Supplementary Material of
Gaining-Sharing Knowledge Based Algorithm Assisted Differential Evolution

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1. Algorithms

1.1. AMCDE

The pseudo-Code of AMCDE is given in Algorithm S1. In Algorithm S1, Algorithm S2 is called. The algorithms are followed by more details.

$$\vec{v}_{i,g} = \vec{x}_{i,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{i,g} + \vec{x}_{r1,g} - \vec{x}_{r3,g}), \tag{1}$$

$$\vec{v}_{i,g} = \vec{x}_{i,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{i,g} + \vec{x}_{r1,g} - \vec{x}_{r2,g}), \tag{2}$$

$$\vec{v}_{i,g} = F_i \cdot \vec{x}_{r1,g} + F_i \cdot (\vec{x}_{\phi,g} - \vec{x}_{r2,g}). \tag{3}$$

In the three equations, $r1 \in \{1, 2, ..., NP\}$, $r2 \in \{1, 2, ..., NP\}$, $r3 \in \{1, 2, ..., NP + |A|\}$, and $r1 \neq r2 \neq r3 \neq i$. In fact, A is used to store individuals eliminated from the population. $|A|_{min} = 0$, while $|A|_{max} = NP$. Moreover, $\vec{x}_{\phi,g}$ is among the best ϕ individuals in the gth generation. Here, F_i is F for the ith position of the population.

$$u_{j,i,g} = \begin{cases} \begin{cases} v_{j,i,g}, & if\ rand(0,1) \leq CR\ or\ j = randn(i), \\ x_{j,i,g}, & otherwise, \end{cases} & if\ rand(0.1) \leq p \\ \begin{cases} v_{j,i,g}, & for\ j = \langle l \rangle_D, \langle l+1 \rangle_D, ..., \langle l+L-1 \rangle_D \\ x_{j,i,g}, & for\ all\ other\ j \in [1, D], \end{cases} & otherwise, \end{cases} \tag{4}$$

where p is the probability of the binomial manner. That is, the exponential manner has the probability $1 - p$. Moreover, $l \in \{1, 2, ..., D\}$ is randomly decided during execution, while $L \in \{1, 2, ..., D\}$, as a parameter, requires to be set before execution.

Let $\overline{\Delta f}_k$ be the average improvement from target vector to trial vector among all the positions in the population controlled by the kth mutation strategy. The controlling ratio of the kth mutation

Algorithm 1 Pseudo-Code of AMCDE

Input: NP_{init}, NP_{min}, $MaxFES$, G_n, p_{bc1}, p_{bc2}, p_w, and p_r;

Parameter: g_n, g, FES;

Output: S

1: Obtain the initial generation of the population P_0
2: $FES = NP_{init}$, $g_n = 0$, $g = 0$
3: Evaluate individuals in the population
4: $FES = FES + NP_{init}$ and $g = g + 1$
5: Choose DE/current-to-ϕbest/1 with archive shown in Equation S1 as the monopolizer
6: **while** $FES < MaxFES$ **do**
7:    **if** $g_n < G_n$ **then**
8:       Execute the chosen mutation strategy
9:       Execute the binomial or exponential crossover in Equation S4 where $p = p_{bc1}$
10:      Evaluate offspring
11:      Execute Equation 2 for selection
12:      **if** the best fitness is not improved **then**
13:         $g_n = g_n + 1$
14:      **else**
15:         $g_n = 0$
16:      **end if**
17:    **else**
18:      Execute Algorithm S2 with parameters, (P_g, p_{bc2}, p_w, p_r), for a generation
19:      **if** the best fitness is improved **then**
20:         $g_n = 0$ and $G_n = G_n + 1$
21:         Choose the mutation strategy obtaining the new best fitness among Equations S1-S3 as the new monopolizer
22:      **end if**
23:    **end if**
24:    $FES = FES + NP$ and $g = g + 1$
25:    **if** $FES \geq 0.85 \cdot MaxFES$ **then**
26:      Apply the local search method based on sequential quadratic programming
27:      Update FES
28:    **end if**
29:    Decrease NP according to the linear population size reduction based on NP_{init} and NP_{min}
30: **end while**
31: Report the final solution S

