Empirical Cyclic Frequency Sieve

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1. Introduction

The study of prime numbers has been a constant theme in the history of mathematics, not only due to its theoretical significance, but also because of its applications in fields such as cryptography, information theory, and computing. Over time, various methods have been proposed for generating or verifying prime numbers, notably including the Sieve of Eratosthenes, probabilistic primality tests, and factorization algorithms.

This work presents an alternative approach, based on empirical observation of numerical patterns, that allows the detection of prime numbers greater than 3 through cyclic properties associated with numbers of the form $6n \pm 1$. The core of the method lies in the appearance of **regular frequencies** within a sequence generated exclusively by simple arithmetic operations, without the need for prior factorization or explicit knowledge of previous primes.

By analyzing these frequencies, it is possible to **generate complete lists of primes** as well as **directly evaluate the primality of a given number**, by comparing its behavior within the pattern to that of other previously defined elements. The model is based on a modular and repetitive structure, suggesting the potential for **large-scale computational optimization** and reuse of patterns.

2. Theoretical Foundation

Every natural number greater than 1 can be classified according to its remainder modulo 6. This elementary observation allows the grouping of natural numbers into six congruence classes:

$$\begin{array}{ll} \text{Class 0:} & 6n \\ \text{Class 1:} & 6n+1 \\ \text{Class 2:} & 6n+2 \\ \text{Class 3:} & 6n+3 \\ \text{Class 4:} & 6n+4 \\ \text{Class 5:} & 6n+5 \end{array}$$

Of these, only numbers of the forms $6n+1$ and $6n-1 = 6n+5$ can be prime when $n > 0$, since the remaining classes correspond to multiples of 2 or 3:

- Those of the form $6n$, $6n+2$, and $6n+4$ are even (multiples of 2).
- Those of the form $6n+3$ are divisible by 3.

Therefore, **all primes greater than 3 are found exclusively within the sequence generated by numbers of the form** $6n \pm 1$.

This property allows the search for primes to be restricted to a single alternating sequence. Starting from the number 5, it is possible to systematically generate all prime candidates using the following arithmetic rule:

$$5, \quad 5+2=7, \quad 7+4=11, \quad 11+2=13, \quad 13+4=17, \quad \ldots$$

This alternating pattern of additions $+2, +4$ produces the exact sequence of numbers of the form $6n \pm 1$, in increasing order and without omissions. This structural regularity constitutes the foundation of the proposed method.

Once this sequence is established, its elements can be organized into a two-dimensional matrix and their behavior studied through simple division operations. It is observed that when dividing the elements of the sequence by integer exponents, coincidences appear in specific positions, giving rise to what are referred to as **frequencies**.

The analysis of these frequencies, their generation, and their cyclic behavior is what enables the establishment of an alternative sieve to eliminate composite numbers and reveal true primes.

3. Generation of the Base Sequence

Since all primes greater than 3 are found exclusively in the sequence of numbers of the form $6n \pm 1$, it is possible to construct an ordered set of prime candidates using a simple and arithmetically stable generation rule:

$$a_0 = 5, \quad a_{n+1} = \begin{cases} a_n + 2 & \text{if } a_n \equiv 5 \pmod 6 \\ a_n + 4 & \text{if } a_n \equiv 1 \pmod 6 \end{cases}$$

This alternating procedure ensures that all generated terms belong to the sequence $6n \pm 1$, in the correct order. The resulting sequence is:

$$5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 25,\ 29,\ 31,\ \ldots$$

Note that this sequence contains both primes and composites. For example, 25 and 35 are included, but they are not prime. This is expected, as not all numbers of the form $6n \pm 1$ are prime; however, all primes greater than 3 are included in this sequence.

Once this initial list is obtained, its elements can be organized into a table or two-dimensional matrix, where each position corresponds to a natural index, called the *consecutive*, and is associated with certain *frequency* values, defined below.

During the empirical development of this research, it was observed that some quotients generated by dividing elements of the sequence by others also belonged to the same sequence and appeared periodically. This regularity led to the construction of a frequency table, where each row contains:

- The consecutive index k,
- Frequency 2 (f_2), associated with modified multiples of 4,
- Frequency 1 (f_1), derived from f_2 plus the multiple used,
- The average of both frequencies, which is the base number to evaluate.

