Firm	1	2	4	3	8	9	10	6	7	5		
Tank 1	Type 2	—	—	—	—	—	—	100	—	—	—	100	—	—	88.6%
Tank 2	Type 1	—	100	100	100	—	—	100	100	100	—	100	—	—	94.2%
Tank 3	Type 2	—	—	—	—	45.2	—	—	—	—	—	—	—	—	78.4%
Tank 4	Type 2	—	—	—	—	54.8	—	—	—	—	—	—	—	—	95.0%

**Table 10**  
Computational results for scenarios.

Scenario	Number of Product	Number of Tankers	Best Total Cost	Total Fuel Consumption Cost (TL)	Total Tanker Usage Cost (TL)	Total Route Duration	Gap (%)		Time (sec.)
							Total Cost	Total Duration	
Scenario 0	1	8	7581.76	4890.76	2691.00	2077.37	—	—	1389.17
Scenario 1	2	8	7942.31	5196.31	2746.00	2170.65	4.76%	4.49%	1471.92
Scenario 2	3	8	8176.59	5430.59	2746.00	2242.96	7.85%	7.97%	2942.76
Scenario 3	4	9	8652.30	5594.30	3058.00	2344.28	14.12%	12.85%	5942.04
Scenario 4	2	9	8552.89	5494.89	3058.00	2318.58	12.81%	11.61%	2728.91

iterations, and  $p_{max}$ , the maximum number of iterations between two improvements. After several preliminary tests, the following settings were chosen to provide the most promising initial solutions for general subsets of the VNS framework:  $N \times 50$  non-improving loops ( $p_{max}$ ), and the global search stopping after  $N \times 1000$  ( $s_{max}$ ) iterations. Table 8 summarizes the performance of the mathematical model and the VNS framework.

When the results of the mathematical model are examined, it is observed that the mathematical model can reach the proven optimum result for up to 11 nodes. The VNS framework achieved optimum results for data sets with 9 to 11 nodes. When the number of nodes is 12 or more, GAPS form between the results of the algorithm and the estimated lower bound solution. The mathematical model cannot reach any solution when the number of nodes is 18.

The tank assignment is explained in the 11-node test set in detail as an example. The tanker (T3), which started its route from the dairy company empty, first visited producer number one. 405 L of type 1 milk was loaded into the second tank of the tanker. The loading rates of the tanks of the tanker and the total usage rates of these tanks are presented in Table 9.

The total cost for 11 nodes is 847.97 TL, of which 420 TL is the fuel consumption cost when the tanker was empty, 60 TL is the fuel consumption cost when the tanker was full, and 367 TL is the tanker usage cost.

After the validation of the VNS framework, five scenarios are solved. The best-found solutions for the scenarios in ten replications are presented in Table 10. Since there is one milk type in Scenario 0, the structure of the problem can be considered as a split-delivery, single-product, multi-compartment vehicle routing problem. Therefore, the total cost calculated is the lowest of all the scenarios. For Scenario 0, the milk collection

used eight tankers at a total cost of 7581.76 TL, of which 4890.76 TL is the fuel consumption cost and 2691 TL the tanker usage cost.

Because the number of milk types in Scenario 1 is two, the cost of the best route created is different from Scenario 0. The cost was 7942.31 TL, an increase of 4.76% over Scenario 0. In Scenario 2, the number of milk types is three, and the cost increases by 7.85% and the route time by 7.97% compared to Scenario 0. In Scenario 3, collecting four different types of milk from 87 nodes results in an increase of 14.12% in the total cost and 12.85% in the travel time.

The developed VNS framework was designed in a structure that can perform assignment and routing operations for multiple products of the customers at the same time. In Scenario 4, prepared in line with this structure, a variety of types of products is provided for the producers. The total cost is 8552.89 TL and the total route duration is 2318.58 min.

Information about the tankers used for the routes created in the scenarios and the utilization rates using these tankers are presented in Table 11. In Table A2 Scenario 0, the milk collection operation was completed using 67.13% of the total capacity of the fleet. As expected, with an increase in the number of product types, tanker usage rates increase compared to Scenario 0. In each scenario, 50.893 L of milk must be collected from 87 nodes. While the total capacity of the vehicles used in Scenario 0 is 31.65% greater than the amount of milk to be collected, this rate increases gradually due to the increase in the number of product types in the other scenarios.

In summary, when the results obtained are analyzed, it is seen that as the number of milk types in the scenarios increases, vehicle costs and route durations increase as expected.

