1

Prove that $B, D \to A, C$

$R = \{A, B, C, D\}$

$FD = \{A \to D, C \to B, B \to A, D \to C\}$

① ② ③ ④

Prove 1.

1). $B \to A$ ③

2). $B, D \to A, D$ augmentation from 1)

3). $A, D \to C$ reflexibility with ④

4). $B, D \to C$ transitivity with 2). 3).

5. $B, D \to A, D, C$ union 2). 4).

6. $B, D \to A, C$ 5). decomposition

$B, D \to A, C$ proven!

Prove 2.

$B, D \to A$

1) $B \to A$ ③

$\left.\begin{array}{l} B \subseteq B, D \\ B, D \to B^+ \\ B, D \to A \end{array}\right]$ reflexivity

2) $B, D \to A$

$B, D \to C$

1) $D \to C$ ④

$\left.\begin{array}{l} D \subseteq B, D \\ B, D \to D^+ \\ B, D \to C \end{array}\right]$ reflexivity

2) $B, D \to C$

union

$B, D \to A, C$ proven!

9.

$R = \{M, N, O, P, Q, S\}$

$F = \{MN \rightarrow OP, O \rightarrow Q, QS \rightarrow N, S \rightarrow P\}$

$MN^+ = MNOPQ$ ✗ sk
$O^+ = O$ ✗ sk
$QS^+ = QSNP$ ✗ sk
$S^+ = SP$ ✗ sk

<u>Not BCNF</u>

$MNS^+ = MNOPQS$ ✓ sk
$MQS^+ = MQSNOP$ ✓ sk
$MOS^+ = MOSQNP$ ✓ sk

prime attributes = $\{M, N, Q, S, O\}$

RHS	
OP	✓ in Prime Atr.
Q	✓ in Prime Atr.
N	✓ In Prime Atr.
P	✗ Not in prime atr.

<u>Not 3NF</u>

∴ conditions for both BCNF and 3NF are not satisfied

<u>R is not in 3NF and BCNF</u>

3. $R = \{A, B, C, D, E, G, H\}$

$F = \{AB \to C,\ AE \to CG,\ AC \to B,\ A \to H,\ BC \to EG,\ BD \to H\},\ AD \to E,\ B \to DG,\ BE \to A,$

① decompose into single LHS FDs
② eliminate redundant FDs
③ eliminate composite redundant FD.

①

$AB \to C$
$AC \to B$
$AD \to E$
$BE \to A$
$A \to H$
$BD \to H$
$BC \to E$, $BC \to G$] $\to BC \to EG$
$B \to D$, $B \to G$] $\to B \to DG$
$AE \to C$, $AE \to G$] $\to AE \to CG$

②

$AE^+ = AEHCB\underline{G}$ $\cancel{AE \to G}$
$AE^+ = AEH$
$B^+ = BDH$
$B^+ = BG$
$BC^+ = BCG\underline{G}$ $\cancel{BC \to G}$
$BC^+ = BCDGH$
$BD^+ = BDG$
$A^+ = A$
$BE^+ = BEDGH$
$AD^+ = ADH$
$AC^+ = ACH$
$AB^+ = ABHDGE\underline{C}$ $\cancel{AB \to C}$

③

$AC \to B$
$AD \to E$
$BE \to A$
$A \to H$
$\cancel{BD \to H}$ $B \to H$
$BC \to E$
$B \to D$
$B \to G$
$AE \to C$

$BD \to H$
since $B \to D$
$BD \to H$ is redundant

Minimal cover is $= \{AC \to B,\ AD \to E,\ BE \to A,\ A \to H,\ BC \to E,\ B \to D,\ B \to G,\ AE \to C\}$

4.

$r_2(x)\ r_3(y)\ r_1(x)\ w_3(y)\ w_1(x)\ r_4(y)\ r_1(z)\ w_4(y)\ r_2(y)\ r_3(z)\ w_2(y)$

x: $r_2(x)\ r_1(x)\ w_1(x)$

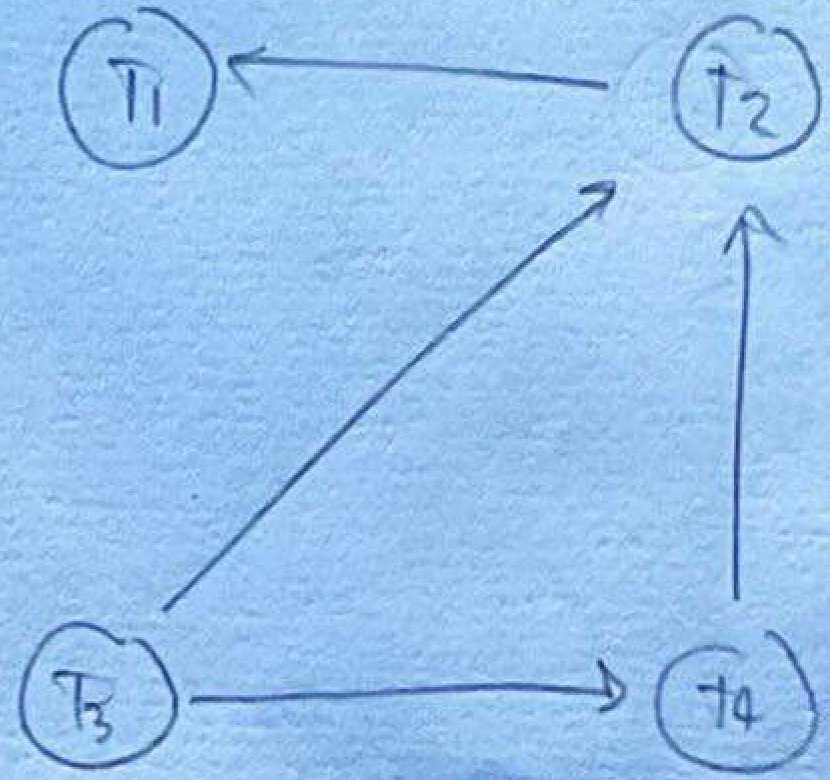
y: $r_3(y)\ w_3(y)\ r_4(y)\ w_4(y)\ r_2(y)\ w_2(y)$

z: $r_1(z)\ r_3(z)$

Conflict pairs

x: $r_2(x)\ w_1(x)$

y: $r_3(y)\ w_4(y)$
$r_3(y)\ w_2(y)$
$w_3(y)\ r_4(y)$
$w_3(y)\ w_4(y)$
$w_3(y)\ r_2(y)$
$w_3(y)\ r_2(y)$
$r_4(y)\ w_2(y)$
$w_4(y)\ r_2(y)$
$w_4(y)\ w_2(y)$



∴ graph is apparently acyclic, so it is serializable

Equivalent serial history:

$T_3 \rightarrow T_4 \rightarrow T_2 \rightarrow T_1$ (used manual topological sort)

Bucket T_1: $r_1(x)\ w_1(x)\ r_1(z)$

Bucket T_2: $r_2(x)\ r_2(y)\ w_2(y)$

Bucket T_3: $r_3(y)\ w_3(y)\ r_3(z)$

Bucket T_4: $r_4(y)\ w_4(y)$

Equivalent serial schedule: $r_3(y)\ w_3(y)\ r_3(z)\ r_4(y)\ w_4(y)\ r_2(x)\ r_2(y)\ w_2(y)\ r_1(x)\ w_1(x)\ r_1(z)$