$$S_{xy} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$\begin{aligned}
S_{\boldsymbol{x}_1,\ldots,\boldsymbol{x}_l} &= \frac{1}{n-1} \sum_{i=1}^{n} (x_{k,i} - \overline{x}_k)(x_{l,i} - \overline{x_e}) \\
&= \frac{n}{n-1} \left(\overline{x_k x_l} - \bar{x}_k \bar{x}_l\right) \\
\text{mit } \overline{x_k x_l} &= \frac{1}{n} \sum_{i=1}^{n} (x_{k,i} \cdot x_{l,i}) \\
&= \frac{n}{n-1} \left(\overline{\boldsymbol{x}\boldsymbol{x}^T} - \bar{\boldsymbol{x}}\bar{\boldsymbol{x}}^T\right) && \text{wahrscheinlich ist das die wichtigste der Formeln hier}
\end{aligned}$$

$$S_{\boldsymbol{x}_1,\ldots,\boldsymbol{x}_n} = \begin{pmatrix} S_{x_1,x_1} & \cdots & S_{x_1,x_d} \\ S_{x_2,x_1} & \cdots & \vdots \\ \vdots & \ddots & \vdots \\ S_{x_d,x_1} & \ldots & S_{x_d,x_d} \end{pmatrix}$$

$$Sx = S_x^2$$

Monat	1	2	3	4	5	6	7	8	9	10	11	12
Werbekosten	2	4	5	10	10	11	15	15	15	16	17	20
Umsatz	6	8	10	12	15	18	19	20	22	24	28	30

Kovarianzmatrix:

$$\bar{x} = \begin{pmatrix} 35/3 \\ 53/3 \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 2+4+5+\ldots+20 \\ 6+8+10+\ldots+30 \end{pmatrix}$$

$$\bar{x}^T = \begin{pmatrix} 35/3 & 53/3 \end{pmatrix}$$

$$\bar{x} \cdot \bar{x}^T = \begin{pmatrix} 35/3 \cdot 35/3 & 35/3 \cdot 53/3 \\ 53/3 \cdot 35/3 & 53/3 \cdot 53/3 \end{pmatrix} = \begin{pmatrix} 1225/9 & 1855/9 \\ 1855/9 & 2809/9 \end{pmatrix}$$

$$\begin{aligned} \overline{\boldsymbol{x}\boldsymbol{x}^T} &= \frac{1}{12}\boldsymbol{x}\boldsymbol{x}^T \\ &= \frac{1}{12} \begin{pmatrix} 1986 & 2937 \\ 2937 & 4398 \end{pmatrix} = \begin{pmatrix} 331/2 & 979/4 \\ 979/4 & 733/2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} S_x &= \frac{12}{11} \left(\begin{pmatrix} 331/2 & 979/4 \\ 979/4 & 733/2 \end{pmatrix} - \begin{pmatrix} 1225/9 & 1885/9 \\ 1855/9 & 2809/9 \end{pmatrix} \right) \\ &= \begin{pmatrix} 1058/33 & 1391/33 \\ 1391/33 & 178/3 \end{pmatrix} => \begin{pmatrix} S_{xx} & S_{xy} \\ S_{yx} & S_{yy} \end{pmatrix} \end{aligned}$$

Regressionsgerade:
$y = \beta_0 + x \cdot \beta$ mit $\beta_0 = \bar{y} - \frac{S_{xy}}{S_{x^2}} \cdot \bar{x}$ und $\beta = \frac{S_{xy}}{S_x}$

$$\begin{aligned} \bar{y} &= \frac{53}{3} \\ \bar{x} &= \frac{35}{3} \\ S_x^2 &= \frac{n}{n-1}(\overline{xx} - \bar{x}\bar{x}) = \frac{n}{n-1}\left(\frac{1}{n}\sum_{i=1}^{n}(x_i)^2 - \bar{x} \cdot \bar{x}\right) \\ S_{xy} &= \frac{n}{n-1}(\overline{xy} - \bar{x}\bar{y}) = \frac{n}{n-1}\left(\frac{1}{n}\sum_{i=1}^{n}(x_i \cdot y_i) - \bar{x} \cdot \bar{y}\right) \\ &\Rightarrow y = 2,328 + x \cdot 1,315 \end{aligned}$$