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MATHEMATICS

Over all real numbers, find the minimum value of a positive number, y such that

$$y = \sqrt{(x+6)^2 + 25} + \sqrt{(x-6)^2 + 121}$$

NB: The minimum value of a parabola occurs where $f'(x) = \frac{\partial y}{\partial x} = 0$; $\lim \to 0$

where the slope $= 0$ or where the turning / critical point is 0.

Therefore, when we differentiate the above function, we find it's x-value @ 0 then we find it's value for y.

$$f'(x) = \frac{d}{dx}\left[\sqrt{(x+6)^2+25} + \sqrt{(x-6)^2+121}\right]$$

$$= \frac{d}{dx}\sqrt{(x+6)^2+25} + \frac{d}{dx}\sqrt{(x-6)^2+121}$$

Since differentiation is linear;

Using Chain Rule; if u is a function of x and $y = u$; then $dy/dx = dy/du \times du/dx$

$\therefore$ For the first derivative:
where $u = (x+6)^2 + 25$

$$\frac{d}{dx}\sqrt{(x+6)^2+25} = \frac{d}{dx}\sqrt{u} = \frac{1}{2}u^{1/2 - 1} \times \frac{d}{dx}(x+6)^2+25$$

$$= \frac{1}{2u^{1/2}} \times \left[\frac{d}{dx}(x+6)^2 + \frac{d}{dx}[25]\right]$$

$$= \frac{2(x+6) \cdot d/dx[x+6] + 0}{2u^{1/2}}$$ (another Chain rule)

$$= \frac{2(x+6) \times \frac{d}{dx}(x) + d/dx(6)}{2u^{1/2}}$$

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$$= \frac{2(x+6)\times(1+0)}{2\sqrt{(x+6)^2+25}}$$

$\therefore$ The derivative of the first term is

$$\frac{\cancel{2}(x+6)}{\cancel{2}\sqrt{(x+6)^2+25}} = \frac{x+6}{\sqrt{(x+6)^2+25}}$$

Following the same methodology for the second derivative.

$$\frac{d}{dx}\sqrt{(x-6)^2+121}$$

$$= \frac{1}{2u^{1/2}} \times \frac{d}{dx}\left[(x-6)^2+121\right]$$

$$= \frac{1}{2u^{1/2}} \times \left[\frac{d}{dx}(x-6)^2 + \frac{d}{dx}(121)\right]$$

$$= \frac{2(x-6)\cdot \frac{d}{dx}(x-6) + 0}{2u^{1/2}}$$

$$= \frac{2(x-6)\times\left[\frac{d}{dx}(x) - \frac{d}{dx}(6)\right]}{2u^{1/2}}$$

$$= \frac{2(x-6)\times(1-0)}{2u^{1/2}}$$

$$= \frac{2(x-6)}{2\sqrt{(x-6)^2+121}}$$

$$= \frac{x-6}{\sqrt{(x-6)^2+121}}$$

Adding the two derivatives

$$y' = \frac{x+6}{\sqrt{(x+6)^2+25}} + \frac{x-6}{\sqrt{(x-6)^2+121}}$$

Now, we find x @ $y' = 0$

$$0 = \frac{x+6}{\sqrt{u}} + \frac{x-6}{\sqrt{v}}$$

multiply through by $\sqrt{uv}$

$$0 = (x+6)\sqrt{v} + (x-6)\sqrt{u}$$

$$-(x+6)\sqrt{v} = (x-6)\sqrt{u}$$

$$-\frac{(x+6)}{x-6} = \frac{\sqrt{u}}{\sqrt{v}}$$

square both sides

$$-\frac{(x+6)^2}{(x-6)^2} = \left(\frac{\sqrt{u}}{\sqrt{v}}\right)^2$$

$$-\frac{(x+6)^2}{(x-6)^2} = \frac{u}{v}$$

$$-(x+6)^2\left[(x-6)^2+121\right] = (x-6)^2\left[(x+6)^2+25\right]$$

Open brackets

$$-(x+6)^2(x-6)^2 - 121(x+6)^2 = (x-6)^2(x+6)^2 + 25(x-6)^2$$

Collect like terms

$$-(x+6)^2(x-6)^2 - (x-6)^2(x+6)^2 = 25(x-6)^2 + 121(x+6)^2$$

$$-2\left[(x^2+12x+36)(x^2-12x+36)\right] = 25(x^2-12x+36)+121(x^2+12x+36)$$

$$-2\left[x^4-12x^3+36x^2+12x^3-144x^2+432x+36x^2-432x\right.$$

$$\left.+1296\right] = 25x^2-300x+900+121x^2+1452x+4356$$

$\therefore$

$$-2\left[x^4-72x^2+1296\right]=146x^2+1152x+5256$$

$$;-2x^4+144x^2-2592=146x^2+1152x+5256$$

; Collect like terms

$$;-2x^4-2x^2-1152x-7848=0$$

divide through by -2

$$x^4+x^2+576x+3924=0$$

This is the resulting equation to find x from.

For a quartic equation in the form:

$$ax^4+bx^3+cx^2+dx+e=0$$

$$x_{1,2}=\frac{-b}{4a}-S\pm\frac{1}{2}\sqrt{-4S^2-2p+q/s}$$

$$x_{3,4}=\frac{-b}{4a}+S\pm\frac{1}{2}\sqrt{-4S^2-2p-q/s}$$

Where $p=\dfrac{8ac-3b^2}{8a^2}$; $q=\dfrac{b^3-4abc+8a^2d}{8a^3}$

and $S=\dfrac{1}{2}\sqrt{\dfrac{-2}{3}p+\dfrac{1}{3a}\left(Q+\dfrac{\Delta_0}{Q}\right)}$

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According to the above equations

$$x_1 = 5.78394 + 7.67141i$$
$$x_2 = 5.78394 - 7.67141i$$
$$x_3 = -5.78394 + 3.00956i$$
$$x_4 = -5.78394 - 3.00956i$$

The value of x are all complex, this means there is no minimum value of y in real numbers.