Chapter 3 : Basic Nodal and Mesh Analysis

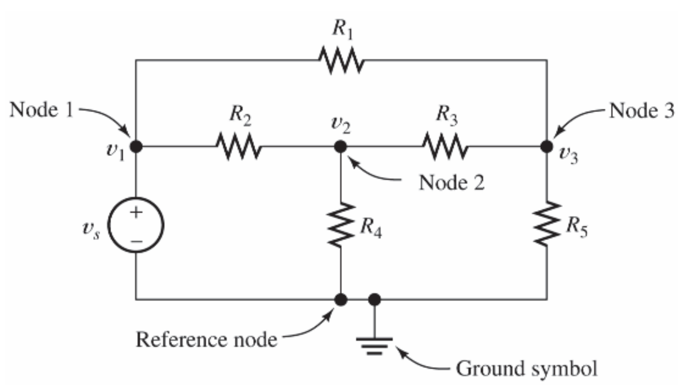
Course Objectives

After completed study this chapter, students will be able to :

- Applying KCL , KVL and Ohm's Law for basic circuit analysis.
- Nodal and Mesh analysis techniques are introduced .
- Supernode and Loop for some circuits analysis.

3.1 Circuit Analysis

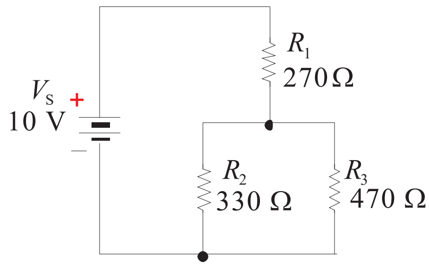
Most practical circuits have combinations of series and parallel components. You can frequently simplify analysis by combining series and parallel components.



- as circuits get more complicated, we need an organized method of applying KVL, KCL, and Ohm's
- *nodal* analysis assigns *voltages* to each node, and then we apply *KCL*
- *mesh* analysis assigns *currents* to each mesh, and then we apply *KVL*

Combination circuits

Kirchhoff's voltage law and **Kirchhoff's current law** can be applied to any circuit, including combination circuits



I_1= 21.6 mA	R_1= 270 Ω	V_1 = 5.82 V	P_1 = 126 mW
I_2= 12.7 mA	R_2= 330 Ω	V_2 = 4.18 V	P_2 = 53.1 mW
I_3= 8.9 mA	R_3= 470 Ω	V_3 = 4.18 V	P_3 = 37.2 mW
I_T= 21.6 mA	R_T = 464 Ω	V_S = 10 V	P_T = 216 mW

- General approach to circuit analysis:
  - Study the problem in total and make a brief mental sketch of the overall approach you plan to use.
  - Examine each region of the network independently before tying them together in series-parallel combinations.
  - Redraw the network as often as possible with reduced branches and undisturbed unknown quantities to maintain clarity.
  - When you have a solution, check to see that it is reasonable by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve using another approach or check over your work very carefully

3.2 The Nodal Analysis Method

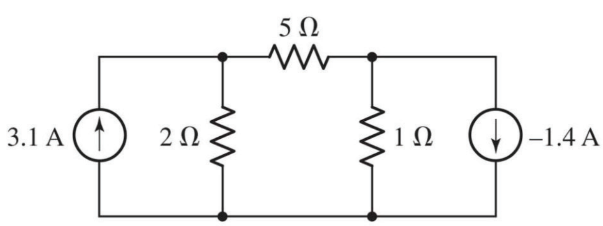
In the **node voltage method**, you can solve for the unknown voltages in a circuit using KCL.

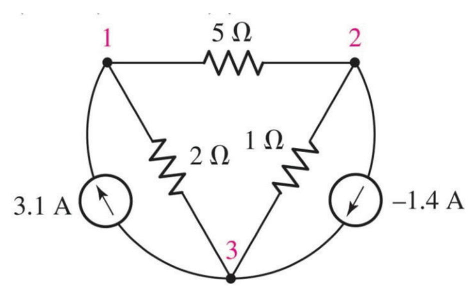
Steps:

1. Determine the number of nodes.
2. Select one node as a reference. Assign voltage designations to each unknown node.
3. Assign currents into and out of each node except the reference node.
4. Apply KCL at each node where currents are assigned.
5. Express the current equations in terms of the voltages and solve for the unknown voltages using Ohm's law.

