Analysis of influence factors in Quantum Approximate Optimization Algorithm for Solving Max-cut Problem

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2023 21st International Conference on Optical Communications and Networks (ICOCN) | 979-8-3503-4350-2/23/$31.00 ©2023 IEEE | DOI: 10.1109/ICOCN59242.2023.10236124

***Abstract*—In recent years, quantum algorithm research has developed rapidly, and it has become an important mode to use quantum computing cloud platform to carry out algorithm application exploration. Quantum approximate optimization algorithm can be used to solve the max-cut problem. In the process of algorithm execution, optimizer and various noise had obvious influence on the algorithm execution result. For the quantum computing simulator of HiQ platform, quantum approximate optimization algorithm was used for experimental verification, to study the types of optimizer, different noise scenarios, and the impact of noise error parameters on the maximum number of cutting edges. The prospect of quantum algorithm research was also discussed.**

Keywords—quantum computing, quantum approximate optimization algorithm, quantum noise, cloud platform

I. Introduction

Quantum computing has the potential to provide a powerful parallel computing capability that far exceeds classical computing in principle, and may offer potential solutions to large-scale computing problems in artificial intelligence, quantitative finance, and cryptography. With demonstration of quantum computing superiority [1,2,3], killer applications that combine clear practical value and significant acceleration effects have not yet been developed. Exploring and practicing quantum algorithms using tools such as quantum computing cloud platforms can help better understand which practical problems can be solved by quantum algorithms. During NISQ, quantum-classical hybrid algorithms have been a focus of attention in the industry.

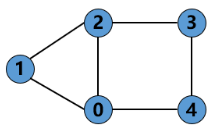
Quantum approximate optimization algorithm, as a typical quantum-classical hybrid algorithm [4,5], has used a classical optimizer in the classical processor to train and optimize the parameters of the neural network. QAOA has been mainly used to find the ground state of the target Hamiltonian and can be applied to solve the max-cut problem. Max-cut problem is a common NP-complete problem in graph theory and has had important applications in image processing, statistical physics, and circuit design. During NISQ, the performance of QAOA may inevitably be affected by quantum noise, classical optimizer, and other factors. In Ref. [6,7], researchers had studied the influence of quantum noise on quantum-classical hybrid algorithms. These studies were very important, not only for exploring the optimization methods for quantum-classical hybrid algorithms but also for supporting the design of quantum-classical hybrid algorithms.

With this background, this paper mainly analyzed the effects of different classical optimizers, learning rates, and quantum noise on the results (maximum number of cutting edges) of QAOA, and conducted experimental verification. In addition, this paper discussed the research needs, limitations, and future possible research directions of quantum algorithm influence factors.

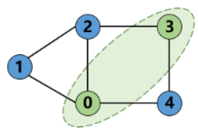
II. Experiments on Solving the Max-cut Problem Based on QAOA

A. Experimental Setup

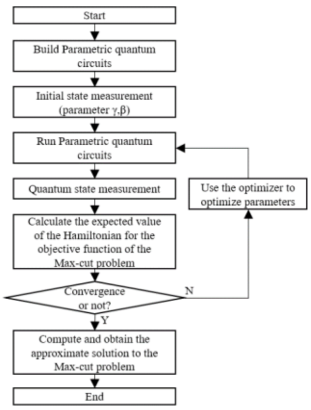
The Max-Cut graph and algorithm implementation were shown in Fig.1. HiQ quantum computing cloud platform [6] and quantum computing open source suite MindQuantum [7] were used. The quantum simulator was configured with three experimental scenarios. Noise-free was set as noise scenario one, with optimizer used Adam optimizer and Adagrad optimizer. Pauli noise was set as noise scenario two, and the effect of such noise on the quantum circuit was reflected in a certain probability of being affected by a Pauli gate, with error parameters included bit flip, phase flip, bit-phase flip, depolarizing probability. Damping noise was set as noise scenario three, where amplitude damping noise caused the dissipation of energy in the quantum system and phase damping channel caused the loss of quantum information in the quantum system, and the error parameters included amplitude damping coefficient and phase damping coefficient.



(a)



(b)



(c)

Fig. 1. Max-cut graph with five vertices and six edges(a), one possible maximum cut method(b) and algorithm implementation(c).

Transforming max-cut problem into a ground state energy solution problem with Hamiltonian, the Hamiltonian H_C for max-cut problem could be constructed using the Pauli Z-gate to obtain the Hamiltonian corresponding to the objective function of this problem as:

$$H_C = \sum_{(i,j)\in C}(Z_i Z_j - 1)/2 \tag{1}$$

where the set of all edges was C.

