華中科技大學

数据结构

第6章　树和二叉树

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线性结构：　　线性表, 栈, 队列
串, 数组, 广义表

非线性结构：　　树和二叉树
图，网

6.1树的定义

6.1.1 定义和术语

1.树（tree）:

树是n(n≥0)个结点的有限集T,

当n=0时，T为空树；

当n>0时,

(1)有且仅有一个称为T的根的结点,

(2)当n>1时,余下的结点分为m(m>0)个互不相交的有限集T1,T2,...,Tm ,每个Ti(1≤i≤m)也是一棵树,且称为根的子树。

例1. 一个结点的树

T＝{A}

T1

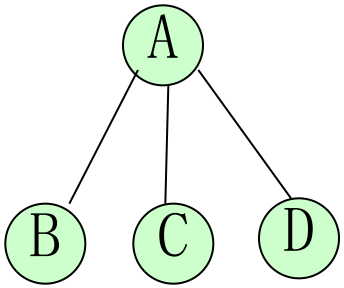
例2. 四个结点的树

T＝{A, B, C, D}

T1={B}

T2={C}

T3={D}



T2

例3：有16个结点的树

T={A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P}

T1={B, C, D, E, F}

T11={C, D, E}

T111={D}

T112={E}

T12={F}

T2={G, H}

T21={H}

T3={I, J, K, L, M, N, O, P}

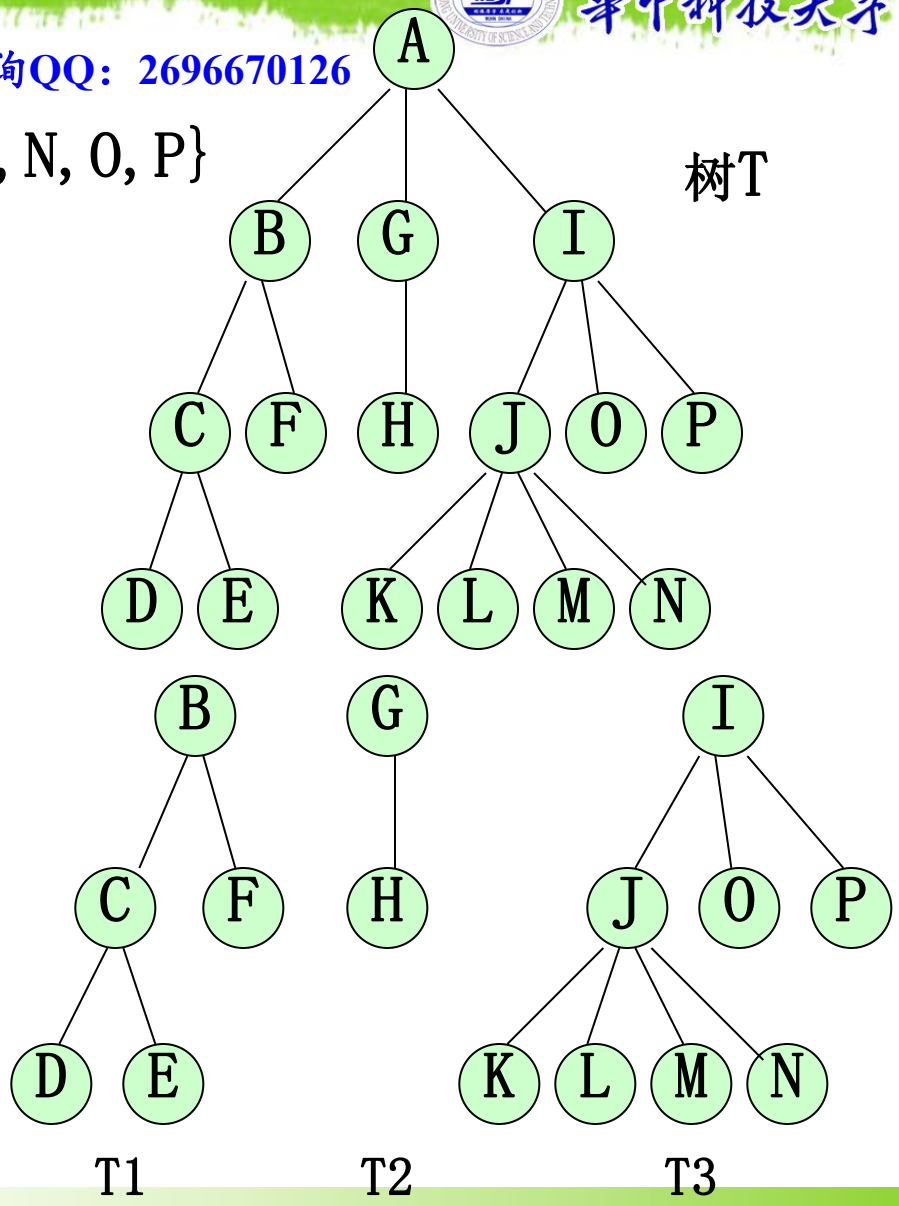
T31={J, K, L, M, N}

T32={O}

T33={P}

T312={L} ...

T311={K}



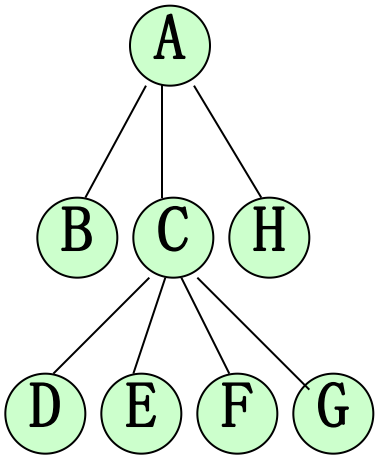
2. 结点的度(degree)：
结点的子树数目

3. 树的度：
树中各结点的度的最大值

4. n度树：　度为n的树

5. 叶子(终端结点)：度为0的结点

6. 分枝结点(非终端结点,非叶子)：
度不为0的结点



4度树

7. 双亲(父母,parent)和孩子(儿子,child)：
若结点C是结点P的子树的根,称P是C的双亲，C是P的孩子。

8. 结点的层(level)：

规定树T的根的层为1，其余任一结点的层等于其双亲的层加1。

9. 树的深度(depth, 高度)：

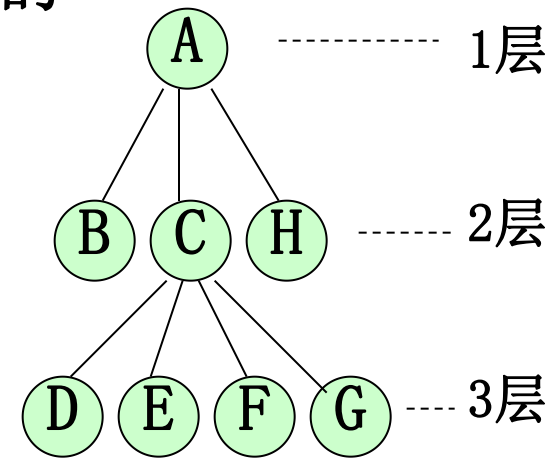
树中各结点的层的最大值。

10. 兄弟(sibling)：

同一双亲的结点之间互为兄弟。

11. 堂兄弟：

同一层号的结点互为堂兄弟。



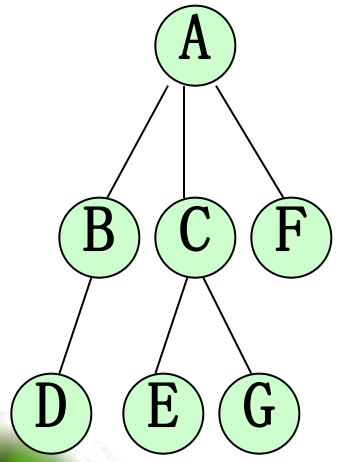
4度树

12. 祖先：从树的根到某结点所经分枝上的所有结点为该结点的祖先。

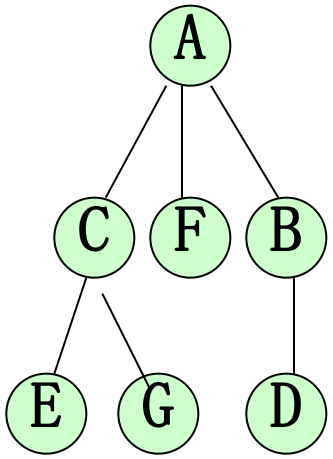
13. 子孙：一个结点的所有子树的结点为该结点的子孙。

14. 有序树：若任一结点的各棵子树，规定从左至右是有次序的，即不能互换位置，则称该树为有序树。

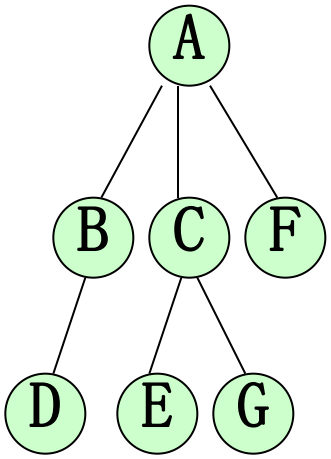
15. 无序树：若任一结点的各棵子树，规定从左至右是无次序的，即能互换位置，则称该树为无序树。



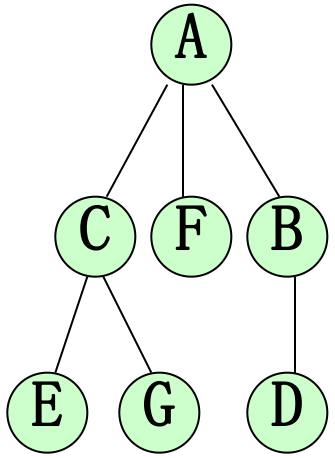
无序树T1



无序树T1



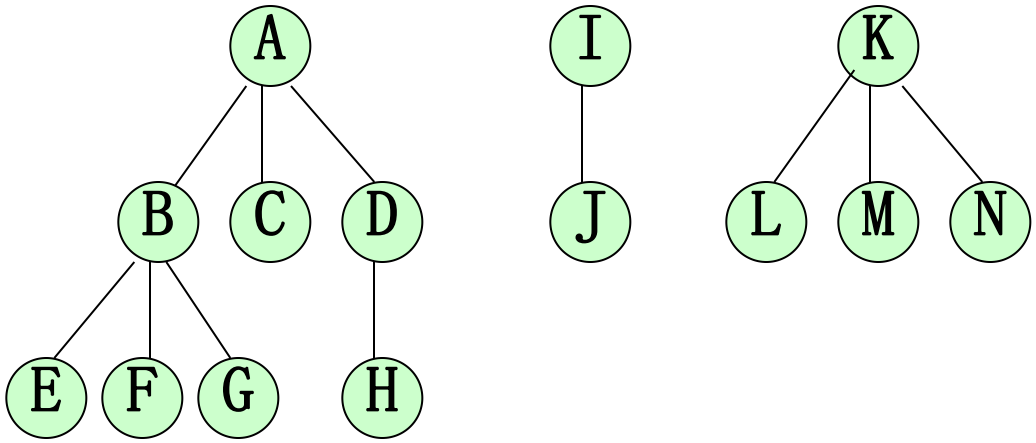
有序树T1



有序树T2

16. 森林：

m(m≥0)棵互不相交的树的集合。



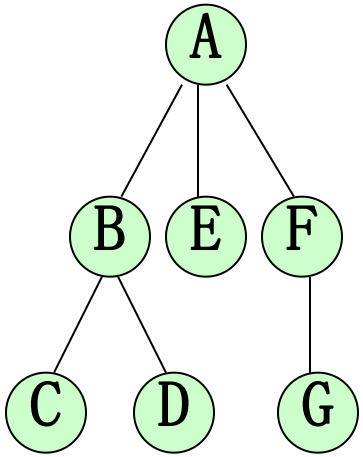
森林F={T1, T2, T3}

6.1.2 树的其它表示形式

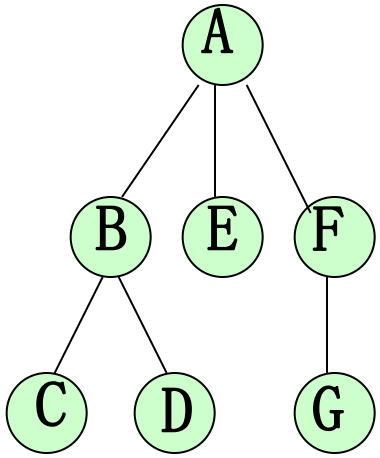
1. 广义表

树T的广义表=(T的根($T_1, T_2, \ldots, T_m$))

其中T_i是T的子树，也是广义表。 ($1 \leqslant i \leqslant m$)



树T

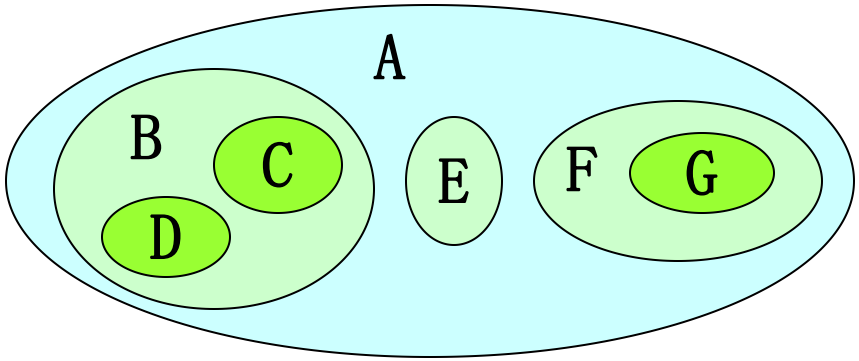


广义表A的树形表示

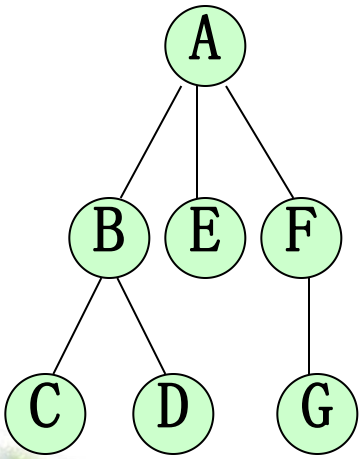
树T的广义表形式=(A(B(C,D),E,F(G,))

广义表A=(B,E,F)=((C,D),( ),(G))=((( ),( )) ,( ),(( )))

2. 嵌套集合：



3. 凹入表/书目表



树T

A ——————————
B ——————————
C ————————
D ————————
E ——————————
F ——————————
G ————————

凹入表

6.1.3 树的基本操作

1. 置T为空树：T={ }
2. 销毁树T
3. 生成树T
4. 遍历树T：按某种规则(次序)访问树T的每一个结点一次且一次的过程。
5. 求树T的深度
6. 求树T的度
7. 插入一个结点
8. 删除一个结点
9. 求结点的层号
10. 求结点的度
11. 求树T的叶子/非叶子
12. ……

6.2 二叉树(binary tree)

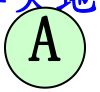
6.2.1 定义和术语

1. 二叉树的递归定义

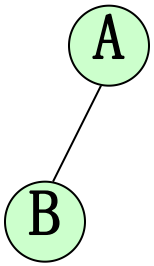
二叉树是有限个结点的集合，它或者为空集；或者是由一个根结点和两棵互不相交的，且分别称为根的左子树和右子树的二叉树所组成。

若二叉树为空集，则为空二叉树。

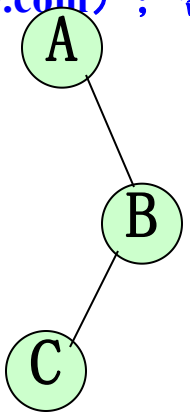
例：



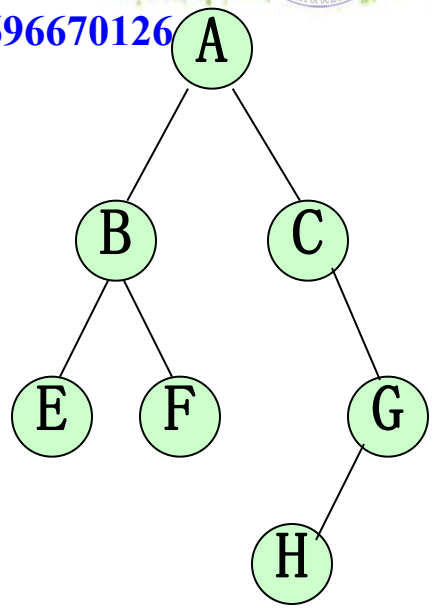
二叉树T1



二叉树T2

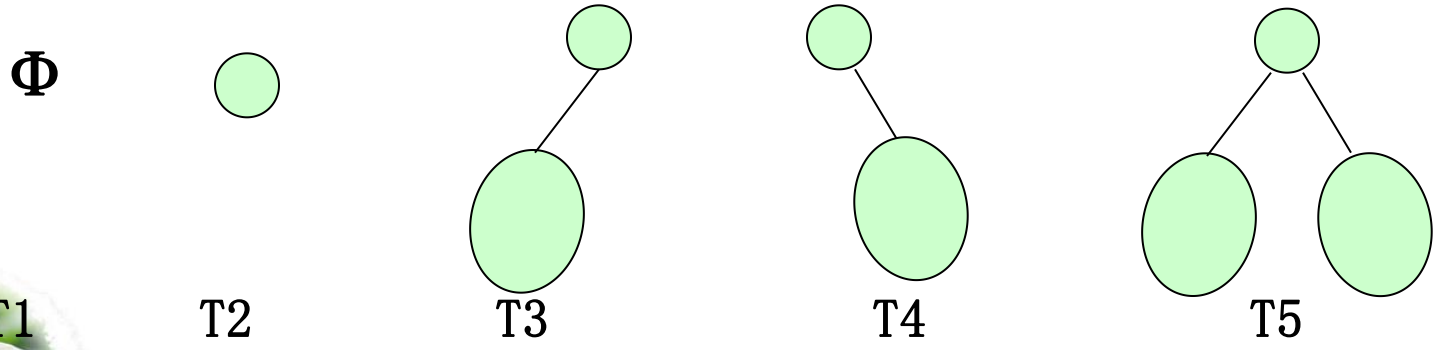


二叉树T3



二叉树T4

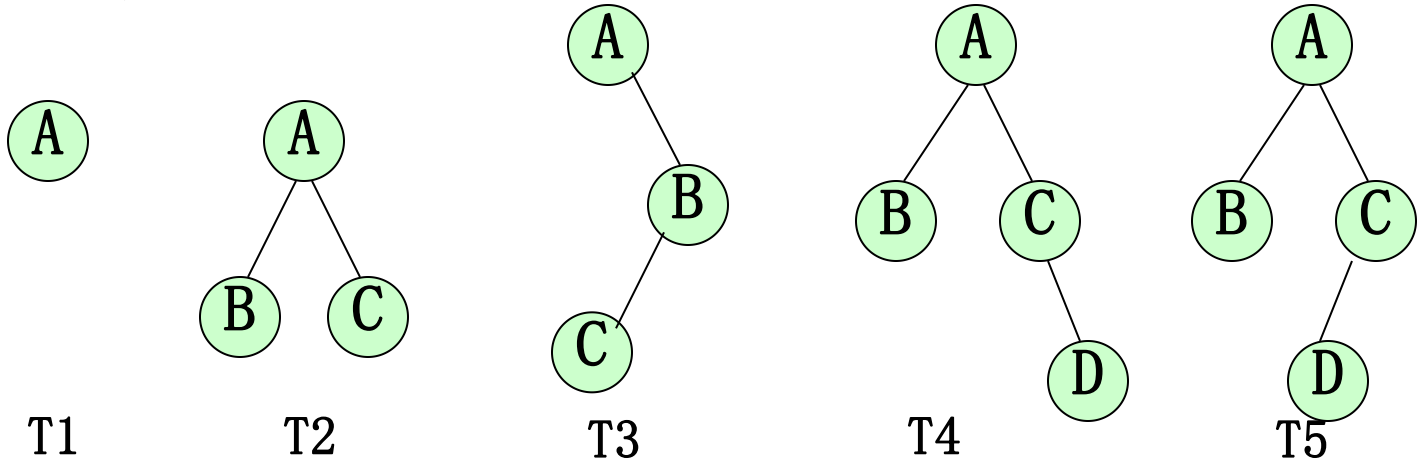
2. 二叉树的5种基本形态：



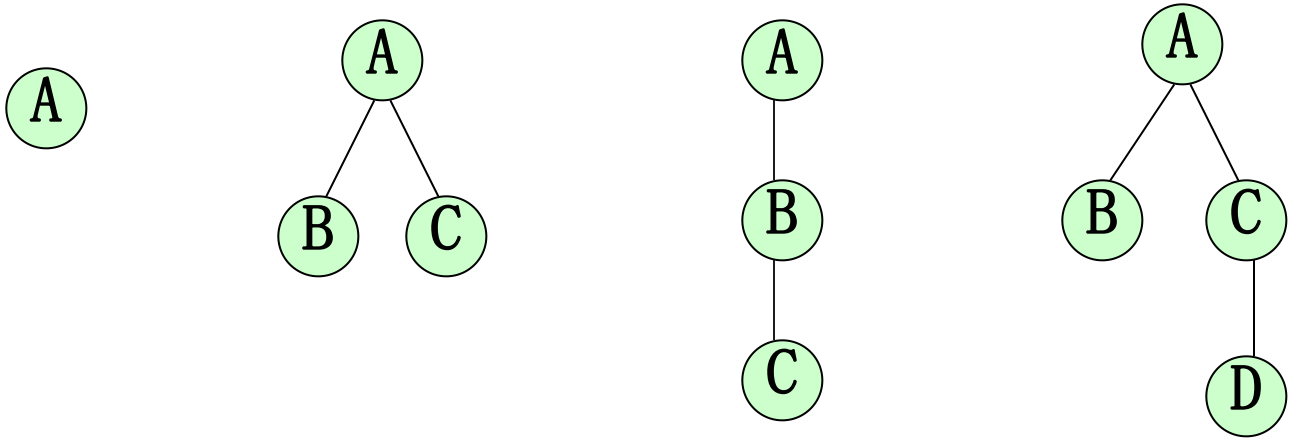
T1　T2　T3　T4　T5

3.二叉树与2度树的区别

(1)二叉树

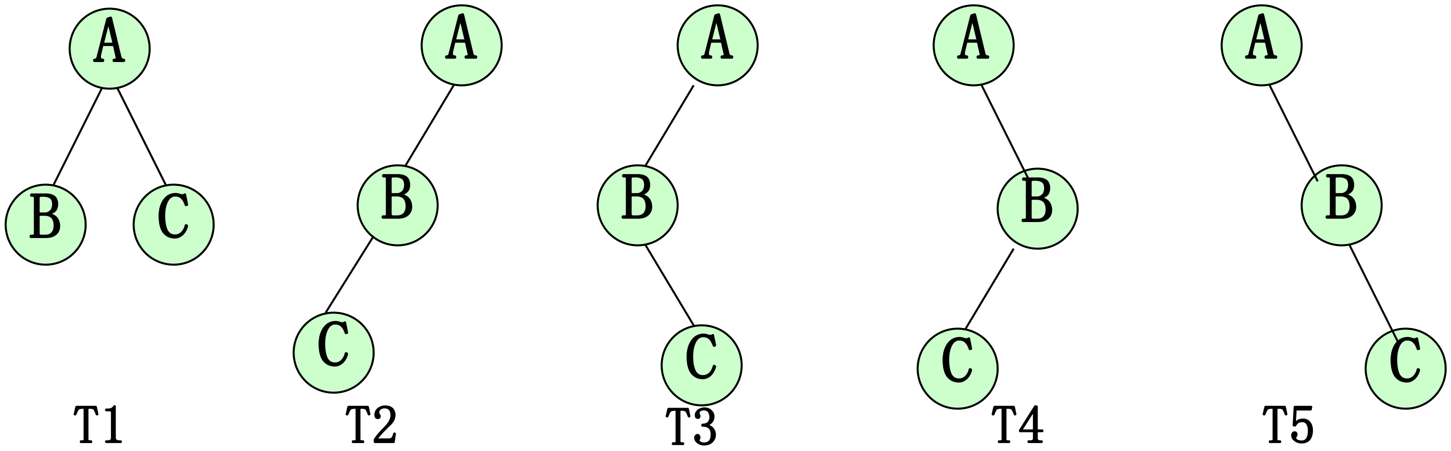


(2)树

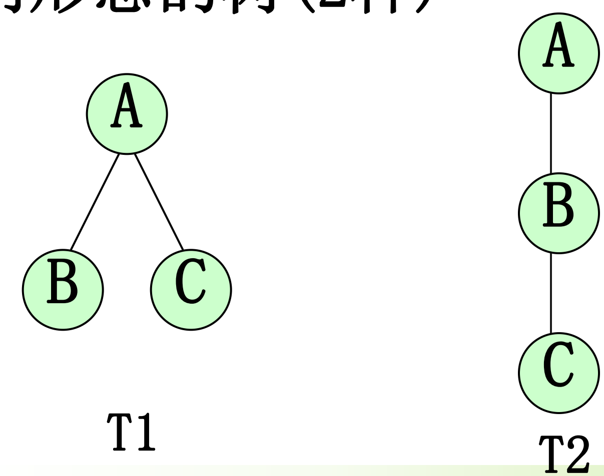


0度树　　2度树　　1度树　　2度树

4. 三个结点不同形态的二叉树(5种)



