## Deep, complex, invertible networks for inversion of transmission effects in multimode optical fibres

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### **Abstract**

We use complex-weighted, deep networks to invert the effects of multimode optical fibre distortion of a coherent input image. We generated experimental data based on collections of optical fibre responses to greyscale input images generated with coherent light, by measuring only image amplitude (not amplitude and phase as is typical) at the output of 1 m and 10 m long, 105 µm diameter multimode fibre. This data is made available as the *Optical fibre inverse problem* Benchmark collection. The experimental data is used to train complex-weighted models with a range of regularisation approaches. A *unitary regularisation* approach for complex-weighted networks is proposed which performs well in robustly inverting the fibre transmission matrix, which is compatible with the physical theory. A benefit of the unitary constraint is that it allows us to learn a forward unitary model and analytically invert it to solve the inverse problem. We demonstrate this approach, and outline how it has the potential to improve performance by incorporating knowledge of the phase shift induced by the spatial light modulator.

### **Problem Description and Experimental Setup**

Optical fibres are widely used for communication and medical purposes, for their capacity to transport light over long distances or into otherwise inaccessible places.

The most commonly used fibres are single-mode fibres which cannot be used for imaging purposes. Multi-mode fibres would be preferable, but images undergo distortions when transmitted via the multi-mode fibres.

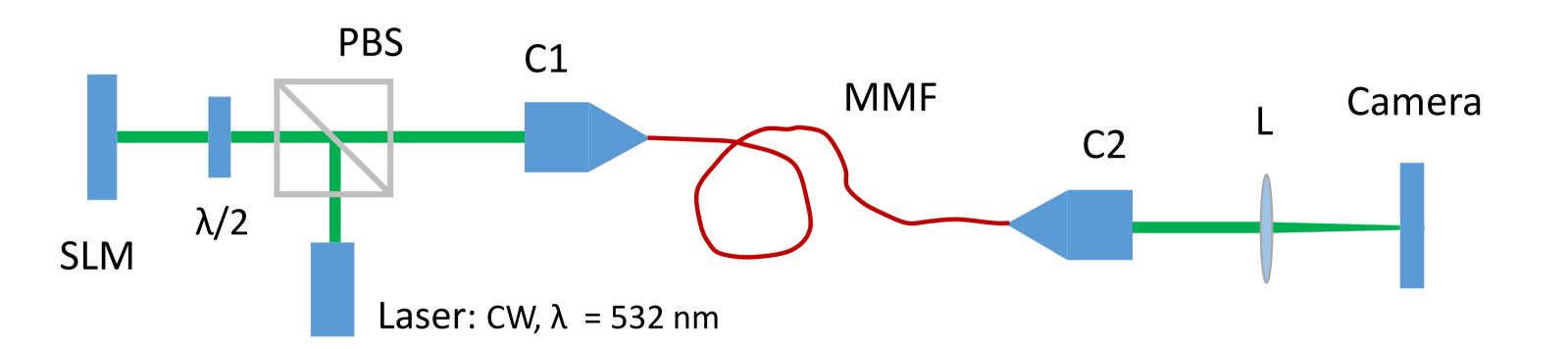


Figure: Experimental Setup. C1, C2: collimator lens; L: lens; PBS: polarising beam splitter; SLM: spatial light modulator; MMF: multimode fibre (1 m & 10 m,  $105 \,\mu\text{m}$  diameter,  $0.22 \,\text{NA}$ ); Laser, CW: continuous wave;  $\lambda/2$ : half-wave plate.

