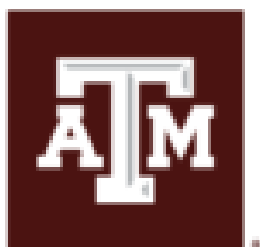


AERO 321 Term Project Report

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AEROSPACE ENGINEERING
TEXAS A & M UNIVERSITY

Submitted By:

Chenliang Zhang	UIN: 827008031
Daniel Duhon	UIN: 827004077
Russtam Wafayee	UIN: 626006337
Steven Mai	UIN: 826007930

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1. Problem Statement

A manufacturer of a small twin-engine turboprop aircraft contacted our engineering firm for a simple analysis of their aircrafts flying qualities. Our firm is a team of 4 engineers which doesn't possess any CFD (computational fluid dynamics) software, tables from AAA, or empirical equations from DATCOM. The following information is given:

1. The vehicle will cruise at Mach numbers below 0.3 and the lifting surfaces can be assumed to be rectangular (no sweep or taper). The wing aspect ratio will be high.
2. The estimated weight of the vehicle is 12,000 lb.
3. A stability augmentation system is not planned for the vehicle.
4. Constant horsepower propulsive system.
5. Weight and Balance Table (shown in results section)

1.1 Requirements

We are required to perform a trim analysis, modal analysis, and determine the flying qualities of the aircraft at the given cruise speed and altitude. Because our engineering firm doesn't have CFD or information from AAA or DATCOM we will conduct our analysis with non empirically derived equations.

1.2 Scope of Analysis

The scope of our analysis of the manufacturers aircraft will be brief and a good predictor of the flight dynamics for it. Our analysis will be less accurate than using CFD, wind tunnel testing, or data from AAA or DATCOM but will be a good surface level analysis.

2. Problem Formulation

2.1 Assumptions and Knowns

Assumptions:

1. Rectangular wing (no sweep or taper ratio = 1), high aspect ratio
2. Standard atmosphere
3. Zero Sideslip, straight and level flight
4. Assume C_{D_u} is zero

Knowns

Flight Altitude: 10,000 *ft*

Air Density: 0.001755 $\frac{\text{slugs}}{\text{ft}^3}$

Temperature: 483.03 °R

Speed of Sound, a: 1077.233 *ft/s*

Speed of air flow: $U_0 = 278 \text{ ft/s}$

2.2 Analysis Outline

2.2.1 Moment of Inertias

$$I_{yy} = 20,399.534 \text{ slugs} \cdot \text{ft}^2$$

2.2.2 Trim Analysis

Under trim condition, 3 relations can be obtained:

$$C_{L_{Trim}} q_{\infty} S_w = W \text{ \& } C_{D_{Trim}} = C_{D_{Parasite}} + \frac{(C_{L_{Trim}})^2}{\pi AR_w e_{Eff}} \text{ \& } C_{M_{Trim}} = 0$$

For calculation of $C_{L_{awf}}$, two equations are utilized:

$$C_{L_{awf}} = K_{wf} C_{L_{aw}} \text{ \& } K_{wf} = 1 + 0.025\left(\frac{d_f}{b}\right) - 0.25\left(\frac{d_f}{b}\right)^2$$

Downwash gradient at subsonic field can be expressed as:

$$\frac{d\varepsilon}{d\alpha_w} = \frac{C_{L_{aw}}}{\pi e AR} \text{ (at wing a.c.; 1 at wing trailing edge)}$$

$$\frac{d\varepsilon}{d\alpha_w} = \frac{2C_{L_{aw}}}{\pi e AR} \text{ (at infinity behind the wing)}$$

Lift coefficient derivatives with respect to α & δ_E & i_H and lift coefficient at $\alpha = \delta_E = 0$:

$$C_{L_{\alpha}} = C_{L_{\alpha_{WF}}} + C_{L_{\alpha_H}} \eta_H \frac{S_H}{S_W} \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$C_{L_{\delta_E}} = C_{L_{\alpha_H}} \eta_H \alpha_{\delta} \frac{S_H}{S_W}$$

$$C_{L_{i_H}} = C_{L_{\alpha_H}} \eta_H \frac{S_H}{S_W}$$

$$C_{L_{\alpha=0, \delta_E=0}} = C_{L_{Trim}} - C_{L_{\alpha}} \alpha - C_{L_{\delta_E}} \delta_E - C_{L_{i_H}} i_H$$