5. 三个结点不同形态的树(2种)

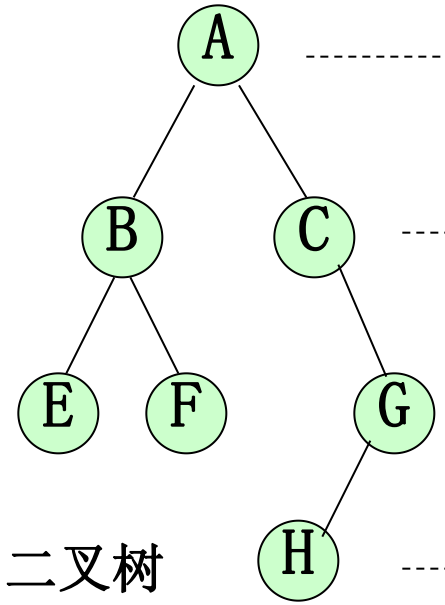


6. 二叉树的基本操作

1. 置T为空二叉树：T={ }
2. 销毁二叉树T
3. 生成二叉树T：生成哈夫曼树、二叉排序树、平衡二叉树、堆
4. 遍历二叉树T：
   按某种规则访问T的每一个结点一次且仅一次的过程。
5. 二叉树 ←→ 树
6. 二叉树 → 平衡二叉树
7. 求结点的层号
8. 求结点的度
9. 求二叉树T的深度
10. 插入一个结点
11. 删除一个结点
12. 求二叉树T的叶子/非叶子 ……

6.2.2 二叉树的性质和特殊二叉树

1. 二叉树的第i(i≥1)层最多有2^{i-1}个结点(性质1)



二叉树

$\leqslant 2^0 = 1$个

$\leqslant 2^1 = 2$个

$\leqslant 2^2 = 4$个

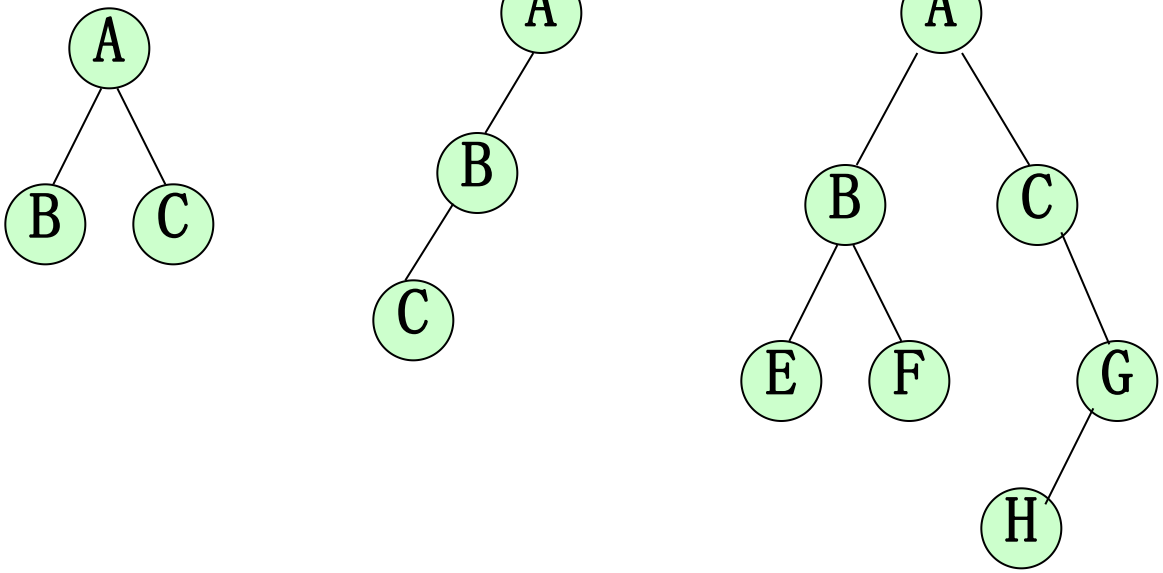
$\leqslant 2^3 = 8$个

2. 深度为k的二叉树最多有2^k-1个结点(性质2)

$$2^0+2^1+\ldots+2^{k-1}=\frac{2^0(1-2^k)}{1-2}=2^k-1$$

3. 叶子的数目=度为2的结点数目+1（性质3）

n0 = n2 + 1

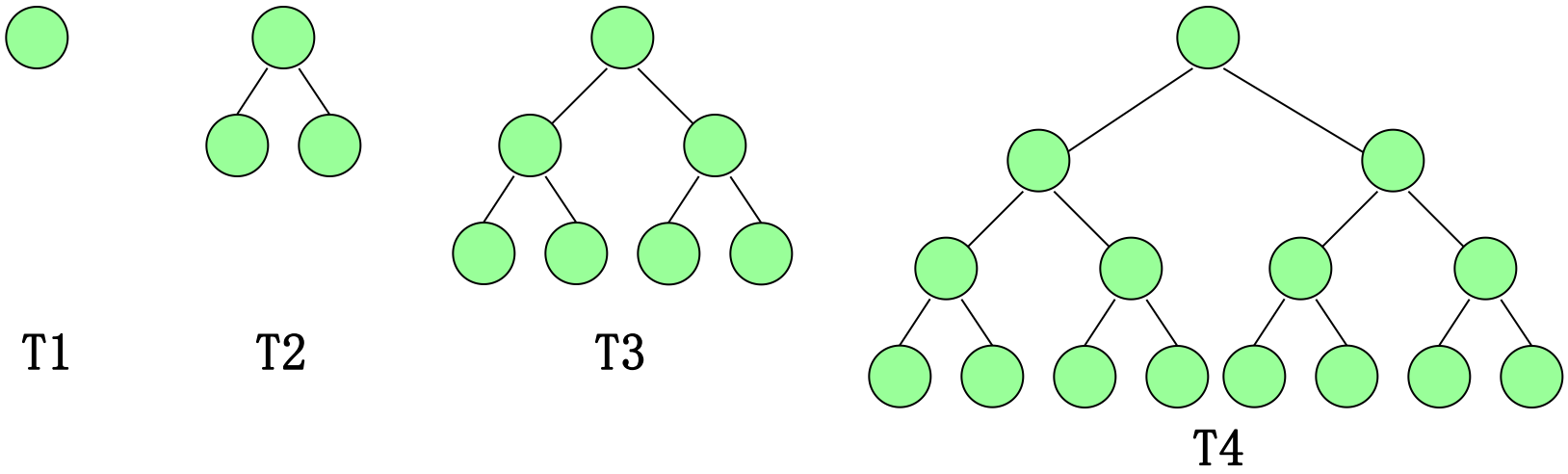


T1　　T2　　T3

T1：n0=2,　n2=1,　2=1+1
T2：n0=1,　n2=0　1=0+1
T3：n0=3,　n2=2　3=2+1

4. 满二叉树（full binary tree）——

深度为k且有2^k-1个结点的二叉树。



（1）n个结点的满二叉树的深度=$\log_2(n+1)$（性质4）

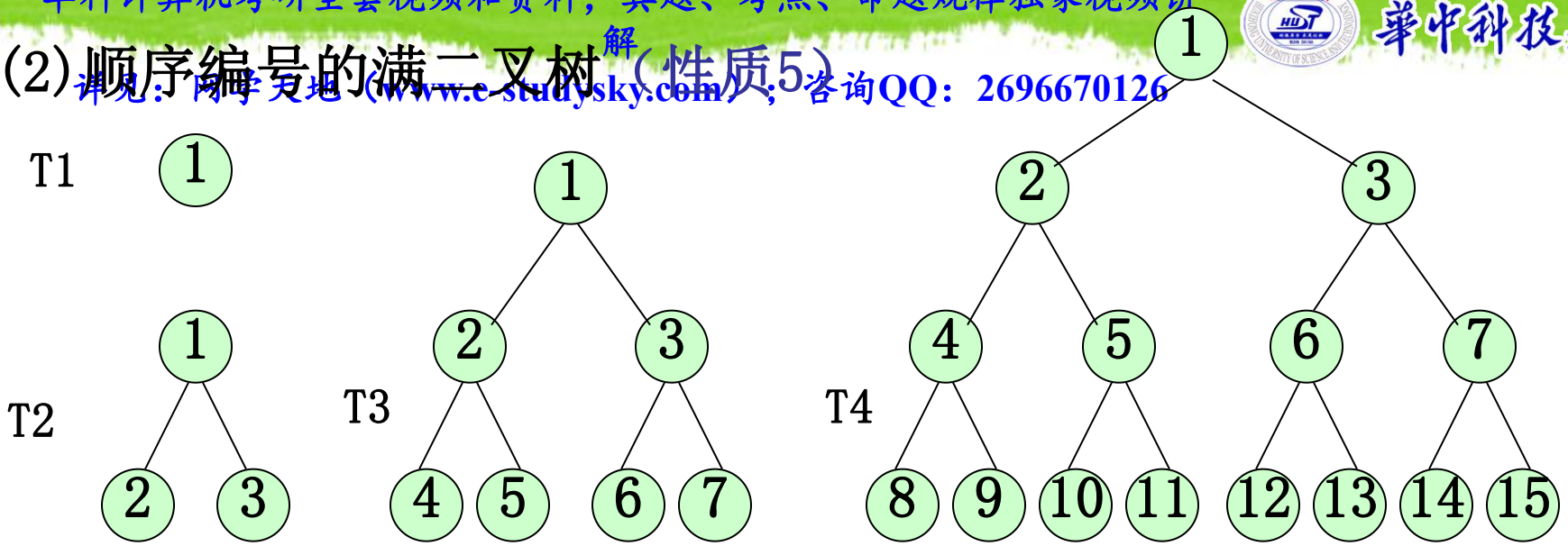
设深度为k

∵ $2^k - 1 = n$

$2^k = n + 1$

∴ $k=\log_2(n+1)$

(2)顺序编号的满二叉树（性质5）



设满二叉树有n个结点，编号为1，2，...，n

- 左小孩为偶数，右小孩为奇数；
- 结点i的左小孩是2i，　　$2i \leqslant n$

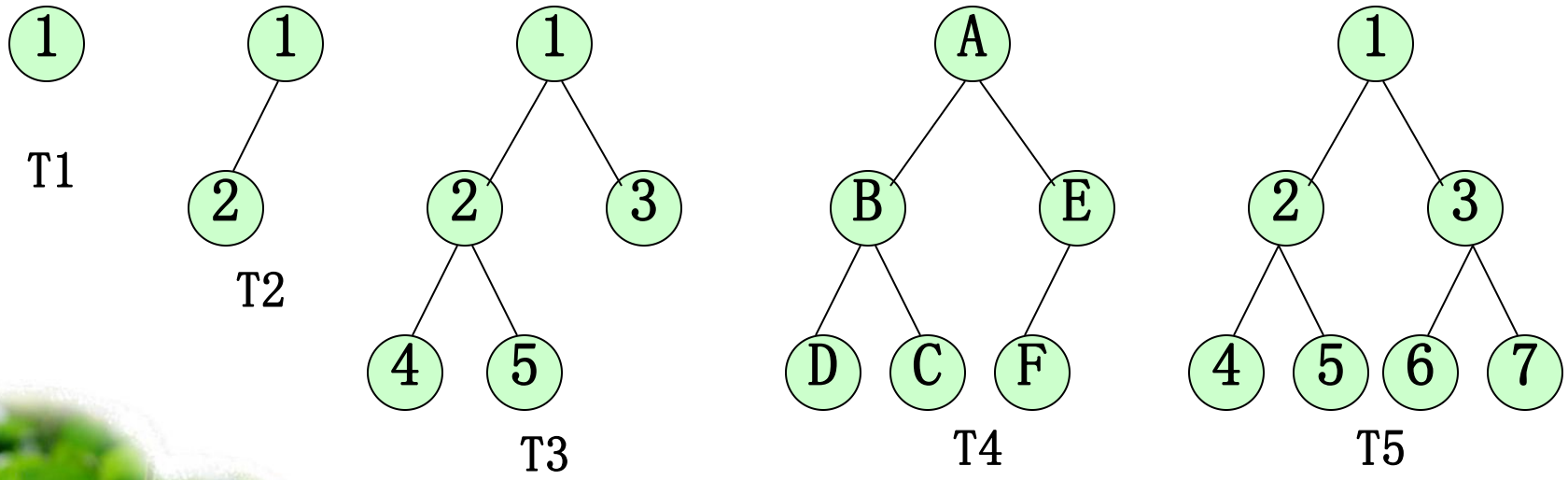
  结点i的右小孩是2i+1，　　$2i+1 \leqslant n$

  结点i的双亲是$\lfloor i/2 \rfloor$；　　$2 \leqslant i \leqslant n$
- 结点i的层号= $\lfloor \log_2 i \rfloor + 1 = \lceil \log_2 (i+1) \rceil$　　$1 \leqslant i \leqslant n$

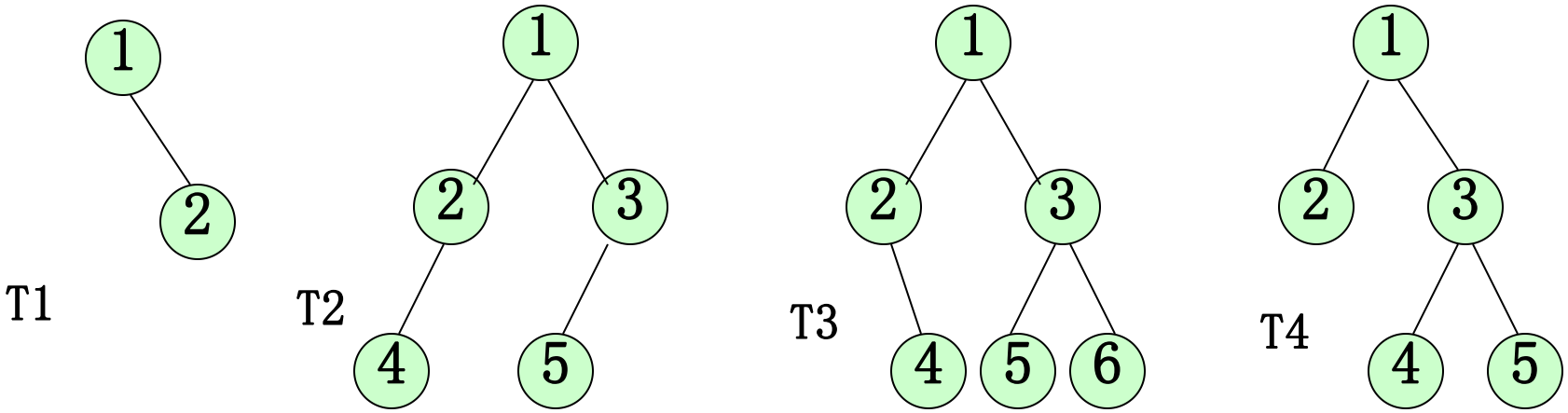
5. 完全二叉树（full binary tree）：

深度为k的有n个结点的二叉树，当且仅当每一个结点都与同深度的满二叉树中编号从1至n的结点一一对应，称之为完全二叉树（其它教材称为“顺序二叉树”）。

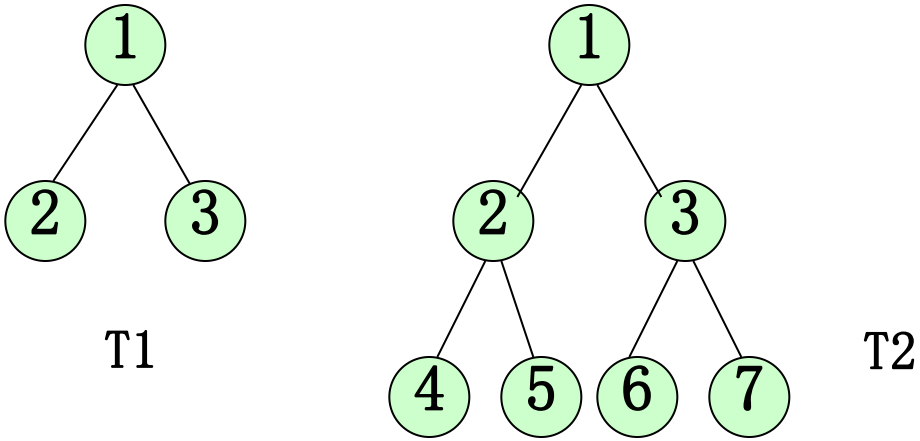
例. 完全二叉树：



例 非完全二叉树：



例　满二叉树：



n(n>0)个结点的完全二叉树的深度=$\lfloor \log_2 n \rfloor + 1$

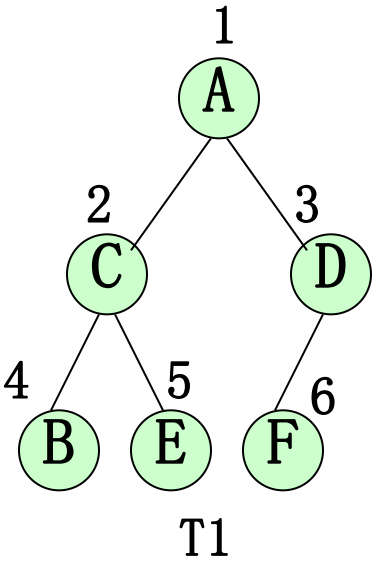
$= \lceil \log_2 (n+1) \rceil$

6.2.3 二叉树的存储结构

1. 顺序结构

（1）n个结点的完全二叉树，使用一维数组：

例. ElemType tree[n+1]; char t[7];



T1

t[1..6]

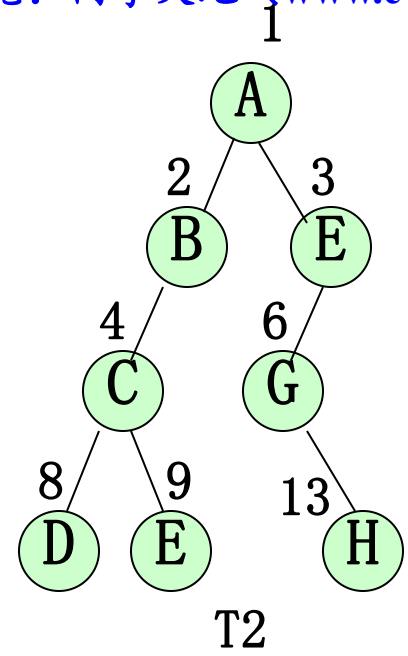
//	A	C	D	B	E	F
0	1	2	3	4	5	6

T1的顺序结构

父子关系

- 元素(结点)t[i]的双亲是t[i/2], $2 \leqslant i \leqslant n$
- 元素(结点)t[i]的左小孩是t[2*i], $2i \leqslant n$
- 元素(结点)t[i]的右小孩是t[2*i+1], $2i+1 \leqslant n$

(2) 一般二叉树



T2

t[1..13]

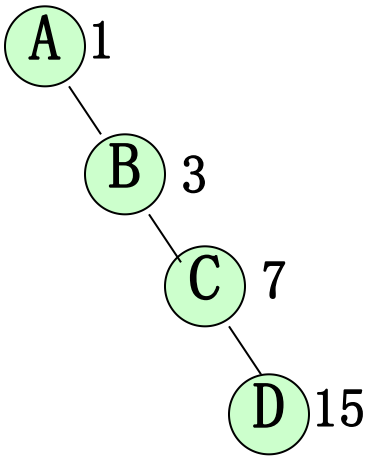
A	B	E	C		G		D	E				H
1	2	3	4	5	6	7	8	9	10	11	12	13

T2的顺序结构

父子关系

- 若t[i]存在,t[i]的双亲是t[i/2]； $2 \leqslant i \leqslant n$
- 若t[2*i]存在,t[i]的左小孩是t[2*i]； $2i \leqslant n$
- 若t[2*i+1]存在,t[i]的右小孩是t[2*i+1]。 $2i+1 \leqslant n$

(3)右单枝树



T3

t[1..15]

A		B				C								D
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

T3的顺序结构

深度为k的二叉树，最多需长度为2^k-1的一维数组。
若是右单枝树，空间利用率为:

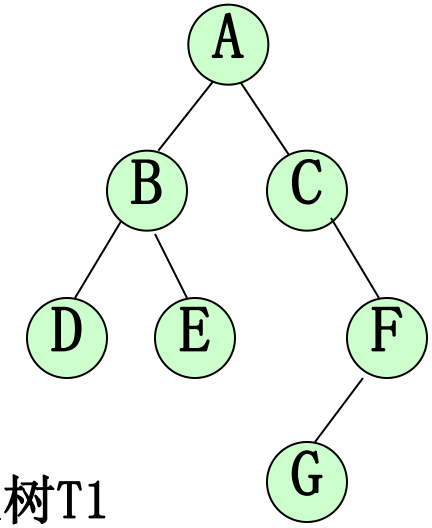
$$\alpha = \frac{k}{2^k - 1}$$

k=4, α=4/15 ；
k=10, α=10/1023≈0.0098

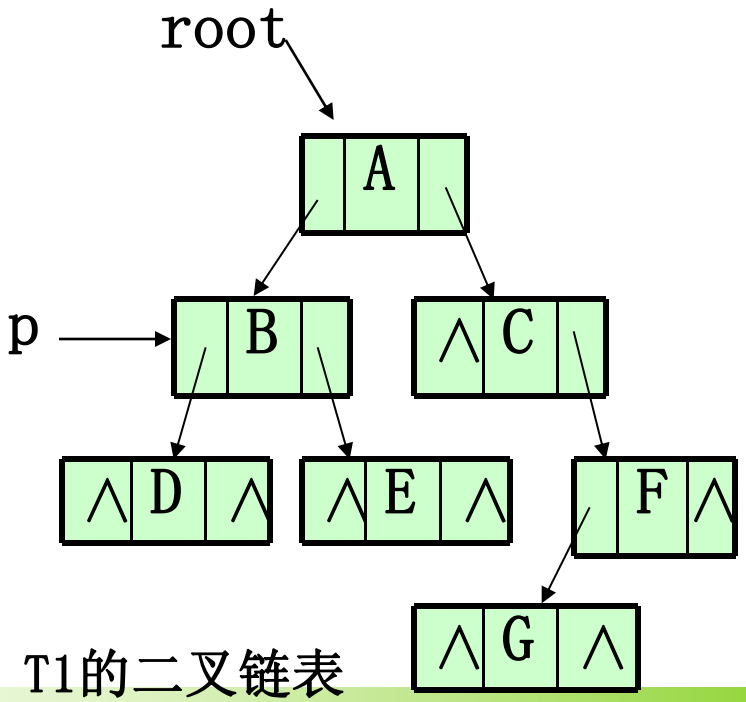
2.链式存储结构

(1)二叉链表

```
struct BiTNode                          //结点类型
{ struct BiTNode * lchild;              //左孩子指针
  ElemType data;                        //抽象数据元素类型
  struct BiTNode * rchild;              //右孩子指针
 } *root,*p;
```



