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A Matlab toolbox to make optimization on manifolds feel as simple as unconstrained optimization

A project of the RANSO group

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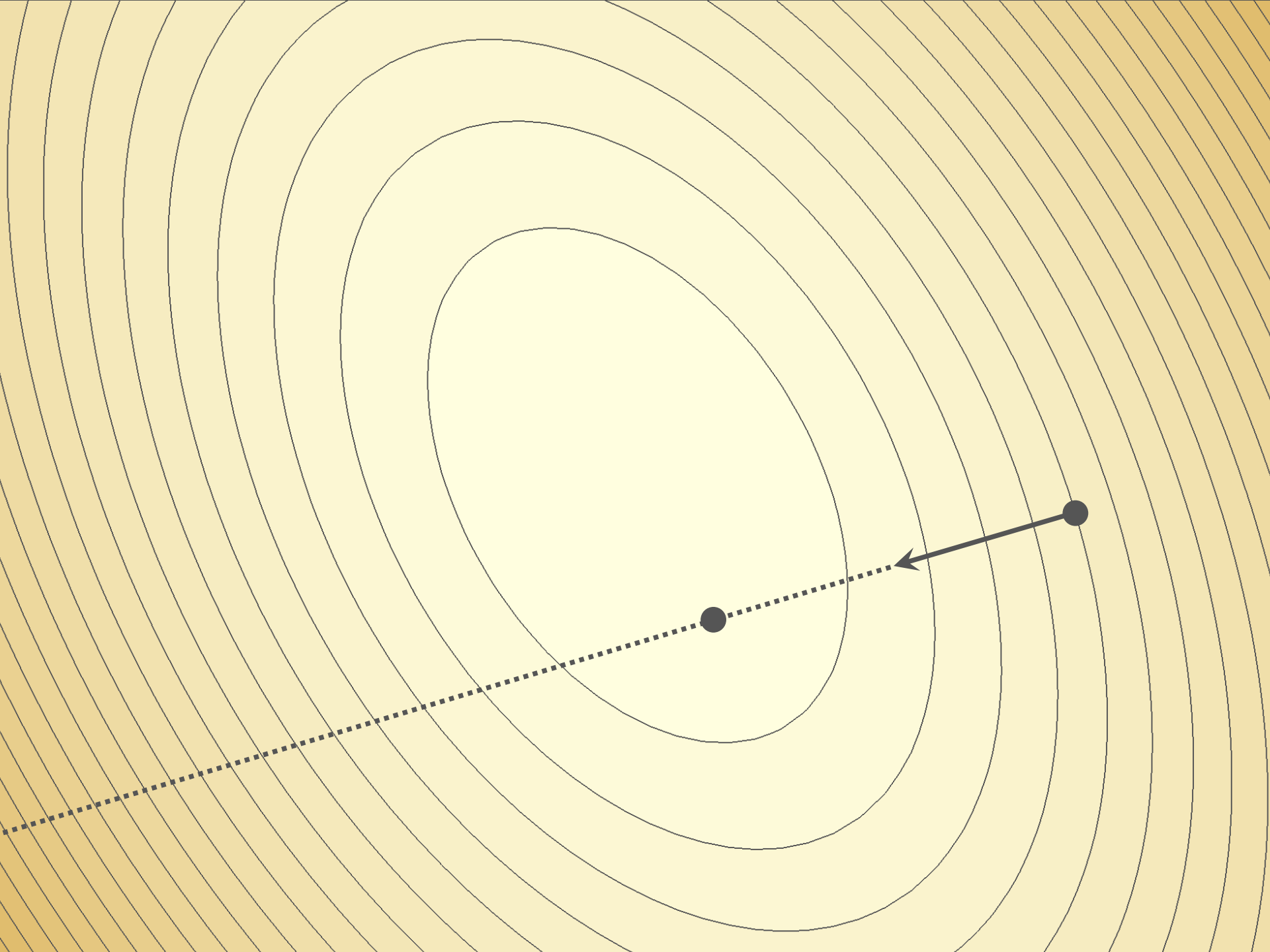