For some distance parameters:

$$\overline{\Delta X}_{AC_H} = \frac{X_{AC_H} - X_{CG}}{\overline{C}_w} \text{ \& } \Delta X_{AC_f} = X_{AC_f} - X_{CG}$$

$$X_{AC_{WF}} = X_{AC_w} + \Delta X_{AC_f} \text{ \& } \overline{\Delta X}_{AC_{WF}} = \frac{X_{CG} - X_{AC_{WF}}}{\overline{C}_w}$$

Under trim condition:

$$T_{trim} = D_{trim} = C_{D_{trim}} S_w q_{\infty}$$

$$\overline{Z}_T = \frac{Z_T}{\overline{c}_w}$$

$$C_{M_T} = C_p = \frac{T_{trim}}{q_{\infty} S_w} \overline{Z}_T$$

Pitching moment coefficient derivatives with respect to α & δ_E & i_H and pitching moment coefficient at $\alpha = \delta_E = 0$

$$C_{M_\alpha} = C_{L_{\alpha_{WF}}} \Delta \bar{X}_{AC_{WF}} - C_{L_{\alpha_H}} \eta_H \left(\frac{S_H}{S_W} \Delta \bar{X}_{AC_H} \right) \left(1 - \frac{d\epsilon}{d\alpha} \right)$$

$$C_{M_{i_H}} = -C_{L_{\alpha_H}} \eta_H \left(\frac{S_H}{S_W} \Delta \bar{X}_{AC_H} \right)$$

$$C_{M_{\delta_E}} = -C_{L_{\alpha_H}} \eta_H \alpha_\delta \left(\frac{S_H}{S_W} \Delta \bar{X}_{AC_H} \right)$$

$$C_{M_{\alpha=0, \delta E=0}} = C_P - C_{M_\alpha} \alpha - C_{M_{\delta_E}} \delta E - C_{M_{i_H}} i_H$$

The Neutral Point can be calculated using the following equation:

$$\bar{X}_{NP} = \frac{C_{L_{\alpha_{WF}}} \bar{X}_{AC_{WF}} + C_{L_{\alpha_H}} \eta_H \left(\frac{S_H}{S_W} \bar{X}_{AC_H} \right) \left(1 - \frac{d\epsilon}{d\alpha} \right)}{C_{L_\alpha}}$$

From the Neutral Point locating and the location of the C.G., we can compute the Static Margin:

$$SM = \bar{X}_{cg} - \bar{X}_{NP}$$

$$X_{CG} = 2.798 \text{ ft}$$

$$\bar{X}_{CG} = X_{CG} / \bar{c}_W = 0.430$$

Mach number can be calculated from:

$$M = \frac{U_0}{a} = \frac{278 \text{ ft/s}}{1077.233 \text{ ft/s}} = 0.258$$

Table 1:

Parameter	Equation	Value
q_∞ at $u=278 \text{ ft/s}$ and $h = 10,000 \text{ ft}$	$q_\infty = 0.5 \rho_\infty u^2$	$67.81 \frac{\text{lbf}}{\text{ft}^2}$
Aspect ratio of wing	$AR_w = \frac{b_w^2}{S_w}$	10
Wingspan (for rectangular wing)	$b_w = \frac{S_w}{c_w}$	65 ft
C_{L_0} (Trim)	$C_{L_0} = C_{L_{Trim}} = \frac{W}{q_\infty S_W}$	0.419
C_{D_0} (Trim)	$C_{D_0} = C_{D_{Trim}} = C_{D_{Parasite}} + \frac{(C_{L_{Trim}})^2}{\pi AR_w e_{Eff}}$	0.0469
C_{M_0} (Trim,Aero)	$C_{M_0} = C_{M_{Trim}} = 0$	0

