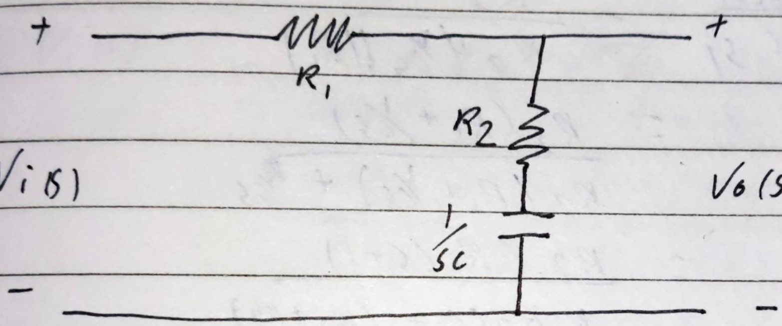
511885 - Vishal [Cs - lab]

Experiment one

→ Design and study of Lead, Lag and Lag-Lead compensator network

1) Compute the transfer function and Pole-Zero plot of the following.

a) Lag compensator



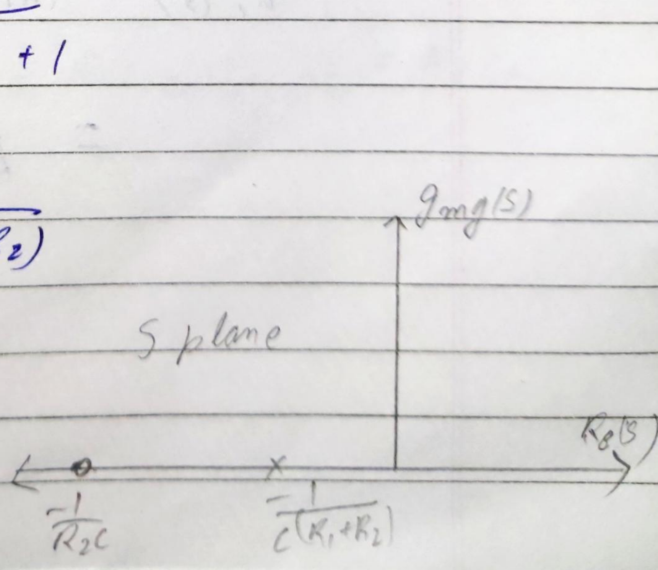
$$\rightarrow \quad V_o(s) = \frac{V_i(s)\left(R_2 + \frac{1}{sC}\right)}{\left(R_1 + R_2 + \frac{1}{sC}\right)}$$

$$T(s) \Rightarrow \frac{V_o(s)}{V_i(s)}$$

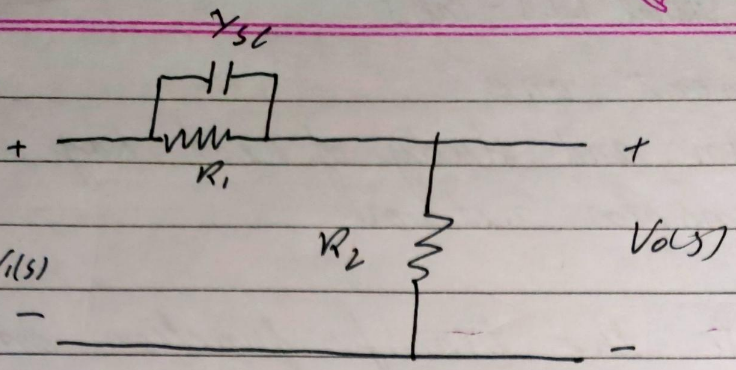
$$T(s) = \frac{1 + R_2 sC}{C(R_1+R_2)s + 1}$$

$$\text{Pole} \Rightarrow s = \frac{-1}{C(R_1+R_2)}$$

$$\text{Zero} \Rightarrow s = \frac{-1}{R_2 C}$$



b)



Lead compensator

→

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + (1/Cs \,||\, R_1)}$$

$$= \frac{R_2 (R_1 + 1/Cs)}{R_2 (R_1 + 1/Cs) + R_1/Cs}$$

$$= \frac{R_2 (R_1 Cs + 1)}{R_1 R_2 Cs + (R_1 + R_2)}$$

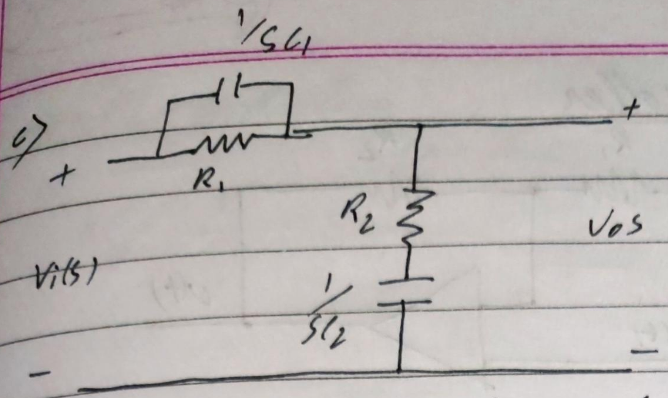
$$\beta = \frac{R_2}{R_1 + R_2}$$

$$T = R_1 C$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 (R_1 Cs + 1)}{(R_1 + R_2)\left[\left(\frac{R_1 R_2 Cs}{R_1 + R_2}\right) + 1\right]}$$

$$= \beta \left(\frac{sT + 1}{\beta sT + 1}\right)$$

c)