Ex 3.1 : Find the nodes voltage v_1 and v_2

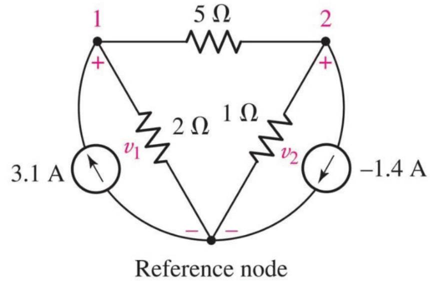
Step 1: Assign voltages to every node relative to a reference node.

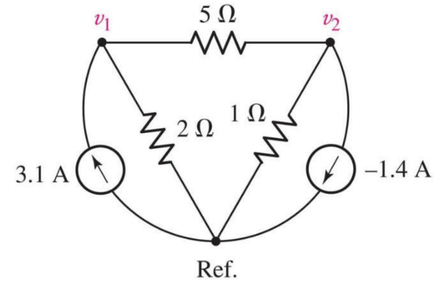




in this example, there are three nodes

Step 2 : Choosing the Reference Node





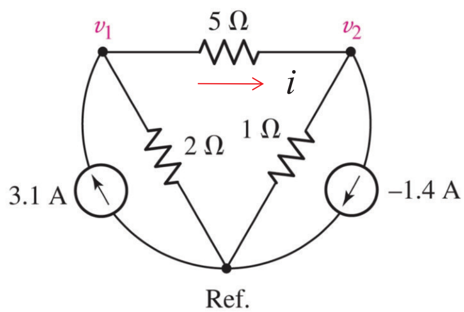
- as the bottom node, or
- as the ground connection, if there is one, or
- a node with many connections
- assign voltages relative to reference

Step 3 : Apply KCL to find node voltages

Apply KCL to node 1 (Σ out = Σ in) and Ohm's law to each resistor:

$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1 \qquad \text{or}$$

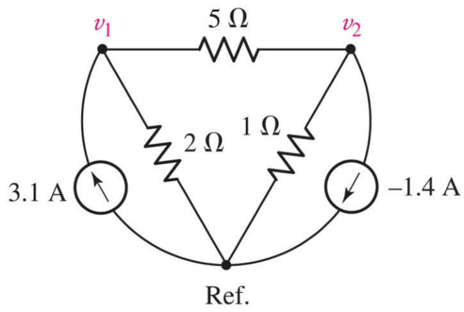
$$7v_1 - 2v_2 = 31 \qquad \text{------- (1)}$$



Note: the current flowing out of node 1 through the 5 Ω resistor is

$$v_1 - v_2 = i \times 5$$

Apply KCL to node 2 (Σ out = Σ in) and Ohm's law to each resistor:



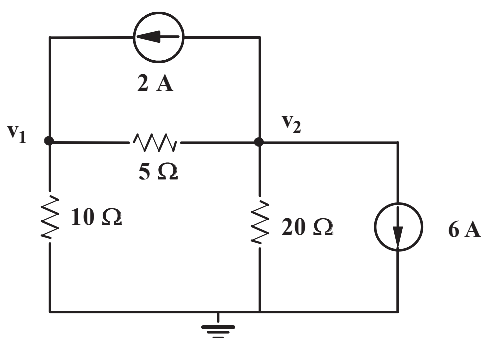
$$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4) \quad \text{or}$$

$$-v_1 + 6v_2 = 7 \qquad \text{------- (2)}$$

We now have two equations for the two unknowns v_1 and v_2 and can solve.

Ans [$v_1 = 5$ V and $v_2 = 2$V]

Ex 3.2 : Find the nodes voltage v_1 and v_2



KCL @ nodes v_1

$$\frac{V_1}{10} + \frac{V_1 - V_2}{5} = 2$$

$$V_1 + 2V_1 - 2V_2 = 20$$

or $3V_1 - 2V_2 = 20$ ------- (1)

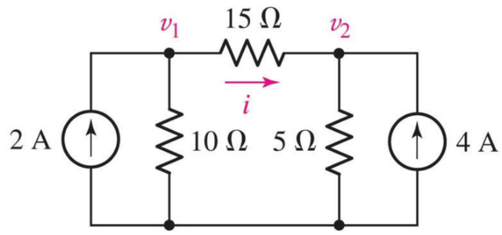
KCL @ nodes v_2 $\dfrac{V_2 - V_1}{5} + \dfrac{V_2}{20} = -6$ => $4V_2 - 4V_1 + V_2 = -120$

or $-4V_1 + 5V_2 = -120$ ------- (2)

Equation (1) and (2) giving =>

Solution: $V_1 = -20$ V, $V_2 = -40$ V

Ex 3.3 : Find the current i in the circuit.



