ICMA 223 Linear Algebra A

Problem Set 6

General Information

Important Note! Please do write a list of collaborators (friends you work with) and sources you consult for this assignment (e.g. lecture notes, specific pages of a book, specific links to Wikipedia, etc., but do not write just "YouTube" or "Google" without further information). Even if you work on this assignment alone and do not consult any source, please write "**Collaborators:** None. **Sources consulted:** None." in your submission.

Collaboration on problem sets is allowed, and is in fact encouraged. Working with friends can be an enjoyable way to learn mathematics!

Information: This problem set is due at **11:59 pm (Thai time), Friday, June 6, 2025**. You should submit your work on Canvas. See the syllabus for the homework policy.

For each problem, please show your work! For correct answers alone without proper explanations or derivations, you might be awarded only very few, or even zero, points. On the other hand, for incorrect answers with proper explanations or derivations, you might be awarded a lot of points.

Problem 0 (10 points)

Please provide the following information. Please refer to the "General Information" section above for details.

(a) What is your full name (first name and last name)?

(b) What is your student ID number?

(c) Which section are you a student of, Section 1 or Section 2?

(d) Please write the list of your collaborators for this problem set.

(e) Please write the list of sources you consult for this problem set.

Optional: what is your nickname (if you have one)?

Problem 1 (30 points)

Read the lecture notes for the sixth week.

Write one short paragraph (containing approximately 3 - 5 sentences) about the reading. Your paragraph can be, for example, something you found interesting or confusing about what you read, or it can be where you work out an explicit example of some result from the notes, or it can even be other things you would like to write about which are related to the reading!

Problem 2 (20 points)

Consider the following matrix.

$$A := \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 3 & 4 \\ 0 & 0 & 5 & 0 \end{bmatrix}.$$

(a) Perform Gaussian elimination on the matrix $[A|I_4]$. Please show every step, and keep track of the determinant of the elementary matrix involved in every step.

(b) Is A invertible? If A is invertible, what is the inverse A^{-1} of A?

(c) Use your result from part (a), compute the determinant $\det(A)$.

Problem 3 (20 points)

Consider the following matrix.

$$B := \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & -2 \\ 1 & 2 & -1 \end{bmatrix}.$$

(a) Perform Gaussian elimination on the matrix $[B|I_3]$. Please show every step, and keep track of the determinant of the elementary matrix involved in every step.

(b) Is B invertible? If B is invertible, what is the inverse B^{-1} of B?

(c) Use your result from part (a), compute the determinant $\det(B)$.

Problem 4 (20 points)

Let

$$C := \begin{bmatrix} 0 & 3 & 4 \\ -1 & 0 & 1 \\ 2 & 5 & 2 \end{bmatrix}.$$

(a) Use the rule of Sarrus to compute $\det(C)$.

(b) Find matrices M and N such that M is a symmetric matrix, N is an antisymmetric matrix, and $C^2 = M + N$.