Final exam n°1

Duration : three hours
Documents and calculators not authorized

Name :

First name :

Instructions :

– ***no sheets other than the stapled ones provided for answers shall be corrected.***

– answers written using lead penils shall not be corrected.

Exercise 1 (4 points)

Write the negation of the following sentences :

1. « No graduate of EPITA will have a first gross annual salary below 40 k€ ».

2. « If I join the research lab of EPITA, I'll be in a position to work in the medical imaging sector ».

3. « Some MiMo are complicated ».

4. « All your movements on IONISx are analyzed ».

Exercise 2 (2 points)

Let $x \in \mathbb{R}_+^*$. Prove by induction that for all $n \in \mathbb{N}^*$, $(1+x)^n \geqslant 1+nx$.

[the answer frame continues on the next page]

Exercise 3 (2 points)

Write in mathematic language (using quantifiers) the following sentences (disregard about the validity of the sentences, they may be true or false) :

1. « Any real number is the cube of a real number ».

2. « There exists a real number which is the cube of all the real numbers ».

3. « Any natural number is even or odd ».

4. « Between two distinct real numbers, one can always find a rational number ».

Exercise 4 (2 points)

For each of the following questions, CIRCLE the correct answers.

1. Let $f : \left\{ \begin{array}{ccc} \mathbb{R}_+ & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^2 \end{array} \right.$. Then,

   a. f is injective
   b. f is not injective
   c. f is surjective
   d. f is not surjective

2. Let $f : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x^2 \end{array} \right.$. Then,

   a. f is injective
   b. f is not injective
   c. f is surjective
   d. f is not surjective

3. Let $f : \left\{ \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R}_+ \\ x & \longmapsto & x^2 \end{array} \right.$. Then,

   a. f is injective
   b. f is not injective
   c. f is surjective
   d. f is not surjective

4. Let $f : \left\{ \begin{array}{ccc} \mathbb{R}_+ & \longrightarrow & \mathbb{R}_+ \\ x & \longmapsto & x^2 \end{array} \right.$. Then,

   a. f is injective
   b. f is not injective
   c. f is surjective
   d. f is not surjective

Exercise 5 (3 points)

1. Using Euclid's algorithm, determine a particular solution of the equation $524x + 144y = 4$.

2. Using imperatively Gauss's theorem, determine the set of all the ordered pairs $(x, y) \in \mathbb{Z}^2$ such that $524x + 144y = 4$.

Exercise 6 (2 points)

Let a and b be two non-zero natural numbers and $d = a \wedge b$.

1. Show that there exists $(a', b') \in \mathbb{N}^{*2}$ such that $a = da'$, $b = db'$ and $a' \wedge b' = 1$.

2. Using the previous question and Bézout's theorem, show that there exists $(u, v) \in \mathbb{Z}^2$ such that $au + bv = d$.

Exercise 7 (2 points)

Determine the order of multiplicity of the root 1 of the polynomial $P(X) = X^4 - X^3 - 3X^2 + 5X - 2$.

Exercise 8 (3 points)

Let $n \geqslant 2$.

1. Show that the polynomial $P(X) = (X-2)^{2n} + (X-1)^n - 1$ is divisible by $X^2 - 3X + 2$.

2. Determine the remainder of the euclidean division of $Q(X) = (X-2)^{2n} + (X-1)^n - 2$ by :

   a. $(X-2)(X-1)$

   b. $(X-1)^2$

Exercise 9 (2 points)

For which value(s) of $a \in \mathbb{R}$ does the polynomial $Q(X) = (X+1)^7 - X^7 - a$ have a real root which is at least of order two?