Using Quantum Adiabatic Evolution and according to Hamiltonian $H_B = \sum_i X_i$ and H_C, respectively defined unitary transformation:

$$U_B(\beta) = e^{-i\beta \sum_i X_i} \tag{2}$$

$$U_C(\gamma) = e^{-i\gamma \sum_{\langle i,j\rangle} Z_i Z_j} \tag{3}$$

γ, β were trainable parameters. The definition was:

$$|\psi(\gamma,\beta)\rangle = \prod_{l=1}^{p} e^{-i\beta_l H_B} e^{-i\gamma_l H_C} |\psi_{in}\rangle \tag{4}$$

The expected value of the objective function Hamiltonian H_C was the expected value of the Hamiltonian H_C in the quantum state of (4).

$$\langle\psi|H_C|\psi\rangle = \langle\psi(\gamma,\beta)|H_C|\psi(\gamma,\beta)\rangle \tag{5}$$

The minimum expected value of the Hamiltonian corresponded to the maximum number of cut edges, which was the result measurement used in this paper's experiments.

B. *Test Results*

Different types of classical optimizers and learning rates might affect the output results of using QAOA to solve the max-cut problem. Therefore, experiments were conducted in a noise-free scenario, using different optimizer types and learning rates to analyze the impact of these factors on the output results. Adam optimizer and Adagrad optimizer were used, with learning rates set to 0.025, 0.05, and 0.075. The experimental results were shown in Fig.2. In the case of the same number of quantum circuit layers (p), with increasing training iterations N, the results tended to increase, and when N reached around 400, the change in results became flattening, and the results remained stable if N continued to increase. This might be because for gradient descent optimization algorithms, it took several training iterations to get the optimal results, and the larger the N was, the more accurate the experimental results became before the optimal results were obtained. For the same optimizer, the more layers of the quantum circuit, the greater the experimental results. This might be because of the complexity of the maximum cut problem studied in this paper, the more layers in the neural network, the better it could learn data features, and the more accurate the results it could obtain. For the same optimizer, when the learning rates were set to 0.025 and 0.05, with increasing N, the experimental results gradually increased. When the learning rate was 0.05, the experimental results tended to approach the optimal solution at smaller values of N. On the other hand, the results increased but were unstable as N increased when the learning rate was 0.075. This might be because the learning rate was too large, which might cause the results to oscillate near the extreme values, resulting in unstable experimental results. Regarding the max-cut problem studied in this paper, the learning rate around 0.05 was more appropriate. For the two optimizers, when the same number of quantum circuit layers (p) was used, the results obtained from the Adam optimizer were better than those obtained from the Adagrad optimizer. This might be because although both optimizers used gradient descent optimization algorithms, the Adam optimizer optimized on the basis of the Adagrad optimizer and was more suitable for solving the max-cut problem studied in this paper.

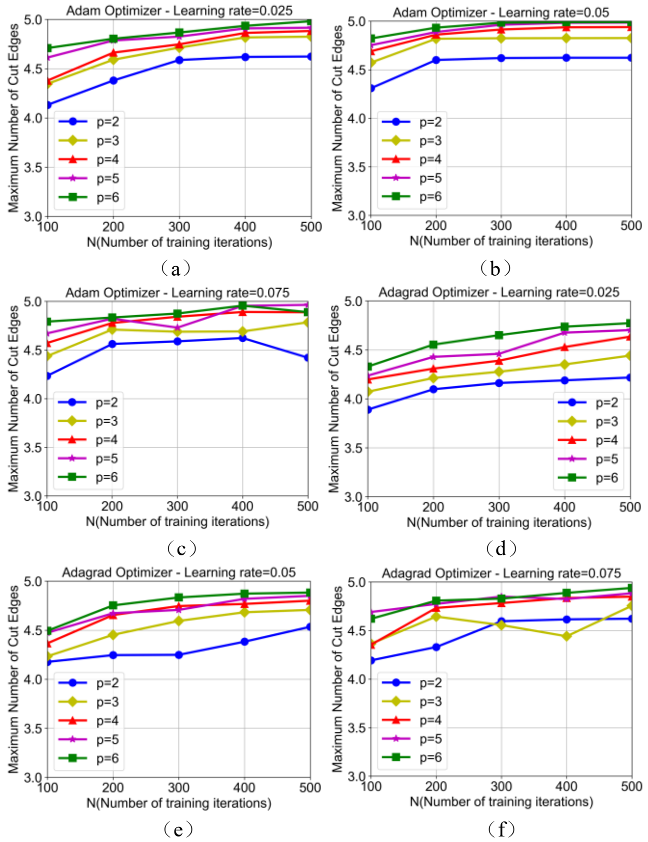


Fig. 2. Experiment with QAOA using the Adam optimizer (a)(b)(c) and Adagrad optimizer (d)(e)(f).

