Cheat Sheet – Bias-Variance Tradeoff

What is Bias?

$$bias = \mathbb{E}[f'(x)] - f(x)$$

- Error between average model prediction and ground truth
- The bias of the estimated function tells us the capacity of the underlying model to predict the values

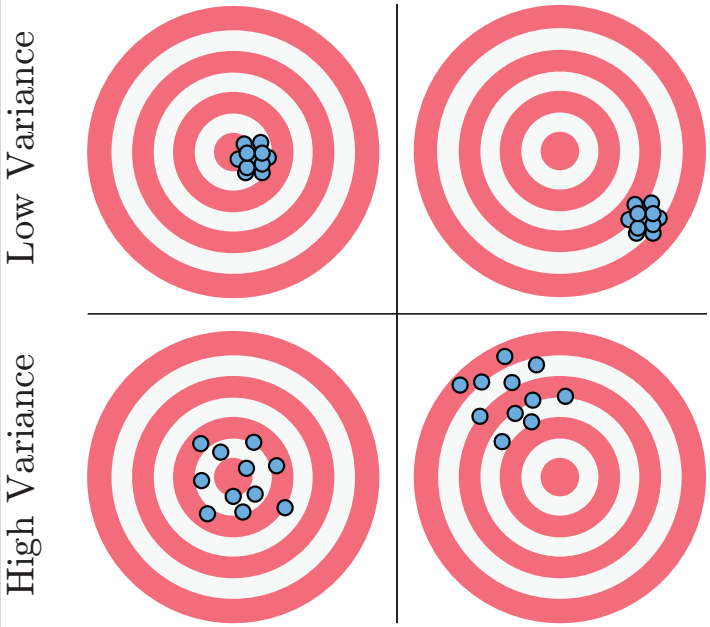
What is Variance?

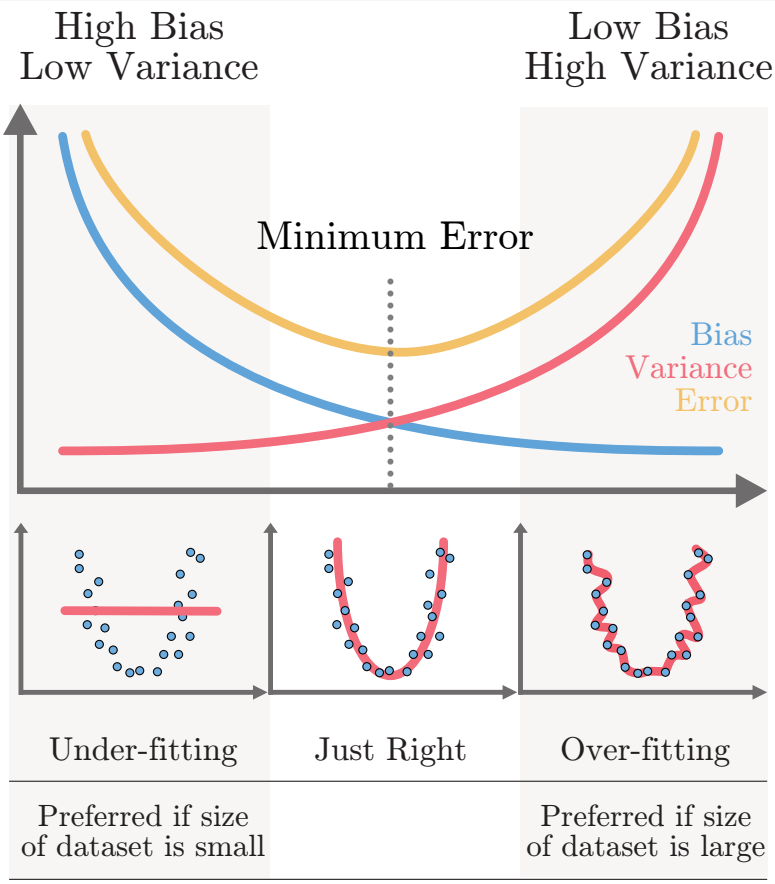
$$variance = \mathbb{E}\left[\left(f'(x) - \mathbb{E}[f'(x)]\right)^2\right]$$

- Average variability in the model prediction for the given dataset
- The variance of the estimated function tells you how much the function can adjust to the change in the dataset

High Bias
- → Overly-simplified Model
- → Under-fitting
- → High error on both test and train data

High Variance
- → Overly-complex Model
- → Over-fitting
- → Low error on train data and high on test
- → Starts modelling the noise in the input

