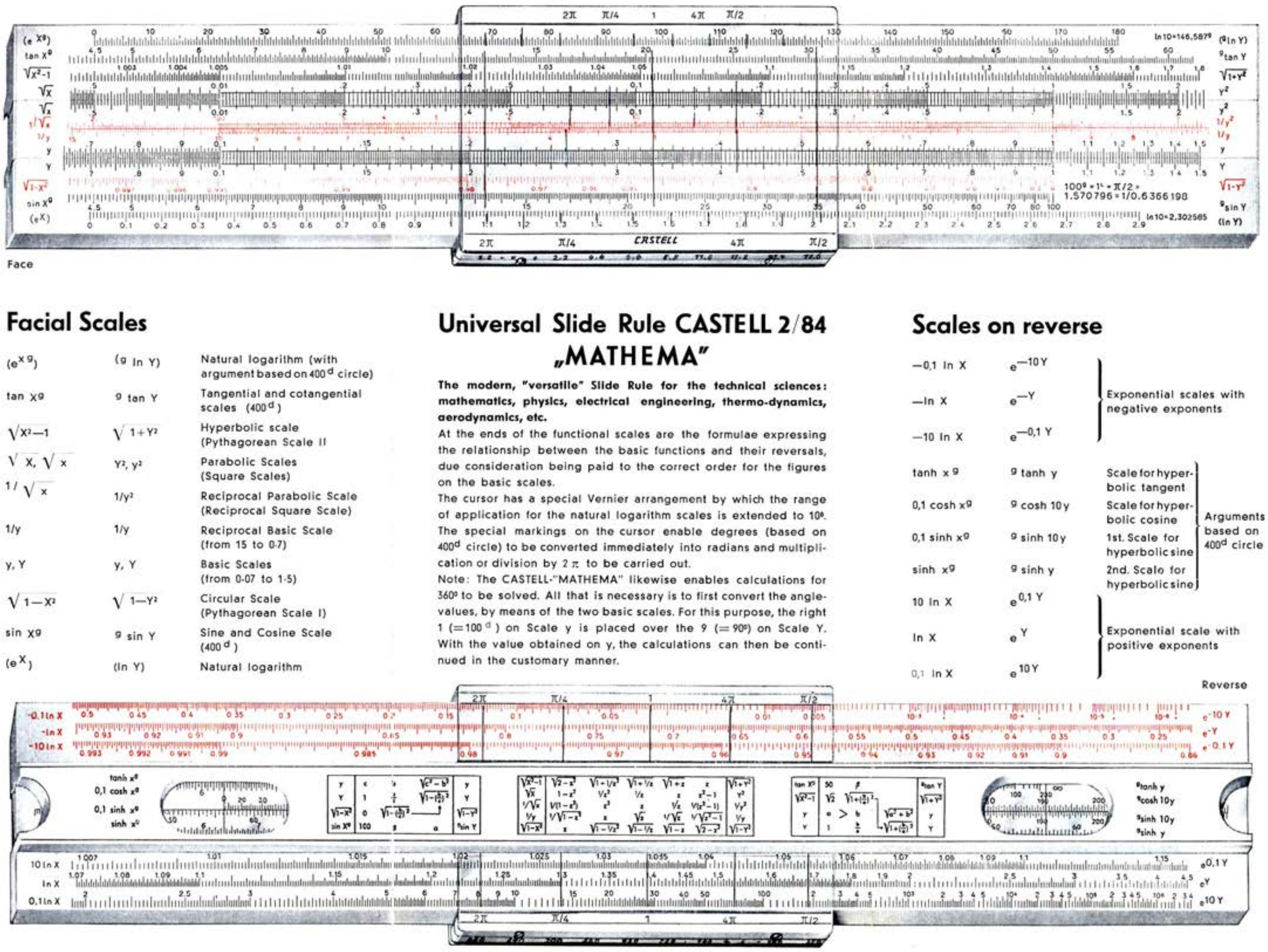
Some Comments Regarding the Faber-Castell Slide Rule Mathema 2/84[1]

Peter Wutsdorff



Face

Facial Scales

$(e^{x\,g})$	$(g \ln Y)$	Natural logarithm (with argument based on 400^d circle)
$\tan x^g$	$g \tan Y$	Tangential and cotangential scales (400^d)
$\sqrt{x^2-1}$	$\sqrt{1+Y^2}$	Hyperbolic scale (Pythagorean Scale II
$\sqrt{X}, \sqrt{x}$	Y^2, y^2	Parabolic Scales (Square Scales)
$1/\sqrt{x}$	$1/y^2$	Reciprocal Parabolic Scale (Reciprocal Square Scale)
$1/y$	$1/y$	Reciprocal Basic Scale (from 15 to 0·7)
y, Y	y, Y	Basic Scales (from 0·07 to 1·5)
$\sqrt{1-x^2}$	$\sqrt{1-Y^2}$	Circular Scale (Pythagorean Scale I)
$\sin x^g$	$g \sin Y$	Sine and Cosine Scale (400^d)
(e^X)	$(\ln Y)$	Natural logarithm