$\frac{d\epsilon}{d\alpha}$	$\frac{2C_{L_{\alpha w}}}{\pi e AR}$	0.3247
$C_{L_{\alpha_{wf}}}$	$C_{L_{\alpha_w}} [1 + 0.025(\frac{df}{b}) - 0.25(\frac{df}{b})^2]$	5.1
$C_{L_{\alpha}}$	$C_{L_{\alpha}} = C_{L_{\alpha_{wf}}} + C_{L_{\alpha_H}} \eta_H \frac{S_H}{S_W} (1 - \frac{d\epsilon}{d\alpha})$	5.687
$C_{D_{\alpha}}$	$C_{D_{\alpha}} = \frac{2}{\pi} (C_{L_{\alpha w}} \frac{C_{L_w}}{AR_w e_w} + C_{L_{\alpha H}} \frac{C_{L_H}}{AR_H e_H} \eta_H \frac{S_H}{S_W} (1 - \frac{d\epsilon}{d\alpha}))$	
$C_{L_{\delta_E}}$	$C_{L_{\delta_E}} = C_{L_{\alpha_H}} \eta_H \alpha_{\delta} \frac{S_H}{S_W}$	0.478
$C_{L_{i_H}}$	$C_{L_{i_H}} = C_{L_{\alpha_H}} \eta_H \frac{S_H}{S_W}$	0.869
$\Delta \bar{X}_{AC_H}$	$\frac{X_{AC_H} - X_{CG}}{\bar{c}_w}$	3.97
ΔX_{AC_f}	$X_{AC_f} - X_{CG}$	-1.59 ft
$X_{AC_{wf}}$	$X_{AC_w} + \Delta X_{AC_f}$	0.035 ft
$\Delta \bar{X}_{AC_{w\&f}}$	$\frac{X_{CG} - X_{AC_{wf}}}{\bar{c}_w}$	0.424
$C_{M_{\alpha}}$	$C_{M_{\alpha}} = C_{L_{\alpha_{wf}}} \Delta \bar{X}_{AC_{w\&f}} - C_{L_{\alpha_H}} \eta_H (\frac{S_H}{S_W} \Delta \bar{X}_{AC_H}) (1 - \frac{d\epsilon}{d\alpha})$	-0.731
$C_{M_{i_H}}$	$C_{M_{i_H}} = - C_{L_{\alpha_H}} \eta_H (\frac{S_H}{S_W} \Delta \bar{X}_{AC_H})$	-3.45
$C_{M_{\delta_E}}$	$C_{M_{\delta_E}} = - C_{L_{\alpha_H}} \eta_H \alpha_{\delta} (\frac{S_H}{S_W} \Delta \bar{X}_{AC_H})$	-1.89

$C_L _{\alpha=\delta_E=0}$	$C_L _{\alpha=\delta_E=0} = C_{L_{Trim}} - C_{L_\alpha} \alpha - C_{L_{i_H}} - C_{L_{\delta_E}} \delta_E$	0.493
T_{trim}	$T_{trim} = D_{trim} = C_{D_{trim}} S_w q_\infty$	1294.8 lbf
\bar{Z}_T	$\bar{Z}_T = \frac{Z_T}{\bar{c}_w}$	0.4354
C_{M_T}	$\frac{T_{trim}}{q_\infty S_w} \bar{Z}_T$	0.0196
$C_M _{\alpha=\delta_E=0}$	$C_M _{\alpha=\delta_E=0} = C_{M_T} - C_{M_\alpha} \alpha - C_{M_{i_H}} i_H - C_{M_{\delta_E}} \delta_E$	-0.255
i_H Recommended	Need $C_M _{\alpha=\delta_E=0}$ to be negative and $C_L _{\alpha=\delta_E=0}$ to be positive	$i_H \approx -0.0872 \text{ rad}$ $i_H \approx -5 \text{ deg}$
Trim aoa	$\alpha_{trim} = \frac{(C_{L_{trim}} - C_L _{\alpha=\delta_E=0} - C_{L_{i_H}} i_H) C_{M_{\delta_E}} + (C_M _{\alpha=\delta_E=0} + C_{M_{i_H}} i_H) C_{L_{\delta_E}}}{C_{L_\alpha} C_{M_{\delta_E}} - C_{M_\alpha} C_{L_{\delta_E}}}$	$\alpha_{trim} = -0.00209 \text{ rad}$ $\alpha_{trim} = -0.115 \text{ deg}$
Trim elevator angle	$\delta_E(trim) = \frac{(C_{L_{trim}} - C_L _{\alpha=\delta_E=0} - C_{L_{i_H}} i_H) C_{M_{\delta_E}} + (C_M _{\alpha=\delta_E=0} + C_{M_{i_H}} i_H) C_{L_{\delta_E}}}{C_{L_\alpha} C_{M_{\delta_E}} - C_{M_\alpha} C_{L_{\delta_E}}}$	$\delta_{Etrim} = 0.0250 \text{ rad}$ $\delta_{Etrim} = 1.43 \text{ deg}$
Static margin (SM)	$SM = \frac{C_{M_\alpha}}{-C_{L_\alpha}}$	0.125

