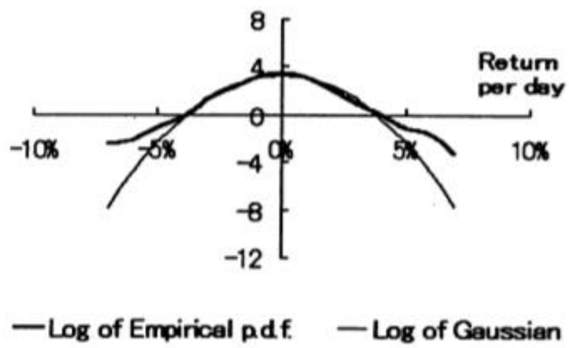
A percolation model of stock price fluctuations: a literature review.

Introduction.

Fat-tail distribution is a statistical distribution phenomenon that happens when data strays widely from the average giving more frequent lower and higher instances of extreme values. This phenomenon can be seen in stock-price fluctuations. Figure 1.1 compares the log of gaussian and the log of empirical pdf for NIKKEI 225 returns per day.



The empirical frequency of return 5 percent per day is higher compared to the gaussian predicts. With actual data, NIKKEI 225 has experience change by more than 7 percent for 6 occasions with 11 years of data, and the gaussian model would need 2000 years to experience 6 of such event to occur. This paper explains the fat-tail distribution as a result of bounded sight between traders. By reproducing the fat-tail distribution, this paper also defines and calculate the floor and ceiling for the stock prices.

This paper assumes that traders have bounded sight, which means that the traders are only interested in price changes in stocks which they hold. Which causes only localized interaction between traders who shares the same stock. Using percolation theory, they modeled the interactions between traders on a 2d lattice. Before this percolation theory has already been applied to stock markets, Stauffer and Penna (1998) model was based on the herding effect. They assumed that traders obtain investment advice from their neighborhood bank and make decisions without considering the economic data. This results in different clusters all sharing the same information. This paper is divided into 4 different section. The first section explains percolation theory and talks about the Stauffer-Penna model, the next section then talks about the model of this paper. The third section then talks about how the model can be used to calculate the floor and ceiling for NIKKEI 225, and lastly it ends with the author's concluding remark. The reason I chose this paper is because I am interested in applying what I learnt from physics to other fields, especially in the field of finance.

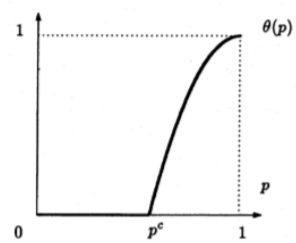
Percolation theory and The Stauffer-Penna model.

Percolation theory was first developed by Broadbent and Hammersley in 1958.Percolation can be explained with following example. Imagine a there is a wildfire, where the trees are located in each site in a $\mathbf{Z}^2$ lattice. The probability of a tree catching on fire, when they are next to a burning tree is $p \in [0,1]$, p is also sometimes referring to as influence rate. The probability of the origin of the fire belongs to an infinite cluster is defined as $\theta(p)$, where $\theta(p)$ is defined as:

$$\theta(p), = P_p(|c| = \infty) = 1 - \sum_{n=1}^{\infty} P_p(|c| = n)$$

The probability of the cluster belongs to an infinite cluster equals to 1 minus the sum of probability of cluster equals to size n from 1 to infinity. If the influence rate is less than the critical probability, p^c, then the cluster size will be finite and if p is larger than p^c, then the cluster size will be infinite.

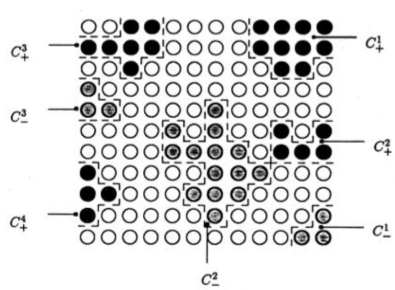
$$\theta(p)\begin{cases} = 0\ (p < p^c) \\ > 0 (p > p^c) \end{cases}$$



In 2-dimension, $p^c = 0.592745$.