Below is a representative portion of this table:

Consecutive	Frequency 2 (f_2)	Frequency 1 (f_1)	Average
0	3	7	5
1	5	9	7
2	7	15	11
3	9	17	13
4	11	23	17
5	13	25	19
6	15	31	23
7	17	33	25
8	19	39	29

Definition of the consecutive index. Let $k \in \mathbb{N}$ denote the *consecutive index*, representing the position of a number within the ordered list of elements of the form $6n \pm 1$, starting from 5.

Each index k corresponds to a row in the frequency table and uniquely determines:

- The number $a_k \in \{6n \pm 1\}$,
- Its associated frequencies $f_1(k)$ and $f_2(k)$,
- The average $f_3(k) = \frac{f_1(k)+f_2(k)}{2}$, which defines the base number.

The index k serves as a reference in both the table-based sieve and the direct collision detection mechanism.

Definition of the base number. Let a_k denote the number of the form $6n \pm 1$ associated with the index k. This number is obtained as the average of the two frequencies at that index:

$$a_k = f_3(k) = \frac{f_1(k) + f_2(k)}{2}$$

The value a_k acts as the *base number* for the cancellation pattern. Each base number defines its own cyclic sequence of composite eliminations within the frequency-based sieve.

In the context of direct primality checking, a_k serves as the anchor from which collision-based evaluation is performed.

The generation of these frequencies is determined by simple operations derived from multiples of 4:

- **Frequency 2:** obtained by subtracting or adding 1 to multiples of 4, i.e., $4n \pm 1$,
- **Frequency 1:** calculated as the frequency 2 value plus the corresponding multiple of 4.

Example:

$$
\begin{aligned}
4-1=3,&\quad 3+4=7\\
4+1=5,&\quad 5+4=9\\
8-1=7,&\quad 7+8=15\\
8+1=9,&\quad 9+8=17
\end{aligned}
$$

This scheme reveals that both the frequencies and the base numbers are deeply linked to the modular structure of 6, and allow the information to be organized in a systematic way for applying a cancellation-based sieve.

4. The Frequency-Based Sieve

The purpose of this sieve is to identify prime numbers within the sequence $6n \pm 1$, eliminating those that are composite using a strategy based on **cyclic frequency cancellation**. Unlike the Sieve of Eratosthenes, this model does not rely on explicit divisions between integers nor does it require prior factorization. It is based on the repeated application of patterns derived from **frequencies 1 and 2**, observed empirically in the quotient matrix.

4.1. Structure of the Sieve

Each base number p in the sequence $6n \pm 1$ is associated with two frequencies:

- f_2: generated as $4n \pm 1$,
- $f_1 = f_2 + 4n$.

These frequencies determine the periodicity with which p will cancel other numbers in the list. Cancellation occurs in an alternating manner: first advance f_1 positions, then f_2, then f_1, and so on.

4.2. Cancellation Procedure

Given a base number p in the list, its corresponding row contains the following elements: the consecutive index k, the frequencies f_1 and f_2, and the associated number $6n \pm 1$. The sieve operates as follows:

1. Starting from the position of p, the frequencies $f_1, f_2, f_1, f_2, \ldots$ are added successively, generating a sequence of indices.
2. At each of these positions, the corresponding number is **canceled**, as it is identified as composite.
3. This process is repeated for each base number in the table, following ascending order.

4.3. Justification of the Cancellation Range

It has been observed that base numbers do not begin to cancel new composites until they reach their square. This means that if the base number is p, the effective cancellation of composites begins from p^2 onward. Before that point, all multiples that are canceled have already been affected by the frequencies of previous numbers in the list.

This property, analogous to what is seen in the Sieve of Eratosthenes, allows computational work to be reduced: when verifying the primality of a number N, it is only necessary to apply the cancellations corresponding to base numbers $p \leq \sqrt{N}$.

4.4. Definition of Collision

Given a target number N belonging to the sequence $6n \pm 1$, a collision occurs when one of the frequencies from a previous base number lands exactly on N. Mathematically, a collision is said to occur if there exists a base number p_i with frequencies f_{1i} and f_{2i}, such that one of the following expressions is an integer:

$$\frac{N - (f_{1i} + i)}{f_{1i} + f_{2i}} \in \mathbb{Z} \quad \text{or} \quad \frac{N - f_{1i} - f_{2i} \cdot i}{f_{1i} + f_{2i}} \in \mathbb{Z}$$

where i is the index of the base number in the frequency table.

In practice, if **at least one** of these expressions is an integer, the number N is considered to have been reached by the cyclic cancellation pattern and is therefore composite.