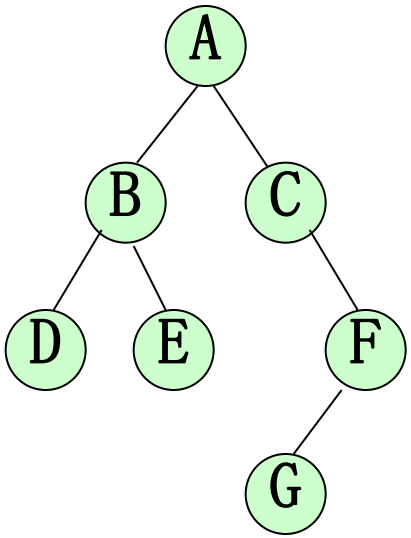
二叉树T1



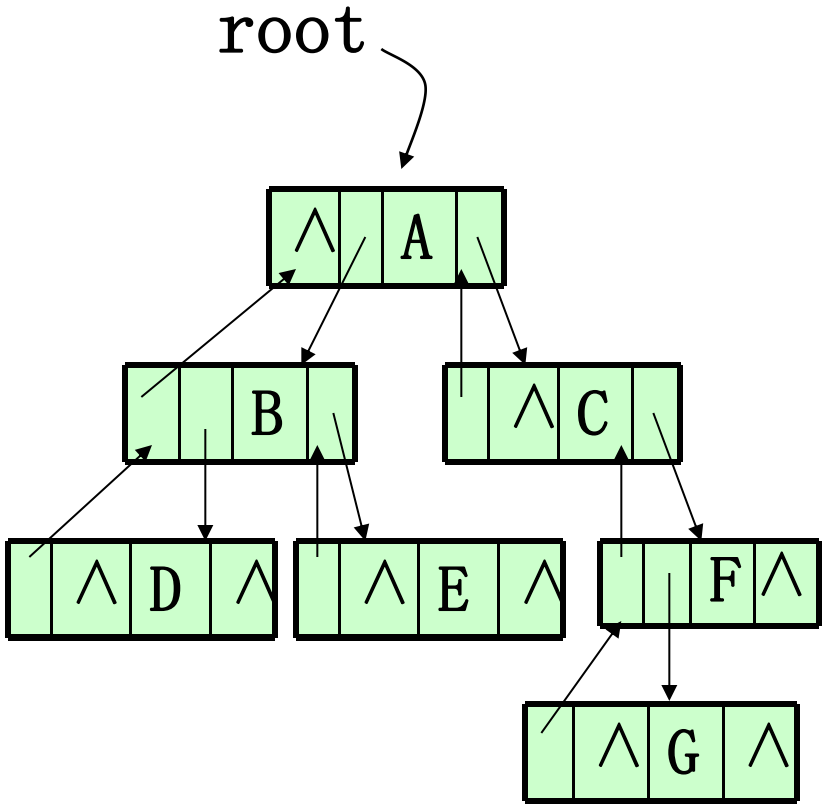
T1的二叉链表

(2)三叉链表

```
struct BiTNode
{ struct BiTNode *parent,*lchild,*rchild ;
  ElemType data;
 } *root,*p;
```



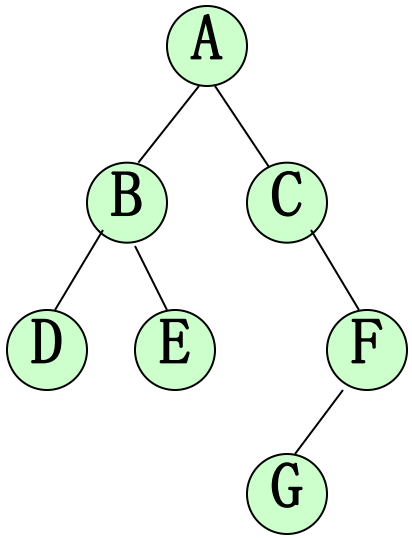
二叉树T2



T2的三叉链表

(3)静态链表

```
struct bnode2
{ ElemType data;
  int lchild,rchild;
 }t[n+1];
```



二叉树

root →

	lchild	data	rchild
0	///	///	///
1	2	A	3
2	4	B	5
3	0	C	6
4	0	D	0
5	0	E	0
6	7	F	0
7	0	G	0

一维数组t[0..7]

6.3 遍历二叉树和线索二叉树

6.3.1 遍历二叉树

按某种规则访问二叉树的每一个结点一次且仅一次的过程。

设：D----访问根结点,输出根结点；
L----递归遍历左二叉树；
R----递归遍历右二叉树。

遍历规则(方案)：

DLR----前序遍历(先根,preorder)
LDR----中序遍历(中根,inorder)
LRD----后序遍历(后根,postorder)
DRL----逆前序遍历
RDL----逆中序遍历
RLD----逆后序遍历

前序遍历

前序遍历二叉树递归定义：

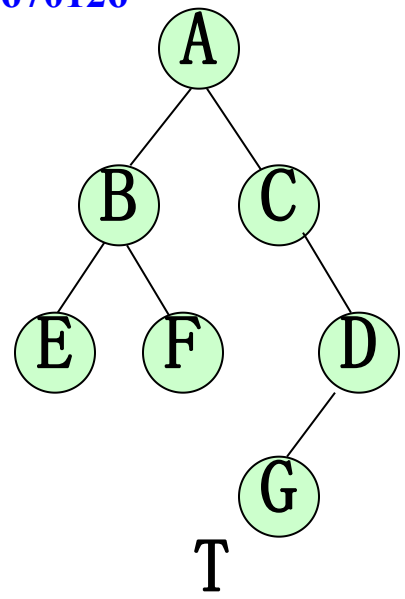
若二叉树为空，则遍历结束；

否则，执行下列步骤：

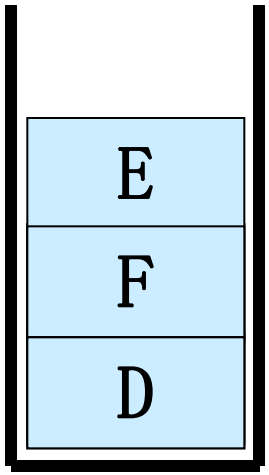
(1) 访问根结点；

(2) 遍历根的左子树；

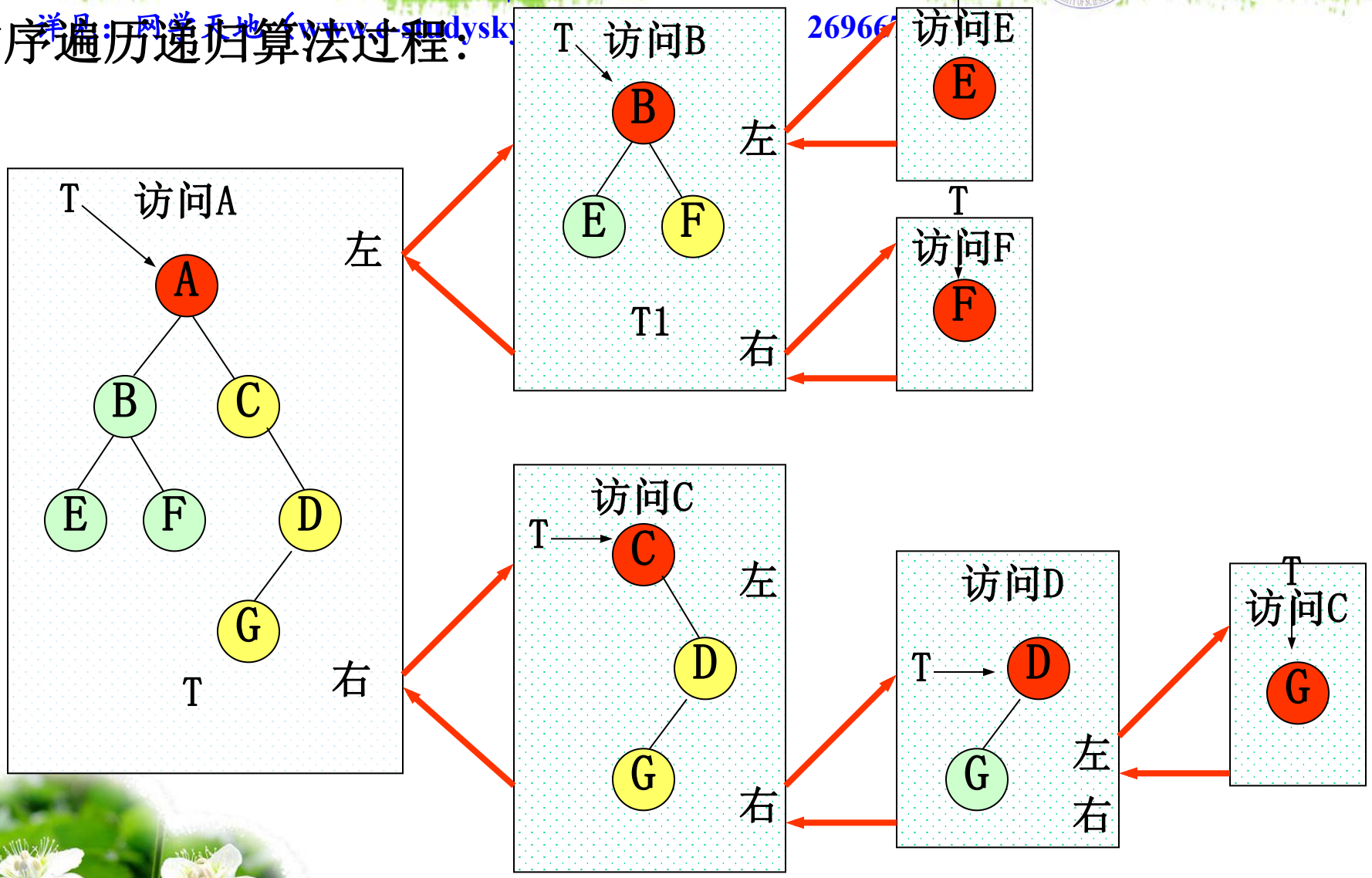
(3) 遍历根的右子树。



A B E F C D G



前序遍历递归算法过程：



先序遍历递归算法：

```
typedef  struct BiTNode *BiTree;  //结点指针类型
void  PreOrderTraverse(BiTree T)
//T是指向二叉链表根结点的指针
{
   if (!T)     return;
   else
  {
   printf(“%c”,T->data);          //访问根指针
   PreOrderTraverse(T->lchild);  //递归访问左子树
   PreOrderTraverse(T->rchild);  //递归访问右子树
   }
  return;
 }
```

先序遍历递归算法

```
typedef  struct BiTNode *BiTree;  //结点指针类型
void  PreOrderTraverse(BiTree   T)
//T是指向二叉链表根结点的指针
{
   if (T)
  {
   printf(“%c”,T->data);          //访问根指针
   PreOrderTraverse(T->lchild);   //递归访问左子树
   PreOrderTraverse(T->rchild);   //递归访问右子树
   }
  return;
}
```

2. 中序遍历

中序遍历二叉树递归定义：

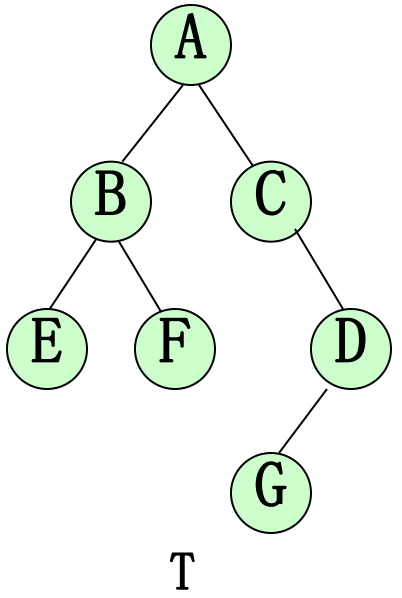
若二叉树为空，则遍历结束；

否则，执行下列步骤：

(1) 遍历根的左子树；

(2) 访问根结点；

(3) 遍历根的右子树。



T

E B F A C G D

中序遍历递归算法

```
typedef  struct BiTNode *BiTree;    //结点指针类型
void  MidOrderTraverse(BiTree   T)
//T是指向二叉链表根结点的指针
{
   if (T)
  {
   MidOrderTraverse(T->lchild);  //递归访问左子树
   printf(“%c”,T->data);         //访问根指针
   MidOrderTraverse(T->rchild);  //递归访问右子树
   }
  return;
 }
```

3. 后序遍历

后序遍历二叉树递归定义：

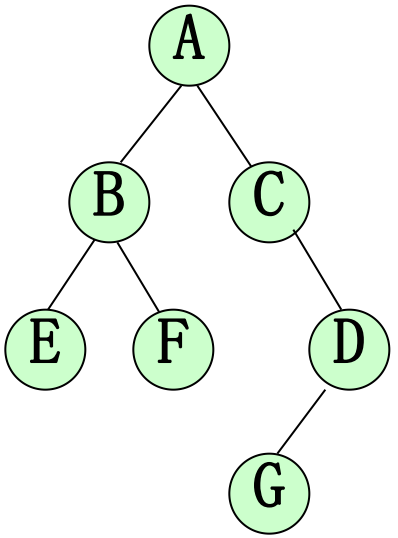
若二叉树为空，则遍历结束；

否则，执行下列步骤：

(1) 遍历根的左子树；

(2) 遍历根的右子树；

(3) 访问根结点。

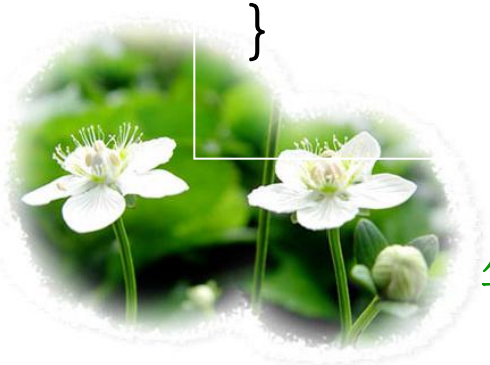


T

E F B G D C A

后序遍历递归算法

```
typedef  struct BiTNode *BiTree;  //结点指针类型
void  PostOrderTraverse(BiTree   T)
//T是指向二叉链表根结点的指针
{
   if (T)
  {
   PostOrderTraverse(T->lchild);  //递归访问左子树
   PostOrderTraverse(T->rchild);  //递归访问右子树
   printf(“%c”,T->data);          //访问根指针
   }
  return;
 }
```



4. 非递归算法(中序遍历)

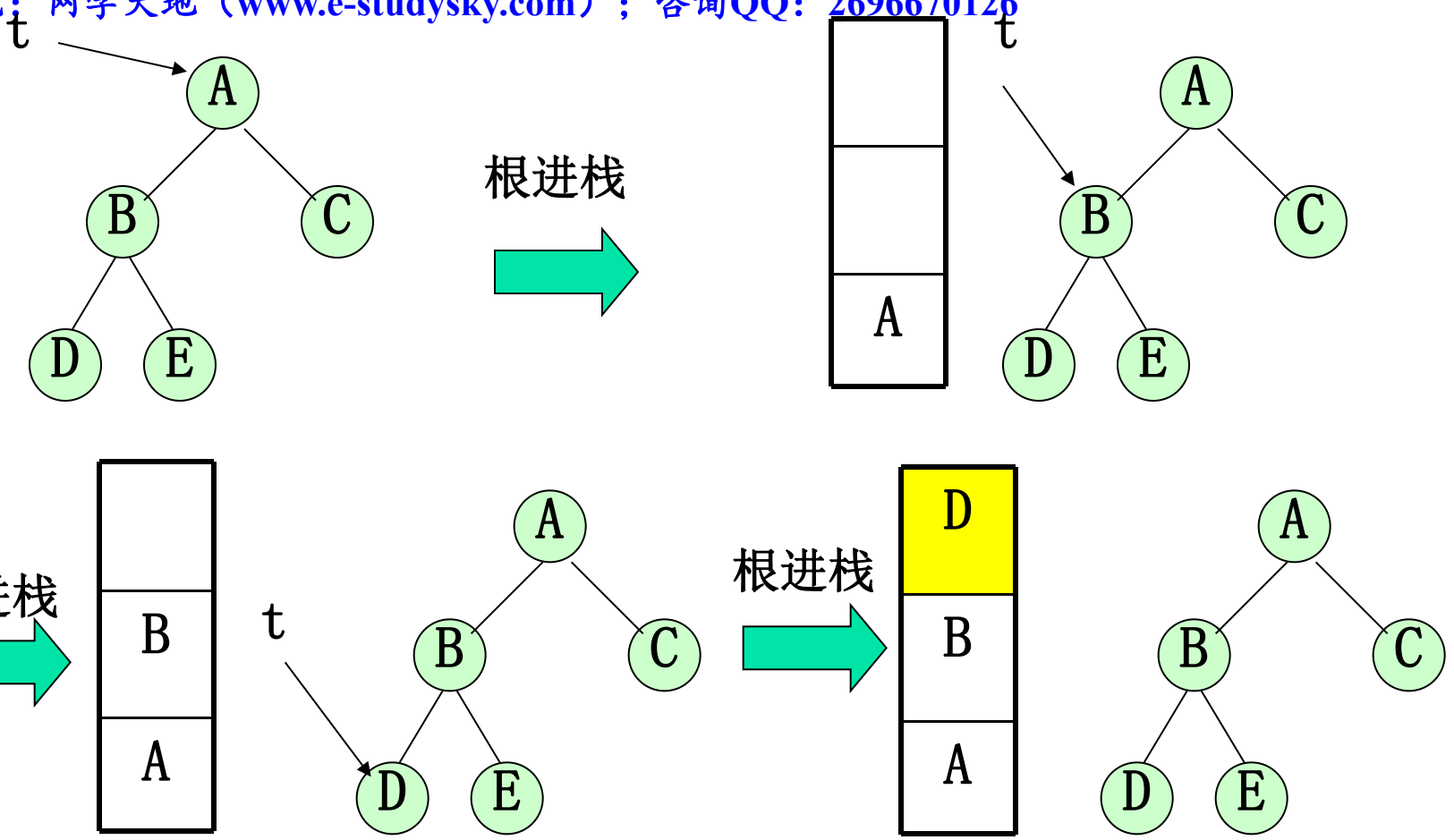
- 递归算法简明精炼，但效率较低，实际应用中常使用非递归；
- 某些高级语言不支持递归；
- 非递归算法思想：

（1）设置一个栈S存放所经过的根结点（指针）信息；初始化S；

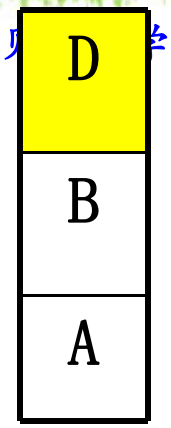
（2）第一次访问到根结点并不访问，而是入栈；

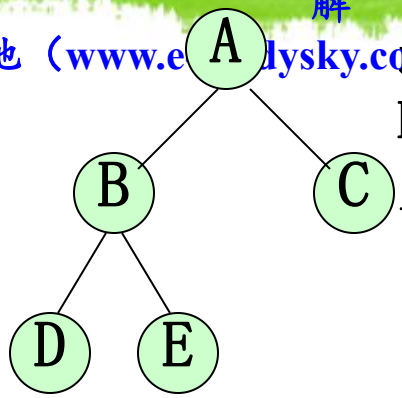
（3）中序遍历它的左子树，左子树遍历结束后，第二次遇到根结点，就将根结点（指针）退栈，并且访问根结点；然后中序遍历它的右子树。

（4） 当需要退栈时，如果栈为空则结束。



t=NULL，表示D的左子树遍历结束

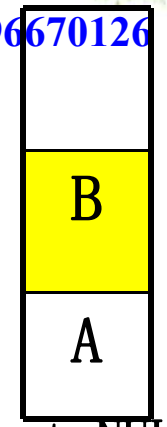


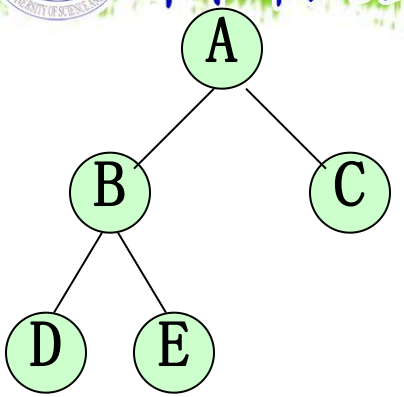


t=NULL,表示D的左子树遍历结束

退栈➔t，访问D，

t->rchild➔t

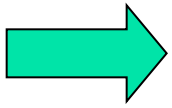


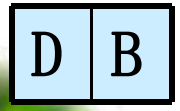


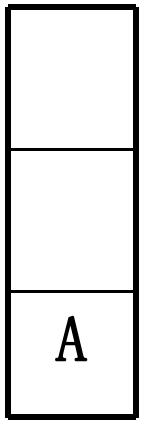
t=NULL,表示B的左子树遍历结束

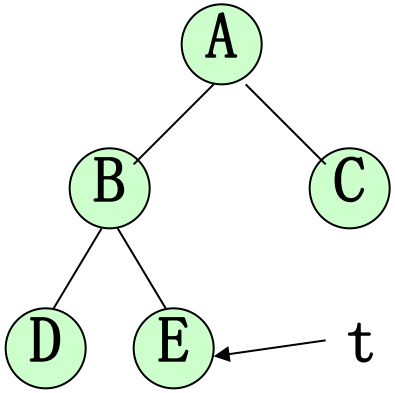
退栈➔t，访问B，

t->rchild➔t

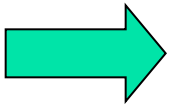


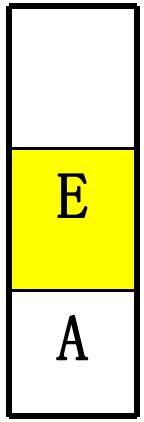


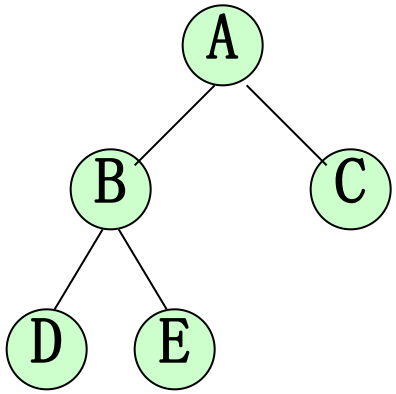




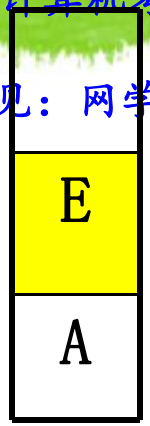
根进栈

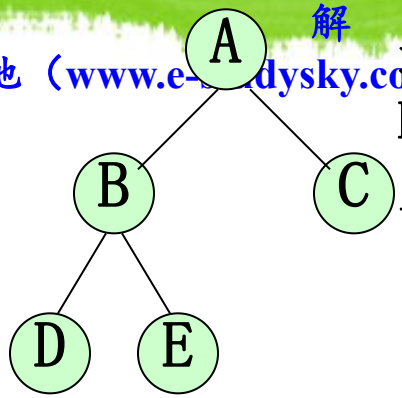






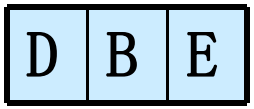
t=NULL,表示E的左子树遍历结束

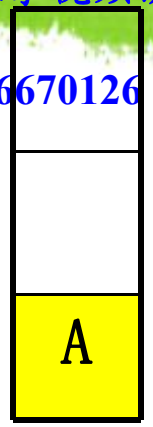


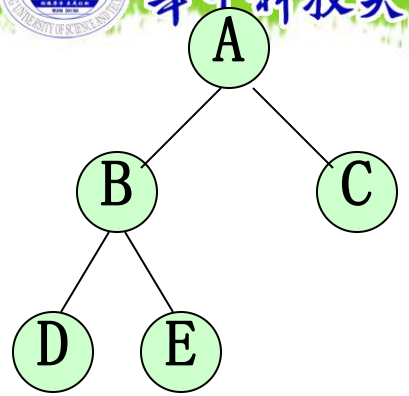


退栈➔t，访问E，

t->rchild➔t





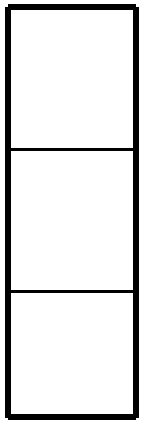


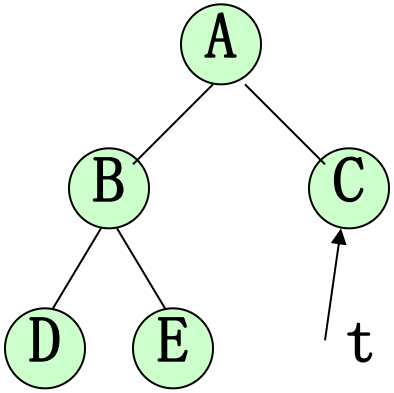
t=NULL,表示E的左子树遍历结束

t=NULL，　t此时为A的左子树最右结点的右孩子。表示A的左子树遍历结束

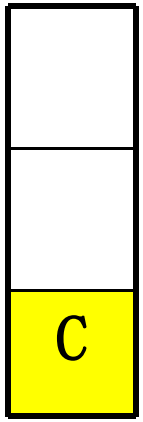
退栈➔t，访问A，

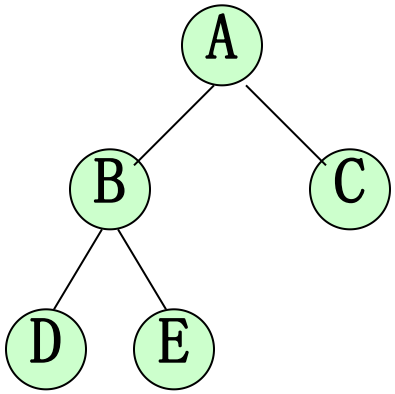
t->rchild➔t



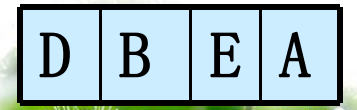


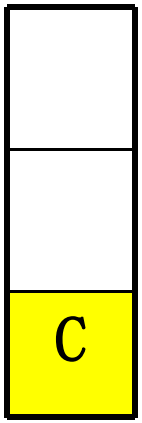
根进栈

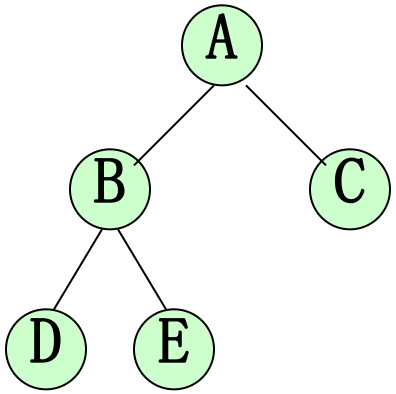




t=NULL,表示C的左子树遍历结束



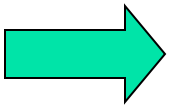


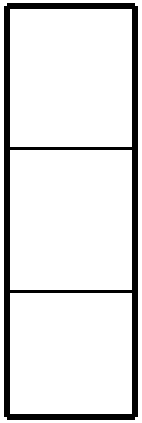


t=NULL，表示C的左子树遍历结束

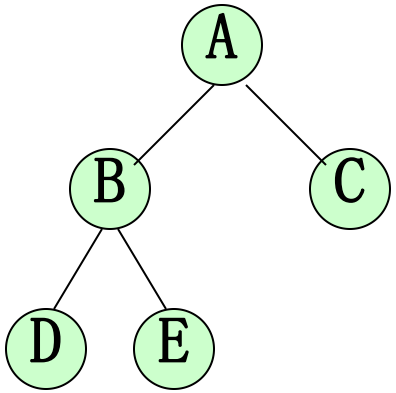
退栈➔t，访问C，

t->rchild➔t





D	B	E	A	C



t=NULL，并且栈S为空，遍历结束

中序遍历非递归算法：

```
void Midorder(struct BiTNode *t)
                //t为根指针
{
  struct BiTNode *st[maxleng];
                        //定义指针栈st[0..maxleng-1]
                        //用于保存访问过的结点，在后续
                        //操作中访问该结点的右孩子
  int top=0;            //置空栈
```