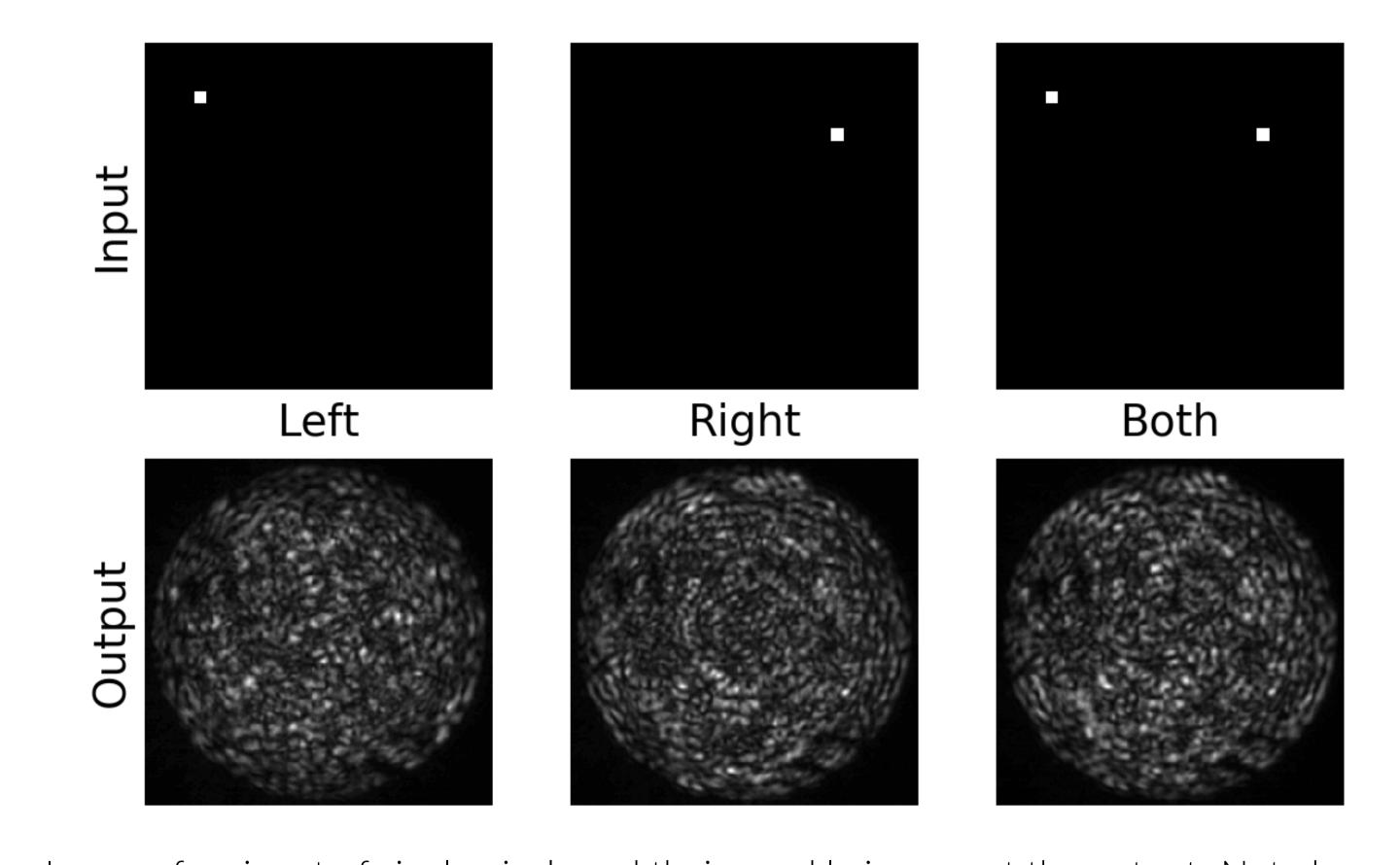


Figure: Image of an input of single pixels and their speckle images at the output. Note how a single pixel generates a response over the full space of the output sensor, and at much finer detail than the resolution of the pixel. Also, the amplitude of output of the two-pixel example is not the sum of amplitudes of the single pixel responses, due to complex-valued interactions of modes in the fibre.

### Image distortion & Retrieving problem

The images are scrambled at the output of the multimode fibre due to:

- Modal dispersion: the different modes in which the image couples travel at different speeds. Interference arises if the light is coherent.
- Mode coupling: due to bending, imperfections and changes in temperature.

