**MATH2070 Project 2016**

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Project completed in MATLAB Live Script File format (.mlx) and exported to PDF format.

All the explanations, plots/images, code, and output are inline and follow a linear order.

# Given information:

```
clear; format compact; format shortG;
info = readtable('tablegiven.csv'); disp(info);
```

```
    Security    Code     Return    Risk
    ________    _____    ______    _____
    'P1'        'BHP'    0.055     0.018
    'P2'        'NAB'     0.07     0.016
    'P3'        'CSR'     0.11     0.022
    'P4'        'AGL'    0.115     0.036
    'P5'        'NCP'    0.125     0.048
```

```
name = char(info.Code);
```

## Fund Returns ($\mu_i$)

```
rBHP = 0.055;
rNAB = 0.070;
rCSR = 0.110;
rAGL = 0.115;
rNCP = 0.125;
vec_r = [rBHP; rNAB; rCSR; rAGL; rNCP];
```

## Fund Risk $(\sigma_i)$

```
sdBHP = 0.018;
sdNAB = 0.016;
sdCSR = 0.022;
sdAGL = 0.036;
sdNCP = 0.048;
vec_sd = [sdBHP; sdNAB; sdCSR; sdAGL; sdNCP];
```

## Fund Variance $(\sigma^2)$

```
vec_var = vec_sd.^2;
```

```
Tvar = table(name, vec_var, 'VariableNames', {'Fund', 'Variance'});
disp(Tvar);
```

```
    Fund    Variance
    ____    ________
    BHP     0.000324
    NAB     0.000256
    CSR     0.000484
    AGL     0.001296
    NCP     0.002304
```

## Correlation Matrix $(\rho_{i,j})$

```
mat_cor = zeros(5,5);
mat_cor(1,3) = -0.25;                % BHP and CSR have negative correlation
mat_cor(3,1) = -0.25;                % BHP and CSR have negative correlation
mat_cor(2,5) = 0.55;                 % NAB and NCP have Positive correlation
mat_cor(5,2) = 0.55;                 % NAB and NCP have Positive correlation
mat_cor(1:6:end) =1;
disp(mat_cor);
```

```
           1           0       -0.25           0           0
           0           1           0           0        0.55
       -0.25           0           1           0           0
           0           0           0           1           0
           0        0.55           0           0           1
```

## Covariance Matrix - S

```
S = diag(vec_sd)*mat_cor*diag(vec_sd);  disp(S);
```

```
    0.000324           0    -9.9e-05           0           0
           0    0.000256           0           0   0.0004224
    -9.9e-05           0    0.000484           0           0
           0           0           0    0.001296           0
           0   0.0004224           0           0    0.002304
```

## Question 1

Investors who short sell in this market are the ones with the risk aversion parameter $(t)$ that satisfies the below inequality:

**Short Sell** $\Longrightarrow x_i(t) = \alpha_i + \beta_i t \leq 0$ **i.e., the values of** $t$ **for which the critical line is negative.**

Equivalently, investors that DO NOT short sell are the ones with the risk aversion parameter $(t)$ that satisfies the below inequality:

**DO NOT Short Sell** $\Longrightarrow$ $x_i(t) = \alpha_i + \beta_i t \geq 0$ **i.e., the values of $t$ for which the critical line is positive.**

```
e = ones(5,1);
```

### Inverse of Covariance Matrix

```
Sinv = inv(S)                         % Inverse of Covariance Matrix
```

```
Sinv =
      3292.2            0        673.4            0            0
           0       5600.4            0            0      -1026.7
       673.4            0       2203.9            0            0
           0            0            0        771.6            0
           0      -1026.7            0            0       622.26
```

```
a = e'*Sinv*e;
b = e'*Sinv*vec_r;
c = vec_r'*Sinv*vec_r;
d = a*c-b^2;
alpha = (1/a)*Sinv*e;
beta = Sinv*(vec_r-((b/a)*e));
con_name = {'a';'b';'c';'d'};
con_vals = [a;b;c;d];
Tcons = table(con_name, con_vals, 'VariableNames',{'Constants','ConstantValues'});
disp(Tcons);
```

```
    Constants    ConstantValues
    _________    ______________
    'a'           11784
    'b'          892.93
    'c'          74.175
    'd'           76715
```

## Alpha $(\alpha)$

```
disp(alpha);
```

```
      0.33653
      0.38813
      0.24417
     0.065481
    -0.034325
```

```
sum(alpha)
```

```
ans =       1
```