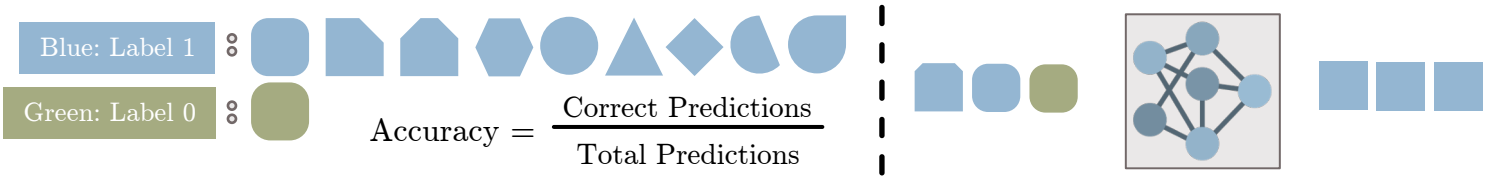




Bias variance Trade-off

- Increasing bias (not always) reduces variance and vice-versa
- Error = bias^2 + variance +irreducible error
- The best model is where the error is reduced.
- Compromise between bias and variance

Cheat Sheet – Imbalanced Data in Classification

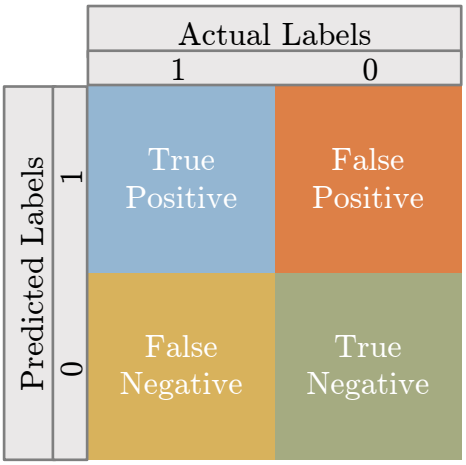


Classifier that always predicts label blue yields prediction accuracy of 90%

Accuracy doesn't always give the correct insight about your trained model

Accuracy: %age correct prediction	Correct prediction over total predictions	One value for entire network
Precision: Exactness of model	From the detected cats, how many were actually cats	Each class/label has a value
Recall: Completeness of model	Correctly detected cats over total cats	Each class/label has a value
F1 Score: Combines Precision/Recall	Harmonic mean of Precision and Recall	Each class/label has a value

Performance metrics associated with Class 1



(Is your prediction correct?) (What did you predict)

True Negative

(Your prediction is correct) (You predicted 0)

$$\text{Precision} = \frac{TP}{TP + FP}$$

$$\text{F1 score} = 2x \frac{(\text{Prec} \times \text{Rec})}{(\text{Prec} + \text{Rec})}$$

$$\text{Specificity} = \frac{TN}{TN + FP}$$

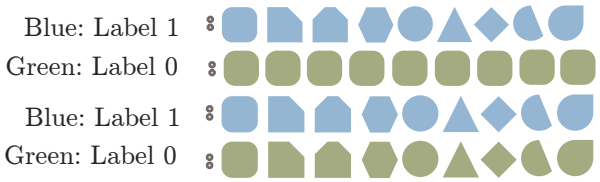
$$\text{False +ve rate} = \frac{FP}{TN + FP}$$

$$\text{Accuracy} = \frac{TP + TN}{TP + FN + FP + TN}$$

$$\text{Recall, Sensitivity} = \text{True +ve rate} = \frac{TP}{TP + FN}$$

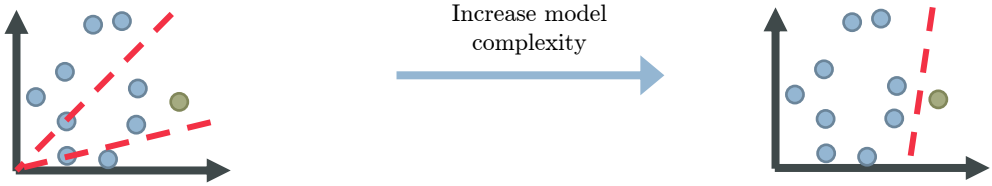
Possible solutions

1. **Data Replication:** Replicate the available data until the number of samples are comparable
2. **Synthetic Data:** Images: Rotate, dilate, crop, add noise to existing input images and create new data
3. **Modified Loss:** Modify the loss to reflect greater error when misclassifying smaller sample set



$$loss = a * loss_{green} + b * loss_{blue} \qquad a > b$$

4. **Change the algorithm:** Increase the model/algorithm complexity so that the two classes are perfectly separable (Con: Overfitting)



No straight line (y=ax) passing through origin can perfectly separate data. **Best solution:** line y=0, predict all labels blue

Straight line (y=ax+**b**) can perfectly separate data. Green class will no longer be predicted as blue

Source: https://www.cheatsheets.aqeel-anwar.com Tutorial: Click here

Cheat Sheet – PCA Dimensionality Reduction

What is PCA?