Answer: $i = 0$ (since $v_1=v_2=20$ V)

solution

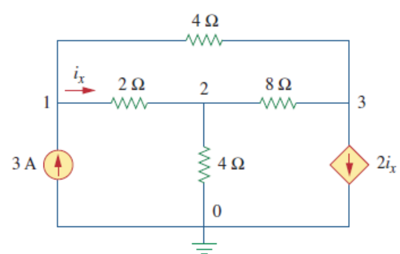
KCL @ node v_1 $\quad 2=\dfrac{v_1}{10}+\dfrac{v_1-v_2}{15} \Rightarrow 5v_1-2v_2=60$ ----- (1)

KCL @ node v_2 $\quad \dfrac{v_2}{5}=4+\dfrac{v_1-v_2}{15} \Rightarrow v_1-4v_2=-60$ ----- (2)

Solving for equation (1) and (2) $\Rightarrow v_1=v_2=20\,V$

Thus $\quad i=\dfrac{v_1-v_2}{15}=\dfrac{20-20}{15}=0\ A$

Ex 3.4 : Determine the voltage at the node in figure below.



At node 1,

$$3=i_1+i_x \quad\Rightarrow\quad 3=\frac{v_1-v_3}{4}+\frac{v_1-v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1-2v_2-v_3=12 \qquad \text{---------- (1)}$$

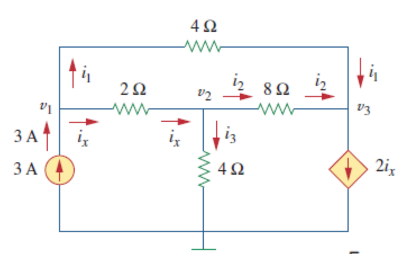
At node 2,

$$i_x=i_2+i_3 \quad\Rightarrow\quad \frac{v_1-v_2}{2}=\frac{v_2-v_3}{8}+\frac{v_2-0}{4}$$

solution

Multiplying by 8 and rearranging terms, we get

$$-4v_1+7v_2-v_3=0 \qquad \text{---------- (2)}$$



At node 3,

$$i_1+i_2=2i_x \quad\Rightarrow\quad \frac{v_1-v_3}{4}+\frac{v_2-v_3}{8}=\frac{2(v_1-v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1-3v_2+v_3=0 \qquad \text{---------- (3)}$$

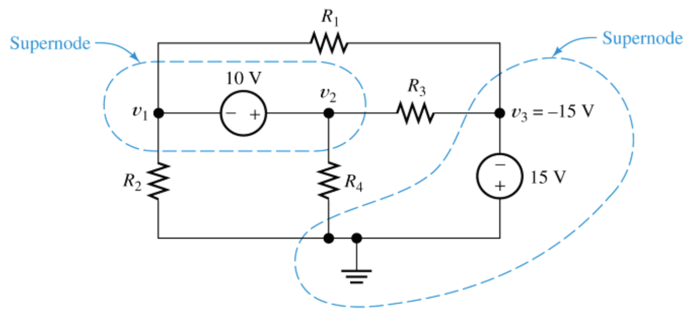
To use Cramer's rule, in equation (1) (2) and (3)

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $v_1 = 4.8$ V, $v_2 = 2.4$ V, $v_3 = -2.4$ V

3.2.1 Voltage Sources and the Supernode

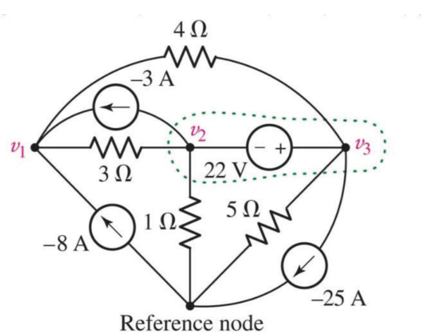
When a voltage source appears between two nodes, an easy way to handle this is to form a super node. The super node encircles the voltage source and the tips of the branches connected to the nodes.

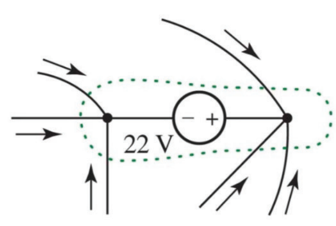


* Can't write *KCL* for node containing voltage source.
* ***Super-node*** combines several node.

