



深蓝学院  
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## 第十章作业讲解



主讲人 Teamo



- 第一部分：补全代码，梳理逻辑
- 第二部分：与EKF进行对比
- 第三部分：不同长度窗口对于结果的影响

# 补全代码梳理逻辑

```
// TODO: add residual blocks:
// b.1. marginalization constraint:
if (
    !residual_blocks_.map_matching_pose.empty() &&
    !residual_blocks_.relative_pose.empty() &&
    !residual_blocks_.imu_pre_integration.empty()
) {
    auto &key_frame_m = optimized_key_frames_.at(N - kWindowSize - 1);
    auto &key_frame_r = optimized_key_frames_.at(N - kWindowSize - 0);

    const ceres::CostFunction *factor_map_matching_pose = GetResMapMatchingPose(
        residual_blocks_.map_matching_pose.front()
    );
    const ceres::CostFunction *factor_relative_pose = GetResRelativePose(
        residual_blocks_.relative_pose.front()
    );
    const ceres::CostFunction *factor_imu_pre_integration = GetResIMUPreIntegration(
        residual_blocks_.imu_pre_integration.front()
    );

    sliding_window::FactorPRVAGMarginalization *factor_marginalization = new sliding_window::FactorPRVAGMarginaliz

    factor_marginalization->SetResMapMatchingPose(
        factor_map_matching_pose,
        std::vector<double *>{key_frame_m.prvag}
    );
    factor_marginalization->SetResRelativePose(
        factor_relative_pose,
        std::vector<double *>{key_frame_m.prvag, key_frame_r.prvag}
    );
    factor_marginalization->SetResIMUPreIntegration(
        factor_imu_pre_integration,
        std::vector<double *>{key_frame_m.prvag, key_frame_r.prvag}
    );
    factor_marginalization->Marginalize(key_frame_r.prvag);
}
```

# 补全代码梳理逻辑

- 雷达位姿单边约束

```
void SetResMapMatchingPose(
    const ceres::CostFunction *residual,
    const std::vector<double *> &parameter_blocks
) {
    // init:
    ResidualBlockInfo res_map_matching_pose(residual, parameter_blocks);
    Eigen::VectorXd residuals;
    std::vector<Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic, Eigen::RowMajor>> jacobians;

    // compute:
    Evaluate(res_map_matching_pose, residuals, jacobians);
    const Eigen::MatrixXd &J_m = jacobians.at(0);

    //
    // TODO: Update H:
    //
    // a. H_mm:
    H_.block<15, 15>(INDEX_M, INDEX_M) += J_m.transpose() * J_m;

    //
    // TODO: Update b:
    //
    // a. b_m:
    b_.block<15, 1>(INDEX_M, 0) += J_m.transpose() * residuals;
}
```

由于是单边约束，所以这里的H矩阵只有最左上角的一部分，同样b也只有最上面一部分。

# 补全代码梳理逻辑

```
void SetResIMUPreIntegration(  
    const ceres::CostFunction *residual,  
    const std::vector<double> *parameter_blocks  
) {  
    // init:  
    ResidualBlockInfo res_imu_pre_integration(residual, parameter_blocks);  
    Eigen::VectorXd residuals;  
    std::vector<Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic, Eigen::RowMajor>> jacobians;  
  
    // compute:  
    Evaluate(res_imu_pre_integration, residuals, jacobians);  
    const Eigen::MatrixXd &J_m = jacobians.at(0);  
    const Eigen::MatrixXd &J_r = jacobians.at(1);  
  
    //  
    // TODO: Update H:  
    //  
    // a. H_mm:  
    H_.block<15, 15>(INDEX_M, INDEX_M) += J_m.transpose() * J_m;  
    // b. H_mr:  
    H_.block<15, 15>(INDEX_M, INDEX_R) += J_m.transpose() * J_r;  
    // c. H_rm:  
    H_.block<15, 15>(INDEX_R, INDEX_M) += J_r.transpose() * J_m;  
    // d. H_rr:  
    H_.block<15, 15>(INDEX_R, INDEX_R) += J_r.transpose() * J_r;  
  
    //  
    // Update b:  
    //  
    // a. b_m:  
    b_.block<15, 1>(INDEX_M, 0) += J_m.transpose() * residuals;  
    // a. b_r:  
    b_.block<15, 1>(INDEX_R, 0) += J_r.transpose() * residuals;  
}
```

```
void SetResRelativePose(  
    const ceres::CostFunction *residual,  
    const std::vector<double> *parameter_blocks  
) {  
    // init:  
    ResidualBlockInfo res_relative_pose(residual, parameter_blocks);  
    Eigen::VectorXd residuals;  
    std::vector<Eigen::Matrix<double, Eigen::Dynamic, Eigen::Dynamic, Eigen::RowMajor>> jacobians;  
  
    // compute:  
    Evaluate(res_relative_pose, residuals, jacobians);  
    const Eigen::MatrixXd &J_m = jacobians.at(0);  
    const Eigen::MatrixXd &J_r = jacobians.at(1);  
  
    //  
    // TODO: Update H:  
    //  
    // a. H_mm:  
    H_.block<15, 15>(INDEX_M, INDEX_M) += J_m.transpose() * J_m;  
    // b. H_mr:  
    H_.block<15, 15>(INDEX_M, INDEX_R) += J_m.transpose() * J_r;  
    // c. H_rm:  
    H_.block<15, 15>(INDEX_R, INDEX_M) += J_r.transpose() * J_m;  
    // d. H_rr:  
    H_.block<15, 15>(INDEX_R, INDEX_R) += J_r.transpose() * J_r;  
  
    //  
    // TODO: Update b:  
    //  
    // a. b_m:  
    b_.block<15, 1>(INDEX_M, 0) += J_m.transpose() * residuals;  
    // a. b_r:  
    b_.block<15, 1>(INDEX_R, 0) += J_r.transpose() * residuals;  
}
```