2.2.3 Non Dimensional and Dimensional Stability / Control Derivatives

Table 2:

Stability Derivative	Equation Used	Value (Units)
C_{D_M}	$C_{D_M} = \frac{M_0}{1-M_0^2} C_{D_0}$	0.01296
C_{D_u}	$C_{D_u} = \frac{C_{D_m}}{a_0}$	0.00001203 ≈ 0
C_{L_M}	$C_{L_M} = \frac{M_0}{1-M_0^2} C_{L_0}$	0.116

C_{L_u}	$C_{L_u} = \frac{C_{L_m}}{a_0}$.0001077
C_{M_u}	$C_{M_u} = \frac{M_u I_{yy}}{q_\infty S_W \bar{c}_w} - \frac{2}{U_0} C_{M_0}, C_{M_0} = 0, M_u = 0$	0
C_{T_u}	$C_{T_u} = -3(C_{D_0} + C_{L_0} \tan(\Theta_0)), \Theta_0 = 5^\circ$	-0.243
X_u	$X_u = \frac{-(C_{D_u} + \frac{2}{U_0} C_{D_0}) q_\infty S_W}{m}$	$-0.02592 \frac{ft}{rad \cdot s}$
Z_u	$Z_u = \frac{-(C_{L_u} + \frac{2}{U_0} C_{L_0}) q_\infty S_W}{m}$	$0.359 \frac{ft}{rad \cdot s}$
X_{P_u}	$\frac{dT}{dU}/m$	$-0.000388 \frac{ft}{rad \cdot s}$
Z_α	$Z_\alpha = \frac{-(C_{L_\alpha} + C_{D_0}) q_\infty S_W}{m}$	-451.0828
X_α	$X_\alpha = \frac{-(C_{D_\alpha} + C_{L_0}) q_\infty S_W}{m}$	-32.031
M_α	$M_\alpha = \frac{C_{M_\alpha} q_\infty S_W \bar{c}_w}{I_{yy}}$	-6.2821
M_u	$M_u = \frac{(C_{M_u} + \frac{2}{U_0} C_{M_0}) q_\infty S_W \bar{c}_w}{I_{yy}}$	0
$M_{\alpha_{dot}}$	$M_{\alpha_{dot}} = \frac{C_{M_{\alpha_{dot}}} q_\infty S_W \bar{c}_w}{I_{yy}}$	0.0649
$C_{M_{\alpha_{dot}}}$	$-(C_{L_{\alpha H}} \frac{d\epsilon}{d\alpha} \frac{(X_{ACW} - X_{ACH})(X_{CG} - X_{ACH})}{U_0 \bar{c}_w} \eta_H \frac{S_H}{S_W})$	0.007112
M_q	$M_q = \frac{C_{M_q} q_\infty S_W \bar{c}_w}{I_{yy}}$	-0.06925
C_{M_q}	$C_{M_q} = -\frac{C_{L_{\alpha H}} (X_{CG} - X_{ACH})^2}{U_0 \bar{c}_w} \frac{S_H}{S_W} \eta_H$	-0.007586

