ICMA 223 Linear Algebra A

Problem Set 5

General Information

Important Note! Please do write a list of collaborators (friends you work with) and sources you consult for this assignment (e.g. lecture notes, specific pages of a book, specific links to Wikipedia, etc., but do not write just "YouTube" or "Google" without further information). Even if you work on this assignment alone and do not consult any source, please write "**Collaborators:** None. **Sources consulted:** None." in your submission.

Collaboration on problem sets is allowed, and is in fact encouraged. Working with friends can be an enjoyable way to learn mathematics!

Information: This problem set is **ungraded**. You should work on all the problems, but you do not need to submit your work.

For each problem, please show your work! For correct answers alone without proper explanations or derivations, you might be awarded only very few, or even zero, points. On the other hand, for incorrect answers with proper explanations or derivations, you might be awarded a lot of points.

Problem 0 (10 points)

Please provide the following information. Please refer to the "General Information" section above for details.

(a) What is your full name (first name and last name)?

(b) What is your student ID number?

(c) Which section are you a student of, Section 1 or Section 2?

(d) Please write the list of your collaborators for this problem set.

(e) Please write the list of sources you consult for this problem set.

Optional: what is your nickname (if you have one)?

PROBLEM 1 (30 points)

Read the lecture notes for the fifth week.

Write one short paragraph (containing approximately 3 - 5 sentences) about the reading. Your paragraph can be, for example, something you found interesting or confusing about what you read, or it can be where you work out an explicit example of some result from the notes, or it can even be other things you would like to write about which are related to the reading!

PROBLEM 2 (30 points)

Let

$$X := \begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

Let us perform Gaussian elimination on X using three elementary row operations as follows.

- $R_2 \mapsto R_2 + a \cdot R_1$,
- $R_2 \mapsto b \cdot R_2$, and
- $R_1 \rightleftharpoons R_2$,

where a and b are real numbers. Let Y be the resulting matrix in reduced row echelon form.

(a) What are a and b?

(b) What is Y? Write out the entries of Y explicitly.

(c) Let E_1, E_2, and E_3 be the elementary matrices corresponding to the three row operations above in the respective order. Compute $\det(E_1)$, $\det(E_2)$, and $\det(E_3)$.

(d) Find elementary matrices L_1, L_2, L_3 such that $X = L_1 \cdot L_2 \cdot L_3 \cdot Y$.

Hint for part (d). The inverse of an elementary matrix is also an elementary matrix.

PROBLEM 3 (30 points)

Let

$$A := \begin{bmatrix} 0 & 3 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

(a) Find elementary matrices E_1, E_2, E_3, E_4, E_5 such that

$$E_5 \cdot E_4 \cdot E_3 \cdot E_2 \cdot E_1 \cdot A$$

is a matrix in reduced row echelon form. (Write out the entries of E_1, E_2, E_3, E_4, E_5 explicitly.)

(b) Compute $\det(E_1)$, $\det(E_2)$, $\det(E_3)$, $\det(E_4)$, and $\det(E_5)$.

(c) Using the result from part (b), compute $\det(A)$.
