Computer Grafik Blatt 4

May 2023

Aufgabe 1.

(a)

$$eye = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \; x = \begin{pmatrix} 0 \\ 0 \\ -32 \end{pmatrix} \; \overrightarrow{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$y_1 = \begin{pmatrix} 0 \\ 8 \\ -24 \end{pmatrix} \; c_{y1} = \begin{pmatrix} 0,5 \\ 0 \\ 0 \end{pmatrix}$$

$$y_2 = \begin{pmatrix} -16 \\ 0 \\ -16 \end{pmatrix} \; c_{y2} = \begin{pmatrix} 0 \\ 0,6 \\ 0 \end{pmatrix}$$

$$C_a = \begin{pmatrix} 0,2 & 0 & 0 \\ 0 & 0,2 & 0 \\ 0 & 0 & 0,2 \end{pmatrix}$$

$$C_d = \begin{pmatrix} 0,8 & 0 & 0 \\ 0 & 0,8 & 0 \\ 0 & 0 & 0,8 \end{pmatrix}$$

$$C_s = \begin{pmatrix} 0,4 & 0 & 0 \\ 0 & 0,4 & 0 \\ 0 & 0 & 0,4 \end{pmatrix}$$

$$\overrightarrow{v} = \frac{eye - x}{||eye - x||} = \frac{1}{32} \cdot \begin{pmatrix} 0 \\ 0 \\ 32 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\overrightarrow{l_1} = \frac{y_1 - x}{||y_1 - x||} = \frac{1}{8\sqrt{2}} \cdot \begin{pmatrix} 0 \\ 8 \\ 8 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix}$$

$$\overrightarrow{r_1} = 2\overrightarrow{n}\,\overrightarrow{n}^T\overrightarrow{l_1} - \overrightarrow{l_1} = 2\overrightarrow{n}\cdot\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\ \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{pmatrix} - \overrightarrow{l_1} = \sqrt{2}\cdot\overrightarrow{n} - \overrightarrow{l_1} = \begin{pmatrix}0\\0\\ \sqrt{2}\end{pmatrix} - \begin{pmatrix}0\\ \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{pmatrix} = \begin{pmatrix}0\\ -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{pmatrix}$$

$$L(x, \overrightarrow{v}) = L_{y1} + L_{y2}$$

$$L_{y1} = (C_a + C_d \cdot (\overrightarrow{l_1}^T\overrightarrow{n}) + C_s \cdot (\overrightarrow{v}^T\overrightarrow{r_1})^s) \cdot c_{y1}$$

$$C_{d1} = C_d\cdot(\overrightarrow{l_1}^T\overrightarrow{n}) = C_d(\begin{pmatrix}0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}) = \begin{pmatrix}0,8 & 0 & 0\\ 0 & 0,8 & 0\\ 0 & 0 & 0,8\end{pmatrix}\cdot\frac{\sqrt{2}}{2} = \begin{pmatrix}\frac{2\sqrt{2}}{5} & 0 & 0\\ 0 & \frac{2\sqrt{2}}{5} & 0\\ 0 & 0 & \frac{2\sqrt{2}}{5}\end{pmatrix}$$

$$C_{s1} = C_s\cdot(\overrightarrow{v}^T\overrightarrow{r_1})^s = C_s\cdot\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}0\\ -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2}\end{pmatrix} = \begin{pmatrix}0,4 & 0 & 0\\ 0 & 0,4 & 0\\ 0 & 0 & 0,4\end{pmatrix}\cdot\frac{\sqrt{2}}{2}^{10} = \begin{pmatrix}\frac{1}{80} & 0 & 0\\ 0 & \frac{1}{80} & 0\\ 0 & 0 & \frac{1}{80}\end{pmatrix}$$

$$L_1 = (C_a + C_{d1} + C_{s1})\cdot\begin{pmatrix}0,5\\0\\0\end{pmatrix} = \begin{pmatrix}\frac{17+32\sqrt{2}}{80} & 0 & 0\\ 0 & \frac{17+32\sqrt{2}}{80} & 0\\ 0 & 0 & \frac{17+32\sqrt{2}}{80}\end{pmatrix}\cdot\begin{pmatrix}0,5\\0\\0\end{pmatrix} = \begin{pmatrix}0,3890927125\\0\\0\end{pmatrix}$$

$$\overrightarrow{l_2} = \frac{y_2 - x}{||y_2 - x||} = \frac{1}{16\sqrt{2}}\cdot\begin{pmatrix}-16\\0\\16\end{pmatrix} = \begin{pmatrix}-\frac{\sqrt{2}}{2}\\0\\ \frac{\sqrt{2}}{2}\end{pmatrix}$$