- Based on the dataset find a new set of orthogonal feature vectors in such a way that the data spread is maximum in the direction of the feature vector (or dimension)
- Rates the feature vector in the decreasing order of data spread (or variance)
- The datapoints have maximum variance in the first feature vector, and minimum variance in the last feature vector
- The variance of the datapoints in the direction of feature vector can be termed as a measure of information in that direction.

Steps

1. Standardize the datapoints
2. Find the covariance matrix from the given datapoints
3. Carry out eigen-value decomposition of the covariance matrix
4. Sort the eigenvalues and eigenvectors

$$X_{new} = \frac{X - mean(X)}{std(X)}$$

$$C[i,j] = cov(x_i, x_j)$$

$$C = V\Sigma V^{-1}$$

$$\Sigma_{sort} = sort(\Sigma) \quad V_{sort} = sort(V, \Sigma_{sort})$$

Dimensionality Reduction with PCA

- Keep the first m out of n feature vectors rated by PCA. These m vectors will be the best m vectors preserving the maximum information that could have been preserved with m vectors on the given dataset

Steps:

1. Carry out steps 1-4 from above
2. Keep first m feature vectors from the sorted eigenvector matrix
3. Transform the data for the new basis (feature vectors)
4. The importance of the feature vector is proportional to the magnitude of the eigen value

$$V_{reduced} = V[:, 0:m]$$

$$X_{reduced} = X_{new} \times V_{reduced}$$

Figure 1

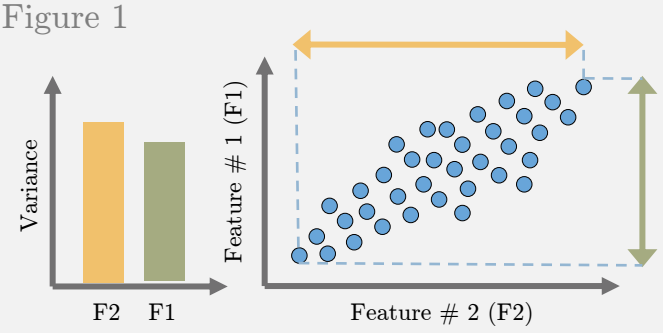


Figure 2

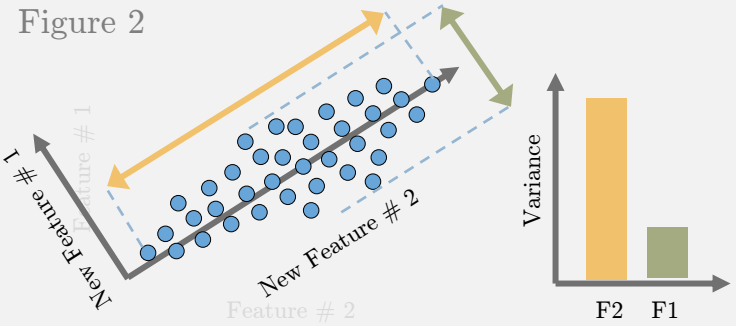


Figure 3

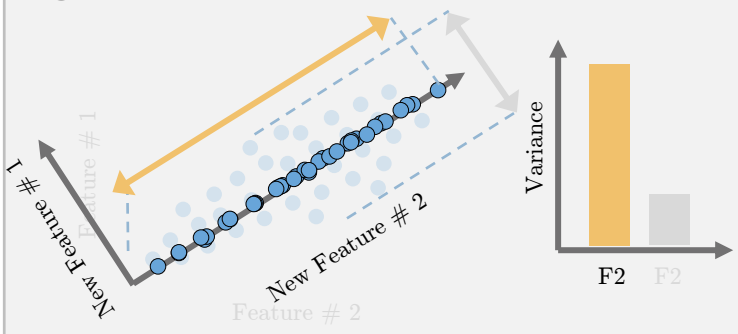


Figure 1: Datapoints with feature vectors as x and y-axis

Figure 2: The cartesian coordinate system is rotated to maximize the standard deviation along any one axis (new feature # 2)

Figure 3: Remove the feature vector with minimum standard deviation of datapoints (new feature # 1) and project the data on new feature # 2

Cheat Sheet – Bayes Theorem and Classifier

What is Bayes' Theorem?