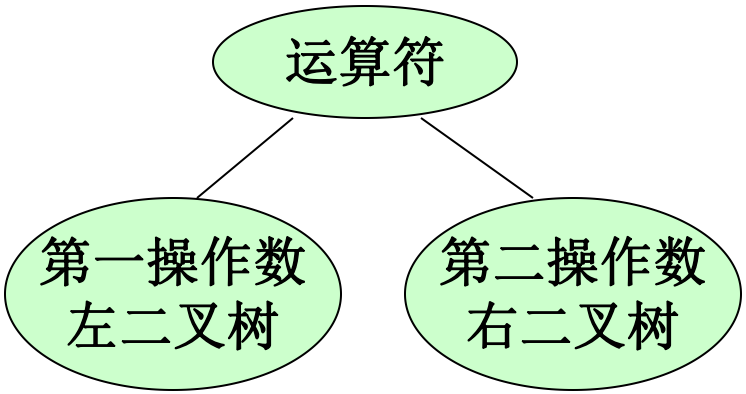
```
do
   { while(t)                          //根指针t表示的为非空二叉树
     { if (top==maxleng) exit(OVERFLOW); //栈已满,退出
       st[top++]=t;                     //根指针进栈
       t=t->lchild;                     //t移向左子树
     }                                  //循环结束表示以栈顶元素的指向为
                                        //根结点的二叉树的左子树遍历结束
    if (top)                            //为非空栈
    { t=st[--top];                      //弹出根指针
      printf("%c",t->data);             //访问根结点
      t=t->rchild;                      //遍历右子树
     }
   } while(top||t);  //父结点未访问，或右子树未遍历
}
```

6.3.2 表达式二叉树

表达式二叉树T =（第一操作数）（运算符）（第二操作数）

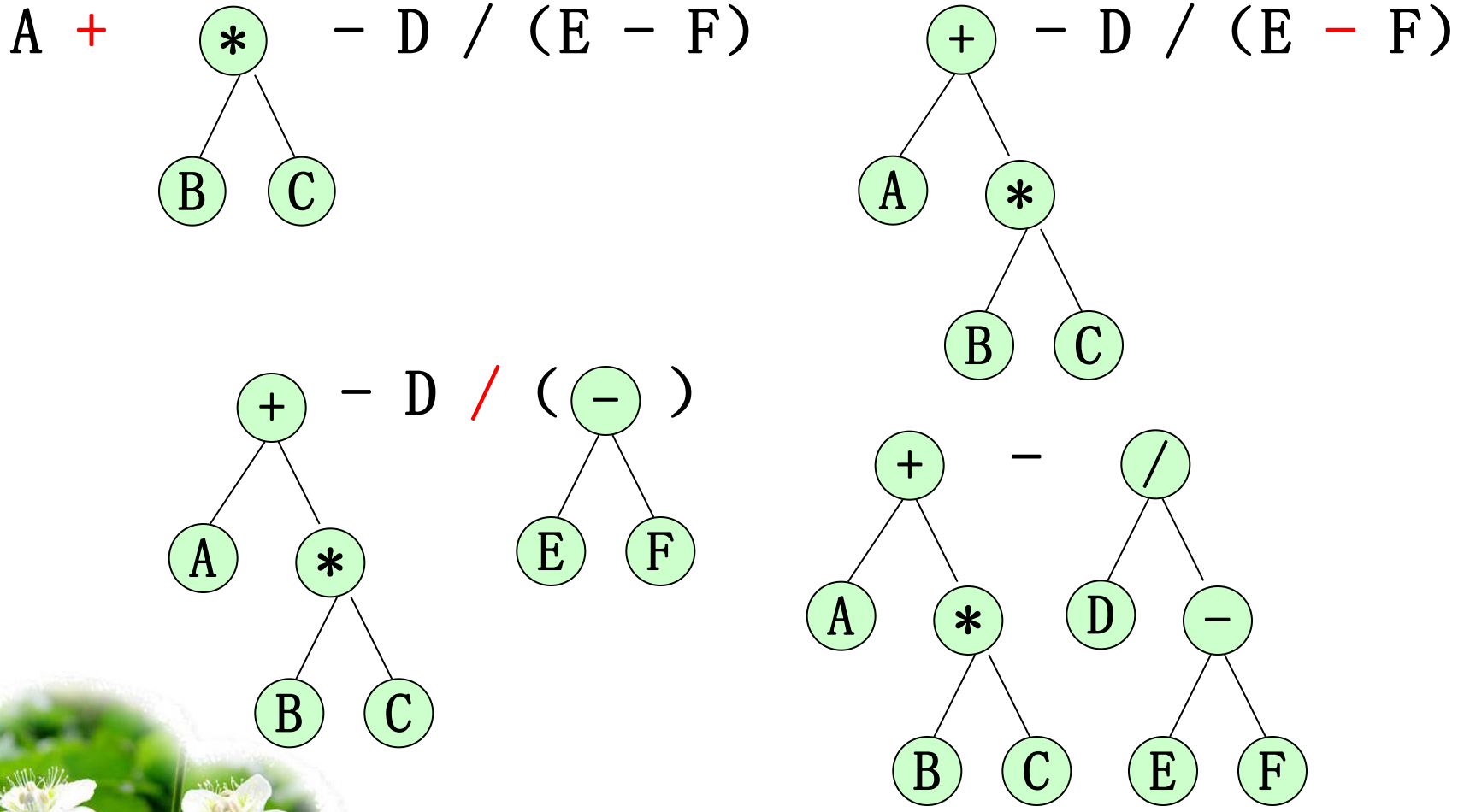
其中：第一操作数、第二操作数也是表达式二叉树，

分别为表达式二叉树T的左子树和左子树

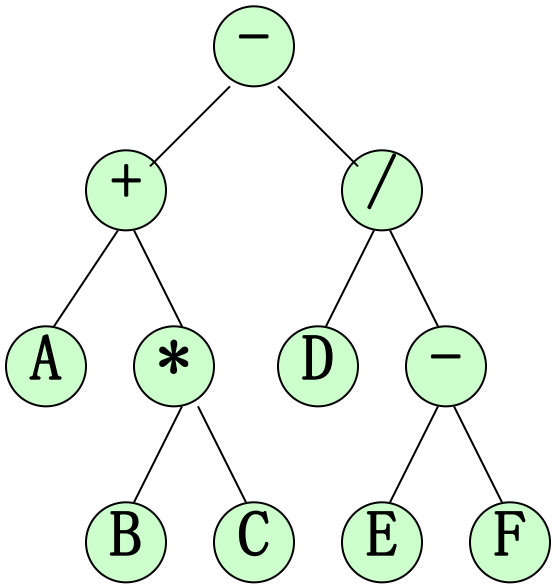


表达式二叉树

例 表达式：　A + B * C - D / (E - F)



例 表达式：A + B * C - D / (E - F)

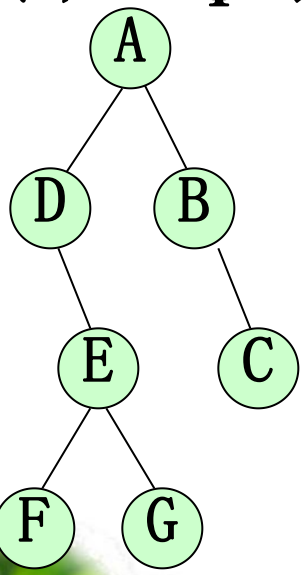


表达式二叉树

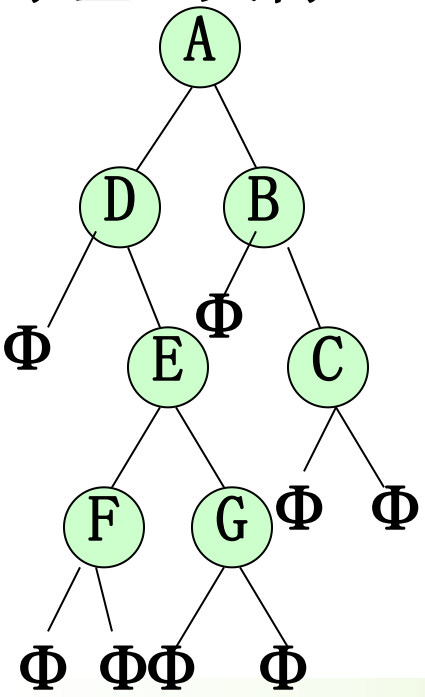
- 前序遍历：- + A * B C / D - E F
  前缀表示，前缀表达式，波兰式
- 中序遍历：A + B * C - D /(E - F)
  中缀表示，中缀表达式
- 后序遍历：A B C * + D E F - / -
  后缀表示，后缀表达式，逆波兰式

6.3.3 创建二叉树（1）

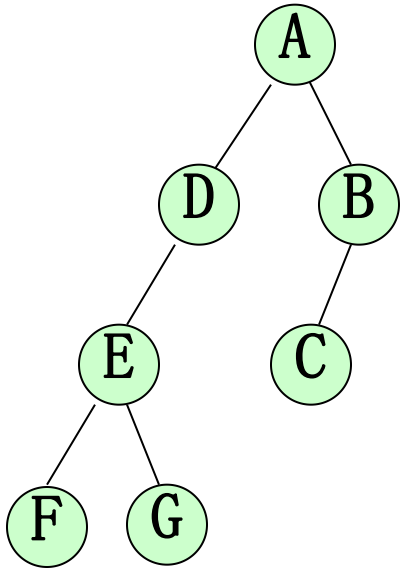
- 二叉树T1的先序序列：
  "ADEFGBC"
  带空二叉树的先序序列：
  "ADΦEFΦΦGΦΦBΦCΦΦ"
  其中：'Φ'表示空二叉树



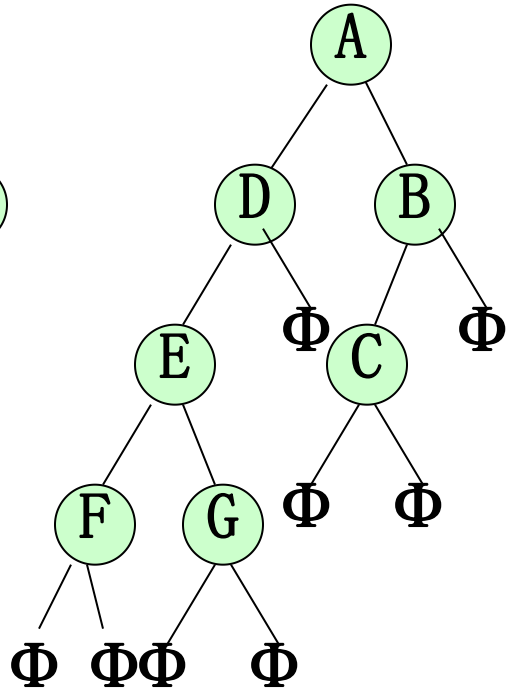
T1



- 二叉树T2的先序序列：
  "ADEFGBC"
  带空二叉树的先序序列：
  "ADEFΦΦGΦΦΦBCΦΦΦ"

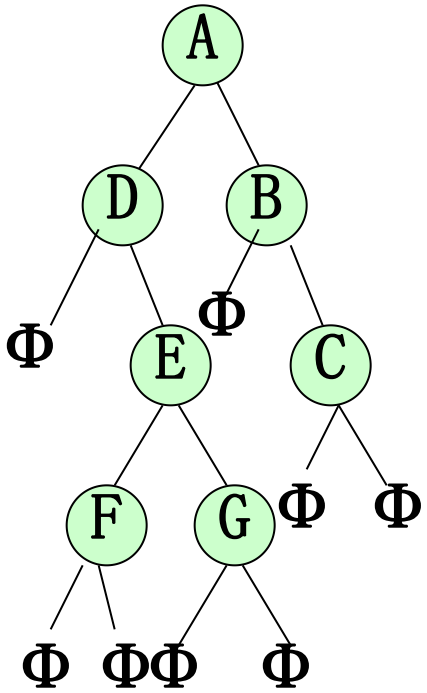


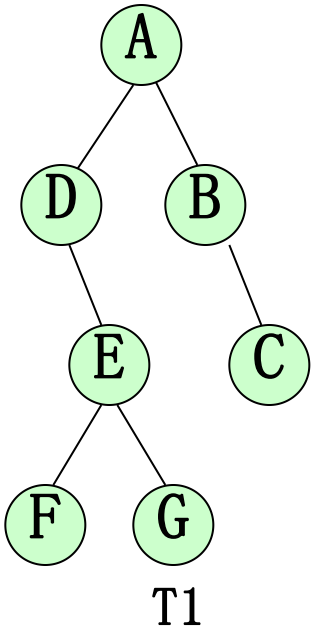
T2



建立方法：

A D Φ E F Φ ΦG Φ ΦB Φ C Φ Φ





A DΦEFΦΦGΦΦ BΦCΦΦ

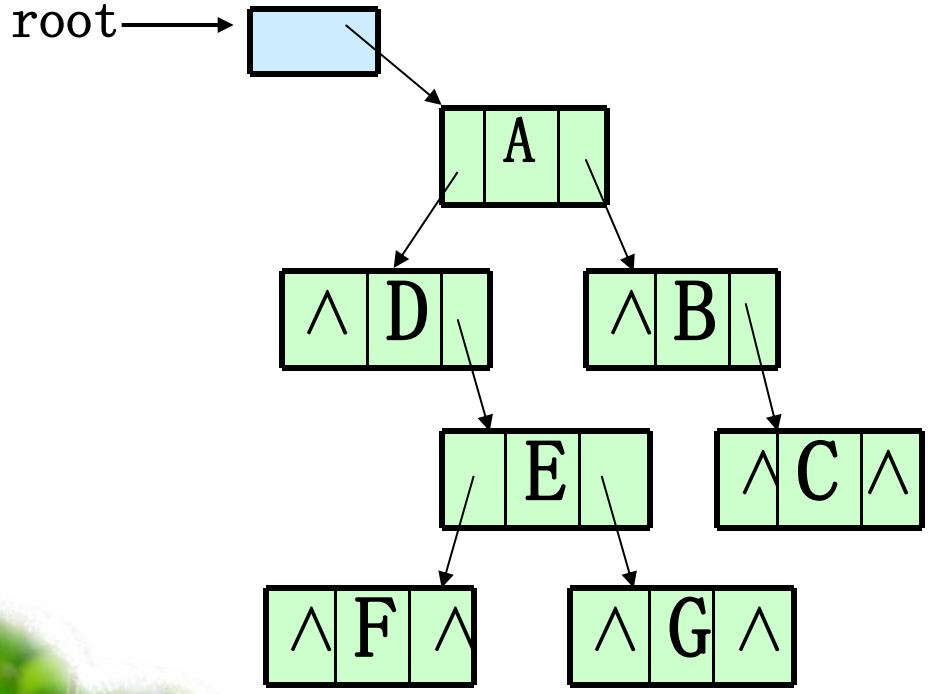
算法：创建二叉树

输入：带空结点的二叉树的先序序列

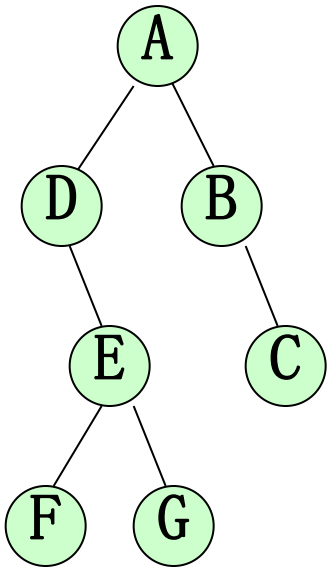
输出：二叉树的根指针

```
#define leng sizeof(BiTNode) //结点所占空间大小
```

假设输入先序序列：ADΦEFΦΦGΦΦBΦCΦΦ



二叉链表



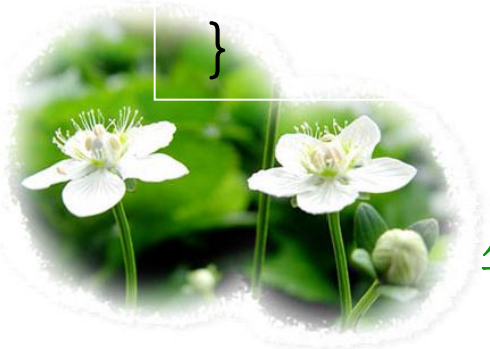
二叉树

实现算法1：

```
void CreatBiTree1(struct BiTNode **root)
//root是指向二叉链表根指针的指针
{ char ch;
  scanf(“%c”,&ch);    /输入一个结点，假定为字符型
  if (ch ==’Φ’) (*root)=NULL;
  else
  {
  (*root)=(struct BiTNode *)malloc(leng);
  (*root)->data=ch;                    //生成根结点
  CreatBiTree1(&(*root)->lchild); //递归构造左子树
  CreatBiTree1(&(*root)->rchild); //递归构造右子树
  }
}
```

实现算法2：

```
BiTree CreatBiTree2()
 { char ch;  BiTree root;  //二叉链表根结点指针
   scanf(“%c”,&ch);        //输入一个结点
   if (ch ==’ Φ ’)
     root=NULL;
   else {
     root=(BiTree) malloc(leng);  //生成根结点
     root->data=ch;
     root->lchild=CreatBiTree2(); //递归构造左子树
     root->rchild=CreatBiTree2(); //递归构造右子树
       }
  return root;
}
```



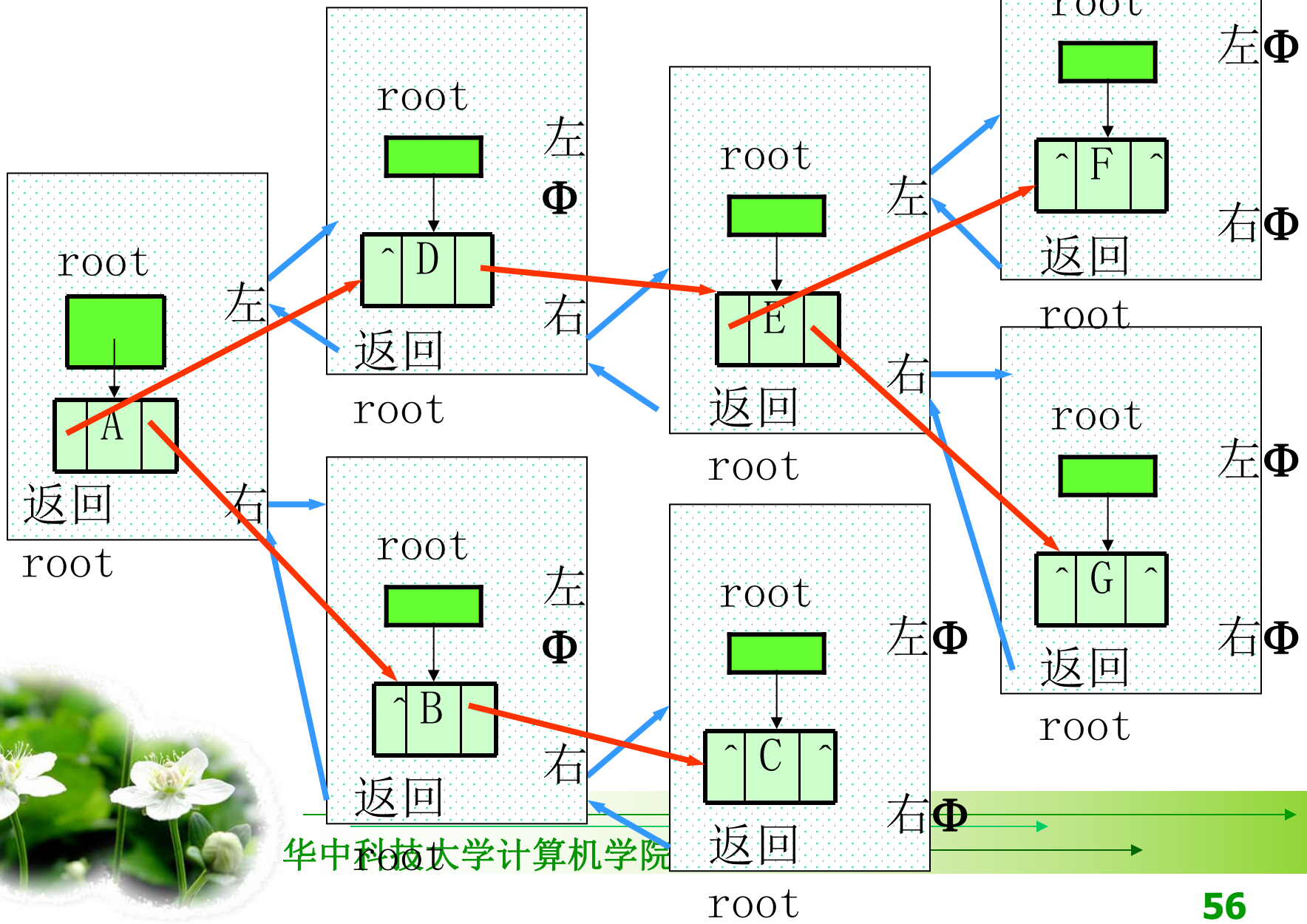
实现算法3:

```
void CreatBiTree3(BiTree  &root)
{ char ch;;
  scanf("%c",&ch);              //输入一个结点
  if (ch ==' Φ ')     root=NULL;
  else
  {
    root=(BiTree) malloc(leng);    //生成根结点
    root->data=ch;
    CreatBiTree3(root->lchild);//递归构造左子树
    CreatBiTree3(root->rchild); //递归构造右子树
  }
}
```