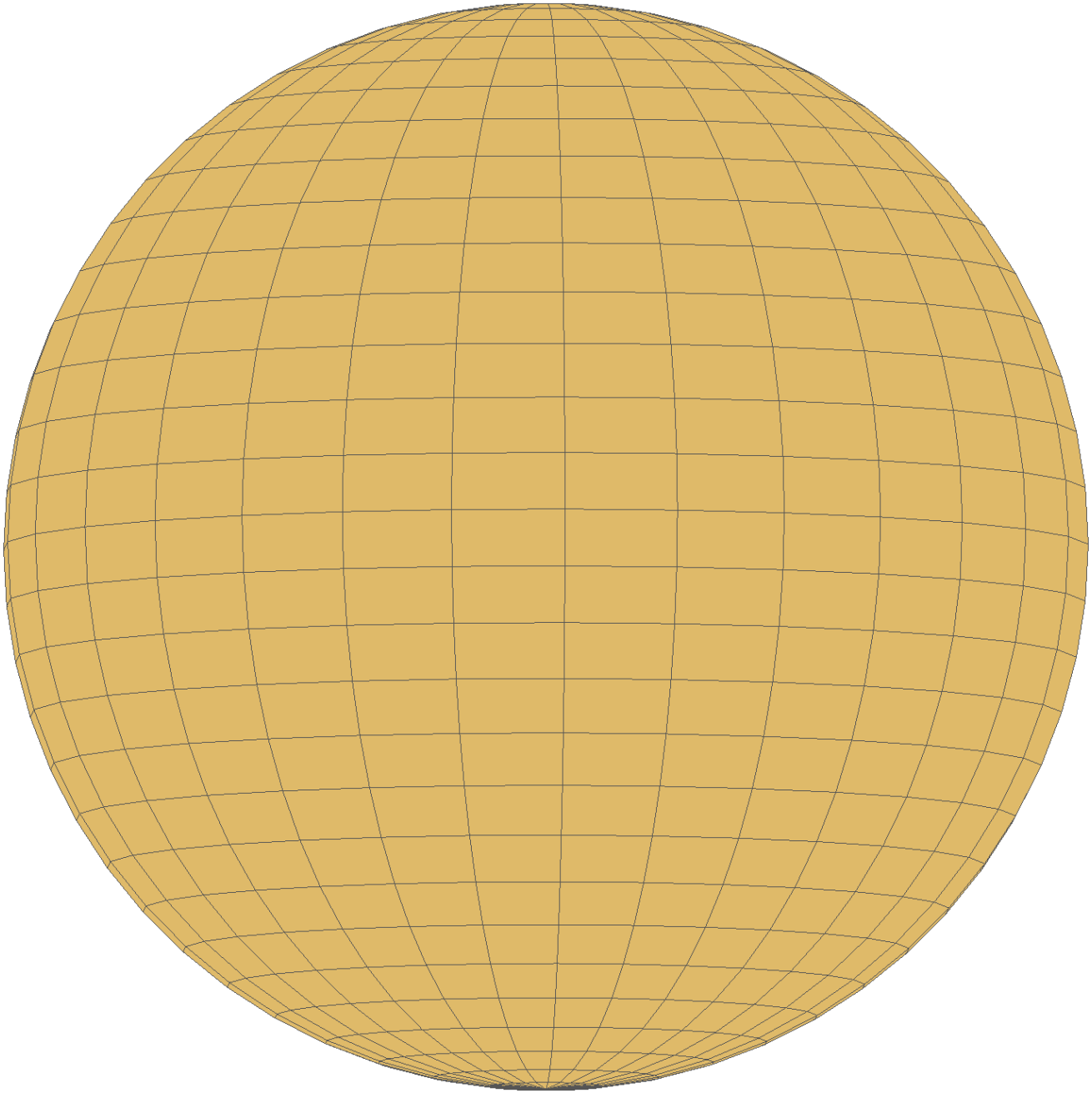
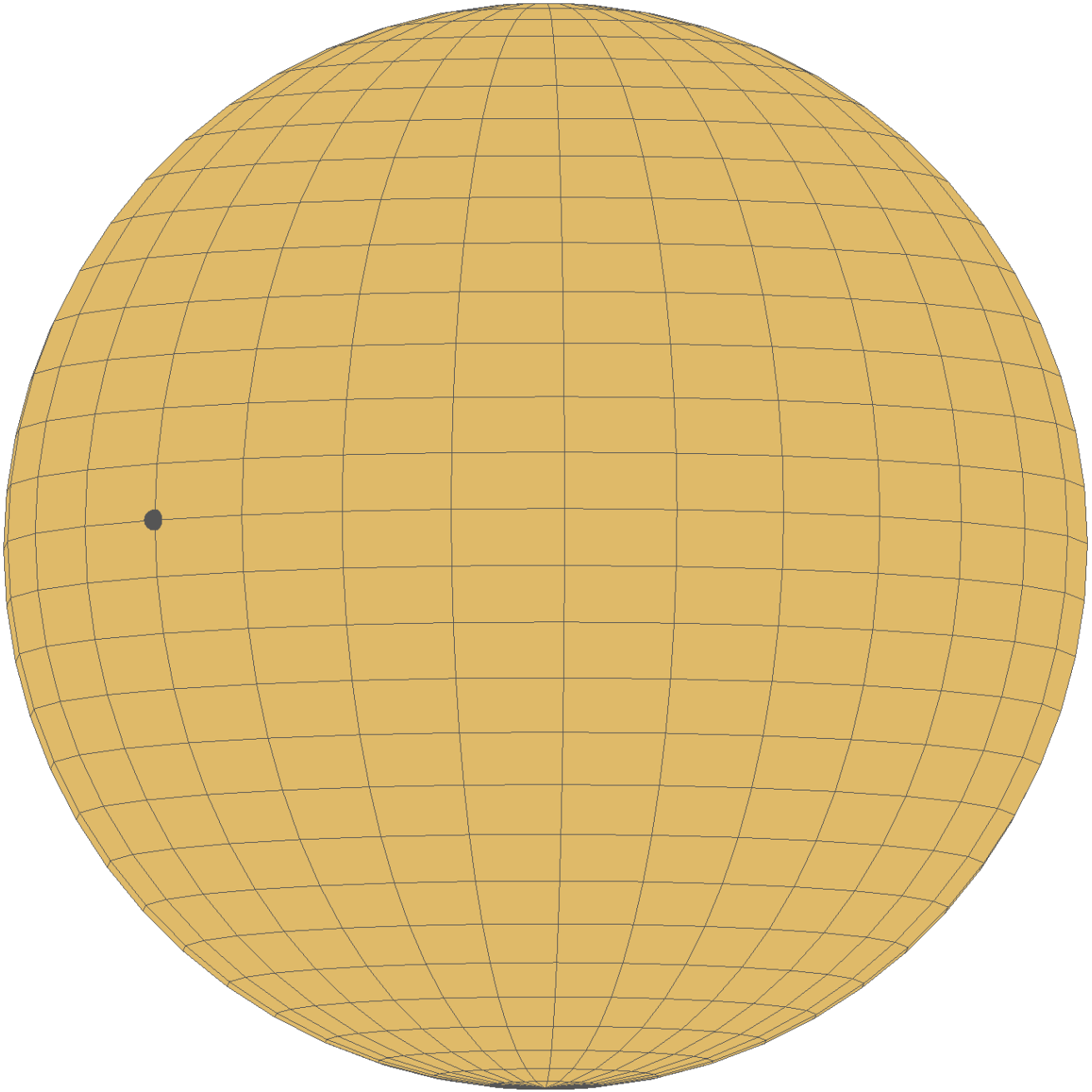
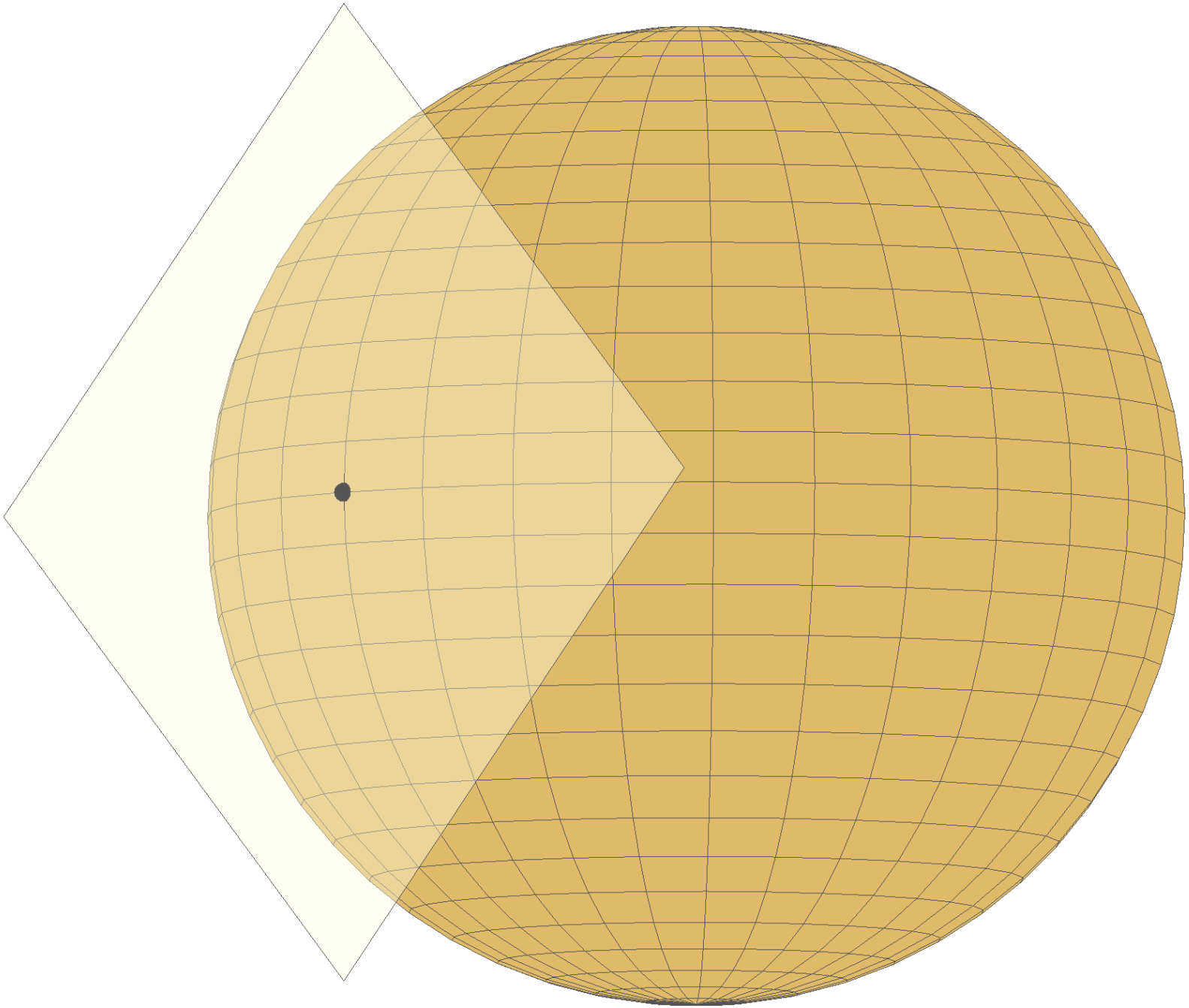