## Beta $(\beta)$

```
disp(beta);
```

```
-45.359
-82.896
 61.429
 30.264
 36.561
```

```
round(sum(beta))
```

```
ans =      0
```

### Finding the values of t, where the critical line is postive, i.e., these $t$ values DO NOT require short-selling.

Therefore, we need to find the values of $t$ that satisfy the following five inequalities:

for BHP: $0.33653 + (-45.359) * (t) \geq 0$

for NAB: $0.38813 + (-82.896) * (t) \geq 0$

for CSR: $0.24417 + (61.429) * (t) \geq 0$

for AGL: $0.065481 + (30.264) * (t) \geq 0$

for NCP: $-\ 0.034325 + (36.561) * (t) \geq 0$

### If $\alpha + \beta(t) \geq 0$, then $t \geq \frac{-\alpha}{\beta}$, solving for t, we get:

```
lb =(-alpha)./beta; %find lower bound for t parameter when x(t) = 0
ub = repelem(inf,5)';
```

As Beta for BHP and NAB are negative, we flip the bounds.

```
ub(1,1) = lb(1,1);
lb(1,1) = -inf;
ub(2,1) = lb(2,1);
lb(2,1) = -inf;
T2 = table(name,lb,ub,'VariableNames',{'Fund','LowerBound','UpperBound'});
```

**Upper and lower bounds of $t$ for each Fund with NO short selling.**

```
disp(T2);
```

| Fund | LowerBound | UpperBound |
|---|---|---|
| BHP | -Inf | 0.0074194 |
| NAB | -Inf | 0.0046822 |
| CSR | -0.0039749 | Inf |
| AGL | -0.0021637 | Inf |
| NCP | 0.00093883 | Inf |

```
lbt = max(lb);   ubt= min(ub);
```

**Range of $t$ that satisfy all five inequalities stated above are, i.e.,$t$ values for which there will be NO short selling:**

```
fprintf('Lower Bound = %.4e \nUpper Bound =  %.4e\n', lbt,ubt);
```

```
Lower Bound = 9.3883e-04
Upper Bound =  4.6822e-03
```

**Comments:**

**Investors with the risk aversion parameter $(t)$ within the above range $[0.0009388, 0.004682]$ will NOT short sell any of the funds but investors with $t$ outside this range will short sell.**

For $t < 0.0009388$, the investor will not short sell any funds except NCP as we only consider $t \geq 0$.

For $t > 0.004682$, the investor can short sell NAB and BHP but not CSR, AGL or NCP.

**The only funds that no one will short sell are CSR and AGL due to the requirement that $t$ needs to be $\geq 0$, i.e., the critical line is positive $\forall t \in [0, \infty)$ for CSR and AGL.**

# Minimum risk Portfolio (i.e., when t = 0):

```
mu_mvp = b/a          % mu for t = 0 on the MVF, or expected return from MVP
```

```
mu_mvp =      0.075778
```

```
sig_mvp = 1/sqrt(a)        % corresponding sigma for t = 0, risk for the MVP
```

```
sig_mvp =     0.0092122
```

# Question 2 - Hypatia's optimal portfolio - *P**

## Q2 (a)

**Optimal allocation for Hypatia is given by**$x_i(0.007) = \alpha_i + \beta_i(0.007)$

```
cap = 200000;                       % Capital
t = 0.007;                          % Risk Aversion Parameter of Hypatia
x_hy = alpha + beta*(t);            % Optimal allocation
c_hy = cap*x_hy;                    % Capital allocation
```

**Expected return on Hypatia's optimal portfolio $(\mu_{Hypatia})$:**

```
mu_hy = x_hy'*vec_r                 % Expected return on Hypatia's portfolio
```

```
mu_hy =       0.12135
```

**Hypatia's optimal portfolio variance** $(\sigma^2_{Hypatia})$**:**

```
var_hy = x_hy'*S*x_hy                  % Hypatia's portfolio variance
```

```
var_hy =    0.00040387
```

**Hypatia's optimal portfolio risk** $(\sigma_{Hypatia})$:

```
sig_hy = sqrt(var_hy)                   % Hypatia's portfolio risk (SD)
```

```
sig_hy =      0.020096
```

```
format long G;
Txhy = table(name, x_hy, c_hy, ...
        'VariableNames',{'Fund', 'Optimal_Allocation','Capital_Allocation'});
```