The Supernode

We can eliminate the need for introducing a current variable by applying KCL to the *supernode.*





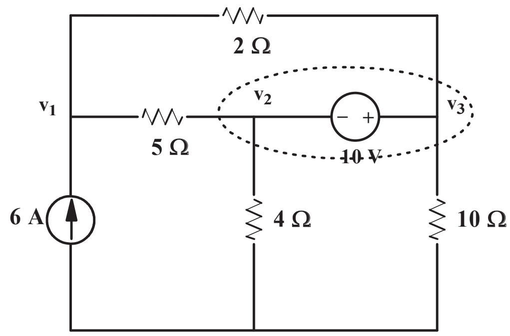
- Apply KCL at Node 1.
- Apply KCL at the supernode.
- Add the equation for the voltage source inside the supernode.

$$\frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{3} = -3 - 8$$

$$\frac{v_2}{1} + \frac{v_2 - v_1}{3} + \frac{v_3}{5} + \frac{v_3 - v_1}{4} = -(-25) - (-3)$$

$$v_3 - v_2 = 22$$

Ex 3.5 : Determine the nodes voltage v_1 v_2 and v_3



Constraint Equation

$V_2 - V_3 = -10$ ------ (1)

KCL @ nodes v_1

$$\frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2} = 6 \quad \text{------ (2)}$$

KCL @ supernode

$$\frac{V_2 - V_1}{5} + \frac{V_2}{4} + \frac{V_3}{10} + \frac{V_3 - V_1}{2} = 0 \quad \text{------ (3)}$$

Rearrange equation (1) (2) and (3)

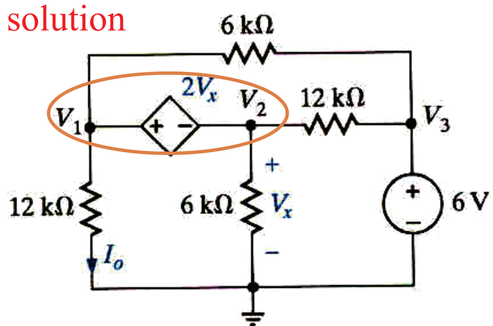
$$7V_1 - 2V_2 - 5V_3 = 60$$
$$-14V_1 + 9V_2 + 12V_3 = 0$$
$$V_2 - V_3 = -10$$

Solving gives =>

$V_1 = 30$ V, $V_2 = 14.29$ V, $V_3 = 24.29$ V

Ex 3.6 : Determine the current I_0

solution



Voltage source connected to reference

$V_3 = 6V$

Supernode Constraint Equation

$V_1 - V_2 = 2V_x$

KCL @ supernode $V_x = V_2$ => $V_1 = 3V_x = 3V_2$

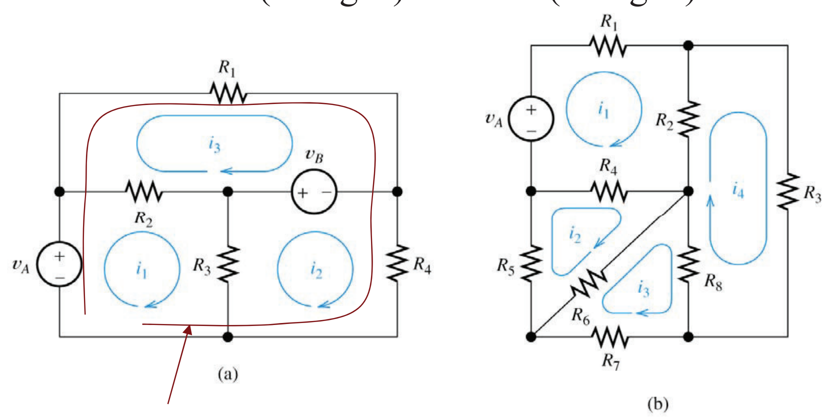
$$\boxed{\frac{V_1 - V_3}{6k} + \frac{V_1}{12k} + \frac{V_2}{6k} + \frac{V_2 - V_3}{12k} = 0}$$

$$2(V_1 - 6) + V_1 + 2V_2 + V_2 - 6 = 0 \implies 3V_1 + 3V_2 = 18 \quad \Rightarrow 4V_1 = 18$$

