

Brief correction of midterm n°1 in Physics

Exercise I

Part A

- 1) You had to derive with respect to time the functions $\begin{cases} x(t) = A(\omega t - \sin(\omega t)) \\ y(t) = A(1 - \cos(\omega t)) \end{cases}$

Just few comments about notations for velocity and acceleration components. \vec{v} is a vector which in Cartesian basis can be written as $\vec{v} = v_x \vec{u}_x + v_y \vec{u}_y$ where (v_x, v_y) are the components.

Another notation is the system $\begin{cases} v_x = \dot{x}(t) = A\omega(1 - \cos(\omega t)) \\ v_y = \dot{y}(t) = A\omega \sin(\omega t) \end{cases}$. Note that there no arrow on vector components!

The acceleration reads $\begin{cases} a_x = \ddot{x}(t) = A\omega^2 \sin(\omega t) \\ a_y = \ddot{y}(t) = A\omega^2 \cos(\omega t) \end{cases}$.

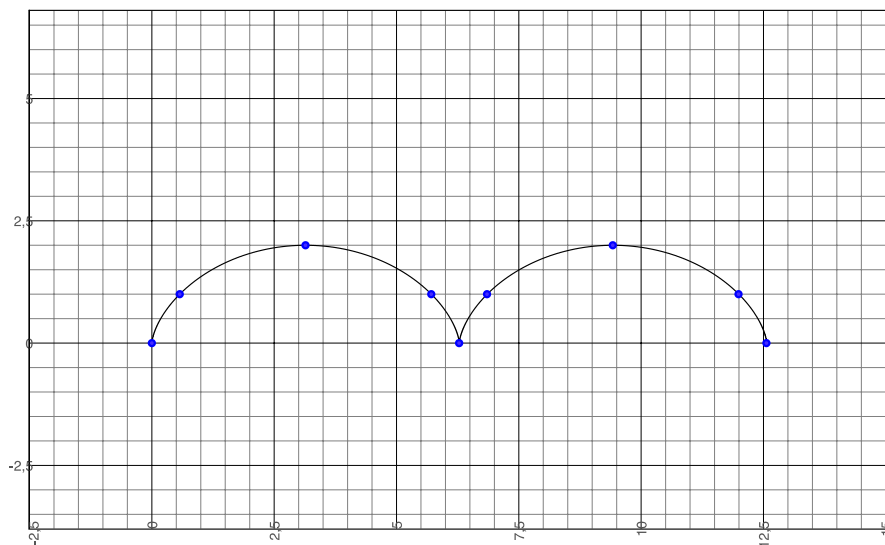
- 2) The norm of velocity vector reads $\|\vec{v}\| = \sqrt{v_x^2 + v_y^2} = \sqrt{A^2\omega^2(1 - \cos(\omega t))^2 + A^2\omega^2 \sin^2(\omega t)}$

Then after simplification $\|\vec{v}\| = A\omega\sqrt{1 - 2\cos(\omega t) + \cos^2(\omega t) + \sin^2(\omega t)} = A\omega\sqrt{4\sin^2(\frac{\omega t}{2})}$

$$\|\vec{v}\| = 2A\omega \sin\left(\frac{\omega t}{2}\right)$$

For acceleration, it's easier $\|\vec{a}\| = A\omega^2$.

- 3) Here pay attention on variables. We were looking for graph ($y = f(x)$), the so-called cycloid. Just brief notation: $t_k = k \cdot \frac{T}{4} = \frac{2\pi k}{\omega} \frac{1}{4}$ and for the question $k \in \llbracket 0, 8 \rrbracket$.