**Optimal allocation and dollar investment in the five funds for Hypatia :**

Note - Capital allocation is in $'s.

```
disp(Txhy);
```

| Fund | Optimal_Allocation | Capital_Allocation |
|---|---|---|
| BHP | 0.0190216325228673 | 3804.32650457345 |
| NAB | -0.192134592789243 | -38426.9185578486 |
| CSR | 0.674178645895665 | 134835.729179133 |
| AGL | 0.277329724532205 | 55465.9449064411 |
| NCP | 0.221604589838504 | 44320.9179677009 |

```
sum(Txhy.Capital_Allocation)
```

```
ans =         2e+05
```

```
format shortG;
```

## Q2 (b) - $\mu\sigma$ - plane plots

```
t_EF = (0:0.001:0.02);                 % 0 < t < 0.02
t_IEF = (-0.02:0.001:0);                 % -0.02 < t < 0
mu_EF = (b+d*t_EF)/a;
mu_IEF = (b+d*t_IEF)/a;
sig_EF = sqrt((1+d*t_EF.^2)/a);
sig_IEF = sqrt((1+d*t_IEF.^2)/a);
```

## Q2 (c) - 1000 random feasible portfolios satisfying $|x_i| \leq 2.5$:

First, initializing the random number generator with seed = 0 to make the results repeatable.

```
rng(0,'twister');
```

**Generating N random numbers in the interval (a,b) with the formula r = a + (b-a).*rand(N,1).**

Note - *rand* function draws the values from a uniform distribution in the open interval (a, b), it does not include the end points!

```
count = 0;
j = 0;
x = zeros(5,1000);
while j<1000
    count = count+1;
    xtemp = -2.5 + (5).*rand(5,1);    ...
        % generate 5 random numbers in the interval (-2.5, 2.5)
    xtemp = xtemp/sum(xtemp);         ...
        % Making each column sum to 1:
        if (sqrt(xtemp'*S*xtemp) <= 0.05);    ...
                % calculate sigma and  check if sigma <= 0.05
            j=j+1;                        ...
                % if success increment j and accept x values
            x(:,j) = xtemp;
        end
    end
% iterations required to generate 1000 feasible portfolios with sigam <= 0.05
count
```

```
count =        1685
```

```
size(x)
```

```
ans =          5        1000
```

```
sum(sum(abs(x) > 2.5))                        % checking |x_i| < 2.5
```

```
ans =      0
```

**Calculating $\mu_i \quad and \quad \sigma_i$ for the 1000 portfolios:**

```
for j=1:1000
    mu_i(j) = x(:,j)'*vec_r;
    sig_i(j) = sqrt(x(:,j)'*S*x(:,j));
end
sum(sig_i > 0.05)       % checking to make sure sigma <= 0.05
```

```
ans =      0
```

### Hypatia's Indifference Curve:

Using the Utility function: $U(\mu, \sigma) = F(t\mu - \frac{1}{2}\sigma^2)$ to find the indifference curve.

Negative of $U(\mu, \sigma)$ is $Z(\mu, \sigma) = -t\mu + \frac{1}{2}\sigma^2$ and Minimizing $Z(\mu, \sigma)$ maximizes $U(\mu, \sigma)$.

Expressed using the asset allocations, we get $Z(\vec{x}) = -t(\vec{r}\ '\vec{x}) + \frac{1}{2}(\vec{x}\ '\vec{S}\vec{x})$

Using **Hypatia's optimal portfolio variance** $(\sigma^2_{Hypatia})$ and **expected return** $(\mu_{Hypatia})$ to calculate the minimum of the above objective function $Z(\mu, \sigma)$.

```
% minimum of the objective function for Hypatia's optimal portfolio return and variance
z = -t*mu_hy + (1/2)*var_hy
```