Algorithm 2 Pseudo-Code of a generation with competition

Input: P_g, p_{bc2}, p_w, p_r and Cr;

Parameter: $S_{k,g}(k = 1, 2, 3)$, and $S_{win,g}$

Output: P_{g+1}

1: Equal number of positions are randomly allocated to the three mutation strategies and grouped into $S_{k,g}$

2: c_k is computed according to Equation S5

3: $win = 1$

4: **for** $x = 1$ to 3 **do**

5:     **if** $c_x > c_{win}$ **then**

6:         $win = x$

7:     **end if**

8: **end for**

9: **if** g is not the first generation in the current round of competition **then**

10:     **for** $x = 1$ to 3 **do**

11:         **if** $x \neq win$ **then**

12:             $S_{x,g} = S_{x,g} \backslash S_{win,g-1}$

13:         **else**

14:             $S_{x,g} = S_{x,g} \cup S_{win,g-1}$

15:         **end if**

16:     **end for**

17: **end if**

18: Execute mutation strategy of each position

19: Execute the binomial or exponential crossover in Equation S4 where $p = p_{bc2}$ and Cr is set based on Equation S6

20: Evaluate offspring

21: Execute the revised selection method where, at p_r, each of the worst $p_w\%$ target vectors are replaced by its worse trial vector

22: Compute c_k according to Equation S5

23: Find $win \in \{1, 2, 3\}$ to make $c_{win} = max(c_1, c_2, c_3)$

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strategy c_k is computed as below,

$$c_k = \frac{\overline{\Delta f}_k}{\sum_{i=1}^{3} \overline{\Delta f}_i}, \tag{5}$$

where $k = \{1, 2, 3\}$. Then, c_k is adjusted to ensure $c_k \geq 0.1$ on the constant premise of $\sum_{i=1}^{3} c_k = 1$. In detail, if $c_m < 0.1$ ($m \in 1, 2, 3$), c_m is turned to 0.1 by reducing c_{max}.

$$Cr = \begin{cases} 0, & if \ FES < MaxFES \cdot 0.5 \\ (\frac{FES}{MaxFES} - 0.5) \cdot 2, & otherwise. \end{cases} \tag{6}$$

1.2. APGSK

The pseudo-Code of APGSK is given in Algorithm S3. The algorithm is followed by more details.

The process of adapting the control parameters begins by choosing a pool for the two parameters and probability parameter kw_p. The pool used for setting the parameters consists of the following two pairs (K_f, K_r): [(0.1, 0.2), (1.0, 0.1), (0.5, 0.9), and (1.0, 0.9)] which is applied during first 50% of $MaxFES$ while the another pairs: [(-0.15, 0.2), (-0.05, 0.1), (-0.05, 0.9), and (-0.15, 0.9)] will be activated after 50% of $MaxFES$ with probability less than 0.3 for enhancing the diversity of the population to ensure escaping from local optima and to reduce possibility of stagnation. The probability parameter kw_p includes a probability parameter p for each setting of the above-mentioned pool of settings. Therefore, every individual in the population will be assigned only one setting according to its probability parameter p.

The probability parameter adaptation kw_p will start after 10% of the function evaluations. The adaptation of the probability parameter will depend on the performance of each setting via the following formula:

$$\omega_{ps} = \sum_{i=1}^{n} [f(\vec{x}_{i,g}^{new}) - f(\vec{x}_{i,g})], \tag{7}$$

where ω_{ps} represents the sum of the differences between old fitness value and the new fitness value for every individual belonging to parameter setting ps, $f(\cdot)$ represents the fitness function, $\vec{x}_{i,g}^{new}$ is the new solution, $\vec{x}_{i,g}$ is the old solution, and n represents the number of solutions that belong to the parameter setting ps. After that, the improvement rate (Δps) could be calculated for each parameter setting by

$$\Delta_{ps} = max(0.05, \frac{\omega_{ps}}{\sum(omega_{ps})}), . \tag{8}$$

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Algorithm 3 Pseudo-Code of APGSK

