Chapter 2 : Basic Elements and Resistive Circuits

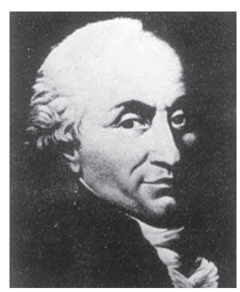
Course Objectives

After completed study this chapter, students will be able to :

- Learn how the basic units and basic electric elements such as voltage sources and current sources.
- Ohm's Law is introduced and its application.
- Basic resistive circuits, calculating the voltage, current and power upon the electric elements.

2.1 Current ,Voltage and Power

Charge



Charge is an electrical property of the atomic particles of which matter consists and is measured in coulombs (Charles Augustin de Coulomb (1736-1806) a French Scientist)

- charge is *conserved*: it is neither created nor destroyed
- symbol: Q or *q;* units are coulomb (C)
- the smallest charge, the *electronic charge*, is carried by an electron (-1.602×10^{-19} C) or a proton ($+1.602\times10^{-19}$ C)
- in most circuits, the charges in motion are electrons

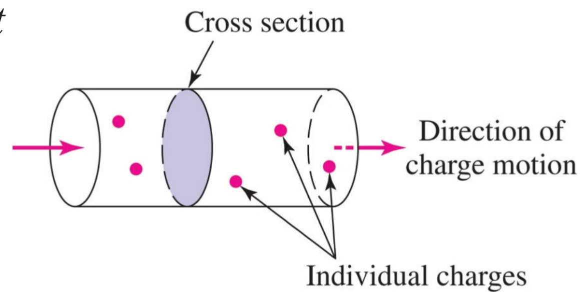
Current

- Current: net flow of charge across any cross section of a conductor, measured in Amperes (Andre-Marie Ampere (1775-1836), a French mathematician and physicist)

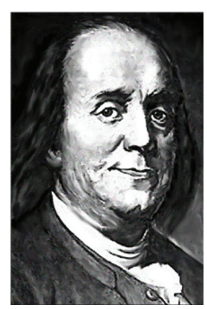
- Current can be thought of as the rate of change of charge

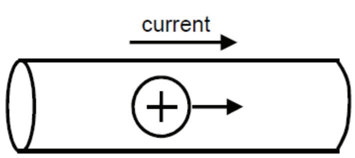
$$i = \frac{dQ}{dt}$$

Current is the rate of charge flow:
1 ampere = 1 coulomb/second
(or 1 A = 1 C/s)



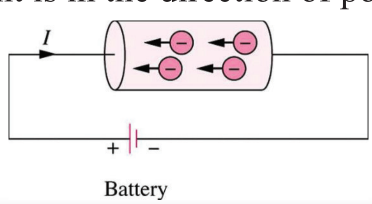
Originally scientists (in particular Benjamin Franklin (1706-1790) an American scientist and inventor) thought that current is only due to the movement of positive charges.



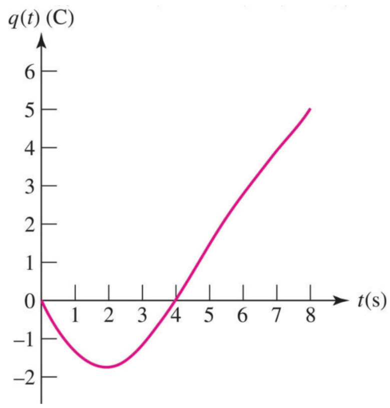


-Thus the direction of the current was considered the direction of movement of positive charges.

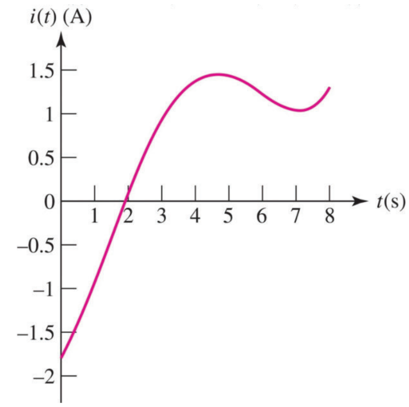
- In reality in metallic conductors current is due to the movement of **electrons, however, we follow the universally accepted** convention that current is in the direction of positive charge movement.



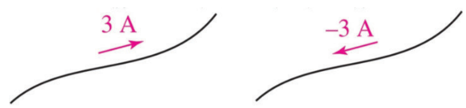
Current (designated by I or i) is the rate of flow of charge



$$i = \frac{dQ}{dt}$$



Current must be designated with both a direction and a magnitude
These two currents are the same:



Voltage (Separation of Charge)

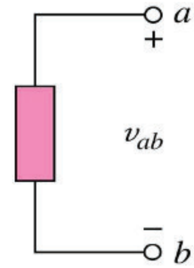
Voltage (electromotive force, or potential) is the energy required to move a unit charge through a circuit element, and is measured in Volts (Alessandro Antonio Volta (1745-1827) an Italian Physicist).

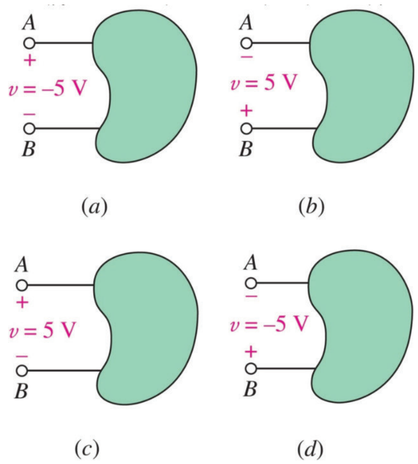
$$v = \frac{dw}{dQ}$$

- When 1 J of work is required to move 1 C of charge from *a* to *b*, there is a voltage of 1 volt between *a* and *b* .

- Voltage (V or *v*) across an element requires both a magnitude and a polarity.

-The assumption is that the potential of the terminal with (+) polarity is higher than the potential of the terminal with (-) polarity by the amount of voltage drop.



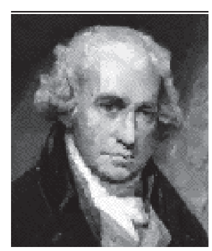


The polarity assignment is somewhat arbitrary! Is this a scientific statement?!! What do you mean by arbitrary?!!!