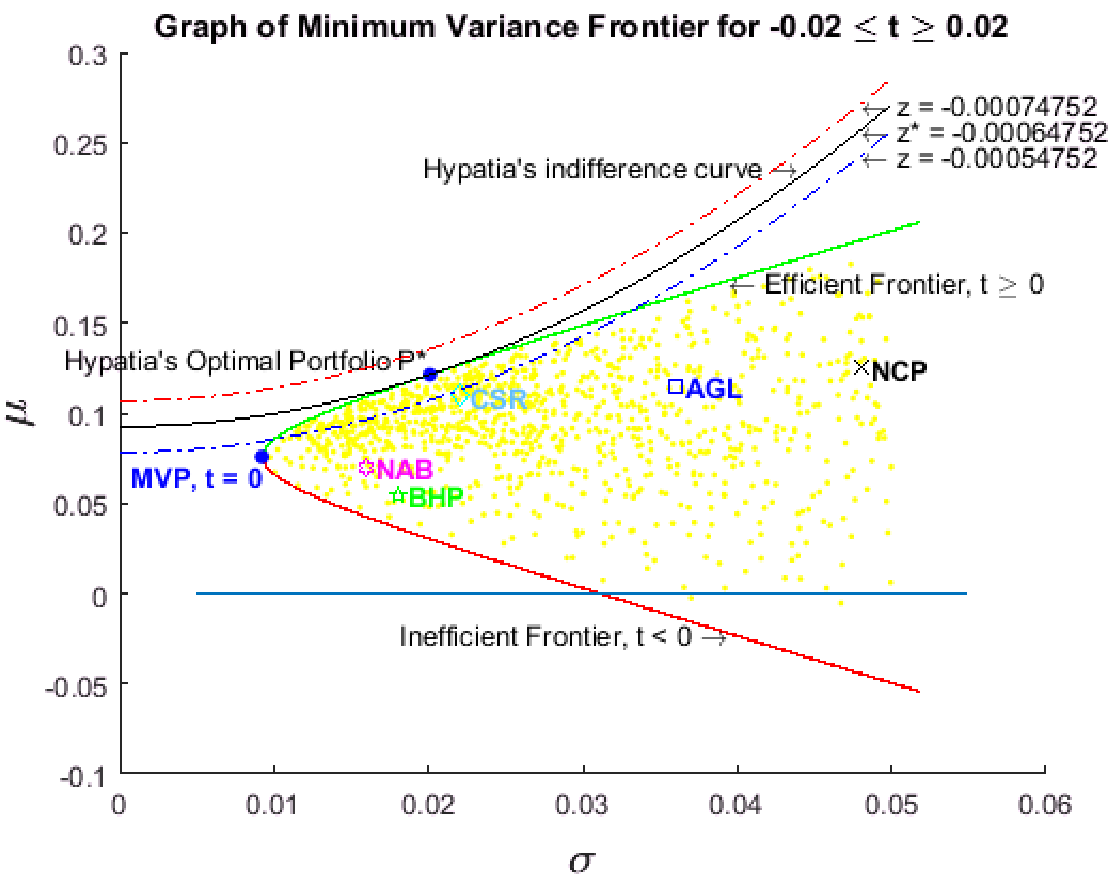
```
z =   -0.00064752
```

```
sd_ind = (0:0.001:0.05);
mu_ind = (-z + (1/2)*(sd_ind.^2))/t;         % points for indifference curves
mu_ind1 = (-(z-0.0001) + (1/2)*(sd_ind.^2))/t;
mu_ind2 = (-(z+0.0001) + (1/2)*(sd_ind.^2))/t;
```

### Plots:

```
figure(2);
hold on
plot(sig_i,mu_i,'.y');     % 1000 feasible portfolios mu and sigma
plot(sdBHP, rBHP, 'pg');
text(sdBHP+0.0001,rBHP,'\color{green} BHP','FontWeight','bold');
plot(sdNAB, rNAB, 'hm');
text(sdNAB+0.0001,rNAB,'\color{magenta} NAB','FontWeight','bold');
plot(sdCSR, rCSR, 'dc');
text(sdCSR+0.0001,rCSR,'\color{lightBlue} CSR','FontWeight','bold');
plot(sdAGL, rAGL, 'sb');
text(sdAGL+0.0001,rAGL,'\color{blue} AGL','FontWeight','bold');
plot(sdNCP, rNCP, 'xk');
text(sdNCP+0.0001,rNCP,'\color{black} NCP','FontWeight','bold');
plot(sig_EF,mu_EF,'g', 'LineWidth', 1);        % Efficient Frontier
txtEF = '\leftarrow Efficient Frontier, t \geq 0';
text(sig_EF(end-5),mu_EF(end-5),txtEF);
plot(sig_IEF,mu_IEF,'r', 'LineWidth', 1);       % Inefficient Frontier
txtIEF = 'Inefficient Frontier, t < 0 \rightarrow';
text(sig_IEF(end-15),mu_IEF(end-15), txtIEF,...
    'HorizontalAlignment','right');
refline(0,0);
xlabel('\sigma','FontSize', 16, 'FontWeight', 'bold');
ylabel('\mu','FontSize', 16, 'FontWeight', 'bold');
title('Graph of Minimum Variance Frontier for -0.02 \leq t \geq 0.02');
plot(sig_hy, mu_hy, '.b', 'MarkerSize', 15);  % Hypatia's optimal portfolio mu and sigma
txtOP = 'Hypatia''s Optimal Portfolio P*';
text(sig_hy, mu_hy+0.01,txtOP,'HorizontalAlignment','right');
plot(sig_mvp, mu_mvp, '.b', 'MarkerSize', 15);   % Minimum Variance Portfolio, t = 0
txtMVP = 'MVP, t = 0';
text(sig_mvp, mu_mvp-0.01, txtMVP, 'color' , 'blue','FontWeight','bold', ...
    'HorizontalAlignment','right')
plot(sd_ind,mu_ind, '-k');
txtIND = 'Hypatia''s indifference curve \rightarrow';
text(sd_ind(end-5)-0.001,mu_ind(end-5), txtIND, 'color', 'black',...
    'HorizontalAlignment','right');
txtINz = ['\leftarrow z* = ',num2str(z)];
text(sd_ind(end-2),mu_ind(end-2), txtINz);
plot(sd_ind,mu_ind1, '-.r');
txtINz1 = ['\leftarrow z = ',num2str(z-0.0001)];
```

```
text(sd_ind(end-2),mu_ind1(end-2), txtINz1);
plot(sd_ind,mu_ind2, '-.b');
txtINz2 = ['\leftarrow z = ',num2str(z+0.0001)];
text(sd_ind(end-2),mu_ind2(end-2), txtINz2);
hold off
```




