Multi-splay trees and tango trees

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Parker Rule

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Dynamic optimality

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A BST data structure is *dynamically optimal* if every sequence *X* requires *O*(OPT(*X*)) operations.

Recall: it is conjectured that splay trees are dynamically optimal (Sleator and Tarjan 1985). However, this conjecture remains unproven.

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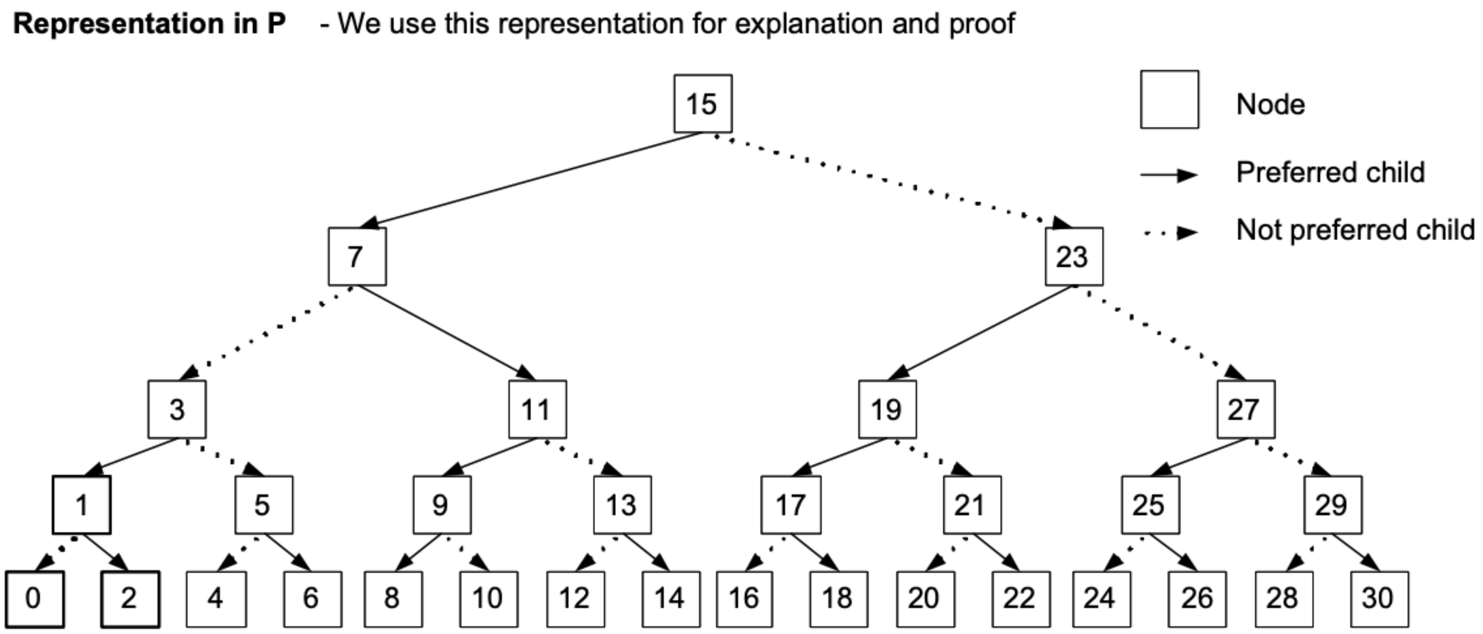
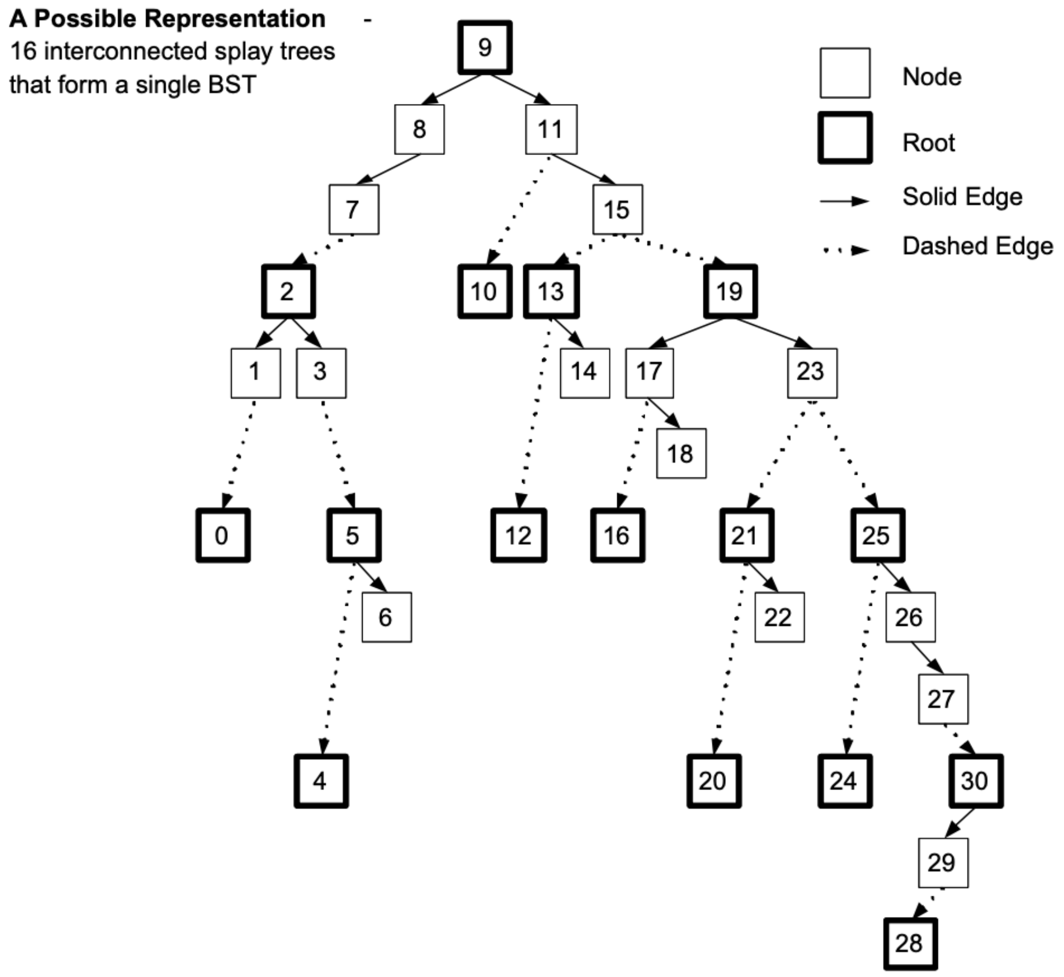
Multi-splay trees (Wang, Derryberry, and Sleator 2006): Somewhat similar to tango trees, but preferred paths are represented as splay trees instead. Better (amortized) worst-case performance.

