

A Max K-Cut implementation for QAOA in the measurement based quantum computing formalism

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Abstract—The MaxCut is a combinatorial problem which belongs to the NP-hard class. The algorithm to solve it can be mapped into a quantum system, where the optimal configuration matches the ground state of a target Hamiltonian. It is possible to implement the adiabatic theorem into the circuit model via the QAOA algorithm, by arbitrarily choosing from both the gate-based and the measurement-based quantum computing (MBQC) settings. Recently, the MaxCut problem has been generalized to the Max K-Cut version also in the MBQC frame, where K stands for the number of cuts to implement. Here, the QAOA algorithm for a Max K-Cut is solved for a graph with arbitrary number of vertices V and number of cuts K scaling as a power of 2, along with the circuit depth p. The problem is solved on a emulator based on classical HPC resources. In order to test the performances of a Max K-cut implemented in the MBQC frame, we report how the average runtime and the approximation ratio scale by increasing V, K and p, comparing the results from two different topologies of graphs, 2-regular graphs with null weights on the diagonals versus the the fully-connected graphs. Eventually, the approximation ratio, evaluated as a parameter of success, is shown to yield optimal performances.

Index Terms—QAOA, MBQC, MaxCut, HPC

I. POSTER RELEVANCE

In this actual work, the main focus concerns the classical emulation of a QAOA algorithm, in the frame of measurement-based quantum computing (MBQC). More specifically, we tackled the max K-cut problem, i.e. the maxcut problem generalized to a number K of cluster.

The source of such project rely on Paddle Quantum library, a software for quantum emulation integrated with machine learning tasks, and HPC resources by davinci-1 supercomputer (owned by Leonardo company). In fact, the QAOA is a hybrid algorithm [1], [2], i.e. a combination of classical and quantum computation. In such frame, the HPC infrastructure is

a relevant tool, performing backpropagation and other classical routines in the overall pipeline of the algorithm. MBQC computation quite differs from its most notorious counterpart, the gate-based model. Instead of applying a set of logic gates on the qubits, a huger quantum state of entangled qubits, the so-called source state, is allocated, in order to measure all the ancillary qubits. Such measurements will affect the unmeasured qubits, due to the previous entanglement with the measured ones, the output qubits turning to be the logic ones. Therefore, the simulation of an MBQC process, with respect to the gate model, requires a higher number of qubits, the measured ones needing to be implemented somehow, which may severely hinder the memory capability for a classical device.

Due to the novelty of the topic – the first release of MBQC libraries by Paddle Quantum dates back to August 2021 – the results on classical emulation of an AI algorithm in the MBQC frame are unreleased for the first time. It is therefore possible to compare such results with the actual state of the art in the field of quantum machine learning. In the first place, the actual results can be compared with the performances from the gate model, which is the paramount architecture of computation in quantum information. In the second place, the actual performances of Paddle libraries – in terms of memory usage, global runtime and optimization – provide some hindsight guidelines about how to enhance the library itself. Eventually, the current work could be adapted to specific platforms, in order to test the performances of a Machine Learning algorithm on a quantum hardware. In the MBQC frame, in fact, the hardware implementation mainly relies on photonic devices [3]–[5].

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II. DESCRIPTION

Differently from quantum annealing [6], [7], the QAOA approximates the adiabatic evolution through a set of unitaries generated by a mixing Hamiltonian. The first Section describes the QAOA algorithm, its derivation from the adiabatic theorem and its potential applications in the field of combinatorial problems. Via the adiabatic theorem, it is possible to make $|\Psi(t)\rangle$ evolve from the higher energy state of \hat{H}_M (which is provided at $t = 0$) to the higher energy state of $-\hat{H}_T$ (thus the ground state of \hat{H}_T). In the same section, the maxcut problem is introduced, along with its generalized version, the max K-cut. A figurative example which distinguishes a $K = 2$ and $K = 4$ problem is provided by Figure 1. For a generic K , the Hamiltonian description of the maxcut case ($K = 2$) is formulated as

$$\hat{H}_C = \sum_{ij} \frac{w_{ij}}{2^q} \sum_{n=1}^q \left(C_n \hat{I} - \hat{C}_n [\{\hat{Z}_{il}\hat{Z}_{jl}\}_{l=1}^q] \right) \quad (1)$$

The \hat{C}_n operator takes into account all the possible combinations of n elements from the $\{\hat{Z}_{il}\hat{Z}_{jl}\}_{l=1}^q$ set, while C_n is the integer number counting all the possible combinations for n fixed. Next, the embedding of the QAOA algorithm into the MBQC architecture is reported in Figure 2. In Figure 2, the same transformation is illustrated to be repeated p times, p being the number of repeated layers. The core of the algorithm consists of implementing the rotations by the $\{\gamma_i^*, \beta_i^*\}_{i=1}^p$ angles, and therefore tuning such angles by backpropagating via gradient descent. The cost function is provided by $E = -\langle \Psi | \hat{H}_C | \Psi \rangle$, i.e. the lesser the energy, the better the approximation of the final state $|\Psi_f\rangle$. A good flag to assess the proximity of the $|\Psi\rangle$ state to the goal state $|\Psi_{goal}\rangle$ is provided by the approximation ratio [8], [9], i.e. the ratio between the actual energy value of the system by the $-\hat{H}_C$ Hamiltonian and the maximum eigenvalue of $-\hat{H}_C$ itself, $r = \frac{E}{E_{max}}$. Here $E = -\langle \Psi | \hat{H}_C | \Psi \rangle$ and $E_{max} = -\langle \Psi_{goal} | \hat{H}_C | \Psi_{goal} \rangle$. The global pipeline of the algorithm implementation – installing the graph state to be measured, evaluate the cost function and fix the values of the $\{\gamma_i^*, \beta_i^*\}_{i=1}^p$ angles via backpropagation – is illustrated in Figure 2.

The last Section of the poster describes the code implementation and the achieved results. The maxcut problem has been applied to a set of vertices arranged in a regular polygonal fashion. The number of vertices varies with respect to K and the available RAM memory on the GPUs of davinci-1 supercomputer.

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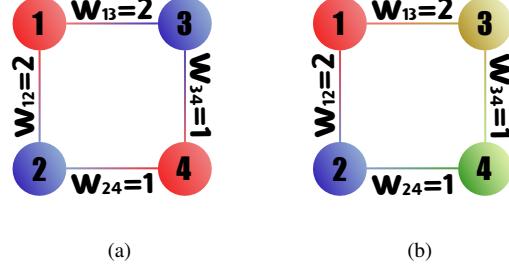


Fig. 1

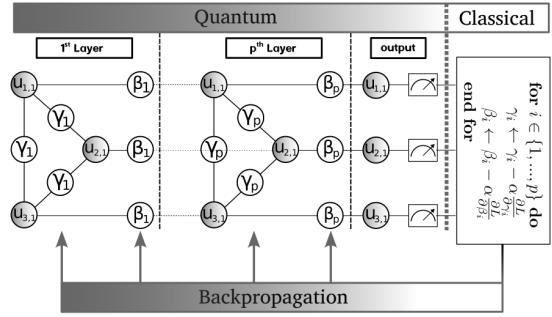


Fig. 2

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