雷达相对位姿和IMU预积分为双边约束，所以对于第一帧和第二帧各15维的状态都有约束，H矩阵为了四块，b矩阵也被分为了两部分

# 补全代码梳理逻辑

```
void Marginalize(  
    const double *raw_param_r_0  
) {  
    // TODO: implement marginalization logic  
    // save x_m_0:  
    Eigen::Map<const Eigen::Matrix<double, 15, 1>> x_0(raw_param_r_0);  
    x_0_ = x_0;  
  
    // marginalize:  
    const Eigen::MatrixXd &H_mm = H_.block<15, 15>(INDEX_M, INDEX_M);  
    const Eigen::MatrixXd &H_mr = H_.block<15, 15>(INDEX_M, INDEX_R);  
    const Eigen::MatrixXd &H_rm = H_.block<15, 15>(INDEX_R, INDEX_M);  
    const Eigen::MatrixXd &H_rr = H_.block<15, 15>(INDEX_R, INDEX_R);  
  
    const Eigen::VectorXd &b_m = b_.block<15, 1>(INDEX_M, 0);  
    const Eigen::VectorXd &b_r = b_.block<15, 1>(INDEX_R, 0);  
  
    //  
    // TODO: shall we improve numeric stability following VIO/LIO-mapping's practice?  
    //  
    Eigen::MatrixXd H_mm_inv = H_mm.inverse();  
    Eigen::MatrixXd H_marginalized = H_rr - H_rm * H_mm_inv * H_mr;  
    Eigen::MatrixXd b_marginalized = b_r - H_rm * H_mm_inv * b_m;  
  
    //  
    // solve linearized residual & Jacobian:  
    //  
    Eigen::SelfAdjointEigenSolver<Eigen::MatrixXd> saes(H_marginalized);  
    Eigen::VectorXd S = Eigen::VectorXd(  
        (saes.eigenvalues().array() > 1.0e-5).select(saes.eigenvalues().array(), 0)  
    );  
    Eigen::VectorXd S_inv = Eigen::VectorXd(  
        (saes.eigenvalues().array() > 1.0e-5).select(saes.eigenvalues().array().inverse(), 0)  
    );  
  
    Eigen::VectorXd S_sqrt = S.cwiseSqrt();  
    Eigen::VectorXd S_inv_sqrt = S_inv.cwiseSqrt();  
  
    // finally:  
    J_ = S_sqrt.asDiagonal() * saes.eigenvectors().transpose();  
    e_ = S_inv_sqrt.asDiagonal() * saes.eigenvectors().transpose() * b_marginalized;  
}
```

# 补全代码梳理逻辑

这里面的具体逻辑与ppt一致，需要注意的是如何从H和b矩阵中获得相应的雅各比和残差。这里我们使用奇异值分解

$$H = J^T J = Q S Q^T$$

其中 $S$ 为奇异值， $Q$ 为奇异值对应的右奇异向量，那么

$$J = \sqrt{S} Q^T$$

根据 $b = J^T e$ 可以得到

$$e = J^{T-1} b = \sqrt{S^{-1}} Q^T$$

# 补全代码梳理逻辑

```
// add marginalization factor into sliding window
problem.AddResidualBlock(
    factor_marginalization,
    NULL,
    key_frame_r.prvag
);

residual_blocks_.map_matching_pose.pop_front();
residual_blocks_.relative_pose.pop_front();
residual_blocks_.imu_pre_integration.pop_front();
```