Universal Slide Rule CASTELL 2/84 „MATHEMA"

The modern, "versatile" Slide Rule for the technical sciences: mathematics, physics, electrical engineering, thermo-dynamics, aerodynamics, etc.

At the ends of the functional scales are the formulae expressing the relationship between the basic functions and their reversals, due consideration being paid to the correct order for the figures on the basic scales.

The cursor has a special Vernier arrangement by which the range of application for the natural logarithm scales is extended to 10^8. The special markings on the cursor enable degrees (based on 400^d circle) to be converted immediately into radians and multiplication or division by 2π to be carried out.

Note: The CASTELL-"MATHEMA" likewise enables calculations for 360° to be solved. All that is necessary is to first convert the angle-values, by means of the two basic scales. For this purpose, the right 1 ($=100^d$) on Scale y is placed over the 9 ($=90^\circ$) on Scale Y. With the value obtained on y, the calculations can then be continued in the customary manner.

Scales on reverse

$-0{,}1 \ln X$	e^{-10Y}	Exponential scales with negative exponents	
$-\ln X$	e^{-Y}		
$-10 \ln X$	$e^{-0{,}1Y}$		
$\tanh x^g$	$g \tanh y$	Scale for hyperbolic tangent	Arguments based on 400^d circle
$0{,}1 \cosh x^g$	$g \cosh 10y$	Scale for hyperbolic cosine	
$0{,}1 \sinh x^g$	$g \sinh 10y$	1st. Scale for hyperbolic sine	
$\sinh x^g$	$g \sinh y$	2nd. Scale for hyperbolic sine	
$10 \ln X$	$e^{0{,}1Y}$	Exponential scale with positive exponents	
$\ln X$	e^{Y}		
$0{,}1 \ln X$	e^{10Y}		

Reverse

FIGURE 1.
Faber-Castell MATHEMA 2/84 Leaflet in English

This contribution was received through the kind help of Dieter von Jezierski, who notes that:

it is especially intended for all current Mathema fans, and those yet to come.

We know that the Faber-Castell Mathema models 2/84 and 2/84N are highly regarded.

Many have Mathemas; many seek them; but not everyone understands them.

Professor Peter Wutsdorff has a 2/84, understands it, and can't "accept" it completely.

General

As a certified engineer I have used the slide rule to carry out many calculations relating to structural engineering and the construction of steam and gas turbines. As a student I was already using the slide rule in mathematics classes. In my lectures, as a kind of loosening up exercise, I would explain to my students (who were "born with electronic calculators") how calculations used to be carried out. In the process I would get into the proportional dividers used by the old master-builders, the tables of logarithms, the mechanical calculators (e.g., Walther and Curta), and the slide rule. I gave an extensive lecture on this subject at the observatory at Heppenheim.

The following is intended to raise a few questions regarding some points about the Faber-Castell Mathema 2/84 (FC 2/84) that are not clear. Perhaps some reader can help me so that "I don't die dumb." The slide rules Faber-Castell Novo Duplex 2/83N and the Reiss duplex 3227 serve for purposes of comparison.

The first thing to be noticed is that the effective scale length is only 20 cm long, which makes it more difficult to read the scales. However, the extensions on the right and left are wide and are very useful in some cases. At this point we should consider the special features of the FC 2/84.

Scales

The mathematically inclined engineer is accustomed to seeing a mathematical function in the form $y = f(x)$. Why the identical C and D scales are designated Y and y respectively and the inverse functions on the left designated $x = f(y)$ is difficult to understand. Why are the identical C and D scales designated differently with *y* and Y. Hughes [1] praises this labeling. In the instruction manual it is certainly an advantage.

Cursor

The very wide cursor is striking. It allows the 4π, 2π, $\pi/2$, and $\pi/4$ marks to be displayed over the Y/y and Y2/y2 scales, an undeniable advantage. Also the availability of the 4π, 2π, $\pi/2$, and $\pi/4$ marks on the reverse of the slide rule is often very useful for calculations that are continued on the reverse and involve the functions $e^{\pm Y}$.