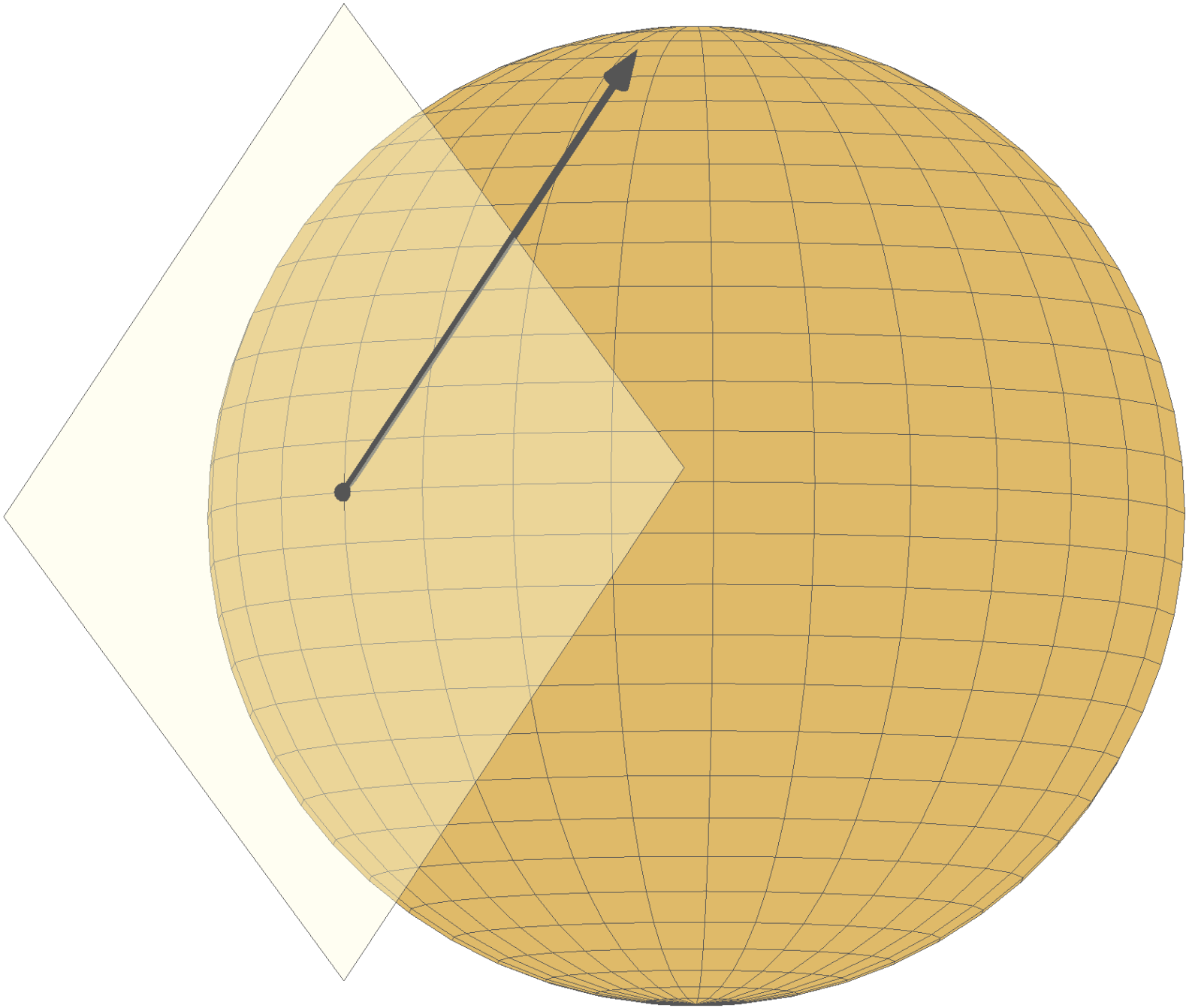
What is the minimal framework you need for steepest descent optimization?