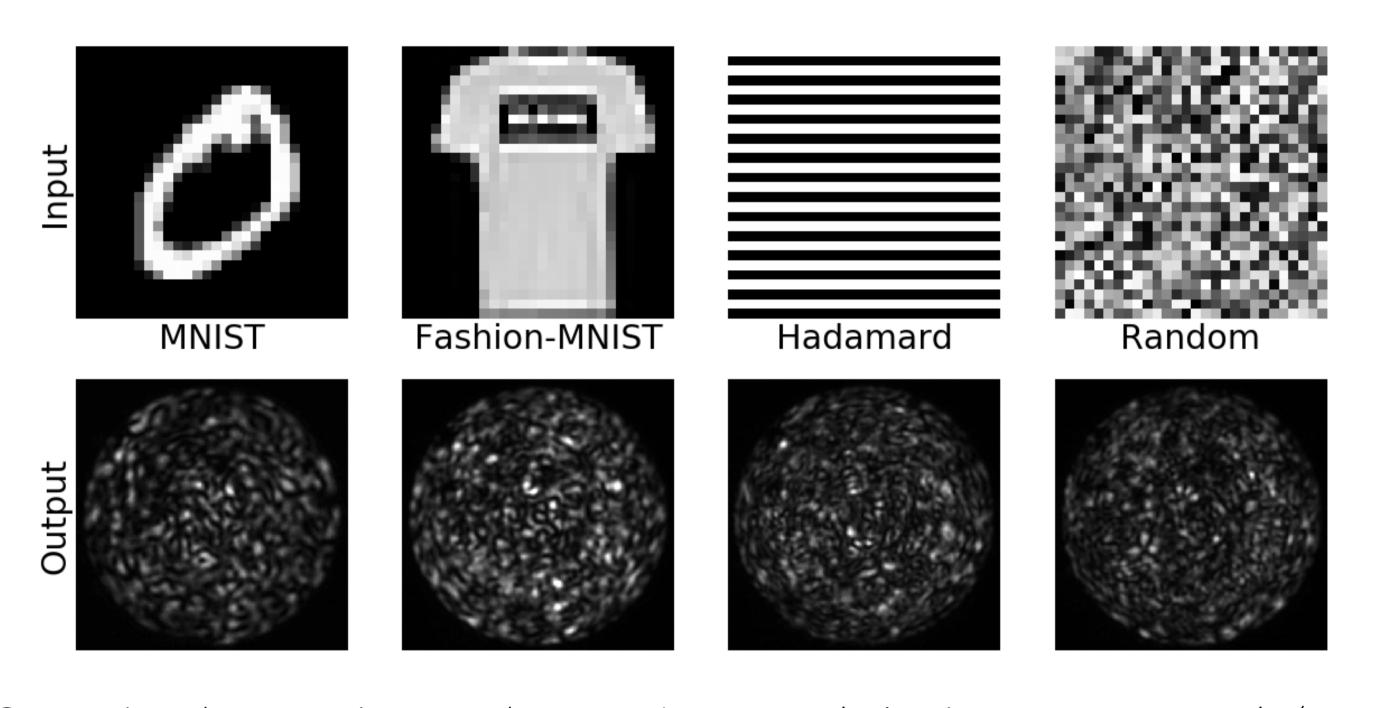


Figure: Comparison between input and output. Input: resolution is  $28 \times 28$ , grey-scale (0–100). Output: resolution is  $350 \times 350$ , grey-scale (0–255)

But, propagation is still:

- Linear
- Deterministic

**Linearity**  $\rightarrow$  it is always possible to define an input basis and measure the relative output for each element of the basis. In this way it is possible to build up the so called **transmission matrix (TM)**:

 $\mathsf{Output} = \mathsf{TM} \times \mathsf{Input}$ 

The propagation is complex, so the **transmission matrix is complex** as well. Challenges:

- Non-locality: Makes standard convolutional approaches difficult;
- No phase measurements: Makes our system under-constrained;
- Non-scanning measurement: common techniques scan through the input basis in order to retrieve the TM

Is it possible to build up a good approximation to the TM in these conditions?

### Inversion challenge

Two approaches to using machine learning to solve the inverse problem:

- 1. Identify the `causal' or `forward' model from image to speckle, then numerically invert the forward model and optimise to find the input most likely to have generated that image.
- 2. Directly learn an inverse model from speckle to image.

### Complex networks and model pipeline

We learn an inverse model to transform the speckle image back towards the original image space with a complex layer.

We use activation functions with separable real and imaginary parts and complex-valued analogues of the ReLU and PReLU.

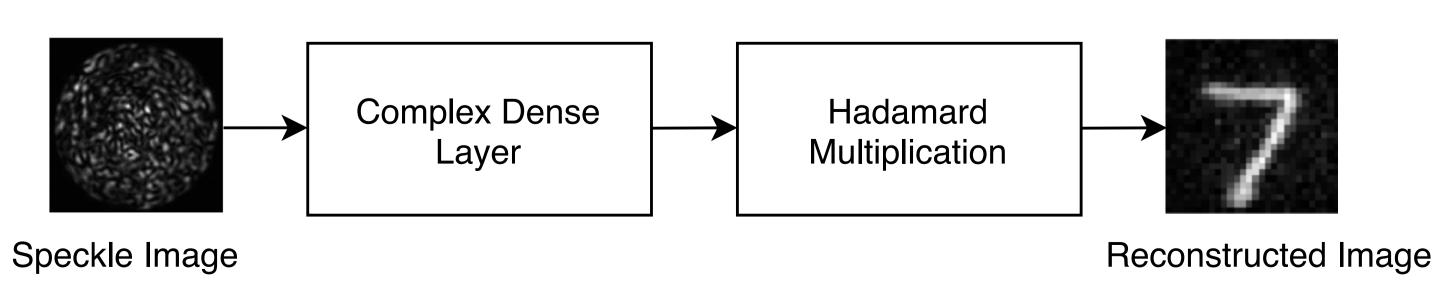


Figure: Inversion pipeline. A speckled image is transformed to the inferred input image that generated it, via an initial complex affine transformation.