The small hairlines on the upper and lower border of the cursor (which are related to the numbers on the two side edges of the cursor) will be treated below.

The Scales in Detail

The "ln" scales above and below, the $\sqrt{(Y^2+1)}$ scale, and of course the hyperbolic function scales are all new.

Trigonometric Functions

It is understandable that the angle argument of the trigonometric functions is based on *Neugrad* (New Degrees). For example, modern theodolites are known to be graduated in Neugrad. It is simply easier to work in Neugrad. However, in most cases engineers still think in old degrees, something that Hughes [1] mentions too. The angle in old degrees is converted to the corresponding angle in Neugrad by multiplying by 10/9=1.111 because 100 Neugrad corresponds to 90 old degrees. Hughes provides a very nice table for converting between the various measurements of angles, including the conversion of old degrees and Neugrad to radians (multiplying by $\pi/180$ and $\pi/200$ respectively).

"ln" Functions on the Front Scale Face[2]

It takes some time to get used to the "ln" scales on the front of the slide rule. The lower "ln" scale (black) is for arguments larger than 1, and the upper "ln" scale (red) is for arguments less than 1. In addition it is difficult to understand why the upper "ln" scale is multiplied by the factor $200/\pi \approx 63.7$.

The instruction book does not explain the use of these scales in sufficient detail, at least as far as the non-mathematician is concerned. In the case of these scales one must use the cursor hairlines positioned over the "ln" scales. On the edge of the cursor one finds the numbers *x* along the small hairlines on the upper and lower border of the cursor: on the lower side the values $(2.2)n$, where $n = -1, 1, 2 \ldots 8$, and on the upper side $(140)n$, where $n = -2, -1, 1, 2 \ldots 7$. On the upper and lower border of the cursor one finds the hairlines (corresponding to the numbers on the cursor edges) under which the required "ln" values can be read. But the direct value is only at *x*. The numbers on the upper and lower edge of the cursor have to be added to, respectively subtracted from the corresponding values on the "(ln Y)"-scale, as the "ln" of the chosen hairline value increases/decreases by a power of ten at each step. Hughes also [1] supplies clear and understandable tables for this function. Anyhow he is not quite satisfied with the FC manuals.

It sounds complicated, when phrased in these general terms. Examples will be used to explain further:

ln (3) = ?

The middle hairline is set to 0.3 on the Y-scale, and the "ln" value 1.1 is read under the small hairline *x*.

If however we require ln (30), then the middle hairline is set again to 0.3 on the Y-scale and now the value 1.2 is read under the small hairline corresponding to the number 2.2 on the edge of the cursor. This number has to be added to 1.2, so the result is ln (30) = 1.2 + 2.2 = 3.4.

Likewise ln (3000) = 1.41 + 6.6 = 8.01, now to be read under the hairline at edge number 6.6.

Or ln (0.3) = 1.0, to be read at hairline –2.2, so 1.0 – 2.2 = –1.2.

The upper $(\text{-lnY})^g$ scale in red is used in a similar way. Here too, some examples:

$(\ln 0.5)^g = -44.1$. But, if ln (0.5) is required, the value –44.1 has to be divided by 63.7, therefore

–44.1 : 63.1 = - 0.69.

$(\ln 0.05)^g = -50.7$; in this case one has to add –140, therefore – 190.7.

Then ln (0.05) is – 190.7 : 63.7 = – 2.99.

Or $(\ln 0.005)^g = 57.2 - 280 = -337.2$, respectively ln (0.005) = – 337.2 : 63.7 = 5.3.

$(\ln 5)^g = -37.5 + 140 = 102.5$, and ln (5) again is 102.5 : 63.7

= 1.6.

It is therefore achieved by subtraction of the constant 140 at the lower scale, but conclusively by dividing by 63.7.

Using these “ln”-scales the precision is in some ranges a bit higher than with the regular $e^{\pm Y}$ - scale. The real need for this is determined by the specific problem addressed. The advantage of the lower “ln”-scale is that “ln” of considerably larger values (in the order of magnitude of 10^8) can be determined than with the regular e^x – scale which only runs up to about 10^5.

On the upper ln-scale one can determine “ln” values of $(1–\varepsilon)$, with $\varepsilon <<1$, down to about 10^{-7}.

The Hyperbolic functions

These functions on the back of the slide need as argument the angle in grads.

Hyperbolic functions are known to be defined as:

$\sinh(x) = (e^x – e^{-x}) / 2$, and $\cosh(x) = (e^x + e^{-x}) / 2$

Why the 2/84 requires the argument in grads, is not clear to me. Every argument x thus has to be multiplied by 63.7.