Example: (a)=(b), (c)=(d)

Power

The rate of change of (expending or absorbing) energy per unit time, measured in Watts (James Watt (1736-1819) a Scottish inventor and mechanical engineer)

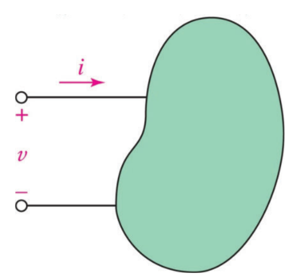


$$p = \frac{dw}{dt} = \frac{dQ}{dt} \times \frac{dw}{dQ} = i \times v$$

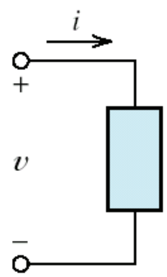
- The power required to push a current i (C/s) into a voltage v (J/C) is $p = vi$ (J/s = W).

- When power is positive, the element is *absorbing* energy.

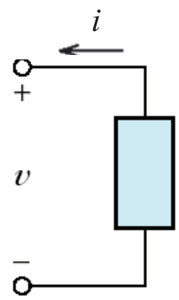
- When power is negative, the element is *supplying* energy.



- One common classification for circuit components is to group them in two major groups:
  1) Passive components or passive elements
     - Components or elements that absorb power.
  2) Active components or active elements
     - Components that are not passive! that is, components that deliver power.



passive elements (loads)



$$p = -vi$$

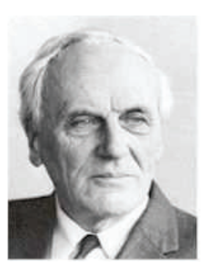
active elements (sources)

Note: a positive p for a battery means: *charging the battery*.

Tellegan's Theorem

- **Principle of Conservation of the Power: The algebraic sum of** the powers absorbed by all elements in a circuit is zero at any instance of time **($\Sigma P=0$). That is, the sum of absorbed powers is** equal to the sum of generated powers at each instance of time.

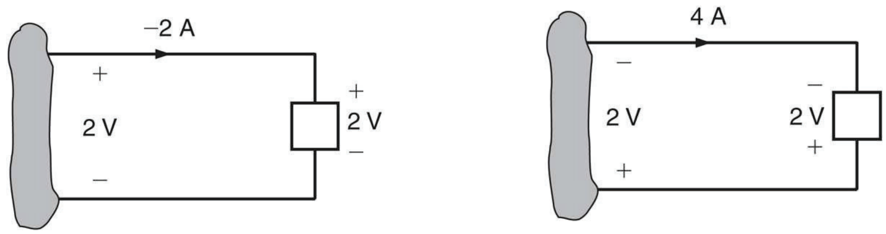
- This principle is also known as Tellegan's theorem. (Bernard D.H. Tellegan (1900-1990), a Dutch electrical engineer)



- Similarly, one can write the principle of conservation of energy.

- Calculate the power absorbed or supplied by each of the following elements:

- Given the two diagrams shown below, determine whether the element is absorbing or supplying power and how much.



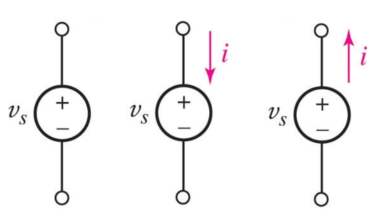
2.2 Voltage Sources and Current Sources

2.2.1 Independent Sources

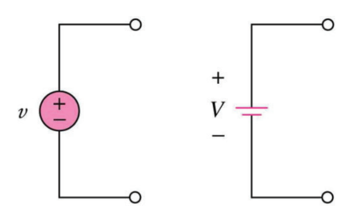
Independent sources: An (ideal) independent source is an active element that provides a specified voltage or current that is independent of other circuit elements and/or how the source is used in the circuit.

voltage source

- An ideal voltage source is a circuit element that will maintain the specified voltage v_s across its terminals.
- The current will be determined by other circuit elements.

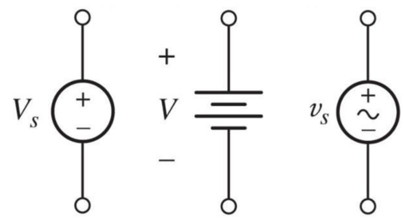


(*a*) (*b*) (*c*)



(a) (b)

- Any value of the current can go through the voltage source, in any direction. The current can also be zero. The voltage source does not "care about" current. It "cares" only about voltage.
- A voltage source is an idealization (no limit on current) and generalization (voltage can be time-varying) of a battery.
- A battery supplies a constant "dc" voltage V but in practice a battery has a maximum power.



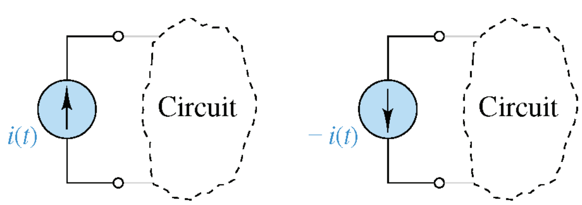
(*a*) (*b*) (*c*)

(a) Constant or dc voltage source

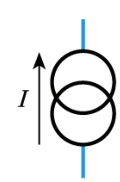
(b) Ac voltage source

current source

- An ideal current source is a circuit element that maintains the specified current flow i_s through its terminals.
- The voltage is determined by other circuit elements.