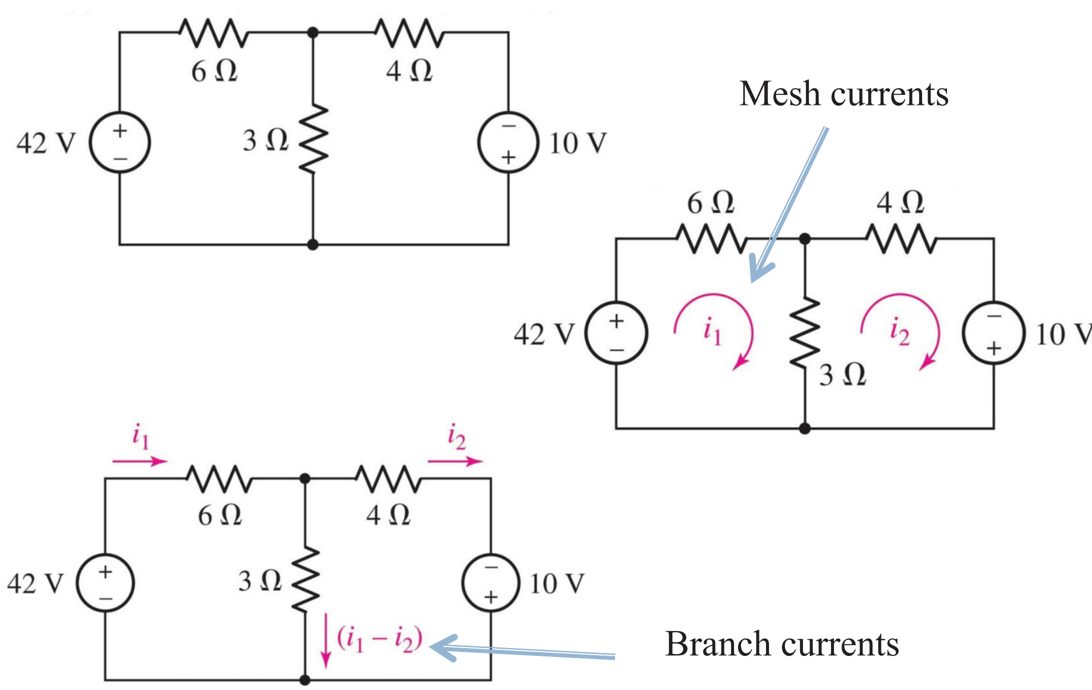
$$\boxed{I_o = \frac{V_1}{12k} = \frac{3}{8}\text{mA}}$$

3.2 The Mesh and Loop Analysis Method

- a mesh is a loop which does not contain any other loops within it
- in mesh analysis, we assign currents and solve using KVL
- assigning mesh currents automatically ensures KCL is followed
- this circuit has three (in Fig. a) and four (in Fig. b) meshes:



This is a loop but not a mesh (current)



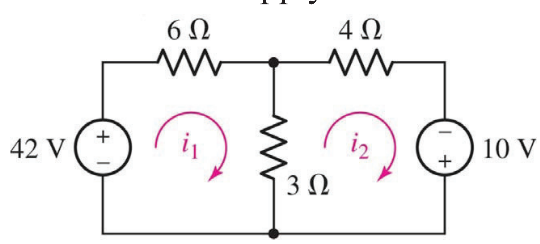
Mesh current method

In the **mesh current method**, you can solve for the currents in a circuit using simultaneous equations.

Steps:

1. Assign a current in each nonredundant mesh in an arbitrary direction.
2. Show polarities according to the assigned direction of current in each mesh.
3. Apply KVL around each closed mesh.
4. Solve the resulting equations for the mesh currents.

Step 1 : Assign mesh current and apply KVL



Apply KVL to mesh 1 (Σ drops=0):

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

Apply KVL to mesh 2 (Σ drops=0):

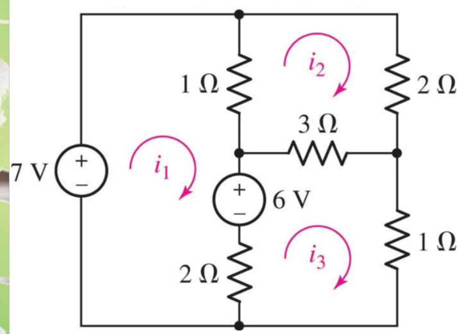
$$3(i_2 - i_1) + 4i_2 - 10 = 0$$

Step 2 : Rearrange the simultaneous and solving them

$-42 + 6i_1 + 3(i_1 - i_2) = 0 \quad \Rightarrow \quad 9i_1 - 3i_2 = 42$ and

$3(i_2 - i_1) + 4i_2 - 10 = 0 \quad \Rightarrow \quad -3i_1 + 7i_2 = 10$ yield $i_1 = 6$; $i_2 = 4$

Ex 3.7 : Determine the mesh current i_1 i_2 *and* i_3



solution

Follow each mesh clockwise

$$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$$
$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$
$$2(i_3 - i_1) - 6 + 3(i_3 - i_2) + 1i_3 = 0$$

