

**Physics Midterm n°1***Calculators and documents are not allowed.**Please answer only on exam sheets***Exercise 1**      *Cycloidal motion*    (7 points)**Part A**

One considers Cartesian basis (Oxyz). One studies a wheel of radius R and center C which is rolling without gliding on plan (xOy) : it is admitted that the position of wheel center is linked to the angle  $\theta$  describing the wheel rotation.

Coordinates of vector position can be expressed as :

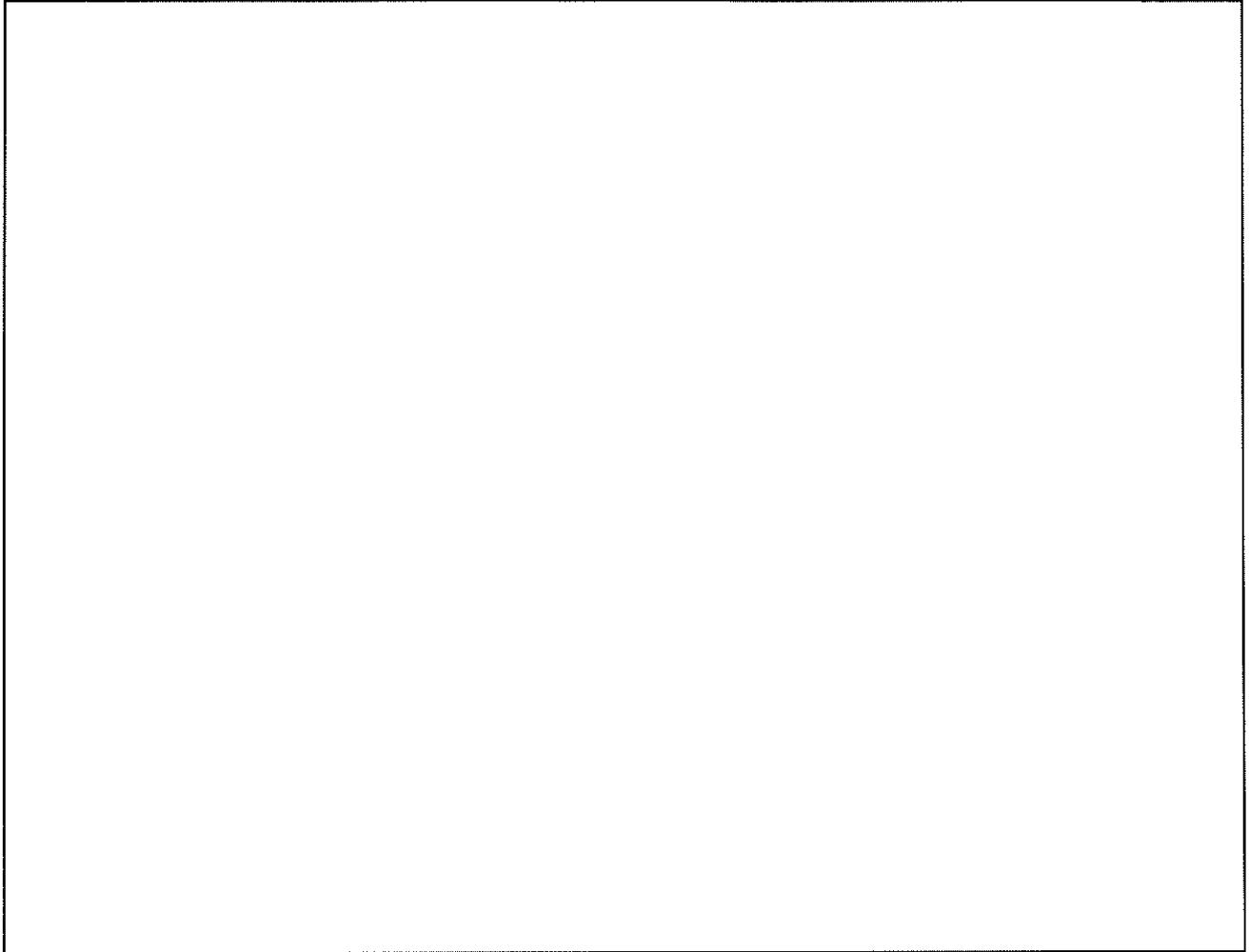
$$\begin{cases} x(t) = A(\omega t - \sin(\omega t)) \\ y(t) = A(1 - \cos(\omega t)) \end{cases} \quad (\theta = \omega t) ; \text{ where } A \text{ and } \omega \text{ are positive constants.}$$

1- Write the Cartesian components of velocity and acceleration vectors.

2- Deduce norms of both vectors. Useful formula :  $1 - \cos(\alpha) = 2 \cdot \sin^2(\alpha/2)$ .

3-Draw the cycloid ( $y = f(x)$ ) over a time interval of 2 periods ( $2T$ ). Remember that  $\omega$  is linked to the period  $T$  by  $\omega = 2\pi/T$ .

(Consider the values :  $t = 0$  ;  $t = T/4$  ;  $t = T/2$  ;  $t = 3T/4$  ;  $t = T$ ).