### Unitary, complex dense layer regularisation

Unitary regularisation is motivated by the physics of the problem.

Transmission channel can be represented by a limited number of orthogonal modes (9000 modes for this fibre).

We propose a unitary regularisation, where the complex weight matrix W of the complex layer is pushed towards a unitary matrix for  $W \in \mathbb{C}^{m \times m}$ , or more generally toward a semi-unitary matrix for rectangular W, by a regularising term

 $\mathcal{L}_{unitary}(W) = \|WW^* - I\|_1 \text{ for } W \in \mathbb{C}^{m \times n},$ 

where  $W^*$  is the conjugate transpose of W.

# Nonlinear compensation for SLM effects and laser power drop-off

The spatial light modulator (SLM) in the experimental configuration introduces non-linear effects as a function of pixel value to intensity and phase.

We can compensate for these effects either in advance or by adding a layer to the network.

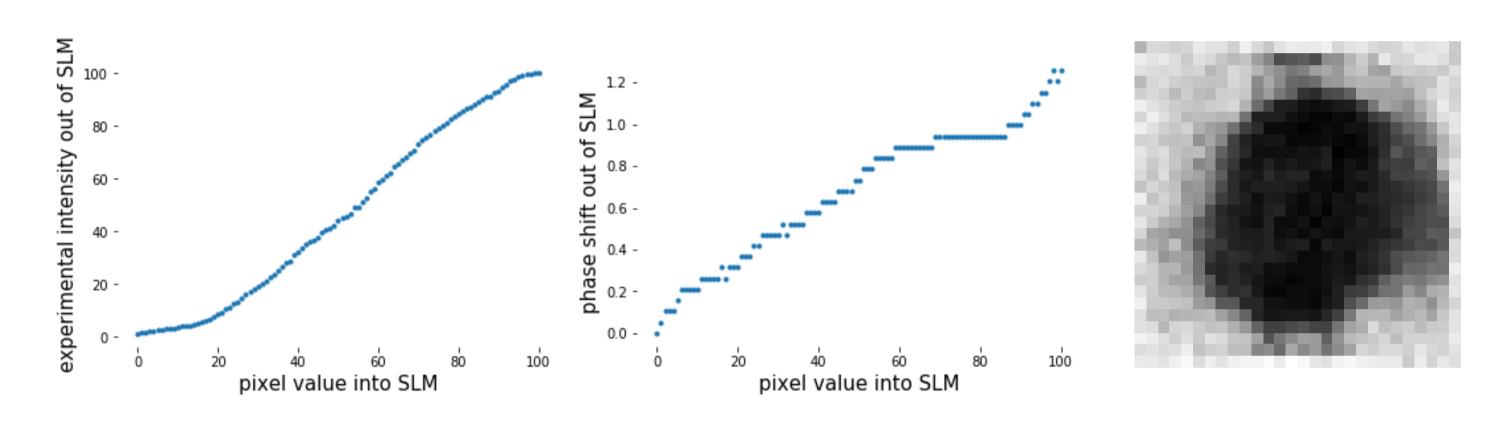


Figure: Optical nonlinearities. Graphs of Experimental Intensity and Phase mappings induced by Spatial-light modulator as a function of pixel value. Right, parameters of the Hadamard layer show the learned function of intensity drop-off of the laser

### Inversion of the transmission matrix

We show the results of the direct approach to inversion with different regularisation approaches.