Multi-splay trees



Source: Sleator and Wang 2004

Multi-splay trees



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Multi-splay implementation

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Augmented tree (of splay trees) with Query method implemented

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Only support static trees, augmentations depend on a perfect binary search tree

Multi-splay Query implementation

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Left2Right(node) and Right2Left(node)
change the preferred child of node

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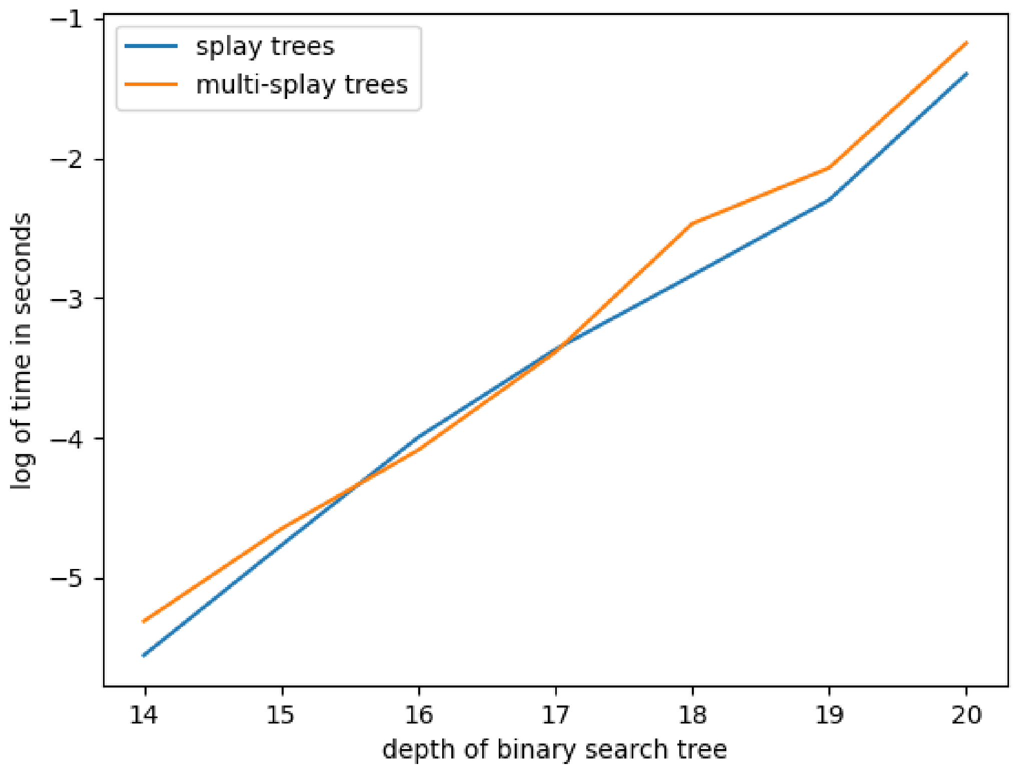
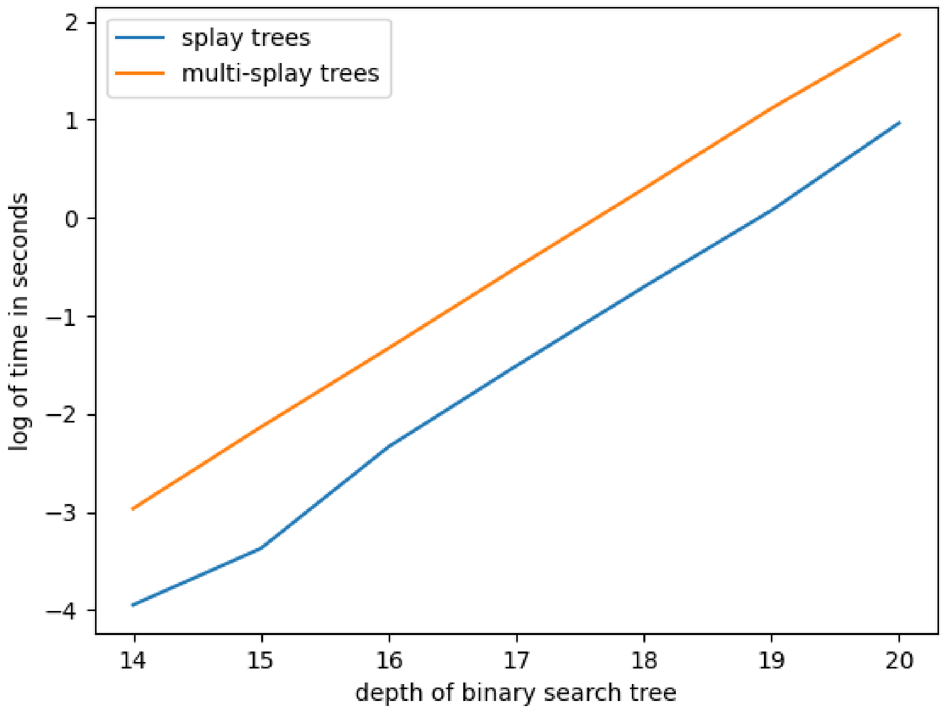
Left2Right(node) and Right2Left(node)
change the preferred child of node

Change the preferred children top-down
and splay such nodes

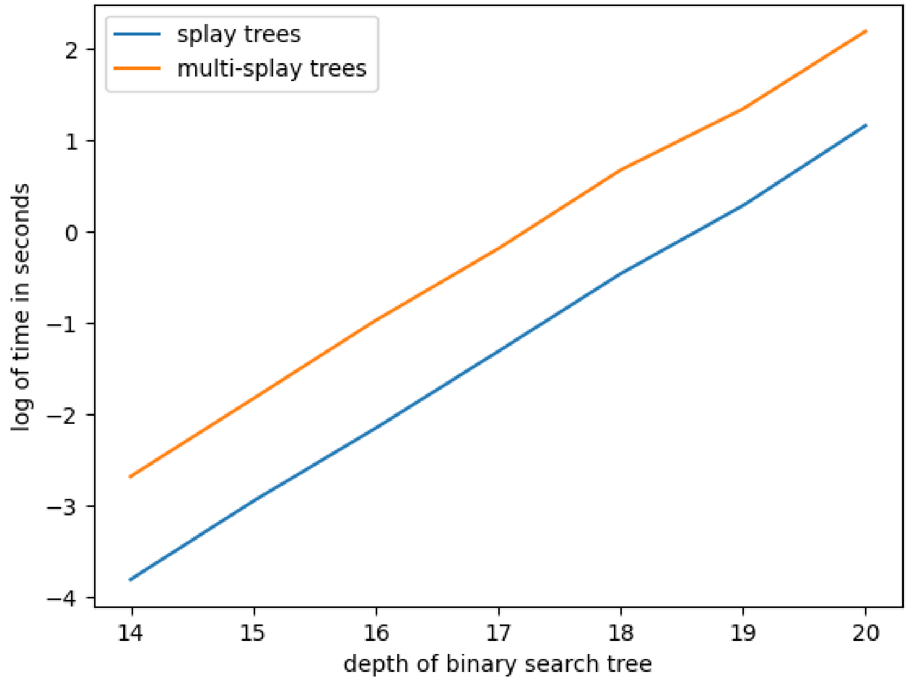
Multi-splay benchmarks: sequential access



