a)

$$\beta = S^{-1}_{x_1,\dots,x_n} \cdot (\overline{xy} - \bar{x}\bar{y})$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \cdot (\overline{xy} - \bar{x}\bar{y})$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \cdot \left(\left(\frac{1}{n}\sum_{i=1}^{n} x_i \cdot (b + x_i a)\right) - \left(\bar{x} \cdot \frac{1}{n}\sum_{i=1}^{n} b + x_i a\right)\right)$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \cdot \left(\left(\frac{1}{n}\sum_{i=1}^{n} x_i b + x_i^2 a\right) - \left(\frac{1}{n}\sum_{i=1}^{n} x_i \cdot \frac{1}{n}\sum_{i=1}^{n} x_i a + b\right)\right)$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \cdot \left(\frac{1}{n}\left(\sum_{i=1}^{n} x_i b + \sum_{i=1}^{n} x_i^2 a\right) - \left(\bar{x} \cdot \frac{1}{n}\left(\sum_{i=1}^{n} x_i a + \sum_{i=1}^{n} b\right)\right)\right)$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \cdot \left(\bar{x}b + a\cdot\overline{x^2} - \bar{x}\cdot a\cdot\bar{x} - \bar{x}b\right)$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \left(a\overline{x^2} - a\bar{x}^2\right)$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \cdot \left(a\left(\overline{x^2} - \bar{x}^2\right)\right)$$

$$= \left(\frac{n}{n-1}\left(\overline{xx^T} - \bar{x}\bar{x}^T\right)\right)^{-1} \cdot \left(a\left(\overline{xx} - \bar{x}\bar{x}\right)\right)$$

$$= a$$

b) $\beta_0 = \bar{y} - \beta\bar{x}$

$$= \frac{1}{n}\sum_{i=1}^{n} y_i - a\bar{x}$$

$$= \frac{1}{n}\sum_{i=1}^{n} b + x_i a - a\cdot\bar{x}$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n} x_i a + \sum_{i=1}^{n} b\right) - a\cdot\bar{x}$$

$$= \frac{1}{n}\cdot(a\cdot\bar{x}n) + \frac{1}{n}\cdot n\cdot b - a\bar{x}$$

$$= a\bar{x} - a\bar{x} + b = \underline{\underline{b}}$$