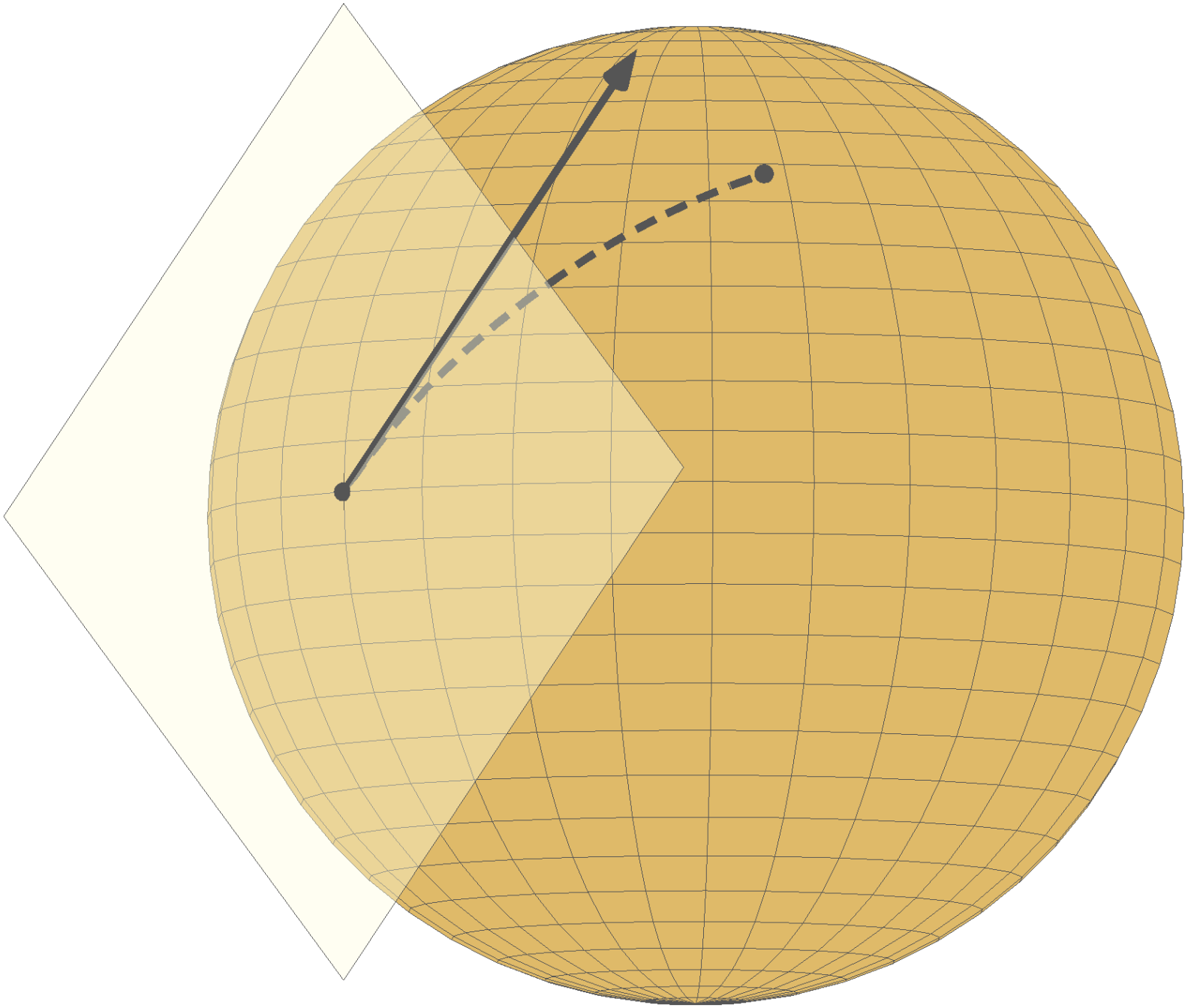










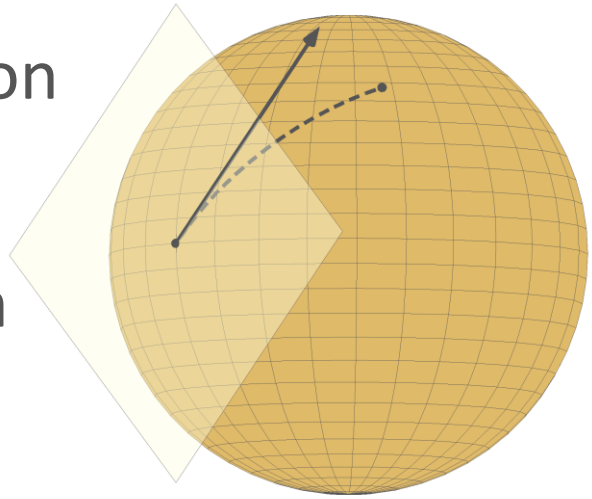


We only need the search space to be a Riemannian manifold

$$\min_{x \in M} f(x)$$

We need…

- A notion of directions along which we can move
  tangent space, tangent vector
- A notion of steepest descent direction
  inner product, gradient
- A means of moving along a direction
  Geodesics, retractions