Input: NP_{init}, NP_{min}, $MaxFES$;

Parameter: NP, FES, k_f, k_r, and kw_p

Output: P_{g+1}

  1: Obtain the initial generation of the population with individuals x_i $(i = 1, 2, ..., NP)$

  2: $g = 0$

  3: Initialize parameters: N, k_f, k_r, K and P

  4: Evaluate the individuals

  5: **while** $FES < MaxFES$ **do**

  6:     **if** $FES > 0.1 \cdot MaxFES$ **then**

  7:        Update kw_p

  8:     **end if**

  9:     Compute The number of dimensions for junior action D_j and that for senior action D_s based on k_r according to Equations 4 and 5, respectively

10:     Junior action is executed at the probability k_r in the D_j dimensions according to Equations 6 or 7

11:     Senior action is executed at the probability k_r in the D_s dimensions according to Equations 8 or 9

12:     **for** i=1 to NP **do**

13:        Assign value of k_f and k_r for $\vec{x}_{i,g}$

14:        **if** $f(\vec{x}_{i,g}^{new}) < f(\vec{x}_{i,g})$ **then**

15:           $\vec{x}_{i,g} = \vec{x}_{i,g}^{new}$

16:        **end if**

17:     **end for**

18: **end while**

Here, 0.05 is used to express the minimum probability that could be assigned for each parameter setting in order to guarantee that all settings have a probability of being selected. The improvement rate (Δps) for each parameter setting is used for updating kw_p due to the following formula:

$$kw_{p,g+1} = (1-c)kw_{p,g} + c \cdot \Delta_{ps}, . \tag{9}$$

where c represents the learning rate. A constant learning rate c is used in order to make a benefit from the cumulative knowledge about each parameter setting's performance.

2. Tables

Table 1: Classification of the functions in the two CEC benchmark test suites

Type	CEC 2020	CEC 2022
Unimodal functions	F1	F1
Basic functions	F2-F4	F2-F5
Hybrid functions	F5-F7	F6-F8
Composition functions	F8-F10	F9-F12

Table 2: Setting of the CEC 2020 and CEC 2022 benchmark test suites

D	MaxFES	
	2020	2022
5	5.00E+04	-
10	1.00E+06	1.00E+06
15	3.00E+06	-
20	1.00E+07	1.00E+07

Table 3: The problems selected from the CEC 2011 suite

Problem number	Problem name	Dimensionality
P1	Parameter estimation for frequency modulated sound waves	6
P3	Bifunctional catalyst blend optimal control problem	1
P4	Optimal control of a non-linear stirred tank teacto	1
P8	Transmission network expansion planning problem	7
P10	Circular antenna array design problem	12
P11.3	Static economic load dispatch instance 1	6
P11.4	Static economic load dispatch instance 2	13
P11.5	Static economic load dispatch instance 3	15

Table 4: Value of $MaxFES$ for the problems selected from the CEC 2011 suite

D	$MaxFES$
$D \leq 5$	5.00E+04
$5 < D \leq 10$	1.00E+06
$10 < D \leq 15$	3.00E+06

Table 5: Settings of the involved Peers

Algorithm	Parameters
IMODE	$NP_{max} = D^2 \cdot 6$, $NP_{min} = 4$, $A_{rate} = 2.60$, $H = D \cdot 20$, $FES_{LS} = MaxFES \cdot 0.85$, and $p = 0.3$
APGSK	$NP_{max} = D \cdot 200$, $NP_{min} = 4$, $p = 0.05$, $r_{super} = 5\%$, $r_{medium} = 90\%$, $r_{inferior} = 5\%$, and $c = 0.05$
APGSK-IMODE	$NP_2^{max} = \frac{D \cdot 30}{4}$, $NP_2^{min} = 12$, $NP_1^{max} = D \cdot 30 - NP_2^{max}$, $NP_1^{min} = 4$, and $CS = 50$
NL-SHADE-RSP	$NP_{max} = 30D$, $M_{f,r} = 0.2$, $M_{CR,r} = 0.2$, and $n_A = 0.5$
MLS-L-SHADE	$NP_{max} = D \cdot 18$, $NP_{min} = 4$, $A_{rate} = 2.60$, and $H = D^2 \cdot 0.36$,
EA4eig	$NP_{max} = 100$ and $NP_{min} = 10$
AMCDE	$NP_{max} = D^2 \cdot 6$, $NP_{min} = 4$, $A_{rate} = 2.60$, $H = D \cdot 20$, $FES_{LS} = MaxFES \cdot 0.85$, and $p = 0.3$, $G_{n\_init} = 5$, $p_{bc1} = 0.4$, $p_{bc2} = 0.4$, $p_r = 0.01$, and $p_w = 0.2$

