问题1

关于系统初始化的讨论。对于一个lidar+imu+gnss接收机（没做组合导航）的系统，老师说可以按照vins的思路去初始化。我查阅了相关论文，总结了一下方法，老师请帮忙看看，打算在之后大作业中实现。

通过vins-mono论文中的方法，我们可以得到：陀螺仪的bias，重力，速度，雷达坐标系相对于世界坐标系（ENU系）的旋转（其中roll和pitch是准的，yaw不准）。所以雷达坐标系与真正的ENU系对齐**还差一个平移向量和yaw角**，参考黄国权老师的《Intermittent GPS-aided VIO: Online Initialization and Calibration》论文，我找到了解决思路：

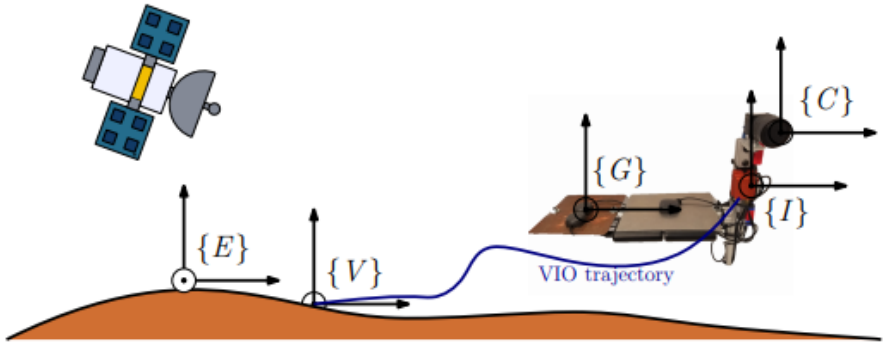

Fig. 2: Our integrated sensor system is composed of five different frames: ENU frame $\{E\}$, VIO frame $\{V\}$, IMU sensor frame $\{I\}$, camera sensor frame $\{C\}$, and GPS sensor frame $\{G\}$. $\{E\}$ is the frame of the reference of the GPS measurements and $\{V\}$ is the local frame set up by VIO whose orientation is aligned with gravity.

Given a set of GPS position measurements in the ENU frame $\{{}^E\mathbf{p}_{G_1}, \cdots, {}^E\mathbf{p}_{G_n}\}$ within the keyframe window and the corresponding interpolated positions in the VIO frame $\{{}^V\mathbf{p}_{G_1}, \cdots {}^V\mathbf{p}_{G_n}\}$, we use the following geometric constraints to derive the frame initialization:

$$ {}^E\mathbf{p}_{G_i} = {}^E\mathbf{p}_V + {}^E_V\mathbf{R}\,{}^V\mathbf{p}_{G_i}, \forall i = 1 \cdots n \Rightarrow \quad (21) $$

$$ {}^E\mathbf{p}_{G_j} - {}^E\mathbf{p}_{G_1} = {}^E_V\mathbf{R}({}^V\mathbf{p}_{G_j} - {}^V\mathbf{p}_{G_1}), \forall j = 2 \cdots n \quad (22) $$

As mentioned earlier, there is a 4 d.o.f (instead of 6 d.o.f) transformation including 3 d.o.f translation and 1 d.o.f for yaw between the ENU and VIO frames due to the fact that both frames are gravity aligned, which entails that we can simply use the rotation about the global z-axis with yaw

angle θ:

$$ {}^{E}_{V}\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{23} $$

With (23) we can re-write (22) as the following linear constraint:

$$ \mathbf{A}_j \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} := \mathbf{A}_j \mathbf{w} = \mathbf{b}_j, \forall j = 2 \cdots n \tag{24} $$

Stacking all these constraints yields the following linear least-squares with quadratic constraint, which can be solved for $\mathbf{w}$, e.g., by Lagrangian multipliers [39]:

$$ \min_{\mathbf{w}} \ ||\mathbf{A}\mathbf{w} - \mathbf{b}||^2, \quad \text{s.t. } ||\mathbf{w}||^2 = 1 \tag{25} $$

The solution of (25) immediately provides the sought rotation ${}^{E}_{V}\hat{\mathbf{R}}$. We substitute it into (21) and solve for ${}^{E}\mathbf{p}_V$ as:

$$ {}^{E}\hat{\mathbf{p}}_V = \frac{1}{n} \sum_{i=1}^{n} \left[{}^{E}\mathbf{p}_{G_i} - {}^{E}_{V}\hat{\mathbf{R}}^{V}\mathbf{p}_{G_i} \right] \tag{26} $$

The resulting $\{{}^{E}_{V}\hat{\mathbf{R}}, {}^{E}\hat{\mathbf{p}}_V\}$ initial guess of the GPS-VIO frame transformation is further corrected using delayed initialization [42], [45], which appends the transform to the state in a probabilistic fashion. Specifically, by augmenting the state vector with the transformation along with an *infinite* covariance prior for these new variables, we perform the standard EKF update using all collected GPS measurements. After initialization, we marginalize all the keyframes to reduce the state to the original state size (see Fig. 3).

按照论文中的（25）（26）公式我们就能得到一个**比较准确**的雷达原点坐标系到ENU系的变化矩阵

问题：

1. 我们获得上述这个**相对比较准的变换矩阵**对于gnss轨迹与lidar轨迹对齐已经够用了吧？
2. 公式（25）为什么要优化一个带约束的非线性最小二乘问题？这使得优化变得非常复杂，本身R_V^E就是关于θ的函数。我可不可以直接优化θ：

$$ \arg\min_{\theta} \ ||A\theta - b||^2 $$

把公式（25）变成无约束的最小二乘问题去做，这样会不会有影响？

问题2

关于编码器测量建模的问题。在推导imu+odom做预测，gnss做观测的ESKF中，我速度误差这样推导有没有问题？

位置误差

1）不考虑误差时的微分方程

$$\dot{p} = R_{wb} v^b$$

2）考虑误差时的微分方程

$$\dot{\tilde{p}} = \tilde{R}_{wb} \tilde{v}^b$$

3）真实值与理想值的关系（有“~”为估计值）

$$\tilde{p} = p + \delta p$$
$$\tilde{R}_{wb} = R_{wb}(I + [\delta\theta]_\times)$$
$$\tilde{v}^b = v^b + n_v$$

4）代入

$$\dot{p} + \delta\dot{p} = R_{wb}(I + [\delta\theta]_\times)(v^b + n_v)$$

5）移项并忽略二阶小量

$$\delta\dot{p} = -R_{wb}[v^b]_\times \delta\theta + R_{wb} n_v$$

老师讲编码器预积分时提到：编码器测量也是有bias的，那考虑编码器测量速度的bias则是否可以建模为：

$$\tilde{v} = v^b + n_v + b_v$$

问题3

预积分每次把bias当0去计算感觉有点不合理。老师你也说过，bias是一个很大的量，同时预积分也是一个很强非线性的函数。我们用这个在0工作点处算的雅克比去更新预积分是不是有点不合理？已经明显不满足局部近似了

kitti数据中imu比较好，可能影响没那么大；在低成本imu中我们是不是不应该这样做？