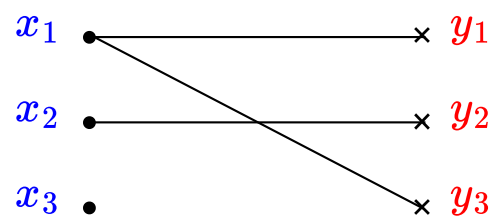

$R = \{(x_1, y_1), (x_1, y_3), (x_2, y_2)\}$

Start with Hausdorff

$$d_H\big((X,d),(Y,d)\big) = \inf_{R\in\mathcal{R}(X,Y)} \sup_{(x,y)\in R} d(x,y)$$

$\downarrow$ $R \to \pi$ $\downarrow$

$$W_\infty\big((X,\mu_X,d),(Y,\mu_Y,d)\big) = \inf_{\pi\in U(\mu_X,\mu_Y)} \sup_{(x,y)\in \mathrm{supp}(\pi)} d(x,y)$$

$\downarrow$ $L^\infty \to L^p$

$$W_p^p\big((X,\mu_X,d),(Y,\mu_Y,d)\big) = \inf_{\pi\in U(\mu_X,\mu_Y)} \int d(x,y)^p \,\mathrm{d}\pi(x,y)$$

Now apply to GH

$$\mathrm{GH}\big((X,d_X),(Y,d_Y)\big) = \frac{1}{2} \inf_{R\in\mathcal{R}(X,Y)} \sup_{\substack{(x_1,y_1)\in R\\(x_2,y_2)\in R}} \big|d_X(x_1,x_2) - d_Y(y_1,y_2)\big|$$

$\downarrow$ $R \to \pi$ $\downarrow$

$$\mathrm{GW}_\infty\big((X,\mu_X,d_X),(Y,\mu_Y,d_Y)\big) = \inf_{\pi\in U(\mu_X,\mu_Y)} \sup_{\substack{(x_1,y_1)\in \mathrm{supp}(\pi)\\(x_2,y_2)\in \mathrm{supp}(\pi)}} \big|d_X(x_1,x_2) - d_Y(y_1,y_2)\big|$$

$\downarrow$ $L^\infty \to L^p$

$$\mathrm{GW}_p^p\big((X,\mu_X,d_X),(Y,\mu_Y,d_Y)\big) = \inf_{\pi\in U(\mu_X,\mu_Y)} \iint \big|d_X(x_1,x_2) - d_Y(y_1,y_2)\big|^p \,\mathrm{d}\pi(x_1,y_1)\,\mathrm{d}\pi(x_2,y_2)$$