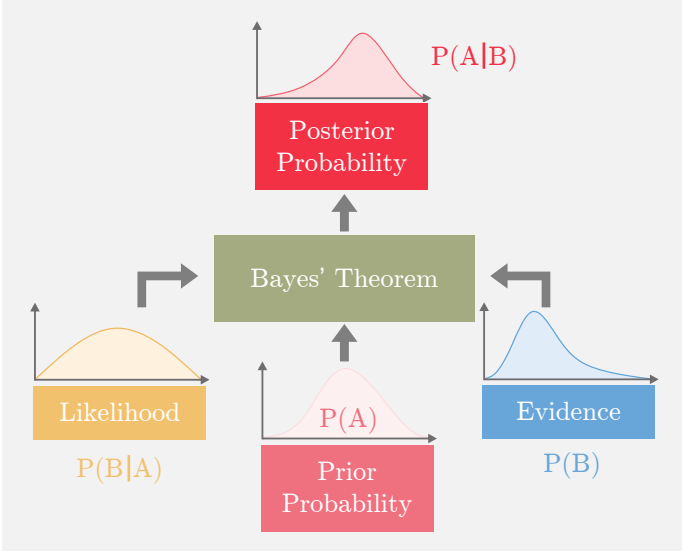
- Describes the probability of an event, based on prior knowledge of conditions that might be related to the event.

$$P(A|B) = \frac{P(B|A)(likelihood) \times P(A)(prior)}{P(B)(evidence)}$$

- How the probability of an event changes when we have knowledge of another event

P(A) ⟶ P(A|B)

↳ Usually, a better estimate than P(A)



Example

- Probability of fire P(F) = 1%
- Probability of smoke P(S) = 10%
- Prob of smoke given there is a fire P(S|F) = 90%
- What is the probability that there is a fire given we see a smoke P(F|S)?

$$P(F|S) = \frac{P(S|F) \times P(F)}{P(S)} = \frac{0.9 \times 0.01}{0.1} = 9\%$$

Maximum Aposteriori Probability (MAP) Estimation

The MAP estimate of the random variable y, given that we have observed iid (x_1, x_2, x_3, …), is given by. We try to accommodate our prior knowledge when estimating.

$$\hat{y}_{MAP} = argmax_y \;\; P(y) \prod_i P(x_i|y)$$

y that maximizes the product of prior and likelihood

Maximum Likelihood Estimation (MLE)

The MAP estimate of the random variable y, given that we have observed iid (x_1, x_2, x_3, …), is given by. We assume we don't have any prior knowledge of the quantity being estimated.

$$\hat{y}_{MLE} = argmax_y \prod_i P(x_i|y)$$

y that maximizes only the likelihood

MLE is a special case of MAP where our prior is uniform (all values are equally likely)

Naïve Bayes' Classifier (Instantiation of MAP as classifier)

Suppose we have two classes, $y=y_1$ and $y=y_2$. Say we have more than one evidence/features (x_1, x_2, x_3, …), using Bayes' theorem

$$P(y|x_1, x_2, x_3, \ldots) = \frac{P(x_1, x_2, x_3, \ldots|y) \times P(y)}{P(x_1, x_2, x_3, \ldots)}$$

Naïve Bayes' theorem assumes the features (x_1, x_2, …) are i.i.d. i.e $P(x_1, x_2, x_3, \ldots|y) = \prod_i P(x_i|y)$

$$P(y|x_1, x_2, x_3, \ldots) = \prod_i P(x_i|y) \frac{P(y)}{P(x_1, x_2, x_3, \ldots)}$$

$$\hat{y} = y_1 \; if \; \frac{P(y_1|x_1, x_2, x_3, \ldots)}{P(y_2|x_1, x_2, x_3, \ldots)} > 1 \;\; else \; \hat{y} = y_2$$

Cheat Sheet – Regression Analysis

What is Regression Analysis?
Fitting a function f(.) to datapoints y_i=f(x_i) under some error function. Based on the estimated function and error, we have the following types of regression

1. Linear Regression:
Fits a line minimizing the sum of mean-squared error for each datapoint.

$$min_\beta \sum_i \|y_i - f_\beta^{linear}(x_i)\|^2$$
$$f_\beta^{linear}(x_i) = \beta_0 + \beta_1 x_i$$

2. Polynomial Regression:
Fits a polynomial of order k (k+1 unknowns) minimizing the sum of mean-squared error for each datapoint.

$$min_\beta \sum_{i=0}^{m} \|y_i - f_\beta^{poly}(x_i)\|^2$$
$$f_\beta^{poly}(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \ldots + \beta_k x_i^k$$