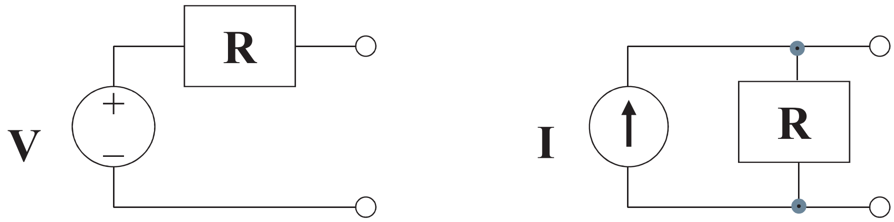


(a) (b)

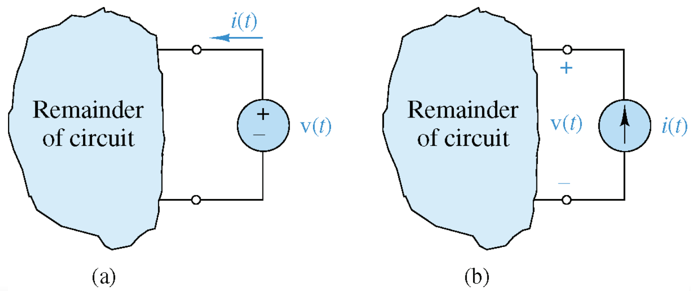


Equivalent representation of ideal independent current sources whose current *i(t) is maintained under all voltage requirements* of the attached circuit:

• While an ideal voltage source has *zero* output resistance, an ideal current source has *infinite* output resistance



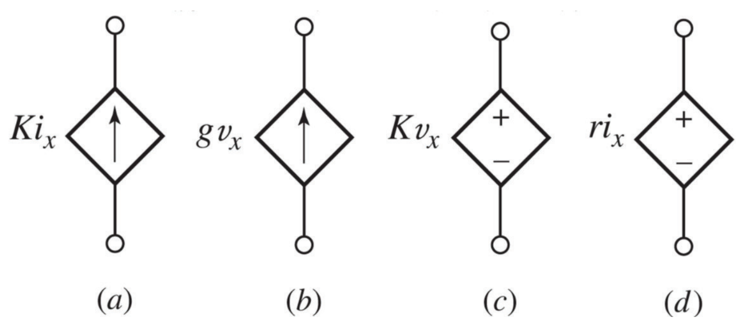
• Is this different from passive sign convention?
• Can we use the passive convention for sources



2.2.2 Dependent Sources

• Dependent current sources (a) and (b) maintain a *current* specified by another circuit variable.

• Dependent voltage sources (c) and (d) maintain a *voltage* specified by another circuit variable.

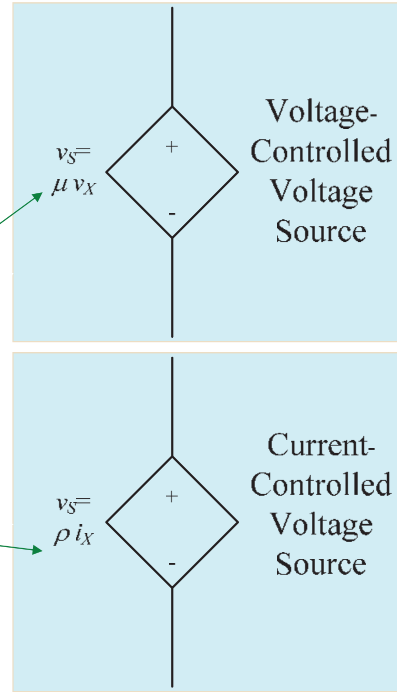


Dependent Voltage Sources

The schematic symbols that we use for dependent voltage sources are shown here, of which there are 2 forms:

1) Voltage-Controlled Voltage Sources (VCVS). The symbol μ is the coefficient of the voltage v_X. It is dimensionless.

2) Current-Controlled Voltage Sources (CCVS). The symbol ρ is the coefficient of the current i_X. It has dimensions of [voltage/current].

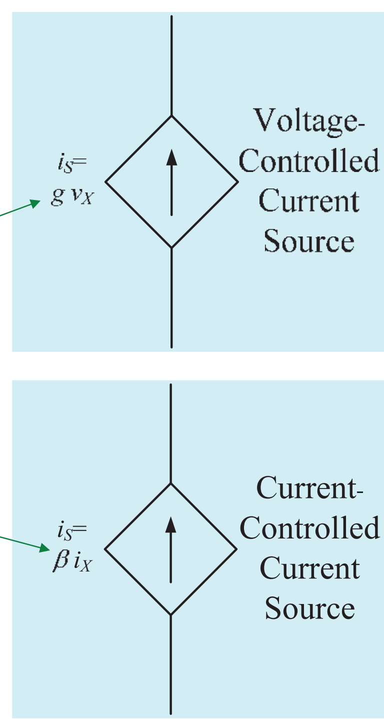


Dependent Current Sources

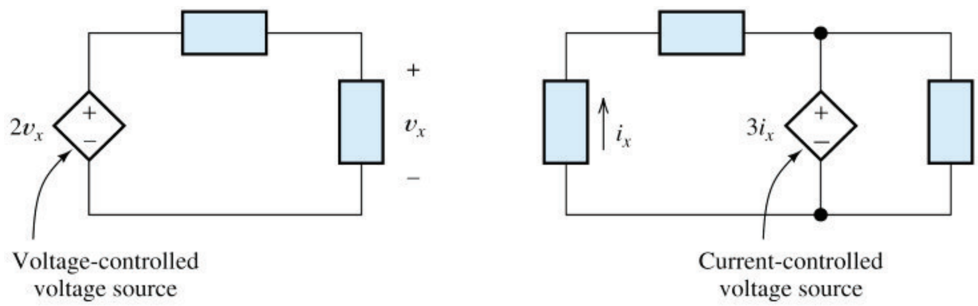
The schematic symbols that we use for dependent current sources are shown here, of which there are 2 forms:

1) Voltage-Controlled Current Sources (VCCS). The symbol g is the coefficient of the voltage v_X. It has dimensions of [current/voltage].

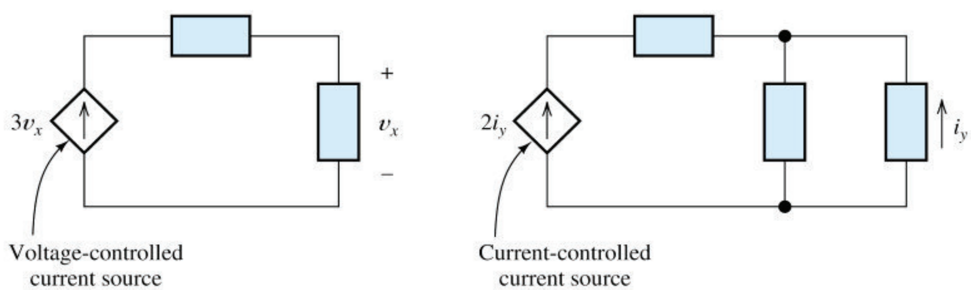
2) Current-Controlled Current Sources (CCCS). The symbol β is the coefficient of the current i_X. It is dimensionless.