Table 6: The results for the CEC 2020 benchmark test functions when $D = 10$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average (standard deviation)							
	IMODE	APGSK	APGSK-IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	APGSK-A-AMCDE
F1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)
F2	3.93E+00	2.06E+01	6.04E+00	7.03E-01	8.73E+00	4.42E+00	5.52E+00	5.62E+00
	(3.41E+00)+	(1.11E+01)−	(4.80E+00)≈	(2.06E+00)+	(5.07E+00)−	(5.10E+00)+	(3.11E+00)≈	(3.11E+00)
F3	1.24E+01	9.51E+00	1.20E+01	7.97E+00	1.02E+01	1.07E+01	9.91E+00	1.08E+01
	(7.58E-01)−	(3.77E+00)≈	(7.73E-01)≈	(1.85E+01)+	(3.15E+00)≈	(1.22E+00)+	(3.05E+00)+	(2.56E+00)
F4	2.96E-03	4.09E-02	1.14E-01	0.00E+00	0.00E+00	3.08E-01	0.00E+00	0.00E+00
	(4.60E-03)−	(2.24E-02)−	(7.40E-02)−	(0.00E+00)≈	(0.00E+00)≈	(7.35E-02)−	(0.00E+00)≈	(0.00E+00)
F5	4.23E-01	4.27E-01	5.51E-01	2.47E-01	2.53E+00	5.55E-02	1.87E-01	3.51E-01
	(3.88E-01)≈	(3.08E-01)≈	(4.05E-01)−	(7.64E-02)+	(2.13E+00)−	(9.36E-02)+	(1.38E-01)+	(2.86E-01)
F6	1.20E-01	7.43E-02	7.30E-02	6.90E-02	3.94E-01	4.78E-02	3.97E-02	6.05E-02
	(7.22E-02)−	(6.92E-02)≈	(9.29E-02)≈	(5.83E-03)≈	(2.13E-01)−	(9.67E-02)+	(2.77E-02)≈	(6.21E-02)
F7	6.76E-04	7.49E-02	8.52E-04	1.10E-03	1.36E-01	2.09E-02	1.28E-04	6.93E-04
	(7.30E-04)≈	(1.58E-01)−	(1.53E-03)≈	(1.37E-06)−	(1.50E-01)−	(7.96E-02)−	(1.61E-04)+	(1.73E-03)
F8	2.72E+00	0.00E+00	3.20E+01	3.29E+00	1.28E+01	1.00E+02	2.45E+00	1.15E+00
	(7.46E+00)≈	(0.00E+00)≈	(3.62E+01)−	(5.70E+01)−	(1.95E+01)−	(0.00E+00)−	(7.27e+00)≈	(3.51E+00)
F9	4.15E+01	1.33E+01	8.85E+01	6.44E+01	7.42E+01	2.56E+02	6.43E+01	5.40E+01
	(4.38E+01)≈	(3.46E+01)+	(3.11E+01)−	(2.28E+03)≈	(3.77E+01)−	(9.65E+01)−	(4.80E+01)−	(5.02E+01)
F10	3.98E+02	1.00E+02	3.58E+02	3.68E+02	3.68E+02	4.09E+02	3.68E+02	2.89E+02
	(3.42E-12)−	(8.92E-05)+	(1.03E+02)−	(8.25E+03)−	(8.92E+01)−	(1.96E+01)−	(9.08E+01)−	(1.46E+02)
Wilcoxon −/+/≈	4/1/5	3/2/5	5/0/5	2/3/5	7/0/3	5/4/1	1/3/6	-/-/-
Friedman ranking	4.65	4.25	5.75	3.60	5.90	5.35	3.00	3.50