2.2.4 Modal Analysis

$$\begin{bmatrix} \dot{u} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_\alpha & 0 & -g \\ Z_u / U_0 & Z_\alpha / U_0 & 1 & 0 \\ M_u + M_{\dot{\alpha}} Z_u / U_0 & M_\alpha + (M_{\dot{\alpha}} Z_\alpha / U_0) & M_q + M_{\dot{\alpha}} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ \alpha \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_E} \\ Z_{\delta_E} / U_0 \\ M_{\delta_E} + (M_{\dot{\alpha}} Z_{\delta_E} / U_0) \\ 0 \end{bmatrix} \delta_E$$

And this matrix can be written as the form:

$$\dot{X}_{dot} = AX + BU \text{ (through this form we can obtain matrix A and B)}$$

And eigenvalues and eigenvectors matrix can be obtained through:

$$AM = M\Lambda$$

$$M^{-1}AM = \Lambda$$

M: matrix of eigenvectors

Λ : diagonal matrix of eigenvalues

Characteristic equation: $\det(sI - A) = 0$ which can be used to solve for s and get matrix Λ

Then using matrix relations to solve for M

A matrix

-0.02459	-32.031	0	-32.174
0.001291	-1.6226	1	0
0.000083844	-6.3874	-0.00435	0
0	0	1	0

Eigenvectors

Vector 1	Vector 2	Vector 3	Vector 4
0.9929	0.9929	1	1
0.0249-0.0323i	0.0249+0.0323i	0	0
0.0982+0.0331i	0.0982-0.0331i	0.0013-0.0001i	0.0013+0.0001i
-0.0003-0.0412i	-0.0003+0.0412i	-0.0005-0.0064i	-0.0005+0.0064i

Eigenvalues

Value 1	-0.8184+2.3784i
Value 2	-0.8184-2.3784i
Value 3	-0.0073+0.2063i
Value 4	-0.0073-0.2063i

Table 3

Mode	eigenvalues	eigenvectors
Phugoid	$-0.0073 \pm 0.2063j$	$\begin{bmatrix} 1 \\ 0 \\ 0.0013 \mp 0.2063j \\ -0.0005 \mp 0.0064j \end{bmatrix}$
Short Period	$-0.8184 \pm 2.3784j$	$\begin{bmatrix} 0.9929 \\ 0.0249 \mp 0.0323j \\ 0.0982 \pm 0.0331j \\ -0.0003 \mp 0.0412j \end{bmatrix}$

2.2.5 Modal Characteristics

Characteristic equation: $\det(sI - A) = 0$

$$sI - A =$$

$$\begin{pmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{pmatrix} - \begin{pmatrix} -0.02459 & -32.031 & 0 & -32.174 \\ 0.001291 & -1.6226 & 1 & 0 \\ 0.000083844 & -6.3874 & -0.00435 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} s+0.02459 & 32.031 & 0 & 32.174 \\ -0.001291 & s+1.6226 & -1 & 0 \\ -0.000084 & 6.3874 & s+0.00435 & 0 \\ 0 & 0 & -1 & s \end{pmatrix}$$

$$\det(sI - A) = s^4 + 1.651s^3 + 6.475s^2 + 0.1628s - 0.2609 = 0$$

Our characteristic equation:

$$s^4 + 1.651s^3 + 6.475s^2 + 0.1628s - 0.2609 = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

Natural frequency: ω_n

Damping ratio: ζ

Then s values are solved from Characteristic equation:

$$s = \eta \pm j\omega = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$\text{Damped frequency: } \omega = \omega_n\sqrt{1-\zeta^2}$$

$$\text{Period of oscillation: } t_p = \frac{2\pi}{\omega}$$

$$\text{time -to-half: } t_{half} = \frac{\log_{natural} 2}{|\eta|}$$

$$\text{Cycles-to-half: } \frac{t_{half}}{t_p}$$

$$\text{Time constant: } \frac{1}{|\eta|}$$

Table 4

Mode	Natural Frequency (rad/s)	Damping Ratio	Damped Frequency (rad/s)	Time-to-half (s)	Cycles-to-half	Time constant (s)
Phugoid	0.206	0.035	0.2063	94.9517	3.1176	136.9863
Short Period	2.515	0.325	2.3784	0.8470	0.3390	1.2219