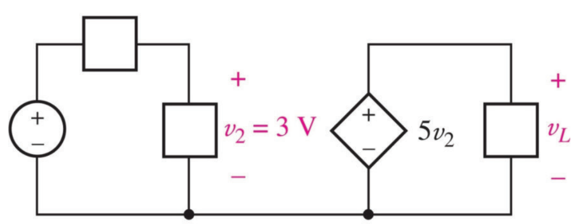
Dependent (or Controlled) Voltage Sources



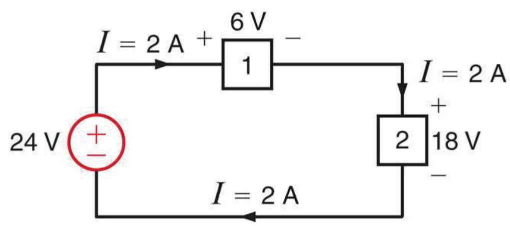
Dependent (or Controlled) Current Sources



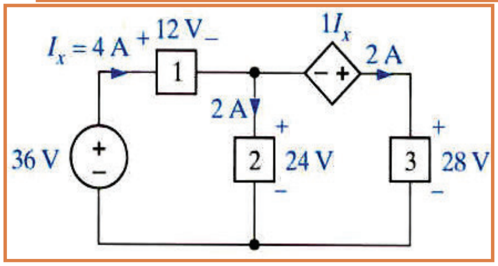
Find the voltage v_L in the circuit below.



Determine the power absorbed or supplied by the elements of the following network:



POWER ABSORBED OR SUPPLIED BY EACH ELEMENT



$P_1 = (12V)(4A) = 48[W]$

$P_2 = (24V)(2A) = 48[W]$

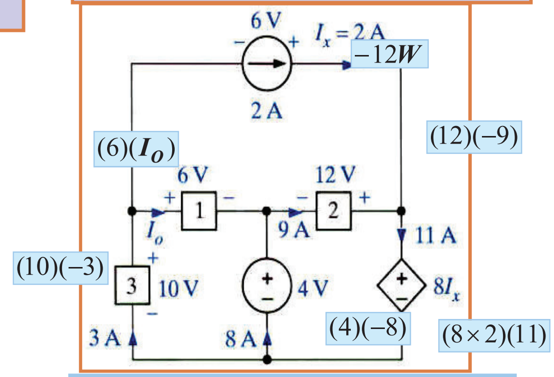
$P_3 = (28V)(2A) = 56[W]$

$P_{DS} = (1I_x)(-2A) = (4V)(-2A) = -8[W]$

$P_{36V} = (36V)(-4A) = -144[W]$

NOTICE THE POWER BALANCE

USE POWER BALANCE TO COMPUTE Io



$P_{2\,A} = (6)(-2) = -12$ W

$P_1 = (6)(I_o) = 6I_o$ W

$P_2 = (12)(-9) = -108$ W

$P_3 = (10)(-3) = -30$ W

$P_{4\,V} = (4)(-8) = -32$ W

$P_{DS} = (8I_x)(11) = (16)(11) = 176$ W

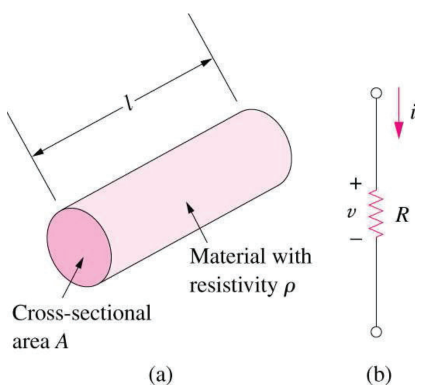
POWER BALANCE

$-12 + 6I_o - 108 - 30 - 32 + 176 = 0$ $\quad I_O = 1[A]$

2.3 Ohm's Law

Resistance

• Different material allow charges to move within them with different levels of ease. This physical property or ability to resist current is known as resistance.

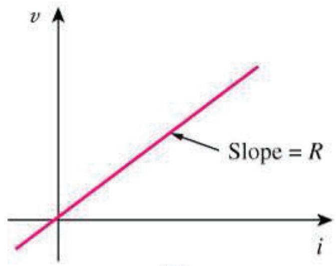


$$R = \rho\frac{L}{A}$$

The constant of the proportionality is the resistivity of the material, i.e., ρ

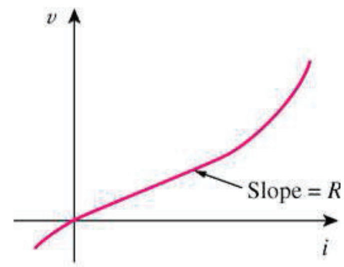
• The resistance of any material with a uniform cross-sectional area *A and length l is inversely proportional to A and directly* proportional to *l*.

Linear resistor



(a)

Nonlinear resistor



(b)

• In this course, we assume that all the elements that are designated as resistors are linear (unless mentioned otherwise)

Conductance

G=1/R is called the conductance of the element and is measured in siemens (S) or mho (℧) .

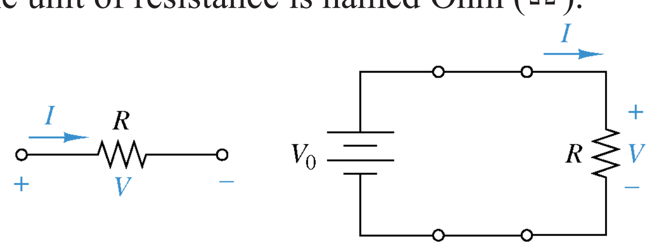
$$G = \frac{1}{R}$$

German inventor Ernst Werner von Siemens (1816-1892)

Georg Simon Ohm (1787-1854) studied the relationship between voltage, current, and resistance and formulated the equation that bears his name.