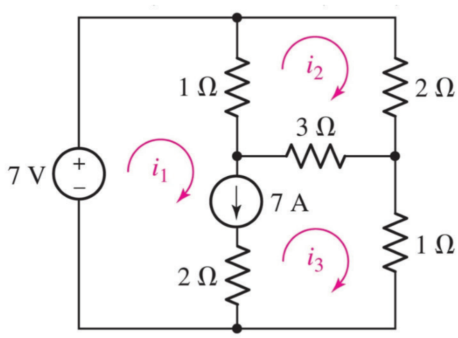
$$3i_1 - i_2 - 2i_3 = 1$$
$$-i_1 + 6i_2 - 3i_3 = 0$$
$$-2i_1 - 3i_2 + 6i_3 = 6$$

Solve the equations:

$i_1 = 3$ A, $i_2 = 2$ A, and $i_3 = 3$ A.

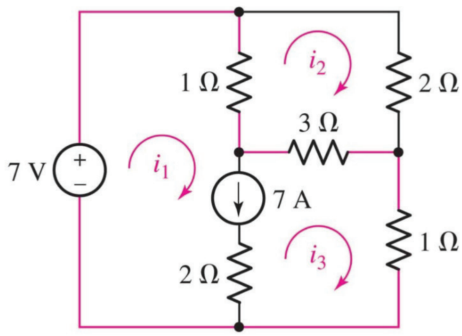
3.3.1 Current Sources and the Supermesh

What is the voltage across a current source in between two meshes?



We can eliminate the need for introducing a voltage variable by applying KVL to the *supermesh* formed by joining mesh 1 and mesh 3

The Supermesh



Apply KVL to mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

Apply KVL supermesh 1/3:

$$-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$$

Add the current source:

$$7 = i_1 - i_3$$

Rearrange the simultaneous equations and solving them :

$$-i_1 + 6i_2 - 3i_3 = 0$$

$$i_1 - 4i_2 + 4i_3 = 7$$

$$i_1 + 0i_2 - i_3 = 7$$

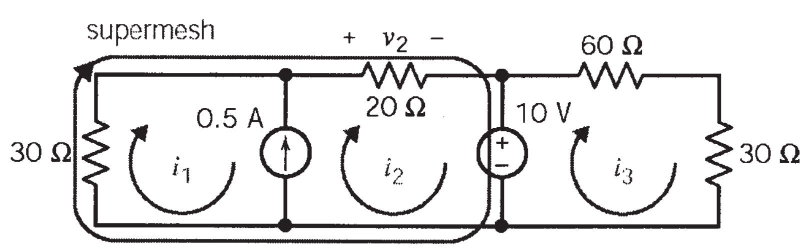
Cramer's Rule =>

$$i_1 = 9$$

$$i_2 = 2.5$$

$$i_3 = 2$$

Ex 3.8 : Determine the mesh current i_1 i_2 and voltage v_2



solution

Express the current source current as a function of the mesh currents:

$$i_1 - i_2 = -0.5 \Rightarrow i_1 = i_2 - 0.5$$

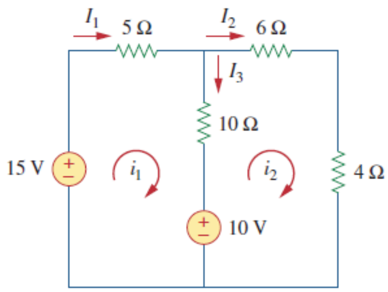
Apply KVL to the supermesh:

$$30\, i_1 + 20\, i_2 + 10 = 0 \Rightarrow 30(i_2 - 0.5) + 20 i_2 = -10$$

$$50\, i_2 - 15 = -10 \Rightarrow i_2 = \frac{5}{50} = .1 \text{ A}$$

$$i_1 = -.4 \text{ A} \quad \text{and} \quad v_2 = 20\, i_2 = \underline{2 \text{ V}}$$

Ex 3.9 : Find the branch currents I_1 I_2 *and* I_3 using mesh analysis.



solution

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \qquad \text{----------} (1)$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

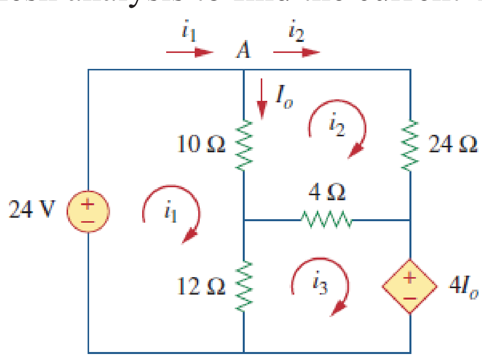
$$i_1 = 2i_2 - 1 \qquad \text{----------} (2)$$

Using the substitution method, we substitute equation (2) in (1)

