Cyclic Frequency Sieve

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1. Introduction

The study of prime numbers has been a constant theme in the history of mathematics, not only due to its theoretical significance, but also because of its applications in fields such as cryptography, information theory, and computing. Over time, various methods have been proposed for generating or verifying prime numbers, notably including the Sieve of Eratosthenes, probabilistic primality tests, and factorization algorithms.

This work presents an alternative approach, based on empirical observation of numerical patterns, that allows the detection of prime numbers greater than 3 through cyclic properties associated with numbers of the form $6n \pm 1$. The core of the method lies in the appearance of **regular frequencies** within a sequence generated exclusively by simple arithmetic operations, without the need for prior factorization or explicit knowledge of previous primes.

By analyzing these frequencies, it is possible to **generate complete lists of primes**, based on the systematic cancellation of non-primes within the frequency pattern. The model is based on a modular and repetitive structure, suggesting the potential for **large-scale computational optimization** and reuse of patterns.

2. Theoretical Foundation

Every natural number greater than 1 can be classified according to its remainder modulo 6. This elementary observation allows the grouping of natural numbers into six congruence classes:

$$\begin{array}{ll} \text{Class 0:} & 6n \\ \text{Class 1:} & 6n+1 \\ \text{Class 2:} & 6n+2 \\ \text{Class 3:} & 6n+3 \\ \text{Class 4:} & 6n+4 \\ \text{Class 5:} & 6n+5 \end{array}$$

Of these, only numbers of the forms $6n+1$ and $6n-1 = 6n+5$ can be prime when $n > 0$, since the remaining classes correspond to multiples of 2 or 3:

- Those of the form $6n$, $6n+2$, and $6n+4$ are even (multiples of 2).
- Those of the form $6n+3$ are divisible by 3.

Therefore, **all primes greater than 3 are found exclusively within the sequence generated by numbers of the form** $6n \pm 1$.

This property allows the search for primes to be restricted to a single alternating sequence. Starting from the number 5, it is possible to systematically generate all prime candidates using the following arithmetic rule:

$$5, \quad 5+2=7, \quad 7+4=11, \quad 11+2=13, \quad 13+4=17, \quad \ldots$$

This alternating pattern of additions $+2, +4$ produces the exact sequence of numbers of the form $6n \pm 1$, in increasing order and without omissions. This structural regularity constitutes the foundation of the proposed method.

Once this sequence is established, its elements can be organized into a two-dimensional matrix and their behavior studied through simple division operations. It is observed that when dividing the elements of the sequence by integer exponents, coincidences appear in specific positions, giving rise to what are referred to as **frequencies**.

The analysis of these frequencies, their generation, and their cyclic behavior is what enables the establishment of an alternative sieve to eliminate composite numbers and reveal true primes.

3. Generation of the Base Sequence

Since all primes greater than 3 are found exclusively in the sequence of numbers of the form $6n \pm 1$, it is possible to construct an ordered set of prime candidates using a simple and arithmetically stable generation rule:

$$a_0 = 5, \quad a_{n+1} = \begin{cases} a_n + 2 & \text{if } a_n \equiv 5 \pmod 6 \\ a_n + 4 & \text{if } a_n \equiv 1 \pmod 6 \end{cases}$$

This alternating procedure ensures that all generated terms belong to the sequence $6n \pm 1$, in the correct order. The resulting sequence is:

$$5,\ 7,\ 11,\ 13,\ 17,\ 19,\ 23,\ 25,\ 29,\ 31,\ \ldots$$

Note that this sequence contains both primes and composites. For example, 25 and 35 are included, but they are not prime. This is expected, as not all numbers of the form $6n \pm 1$ are prime; however, all primes greater than 3 are included in this sequence.

Once this initial list is obtained, its elements can be organized into a table or two-dimensional matrix, where each position corresponds to a natural index, called the *consecutive*, and is associated with certain *frequency* values, defined below.

During the empirical development of this research, it was observed that some quotients generated by dividing elements of the sequence by others also belonged to the same sequence and appeared periodically. This regularity led to the construction of a frequency table, where each row contains:

- The consecutive index k,
- Frequency 2 (f_2), associated with modified multiples of 4,
- Frequency 1 (f_1), derived from f_2 plus the multiple used,
- The average of both frequencies, which is the base number to evaluate.

Below is a representative portion of this table:

Consecutive	Frequency 2 (f_2)	Frequency 1 (f_1)	Average
0	3	7	5
1	5	9	7
2	7	15	11
3	9	17	13
4	11	23	17
5	13	25	19
6	15	31	23
7	17	33	25
8	19	39	29

The generation of these frequencies is determined by simple operations derived from multiples of 4:

- **Frequency 2:** obtained by subtracting or adding 1 to multiples of 4, i.e., $4n \pm 1$,
- **Frequency 1:** calculated as the frequency 2 value plus the corresponding multiple of 4.