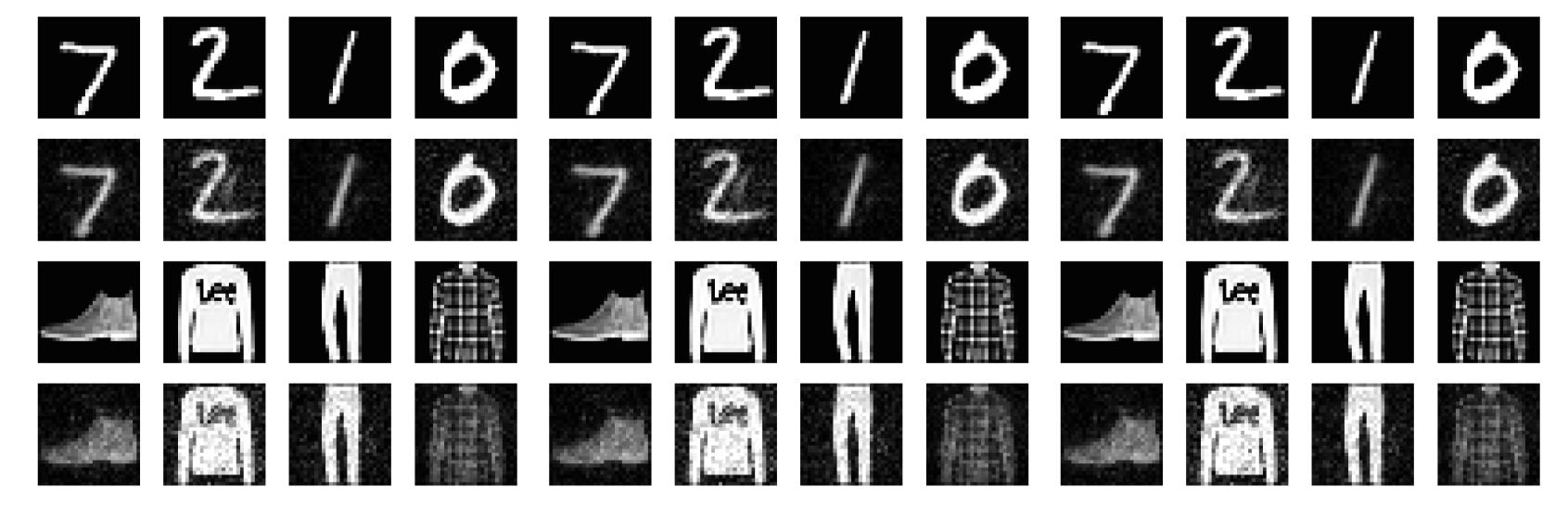


Figure: No regularisation  $l_2$  weight regularisation Unitary regularisation The impact of regularisation on inference by a single complex-valued layer with various regularisation methods. Speckle images of  $112 \times 112$  resolution were used on 1 m data.

Table: Model comparisons where each single layer model with 19,669,776 parameters (9,835,280 for real-valued models) was trained for 300 epochs, or until convergence, on a speckle resolution of  $112 \times 112$ , with  $\lambda = 0.03$  where the model was  $l_2$  regularised.

Name	Regularisation	MSE
Real-valued	$\it l_2$ weight regularisation	1034.62
Real-valued	None	1025.96
Complex	Unitary regularisation	989.28
Complex	Multiscale no regularisation	988.10
Complex	$l_2$ weight regularisation	962.33
Complex	None	960.31