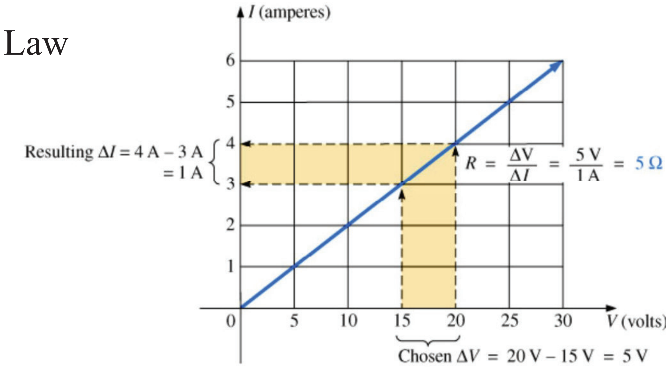
$$V = IR$$

A conductor designed to have a specific resistance is called a resistor, the unit of resistance is named Ohm (Ω).



IMPORTANT: use Ohm's Law only on resistors. It does not hold for sources.

Plotting Ohm's Law



The voltage *v across a resistor is directly proportional to the* current *i flowing through the resistor. The proportionality* constant is the resistance of the resistor, i.e., $v(t) = Ri(t)$

- One can also write: $$i(t) = \frac{v(t)}{R} = Gv(t)$$
- Instantaneous power dissipated in a resistor

$$p(t) = i(t) \times v(t) = \frac{v^2(t)}{R} = i^2(t) \times R$$

Example: Resistor Power

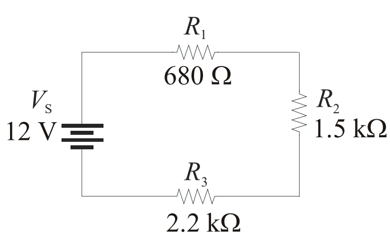
Ex 2.1 A 560 Ω resistor is connected to a circuit which causes a current of 42.4 mA to flow through it. Calculate the voltage across the resistor and the power it is dissipating.

Solution

$v = iR = (0.0424)(560) = 23.7$ V

$p = i^2R = (0.0424)^2(560) = 1.007$ W

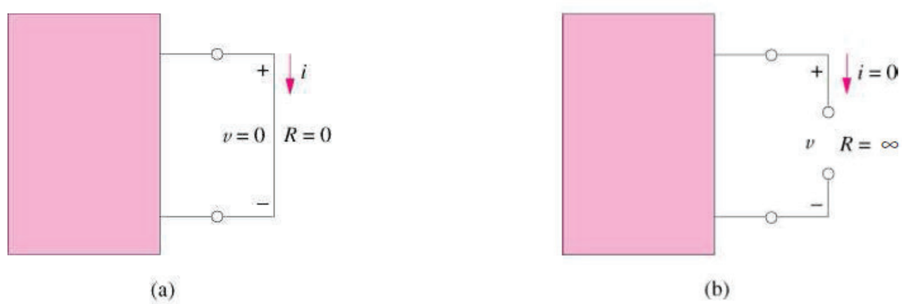
Ex 2.2 From the circuit below, complete the parameters listed in the Table.

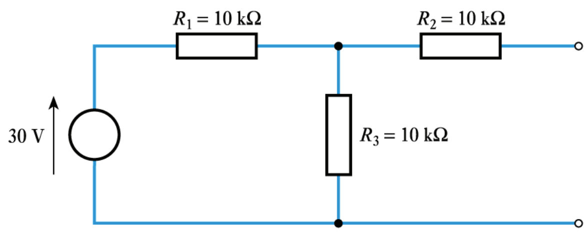


I_1= 2.74 mA	R_1= 0.68 kΩ	V_1= 1.86 V	P_1= 5.1 mW
I_2= 2.74 mA	R_2= 1.50 kΩ	V_2= 4.11 V	P_2= 11.3 mW
I_3= 2.74 mA	R_3= 2.20 kΩ	V_3= 6.03 V	P_3= 16.5 mW
I_T = 2.74 mA	R_T= 4.38 kΩ	V_S= 12 V	P_T= 32.9 mW

Short and Open Circuits

A device with zero resistance is called short circuit and a device with zero conductance (i.e., infinite resistance) is called open circuit.

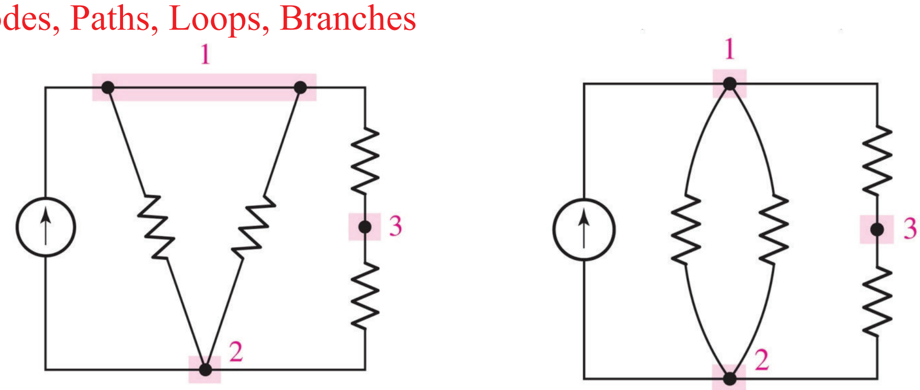




Find the voltage and current passes through R_1 R_2 and R_3.

2.4 Resistive Circuits , KVL and KCL

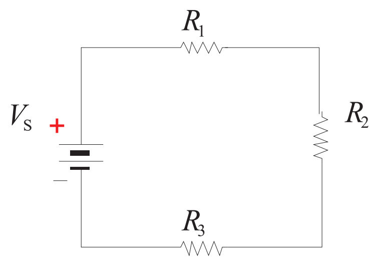
Nodes, Paths, Loops, Branches

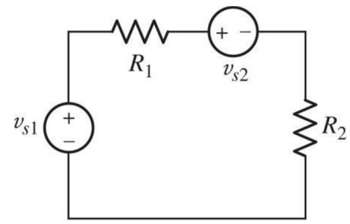


- these two networks are equivalent
- there are three *nodes* and five *branches*
- a *path* is a sequence of nodes
- a *loop* is a closed (circular) path

Series Circuits

A *series circuit* is one that has only one current path.

