Lesson 4 比例控制器

$$\dot{m} = \frac{1}{7000}(u - 100\alpha m + d)$$

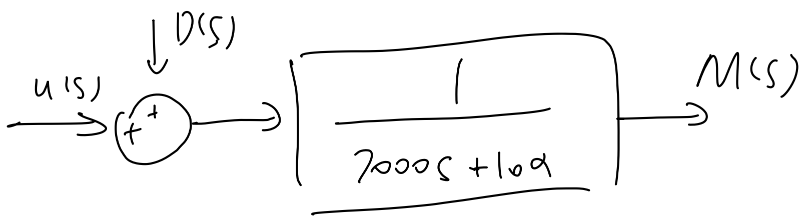
$$7000\dot{m} + 100\alpha m = u + d$$

两边拉氏变换

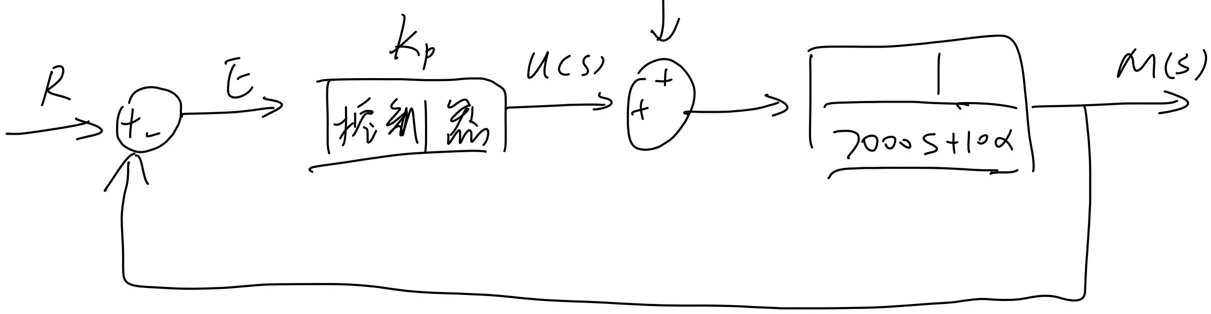
$$7000 s M(s) + 100\alpha M(s) = U(s) + D(s)$$

$$(7000s + 100\alpha) M(s) = U(s) + D(s)$$

$$\frac{M(s)}{U(s) + D(s)} = \frac{1}{7000s + 100\alpha}$$



加入控制



$$U = K_p e$$

$$E = R - M$$

$$U(s) = K_p(R - M)$$

$$[K_p(R - M) + D] \cdot \frac{1}{7000s + 100\alpha} = M$$

$$K_p R - K_p M + D = (7000s + 100\alpha) M$$

$K_pR + D = (7000s + 10\alpha + K_p)M$

$$M = \frac{K_pR + D}{7000s + 10\alpha + K_p} \implies R.D 稳定则 M 稳定$$

$R = \mathcal{L}[r] = \frac{r}{s}$　　$D = \mathcal{L}[r] = \frac{d}{s}$

$$M = \frac{\frac{K_p r}{s} + \frac{d}{s}}{7000s + 10\alpha + K_p} = \frac{K_p r + d}{s(7000s + 10\alpha + K_p)} \implies 极点\ p_1 = 0$$

$$p_2 = -\frac{10\alpha + K_p}{7000}$$

拉氏反变换 $m(t) = C_1 e^{0t} + C_2 e^{-\frac{10\alpha + K_p}{7000}}$

$= C_1 + \underbrace{C_2 e^{-\frac{10\alpha + K_p}{7000}}}$ 决定稳定性：$-\frac{10\alpha + K_p}{7000} < 0$ 即 $-10\alpha - K_p < 0$

$\underbrace{K_p > -10\alpha}$

仅用比例控制P会产生稳态误差

因此还需要引入积分控制

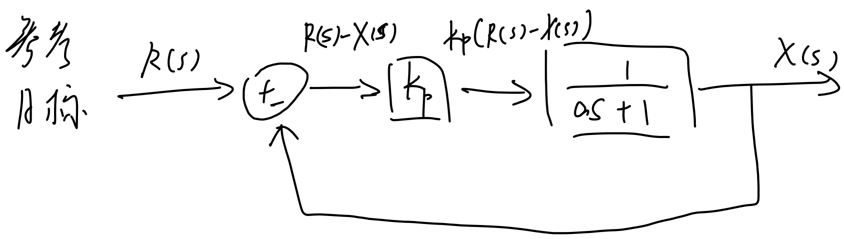
Lesson 5 终值定理和稳态误差

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) \qquad X(s) = \mathcal{L}[x(t)]$$