Lag-Lead compensator

$$\rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2 + 1/sC_2}{R_2 + 1/sC_2 + \left(\frac{R_1/sC_1}{R_1 + 1/sC_1}\right)}$$

$$= \frac{(sC_2R_2+1)(1+sC_1R_1)}{(1+sC_2R_2)(1+sC_1R_1) + R_1sC_2}$$

$$= \frac{R_1C_1\left(\frac{1}{R_1C_1}+s\right)R_2C_2\left(\frac{1}{R_2C_2}+s\right)}{R_1C_1R_2C_2\left[s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)+\frac{1}{R_1R_2C_1C_2}\right]}$$

$$= \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\beta/T_1\right)\left(s+1/\beta T_2\right)}$$

$$\beta = \frac{R_2}{R_1+R_2}, \quad T_1 = R_1C_2, \quad T_2 = R_2C_2$$

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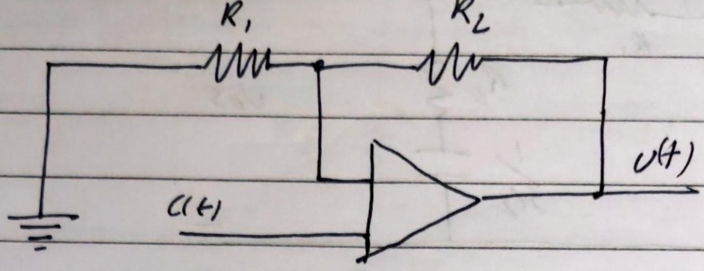
d> P-controller



→

∴ It is a Non-Inverting opam
By virtual ground condition
$v^+ = v^-$
so Node A voltage is '$c(s)$'
→ By nodal analysis

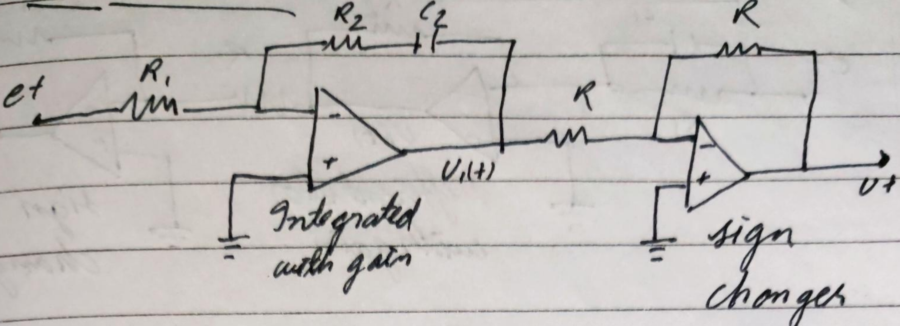
$$\frac{C(s) - 0}{R_1} + \frac{C(s) - U(s)}{R_2} = 0$$

$$\frac{C(s)}{R_1} + \frac{C(s)}{R_2} = \frac{U(s)}{R_2}$$

$$\frac{R_2}{R_1} + 1 = T(s)$$

$$T(s) = \frac{R_2}{R_1} + 1$$

e) PI-controller



→ By laplace transform

$$\frac{0 - e(s)}{R_1} + \frac{0 - u_1(s)}{R_2 + 1/C_2 s} = 0$$

$$V_1(s) \Rightarrow -e(s)\frac{(1+R_2C_2s)}{R_1C_2s} \quad \text{—— ①}$$

Now,

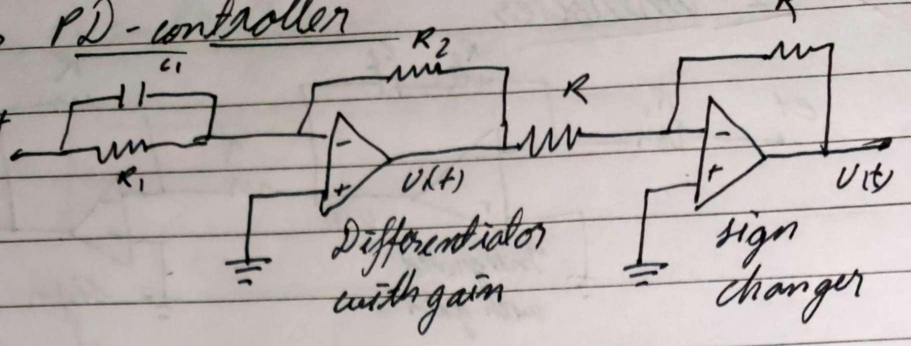
$u(t) = -v_1(t)$ [inverting opam]