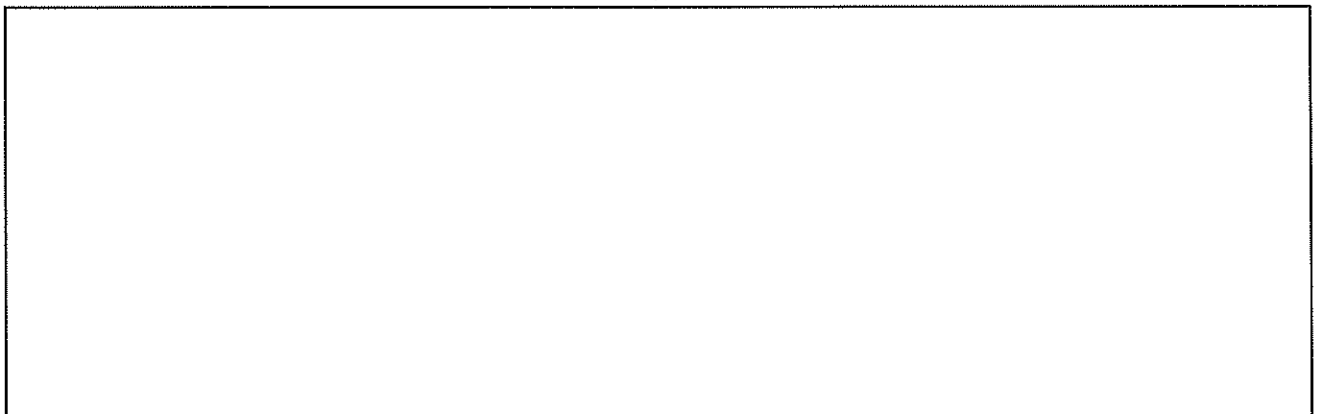
### **Part B**

One considers spiral motion described by the following equations :

$$\begin{cases} \rho(t) = \rho_0 e^{\omega t} \\ \theta(t) = \omega t \end{cases} ; \quad \text{where } \rho_0 \text{ and } \omega \text{ are positive constants.}$$

1- Express the velocity vector of this motion in polar coordinates. One reminds you that :

$$\vec{V} = \dot{\rho} \vec{u}_\rho + \rho \dot{\theta} \vec{u}_\theta$$



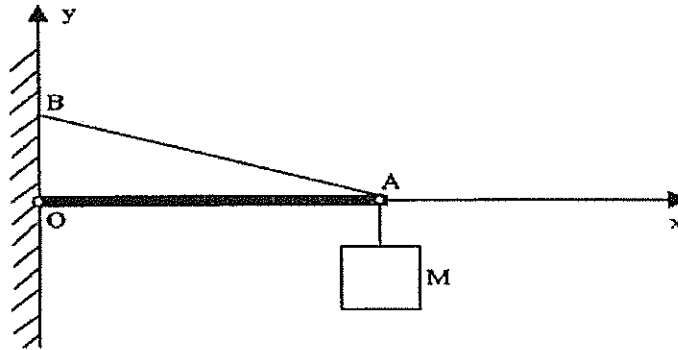
2- Write the norm of velocity vector.

3-a) Remembering that in Frenet's basis  $\vec{V} = V\vec{u}_T = R(t)\dot{\theta}\vec{u}_T$ , express the radius  $R(t)$  of that trajectory.

b) Deduce from it the components of acceleration vector  $\vec{a}(a_T, a_N)$  in Frenet's basis  $(\vec{u}_T, \vec{u}_N)$ .

**Exercise 2**      System at equilibrium    (6 points)

A homogeneous horizontal beam OA of length  $L$  and mass  $m = 40 \text{ kg}$  is fixed to a wall at its tip O. A cable AB, whose mass is neglected and length is fixed, connects the wall and the tip A of the beam. A mass  $M = 150 \text{ kg}$  is hung at point A. Given data :  $\text{BAO} = 30^\circ$  and  $g = 10 \text{ m.s}^{-2}$ .



1- Write explicitly which exterior forces are acting on the beam. Sketch them.

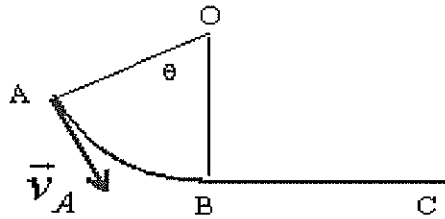
2- a) Write the rotation equilibrium condition with respect to point O and deduce then the norm of cable stress.

b) Use translation equilibrium condition to write the components  $(R_x, R_y)$  of  $\vec{R}_{\text{wall}}$ .

c) Compute the norm of the reaction  $\vec{R}_{wall}$ .

**Exercise 3** (7 points)

A pointlike solid of mass  $m$  is moving on the path sketched below. Part AB is circular with radius  $R$ , angle  $\theta$  and center  $O$  while BC is horizontal. The solid is thrown from point A with a velocity  $V_A$  tangential to the circle.



1-a) Write all exterior forces acting on solid between A and B by assuming that frictions over part AB can be modeled by a constant force  $f$ . Sketch them.

b) Use kinetic energy theorem between A and B in order to express the friction force  $f$  as function of  $R$ ,  $g$ ,  $V_A$ ,  $V_B$ ,  $m$  and  $\theta$ . Compute numerically with  $m = 0,1\text{kg}$ ,  $g = 10\text{ms}^{-2}$ ,  $R = 1,5\text{m}$ ,  $V_A = 2\text{ms}^{-1}$ ,  $V_B = 3\text{ms}^{-1}$ ;  $\theta = 60^\circ \approx 1 \text{ rad}$ .

2- a) Frictions over path BC can be modeled by a force  $f = 0,1 \text{ N}$ . Compute the speed at point C with  $BC = 2\text{m}$ .

b) Compute the norm of the total reaction  $\vec{R}$  which is acting on solid over path BC.