$$\overrightarrow{r_2} = 2\overrightarrow{n}\,\overrightarrow{n}^T\overrightarrow{l_2} - \overrightarrow{l_2} = 2\overrightarrow{n}\cdot\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}-\frac{\sqrt{2}}{2}\\0\\ \frac{\sqrt{2}}{2}\end{pmatrix} - \overrightarrow{l_2} = \sqrt{2}\cdot\overrightarrow{n} - \overrightarrow{l_2} = \begin{pmatrix}0\\0\\ \sqrt{2}\end{pmatrix} - \begin{pmatrix}-\frac{\sqrt{2}}{2}\\0\\ \frac{\sqrt{2}}{2}\end{pmatrix} = \begin{pmatrix}\frac{\sqrt{2}}{2}\\0\\ \frac{\sqrt{2}}{2}\end{pmatrix}$$

$$L_{y2} = (C_a + C_d \cdot (\overrightarrow{l_2}^T\overrightarrow{n}) + C_s \cdot (\overrightarrow{v}^T\overrightarrow{r_2})^s) \cdot c_{y2}$$

$$C_{d2} = C_d\cdot(\overrightarrow{l_2}^T\overrightarrow{n}) = C_d(\begin{pmatrix}-\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2}\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}) = \begin{pmatrix}0,8 & 0 & 0\\ 0 & 0,8 & 0\\ 0 & 0 & 0,8\end{pmatrix}\cdot\frac{\sqrt{2}}{2} = \begin{pmatrix}\frac{2\sqrt{2}}{5} & 0 & 0\\ 0 & \frac{2\sqrt{2}}{5} & 0\\ 0 & 0 & \frac{2\sqrt{2}}{5}\end{pmatrix}$$

$$C_{s2} = C_s\cdot(\overrightarrow{v}^T\overrightarrow{r_2})^s = C_s\cdot\begin{pmatrix}0 & 0 & 1\end{pmatrix}\begin{pmatrix}\frac{\sqrt{2}}{2}\\0\\ \frac{\sqrt{2}}{2}\end{pmatrix} = \begin{pmatrix}0,4 & 0 & 0\\ 0 & 0,4 & 0\\ 0 & 0 & 0,4\end{pmatrix}\cdot\frac{\sqrt{2}}{2}^{10} = \begin{pmatrix}\frac{1}{80} & 0 & 0\\ 0 & \frac{1}{80} & 0\\ 0 & 0 & \frac{1}{80}\end{pmatrix}$$

$$L_2 = (C_a + C_{d2} + C_{s2})\cdot\begin{pmatrix}0\\0,6\\0\end{pmatrix} = \begin{pmatrix}\frac{17+32\sqrt{2}}{80} & 0 & 0\\ 0 & \frac{17+32\sqrt{2}}{80} & 0\\ 0 & 0 & \frac{17+32\sqrt{2}}{80}\end{pmatrix}\cdot\begin{pmatrix}0\\0,6\\0\end{pmatrix} = \begin{pmatrix}0\\0,466911255\\0\end{pmatrix}$$

$$L = \begin{pmatrix}0,3890927125\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0,466911255\\0\end{pmatrix} = \begin{pmatrix}0,3890927125\\0,466911255\\0\end{pmatrix}$$

(b)

$$\overrightarrow{h_1} = \frac{\overrightarrow{v} + \overrightarrow{l_1}}{||\overrightarrow{v} + \overrightarrow{l_1}||} = \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{2+\sqrt{2}}{2} \end{pmatrix} \div \sqrt{\frac{4+2\sqrt{2}}{2}} \cdot = \begin{pmatrix} 0 \\ 0,3826834324 \\ 0,9238795325 \end{pmatrix}$$

$$L_{y1} = (C_a + C_d \cdot (\overrightarrow{l_1}^T \overrightarrow{n}) + C_s \cdot (\overrightarrow{h1}^T \overrightarrow{n})^s) \cdot c_{y1}$$

$$C_{s1} = C_s \cdot (\overrightarrow{h1}^T \overrightarrow{n})^s = C_s \cdot \begin{pmatrix} 0 & 0,3826834324 & 0,9238795325 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} 0,4 & 0 & 0 \\ 0 & 0,4 & 0 \\ 0 & 0 & 0,4 \end{pmatrix} \cdot (\frac{2+\sqrt{2}}{2} \div \sqrt{\frac{4+2\sqrt{2}}{2}})^{20} = \begin{pmatrix} 0,08210449037 & 0 & 0 \\ 0 & 0,08210449037 & 0 \\ 0 & 0 & 0,08210449037 \end{pmatrix}$$