**Legend:**

- P* - Hypatia's Optimal Portfolio
- MVP - Minimum Variance Portfolio, where risk aversion parameter $(t) = 0$.
- Yellow dots - 1000 random feasible portfolios

**Comments:**

- Set of all feasible portfolios for the given five funds fall within the Mean variance Frontier (the hyperbola formed by the efficient and inefficient frontiers).
- All the efficient portfolios fall on the top half of the Mean Variance Frontier (shown in green) and as such, Hypatia's optimal portfolio $(P^*)$ is on the Efficient Frontier.
- Hypatia's indifference curve (z = -0.00064752) is convex and the Efficient Frontier is concave, the unique point at which both these curves are tangential is given by the optimum portfolio $P^* = (\mu^*, \sigma^*)$ = (0.12135 , 0.020096) and this portfolio offers the maximum expected utility of all the feasible portfolios.

## Question 3 - Adding Riskless Cash Fund - CAPM

```
r0 = 0.04;                      % risk-free rate
r_new = vec_r - r0              % modified rate of return
```

```
r_new =
                    0.015
                     0.03
                     0.07
                    0.075
                    0.085
```

**New Optimal Allocation in risky assets under CAPM given by $\vec{x} = t(S^{-1}\bar{r})$**

```
x_hy_new = t*(Sinv*r_new)       % New Optimal Allocation under CAPM
```

```
x_hy_new =
        0.675645342312009
        0.565169504181601
         1.15059687786961
        0.405092592592593
        0.154632118677818
```

```
sum(x_hy_new)    % Sum of risky assest is  > 1, meaning the investor needs to borrow
```

```
ans =           2.95113643563363
```

c_bar is defined as: $\bar{C} = \bar{r}^T S^{-1}\bar{r}$

```
c_bar = r_new'*Sinv*r_new
```

```
c_bar =           21.5938887490085
```

**Expected return under CAPM, i.e., when riskless fund is added to the portfolio, is given by $\hat{\mu} = r_0 + \bar{c}t$**

```
mu_new = r0 + t*c_bar           % New Expected return
```

```
mu_new =           0.19115722124306
```

**Variance is given by $\hat{\sigma}^2 = \bar{c}t^2 = (\hat{\mu} - r_0)t$**

```
var_new = t^2*c_bar             % New Variance
```

```
var_new =        0.00105810054870142
```

```
check_var = (mu_new-r0)*t        % Gives the same value
```

```
check_var =       0.00105810054870142
```

```
sig_new = sqrt(var_new)        % new Portfolio risk
```

```
sig_new =        0.0325284575210909
```

### Allocation in Riskless cash fund (borrowing to invest in risky funds):

```
x0 = 1- sum(x_hy_new)             % Allocation in Riskless cash fund
```

```
x0 =          -1.95113643563363
```

```
names = cellstr(name);  names{end+1} = 'Riskless';
x_tot = x_hy_new;     x_tot(end+1) = x0;    sum(x_tot)
```

```
ans =      1
```

```
format longG;
cap_tot = x_tot.*cap;
```

### Total amount of capital invested in risky assets (including the borrowed funds):

```
inv_risk = sum(x_hy_new.*cap);  % Total amount of capital invested in risky assets
```