主函数(调用方法)：

```
main( )
{
   struct BiTNode *root1,*root2;
   BiTree          root3;
   CreatBiTree1(&root1);              /*算法1*/
   root2=CreatBiTree2();              /*算法2*/
   CreatBiTree3(root3);               /*算法3*/
}
```

先序序列: ADΦEFΦΦGΦΦBΦCΦΦ



6.3.3 创建二叉树（2）

算法4：用非递归算法创建二叉树
输入：　各结点的值及其在满二叉树中的编号
输出：　二叉树根指针

```
#define MAXSIZE 100
BiTree CreatTree()
   {
     BiTTree    s[MAXSIZE+1],root,q;
     int i,j;
     ElemType x;
     printf("i,x=");
     scanf("%d%d",&i,&x);
```

```
while(i!=0)
    { q=(BiTNode *) malloc(sizeof(BiTNode));
      q->data=x;q->lchild=q->rchild=NULL;
      s[i]=q;
      if (i==1) root=q;
      else  { j=i/2;
              if (i%2)    s[j]->rchild=q;
              else        s[j]->lchild=q; }
      printf("i,x=");  scanf("%d%d",&i,&x);
    }
  return root;
}
```

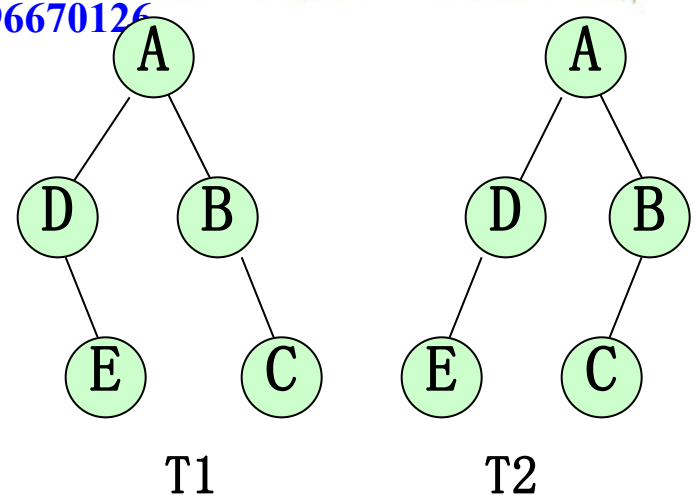
6.3.3 创建二叉树（3）

根据中序遍历序列和前(或后)序序列确定唯一棵二叉树

由前序序列：A D E B C

和(或)后序序列：E D C B A

不能确定唯一棵二叉树

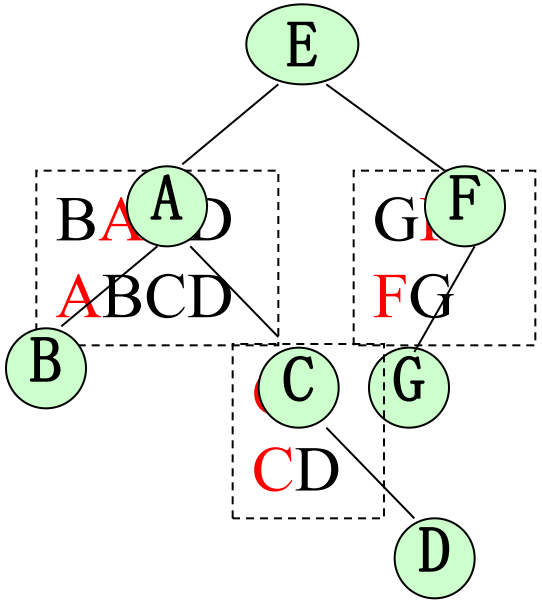


例1. 给定二叉树T的

中序序列：B A C D E G F

前序序列：E A B C D F G

如何求二叉树T?

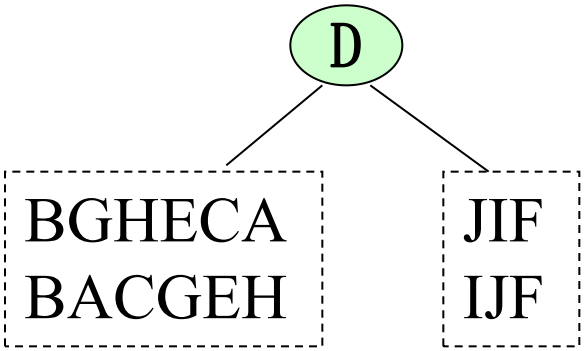


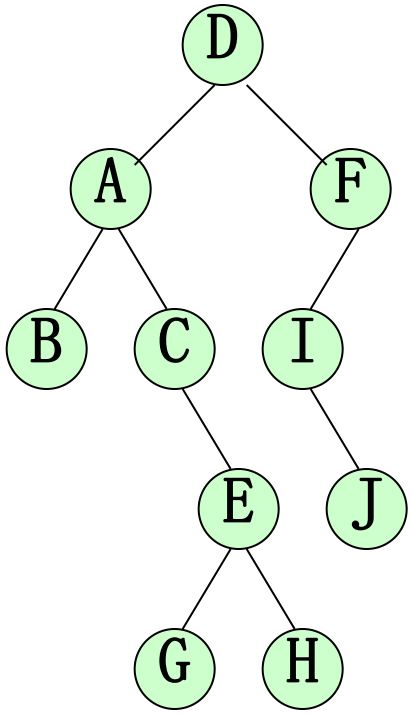
例2（练习）．给定二叉树T的

后序序列：B G H E C A J I F D

中序序列：B A C G E H D I J F

如何求二叉树T？





二叉树T

6.3.4 线索二叉树

- 遍历二叉树是按某种规则将非线性结构的二叉树结点线性化。
- 二叉树结点中没有相应前驱和后继的信息。每次遍历时需按规则动态产生。
- n各节点的二叉树，有：

  n*2 个指针域
- 使用： n-1 个指针，除根以外，每个结点被一个指针指向空指针域，空指针域数： n*2-(n-1)=n+1

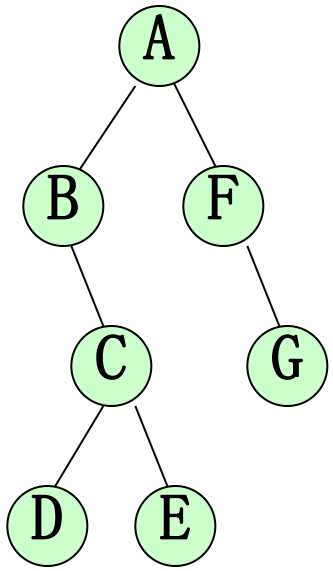
- 当对某二叉树经常按某种规则遍历访问时，可利用空指针域。　将空的左指针域指向直接前驱，空的右指针域指向直接后继，非空指针不需要改变。称该处理过程称为二叉树线索化。由此可分别得到：

- 前序线索二叉树，中序线索二叉树，　后序线索二叉树

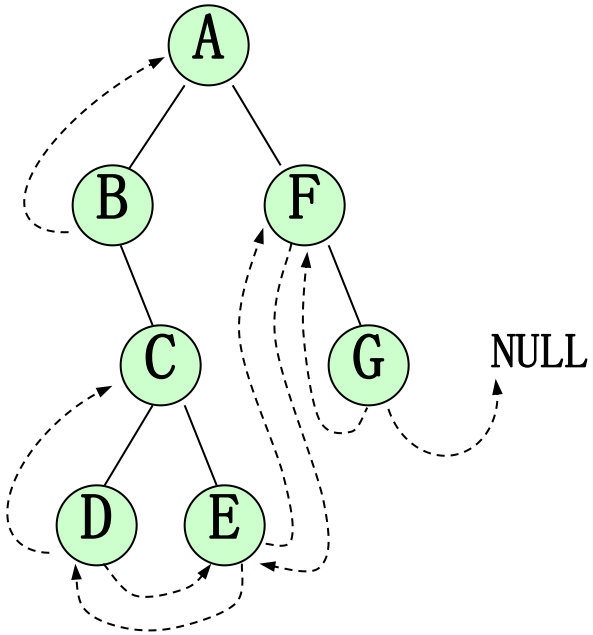
1. 前序线索二叉树：

线索指向前序遍历中前趋、后继的线索二叉树。

例. T的前序序列：A, B, C. D, E, F, G



二叉树T

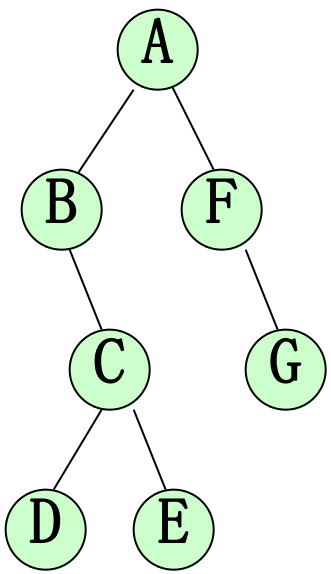


T的前序线索二叉树

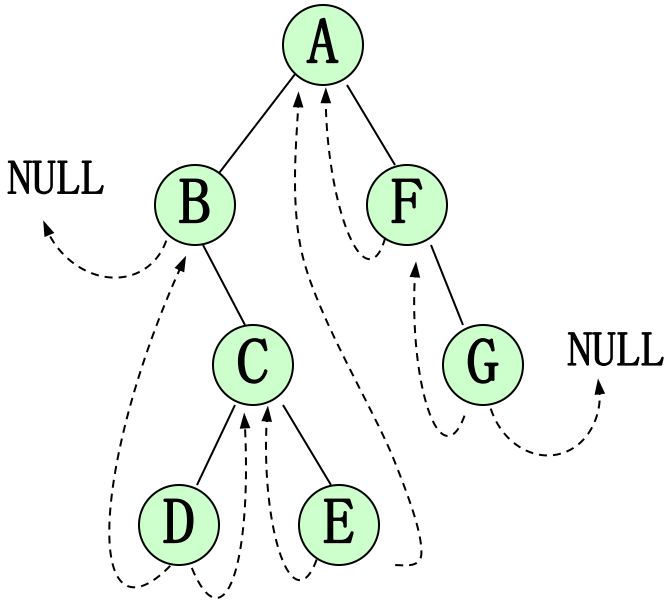
2. 中序线索二叉树：

线索指向中序遍历中前趋、后继的线索二叉树。

例. T的中序序列：B, D, C, E, A, F, G



二叉树T

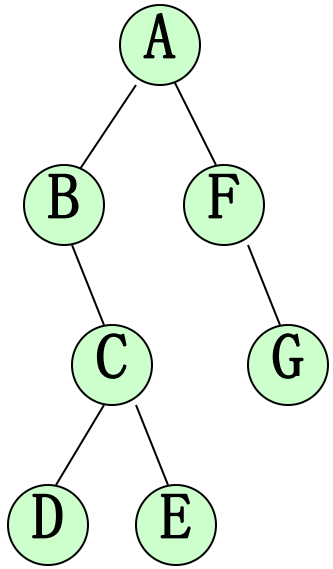


T的中序线索二叉树

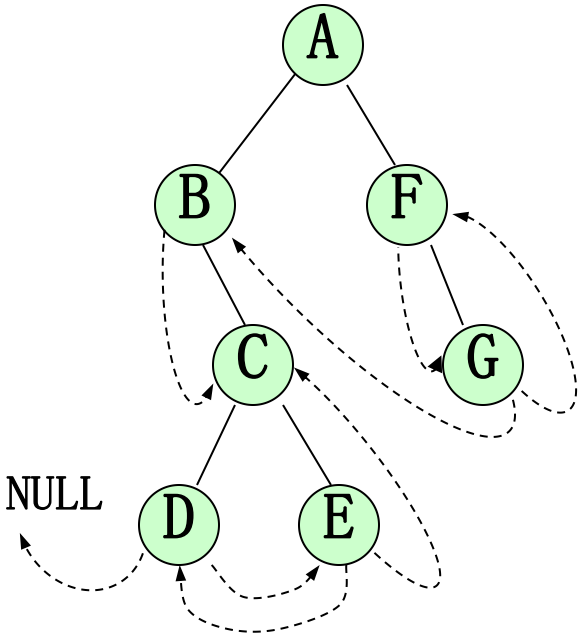
3. 后序线索二叉树：

线索指向后序遍历中前趋、后继的线索二叉树。

例. T的后序序列：D, E, C, B, G, F, A



二叉树T

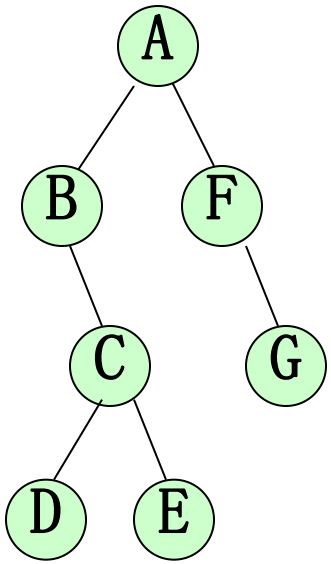


T的后序线索二叉树

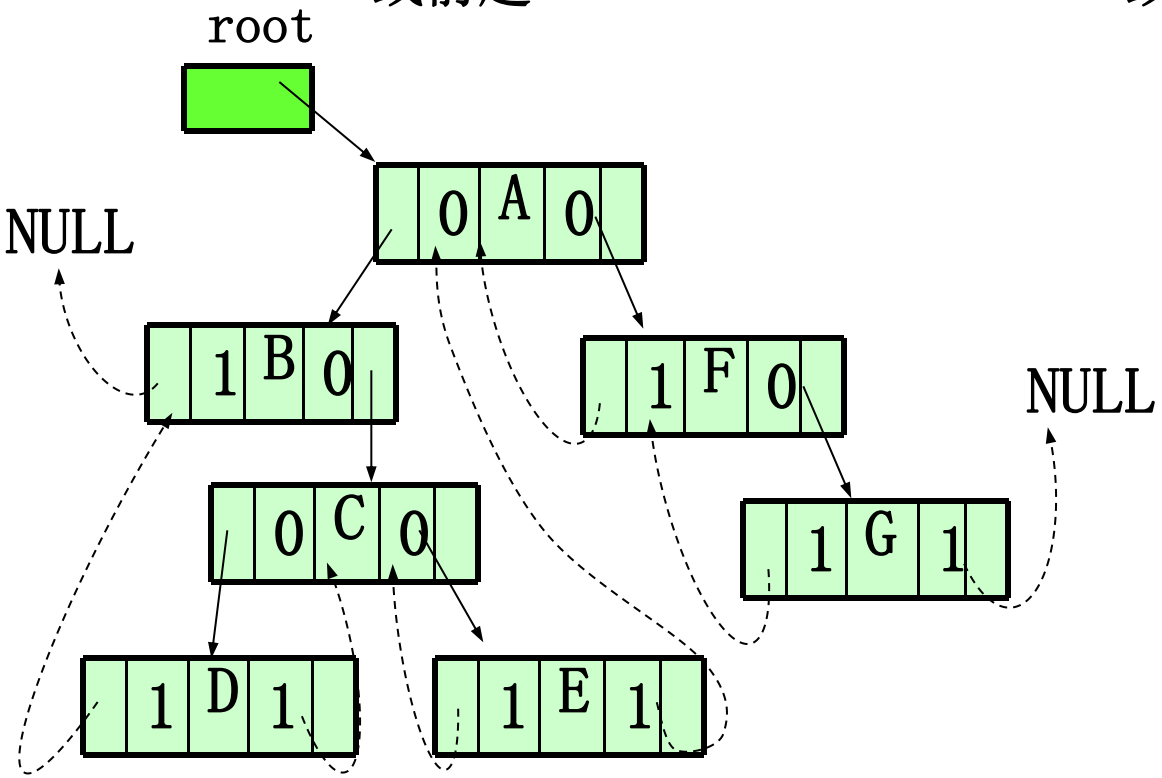
4. 线索二叉树的存储结构

(1). 结点结构：

lchild	ltag	data	rtag	rchild
	0/1		0/1	
左小孩或前趋	左标志	结点值	右标志	右小孩或后继

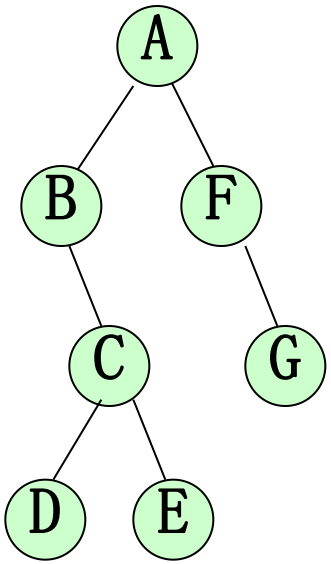


二叉树

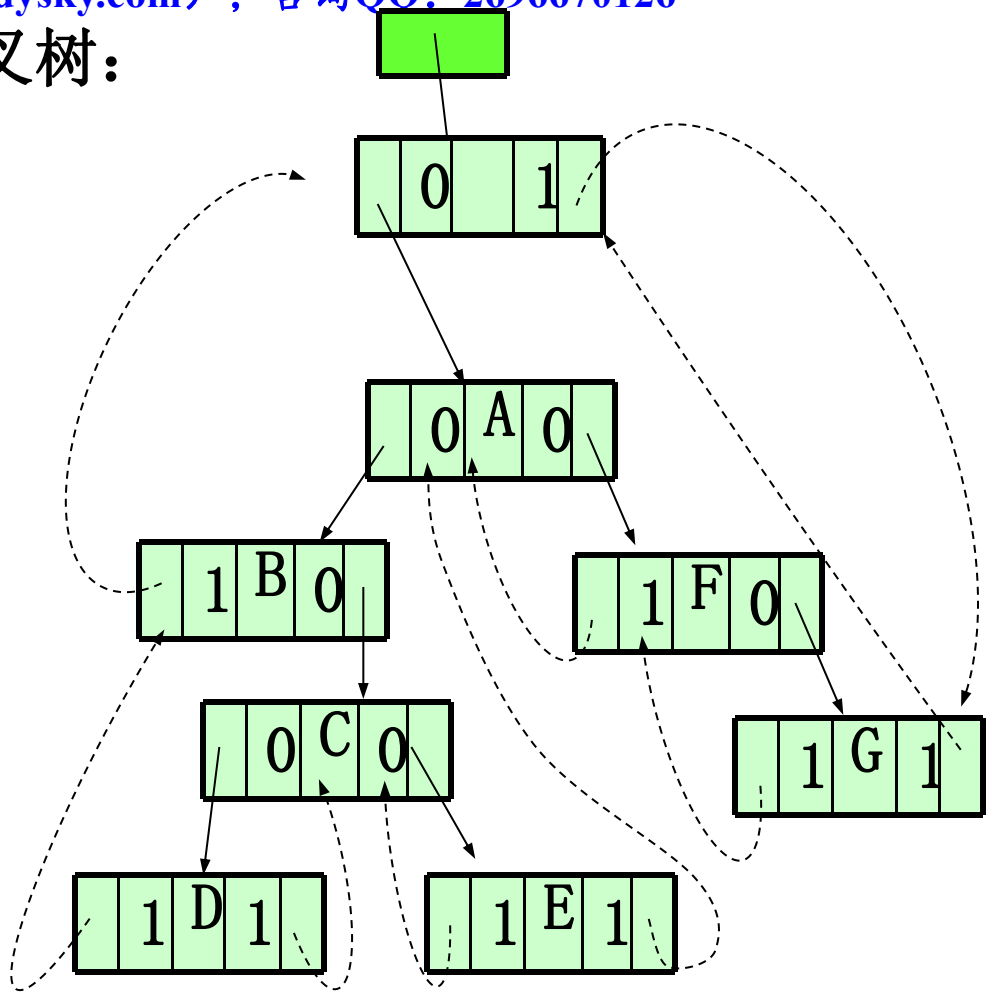


中序线索二叉树链表

带头结点的线索二叉树：



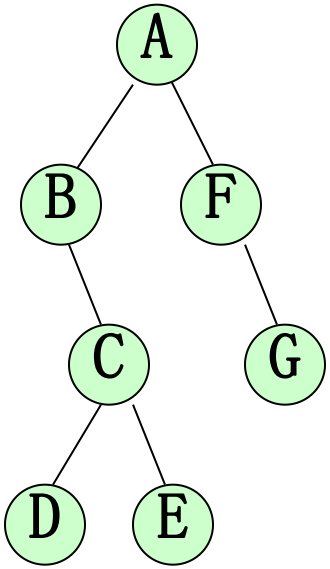
二叉树



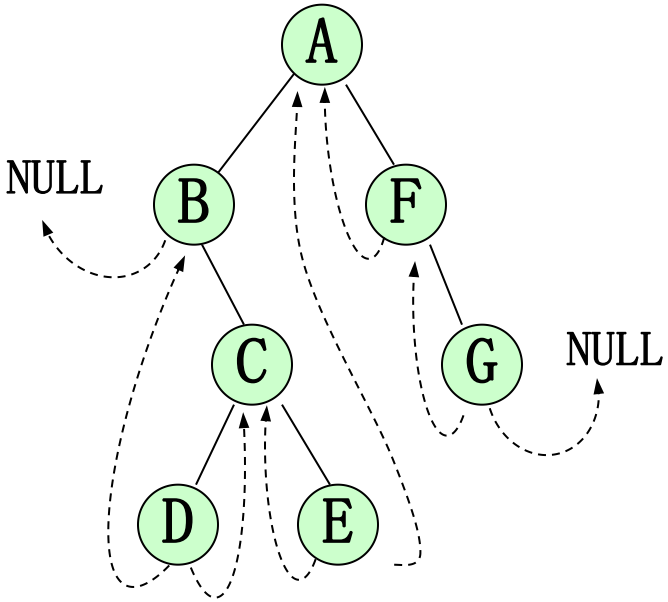
中序线索二叉树链表

(2) 将二叉数线索化（中序）：

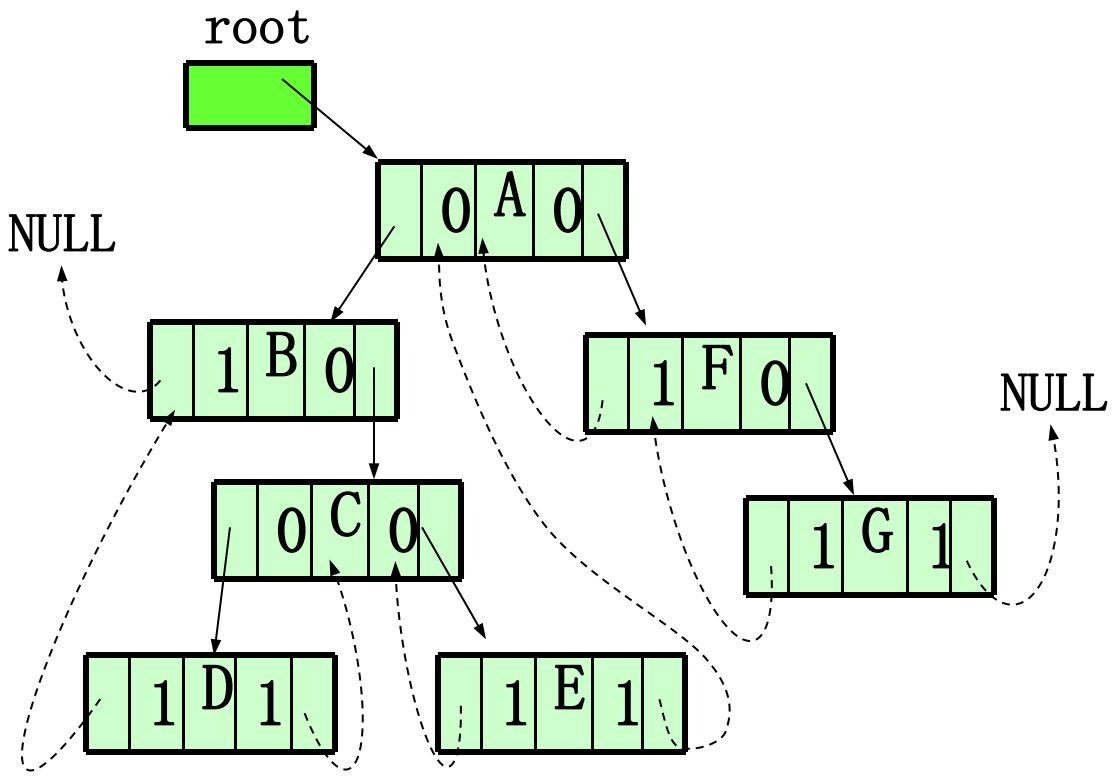
T的中序序列：B, D, C, E, A, F, G



二叉树T



T的中序线索二叉树



中序线索二叉树链表

```
void creat_thread(struct BiTNode *t)
{
  struct BiTNode *st[maxleng+1]; //指针栈
  int top=0;                     //置空栈
  struct BiTNode *pre=NULL;      //前驱结点指针
  do{
    while(t)          //根指针t表示的为非空二叉树
      {
      if (top==maxleng)
            exit(OVERFLOW);      //栈已满,退出
      st[top++]=t;               //根指针进栈
      t=t->lchild;               //t移向左子树
      }
    if (top)                     //为非空栈
     {
      t=st[--top];               //弹出根指针
      printf("%c",t->data);      //访问根结点
      if (t->lchild!=NULL)       //右指针为孩子
```