Part B

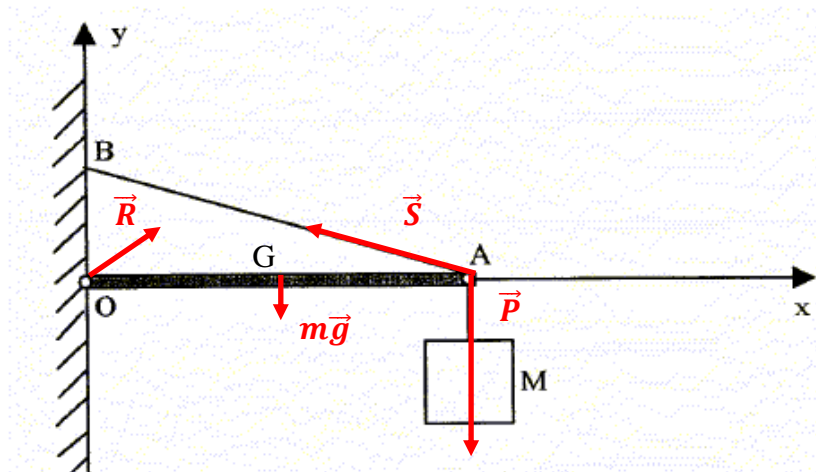
1) We were studying the polar coordinates $\begin{cases} \rho(t) = \rho_0 e^{\omega t} \\ \theta(t) = \omega t \end{cases}$. The velocity vector can be written as $\vec{v} = \dot{\rho}\vec{u}_\rho + \rho\dot{\theta}\vec{u}_\theta = \rho_0\omega e^{\omega t}\vec{u}_\rho + \rho_0\omega e^{\omega t}\vec{u}_\theta = \rho(t)\omega(\vec{u}_\rho + \vec{u}_\theta)$.

2) By definition the norm of \vec{v} is $\|\vec{v}\| = \sqrt{v_\rho^2 + v_\theta^2} = \rho(t)\omega\|\vec{u}_\rho + \vec{u}_\theta\| = \rho(t)\omega\sqrt{2} = v$.

3) We recall that $\vec{v} = v\vec{u}_T = R(t)\dot{\theta}\vec{u}_T = R(t)\omega\vec{u}_T$ where (\vec{u}_T, \vec{u}_N) is the Frenet's basis. Please here do NOT divide by unitary vector \vec{u}_T ! It doesn't exist, no meaning! Then one deduces $R(t) = \frac{v}{\omega} = \rho(t)\sqrt{2}$.

4) In Frenet's basis acceleration components reads $\begin{cases} a_T = \frac{dv}{dt} = \rho(t)\omega^2\sqrt{2} \\ a_N = \frac{v^2}{R} = \rho(t)\omega^2\sqrt{2} \end{cases}$ by using expression of v found in question 2.

Exercise II



1) First we define the studied system, namely here the beam. Four forces are acting on beam:

- 1° wall reaction \vec{R} at point O
- 2° weight of beam $m\vec{g}$ acting at center-of-mass G
- 3° weight \vec{P} of mass M acting at point A
- 4° cable stress \vec{S} pointing from A to B

2) a) The rotation equilibrium with respect to O reads

$$\vec{0} = \sum \vec{M}_O(\vec{F}_{ext}) = \vec{M}_O(\vec{P}) + \vec{M}_O(\vec{S}) + \vec{M}_O(\vec{R}) + \vec{M}_O(m\vec{g})$$

By definition $\vec{M}_O(\vec{R}) = \vec{OO} \wedge \vec{R} = \vec{0}$ and using same formula one gets

$$\vec{0} = OG \cdot mg \cdot \sin\left(-\frac{\pi}{2}\right)\vec{u}_z + OA \cdot Mg \cdot \sin\left(-\frac{\pi}{2}\right)\vec{u}_z + OA \cdot S \cdot \sin\left(\pi - \frac{\pi}{6}\right)\vec{u}_z$$

Beware that this is a vector equality! After projection on \vec{u}_z and simplification ($OA = L$)

$$0 = -\frac{L}{2} \cdot mg - MgL + L \cdot S \cdot \sin\left(\frac{\pi}{6}\right)$$

$$S = mg + 2Mg = 3400 \text{ N}$$

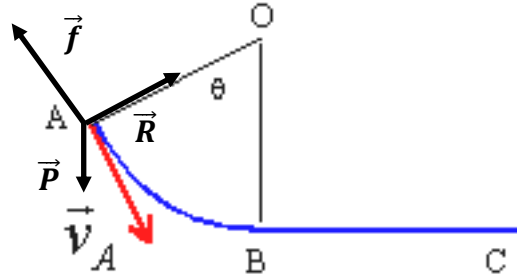
b) The translation equilibrium is the usual Newton's second law $\vec{0} = \sum \vec{F}_{ext}$.

