imu RK4积分

RK4通式

$$
\begin{aligned}
x_{n+1} = x_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k_4) \\
k_1 = f(t_n, x_n) \\
k_2 = f(t_n + \frac{1}{2}\Delta t, x_n + \frac{\Delta t}{2}k_1) \\
k_3 = f(t_n + \frac{1}{2}\Delta t, x_n + \frac{\Delta t}{2}k_2) \\
k_4 = f(t_n + \Delta t, x_n + \Delta t \cdot k_3)
\end{aligned}
$$

姿态

四元数的微分方程：

$$
f(t,q) = \dot{q} = \frac{1}{2} q \otimes \begin{bmatrix} 0 \\ \omega(t) \end{bmatrix}
$$

预备知识，imu在t(n)和t(n+1)这两个时间段测量的角速度为w1和w2，即t(n)~t(n+1)时刻的角加速度为：

$$
a = \frac{w_2 - w_1}{\Delta t}
$$

即我们可以得到：

$$
\omega(t_n + \frac{\Delta t}{2}) = w_1 + \frac{\Delta t}{2} a = \frac{w_1 + w_2}{2}
$$

下面正式推导基于RK4的姿态解算：

$$
k_1 = f(t_n, x_n) = \frac{1}{2} q_0 \otimes \begin{bmatrix} 0 \\ \omega(t_n) \end{bmatrix}
$$

其中q_0为**单位四元数**

$$
\begin{aligned}
k_2 = f(t_n + \frac{1}{2}\Delta t, x_n + \frac{\Delta t}{2}k_1) \\
= \frac{1}{2}(q_0 + \frac{\Delta t}{2}k_1) \otimes \begin{bmatrix} 0 \\ \omega(t_n + \frac{\Delta t}{2}) \end{bmatrix} \\
= \frac{1}{2} q_1 \otimes \begin{bmatrix} 0 \\ \omega(t_n + \frac{\Delta t}{2}) \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
k_3 = f(t_n + \frac{1}{2}\Delta t, x_n + \frac{\Delta t}{2}k_2) \\
= \frac{1}{2}(q_0 + \frac{\Delta t}{2}k_2) \otimes \begin{bmatrix} 0 \\ \omega(t_n + \frac{\Delta t}{2}) \end{bmatrix} \\
= \frac{1}{2} q_2 \otimes \begin{bmatrix} 0 \\ \omega(t_n + \frac{\Delta t}{2}) \end{bmatrix}
\end{aligned}
$$

$$\begin{aligned}k_4 &= f(t_n+\Delta t, x_n+\Delta t\cdot k_3)\\ &= \frac{1}{2}(q_0+\Delta t\cdot k_3)\otimes\begin{bmatrix}0\\ \omega(t_n+\Delta t)\end{bmatrix}\\ &= \frac{1}{2}q_3\otimes\begin{bmatrix}0\\ \omega(t_n+\Delta t)\end{bmatrix}\end{aligned}$$

最终：

$$dq = q_0+\frac{\Delta t}{6}(k_1+2k_2+2k_3+k_4)$$
$$q^w_{b_{n+1}} = q^w_{b_n}\otimes dq$$

速度

速度的微分方程：

$$\dot{v} = f(t,v) = R\cdot a - g$$

t(n)和t(n+1)加速度计测量值为a1和a2，则a(t+dt)的测量值为：

$$a(t+\frac{\Delta t}{2}) = \frac{a_1+a_2}{2}$$

下面的q0、q1、q2、q3请参考上面姿态解算的公式，C( )表示四元数转成旋转矩阵

$$k_1 = f(t_n, x_n) = C(q^w_{b_n}q_0)a(t_n) - g$$

$$k_2 = f(t_n+\frac{1}{2}\Delta t, x_n+\frac{\Delta t}{2}k_1)$$

$$= C(q^w_{b_n}q_1)a(t_n+\frac{\Delta t}{2}) - g$$

$$k_3 = f(t_n+\frac{1}{2}\Delta t, x_n+\frac{\Delta t}{2}k_2)$$

$$= C(q^w_{b_n}q_2)a(t_n+\frac{\Delta t}{2}) - g$$

$$\begin{aligned}k_4 &= f(t_n+\Delta t, x_n+\Delta t\cdot k_3)\\ &= C(q^w_{b_n}q_3)a(t_n+\Delta t) - g\end{aligned}$$

最终:

$$v_{n+1} = v_n+\frac{\Delta t}{6}(k_1+2k_2+2k_3+k_4)$$

位置

位置的微分方程:

$$f(t,p) = \dot{p} = v$$

$$k_1 = f(t_n, x_n) = v_n$$
$$k_2 = f(t_n+\frac{1}{2}\Delta t, x_n+\frac{\Delta t}{2}k_1) = v_n+\frac{\Delta t}{2}k_1$$
$$k_3 = f(t_n+\frac{1}{2}\Delta t, x_n+\frac{\Delta t}{2}k_2) = v_n+\frac{\Delta t}{2}k_2$$
$$k_4 = f(t_n+\Delta t, x_n+\Delta t\cdot k_3) = v_n+\Delta t k_3$$