Aufgabe 2

$$\mathbb{P}\left(\bigcup_{i=1}^{m} A_i\right) = \sum_{k=1}^{m} (-1)^{k+1} S_k$$

$$S_k = \sum_{\{i_1,i_2,\dots,i_k\}\subseteq\{1,\dots,m\}} \mathbb{P}\left(\bigcap_{l=1,\dots,k} A_{i_l}\right)$$

Induktionsanfang mit $m = 2$, gegeben durch Skript 4.1 (6)

Induktionsschritt mit $m + 1$

$$\begin{aligned}
\mathbb{P}\left(\bigcup_{i=1}^{m+1} A_i\right) &= \mathbb{P}\left(\left(\bigcup_{i=1}^{m} A_i\right) \cup A_{m+1}\right) \\
&= \mathbb{P}\left(\bigcup_{i=1}^{m} A_i\right) + \mathbb{P}(A_{m+1}) - \mathbb{P}\left(\bigcup_{i=1}^{m} A_i \cap A_{m+1}\right) \\
&= \sum_{k=1}^{m} (-1)^{k+1} S_k + \mathbb{P}(A_{m+1}) + \sum_{j=1}^{m} (-1)^{j} S'_j \\
&\text{mit } S'_j = \sum_{\{i_1,i_2,\dots,i_k\}\subseteq\{1,\dots,m\}} \mathbb{P}(A_{i_1} \cap \dots \cap A_{i_j} \cap A_{m+1}) \\
&= S_1 + \mathbb{P}(A_{m+1}) + \sum_{k=2}^{m} \left((-1)^{k+1}(S_k + S'_{k-1})\right) + (-1)^m S'_m \\
&\text{mit } S_k + S'_{k-1} = \sum_{\{i_1,i_2,\dots,i_k\}\subseteq\{1,\dots,m+1\}} \mathbb{P}\left(\bigcap_{l=1,\dots,k} A_{i_l}\right) \\
&= S_1 + \mathbb{P}(A_{m+1}) + \sum_{k=2}^{m} \left((-1)^{k+1} \sum_{\{i_1,i_2,\dots,i_k\}\subseteq\{1,\dots,m+1\}} \mathbb{P}\left(\bigcap_{l=1,\dots,k} A_{i_l}\right)\right) + (-1)^m S'_m \\
&= \sum_{k=1}^{m+1} (-1)^{k+1} S_k
\end{aligned}$$