Multi-splay benchmarks: random access



Multi-splay benchmarks: bit reversal sequence



Tango trees

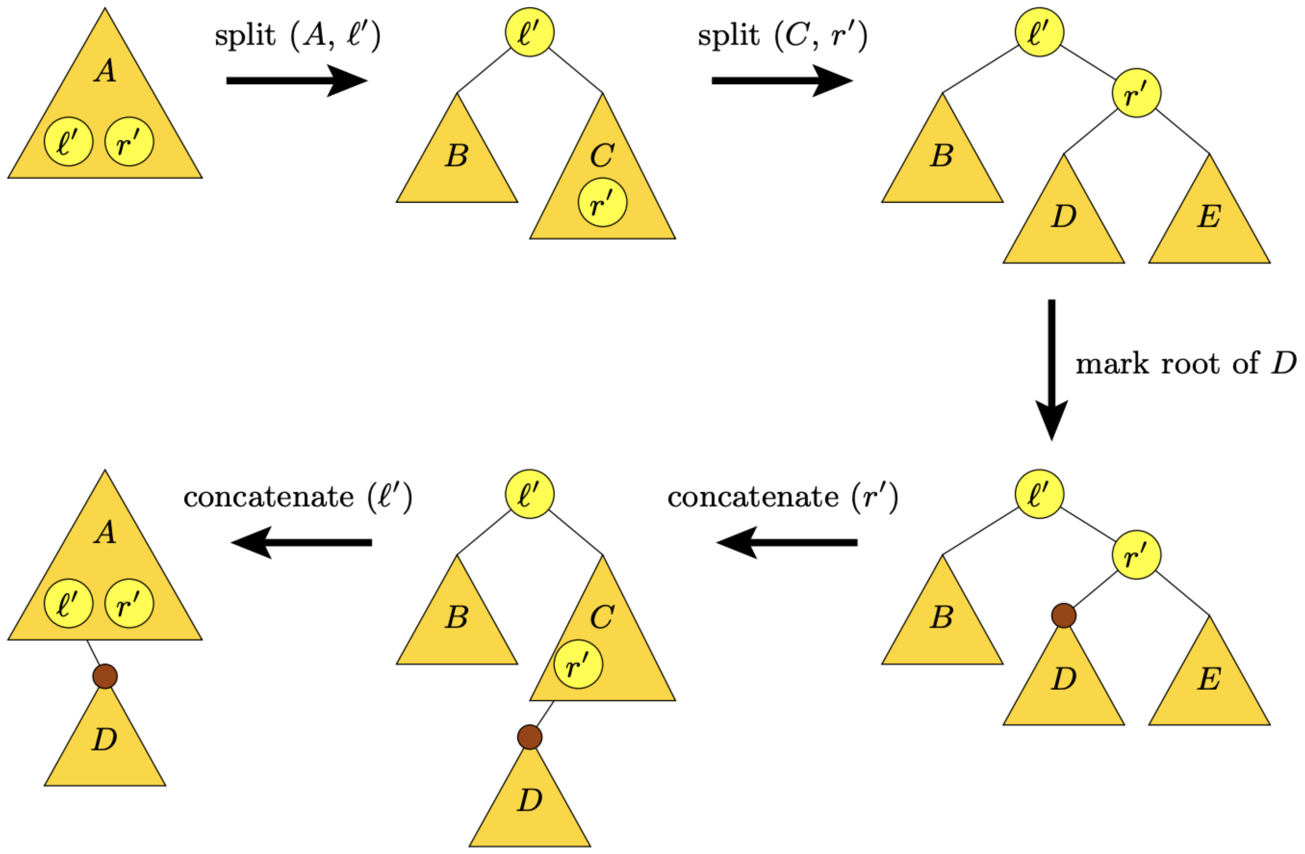


FIG. 3.1. *Implementing cut with split, mark, and concatenate.*

Source: Demaine et al. 2005

Tango implementation progress

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Augmented red-black tree (of trees) with split/concatenate implemented

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API: lock() (read-only) and unlock() (write-only)

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Challenge: the implicit tree-of-trees representation makes everything trickier!

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(Seemingly) need to implement perfect tree → tango tree conversion

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Empirical work: benchmarks against multi-splay implementation and the like

Thank you!