### Capital borrowed at 4 % :

```
borrow = x0*cap;            % Borrowed capital
Tcapm = table(names, x_tot, cap_tot, ...
        'VariableNames', {'Funds', 'Allocation', 'Cap_Allocation'});
```

### New Allocation including the Riskless fund.

```
disp(Tcapm);
```

| Funds | Allocation | Cap_Allocation |
|---|---|---|
| 'BHP' | 0.675645342312009 | 135129.068462402 |
| 'NAB' | 0.565169504181601 | 113033.90083632 |
| 'CSR' | 1.15059687786961 | 230119.375573921 |
| 'AGL' | 0.405092592592593 | 81018.5185185185 |
| 'NCP' | 0.154632118677818 | 30926.4237355635 |
| 'Riskless' | -1.95113643563363 | -390227.287126725 |

```
inv_risk = sum(Tcapm.Cap_Allocation(1:5));
fprintf('Total funds invested in risky assets = $%.2f \nBorrowed Funds = $%.2f\n',...
    inv_risk, abs(borrow));
```

```
Total funds invested in risky assets = $590227.29
Borrowed Funds = $390227.29
```

```
inv_risk - cap              % check borrowed funds
```

```
ans =    3.9023e+05
```

```
format shortG;
```

**Total optimal allocation towards the risky assets is 2.9511 ($590, 227), which means Hypatia has to borrow 1.9511 ($390, 227) at 4% to invest in risky assets.**

## Tangency Portfolio

It is the portfolio given by the point in $\mu\sigma$ -plane that is at the intersection of the Capital Market Line and the risky-asset Efficient Frontier. This portfolio contains no riskless asset and sums to 1.

**Expected return from the tangency portfolio is given by** $\hat{\mu}_T = \frac{c - br_0}{b - ar_0}$ **,**

```
mu_t = (c - b*r0)/(b - a*r0)        % Expected return from the tangency portfolio
```

```
mu_t =       0.09122
```

**Portfolio variance on the Efficient Frontier corresponding to a portfolio return of** $r_o = 4\%$ **is given by** $\sigma_o^2 = \frac{\bar{c}}{d}$

```
% Portfolio Variance on EF for the risky assets only
%   (which corresponds to a portfolio return of 4%)
var_0 = c_bar/d
```

```
var_0 =    0.00028148
```

**Tangency portfolio risk is given by** $\hat{\sigma}_T = \frac{\hat{\mu} - r_0}{\sigma_0 \sqrt{t}}$

```
sig_t = sqrt(var_0*d)/(b - a*r0)    % Risk corresponding to tangency portfolio return
```

```
sig_t =      0.011022
```

**t - value corresponding to the Tangency Portfolio is given by** $t_T = \frac{1}{b - ar_0}$

```
  t_t  = 1/(b - a*r0)                % t value corresponding to tangency portfolio
```

```
t_t =      0.002372
```

```
t_r0 = 1/sqrt(c_bar)                % t value corresponding to r0 or when risk = 0
```

```
t_r0 =        0.2152
```

**Allocation for the Tangency Portfolio (does not include the risk-free cash fund, just the risky funds) is given by $\vec{x} = t(S^{-1}\bar{r})$:**

```
x_t = t_t*(Sinv*r_new)            % Tangency portfolio allocation
```

```
x_t =
      0.22894
      0.19151
      0.38988
      0.13727
     0.052397
```

```
sum(x_t)
```

```
ans =             1
```

## Q4 Capital Market Line

**The Capital Market Line (CML) is given by $\hat{\sigma} = \frac{\hat{\mu}-r_0}{\sigma_0\sqrt{t}}$, where $r_0$ is the riskfree rate, $\sigma_o^2$ is the portfolio variance on the Efficient frontier corresponding to a portfolio return of $r_o$.**

This CML line is the degenerate form of the efficient frontier when one of the assets is riskless.

**Note: from the definition of CML: $\hat{\sigma} = \frac{\hat{\mu}-r_0}{\sigma_0\sqrt{d}} = \sqrt{\bar{c}}\,t$ as $\bar{c} = \bar{r}^{\top}S^{-1}\bar{r} = d\sigma_0^2$, $\bar{r} = r - r_0$ and $\hat{\mu} = r_0 + \bar{c}t$**

There is a linear relation between sigma and t.