Example: sinh (0.5) = sinh (0.5^g) = sinh (0.5 • 63.7) = sinh (31.8) = 0.52, to be read on the *y*-scale at the mark 1 on the Y-scale. Same applies to the inverse function arcsinh etc.

This works easier on the Reiss 3227!

In structural engineering statics the differential equation of the catenary line, or free hanging line, results in a hyperbolic function, as well as the theory of beam bending of the second order (cf. for example I. Szabó: *Repertorium und Übungsbuch der Technischen Mechanik*, Springer 1963). In these cases always the regular angle units (360°) are needed in the hyperbolic function arguments. Hughes [1], in his article on the FC 2/84, proposes to use the hyperbolic function on Euler’s relation for vector and complex number calculations. He considers the formulae presented by him as easy.

I can not agree with that.

The $\sqrt{(Y^2+1)}$ scale

This is very useful, in combination with the $\sqrt{(Y^2+1)}$ scale; it describes, as explained in the instruction manual, the relations of the unity hyperbolic function $x^2 - y^2 = 1$. A very good description of the relation between trigonometric and hyperbolic functions can be found e.g. in Hort-Thoma: *Differentialgleichungen in Technik und Physik*, J.A. Barth, Leipzig 1949.

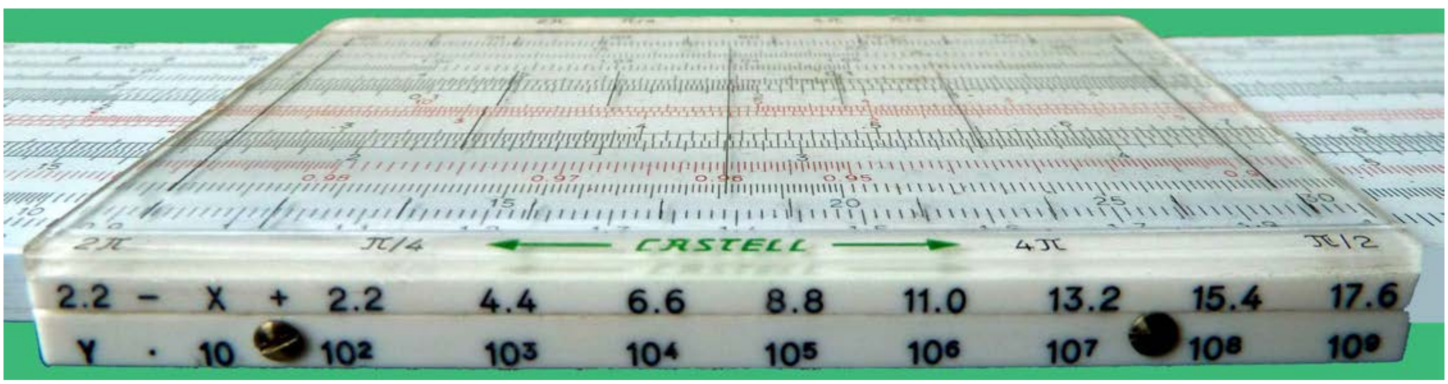


FIG 2.

The hairline and numbering system on the upper side of the cursor is comparable:

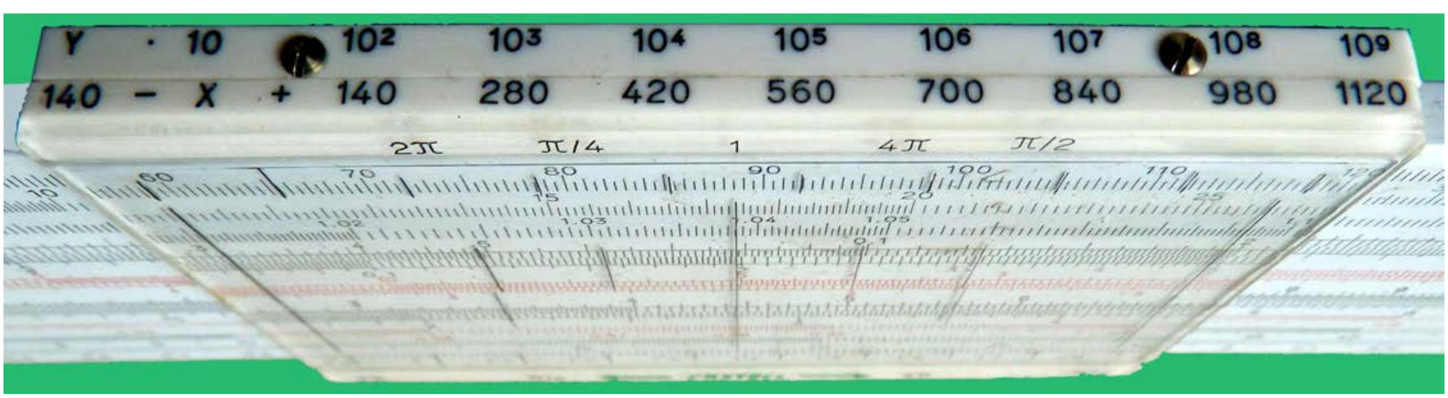


FIG 3.