终值定理

条件: $\lim_{t\to\infty} x(t)$ 存在. 极点在复平面左半边(稳定)

稳态误差 e_{ss}



$$K_p(R(s)-X(s)) \cdot \frac{1}{as+1} = X(s)$$

$$K_p R(s) - K_p X(s) = (as+1)X(s)$$

$$K_p R(s) = (K_p + as + 1)X(s)$$

$$X(s) = \frac{K_p R(s)}{K_p + as + 1}$$

R(s) 稳定

因此考虑分母是否稳定即可

$$K_p + as + 1 < 0 \Rightarrow s = \frac{-1-K_p}{a} < 0 \Rightarrow K_p > -1$$

参考值 $r(t) = r$ 常数

$$R(s) = \mathcal{L}[r] = \frac{r}{s} \qquad X(s) = \frac{K_p \frac{r}{s}}{K_p + as + 1}$$

终值定理: $\lim_{t\to 0} x(t) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{K_p \frac{r}{\cancel{s}}}{K_p + as + 1} = \frac{K_p}{K_p+1} r$ ($x(t)$的终值状态. 稳定值)

$$e_{ss} = r - \frac{K_p}{K_p+1} r = \frac{1}{K_p+1} r$$

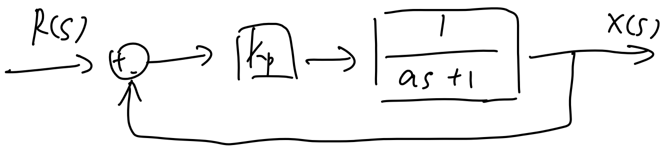
$K_p \downarrow \quad e_{ss} \uparrow$

$K_p \uparrow \quad e_{ss} \downarrow$

$K_p \to \infty \quad e_{ss} = 0$ (实际中不可能让 K_p 很大)

说明仅用比例控制无法消除稳态误差

Lesson 6 比例积分控制器 PI控制

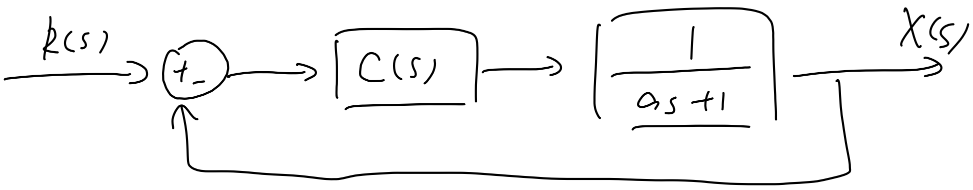


$$X(s)=\frac{K_p\frac{r}{s}}{as+1+K_p}$$

存在 e_{ss}

$\Downarrow$

要引入新的控制器 $C(s)$ $\Rightarrow$ $e_{ss}=0$ $\lim\limits_{t\to\infty}x(t)=r$



$K_p \to C(s)$

$$X(s)=\frac{C(s)\frac{r}{s}}{as+1+C(s)}$$ 设计C(s)使系统稳定

终值定理：$\lim\limits_{t\to\infty}x(t)=\lim\limits_{s\to0}sX(s)=\lim\limits_{s\to0}\frac{C(s)r}{as+1+C(s)}=\lim\limits_{s\to0}\frac{C(s)}{1+C(s)}r$

$$=\lim_{s\to0}\frac{1+C(s)-1}{1+C(s)}r=\lim_{s\to0}\left(1-\frac{1}{1+C(s)}\right)r=r-\lim_{s\to0}\frac{1}{1+C(s)}$$

目标 $e_{ss}=0$ 即 $\lim\limits_{t\to\infty}x(t)=r$ 即 $r-\lim\limits_{s\to0}\frac{1}{1+C(s)}=r$

$$\lim_{s\to0}\frac{1}{1+C(s)}=0 \Rightarrow \lim_{s\to0}C(s)=\infty$$

$\Downarrow$

$C(s)=\frac{1}{s}$ 即可 $C(t)=\int$ 积分 Integral

一般会给 $C(s)=\frac{K_I}{s}$ → 积分增益 Integral Gain

$$X(s)=\frac{C(s)\frac{r}{s}}{as+1+C(s)}=\frac{\frac{K_I}{s}\cdot\frac{r}{s}}{as+1+\frac{K_I}{s}}=\frac{K_I\cdot\frac{r}{s}}{as^2+s+K_I}=\frac{r}{s}\cdot\frac{K_I}{as^2+s+K_I}$$

$$(as^2+s+K_I)X(s)=\frac{rK_I}{s}$$

两边都拉氏反变换 $\mathcal{L}^{-1}$：$a\ddot{x}(t)+\dot{x}(t)+K_I=rK_I$ 二阶系统的阶跃响应

Lesson 7 根轨迹 - 根的作用

Lesson 8 根轨迹 - 手绘技巧

掌握根的变化规律 → 设计控制器/补偿器

Lesson 9 根轨迹 - 分离点/汇合点 & 根轨迹的几何性质

根轨迹的性质：

复数：$z_1 = \sigma_1 + j\omega_1 = \overset{模}{r_1} e^{i\overset{幅角}{\theta_1}}$　　$z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$