Table 7: The results for the CEC 2020 benchmark test functions when $D = 20$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "$+$" or "$-$" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "$\approx$" represents that there is no significant difference

Function		IMODE	APGSK	APGSK-IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	APGSK-A-AMCDE
					Average (standard deviation)				
F1		0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2		3.46E-01 (5.66E-01)+	2.92E+00 (1.54E+00)−	2.63E-01 (5.13E-01)+	1.98E-02 (4.36E-04)+	1.27E+00 (9.83E-01)≈	3.53E+00 (2.30E+00)−	1.25E+00 (1.01E+00)≈	1.71E+00 (1.63E+00)
F3		2.05E+01 (1.24E-01)≈	2.04E+01 (1.73E-01)−	2.04E+01 (0.00E+00)+	2.04E+01 (0.00E+00)+	2.07E+01 (1.37E+00)−	2.22E+01 (8.19E-01)−	2.04E+01 (7.77E-02)≈	2.04E+01 (6.82E-02)
F4		5.00E-01 (8.08E-02)≈	2.07E-01 (7.64E-02)+	5.50E-01 (5.73E-02)≈	0.00E+00 (0.00E+00)+	0.00E+00 (0.00E+00)+	7.24E-01 (1.32E-01)−	4.74E-01 (8.27E-02)≈	5.18E-01 (1.07E-01)
F5		1.05E+01 (4.11E+00)−	3.51E+01 (4.89E+01)−	6.57E+00 (3.41E+00)−	1.37E+01 (9.28E+02)≈	5.05E+00 (6.98E+00)≈	1.26E+01 (2.97E+01)≈	4.24E+00 (2.77E+00)≈	4.13E+00 (2.69E+00)
F6		2.87E-01 (7.40E-02)−	1.79E-01 (5.50E-02)≈	2.03E-01 (6.80E-02)≈	1.38E-01 (1.83E-03)≈	3.39E-01 (7.83E-02)−	1.63E-01 (9.35E-02)≈	1.77E-01 (1.24E-01)≈	2.00E-01 (1.11E-01)
F7		5.10E-01 (1.78E-01)≈	7.64E-01 (7.97E-01)≈	1.43E+00 (2.88E+00)−	2.05E-01 (7.73E-03)+	6.28E-01 (1.51E-01)−	5.72E-01 (1.56E+00)−	4.85E-01 (1.33E-01)≈	5.43E-01 (1.46E-01)
F8		8.42E+01 (1.83E+01)≈	9.39E+01 (1.27E+01)≈	9.82E+01 (6.09E+00)−	8.80E+01 (3.29E-02)≈	9.31E+01 (1.93E+01)≈	1.00E+02 (2.12E-13)−	9.23E+01 (1.59E+01)≈	8.32E+01 (2.57E+01)
F9		9.67E+01 (1.83E+01)≈	8.67E+01 (3.46E+01)≈	1.32E+02 (9.98E+01)≈	1.00E+02 (3.43E-26)≈	1.00E+02 (1.06E-08)≈	4.06E+02 (3.24E+00)−	9.80E+01 (1.07E+01)≈	9.67E+01 (1.83E+01)
F10		4.00E+02 (4.44E-01)+	3.99E+02 (1.56E-02)+	4.05E+02 (5.66E+00)≈	3.99E+02 (1.19E-01)+	4.11E+02 (3.51E+00)−	4.06E+02 (1.53E-02)−	4.08E+02 (7.20E+00)−	4.02E+02 (5.66E+00)
Wilcoxon −/+/≈		2/2/6	3/2/5	3/2/5	0/5/5	4/1/5	7/0/3	1/0/9	-/-/-
Friedman ranking		4.10	4.50	5.35	2.90	5.35	6.35	3.75	3.70

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Table 8: The results for the CEC 2022 benchmark test functions when $D = 10$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