In a paper written by Stauffer and Penna titled Crossover in the Cont-Bouchaud Percolation Model for Market Fluctuations, they were able to show and explain the fat-tail distribution. In their model, traders are distributed on a square lattice. Traders who receives trading information from the same bank will form a cluster, all with the same expectation. Then each cluster randomly decides to buy, with probability equals to $\frac{\alpha}{2}$, to sell at $\frac{\alpha}{2}$, and to not trade at 1-α. Clusters that decided to buy are labeled as $C_+^1, C_+^2, C_+^3 \dots$, and the clusters that decides to sell are labeled as $C_-^1, C_-^2, C_-^3 \dots$, at period t. Then the return is then defined as the return of stock S at t is proportional to difference in size of 2 different types of clusters:

$$\frac{\Delta S_t}{S_t} = \rho\left(\sum_j |C_+^j(t)| - \sum_j |C_-^j(t)|\right)$$



The visualization of the Stauffer and Penna model.

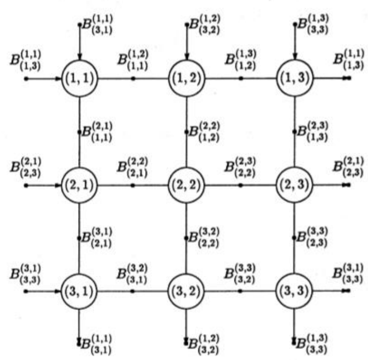
When $\alpha \sim 1$, the generated distribution resembles gaussian, but if $0 < \alpha \ll 1$, the distribution have fat tails. The Stauffer and Penna model does not in modern markets, since information can be shared within seconds around the globe. Another flaw in their model is, that the influence rate is constant.

The Model.

Suppose there are N^2 traders on N X N lattice, and trader i holds M_i kinds of stock $B_i^1 \dots B_i^{M_i}$, and each trader is only supposed to look at only the stock he holds and stock price index S. In this case we take N = 21, on a 2-dimensional torus. Each trader (i,j) holds 4 kinds of stock $B_{(i,j)}^{(i+1,j)}, B_{(i,j)}^{(i-1,j)}, B_{(i,j)}^{(i,j+1)} and\ B_{(i,j)}^{(i,j-1)}$. Each trader now shares one of his four holdings with one of his 4 neighborhood traders. Since it is on a 2-dimensional torus, traders on the edge shares stock with the trader at the opposite edge.

At the beginning of period t, a random trader chosen becomes bull with probability 0.5 or bear with probability 0.5 If the chosen trader becomes bull he will increase his holdings, or tries to sell his holding if he becomes bear. If trader i is trying to buy, the surrounding trader then will decide to buy with probability p^u or do nothing with probability $1-p^u$. If trader i is trying to sell, the surrounding trader then will decide to buy with probability p^d or do nothing with probability $1-p^d$. The influence rate p^u and p^d are assumed to be functions of stock price.

$$p^u(S) = e^{-\alpha S} \quad and \quad p^d(S) = 1 - e^{-\delta S}$$

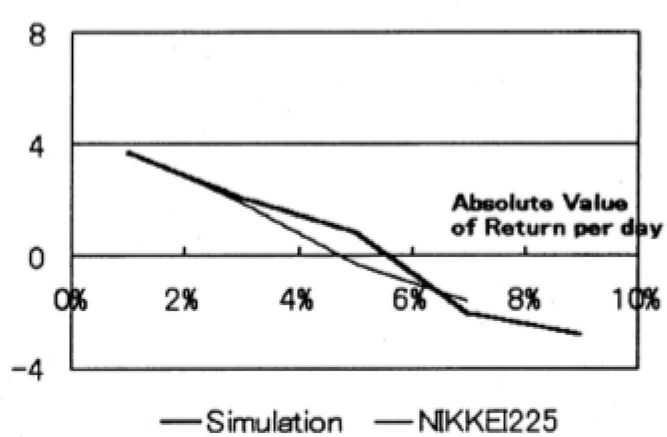


The return is calculated using formula below:

$$\frac{\Delta S_t}{S_t} = \rho \times sgn(C_t) \times |C_t|$$

$$sgn(C_t) = \begin{cases} +1 \ (if \ C_t \ is \ a \ bull \ cluster) \\ -1 \ (if \ C_t \ is \ a \ bear \ cluster) \end{cases}$$