**Table 11**  
Tankers used in the scenarios.

Scenario	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	Total Tanker Usage Capacity	Fleet Capacity Usage Rate	Demand-Usage Capacity Rate
Scenario 0	✓	✓				✓	✓	✓	✓	✓	✓	67,000	67.13%	31.65%
Scenario 1	✓	✓	✓			✓	✓	✓	✓	✓	✓	75,000	75.15%	47.37%
Scenario 2	✓		✓	✓		✓	✓	✓	✓	✓	✓	71,800	71.94%	41.08%
Scenario 3	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	75,800	75.95%	48.94%
Scenario 4	✓	✓	✓			✓	✓	✓	✓	✓	✓	79,000	79.16%	55.23%

## 6. Conclusion

In this study, the milk collection problem, which has been partially studied under different variants of VRP in the literature, was solved by an integrated mathematical model by including various constraints specific to the nature of the problem. In real-life conditions, in addition to providing the shortest path, the solutions that protect the quality of the product and minimize the total cost are more realistic and applicable. With this integrated mathematical model designed in this context, the fuel consumption of tankers and tanker usage costs are minimized, while the safety of the driver and the quality of the milk are better protected.

Optimization problems try to solve real-life problems under specific assumptions. The difficulty of the problems encountered in today's conditions is increasing day by day, and it is not possible to apply exact solution approaches to the solution of related problems. Considering the limitations and assumptions of the problem dealt with in the study, MCP is an NP-hard problem, and it is not possible to solve it in a reasonable time with exact methods. Therefore, a variable neighborhood search metaheuristic optimization framework has been proposed to solve the related problem. The developed VNS framework was evaluated with large-scale scenarios based on real-life data.

Considering the nature of the problem, a possible future research direction is to measure the effect of different milk types which cannot be mixed together on the total cost. Depending on the distance of the producer and the amount of milk produced by these farms, milk is

collected in the morning and evening at some nodes, while at others it is done only in the morning. At the same time, tankers collecting raw milk from producers may deliver the milk to more than one dairy company. Therefore, in future studies, the milk collection problem could be studied with multiple periods and multiple depots.

### CRediT authorship contribution statement

**Olcay Polat:** Conceptualization, Methodology, Software, Investigation, Data curation, Validation, Writing – original draft, Writing – review & editing, Supervision. **Can Berk Kalayci:** Conceptualization, Methodology, Software, Investigation, Data curation, Validation, Writing – original draft, Writing – review & editing, Supervision. **Duygu Topaloglu:** Conceptualization, Methodology, Software, Investigation, Data curation, Validation, Writing – original draft, Writing – review & editing, Supervision.

### Acknowledgments

This work is supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under the 217M578 project. This support is gratefully acknowledged.

### Appendix

**Table A1**

Service time, location requirement, time window information and the type and amount of milk produced by each producer for all scenarios.