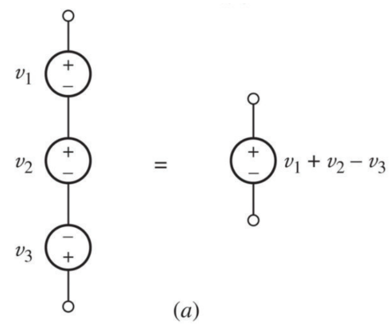




All of the elements in a circuit that carry the same current are said to be connected in **series.**

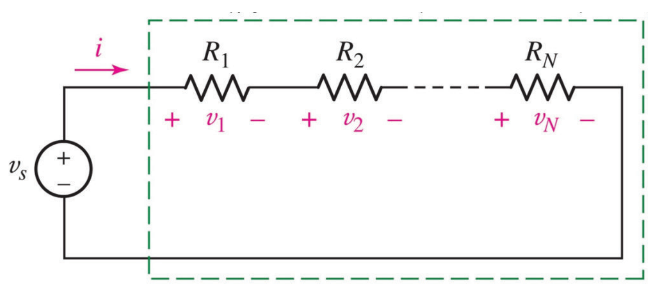
Voltage sources in series

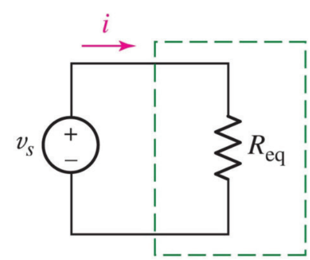
Voltage sources connected in series can be combined into an equivalent voltage source:



Resistors in Series

The total resistance of a series configuration is the sum of the resistance levels





$$R_{eq} = R_1 + R_2 + \ldots + R_N$$

The more resistors we add in series, the greater the resistance (no matter what their value). The polarity of the voltage across a resistor is determined by the direction .

$$v_1 = iR_1 \quad ; \quad v_2 = i\,R_2 \quad ; \; v_N = i\,R_N$$

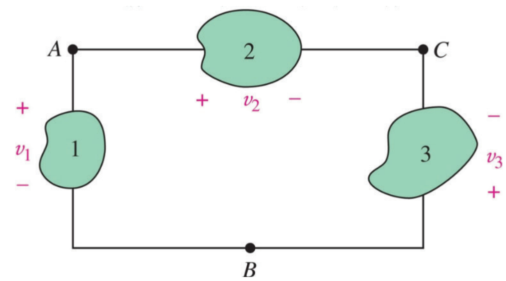
The power applied by the dc supply must equal that dissipated by the resistive elements.

$$P_S = P_{R_1} + P_{R_2} + \ldots + P_{R_N}$$

Kirchhoff's Voltage Law (KVL)

Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero

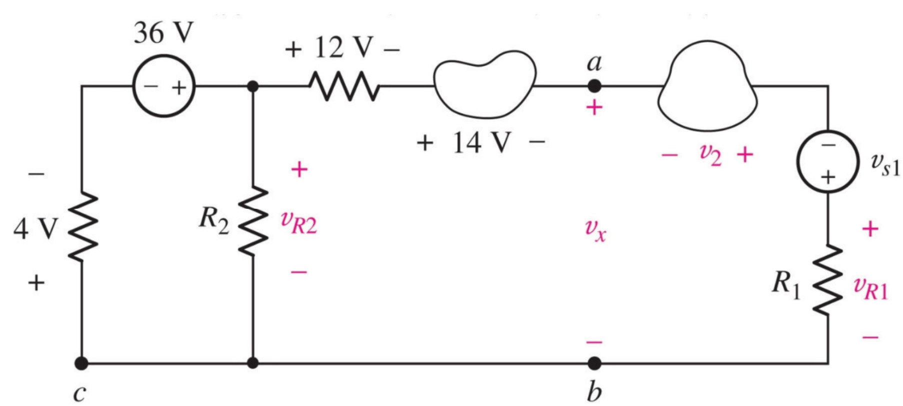
KVL applies to all circuits, but you must apply it to only one closed path. In a series circuit, this is (of course) the entire circuit.



$v_1 + (-v_2) + v_3 = 0$

Example: Applying KVL

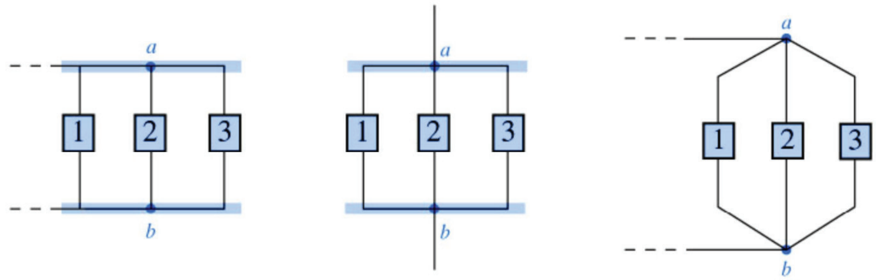
Ex 2.3 Find v_{R2} (the voltage across R_2) and the voltage v_x.



Answer: v_{R2} = 32 V and v_x= 6 V.

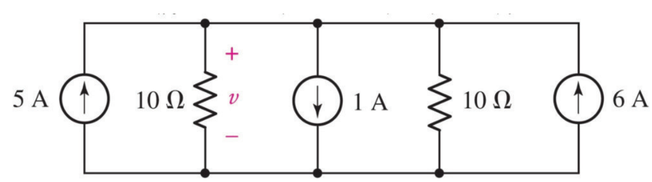
Parallel Circuits

Two elements, branches, or circuits are in parallel if they have two points in common as in the figure below



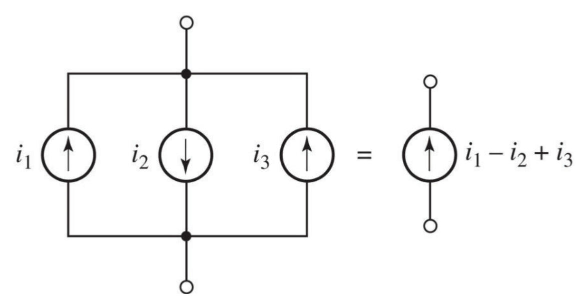
(a) (b) (c)

Elements in a circuit having a common voltage across them are said to be connected in **parallel.**



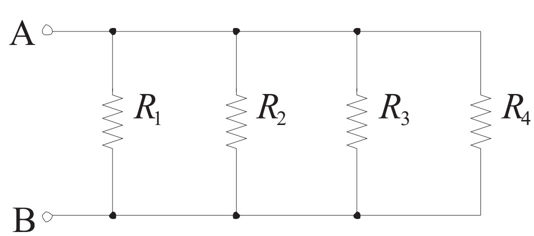
Current sources in parallel

Current sources connected in parallel can be combined into an equivalent current source:

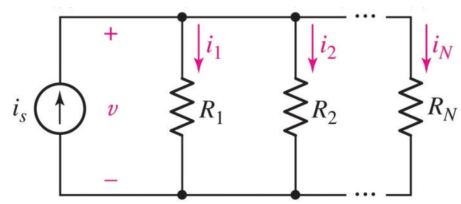


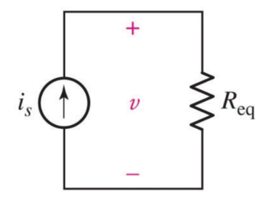
Resistors in Parallel

Resistors that are connected to the same two points are said to be in parallel.