**Few more indifference curves:**

```
sig_ind2 = (0:0.001:0.05);
z_mvp = -t*(mu_mvp) + (1/2)*(sig_mvp^2)    % Indifference curve tangential to MVP
```

```
z_mvp =   -0.00048801
```

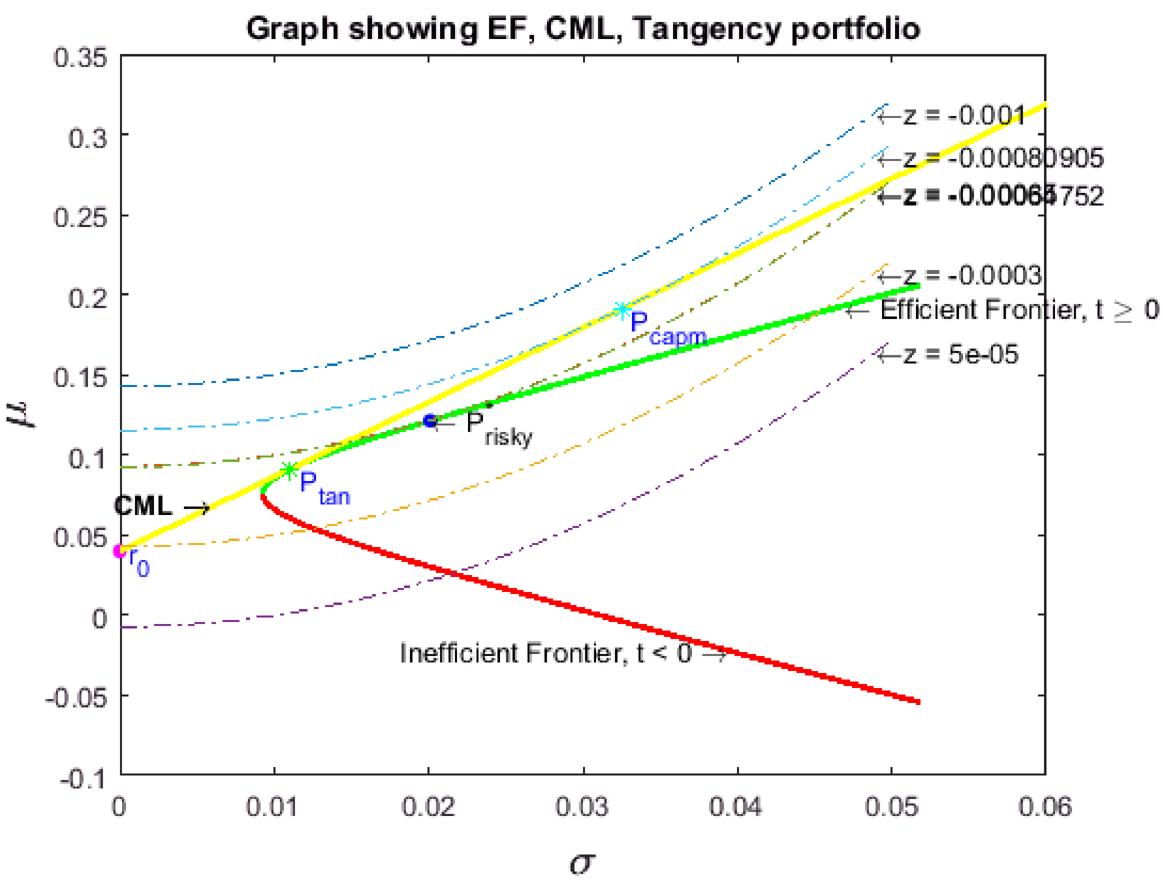
```
z_capm = -t*(mu_new) + (1/2)*(sig_new^2);
z_vals = (-0.001:0.00035:0.0002);
z_vals(end+1) = z;
z_vals(end+1) = z_capm;
```

```
txtz = cell(length(z_vals),1);
for i = 1:length(z_vals)
    mu_ind5{i} = (-z_vals(i) + (1/2)*(sig_ind2.^2))/t;
    txtz{i,:} = ['z = ',num2str(z_vals(i))];
end
txtz;
for i = 1:length(z_vals)
       txtIC = txtz{i,1};
       y = mu_ind5{1,i}(end-1);

end
```