2.2.6 Flying Qualities

Table 5

Parameter	Value (units)	FQ level
Phugoid damping ratio	0.035	Level 2
Short Period damping ratio	0.325	Level 2
Short Period Natural Frequency	2.515	Level 1

Weight and Balance Table

Component	Weight (lbs.)	Mass (slugs)	Distance from leading edge (ft.)	Assumption	lyy $\text{slugs} \cdot \text{ft}^2$	Moment $\text{lbs} \cdot \text{ft}$
Fuselage	1000	31.0809970783863	4.325	Thin Cylinder Radius 3.5 ft, length 41 ft.	5030.50470359089	4325
Wing	1550	48.1755454714987	1.2	Thin flat plate, width 6.5 ft	169.618066347568	1860
H-Tail	165	5.12836451793374	28.625	Point Mass	4202.13380757755	4723.125
V-Tail	200	6.21619941567726	30.325	Point Mass	5716.45194877852	6065
Fuel Tanks	150	4.66214956175794	1.2	Thin rod, length 8ft.	24.864797662709	180
Engines	800	24.864797662709	-2.175	Point Mass	117.626033443153	-1740
Props	350	10.8783489774352	-4.175	Point Mass	189.616421644806	-1461.25
Main Gear	250	7.77024926959657	7.325	Point Mass	416.917580965997	1831.25
Nose Gear	150	4.66214956175794	-10.175	Point Mass	482.675257972276	-1526.25
Instruments	200	6.21619941567726	-2.975	Point Mass	55.0172499533785	-595
Furnishings and Baggage	1075	33.4120718592652	-0.525	Thin rod, length 16.5 ft	758.03638030708	-564.375
Pilots (2)	340	10.5675390066513	-7.175	Point Mass	544.02351277429	-2439.5
Hydraulic Fluid	40	1.24323988313545	-0.675	Neglect	0	-27
Fuel	2500	77.7024926959657	1.2	Thin rod, length 8ft.	414.413294378484	3000
Passengers (19)	3230	100.391620563188	6.175	Thin rod, length 16.5 ft	2277.63489152732	19945.25
Total	12000	372.971964940635	2.798		20399.533946924	33576.25

3. Results Discussion

3.1 Limitations of our Calculations

Our analysis was conducted using non-empirical methods and with no CFD. This will greatly limit the accuracy of our analysis, but these calculations are a good indicator of how the aircraft will perform at cruise. In order to increase the accuracy of these results, we suggest wind tunnel testing or CFD.

3.2 Meaning of Results

In order to determine if an aircraft is statically stable $M_{\alpha} < 0$. In the case of this aircraft M_{α} is -6.821 which means this aircraft is statically stable. This is a good thing because the aircraft will have a tendency to stay trimmed when cruising. For dynamic stability $M_q + M_{\alpha\dot{\alpha}} < 0$. In our case, $M_q + M_{\alpha\dot{\alpha}} = -0.00435$ which means the aircraft will be dynamically stable. This means that if the aircraft is perturbed when it is in equilibrium it will return to a stable state which is a good flight characteristic. The damping ratios of the phugoid and short period are greater than 1 which means the aircraft is overdamped. This is preferred because the system will slowly move towards equilibrium which is preferred for a civilian aircraft. The flying quality level of the aircraft is between 1 and 2 which suggests that the pilot will have to put a little more effort into flying which isn't necessarily a bad thing or a good thing. Our engineers recommend an incidence tail angle of -5 degrees in order to keep the pitching moment coefficient negative and the lift coefficient positive. We want the pitching moment coefficient to be negative in order to keep the aircraft statically stable. The lifting coefficient must be positive for an aircraft if you want to fly. A negative lift coefficient is desirable for cars with aerodynamic surfaces.