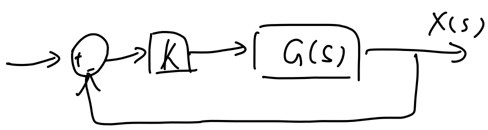
$z_2 = \sigma_2 + j\omega_2 = r_2 e^{i\theta_2}$　　$z_1 / z_2 = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$

$\therefore G(s) = \frac{N(s)}{D(s)}$　　$s = \sigma + j\omega$

$\Downarrow$

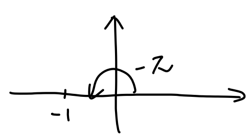
$r = \frac{\Pi \text{ zero length}}{\Pi \text{ pole length}}$　　$\frac{所有零点到复数的积}{所有极点到复数的积}$

$\theta = \sum \text{zero angle} - \sum \text{pole angle}$　　所有零点到复数的夹角之和 − 所有极点到复数的夹角之和



$X(s) = \frac{1}{1+KG(s)}$

$1 + KG(s) = 0$　　$KG(s) = -1$

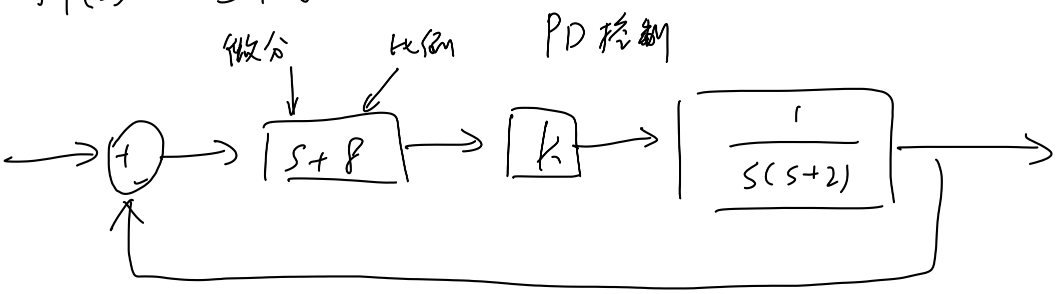


$|KG(s)| = 1 = k \cdot \frac{\Pi \text{ zero length}}{\Pi \text{ pole length}}$

$\angle KG(s) = -\pi(2q+1)$　　$q = 0, \pm 1, \pm 2 \cdots$

Lesson 10 超前补偿器

$H(s) = s + 8$



实际上很少直接使用PD控制：① 无法通过被动元件实现，需要额外能耗

② 对高频噪声敏感

加入超前补偿器

lead compensator

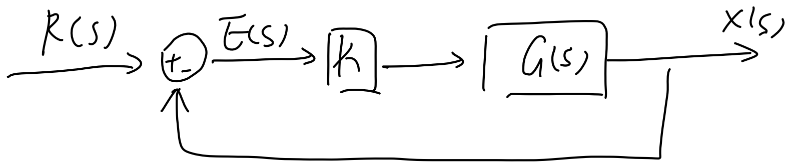
$H(s) = \frac{s+z}{s+p}$ ， $|z| < |p|$

加入零点的同时加入极点

根轨迹向左拉，提高了稳定性，

加快了反应速度。

Lesson 11　滞后补偿器



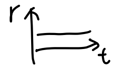
$$G(s) = \frac{N(s)}{D(s)}$$

误差 $E(s) = R(s) - X(s) = R(s) - E(s)KG(s)$

$$E(s)(1+KG(s)) = R(s)$$

$$E(s) = R(s)\frac{1}{1+KG(s)}$$

$R(s)$ 为单位阶跃　$R(s) = \frac{1}{s}$



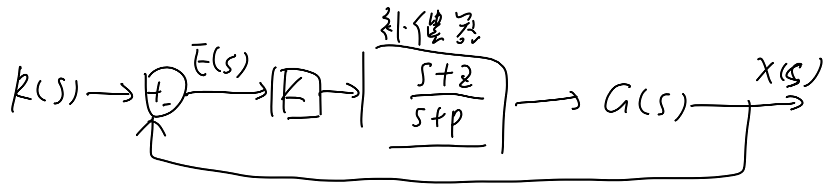
$$E(s) = R(s)\frac{1}{1+KG(s)} = R(s)\frac{1}{1+K\frac{N(s)}{D(s)}} = \frac{1}{s}\,\frac{1}{1+K\frac{N(s)}{D(s)}}$$