5. 线索二叉树的遍历

(1). 先序线索二叉树的遍历：

```
void PreOrder(struct BiTNode *t)        //t为根指针
{ p=t;
  while (p)
   {  printf(“%6c”, p->data);
      if (p->ltag==0) //有左孩子时， p移向左孩子结点
          p=p->lchild;
      else                  //p移向右孩子或右线索指向的结点
          p=p->rchild;
   }
}
```

(2) 中序线索二叉树的遍历：

```
void MidOrder(struct BiTNode *t)        //t为根指针
{ p=t;
  if (p!=NULL) while (p->ltag!=1)
       p=p->lchild;      //查找二叉树的最左结点(第1个结点)
  printf("%6c", p->data);
  while (p->rchild!=NULL)                  // p有后继结点
     {if (p->rtag==1)  p=p->rchild; //p无右孩子时为线索
      else{p=p->rchild;  //p有右孩子时，取右子树最左结点
           while (p->ltag!=1) p=p->lchild;}
      printf("%6c", p->data);
      }
}
```

6.3.5 遍历二叉树的应用

例. 求二叉树中结点的个数

```
int preorder(struct BiTNode *root) //求二叉树中结点的个数
 { int n=0 ;
   if (root)
   { n=1;                                  //根结点计数
    n+=preorder(root->lchild);             //递归计算左子树
    n+=preorder(root->rchild);             //递归计算右子树
    }
  return n; }
main()
{ int n;                    //n为计数器,假定二叉树已生成
  n=preorder(root); //root为指针,执行preorder(root,&n)
          printf("%d",n);          //输出n
}
```

例. 线索化二叉树

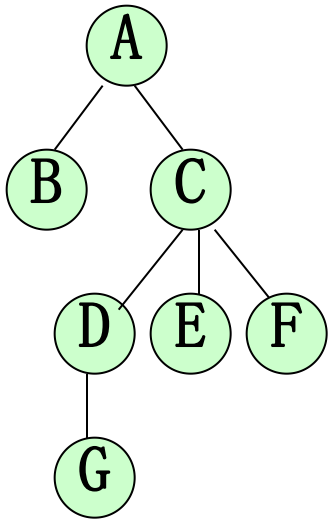
```
void pre(struct BiTNode *T, struct BiTNode &*pre)
 {if(root)
   { l= T->lchild,r= T->rchild
     if (T->lchild!=NULL)          //处理当前结点T
        T->ltag=0;
     else
        {T->ltag=1;  T->lchild=pre;}
    if (pre!=NULL)                 //处理pre
        if (pre->rchild!=NULL)
            pre->rtag=0;
        else
            {pre->rchild=T; pre->rtag=1;}
    pre=T;
    pre(l,pre);
    pre(r,pre);
    }
 }
```

6.4 树和森林

6.4.1 树的存储结构

1. 双亲表示法/数组表示法/顺序表示法

```
struct snode
{ char data;
  int parent;
}t[maxleng+1};
```



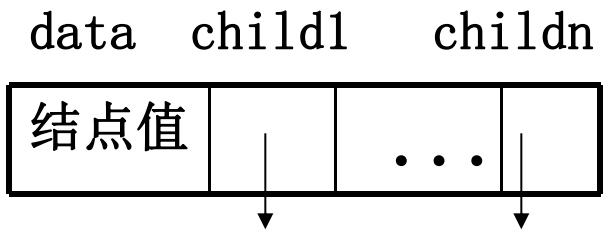
树

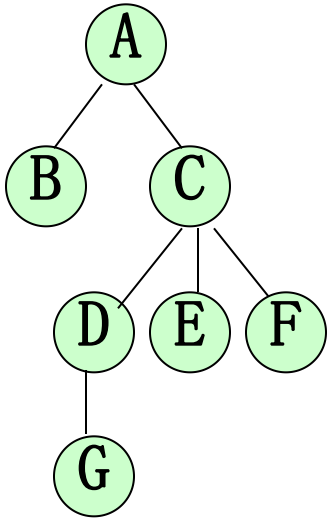
	data	parent
1	A	0
2	B	1
3	C	1
4	D	3
5	E	3
6	F	3
7	G	4

t[1..7]

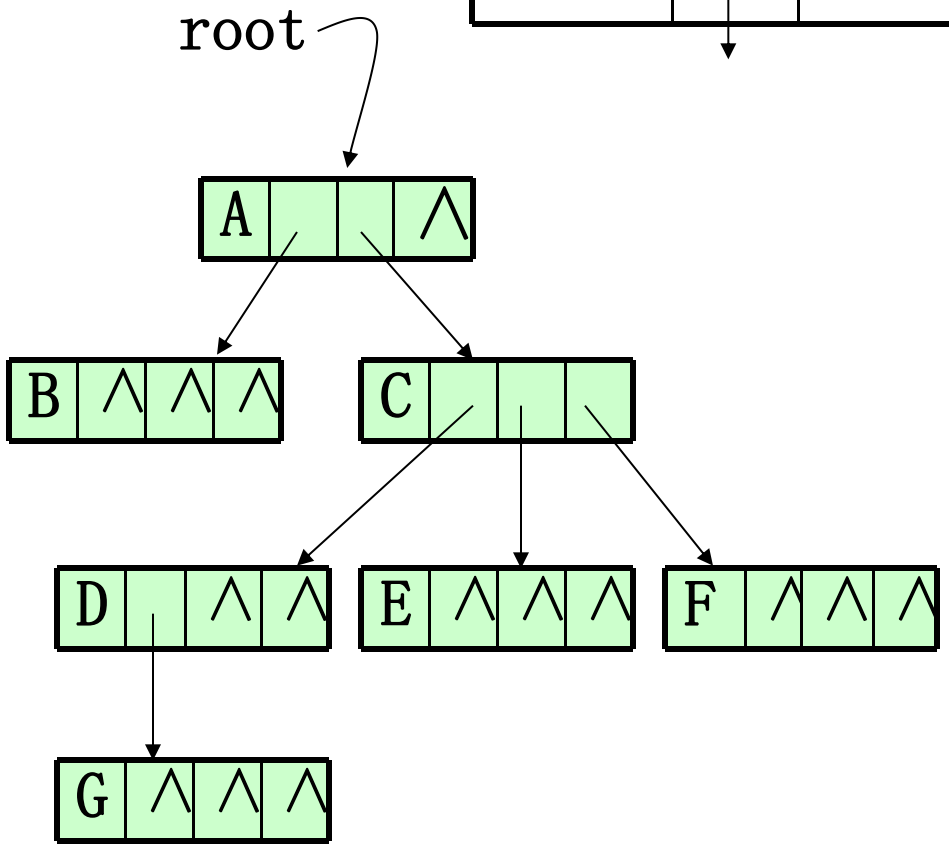
2. 孩子表示法/链接表表示法

(1)固定大小的结点格式，设树T的度为n





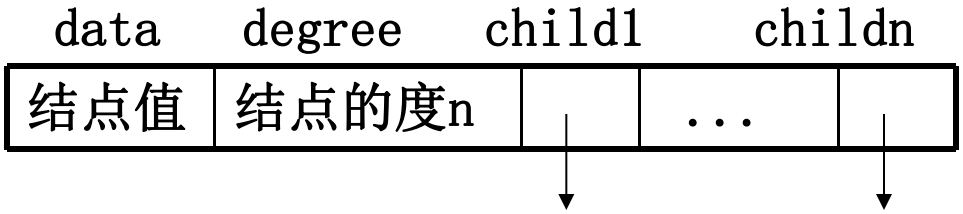
树T

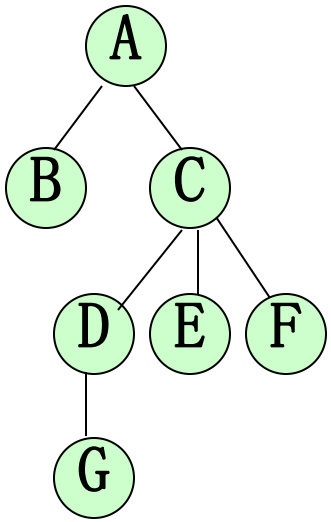


T的链接表

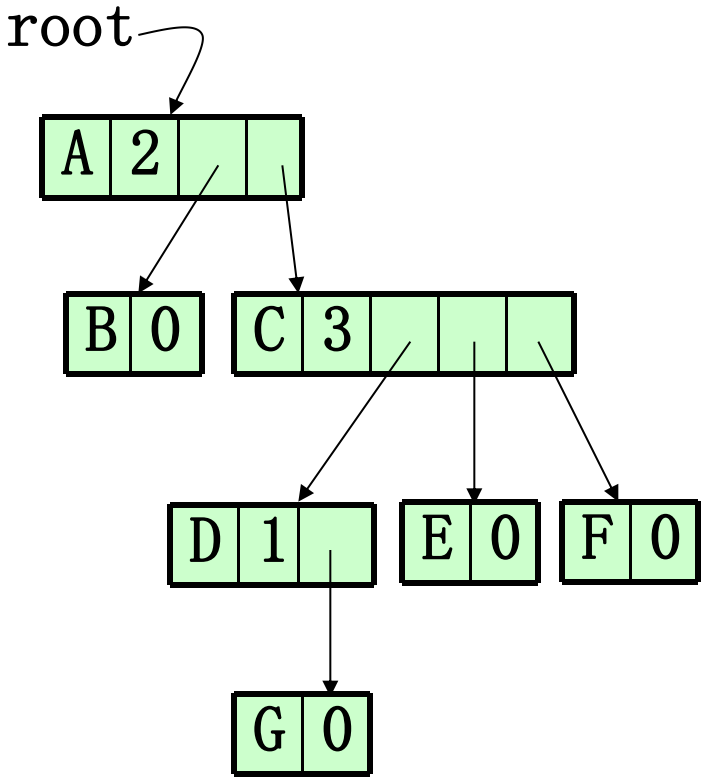
2.孩子表示法/链接表表示法

(2)非固定大小的结点格式





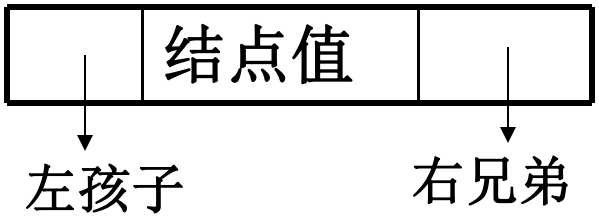
树

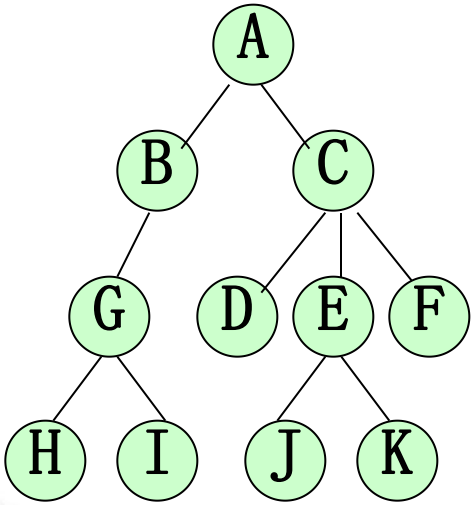


链接表

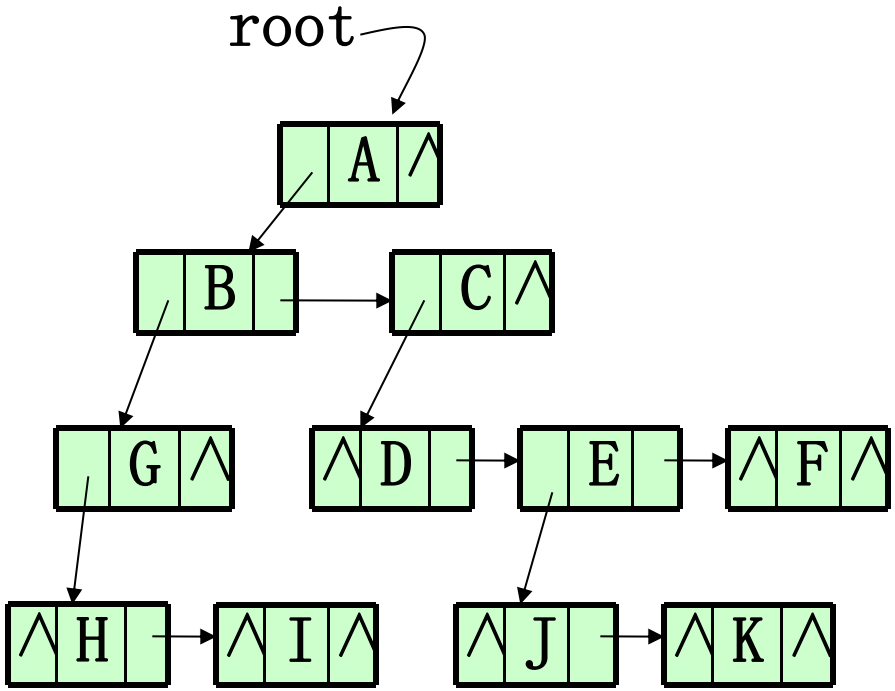
3. 孩子兄弟表示法/二叉树表示法/二叉链表

firstchild data nextbrother





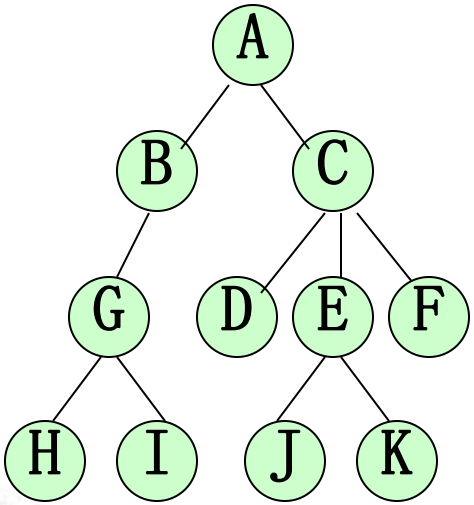
树



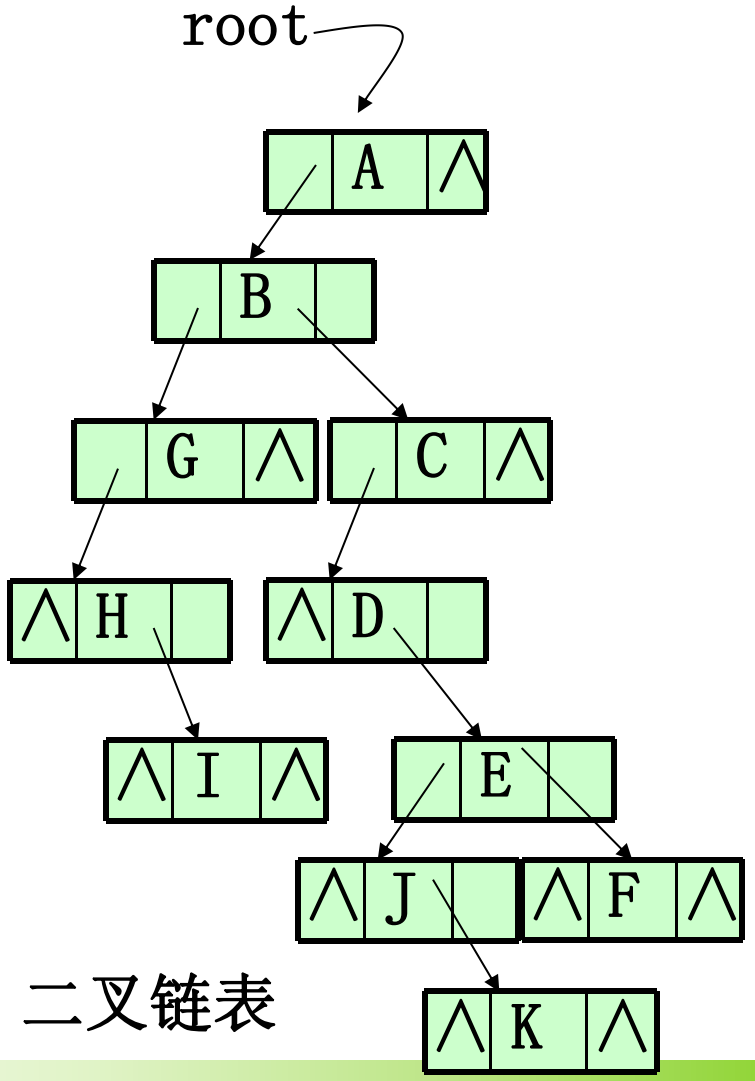
二叉链表

3. 孩子兄弟表示法/二叉树表示法/二叉链表

firstchild data nextbrother



树



二叉链表

4. 孩子链表表示法/单链表表示法

data first

结点值	

→ 第一个孩子

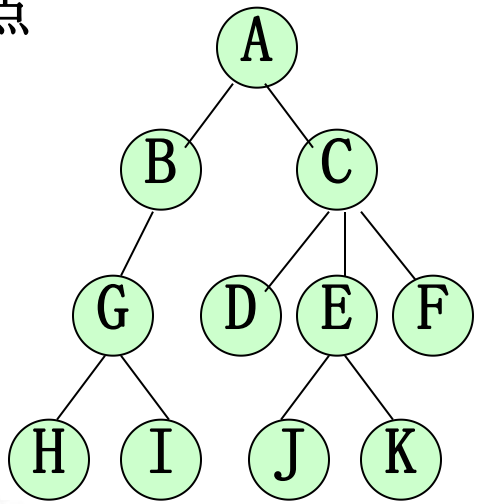
表头结点

child next

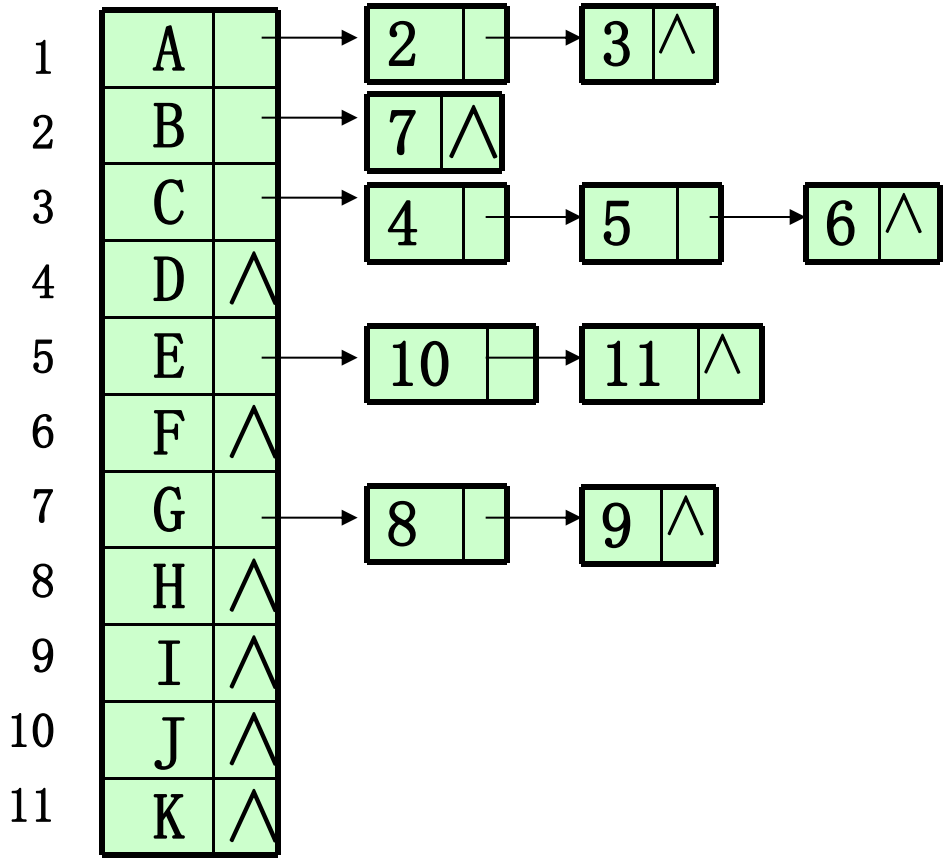
序号值	

→ 下一个孩子

孩子结点/表结点



树



表头结点数组

5. 带双亲的孩子链表表示法

data parent first

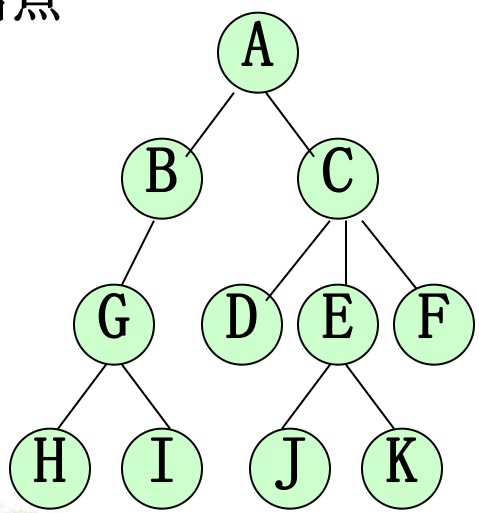
结点值		→ 第一个孩子

表头结点

child next

序号值	→ 下一个孩子

孩子结点/表结点



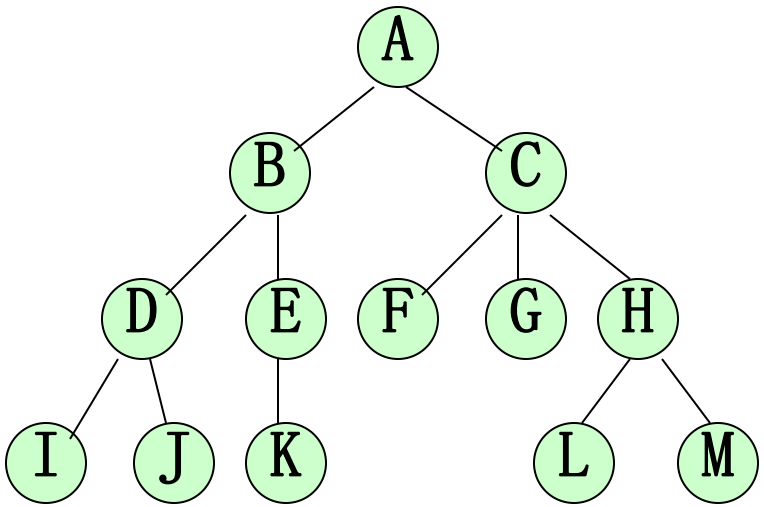
树

	data	parent	first	孩子链表
1	A	0	→	2 → 3 ∧
2	B	1	→	7 ∧
3	C	1	→	4 → 5 → 6 ∧
4	D	3	∧	
5	E	3	→	10 → 11 ∧
6	F	3	∧	
7	G	2	→	8 → 9 ∧
8	H	7	∧	
9	I	7	∧	
10	J	5	∧	
11	K	5	∧	