Symmetry leads to manifold structures

Unit norm vector independent comp. analysis

Orthonormal matrix sparse and robust PCA

Rotation matrix synchronization of rotations

Positive definite matrix diffusion tensor imaging

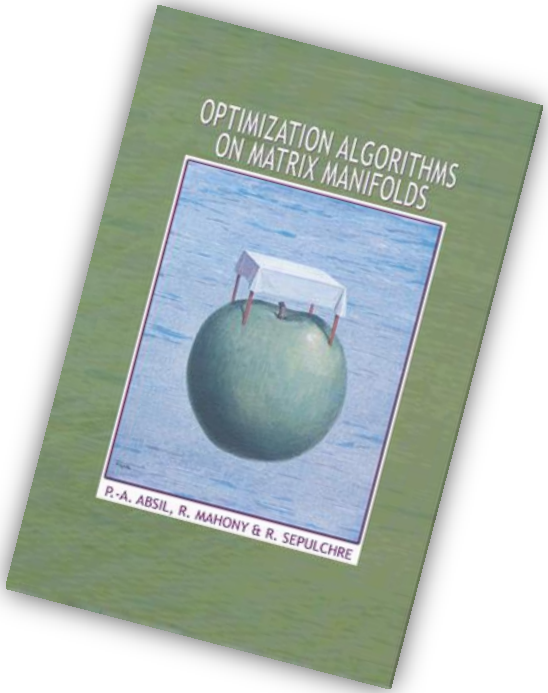
Fixed-rank matrix low-rank matrix completion

Semidefinite fixed-rank matrix covariance estim.

Euclidean distance matrix data visualization

. . .

The theory is mature at this point.



What's been missing is matching software.

Manopt.org

A Matlab toolbox to make optimization on manifolds feel as simple as unconstrained optimization