$6i_2 - 3 - 2i_2 = 1 \quad \Rightarrow \quad i_2 = 1$ A and $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

Ex 3.10 : Using mesh analysis to find the current I_0 in the figure below.



solution

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0 \qquad \text{or}$$

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad \text{----------} (1)$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0 \qquad \text{or}$$

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad \text{----------} (2)$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or $$-i_1 - i_2 + 2i_3 = 0$$ ---------- (3)

Write equation (1) (2) and (3) In matrix form :

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

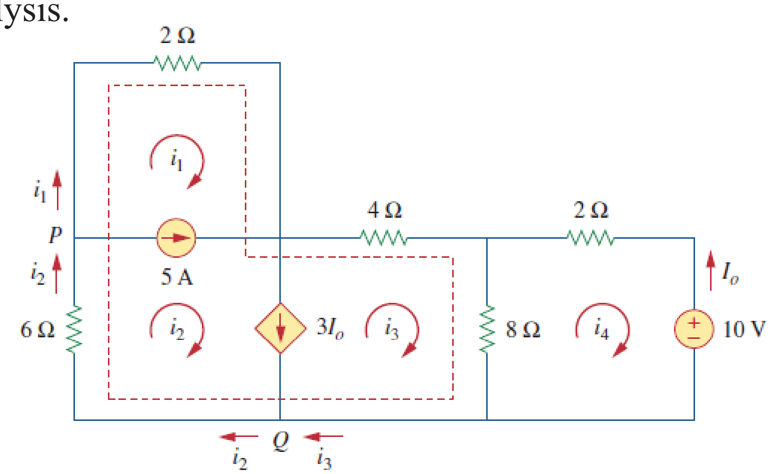
We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25 \text{ A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75 \text{ A},$$

$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5 \text{ A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

Ex 3.11 : For the circuit in figure below , find currents i_1 *to* i_4 using mesh analysis.



solution

Applying KVL to the larger supermesh,

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$ or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$ ---------- (1)

For the independent current source, we apply KCL to node P:

$$i_2 = i_1 + 5 \qquad \text{---------- (2)}$$

For the dependent current source, we apply KCL to node Q:

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4 \qquad \text{---------- (3)}$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

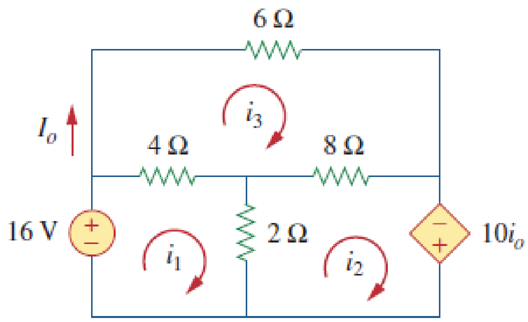
$$5i_4 - 4i_3 = -5 \qquad \text{---------- (4)}$$

For equation (1) to (4) , we can solve for each current :

$i_1 = -7.5$ A, $\quad i_2 = -2.5$ A, $\quad i_3 = 3.93$ A, $\quad i_4 = 2.143$ A

Self-Test : 3.1 Using mesh analysis, find I_0 in the figure below.

(Ans : I_0 = - 4A)



solution : Note that $i_0 = i_3$

Apply KVL to mesh 1 (Σ drops = 0 V) :

$$-16 + 4(i_1 - i_3) + 2(i_1 - i_2) = 0 \quad \text{or}$$

$$6i_1 - 2i_2 - 4i_3 = 16 \quad \text{-------- (1)}$$

Apply KVL to mesh 2 (Σ drops = 0 V) :

$$-10i_0 + 2(i_2 - i_1) + 8(i_2 - i_3) = 0 \quad \text{or}$$

$$-2i_1 + 10i_2 - 18i_3 = 0 \quad \text{-------- (2)}$$

Apply KVL to mesh 3 (Σ drops = 0 V) :

$$+6i_3 + 8(i_3 - i_2) + 4(i_3 - i_1) = 0 \quad \text{or}$$

$$-4i_1 - 8i_2 + 18i_3 = 0 \quad \text{-------- (3)}$$

Write equation (1) (2) and (3) In matrix form :