If no collision occurs after checking all base numbers $p_i \leq \sqrt{N}$, then N is concluded to be prime.

4.5. Theoretical Justification of the Range

As in the list-based sieve, direct primality evaluation does not require considering base numbers greater than $\sqrt{N}$. This is because a composite number $N = ab$, with $a, b > 1$, will always have at least one factor $a \leq \sqrt{N}$, and will therefore be reached by some frequency corresponding to a base number less than or equal to that root.

5. Observed Optimization and Pattern Reuse

One of the most notable features of the cyclic frequency sieve model is the appearance of **reusable cyclic patterns** in the cancellation sequence. This property is not only consistent with the arithmetic regularity of the system, but also opens the door to potential improvements in its computational implementation.

5.1. Cyclic Cancellation Blocks

Each base number of the form $6n \pm 1$ generates its own cancellation sequence through the frequencies f_1 and f_2. These sequences, when represented in a matrix, show a cyclic behavior that repeats as they progress in blocks. In particular, it has been observed that:

- The periodicity of each frequency is preserved when the pattern is replicated over multiples of previous primes.

- As the table advances, certain cancellation patterns repeat in a synchronized manner, forming **structured blocks** that can be reused.

This observation has a direct impact on optimization: **once a base pattern is generated, it can be applied to larger intervals without recalculating the frequencies from scratch**.

5.2. Cyclic Reset and Frequency Coupling

In contexts where multiple frequencies coincide (for example, at positions associated with products of primes such as $5 \times 7 \times 11$), it has been observed that the patterns tend to **reset or recouple** in configurations similar to previous ones, as if there were an underlying modular structure in the cancellation process.

This suggests that the model not only eliminates composites, but does so in a structured and potentially **predictive** way, which significantly distinguishes it from methods such as the Sieve of Eratosthenes, where eliminations do not exhibit such inherent order.

6. Discussion

The cyclic frequency sieve model is presented as a conceptual alternative to the traditional paradigm of prime number detection. While it does not replace methods such as the Sieve of Eratosthenes or modern primality tests, it offers an original approach to the problem from a modular, arithmetic, and structured perspective.

6.1. Advantages of the Model

- **Independence from factorization:** It does not require prior knowledge of divisors nor traditional divisions between integers.

- **Full coverage of the relevant domain:** It includes all prime numbers greater than 3, as it is constructed upon the sequence $6n \pm 1$.

- **Structural regularity:** The alternating frequency pattern reveals an internal organization that allows for the prediction and cancellation of composites through a cyclical, non-random mechanism.

- **Potential for pattern reuse:** Unlike the Sieve of Eratosthenes, which operates through direct multiplicative cancellation, this model shows how cancellation blocks repeat, suggesting a natural optimization based on periodicity.

6.2. Current Limitations

- **Does not cover primes less than 5:** Due to its construction, the model must be complemented with direct verification of cases 2 and 3.
- **Does not operate over the entire set of natural numbers:** The working universe is limited to the sequence $6n \pm 1$, so numbers outside it must be discarded from the start. Some (such as composites of the form $6n \pm 3$) might appear "omitted" without further explanation.
- **Still lacks a complete formal proof:** Although its operation has been empirically validated and its internal logic is coherent, the model does not yet have a rigorous mathematical formalization comparable to established methods.
- **Sensitivity to evaluation range:** Effective cancellation is based on a partial traversal limited by $\sqrt{N}$, which requires careful management of each frequency's scope.

7. Conclusions

The cyclic frequency sieve model constitutes an alternative proposal for primality analysis, built upon empirical arithmetic observations of the sequence $6n \pm 1$. Its operating principle, based on periodic cancellation patterns, enables the identification of composite numbers without resorting to factorization or direct divisions among candidates.

Through a system of frequencies associated with each base number, the model establishes a collision mechanism that determines whether a number should be discarded as non-prime. This methodology, applied in an orderly and cyclical manner, offers a structured approach that respects the modular nature of prime number distribution.

Tests carried out confirm that the system is capable of generating complete lists of primes (greater than 3) and evaluating individual cases with precision. Despite not yet having a complete formal mathematical proof, the model presents a set of coherent, verifiable, and replicable internal rules.

Finally, its operational simplicity, compatibility with programming, and potential for future optimization make it a valuable subject of study for those seeking to explore new approaches in number theory from a practical and structured perspective.