Parameters determine this model is $\alpha, \delta \ and \ \rho$. With $\alpha = 1.6$, $\delta = 0.030 \ and \ \rho = 0.015$, the model was able to produce result below:

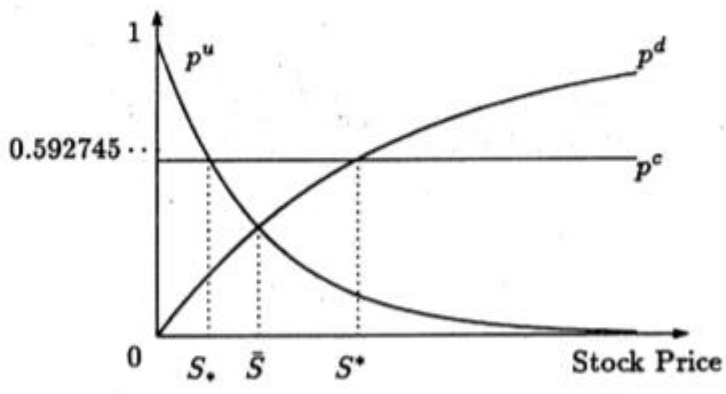


The log of the p.d.f.

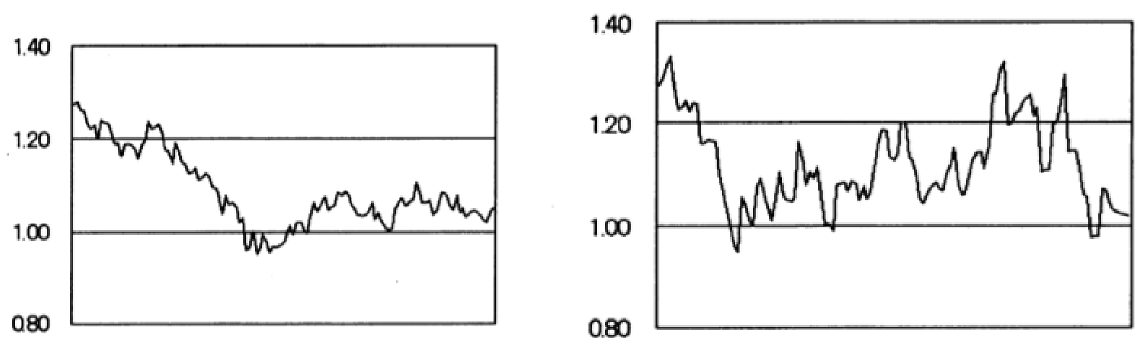
The floor and ceiling for NIKKEI 225.

The floor price is defined as $S_* \; and \; S^* \; for \; the \; ceiling \; price$. It can be calculated using formula below:

$$p^u(S_*) = p^C \quad p^d(S^*) = p^C$$



From the graph above we can see that at price $\bar{S}$, $p^u(\bar{S}) = p^d(\bar{S})$. $\bar{S}$ is the average price over a certain period, and in this case the mean of NIKKEI 225 from July 2001 to December 2001 was used for the simulation. Equating $p^u(\bar{S}) = p^d(\bar{S})$ will produce only a function of δ with α as variable, $\delta = \delta(\alpha)$, not the actual value for the variables. The floor is then calculated to be ¥ 7,471 and ceiling equals to ¥ 15,489, which is close to the actual value of ¥ 9658 and ¥ 12,922.



The graph on the left is the actual NIKKEI 225 from July 2001 to December 2001 and the one on the right is the sample path produced with $\alpha = 0.7, \delta = 0.58 \; and \; \rho = 0.0009$.

Conclusion.

This paper did not provide enough information regarding the 3 variables $\alpha, \delta \; and \; \rho$, the author also did not explain why and how the values for the variable are chosen. Although the author did show the fat-tail distribution on the return per day for NIKKEI 225, the ceiling and floor calculation did not provide a satisfactory result. The author also assumes that the information provided to the traders are local, and not known to other clusters, which might not be an accurate assumption.