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F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.51E-01 (7.19E-01)≈	1.20E+00 (1.86E+00)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F3	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F4	6.70E+00 (1.54E+00)−	4.64E+00 (1.46E+00)−	3.48E+00 (1.19E+00)+	9.06E+00 (5.05E+00)−	1.86E+00 (7.59E-01)+	3.32E-01 (6.58E-01)+	2.26E+00 (1.17E+00)+	3.15E+00 (1.72E+00)
F5	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F6	1.22E-01 (9.16E-02)≈	4.29E-02 (3.07E-02)+	6.29E-02 (6.18E-02)≈	3.48E-02 (5.61E-04)+	1.05E-01 (6.71E-02)≈	8.10E-03 (3.65E-02)+	6.66E-02 (6.93E-02)≈	9.16E-02 (8.58E-02)
F7	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.83E-03 (2.16E-03)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F8	8.53E-03 (1.06E-02)≈	1.00E-01 (8.50E-02)−	5.66E-03 (6.90E-03)≈	2.23E-02 (5.84E-04)−	1.82E+00 (1.64E+00)−	2.57E-03 (4.74E-03)+	8.12E-03 (7.59E-03)≈	6.94E-03 (9.33E-03)
F9	2.22E+02 (4.19E+01)≈	1.48E+02 (1.06E+02)+	2.22E+02 (4.19E+01)≈	2.22E+02 (1.75E+03)≈	2.29E+02 (6.17E-13)≈	1.86E+02 (0.00E+00)+	2.14E+02 (5.82E+01)≈	2.06E+02 (7.00E+01)
F10	1.70E+01 (3.35E+01)−	2.14E+01 (1.12E+01)−	9.46E+01 (1.72E+01)−	0.00E+00 (0.00E+00)+	2.22E-01 (6.22E-01)≈	1.00E+02 (4.02E-02)−	9.50E-01 (1.52E+00)≈	7.04E-01 (1.63E+00)
F11	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F12	1.59E+02 (7.05E-01)≈	1.53E+02 (2.90E+01)+	1.59E+02 (7.65E-01)≈	1.60E+02 (1.33E+03)−	1.59E+02 (2.55e-01)≈	1.46E+02 (2.45E+00)+	1.60E+02 (1.13E+00)−	1.59E+02 (9.16E-01)
Wilcoxon −/+/≈	2/0/10	3/3/6	1/1/10	3/2/7	2/1/9	2/5/5	1/1/10	-/-/-
Friedman ranking	5.08	4.21	4.50	4.67	5.38	3.67	4.42	4.08