Example:

$$\begin{aligned} 4-1=3, &\quad 3+4=7 \\ 4+1=5, &\quad 5+4=9 \\ 8-1=7, &\quad 7+8=15 \\ 8+1=9, &\quad 9+8=17 \end{aligned}$$

This scheme reveals that both the frequencies and the base numbers are deeply linked to the modular structure of 6, and allow the information to be organized in a systematic way for applying a cancellation-based sieve.

4. The Frequency-Based Sieve

The purpose of this sieve is to identify prime numbers within the sequence $6n \pm 1$, eliminating those that are composite using a strategy based on **cyclic frequency cancellation**. Unlike the Sieve of Eratosthenes, this model does not rely on explicit divisions between integers nor does it require prior factorization. It is based on the repeated application of patterns derived from **frequencies 1 and 2**, observed empirically in the quotient matrix.

4.1. Structure of the Sieve

Each base number p in the sequence $6n \pm 1$ is associated with two frequencies:

- f_2: generated as $4n \pm 1$,
- $f_1 = f_2 + 4n$.

These frequencies determine the periodicity with which p will cancel other numbers in the list. Cancellation occurs in an alternating manner: first advance f_1 positions, then f_2, then f_1, and so on.

4.2. Cancellation Procedure

Given a base number p in the list, its corresponding row contains the following elements: the consecutive index k, the frequencies f_1 and f_2, and the associated number $6n \pm 1$. The sieve operates as follows:

1. Starting from the position of p, the frequencies $f_1, f_2, f_1, f_2, \ldots$ are added successively, generating a sequence of indices.
2. At each of these positions, the corresponding number is **canceled**, as it is identified as composite.
3. This process is repeated for each base number in the table, following ascending order.

4.3. Justification of the Cancellation Range

It has been observed that base numbers do not begin to cancel new composites until they reach their square. This means that if the base number is p, the effective cancellation of composites begins from p^2 onward. Before that point, all multiples that are canceled have already been affected by the frequencies of previous numbers in the list.

This property, analogous to what is seen in the Sieve of Eratosthenes, allows computational work to be reduced: when verifying the primality of a number N, it is only necessary to apply the cancellations corresponding to base numbers $p \leq \sqrt{N}$.

5. Observed Optimization and Pattern Reuse

One of the most notable features of the cyclic frequency sieve model is the appearance of **reusable cyclic patterns** in the cancellation sequence. This property is not only consistent with the arithmetic regularity of the system, but also opens the door to potential improvements in its computational implementation.

5.1. Cyclic Cancellation Blocks

Each base number of the form $6n \pm 1$ generates its own cancellation sequence through the frequencies f_1 and f_2. These sequences, when represented in a matrix, show a cyclic behavior that repeats as they progress in blocks. In particular, it has been observed that:

- The periodicity of each frequency is preserved when the pattern is replicated over multiples of previous primes.
- As the table advances, certain cancellation patterns repeat in a synchronized manner, forming **structured blocks** that can be reused.

This observation has a direct impact on optimization: **once a base pattern is generated, it can be applied to larger intervals without recalculating the frequencies from scratch**.

5.2. Cyclic Reset and Frequency Coupling

In contexts where multiple frequencies coincide (for example, at positions associated with products of primes such as $5 \times 7 \times 11$), it has been observed that the patterns tend to **reset or recouple** in configurations similar to previous ones, as if there were an underlying modular structure in the cancellation process.

This suggests that the model not only eliminates composites, but does so in a structured and potentially **predictive** way, which significantly distinguishes it from methods such as the Sieve of Eratosthenes, where eliminations do not exhibit such inherent order.

5.3. Computational Implications

From a programming perspective, this cyclic structure:

- Enables the implementation of **reusable blocks** of frequencies.
- Reduces the number of operations needed over large intervals.
- Opens the possibility of building **parallelizable or vectorized** versions of the algorithm.

These properties make the model particularly promising for **exploration in large prime number ranges**, where computational efficiency is critical.

6. Computational Implementation in Python

To validate and practically apply the cyclic frequency sieve model, a computational implementation has been developed using the Python programming language. This automated version focuses exclusively on the generation of prime number lists through cyclic frequency cancellation, strictly based on the rules defined by the model.

6.1. Code Structure

The algorithm is built around two main modules:

- **Base list generation:** produces the sequence of numbers of the form $6n \pm 1$, along with their associated frequencies.