稳态误差 e_{ss}

终值定理

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s\cdot\frac{1}{s}\,\frac{1}{1+K\frac{N(s)}{D(s)}} = \lim_{s\to 0}\frac{1}{1+K\frac{N(s)}{D(s)}} = \frac{1}{1+K\frac{N(0)}{D(0)}} = \frac{D(0)}{D(0)+KN(0)} = e_{ss}$$

补偿器



误差 $E_c(s) = R(s) - X(s) = R(s) - E(s)KG(s)\frac{s+z}{s+p}$

$$E(s)\left(1+KG(s)\frac{s+z}{s+p}\right) = R(s)$$

$$E(s) = R(s)\frac{1}{1+KG(s)\frac{s+z}{s+p}}$$

$R(s)$ 为单位阶跃　$R(s) = \frac{1}{s}$

$$E(s) = R(s)\frac{1}{1+KG(s)\frac{s+z}{s+p}} = R(s)\frac{1}{1+K\frac{N(s)}{D(s)}\frac{s+z}{s+p}} = \frac{1}{s}\,\frac{1}{1+K\frac{N(s)}{D(s)}\frac{s+z}{s+p}}$$

稳态误差 e_{ss_c}

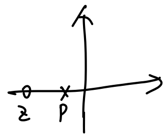
终值定理

$$e_{ss_c} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE_c(s) = \lim_{s\to 0} s\cdot\frac{1}{s}\,\frac{1}{1+K\frac{N(s)}{D(s)}\frac{s+z}{s+p}} = \lim_{s\to 0}\frac{1}{1+K\frac{N(s)}{D(s)}\frac{s+z}{s+p}} = \frac{1}{1+K\frac{N(0)}{D(0)}\frac{z}{p}} = \frac{D(0)}{D(0)+KN(0)\frac{z}{p}} = e_{ss_c}$$

$$e_{ss} = \frac{D(0)}{D(0)+KN(0)} \qquad e_{ss_c} = \frac{D(0)}{D(0)+KN(0)\frac{z}{p}}$$

目标：减小稳态误差 $\Rightarrow KN(0) < KN(0)\dfrac{z}{p} \Rightarrow \dfrac{z}{p} > 1 \Rightarrow |z| > |p|$

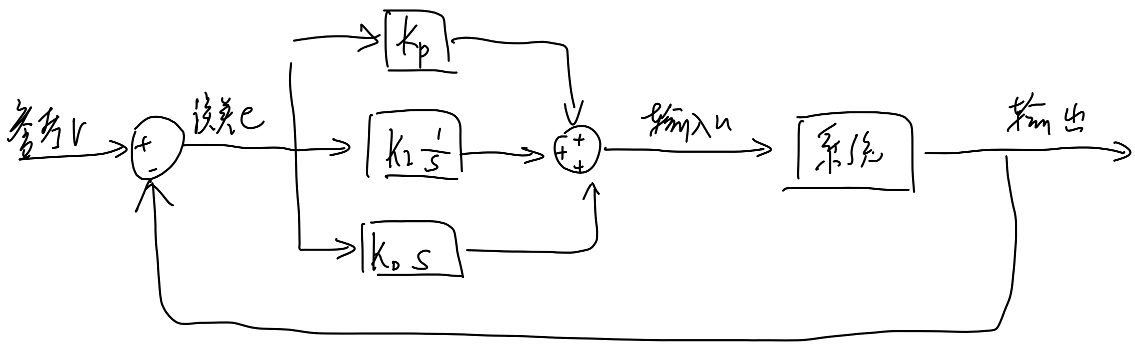
$H(s) = \dfrac{s+z}{s+p}$ $\quad |z| > |p|$



滞后补偿 Lag Compensator

设计滞后补偿器，要尽量靠近虚轴

Lesson 12 PID 控制器



① $\underline{K_p} e$ 当前误差

比例增益

+

② $\underline{K_I} \int e\,dt$ 过去误差，累积

积分增益

+

③ $\underline{K_D} \frac{de}{dt}$ 变化趋势

微分增益

$$u = K_p e + K_I \int e\,dt + K_D \frac{de}{dt}$$

两边同时拉氏变换 L

$$U(s) = \left(K_p + K_I \frac{1}{s} + K_D s\right) E(s)$$

PD 控制：提高稳定性，改善瞬态响应

+ $\Longrightarrow$ PID 控制

PI 控制：改善稳态误差

微分项 D 对噪声十分敏感

Lesson 13 奈奎斯特稳定判据