## Q4 Plot:

```
sig_cml = (0:0.001:0.06);
mu_cml = sig_cml*sqrt(c_bar)+r0;           % calculating CML points
figure(3);
plot(0,0.04, '.m', 'MarkerSize', 15);
text(0, 0.036, '\color{blue} r_{0}');
hold on
title('Graph showing EF, CML, Tangency portfolio');
plot(sig_EF,mu_EF,'g', 'LineWidth', 2);        % Efficient Frontier
txtEF = '\leftarrow Efficient Frontier, t \geq 0';
text(sig_EF(end-2),mu_EF(end-2),txtEF);
plot(sig_IEF,mu_IEF,'r', 'LineWidth', 2);       % Inefficient Frontier
txtIEF = 'Inefficient Frontier, t < 0 \rightarrow';
text(sig_IEF(end-15),mu_IEF(end-15), txtIEF,...
    'HorizontalAlignment','right');
plot(sig_cml,mu_cml, 'y', 'LineWidth', 2);            % CML line
txtCML = 'CML \rightarrow';
text(sig_cml(7), mu_cml(7)+0.002, txtCML,'FontWeight','bold',...
    'HorizontalAlignment','right');
plot(sig_new, mu_new, '*c');
    % Hypatia's new portfolio allocation including the risk-free fund
text(sig_new, mu_new-0.01, '\color{blue} P_{capm}');
plot(sig_t, mu_t, '*g');               % Tangency Portfolio
text(sig_t, mu_t-0.01, '\color{blue} P_{tan}');
plot(sig_hy, mu_hy, '.b', 'MarkerSize', 15);
                        % Hypatia's optimal portfolio without risk-free fund
txtOP = '\leftarrow P_{risky}^*';
text(sig_hy, mu_hy,txtOP);
xlabel('\sigma','FontSize', 16, 'FontWeight', 'bold');
ylabel('\mu','FontSize', 16, 'FontWeight', 'bold');
for i = 1:length(z_vals)
    plot(sig_ind2,mu_ind5{i}, '-.');
    txtIC = ['\leftarrow',txtz{i,1}];
   text(0.049, mu_ind5{1,i}(end-1), txtIC)
end
hold off
```




Legend:

- $P^{*}_{risky}$ – Hypatia's Optimal Portfolio without the riskless fund
- $P_{tan}$ – Tangency Portfolio
- $P_{capm}$ – Hypatia's Optimal Portfolio with the riskless fund
- CML - Capital Market line
- $r_0$ – Risk-free return of 4%

Comments:

- By introducing the risk-free fund the Efficient Frontier degenerates into the Capital Market Line, which dominates all the efficient portfolios available on the efficient frontier except for the Tangency Portfolio.
- If Hypatia did not have the option of risk-free fund, then the optimal portfolio is given by the point $P^{*}_{risky}$, this portfolio offers the maximum Utility (for the specific Utility function $U(\mu, \sigma) = F(t\mu - \frac{1}{2}\sigma^2)$) as the objective function $Z(\mu, \sigma) = -t\mu + \frac{1}{2}\sigma^2$ is minimized for $\mu$ $and\, \beta$ given by the point $P^{*}_{risky}$. Also this point is tangential to the indifference curve z = - 0.0006475, and is where the objective function achieves the minimum for the available set of feasible portfolios.
- The objective function moves northwest as the function decreases.
- z = - 0.001 is smaller than - 0.0006475 but it is outside the feasible set.
- z = - 0.0003 is greater than -0.0006475 and does offer choices of feasible portfolios but they are not optimal
- z = - 0.0006475, offers the optimal portfolio when there is no option of investing in the risk-free fund.

- Z = - 0.00080905, offers a better optimal portfolio when the option of investing in the risk-free fund is available, this indifference curve offers higher expected utility compared to the indifference curve z = - 0.0006475 (without the choice of risk-free asset).
- In summary, if Hypatia had the option of investing in the risk-free fund, then the optimum portfolio is given by $P_{capm}$ on the Capital Market line tangential to the indifference curve z = - 0.00080905, this portfolio offers the maximum expected utility of all the feasible portfolios.

### Estimating the total value of each fund (to the nearest $0.1 million) for the combined net worth of $100 million:

This is calculated using the Tangency Portfolio as weights.

```
net = 100000000;
f_vals = x_t*net;
tvals = num2str(f_vals, '%-.1f\n');
Tval = table(name, x_t, tvals, 'VariableNames',{'Fund','Allocation','Dollar_Value'});
disp(Tval);
```

```
    Fund    Allocation    Dollar_Value
    ____    __________    ____________
    BHP       0.22894     22894412.3
    NAB       0.19151     19150910.7
    CSR       0.38988     38988264.5
    AGL       0.13727     13726664.3
    NCP      0.052397     5239748.2
```

```
sum(f_vals)
```

```
ans =    100000000
```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%