After projecting on x- and y-axes

$$\begin{cases} 0 = R_x - S \cdot \cos\left(\frac{\pi}{6}\right) \\ 0 = R_y + S \cdot \sin\left(\frac{\pi}{6}\right) - mg - Mg \end{cases} \Leftrightarrow \begin{cases} R_x = S \cdot \frac{\sqrt{3}}{2} = 1700\sqrt{3} \text{ N} \\ R_y = -\frac{S}{2} + mg + Mg = 200 \text{ N} \end{cases}$$

c) Finally the norm is equal to $\|\vec{R}_{wall}\| = \sqrt{R_x^2 + R_y^2} = \sqrt{8710000} \text{ N}$

Exercise III



1) a) Please do not consider the velocity vector \vec{v}_A sketch above as a force! The only forces are the weight \vec{P} , the ground reaction \vec{R} and the friction force \vec{f} .

b) The kinetic energy theorem reads for path AB $\Delta_{A \rightarrow B} E_k = \mathcal{W}_{A \rightarrow B}(\vec{R} + \vec{P} + \vec{f})$ (E).

Then using definition of work: $\mathcal{W}_{A \rightarrow B}(\vec{R} + \vec{P} + \vec{f}) = \int_A^B (\vec{R} + \vec{P} + \vec{f}) \cdot \vec{dl}$ where \vec{dl} is an infinitesimal shift along the trajectory.

This means that \vec{dl} is a length element tangent to the local point where we are integrating. Between A and B, the mass has a circular motion which can be described by polar coordinates and basis $(\vec{u}_\rho, \vec{u}_\theta)$, thus $\vec{dl} = R d\theta \vec{u}_\theta$ here.

What's more \vec{R} is orthogonal to motion (we consider this force as the normal ground reaction, the total reaction is given in last question) which implies that $\vec{R} \cdot \vec{dl} = 0$ at any point of the trajectory.

For friction force \vec{f} we have claimed that \vec{f} is collinear to $\vec{v} = R\dot{\theta} \vec{u}_\theta$ but with opposite orientation. So $\vec{f} \cdot \vec{dl} = -f \cdot R \cdot d\theta$ and $\int_A^B d\theta = \theta_B - \theta_A = \theta$.

Last force \vec{P} : conservative force so use the property $\mathcal{W}_{A \rightarrow B}(\vec{P}) = -\Delta_{A \rightarrow B} E_p(\vec{P}) = mg(z_A - z_B)$

Finally, the first equation (E) reads $\frac{1}{2}m(v_B^2 - v_A^2) = mg(z_A - z_B) - fR\theta$

Reference frame is chosen such that $z_B = 0$.

$$\text{So } f = \frac{1}{R\theta} \left(mgR(1 - \cos(\theta)) - \frac{1}{2}m(v_B^2 - v_A^2) \right) = \frac{1}{\frac{3}{2}} \left(\left(1 - \frac{1}{2}\right) \cdot \frac{3}{2} - \frac{1}{2} \cdot 0, 1 \cdot (4 - 9) \right) = 0,34 \text{ N}$$

2) a) Between B and C weight is not working because z is constant. Kinetic energy theorem reads now $\Delta_{B \rightarrow C} E_k = \mathcal{W}_{B \rightarrow C}(\vec{f})$. Please read question before solving any exercise! Here frictions \vec{f} are known and \vec{v}_C must be expressed as function of f .

Writing explicitly the previous equation gives the answer: $\frac{1}{2}m(v_C^2 - v_B^2) = -f \cdot BC$ and numerically $v_C = \sqrt{5} \text{ m} \cdot \text{s}^{-1}$

b) The total reaction \vec{R}_{tot} takes both normal reaction \vec{R} and frictions \vec{f} into account: $\vec{R}_{tot} = \vec{R} + \vec{f}$.

These two forces are orthogonal so the norm is simply given by $\|\vec{R}_{tot}\| = \sqrt{R^2 + f^2} = \sqrt{1,01} \approx 1 \text{ N}$