

$$\begin{aligned}
1.) \quad \mathbb{E}(S_{xy}) &= \mathbb{E}\left(\frac{1}{n-1} \cdot \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})\right) \\
&= \frac{1}{n-1} \sum_{i=1}^n \mathbb{E}((x_i - \bar{x})(y_i - \bar{y})) \\
&= \frac{n}{n-1} (\mathbb{E}(x_1 y_1) - \mathbb{E}(x_1 \bar{y}) - \mathbb{E}(\bar{x} y_1) + \mathbb{E}(\bar{x} \bar{y})) \\
&= \frac{n}{n-1} (\mathbb{E}(x_1 y_1) - (\frac{1}{n} \mathbb{E}(x_1 y_1) + \frac{n-1}{n} \mathbb{E}(x_1 y_2)) - (\frac{1}{n} \mathbb{E}(x_1 y_1) + \frac{n-1}{n} \cdot \\
&\quad \mathbb{E}(x_2 y_1)) + \frac{n^2}{n^2} \mathbb{E}(x_1 y_1) + \frac{n^2-n}{n^2} \mathbb{E}(x_1 y_2)) \\
&= \frac{n}{n-1} (\mathbb{E}(x_1 y_1) \cdot (1 - \frac{1}{n} - \frac{1}{n} + \frac{1}{n}) - \mathbb{E}x_1 \mathbb{E}y_2 (\frac{n-1}{n} + \frac{n-1}{n} + \frac{n-1}{n})) \\
&= \frac{n}{n-1} (\mathbb{E}(x_1 y_1) (\frac{n-1}{n}) - \mathbb{E}x_1 \mathbb{E}y_2 (\frac{n-1}{n})) \\
&= \frac{n}{n-1} \cdot \frac{n-1}{n} \cdot (\mathbb{E}(xy) - \mathbb{E}x \mathbb{E}y) \\
&= \mathbb{E}xy - \mathbb{E}x \mathbb{E}y = K(X, Y)
\end{aligned}$$