$$L_1 = (C_a + C_{d1} + C_{s1}) \cdot \begin{pmatrix} 0,5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0,8477899153 & 0 & 0 \\ 0 & 0,8477899153 & 0 \\ 0 & 0 & 0,8477899153 \end{pmatrix} \cdot \begin{pmatrix} 0,5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.42389496 \\ 0 \\ 0 \end{pmatrix}$$

$$\overrightarrow{h_2} = \frac{\overrightarrow{v} + \overrightarrow{l_2}}{||\overrightarrow{v} + \overrightarrow{l_2}||} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ 0 \\ \frac{2+\sqrt{2}}{2} \end{pmatrix} \div \sqrt{\frac{4+2\sqrt{2}}{2}} \cdot = \begin{pmatrix} -0,3826834324 \\ 0 \\ 0,9238795325 \end{pmatrix}$$

$$C_{s2} = C_s \cdot (\overrightarrow{h2}^T \overrightarrow{n})^s = C_s \cdot \begin{pmatrix} -0,3826834324 & 0 & 0,9238795325 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$\begin{pmatrix} 0,4 & 0 & 0 \\ 0 & 0,4 & 0 \\ 0 & 0 & 0,4 \end{pmatrix} \cdot (\frac{2+\sqrt{2}}{2} \div \sqrt{\frac{4+2\sqrt{2}}{2}})^{20} = \begin{pmatrix} 0,08210449037 & 0 & 0 \\ 0 & 0,08210449037 & 0 \\ 0 & 0 & 0,08210449037 \end{pmatrix}$$

$$L_2 = (C_a + C_{d2} + C_{s2}) \cdot \begin{pmatrix} 0 \\ 0,6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.84778992 & 0 & 0 \\ 0 & 0.84778992 & 0 \\ 0 & 0 & 0.84778992 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0,6 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 7,849262694 \\ 0 \end{pmatrix}$$

$$L = \begin{pmatrix} 0.42389496 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0.50867395 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.42389496 \\ 0.50867395 \\ 0 \end{pmatrix}$$

(c)

$$a = \begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix} b = \begin{pmatrix} -4 \\ 4 \\ -7 \end{pmatrix} c = \begin{pmatrix} 0 \\ 0 \\ -7 \end{pmatrix} p = \begin{pmatrix} 0 \\ 3 \\ -7 \end{pmatrix}$$

Da alle Punkte den gleichen Z-Wert haben, berechnen wir die Flächeninhalte in 2D.

$$A = \frac{1}{2} det[(b-a),(c-a)] = \frac{1}{2}((-4-1)(0-5) - (0-1)(4-5)) = 12$$

$$A_\alpha = \frac{1}{2}det[(b-p),(c-p)] = \frac{1}{2}((-4-0)(0-3)-(0-0)(4-3)) = 6$$
$$A_\beta = \frac{1}{2}det[(c-p),(a-p)] = \frac{1}{2}((0-0)(5-3)-(1-0)(0-3)) = 1,5$$
$$A_\gamma = \frac{1}{2}det[(a-p),(b-p)] = \frac{1}{2}((1-0)(4-3)-(-4-0)(5-3)) = 4,5$$
$$\alpha = \frac{A_\alpha}{A} = 0,5 \;\; \beta = \frac{A_\beta}{A} = 0,125 \;\; \gamma = \frac{A_\gamma}{A} = 0,375$$
$$\alpha\overrightarrow{n_\alpha} = \begin{pmatrix} 0 \\ 0 \\ 0,5 \end{pmatrix} \beta\overrightarrow{n_\beta} = \begin{pmatrix} -0,125 \\ 0 \\ 0 \end{pmatrix} \gamma\overrightarrow{n_\gamma} = \begin{pmatrix} 0 \\ -0,375 \\ 0 \end{pmatrix}$$
$$\frac{\alpha\overrightarrow{n_\alpha} + \beta\overrightarrow{n_\beta} + \gamma\overrightarrow{n_\gamma}}{||\cdot||} = \begin{pmatrix} -0.19611614 \\ -0.58834841 \\ 0.78446454 \end{pmatrix}$$

Hier sind wir uns nicht ganz sicher, ob wir den Foliensatz/die Aufgabe richtig Verstehen. Aber

$$phong(\begin{pmatrix} 0 \\ 3 \\ -7 \end{pmatrix}, \begin{pmatrix} -0.19611614 \\ -0.58834841 \\ 0.78446454 \end{pmatrix}) = \begin{pmatrix} 0.10096444 \\ 0.12290578 \\ 0 \end{pmatrix}$$

Nach unserem Verständnis muss die Phong Beleuchtung nur für p und die aufsummierten und normierten Normalen von a, b und c ausgewertet werden.