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Table 9: The results for the CEC 2022 benchmark test functions when $D = 20$ with outcome of the Wilcoxon rank sum test and that of the Friedman test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	IMODE	APGSK	APGSK-IMODE	NL-SHADE-RSP	MLS-L-SHADE	EA4eig	AMCDE	APGSK-A-AMCDE
				Average (standard deviation)				
F1	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F2	4.27E+01 (1.46E+01)−	0.00E+00 (0.00E+00)+	3.86E+01 (1.64E+01)−	3.62E+01 (3.39E+02)≈	1.34E-01 (7.23e-01)≈	5.32E-01 (1.38E+00)≈	3.46E+01 (1.94E+01)−	6.72E-01 (1.53E+00)
F3	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.89E-14 (4.31E-14)−	3.79E-15 (4.31E-28)≈	0.00E+00 (0.00E+00)≈	7.58E-15 (2.88E-14)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F4	4.87E+01 (6.49E+00)−	3.58E+01 (5.70E+00)−	2.26E+01 (3.64E+00)−	5.19E+01 (7.50E+01)−	5.04E+00 (8.10E-01)≈	8.59E+00 (3.32E+00)−	8.86E+00 (1.35E+01)≈	8.16E+00 (8.82E+00)
F5	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F6	2.88E-01 (7.98E-02)≈	3.14E-01 (9.27E-02)≈	4.72E-01 (4.77E-01)≈	4.44E-01 (1.56E-01)≈	1.88E-01 (6.83E-02)+	1.23E-01 (1.55E-01)+	3.19E-01 (8.74E-02)−	2.65E-01 (1.04E-01)
F7	1.96E+00 (7.58E-01)−	2.64E+00 (2.42E+00)−	1.91E+00 (1.12E+00)≈	5.40E-02 (5.93E-02)+	1.95E+00 (1.14E+00)≈	2.40E+00 (3.56E+00)≈	1.79E+00 (9.29E-01)≈	1.53E+00 (9.04E-01)
F8	1.62E+01 (4.19E+00)≈	1.72E+01 (4.65E+00)≈	1.80E+01 (5.85E+00)−	1.69E+01 (3.56E+01)≈	1.61E+01 (4.46E+00)≈	1.63E+01 (7.56E+00)−	1.35E+01 (6.70E+00)≈	1.57E+01 (4.84E+00)
F9	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)≈	1.81E+02 (7.52E-27)≈	1.81E+02 (8.53E-14)≈	1.65E+02 (0.00E+00)+	1.81E+02 (8.67E-14)≈	1.81E+02 (8.67E-14)
F10	0.00E+00 (0.00E+00)≈	9.44E+01 (1.65E+01)−	1.00E+02 (3.70E-02)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)≈	1.08E+02 (2.93e-01)−	0.00E+00 (0.00E+00)≈	0.00E+00 (0.00E+00)
F11	3.00E+02 (0.00E+00)−	0.00E+00 (0.00E+00)≈	2.40E+02 (1.22E+02)−	2.90E+02 (3.00E+03)−	0.00E+00 (0.00E+00)≈	3.23E+02 (4.30e-01)−	3.00E+02 (0.00E+00)−	0.00E+00 (0.00E+00)
F12	2.34E+02 (1.98E+00)−	2.34E+02 (1.53E+00)−	2.31E+02 (1.80E+00)≈	2.38E+02 (1.22E+01)−	2.32E+02 (8.63E-01)−	2.00E+02 (3.32e-04)+	2.32E+02 (1.68E+00)≈	2.32E+02 (2.22E+00)
Wilcoxon −/+/≈	5/0/7	4/1/5	6/0/6	4/1/7	1/1/10	4/3/5	3/0/9	-/-/-
Friedman ranking	5.17	4.88	5.58	5.33	3.25	4.42	4.13	3.25

Table 10: The results of the selected CEC 2011 real-world optimization problems with the Wilcoxon rank sum test. "+" or "−" denotes that the current result is significantly better or statistical worse than the result of APGSK-A-AMCDE in terms of the Wilcoxon rank sum test at a 0.05 significance level, respectively. Meanwhile, "≈" represents that there is no significant difference

Function	Average (standard deviation)			
	NL-SHADE-RSP	EA4ig	AMCDE	APGSK-A-AMCDE
P1	0.00E+00	0.00E+00	2.81E-01	1.68E-13
	(0.00E+00)≈	(0.00E+00)≈	(1.54E+00)≈	(6.44E-13)
P3	1.15E-05	1.03E-05	1.15E-05	1.15E-05
	(2.18E-19)≈	(3.65E-06)+	(1.90E-19)≈	(2.05E-19)
P4	1.62E+01	1.46E+01	1.65E+01	1.41E+01
	(3.16E+00)−	(1.78E+00)−	(3.13E+00)−	(2.81E-01)
P8	2.20E+02	2.20E+02	2.20E+02	2.20E+02
	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)≈	(0.00E+00)
P10	-2.17E+01	-2.20E+01	-2.16E+01	-2.17E+01
	(9.76E-02)≈	(5.35E-01)+	(1.48E-03)−	(3.61E-02)
P11.3	1.54E+04	1.54E+04	3.08E+04	1.54E+04
	(1.16E+00)≈	(3.53E-04)≈	(2.69E+04)−	(2.13E-07)
P11.4	1.88E+04	1.81E+04	1.82E+04	1.81E+04
	(1.02E+02)−	(2.18E+01)≈	(1.16E+02)−	(6.09E+00)
P11.5	3.30E+04	3.27E+04	2.47E+05	3.29E+04
	(3.51E+01)−	(4.59E-05)+	(1.97E+05)−	(4.27E+01)
Wilcoxon −/ + / ≈	3/0/5	1/3/4	5/0/3	-/-/-