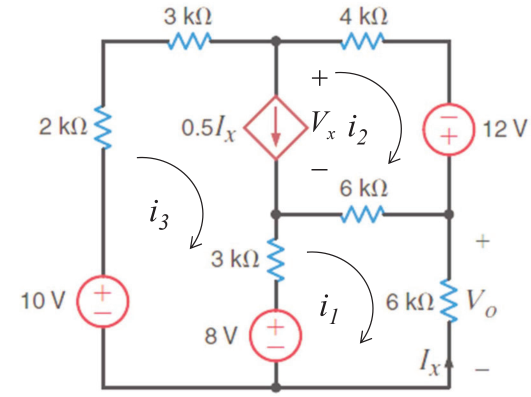
$$\begin{bmatrix} 6 & -2 & -4 \\ -2 & 10 & -18 \\ -4 & -8 & 18 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \\ 0 \end{bmatrix}$$

Using Cramer's rule yield :

$i_1 = -2.57$ A $\qquad i_2 = -7.71$ A

and $i_3 = i_0 = -4$ A

3.2 Find V_o using mesh analysis (Ans : $V_0 = 9$ V)



Solution : Note that $i_1 = -I_x$

$V_0 = -I_x.(6K) = i_1.(6K)$;

$(i_3 - i_2) = 0.5\, I_x = -0.5 i_1$ -------- (1)

Apply KVL to mesh 1 (Σ drops = 0 V) :

$-8 + 3K(i_1 - i_3) + 6K(i_1 - i_2) + 6K.i_1 = 0$

or $15Ki_1 - 6Ki_2 - 3Ki_3 = 8$ -------- (2)

Apply KVL to mesh 2 (Σ drops = 0 V) :

$-12 + 6K(i_2 - i_1) - V_x + 4Ki_2 = 0$ or

$-6Ki_1 + 10Ki_2 - V_x = 12$ -------- (3)

Apply KVL to mesh 3 (Σ drops = 0 V) :

$-10 + 5Ki_3 + V_x + 3K(i_3 - i_1) + 8 = 0$ or

$-3Ki_1 + 8K.i_3 + V_x = 2$ -------- (4)

Adding equation (3) and (4) yield :

$-9Ki_1 + 10Ki_2 + 8K.i_3 = 14$ -------- (5)

Write equation (1) (2) and (5) In matrix form :

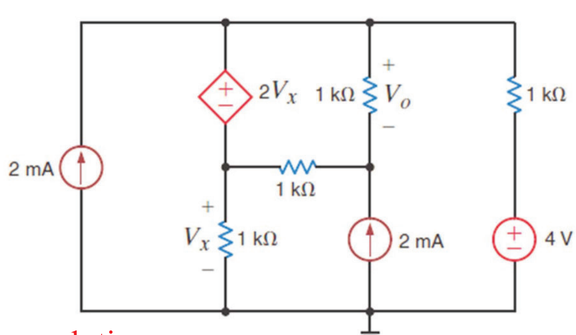
$$\begin{bmatrix} 0.5 & -1 & 1 \\ 15 & -6 & -3 \\ -9 & 10 & 8 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ 14 \end{bmatrix}$$

Using Cramer's rule yield :

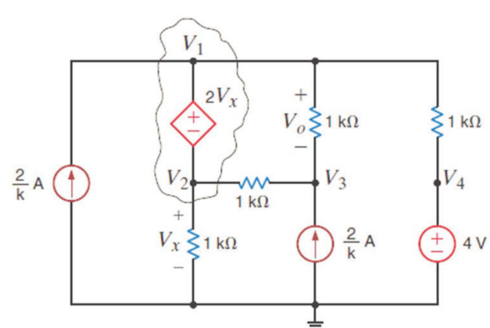
$i_1 = 1.5$ mA , $i_2 = 1.86$ mA , $i_3 = 1.11$ mA ,

and $V_0 = i_1.(6K) = 1.5$ mA*6K = 9 V

3.3 Find V_o and V_x using nodal analysis. (Ans : $V_0 = 1$ V ; $V_x = 2$ V)



solution



Defining node voltage equation is :

$V_1 - V_2 = 2V_x$ $\qquad V_1 = 3V_x$

and

$V_2 = V_x$ $\qquad V_4 = 4$

Applying KCL at the supernode and at the node labeled V_3 The equations are :

$$-\frac{2}{k} + \frac{V_x}{1k} + \frac{V_x - V_3}{1k} + \frac{3V_x - V_3}{1k} + \frac{3V_x - 4}{1k} = 0$$

$$\frac{V_3 - 3V_x}{1k} + \frac{V_3 - V_x}{1k} = \frac{2}{k}$$

Combining the equations yields the two equations

$$8V_x - 2V_3 = 6$$

$$-4V_x + 2V_3 = 2$$

Solving these equations, we obtain

$$V_x = 2\text{ V} \quad \text{and} \quad V_3 = 5\text{ V}$$

$$V_o = 3V_x - V_3 = 1\text{ V}$$