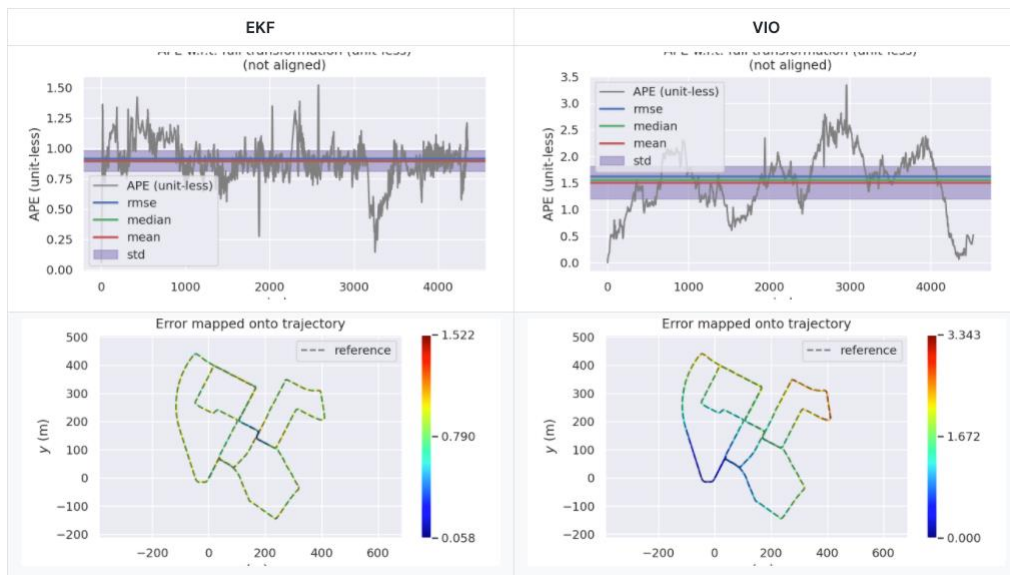


# 补全代码梳理逻辑

```
virtual bool Evaluate(double const *const *parameters, double *residuals, double **jacobians) const {  
    //  
    // parse parameters:  
    //  
    Eigen::Map<const Eigen::Matrix<double, 15, 1>> x(parameters[0]);  
    Eigen::VectorXd dx = x - x_0_;  
  
    //  
    // TODO: compute residual:  
    //  
    Eigen::Map<Eigen::Matrix<double, 15, 1>> residual(residuals);  
    residual = e_ + J_ * dx;  
  
    //  
    // TODO: compute jacobian:  
    //  
    if ( jacobians ) {  
        if ( jacobians[0] ) {  
            // implement computing:  
            Eigen::Map<Eigen::Matrix<double, 15, 15, Eigen::RowMajor> > jacobian_marginalization( jacobians[0] );  
            jacobian_marginalization.setZero();  
  
            jacobian_marginalization = J_;  
        }  
    }  
  
    return true;  
}
```

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# 与EKF进行对比



EKF

APE w.r.t. full transformation (unit-less)  
(not aligned)

max	1.521982
mean	0.897535
median	0.894192
min	0.058380
rmse	0.913938
sse	3635.146581
std	0.172374

VIO

APE w.r.t. full transformation (unit-less)  
(not aligned)

max	3.342999
mean	1.504630
median	1.555986
min	0.000001
rmse	1.623278
sse	11931.417004
std	0.609196

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# 纲要

- WINDOWSIZE = 10

APE w.r.t. full transformation (unit-less)  
(not aligned)

max	3.341343
mean	1.486215
median	1.527445
min	0.000001
rmse	1.599735
sse	11587.833756
std	0.591875

- WINDOWSIZE = 20

APE w.r.t. full transformation (unit-less)  
(not aligned)

max	3.342999
mean	1.504630
median	1.555986
min	0.000001
rmse	1.623278
sse	11931.417004
std	0.609196

- WINDOWSIZE = 30

APE w.r.t. full transformation (unit-less)  
(not aligned)

max	3.349993
mean	1.512396
median	1.562529
min	0.000001
rmse	1.634980
sse	12104.070253
std	0.621142



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感谢各位聆听 !  
Thanks for Listening