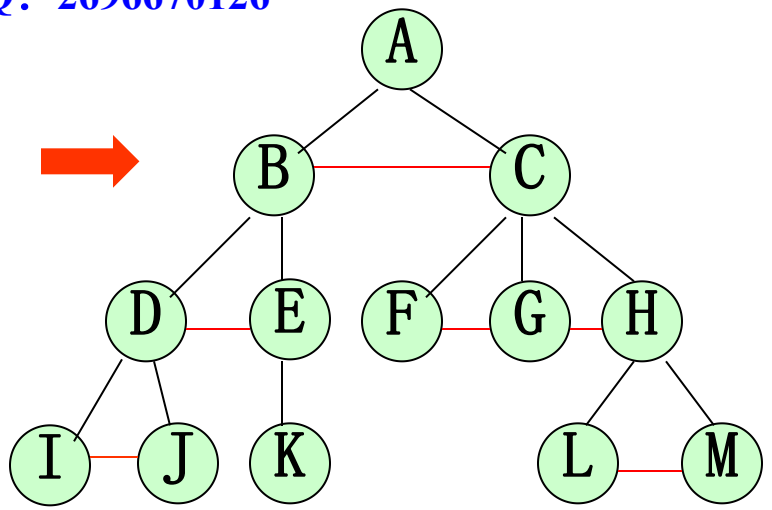
表头结点数组

6.4.2 树与二叉树的转换

1. 树 ➡ 二叉树

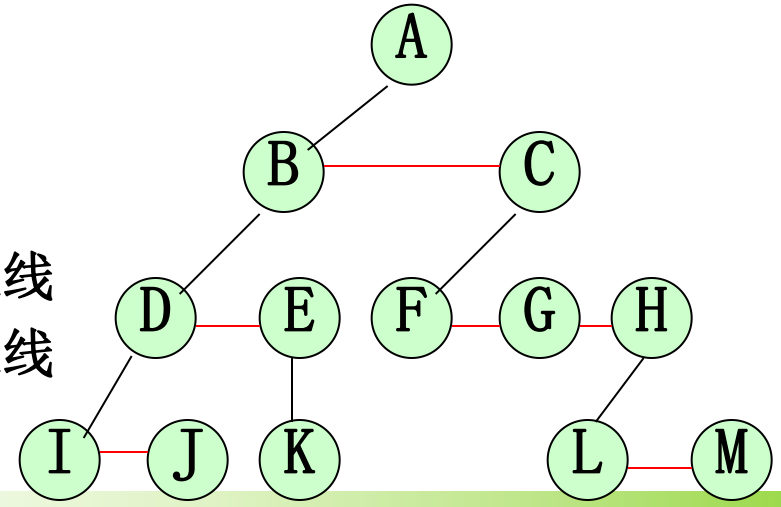


树

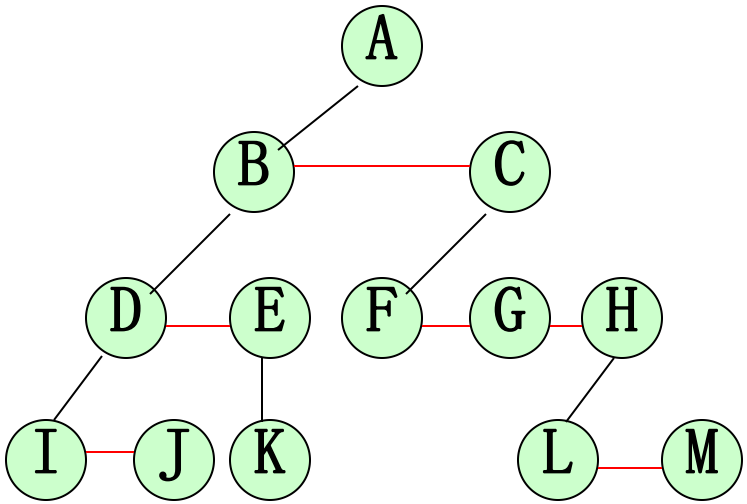


1. 在兄弟之间加连线

2. 保留根与最左孩之间的连线
   删除与其它孩子之间的连线

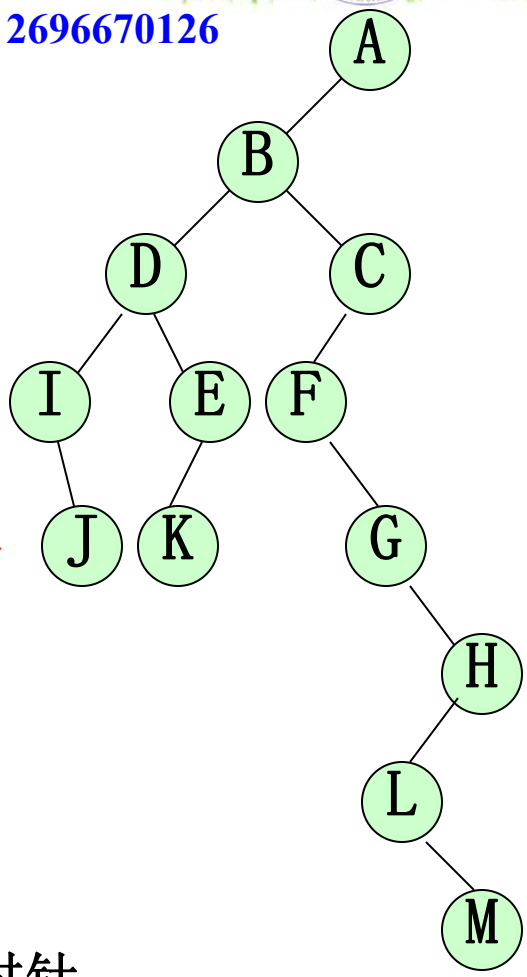


1. 树 ➡ 二叉树



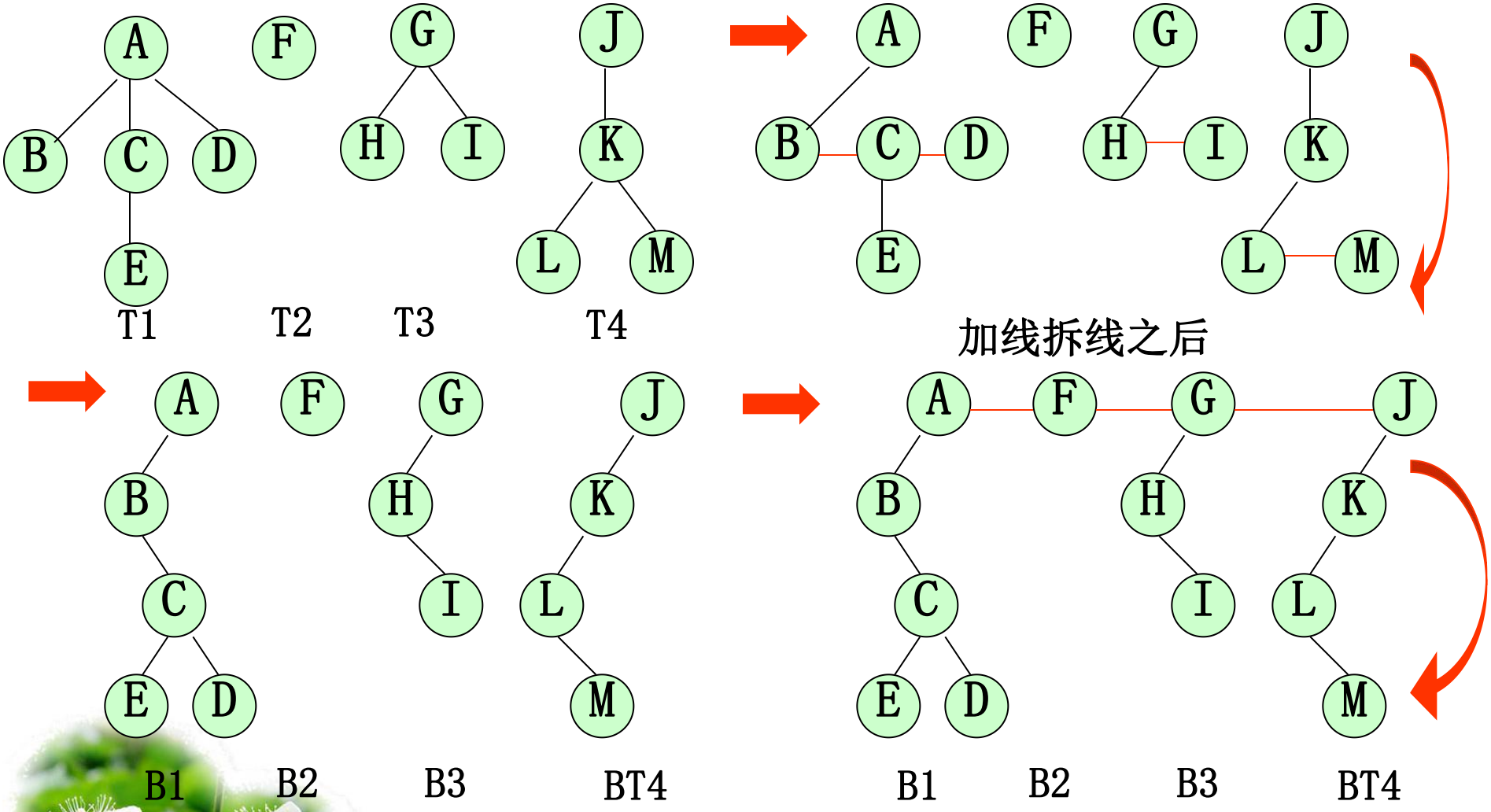
2. 保留根与最左孩之间的连线
   删除与其它孩子之间的连线

3. 以根为轴心顺时针
   方向旋转45度

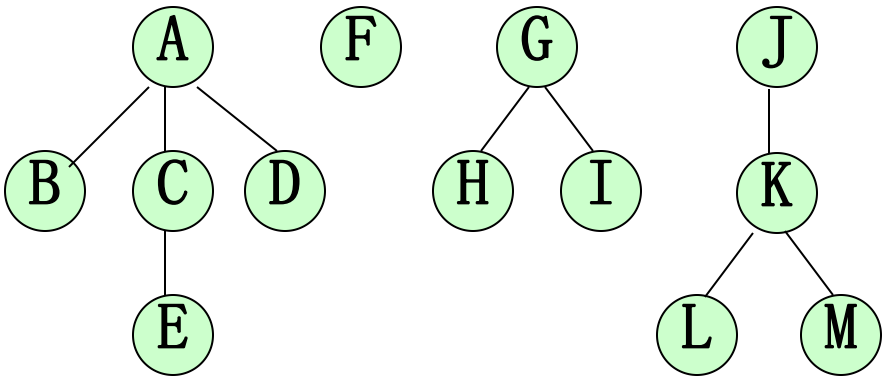


二叉树

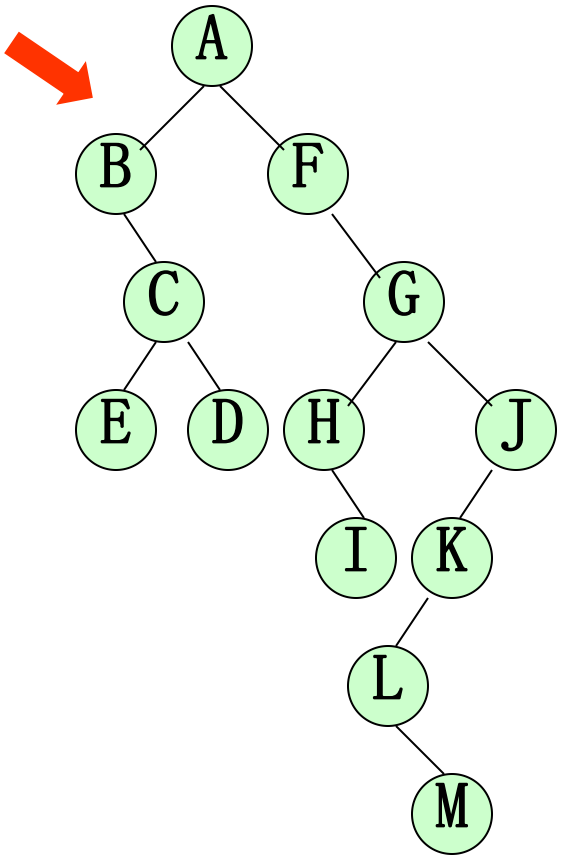
2.森林 → 二叉树



2. 森林 ➡ 二叉树



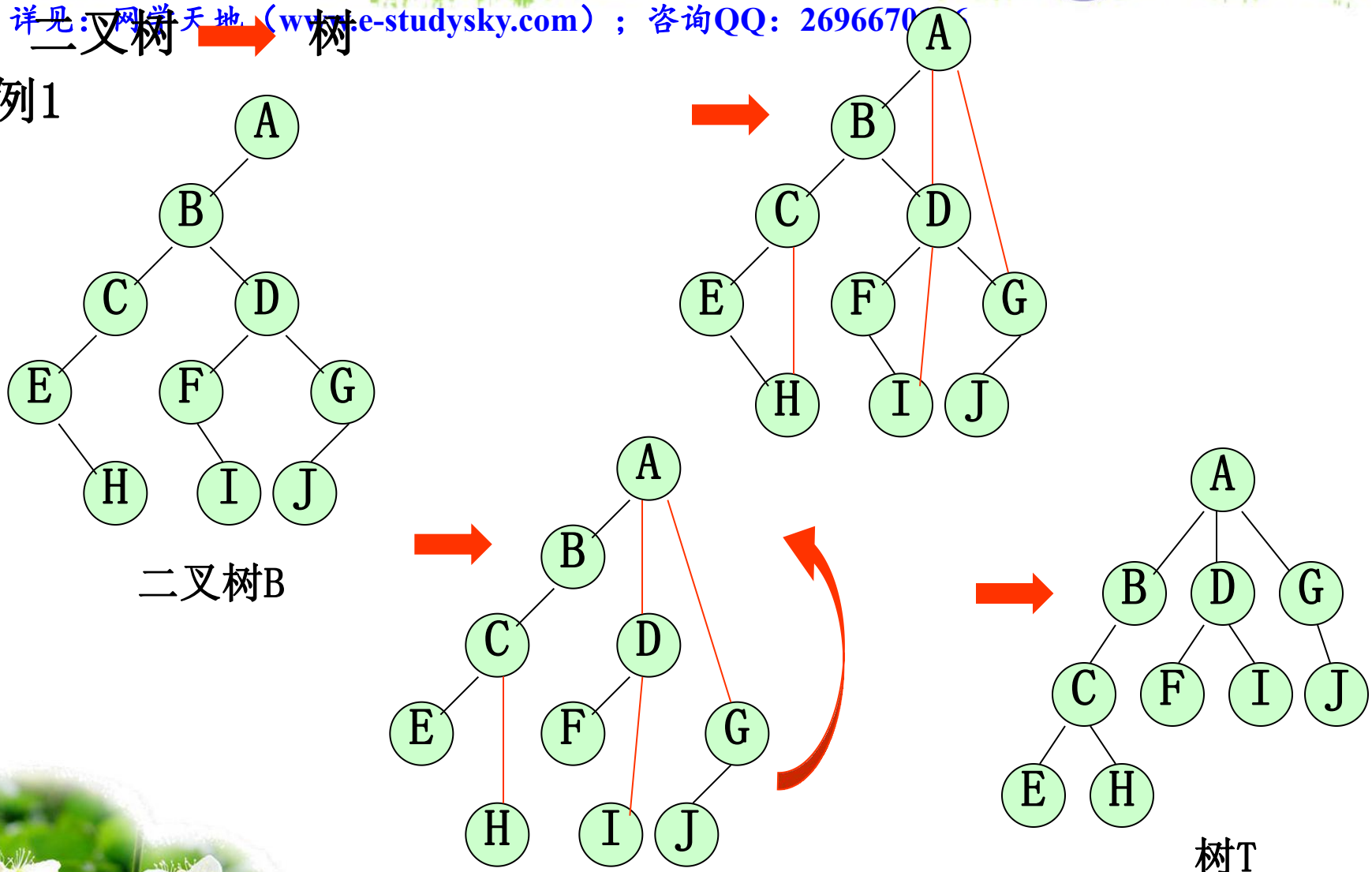
森林F={T1, T2, T3, T4}



旋转后，变为一棵二叉树

3. 二叉树 → 树

例1

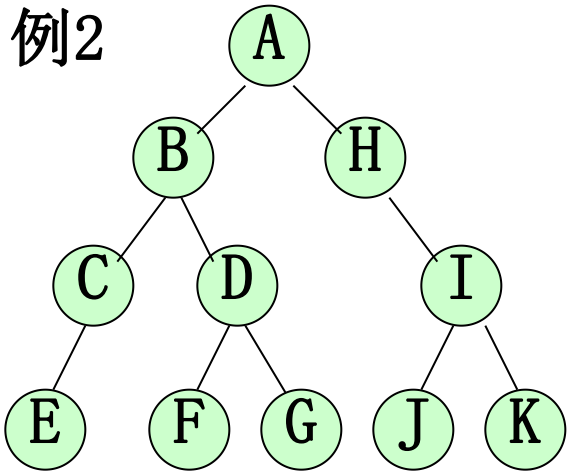


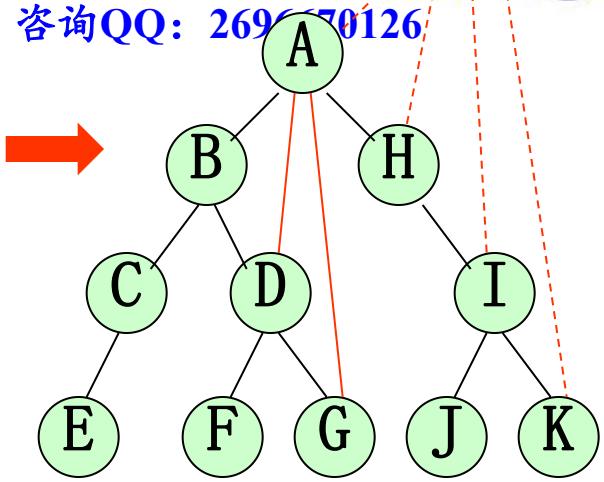
二叉树B

树T

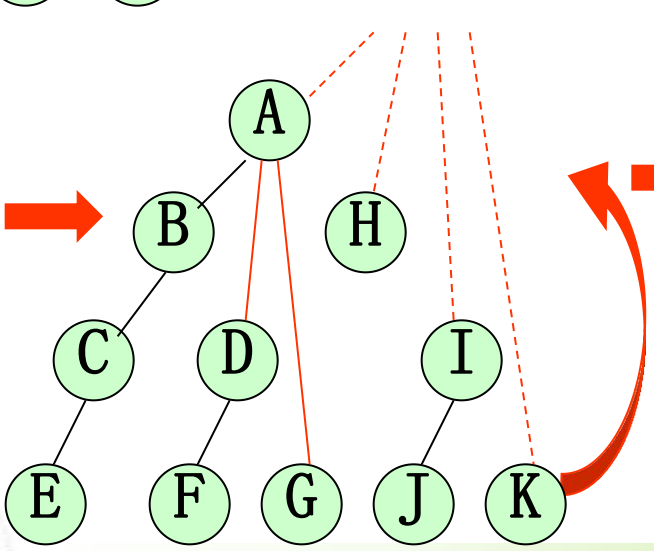
3. 二叉树 → 森林

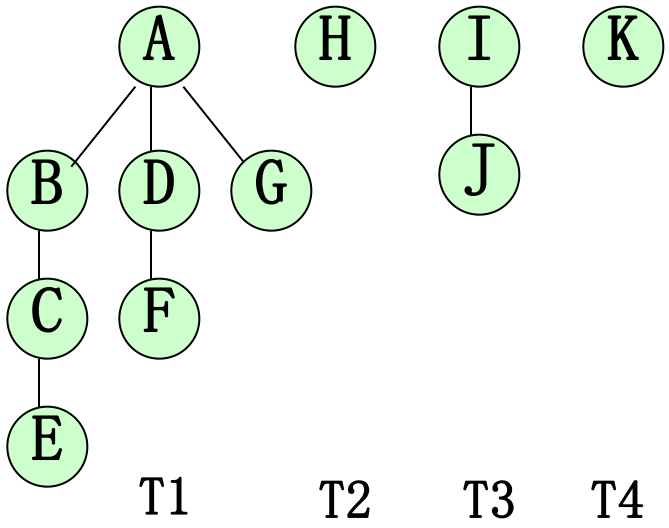
例2





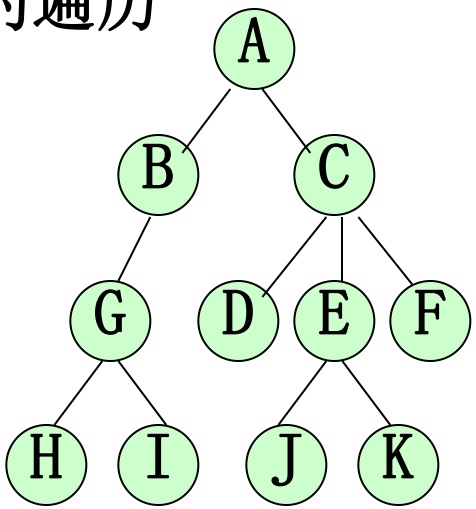
二叉树B





6.4.3 树和森林的遍历

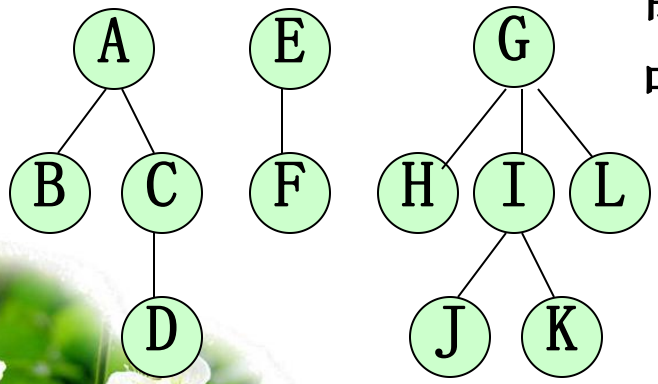
1. 树的遍历



前根遍历：A B G H I C D E J K F

后根遍历：H I G B D J K E F C A

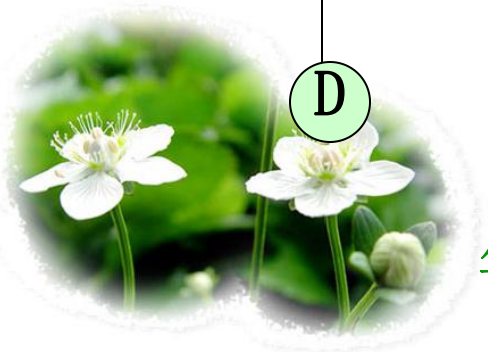
2. 森林的遍历



前序遍历：A B C D E F G H I J K L

中序遍历：B D C A F E H J K I L G

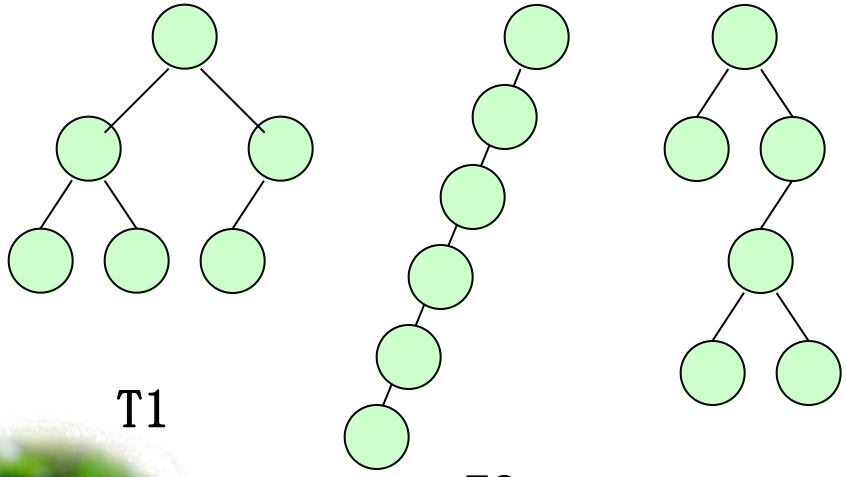
（依次对每一棵树后序遍历）



6.6 哈夫曼（Huffman)树及其应用

1. 路径长度： 路径上分枝的数目(连线的数目)

2. 树T的路径长度： 从树T的根到其余每个结点的路径长度之和,记作PL(T)