### Impact of speckle resolution

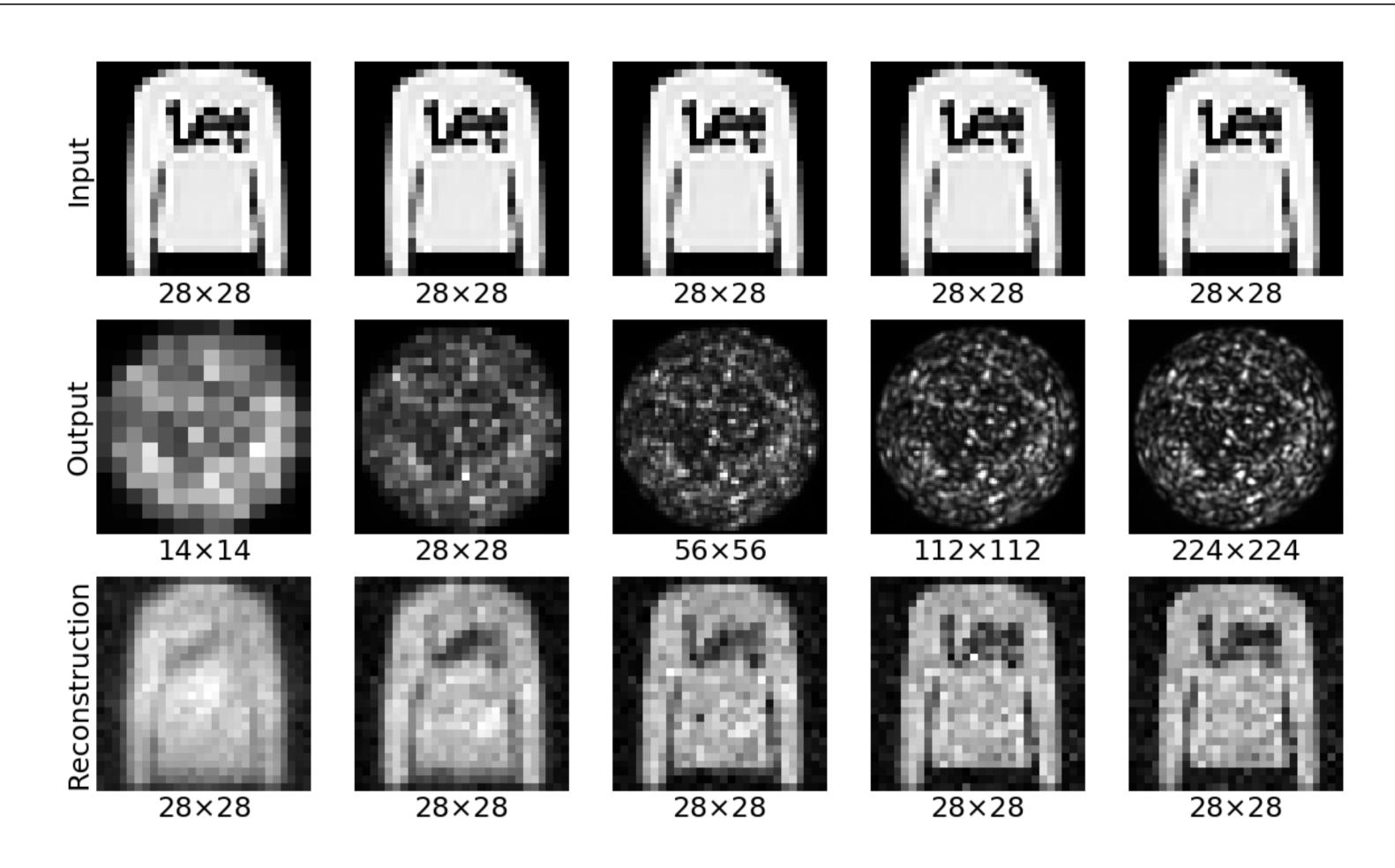
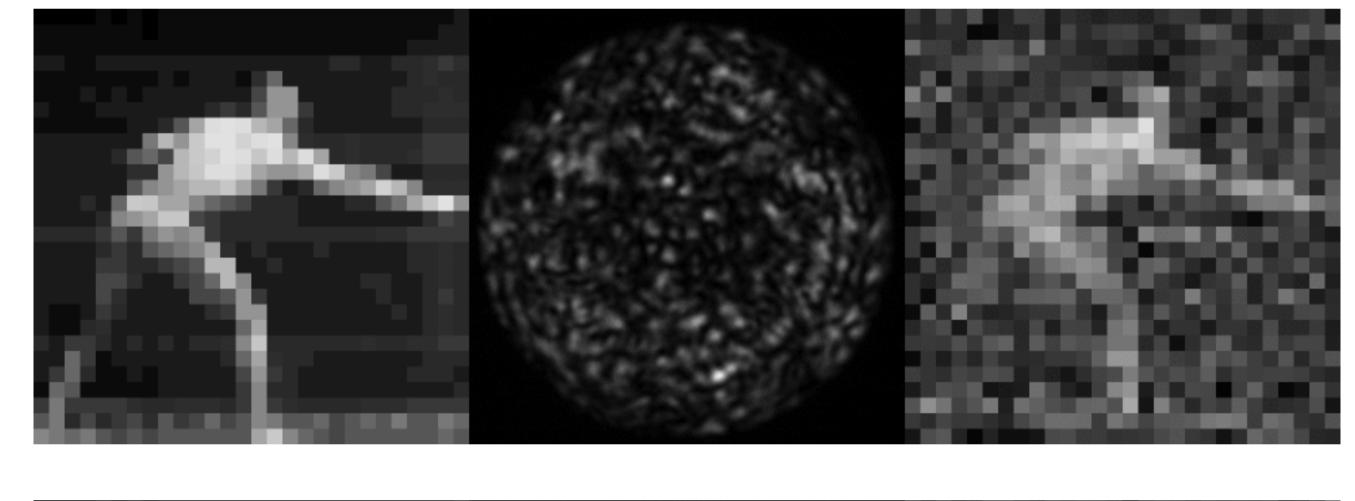


Figure: Impact of speckle resolution on quality of inferred image. From left to right we test on speckle image inputs of  $N_i$  =14, 28, 56, 112, and 224 pixels. This highlights the challenge to the machine learning community—increasing resolution far beyond the target resolution will improve the accuracy of the inverted images (up to the capacity of the fibre), but these cause problems for a straightforward application of the complex-weighted transformation, as the memory requirements scale  $O(N_i^2 N_o^2)$ .

