

ICMA 223 Linear Algebra A

Problem Set 3

GENERAL INFORMATION

Important Note! Please do write a list of collaborators (friends you work with) and sources you consult for this assignment (e.g. lecture notes, specific pages of a book, specific links to Wikipedia, etc., but do not write just “YouTube” or “Google” without further information). Even if you work on this assignment alone and do not consult any source, please write “**Collaborators:** None. **Sources consulted:** None.” in your submission.

Collaboration on problem sets is allowed, and is in fact encouraged. Working with friends can be an enjoyable way to learn mathematics!

Information: This problem set is due at **11:59 pm (Thai time), Friday, May 16, 2025**. You should submit your work on Canvas. See the syllabus for the homework policy.

For each problem, please show your work! For correct answers alone without proper explanations or derivations, you might be awarded only very few, or even zero, points. On the other hand, for incorrect answers with proper explanations or derivations, you might be awarded a lot of points.

PROBLEM 0 (10 points)

Please provide the following information. Please refer to the “General Information” section above for details.

- (a) What is your full name (first name and last name)?
- (b) What is your student ID number?
- (c) Which section are you a student of, Section 1 or Section 2?
- (d) Please write the list of your collaborators for this problem set.
- (e) Please write the list of sources you consult for this problem set.

Optional: what is your nickname (if you have one)?

PROBLEM 1 (20 points)

Since we are in a big class with many students, it is important that we agree on how the logistics of this class works. Please read the syllabus of this class carefully.

After you have finished reading the syllabus, please answer the following questions about our class's logistics. (Respond in your own sentences. Do not simply copy or quote from the syllabus.)

- (i) How is the numerical grade point (100%) for each student computed in this class? How much are Quiz 1, Quiz 2, Midterm, Final, and the homework assignments weighted?
- (ii) What is the cheat sheet policy for this class? Is the cheat sheet policy for each quiz different from the cheat sheet policy for the midterm?
- (iii) What is the homework extension policy for this class? What can a student do if it is less than 20 minutes before the deadline of a problem set, but they are still not half way done with the homework assignment?
- (iv) What is our class's policy on the use of artificial intelligence (AI) tools?

PROBLEM 2 (40 points)

Read the lecture notes for the third week.

Write one short paragraph (containing approximately 3 - 5 sentences) about the reading. Your paragraph can be, for example, something you found interesting or confusing about what you read, or it can be where you work out an explicit example of some result from the notes, or it can even be other things you would like to write about which are related to the reading!

PROBLEM 3 (10 points)

Let $A \in \mathbb{R}^{3 \times 3}$ be a 3×3 -matrix with the following property:

for every $i \in \{1, 2, 3\}$ and for every $j \in \{1, 2, 3\}$, the (i, j) -entry of A is $i^2 - j$.

Let $B \in \mathbb{R}^{3 \times 3}$ be a 3×3 -matrix with the following property:

for every $i \in \{1, 2, 3\}$ and for every $j \in \{1, 2, 3\}$,
the (i, j) -entry is 1 if $i < j$, 0 if $i = j$, and -1 if $i > j$.

Let

$$X := (A + 2 \cdot B) \cdot \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}^T$$

- (a) Compute X explicitly. (Write out the entries of the matrix X .)

- (b) Perform Gaussian elimination to transform X into a matrix in reduced row echelon form. Please show your work in every step.

PROBLEM 4 (10 points)

Consider the following matrices

$$M := \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad N := \begin{bmatrix} -2 & 3 \\ 2 & 4 \\ 3 & 0 \end{bmatrix}.$$

Compute $\text{tr}(M \cdot N)$ and $\text{tr}(N \cdot M)$. Are the two traces equal?

PROBLEM 5 (10 points)

Let

$$Y := \begin{bmatrix} -2 & 1 \\ 1 & 4 \\ 4 & 7 \\ 7 & 10 \end{bmatrix}.$$

- (a) Compute $\text{rank}(Y)$ and $\text{nullity}(Y)$.
(b) Compute $\text{rank}(Y^T)$ and $\text{nullity}(Y^T)$.
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