Node	Time Parameters (m.)			$UB_i$ (cm.)	Scenario 0		Scenario 1		Scenario 2			Scenario 3			Scenario 4		
	$s_i$	$E_i$	$G_i$				Type 1	Type 2	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3	Type 4	Type 1	Type 2
0	0	0	540	2000	0	0	0	0	0	0	0	0	0	0	0	0	0
1	18	0	540	1500	408	408	0	408	0	0	408	0	0	0	0	306	102
2	22	0	540	1500	1116	1116	0	0	1116	0	0	1116	0	0	0	837	279
3	44	50	540	1700	5200	0	5200	0	5200	0	0	5200	0	0	0	3900	1300
4	18	0	540	1200	489	489	0	0	489	0	0	489	0	0	0	367	122
5	16	0	450	1500	65	65	0	0	65	0	0	65	0	0	0	49	16
6	16	0	450	1500	44	44	0	0	44	0	0	44	0	0	0	33	11
7	16	0	450	1500	57	0	57	0	57	0	0	57	0	0	0	43	14
8	30	40	540	1500	2600	0	2600	0	2600	0	0	2600	0	0	0	1950	650
9	16	0	540	1500	122	122	0	0	122	0	0	122	0	0	0	92	31
10	19	0	540	1500	581	581	0	0	581	0	0	581	0	0	0	436	145
11	24	0	540	1500	1613	0	1613	0	0	1613	0	0	0	0	0	1210	403
12	19	0	540	1200	586	586	0	586	0	0	586	0	0	0	0	440	147
13	18	0	540	700	507	507	0	0	507	0	0	507	0	0	0	380	127
14	17	0	540	1500	218	218	0	0	218	0	0	218	0	0	0	164	55
15	24	0	540	1500	1599	0	1599	0	1599	0	0	1599	0	0	0	1199	400
16	21	0	540	1500	1023	1023	0	0	1023	0	0	1023	0	0	0	767	256
17	17	0	540	1000	341	341	0	0	341	0	0	341	0	0	0	256	85
18	23	30	540	1300	1362	0	1362	0	1362	0	0	1362	0	0	0	1022	341
19	17	0	540	1300	296	296	0	0	296	0	0	296	0	0	0	222	74
20	19	0	540	1500	594	594	0	0	594	0	0	594	0	0	0	446	149
21	17	0	540	1500	235	235	0	235	0	0	235	0	0	0	0	176	59
22	23	0	540	1500	1274	1274	0	0	1274	0	0	1274	0	0	0	956	319
23	17	0	540	600	212	0	212	0	0	212	0	0	0	0	0	159	53
24	17	0	540	1300	214	214	0	0	214	0	0	214	0	0	0	161	54
25	16	0	450	1700	16	16	0	16	0	0	16	0	0	0	0	12	4
26	16	0	450	1300	25	25	0	0	25	0	0	25	0	0	0	8	17
27	16	0	450	1300	176	176	0	0	176	0	0	176	0	0	0	59	117
28	16	0	450	1700	28	28	0	0	28	0	0	28	0	0	0	9	19
29	16	0	450	1700	141	141	0	0	141	0	0	141	0	0	0	47	94
30	20	0	540	1000	750	750	0	750	0	0	750	0	0	0	0	250	500
31	16	0	450	1300	108	108	0	0	108	0	0	108	0	0	0	36	72
32	16	0	540	900	118	0	118	0	118	0	0	118	0	0	0	39	79
33	16	0	540	1700	140	0	140	0	0	140	0	0	140	0	0	47	93
34	16	0	540	1300	170	170	0	0	170	0	0	170	0	0	0	57	113
35	16	0	540	1300	141	141	0	0	141	0	0	141	0	0	0	47	94
36	16	0	540	900	55	55	0	0	55	0	0	55	0	0	0	18	37
37	17	0	540	1300	242	242	0	242	0	0	242	0	0	0	0	81	161

(continued on next page)