3. Bayesian Regression:
For each datapoint, fits a gaussian distribution by minimizing the mean-squared error. As the number of data points x_i increases, it converges to point estimates i.e. $n \to \infty, \sigma^2 \to 0$

$$min_\beta \sum_i \|y_i - \mathcal{N}\left(f_\beta(x_i), \sigma^2\right)\|^2$$
$$f_\beta(x_i) = f_\beta^{poly}(x_i) \text{ or } f_\beta^{linear}(x_i)$$
$$\mathcal{N}\left(\mu, \sigma^2\right) \to \text{Gaussian with mean } \mu \text{ and variance } \sigma^2$$

4. Ridge Regression:
Can fit either a line, or polynomial minimizing the sum of mean-squared error for each datapoint and the weighted L2 norm of the function parameters beta.

$$min_\beta \sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{k} \beta_j^2$$
$$f_\beta(x_i) = f_\beta^{poly}(x_i) \text{ or } f_\beta^{linear}(x_i)$$

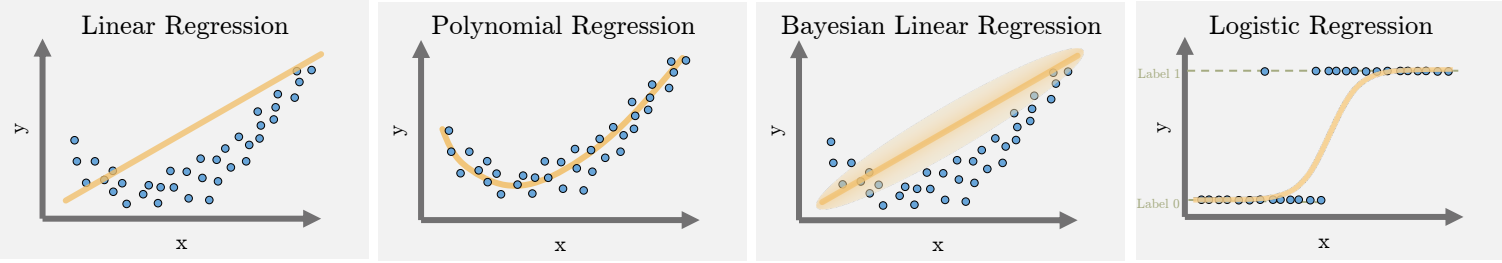
5. LASSO Regression:
Can fit either a line, or polynomial minimizing the the sum of mean-squared error for each datapoint and the weighted L1 norm of the function parameters beta.

$$min_\beta \sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{k} |\beta_j|$$
$$f_\beta(x_i) = f_\beta^{poly}(x_i) \text{ or } f_\beta^{linear}(x_i)$$

6. Logistic Regression:
Can fit either a line, or polynomial with sigmoid activation minimizing the binary cross-entropy loss for each datapoint. The labels y are binary class labels.

$$min_\beta \sum_i -y_i log\left(\sigma\left(f_\beta(x_i)\right)\right) - (1 - y_i) log\left(1 - \sigma\left(f_\beta(x_i)\right)\right)$$
$$f_\beta(x_i) = f_\beta^{poly}(x_i) \text{ or } f_\beta^{linear}(x_i)$$
$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

Visual Representation:



Summary:

	What does it fit?	Estimated function	Error Function
Linear	A line in n dimensions	$f_\beta^{linear}(x_i) = \beta_0 + \beta_1 x_i$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2$
Polynomial	A polynomial of order k	$f_\beta^{poly}(x_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \ldots$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2$
Bayesian Linear	Gaussian distribution for each point	$\mathcal{N}\left(f_\beta(x_i), \sigma^2\right)$	$\sum_i \|y_i - \mathcal{N}\left(f_\beta(x_i), \sigma^2\right)\|^2$
Ridge	Linear/polynomial	$f_\beta^{poly}(x_i)$ or $f_\beta^{linear}(x_i)$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{n} \beta_j^2$
LASSO	Linear/polynomial	$f_\beta^{poly}(x_i)$ or $f_\beta^{linear}(x_i)$	$\sum_{i=0}^{m} \|y_i - f_\beta(x_i)\|^2 + \sum_{j=0}^{n} \|\beta_j\|$
Logistic	Linear/polynomial with sigmoid	$\sigma(f_\beta(x_i))$	$min_\beta \sum_i -y_i log\left(\sigma\left(f_\beta(x_i)\right)\right) - (1 - y_i) log\left(1 - \sigma\left(f_\beta(x_i)\right)\right)$

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