For resistors in parallel, the total resistance is determined from





$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

or

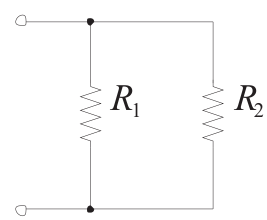
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_N}}$$

The total resistance of parallel resistors is always less than the value of the smallest resistor.

For parallel elements, the total conductance is the sum of the individual conductance values.

$$G_{eq} = G_1 + G_2 + G_3 + \ldots + G_N$$

Special case for resistance of two parallel resistors



The resistance of two parallel resistors can be found by

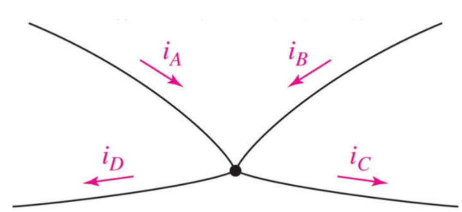
$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

or

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Kirchhoff's Current Law (KCL)

KCL: The algebraic sum of the currents entering any node is zero.



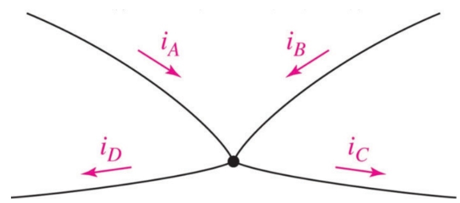
$$i_A + i_B + (-i_C) + (-i_D) = 0$$

The sum of the current entering an area, system or junction must equal the sum of the current leaving the area, system, or junction.

$$\sum I_{in} = \sum I_{out}$$

Current *IN=OUT:*

$i_A + i_B = i_C + i_D$

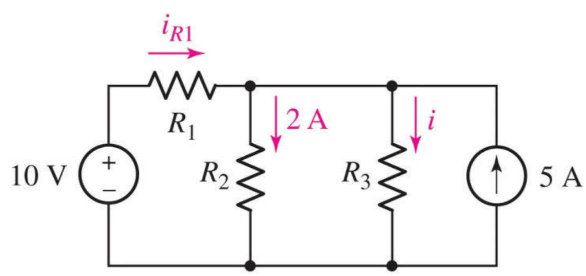


For any resistive circuit, the power applied by the battery will equal that dissipated by the resistive elements.

$$P_S = P_{R_1} + P_{R_2} + P_{R_3} + \ldots + P_{R_N}$$

Example: Applying KCL

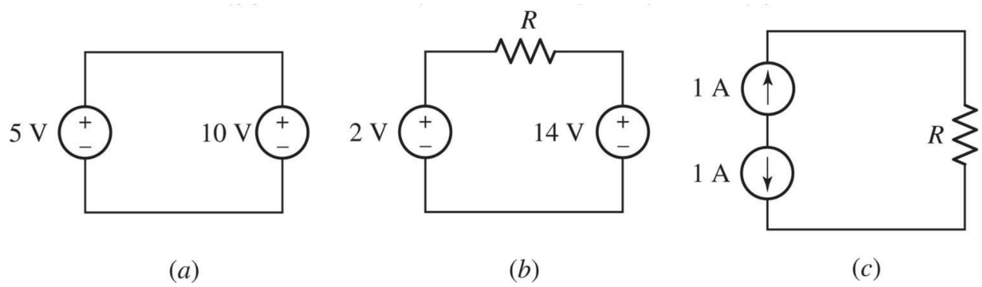
Ex 2.4 Find the current through resistor R_3 if it is known that the voltage source supplies a current of 3 A.



Answer: i =6 A

Impossible Circuits

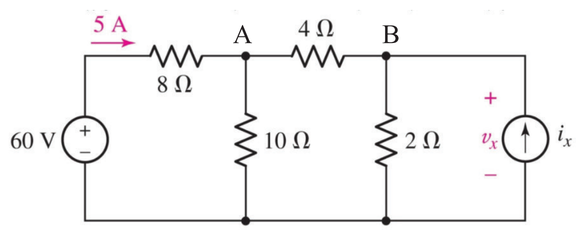
- Our circuit models are idealizations that can lead to apparent physical absurdities:



- V_s in parallel (a) and I_s in series (c) can lead to “impossible circuits”

Applying KVL, KCL, Ohm’s

Ex 2.5 Solve for the voltage v_x and and the current i_x



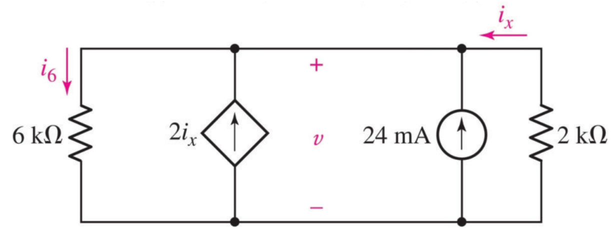
solution KCL @ node A $\Rightarrow \quad 5=\dfrac{v_A}{10}+\dfrac{v_A-v_x}{4} \quad ----(1) \quad ; \quad v_B=v_x$

KCL @ node B $\Rightarrow \quad \dfrac{v_A-v_x}{4}+i_x=\dfrac{v_x}{2} \quad ----(2)$

but $\Rightarrow \quad 5=\dfrac{60-v_A}{8} \quad \Rightarrow \quad v_A=20\,V$ Substitute into (1) yield ;