The algorithm applies the cancellation patterns iteratively across the base list, using the alternating sequence $f_1, f_2, f_1, f_2, \ldots$, in order to identify and discard composite numbers within the predefined range.

6.2. Model Validation

The implementation has been tested using both small and moderately large numbers. While the sieve model is theoretically capable of handling very large ranges, its current computational performance does not support the evaluation of very large numbers, such as Mersenne primes, within practical time limits.

The purpose of the algorithm is not speed, but to illustrate the internal logic of the sieve based on cyclic frequencies with full adherence to the model's structure.

6.3. Optimization Considerations

The code structure was designed to allow for successive improvements:

- Reduction in the number of calculations by leveraging the $\sqrt{N}$ limit as a natural analysis threshold.
- Possibility of reusing already calculated frequencies, storing them in precomputed structures.
- Modular design that allows easy adaptation of the system for both lists and direct analyses.

6.4. Current State and Performance Limits

The current code has been successfully tested within reasonable intervals and has allowed for the theoretical validity of the model to be confirmed across various ranges. However, it does not yet feature advanced optimizations, making it unsuitable for evaluating extremely large numbers, such as Mersenne primes, without incurring significant computation times.

It is worth noting that earlier, lighter versions—developed with different approaches—did manage to correctly evaluate certain large primes, such as $2^{1279} - 1$, in under one second. However, that capability does not apply to the current code, which prioritizes fidelity to the frequency-based model and its complete structure.

6.5. Opportunities for Improvement

Due to its modular and cyclic construction, the model is highly susceptible to optimization in future versions, with strategies such as:

- Reuse of precomputed blocks.
- Evaluation restricted to the domain $p \leq \sqrt{N}$.
- Possible vectorization of the cancellation pattern.
- Parallel implementation of frequency exploration.

These enhancements could enable performance levels comparable to previous implementations, without compromising the conceptual fidelity of the method.

The model's structure, built upon the alternating frequency pattern $f_1, f_2, f_1, f_2, \ldots$, suggests the possibility of deeper algebraic insights into the periodicity and interaction of frequency patterns over extended intervals.

7. Discussion

The cyclic frequency sieve model is presented as a conceptual alternative to the traditional paradigm of prime number detection. While it does not replace methods such as the Sieve of Eratosthenes or modern primality tests, it offers an original approach to the problem from a modular, arithmetic, and structured perspective.

7.1. Advantages of the Model

- **Independence from factorization:** It does not require prior knowledge of divisors nor traditional divisions between integers.
- **Full coverage of the relevant domain:** It includes all prime numbers greater than 3, as it is constructed upon the sequence $6n \pm 1$.
- **Structural regularity:** The alternating frequency pattern reveals an internal organization that allows for the prediction and cancellation of composites through a cyclical, non-random mechanism.
- **Potential for pattern reuse:** Unlike the Sieve of Eratosthenes, which operates through direct multiplicative cancellation, this model shows how cancellation blocks repeat, suggesting a natural optimization based on periodicity.

7.2. Current Limitations

- **Does not cover primes less than 5:** Due to its construction, the model must be complemented with direct verification of cases 2 and 3.
- **Does not operate over the entire set of natural numbers:** The working universe is limited to the sequence $6n \pm 1$, so numbers outside it must be discarded from the start. Some (such as composites of the form $6n \pm 3$) might appear "omitted" without further explanation.
- **Still lacks a complete formal proof:** Although its operation has been empirically validated and its internal logic is coherent, the model does not yet have a rigorous mathematical formalization comparable to established methods.
- **Sensitivity to evaluation range:** Effective cancellation is based on a partial traversal limited by $\sqrt{N}$, which requires careful management of each frequency's scope.

8. Conclusions

The cyclic frequency sieve model constitutes an alternative proposal for primality analysis, built upon empirical arithmetic observations of the sequence $6n \pm 1$. Its operating principle, based on periodic cancellation patterns, enables the identification of composite numbers without resorting to factorization or direct divisions among candidates.

Through a system of frequencies associated with each base number, the model systematically eliminates composite numbers through periodic cancellation. This methodology, applied in an orderly and cyclical manner, offers a structured approach that respects the modular nature of prime number distribution.

Tests carried out confirm that the system is capable of generating complete lists of primes (greater than 3). Despite not yet having a complete formal mathematical proof, the model presents a set of coherent, verifiable, and replicable internal rules.

Finally, its operational simplicity, compatibility with programming, and potential for future optimization make it a valuable subject of study for those seeking to explore new approaches in number theory from a practical and structured perspective.