With generic solvers,
a library of manifolds and
diagnostics tools

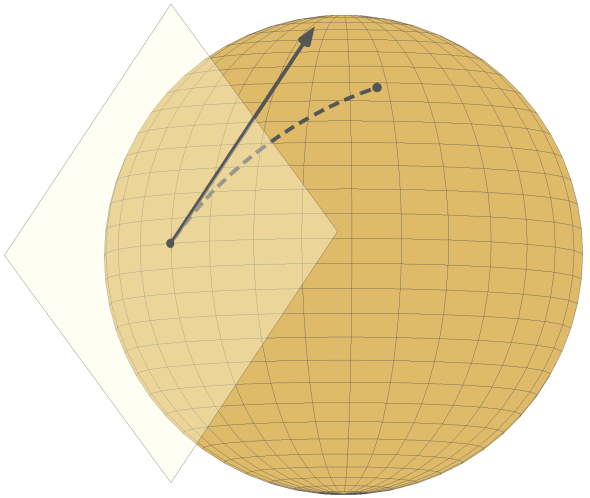
Example code for dominant eigenvectors

$$\max_{\|x\|=1} \; x^T A x$$

$$M = \{x \in \mathbb{R}^n : \|x\| = 1\}$$

$$f(x) = x^T A x$$

$$\nabla f(x) = 2Ax$$



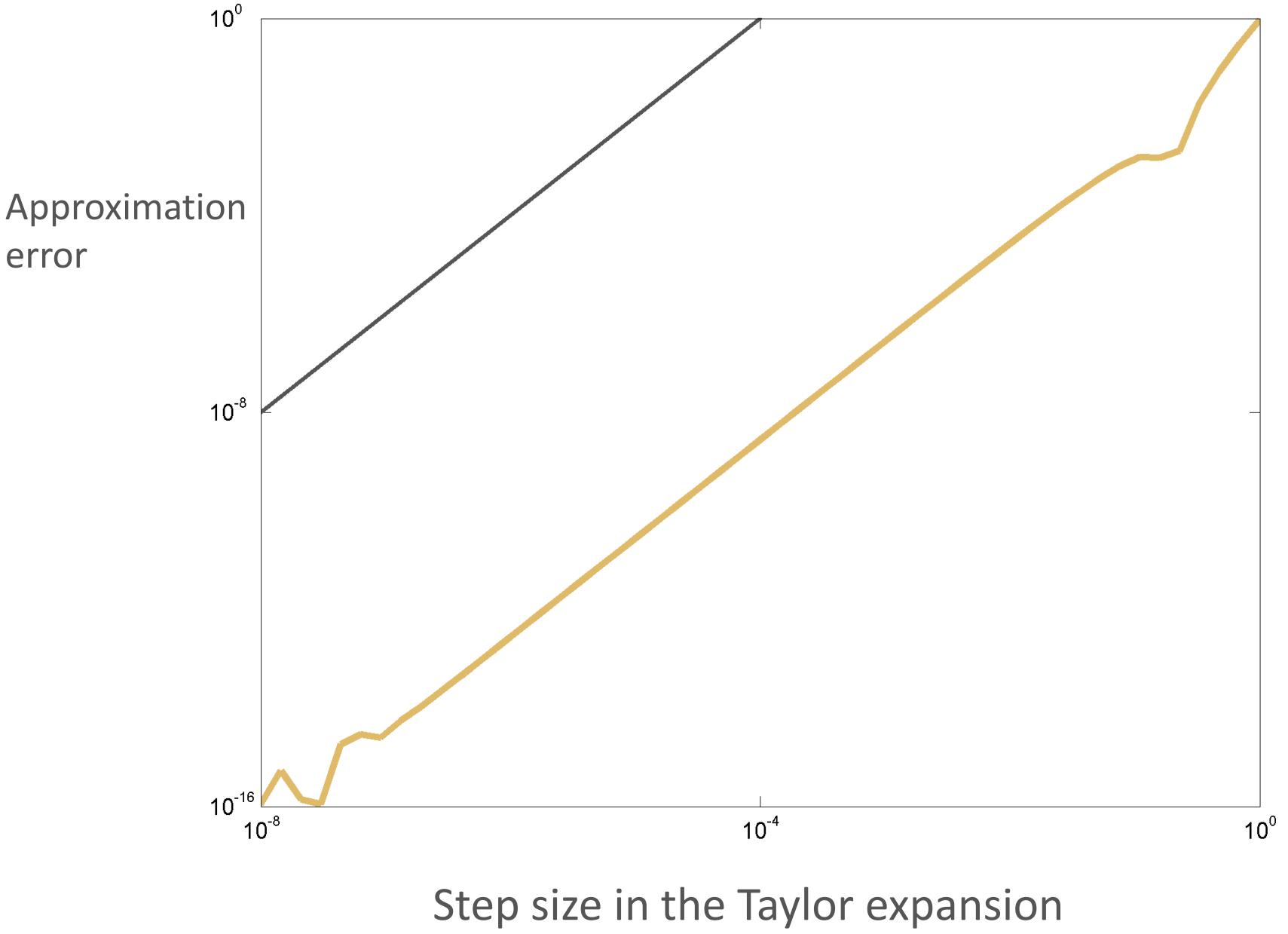
```
% Generate the problem data.
n = 1000;
A = randn(n);
A = .5*(A+A');

% Create the problem structure and specify the manifold.
problem.M = spherefactory(n);

% Define the problem cost function and its gradient.
problem.cost  = @(x) -x'*(A*x);
problem.egrad = @(x) -2*A*x;

% Numerically check gradient consistency.
checkgradient(problem);
```

Gradient check



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% Numerically check gradient consistency.
checkgradient(problem);