Conclusion

As final remark it can be said that the 2/84, to put it mildly, needs much effort to get accustomed to. For daily use by engineers it has to be considered as impractical. Hughes [1] too reaches in his final remarks the conclusion that the FC-2/84 is more geared towards mathematicians. I doubt as to how often mathematicians calculate with numbers, especially with a slide rule; at least I have never heard of it.

The FC-2/84 was probably destined for a very specific class of users.

Reference

1. Hughes, R.S.: FC-2/84 and FC-2/84N, *Journal of the Oughtred Society*, Vol. 15:2, 2006, p. 58.

Notes

1. Translated from Prof. Wutsdorff's article in German by Rodger Shepherd and Otto van Poelje. JOS Plus indicates that additional material for this article is available on our website www.oughtred.org. For this article a German copy of the booklet *Der logarithmische Universal-Rechenstab MATHEMA* (new version) is included.
2. Note by the Editor: When readers compare Fig. 1 with the description and examples of *the upper "ln" scale (red) is for arguments less than 1 …* i.e. "(-lnY)^g", they may be confused because the leaflet's picture is based on a later version of the 2/84 in which the top scale of the front had been changed from a negative exponent to a positive, running in the same left-to-right direction as the bottom exponential scale.

The consequence of this change was that the color of the upper scale, renamed to (^{g}lnY), became black too. Red was consistently used by Faber-Castell only for scales running from right-to-left.

The booklet by Dr. Ing. E. Moeller *The MATHEMA Universal Logarithmic slide rule* in its 1955 version still describes the old version (red scale "(-lnY)^g"), with a remark on page 24 (or at the last page): *The "Mathema Rule" (new design) has a scale ^{g}lnY running to the right*.

Later versions of this booklet only describe the new version. Known digital copies of the English booklet on internet are the old version. The only known new version is in German, see JOS Plus. The known leaflets of the Mathema show the new version, see fig. 1. Most Mathema pictures on internet show the new version – the one in *Herman's Archive* also shows the new version but with a manufacturing error: the color is still red!

Another confusion might arise because most existing pictures of Mathemas are 2-dimensional computer "scans", which do not show the side edges of the cursor. To make the relation clearer between the small hairlines on the front cursor's lower side and the corresponding numbers on the edge of the cursor, the following figure is added - kindly provided by David Rance from a picture of his own Mathema 2/84:

Prof. em. Dr.-Ing. Peter Wutsdorff, Tassilostr. 4,
D 64653 Lorsch. Telephone (0049) (0) 625151978.
E-mail: pdpwff@t-online.de

Astronomy, Logarithms, and the Slide Rule

Bryan Purcell

In examining the history of a topic such as Astronomy, the obvious tool to follow is the telescope. However, this type of focus can leave out of the picture some of the other tools, materials, ideas, and innovations; thus, diminishing the depth of the topic. Another angle is the examination of Astronomy through the lens of the contributions of logarithms and the derived tool, the slide rule.

Astronomy has a recorded history as old as human civilization. The world has numerous temples and sites that have a connection to the skies above (such as Stonehenge in England and the Great Pyramids in Egypt). The regularity of motion of objects, such as the moon, the sun, the planets, and the stars in the skies complemented the human need to recognize and record patterns. With observation, humans measured and developed tables of information about these objects. This information became the input for many mathematical models. The universe seemed to act as a clock for determining the season and as a tool for navigation. The pooled wisdom of the cultures of the Babylonians, the Egyptians, and the Greeks led, 2000 years ago, to a geocentric model of the solar system in which the orbits of the planets were explained by a system of circles within circles (described by Ptolemy ~ 1900 years ago, based on previous Babylonian and Greek works). This model separated the Earth and the rest of the Universe as having essentially two different sets of laws.

The geocentric model was very complicated and had an increasing number of errors for predicting celestial events. The geocentric mode was questioned then overturned in the 17th century through careful observation that lead to a new heliocentric model that was demonstrated using new mathematical ideas. The mathematical models would lead to a new