Homework 2

学号　　姓名

1. Give regular expressions for the following languages.

   i) The set of all strings with an equal number of 0's and l's, such that no prefix has two more 0's than l's, nor two more l's than 0's.

   ii) $L = \{a^n b^m : n < 4, m \leq 3\}$.

   $(01+10)^*$

   $(\varepsilon + a)(\varepsilon + aa)(\varepsilon + b)(\varepsilon + bb)$ or $(a+\varepsilon)^3(b+\varepsilon)^3$

2. Prove $L = \{0^n | n \text{ is a perfect square}\}$ is not regular.

   取 $s = 0^{N^2}$, 则 $|xy^2z| < N^2 + N < (N+1)^2$, 而且 $|xy^2z| > |xyz| = N^2$. 完整证明：

   设该语言为 L，假设 L 是正则的，
   　那么存在正整数 N，对于任意的 $w \in L(|w| > N)$ 满足泵引理。
   取 $w = 0^{N^2}$，显然 $w \in L$；那么有 $w = xyz$，且 $|xy| \leq N$, $|y| > 0$；
   而且 xy^2z 应属于 L，但是
   　由 $|xy^2z| = |xyz| + |y| = N^2 + |y| \leq N^2 + |xy| \leq N^2 + N < N^2 + 2N + 1 = (N+1)^2$
   　　和 $|xy^2z| > |xyz| = N^2$ (因为 $|y| > 0$)
   　得 $|xy^2z|$ 不可能是完全平方数
   所以 $xy^2z \notin L$，所以与假设矛盾，所以 L 不是正则的。

3. If L is a language, and a is a symbol, then L/a, the quotient of L and a, is the set of strings w such that wa is in L. For example, if $L = \{a, aab, baa\}$, then $L/a = \{\varepsilon, ba\}$. Prove that if L is regular, so is L/a. Hint: Start with a DFA for L and consider the set of accepting states.

   令 $L = L(M)$, 其中 $M = (Q, \Sigma, \delta, q_0, F)$

   构造 $M' = (Q, \Sigma, \delta, q_0, F')$, 其中 $F' = \{q | \delta(q, a) \in F\}, q \in Q, a \in \Sigma$

   先证明 $L(M') = L/a$, 再证明 $L(M')$ 正则

   $\because \forall w \in L(M')$ 即 $\delta(q_0, w) \in F'$ 即 $\delta(\delta(q_0, w), a) \in F \therefore w \in L/a$

   又 $\because \forall w \in L/a$ 有 $wa \in L$ 即 $\delta(q_0, wa) \in F$ 即 $\delta(\delta(q_0, w), a) \in F$ 即 $\delta(q_0, w) \in F' \therefore w \in L(M')$

4. Here is a transition table for a DFA:

	0	1
$\rightarrow q_1$	q_2	q_1
q_2	q_3	q_1
$*q_3$	q_3	q_2

$$R_{ij}^k = R_{ik}^{k-1}(R_{kk}^{k-1})^* R_{kj}^{k-1} \cup R_{ij}^{k-1} \tag{1}$$

$$R_{ij}^0 = \begin{cases} \{a | \delta(q_i, a) = q_j\} & i \neq j \\ \{a | \delta(q_i, a) = q_j\} \cup \{\varepsilon\} & i = j \end{cases} \tag{2}$$

   a) Give all the regular expressions $R_{ij}^{(0)}$, $R_{ij}^{(1)}$ and $R_{ij}^{(2)}$. Try to simplify the expressions as much as possible. Note: Think of state q_i as if it were the state with integer number i.

	$k=0$	$k=1$	$k=2$	$k=2$ 等价于
R_{11}^k	$\varepsilon + 1$	1^*	$1^* + 1^*0(11^*0)^*11^*$	$(1+01)^*$
R_{12}^k	0	1^*0	1^*0	$(1+01)^*0$
R_{13}^k	$\varnothing$	$\varnothing$	$1^*0(11^*0)^*0$	$(1+01)^*00$
R_{21}^k	1	11^*	$(11^*0)^*11^*$	
R_{22}^k	ε	$\varepsilon + 11^*0$	$\varepsilon + 11^*0$	
R_{23}^k	0	0	$(11^*0)^*0$	
R_{31}^k	$\varnothing$	$\varnothing$	$1(11^*0)^*11^*$	
R_{32}^k	1	1	$1(11^*0)^*$	
R_{33}^k	$\varepsilon + 0$	$\varepsilon + 0$	$\varepsilon + 0 + 1(11^*0)^*0$	$\varepsilon + 0 + 10 + 11(1+01)^*00$

   b) Give a regular expression for the language of the automaton.

   $(1+01)^*00(0+10+11(1+01)^*00)^*$