% Solve.
[x, xcost, info] = trustregions(problem);
```

```
>> [x, xcost, info] = trustregions(problem);
                                          f:  1.571531e+000   |grad|: 4.456216e+001
REJ TR-   k:     1     num_inner:     1     f:  1.571531e+000   |grad|: 4.456216e+001   negative curvature
acc       k:     2     num_inner:     1     f: -2.147351e+001   |grad|: 3.053440e+001   negative curvature
acc       k:     3     num_inner:     2     f: -3.066561e+001   |grad|: 3.142679e+001   negative curvature
acc       k:     4     num_inner:     2     f: -3.683374e+001   |grad|: 2.125506e+001   exceeded trust region
acc       k:     5     num_inner:     3     f: -4.007868e+001   |grad|: 1.389614e+001   exceeded trust region
acc       k:     6     num_inner:     4     f: -4.237276e+001   |grad|: 9.687523e+000   exceeded trust region
acc       k:     7     num_inner:     6     f: -4.356244e+001   |grad|: 5.142297e+000   exceeded trust region
acc       k:     8     num_inner:     8     f: -4.412433e+001   |grad|: 2.860465e+000   exceeded trust region
acc       k:     9     num_inner:    20     f: -4.438540e+001   |grad|: 3.893763e-001   reached target residual-kappa
acc       k:    10     num_inner:    20     f: -4.442759e+001   |grad|: 4.116374e-002   reached target residual-kappa
acc       k:    11     num_inner:    24     f: -4.442790e+001   |grad|: 1.443240e-003   reached target residual-theta
acc       k:    12     num_inner:    39     f: -4.442790e+001   |grad|: 1.790137e-006   reached target residual-theta
acc       k:    13     num_inner:    50     f: -4.442790e+001   |grad|: 3.992606e-010   dimension exceeded
Gradient norm tolerance reached.
Total time is 2.966843 [s] (excludes statsfun)
```

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checkgradient(problem);

% Solve.
[x, xcost, info] = trustregions(problem);

% Display some statistics.
semilogy([info.iter], [info.gradnorm], '.-');
```
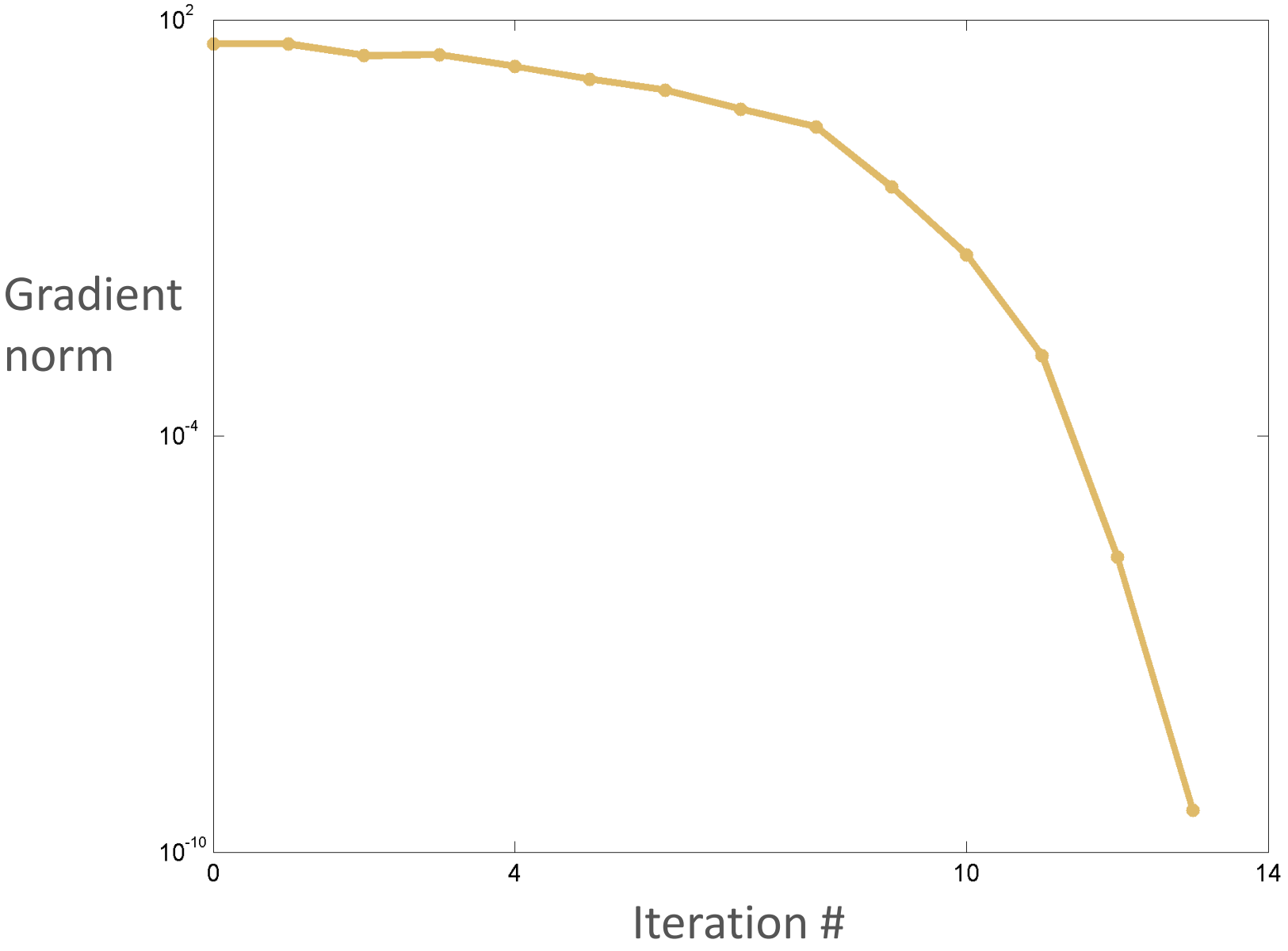

Convergence of the Riemannian trust-regions method



Riemannian optimization is…

Well-understood

Theory is available for robust algorithms

Useful

Leverage the symmetry of your constraints

Easy

With Manopt, you simply provide the cost

Manopt is open source and documented

www.manopt.org