PL(T1)=1+1+2+2+2=8

PL(T2)=1+2+3+4+5=15

PL(T3)=1+1+2+3+3=10

- 当n个结点的二叉树为完全二叉树时，PL(T)具有最小值

  ∵ 结点i的层 $= \lfloor \log_2 i \rfloor + 1$

  树T的根到结点i的路径长度 = 结点i的层-1

  $= \lfloor \log_2 i \rfloor$

  ∴ $PL(T) = \lfloor \log_2 1 \rfloor + \lfloor \log_2 2 \rfloor + \ldots + \lfloor \log_2 n \rfloor$

  $$= \sum_{i=1}^{n} \lfloor \log_2 i \rfloor$$

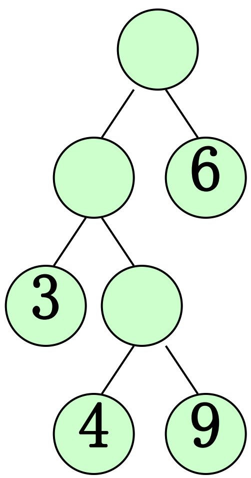
- 当n个结点的二叉树为单枝树时，PL(T)具有最大值：

  $PL(T) = 0+1+2+\ldots+(n-1) = n(n-1)/2$

3. 树T的带权路径长度：每个叶子的权与根到该叶子的路径长度的乘积之和，记作WPL(T)

$$WPL(T) = \sum_{k=1}^{n} w_k l_k$$

其中：n --- 树T的叶子数目　　w_k--- 叶子k的权
l_k--- 树T的根到叶子k的路径长度

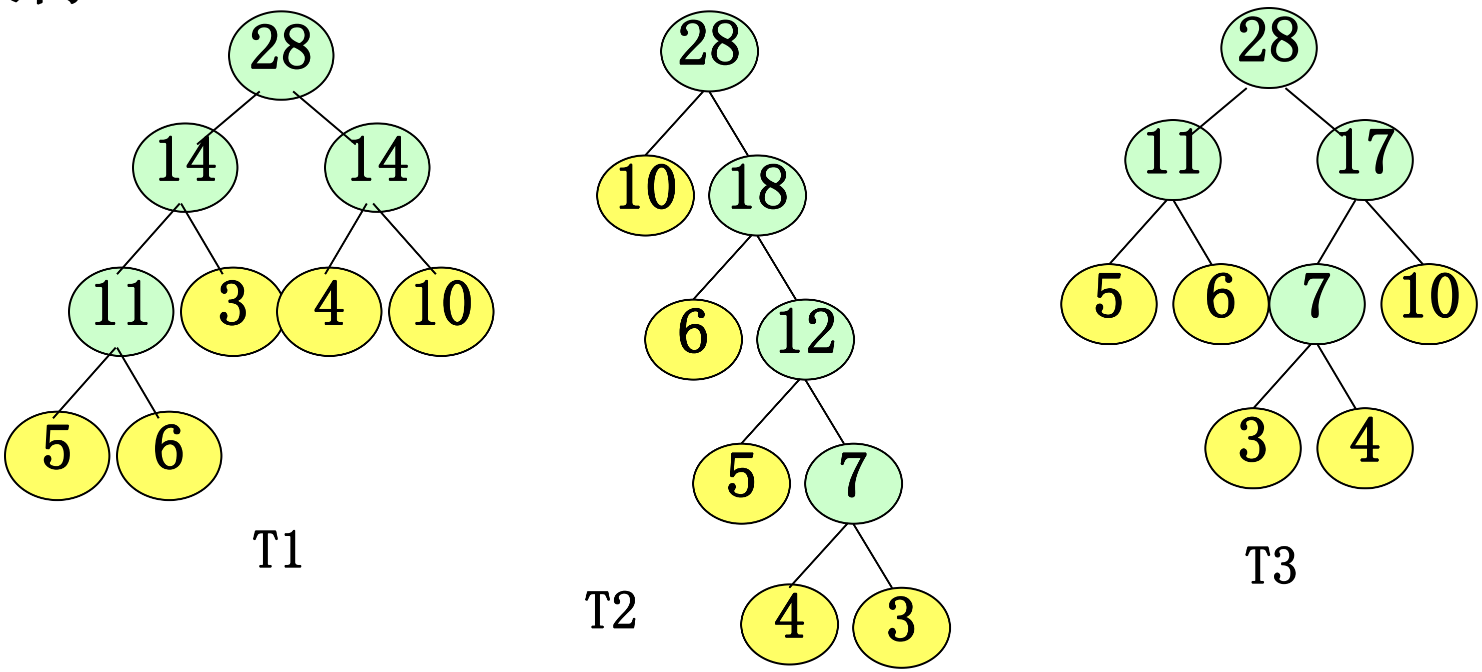


T1

WPL(T)=6*1+3*2+4*3+9*3=51

4. 哈夫曼树/最佳树/最优树

在具有n个相同权值叶子的各二叉树中，WPL(T)最小的二叉树。



WPL(T1)=5*3+6*3+3*2+4*2+10*2=67

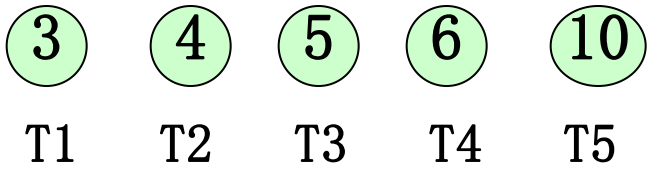
WPL(T2)=10*1+6*2+5*3+4*4+3*4=65

WPL(T3)=5*3+6*3+3*2+4*2+10*2=63

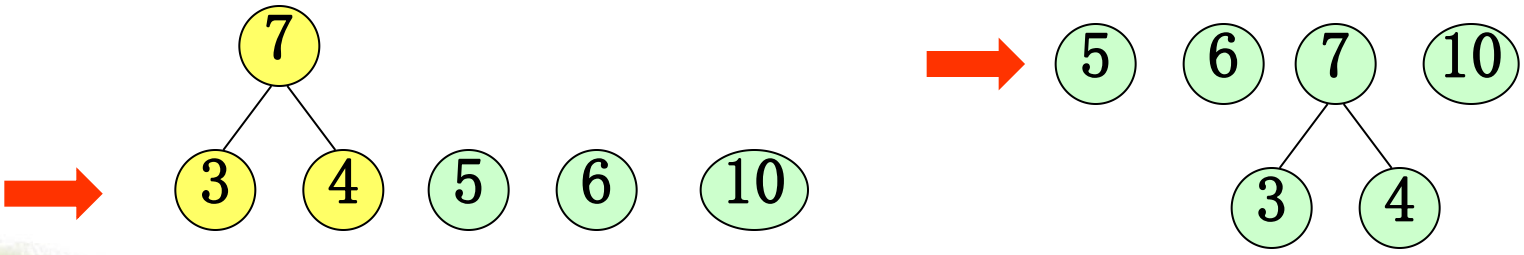
5. 哈夫曼算法

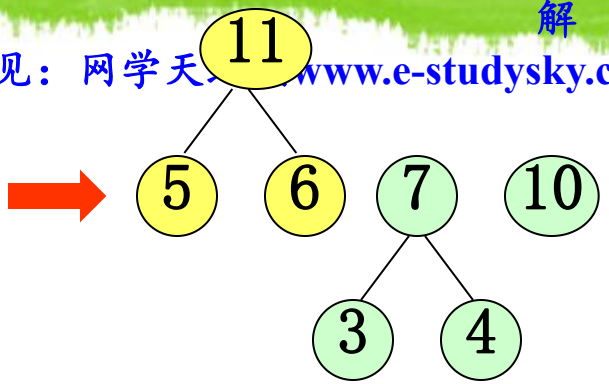
例 给定权集合{4, 5, 3, 6, 10}，构造哈夫曼树

1. 按权值大小排序： 3, 4, 5, 6, 10
2. 生成森林：

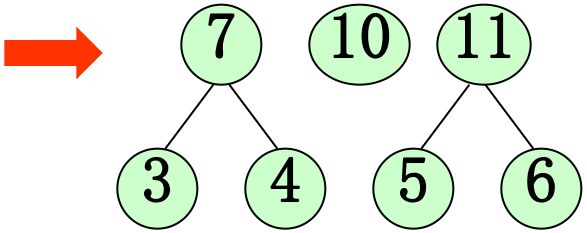


3. 合并两棵权最小的二叉树，并排序，直到为一棵二叉树：

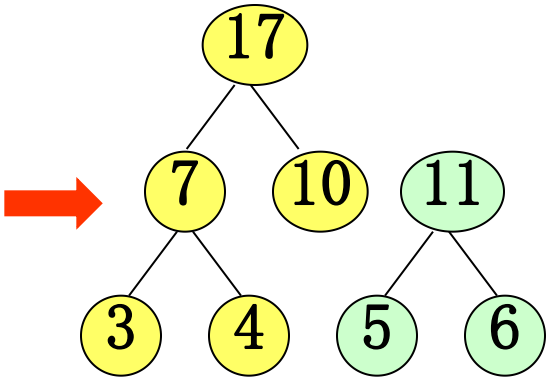




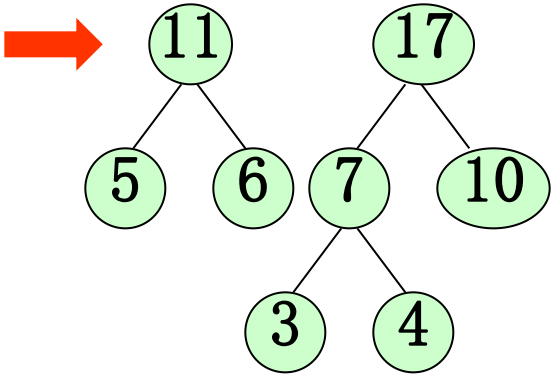
选择合并



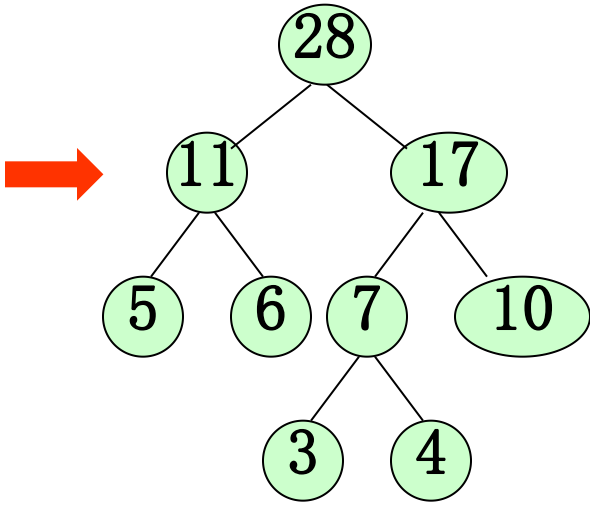
排序



选择合并



排序



哈夫曼树

算法证明：

- 当权值个数为1时，算法显然正确。
- 假定权值个数n=k时，算法对任意k个权值构造出哈夫曼数。则对n=k+1个权值时，不失一般性，假设：

$$w_1 \le w_2 \ldots . w_k \le w_{k+1}$$

设T为这k+1个权值的哈夫曼树，其WPL(T)=W,并设N为最远的内部结点， 如果N的孩子权值不是w_1和w_2，则替换成w_1和w_2，替换后的树还是哈夫曼树。N的权值为w_1+w_2。去掉N的孩子后的树T’，实际上是k个权值$(w_1+w_2,w_3,\ldots,w_{k+1})$的哈夫曼树,$WPL(T')=W-w_1-w_2$，用反证法可以证明。这样，对k+1个权值，首先按算法步骤对w_1和w_2产生一颗子树，其根结点权值为w_1+w_2，然后问题变成构造k个权值$(w_1+w_2,w_3,\ldots,w_{k+1})$的哈夫曼树，由归纳假设，最后得到算法的正确性。

6. 最小冗余码/哈夫曼码

- ASCII码/定长码

  ab12: 01100001 01100010 00110001 00110010

  97 98 49 50

- 哈夫曼码/不定长码

  能按字符的使用频度,使文本代码的总长度具有最小值。

例. 给定有18个字符组成的文本：

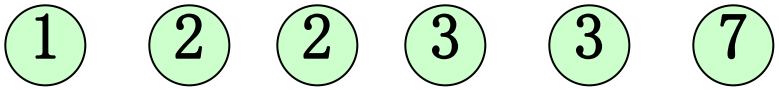
A A D A T A R A E F R T A A F T E R

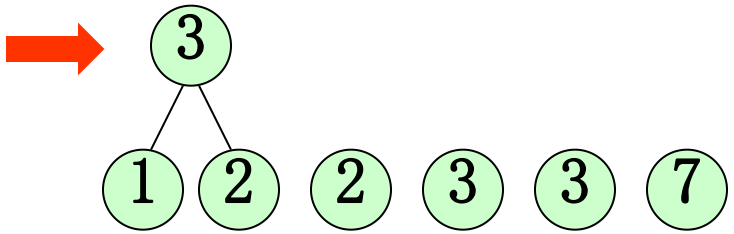
求各字符的哈夫曼码。

(1) 统计：

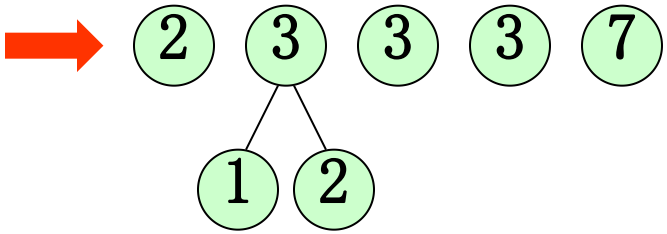
字符	A	D	E	F	T	R
频度	7	1	2	2	3	3

(2) 构造Huffman树：

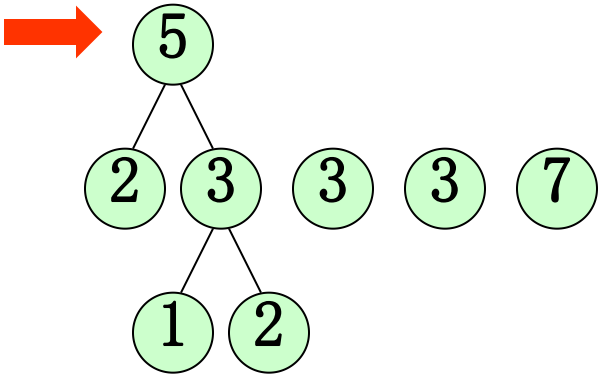




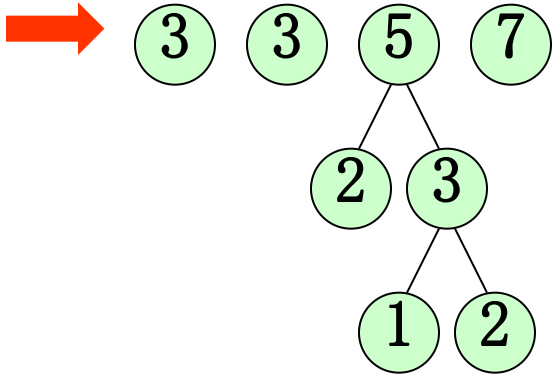
合并1和2



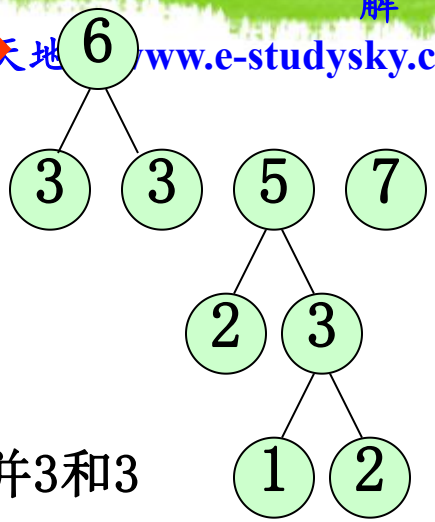
排序



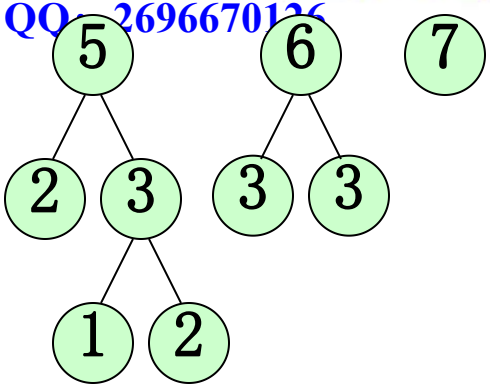
合并2和3



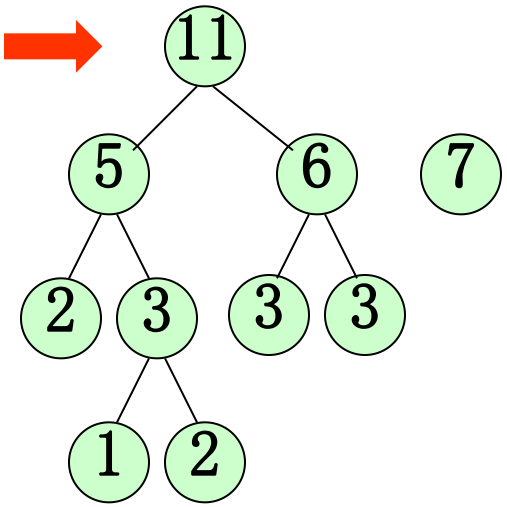
排序



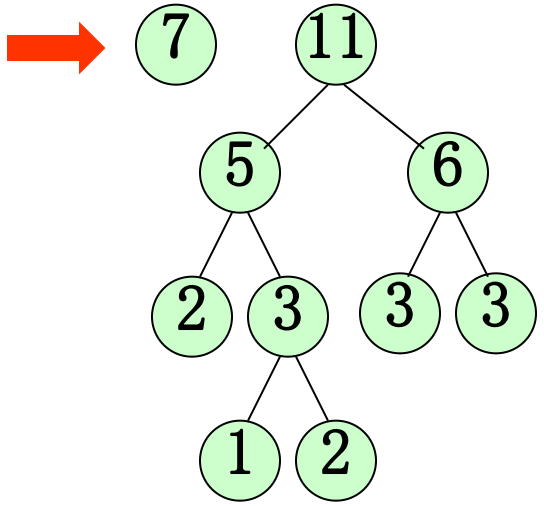
合并3和3



排序

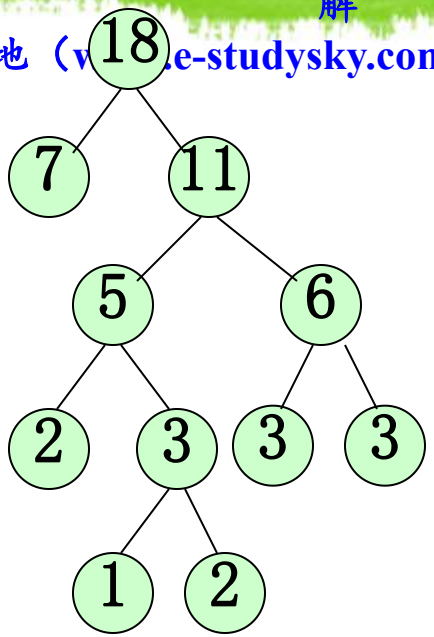


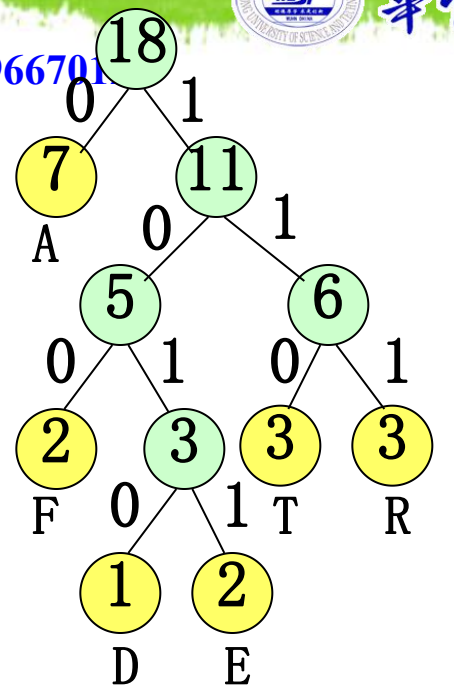
合并5和6



排序

合并7和11得Huffman树





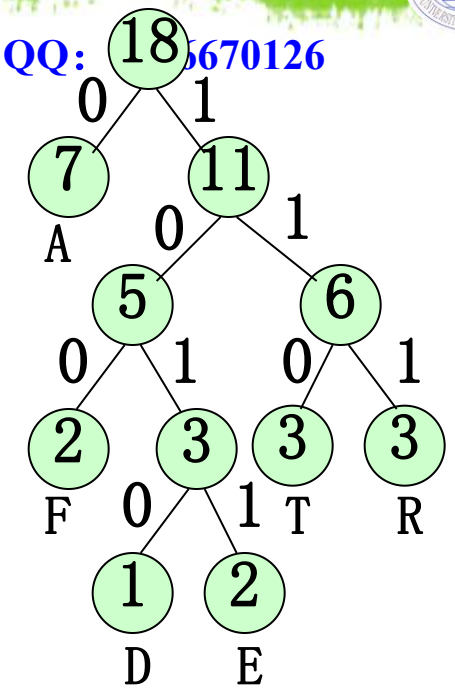
叶结点根据权值附上字符，分支标数的Huffman树

(4) 确定Huffman编码：

字 符	A	D	E	F	T	R
频 度	7	1	2	2	3	3
编 码	0	1010	1011	100	110	111

特点：任一编码不是其它编码的前缀

如何译码？



Huffman树

例. 给定代码序列：

0 0 1 0 0 0 1 1 1 0 1 0 1 0 1 0 1 1 1 10

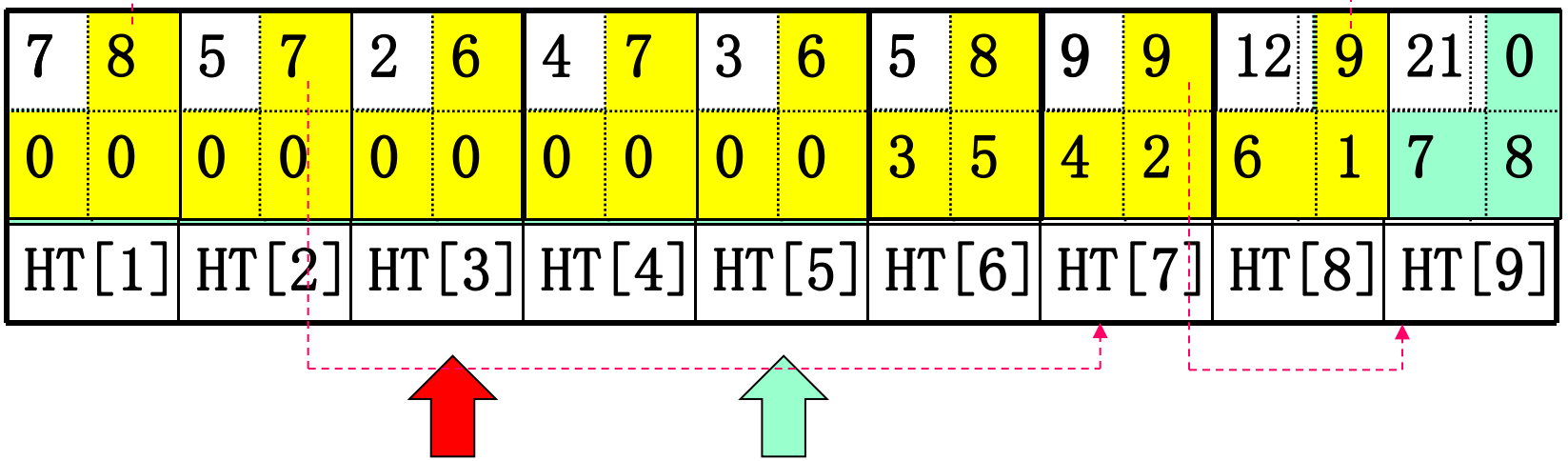
文本为：A A F A R A D E T

算法处理：

结点说明：

data	parent
lchild	rchild

n个权值的哈夫曼树有2n-1个结点



HT[1]的编码反序列为： 11 则编码为：11

HT[2]的编码反序列为： 10 则编码为：01

同理得： HT[3]：100 HT[4]：00 HT[5]：101