Different types of quantum noise and error parameters might have different effects on the output results of using QAOA to solve the max-cut problem. Therefore, experiments were conducted using two different noise scenarios: pauli noise and damping noise. The effects of changing error parameters in the Pauli noise scenario were studied. The error parameters varied in four different noise scenarios: bit flip probability, phase flip probability, bit-phase flip probability, and depolarizing probability. The effects of these noise scenarios on the output results were analyzed. The experimental results were shown in Fig.3. For the same number of layers in the quantum circuit (p), with increasing error parameter, the experimental results gradually decreased. This was because the effect of Pauli noise on the quantum circuit reflected as being influenced by a Pauli gate with certain probabilities. The larger the error parameter, the more the quantum circuit was affected. For the same noise scenario, the more layers in the quantum circuit, the larger the experimental results. Similar to noise-free scenario, this might be because the neural network could better learn the data features with more layers and obtained more accurate results. For the four noise scenarios, the experimental results returned by the bit flip and phase flip noise differed slightly, and the experimental results returned by the bit-phase flip noise were slightly worse, while the experimental results returned by the depolarizing noise were the worst. This might be because for the same flip probability p, the bit-phase flip (Y gate) could be considered as executing the bit flip (X gate) and the phase flip (Z gate) simultaneously, the depolarizing noise simultaneously executed the above three types of flips, thus having the greatest impact on the algorithm execution, followed by the bit-phase flip error.

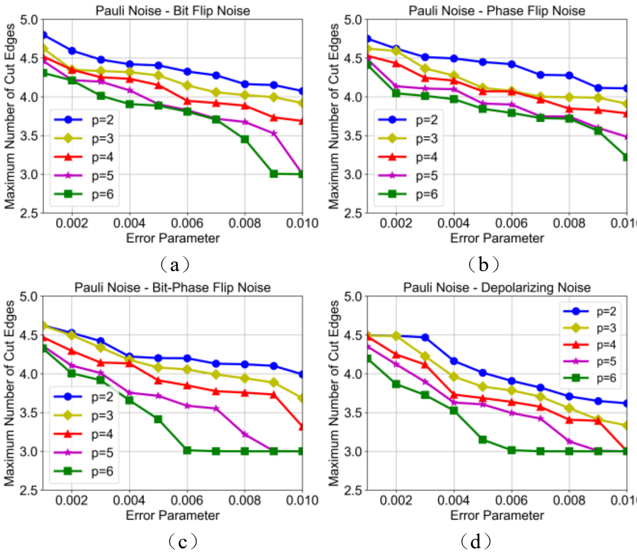


Fig. 3. Experiment with QAOA in pauli noise scenarios of Bit Filp(a), Phase Flip(b), Bit-Phase Flap(c) and Depolarizing(d).

Furthermore, experiments were conducted using damping noise and changing its error parameters. The error parameters varied in two different damping noise scenarios: amplitude damping coefficient and phase damping coefficient. The effects of these noise scenarios on the output results were analyzed. The experimental results were shown in Fig.4. For the same number of layers in the quantum circuit (p), with increasing error parameter, the experimental results gradually decreased. This was because the effect of damping noise on the quantum circuit was reflected in quantum system energy dissipation or information loss. The larger the error parameter, the more the quantum circuit was affected. For the two noise scenarios, the experimental results returned by amplitude damping noise were better than those returned by phase damping noise. This might be because after the quantum bit was affected by phase damping noise, energy was not lost but quantum information was lost, and this loss had greater impact on the returned results.

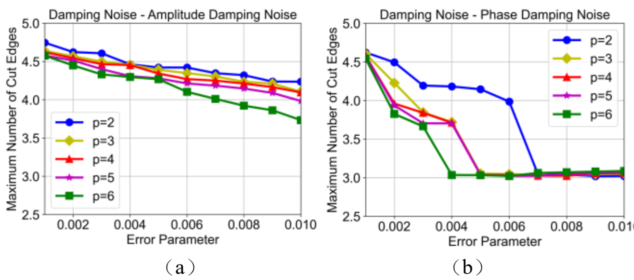


Fig. 4. Experiment with QAOA in damping noise scenarios of Amplitude Damping Noise(a) and Phase Damping Noise Phase Flip(b).

III. Discussion

This article studied the effects of optimizer types, learning rates, and quantum noise on the performance of QAOA. The choice of classical optimizer type and the setting of learning rates were crucial factors to consider when executing QAOA. Choosing an appropriate optimizer required considering various factors such as the characteristics of the optimizer, available computing resources, execution time, etc. Additionally, it was necessary to provide reasonably successful learning rate configurations through continuous experimentation. For the max-cut problem in this paper, Adam optimizer was more suitable than the Adagrad optimizer, and the learning rate should be set around 0.05. In addition, quantum noise scenarios such as pauli noise and damping noise also affect the QAOA output results, and the degree of influence varied with the type of noise and error parameters. The above conclusions not only applied to QAOA but also to other quantum-classical hybrid algorithms that required a classical optimizer. In summary, there are many factors that affect the execution results of QAOA, and studying these factors helps to find quantum algorithms that are robust to noise and provide a basis and reference for designing quantum error correction schemes to reduce the impact of quantum noise. In the future, we will expand the research scope of factors that affect the performance of quantum algorithms and study their effects on other quantum algorithms.

Quantum computing is still in its early stages of development, and the demonstration of quantum computing advantages relies on solving specific problems based on quantum algorithms. Continuous research and breakthroughs in quantum algorithms will strongly drive the development of the quantum computing field.

Acknowledgment

The paper is partly supported by National Science Foundation of China (NSFC) under Grant No.62006248, and National Key R&D Program of China under Grant No.2019YFB1803700.

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