### 1 m and 10 m results

Stills from a video from Muybridge's classic archive at 1 m and 10 m, highlighting generalisation to content quite different from the MNIST and fashion training data.



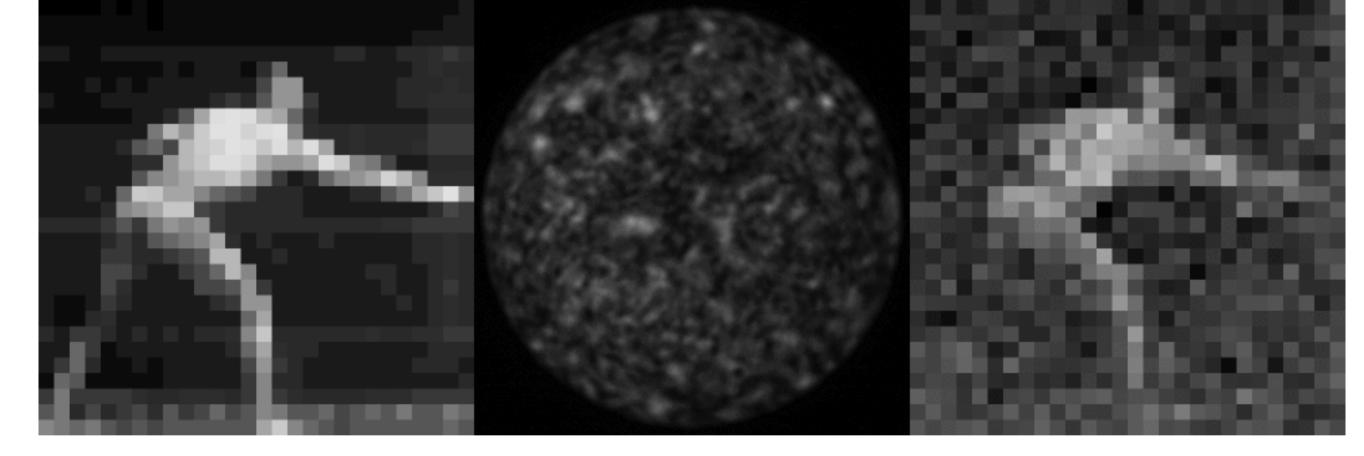


Figure: Comparison of inverse performance for 1 m (upper) and 10 m (lower) fibres, showing input, speckle image, inferred output. Note changes in speckle patterns for the longer distance—an increase in number, and decrease in size of speckles. Inverted images become noisier with increasing distance.

### Unitary matrices& Inversion of forward function

A complex square matrix W is unitary if its conjugate transpose  $W^*$  is its inverse, so if we enforce unitarity in the complex dense layer, we automatically have an analytic inverse at negligible computational cost, and we can directly optimise the parameters in both a *forward* and *inverse* manner, as shown in Figure 9.