**Table A1 (continued)**

Node	Time Parameters (m.)			$UB_i$ (cm.)	Scenario 0	Scenario 1		Scenario 2			Scenario 3				Scenario 4		
	$s_i$	$E_i$	$G_i$			Type 1	Type 2	Type 1	Type 2	Type 3	Type 1	Type 2	Type 3	Type 4	Type 1	Type 2	
38	16	0	540	1300	71	71	0	71	0	0	71	0	0	0	24	47	
39	17	0	540	900	204	204	0	0	204	0	0	204	0	0	68	136	
40	16	0	540	1300	98	98	0	0	98	0	0	98	0	0	33	65	
41	17	0	540	1300	212	212	0	0	212	0	0	212	0	0	71	141	
42	16	0	540	900	152	152	0	0	152	0	0	152	0	0	51	101	
43	16	0	540	1700	82	82	0	82	0	0	0	82	0	0	27	55	
44	16	0	540	900	66	66	0	66	0	0	66	0	0	0	22	44	
45	16	0	540	900	72	72	0	0	72	0	0	72	0	0	24	48	
46	16	0	450	1300	150	150	0	150	0	0	0	150	0	0	50	100	
47	17	0	450	1300	268	268	0	0	268	0	0	268	0	0	89	179	
48	16	0	540	900	15	15	0	0	15	0	0	15	0	0	5	10	
49	17	0	540	1000	295	295	0	295	0	0	295	0	0	0	98	197	
50	16	0	540	600	96	0	96	0	0	96	0	0	96	0	32	64	
51	16	0	540	1300	120	120	0	0	120	0	0	120	0	0	120	0	
52	19	0	540	600	560	0	0	560	0	0	560	0	0	560	0		
53	16	0	540	1700	168	0	168	0	0	168	0	0	168	0	168	0	
54	16	0	540	900	138	138	0	0	138	0	0	138	0	0	138	0	
55	32	30	540	1700	2900	2900	0	2900	0	0	2900	0	0	0	2900	0	
56	18	0	540	1700	539	539	0	539	0	0	539	0	0	0	539	0	
57	18	0	540	1300	419	419	0	0	419	0	0	419	0	0	419	0	
58	21	0	540	1200	1051	1051	0	0	1051	0	0	1051	0	0	1051	0	
59	19	0	540	1300	595	595	0	595	0	0	595	0	0	0	595	0	
60	23	0	540	1500	1333	0	1333	0	0	1333	0	0	1333	0	1333	0	
61	18	0	540	1700	498	498	0	498	0	0	498	0	0	0	498	0	
62	16	0	540	900	80	80	0	80	0	0	80	0	0	0	80	0	
63	16	0	450	900	64	64	0	64	0	0	64	0	0	0	64	0	
64	16	0	450	1200	169	0	0	169	0	0	169	0	0	0	169	0	
65	16	0	450	1200	33	33	0	33	0	0	33	0	0	0	33	0	
66	17	0	540	900	273	273	0	0	273	0	0	273	0	0	273	0	
67	16	0	540	1300	117	117	0	0	117	0	0	117	0	0	117	0	
68	28	30	540	1700	2300	2300	0	2300	0	0	2300	0	0	0	2300	0	
69	49	50	540	1700	6000	6000	0	6000	0	0	6000	0	0	0	6000	0	
70	16	0	540	1500	179	179	0	179	0	0	179	0	0	0	179	0	
71	17	0	540	1500	300	300	0	300	0	0	300	0	0	0	300	0	
72	16	0	540	1500	177	177	0	0	177	0	0	177	0	0	0	177	0
73	16	0	540	1500	158	158	0	158	0	0	158	0	0	0	158	0	
74	17	0	540	1200	294	294	0	0	294	0	0	294	0	0	0	294	0
75	16	0	540	700	68	68	0	68	0	0	68	0	0	0	68	0	
76	16	0	540	1500	150	150	0	0	150	0	0	150	0	0	0	150	0
77	16	0	540	1500	19	19	0	0	19	0	0	19	0	0	0	19	0
78	16	0	540	1500	36	36	0	0	36	0	0	36	0	0	0	36	0
79	17	0	540	1000	272	272	0	272	0	0	272	0	0	0	0	272	0
80	17	0	540	1300	267	267	0	0	267	0	0	267	0	0	0	267	0
81	18	0	540	1300	372	0	372	0	372	0	0	372	0	0	0	372	0
82	29	40	540	1500	2400	0	2400	0	2400	0	0	2400	0	0	0	2400	0
83	17	0	540	1000	217	217	0	217	0	0	217	0	0	0	0	217	0
84	19	0	540	1300	569	569	0	0	569	0	0	569	0	0	0	569	0
85	18	0	540	1300	421	421	0	0	421	0	0	421	0	0	0	421	0
86	30	30	540	1700	2580	0	2580	0	0	2580	0	0	0	0	0	2580	0
87	18	0	540	1300	410	410	0	410	0	0	410	0	0	0	0	410	0

**Table A2**

Tanker fleet information.

Tanker	Number of Tank	Capacity of Tank (L)	Minimum Loading Ratio (L/Capacity)					$TB_k$ (cm)	Vehicle Fuel Consumption Rate (FCR)			$C_k$	$T$
			Tank 1	Tank 2	Tank 3	Tank 4	Tank 5		$\eta_k^b$ (100 km/L)	$\eta_k^d$ (100 km/L)	$\alpha_k$		
T 1	3	4000	0.1	0.1	0.1			1000	22	28	0.0005	339	540
T 2	1	5000	0.1					600	20	26	0.0012	300	540
T 3	4	3000	0.1	0.1	0.1	0.1		1200	22	27	0.0004	367	540
T 4	4	4750	0.1	0.1	0.1	0.1		1300	31	38	0.0004	367	540
T 5	2	900	0.1	0.1				600	20	30.8	0.0060	300	540
T 6	3	3000	0.1	0.1	0.1			900	22	27	0.0006	339	540
T 7	2	2000	0.1	0.1				750	20	26	0.0015	312	540
T 8	5	4000	0.1	0.1	0.1	0.1	0.1	1500	22	28	0.0003	450	540
T 9	3	3000	0.1	0.1	0.1			900	22	27	0.0006	339	540
T10	2	2000	0.1	0.1				600	20	24.8	0.0012	300	540
T11	2	2000	0.1	0.1				700	20	26	0.0015	312	540

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