$U(s) = -U_1(s)$ —— ②

using ① and ②

$$\frac{U(s)}{e(s)} = T(s) = -\frac{(1+R_2C_2s)}{R_1C_2s}$$

f> PD-controller



$$\rightarrow \frac{U_1(s)}{C(s)} = \frac{-R_2}{\dfrac{R_1 \frac{1}{C_1 s}}{\dfrac{R_1 C_1 s + 1}{C_1 s}}}$$

$$\Rightarrow U_1(s) = \frac{-R_2 (R_1 C_1 s + 1) C(s)}{R_1 \cdot s^2}$$

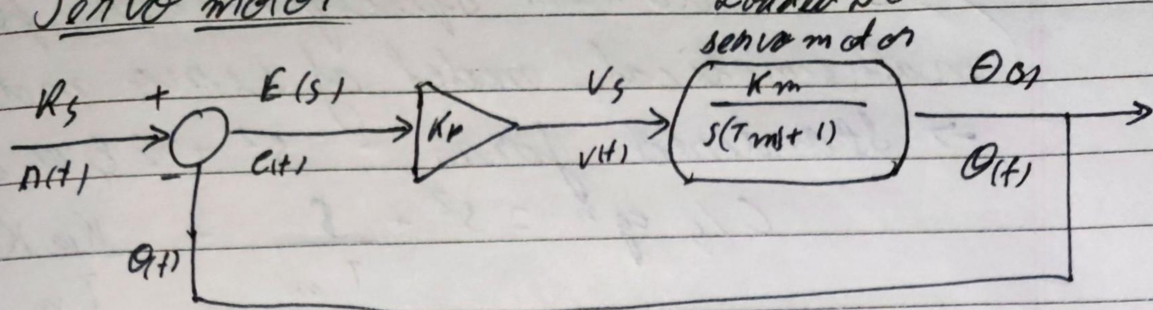
$$u(t) = -u_1(t)$$

$$U(s) = -U_1(s)$$

$$-U(s) = \frac{-R_2 (R_1 C_1 s + 1) \cdot C(s)}{R_1 s^2}$$

$$\frac{U(s)}{C(s)} = \frac{R_2 (R_1 C_1 s + 1)}{R_1 s^2}$$

g> <u>Servo motor</u>



$\rightarrow$

$$\frac{R(s)}{\theta(s)} = \frac{\dfrac{K_p K_m}{s(T_m s+1)}}{1 + \dfrac{K_p K_m}{s(T_m s+1)}}$$

$$= \frac{K_p K_m}{s(T_m s+1) + K_p K_m}$$

Char. equ. : $1 + G(s)H(s) = 0$

$$T_m s^2 + s + K_p K_m = 0$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1 - 4 T_m K_p K_m}}{2 T_m}$$

poles of transfer function depend on the values of K_p & K_m

h> Time domain specification for mathematical model of servo motor.

→ Standered form : $s^2 + 2\xi \omega_n s + \omega_n^2$

$$C/s\ eq^n = s^2 + \frac{s}{T_m} + \frac{K_p K_m}{T_m}$$

natural frequency $\omega_n = \sqrt{\frac{K_p K_m}{T_m}}$

$$2\xi\omega_n = \frac{1}{T_m}$$

$$\xi = \frac{1}{2T_2}\sqrt{\frac{T_m}{K_p K_m}}$$

$$\xi = \frac{1}{2\sqrt{T_m K_p K_m}}$$

raise time $t_r = \frac{\pi - \theta}{\omega d}$

$$\omega d = \omega_n\sqrt{1-\xi^2}$$

$$= \sqrt{\frac{4 K_p K_m T_m - 1}{2 T_m}}$$

$$\theta = \left[(\cos^{-1}\xi)\frac{\pi}{188}\right]$$

$$t_r = \frac{2T_2\left[\pi - \frac{\pi}{180}\cos^{-1}\xi\right]}{\sqrt{4 T_m K_p K_m - 1}}$$

peak time $t_p = \frac{n\pi}{\omega d}$

$n = 1$

$$t_p = \frac{2\pi \quad T_m}{\sqrt{4 T_m K_p K_m - 1}}$$

peak overshoot $M_p = e^{-\pi \frac{\xi}{\sqrt{1-\xi^2}}}$

$$= e^{-\pi \frac{\frac{1}{2}\sqrt{T_m K_p K_m}}{\sqrt{1 - \frac{1}{4 T_m K_p K_m}}}}$$

$$M_p = e^{-\frac{\pi}{\sqrt{4 K_p K_m T_m - 1}}}$$

setting time

2% EB

$$t_{set} = \frac{4}{\xi \omega_n}$$

$$= \frac{4}{\frac{1}{2} T_m} = 8 T_m$$

5% EB

$$t_{set} = \frac{3}{\xi \omega_n} = 6 T_m$$

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1st order system	2nd order system
- 1 independent energy storage element	- 2 independent energy storage element
- it does not exhibit ripples, irrespective of position of poles	- It may or not exhibit oscillatory behavior
- Time constant plays a major role	- Natural frequency and damping ratio plays an imp role.