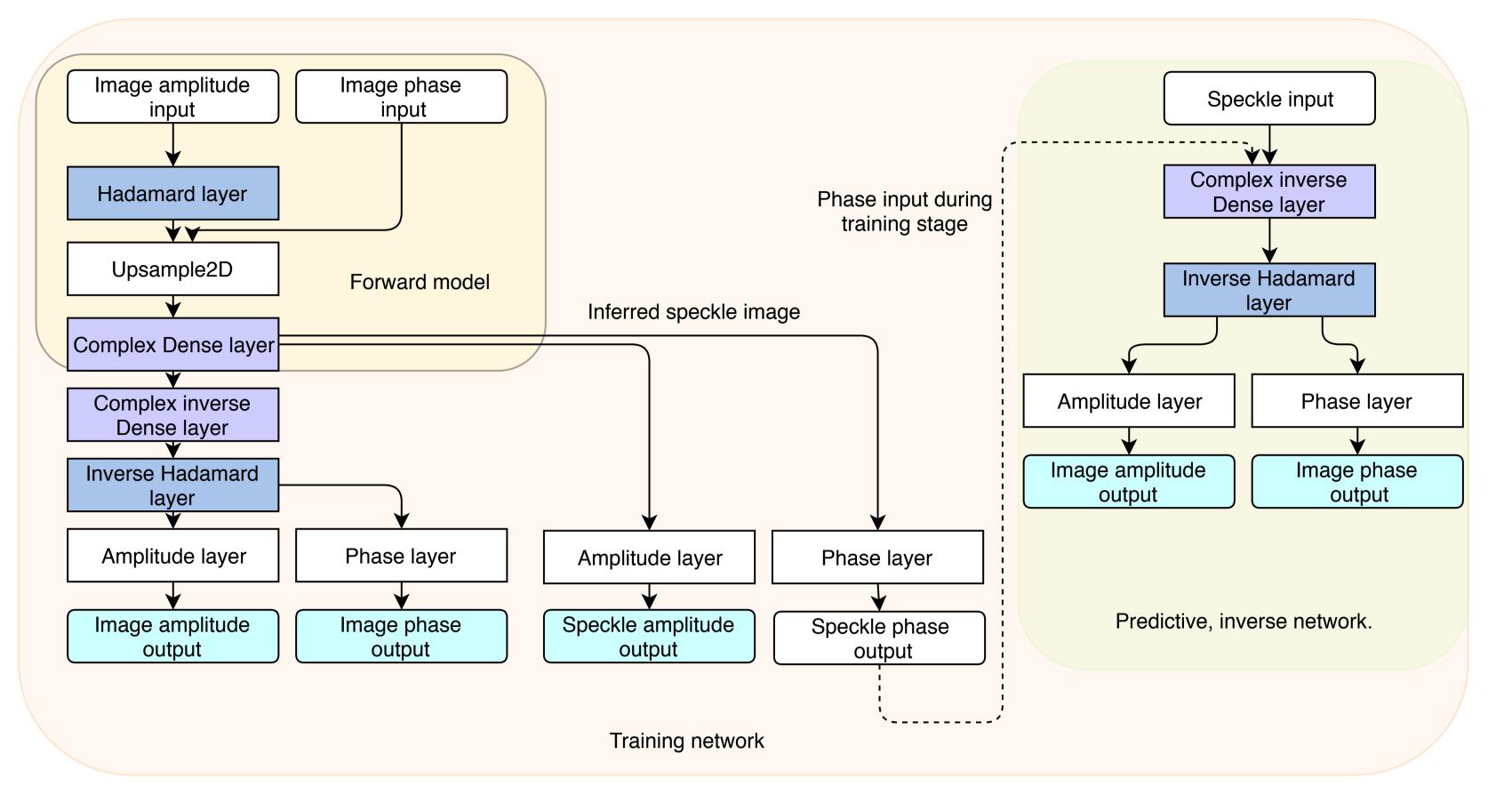


Figure: Model structure of the forward and inverse model. Cyan outputs are those for which we have target training data. Coloured layers indicate tied weights between associated forward and inverse layers. During training, inferred speckle phases from the forward model can be used to augment training via the inverse model.

### Conclusions

We have presented an application of deep, complex-weighted networks to create the best known results on inversion of optical fibre image with a non-scanning, interferometer-free approach to sensing. This allows direct analogue image transfer over bent optical fibre at distances not achieved before without measurement of phase information. One concrete advantage of this approach is that it allows real-time video rate analogue image transmission along very thin (105  $\mu$ m) multimode fibre (scanning approaches would take  $N^2$  longer to communicate each  $N \times N$  image).

We contribute the *Optical fibre inverse problem<sup>a</sup>* benchmark dataset. This can act as an initial challenge set for machine learning researchers looking for an interesting challenge which can not be directly attacked by conventional convolutional networks.

It brings challenging requirements of non-local patches on input and the need for better models of non-local relationships between pixels if we are to be able to work with the smaller speckles associated with longer optical fibres.

We achieved world-leading performance via the use of relatively simple, complex-weighted networks, which proved better than real-weighted networks in representing the inverse transmission matrix, and which can generalise to a wide range of images.

Unitary, complex-weight regularisation, which improved performance compared to real-valued dense layers, is compatible with our physical understanding of the optical fibre inversion problem, and enables analytic invertibility of the trained network.

### Acknowledgements

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aCode and data can be found at https://github.com/rodms/opticalfibreml