$5=\dfrac{20}{10}+\dfrac{20-v_x}{4} \quad \Rightarrow \quad v_x=8$ Substitute v_x into (2) yield ; $i_x = 1$ A

Ex 2.6 Determine the value of v and the power supplied by the independent current source.



solution KCL @ node on the top : $2i_x + i_x + 24 \text{ mA} = i_6$ or

$3i_x + 24 \text{ mA} = i_6$ ------ (1) but $v = -i_x(2K) = i_6(6K)$ => $i_x = -3i_6$ ----- (2)

Substitute into (1) yield ; $3[-3i_6] + 24 = i_6$ => $i_6 = 2.4$ mA

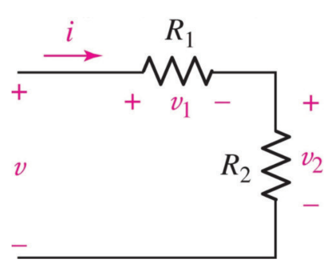
Thus ; $v = i_6 (6K)$ => $v = (2.4 \text{ mA})*(6K) = 14.4$ V

and power from current source is $P_s = I.V = (24 \text{ mA})*(14.4) = 345.6$ mW

2.5 Voltage and Current Division

Voltage Division

The voltage drop across any given resistor in a series circuit is equal to the ratio of that resistor to the total resistance, multiplied by source voltage.

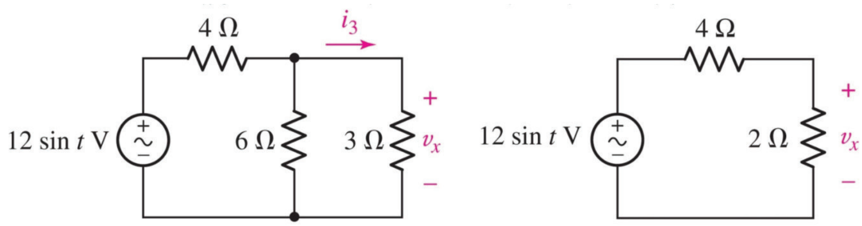


$$v_1 = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_1 + R_2} v$$

Resistors in series “share” the voltage applied to them.

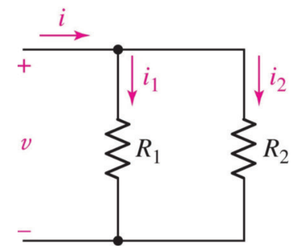
Ex 2.7 Find v_x



Answer: $v_x(t) = 4 \sin t$ V

Current Division

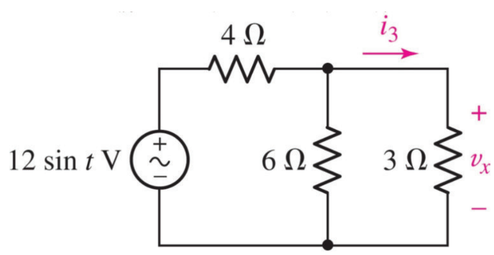
When current enters a junction it divides with current values that are inversely proportional to the resistance values.



$$\boxed{i_1 = i \frac{R_2}{R_1 + R_2}} \qquad \boxed{i_2 = i \frac{R_1}{R_1 + R_2}}$$

Resistors in parallel "share" the current through them.

Ex 2.8 Find i_3(t)



solution

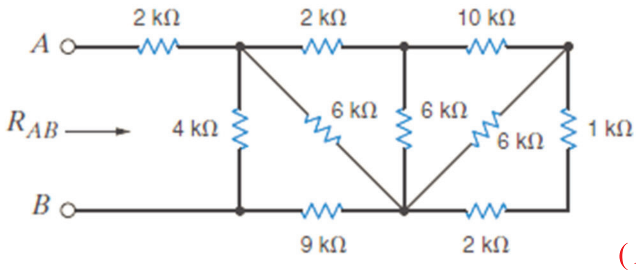
6 // 3 = 2 Ω and the total current is $i = \dfrac{12\sin t}{4+2} = 2\sin t \quad A$

current i_3 is divided from total current i, thus

$$\Rightarrow \quad i_3 = \frac{6}{3+6} \times i = \frac{2 \times i}{3} = \frac{2 \times 2\sin t}{3} = 1.33 \sin t \quad A$$

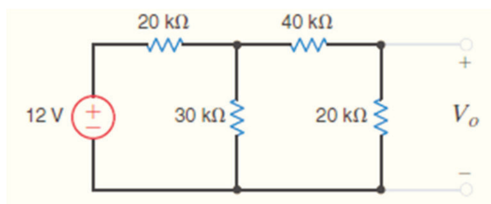
Self Test

1) Determine the resistance at a terminal A – B (R_{AB} = ?).



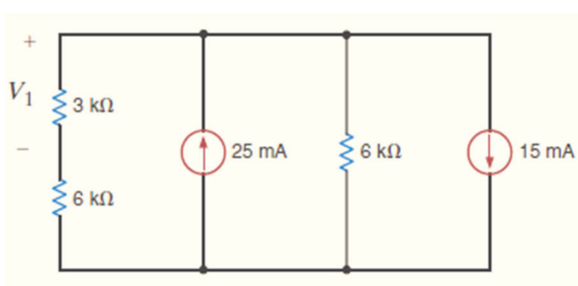
(*Ans* : $R_{AB} = 5$ KΩ)

2) Find V_o in the circuit below.



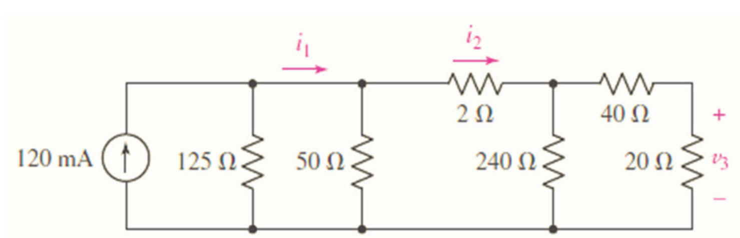
(*Ans* : V_o = 2V)

3) Find V_1 in the circuit below.



(*Ans* : V_1 = 12 V)

4) Use resistance combination methods and current division to find i_1, i_2, *and* v_3.



(*Ans* : i_1 = 100 mA ; i_2 = 50 mA; v_3 = 0.8 V.)