1.a) $\nabla \times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ mxy+3x & 2x^2+3z^2 & 6yz \end{vmatrix}$

$= i(6z-6z) - j(0-0) + k(4x - mx)$

$= x(4-m)k$

$\therefore$ when $m=4$, $\nabla \times F = 0$

$m=4$ //

b) $f = \int \vec{F}\, d\vec{r}$

$= \int (4xy+3x)i + (2x^2+3z^2)j + (6yz)k \; d(xi+yj+zk)$

$= \int 4xy+3x\,dx + 2x^2+z^2\,dy + 6yz\,dz$

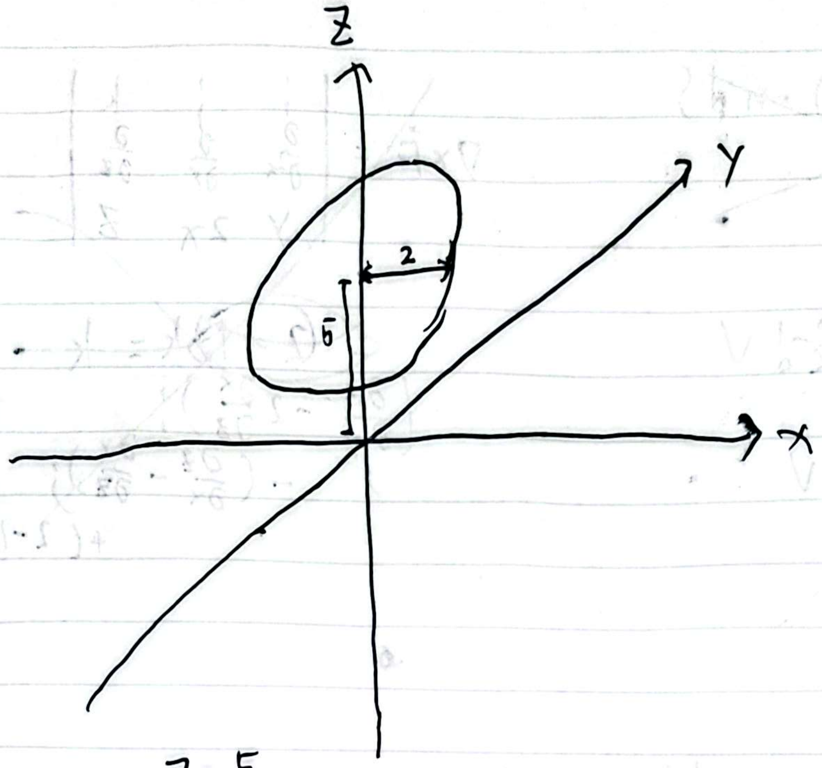
$= [\cancel{2x^2y} + \frac{3}{2}x^2 + 2x^2y + \cancel{z^2y} + 3yz^2]$

$= \frac{3}{2}x^2 + 2x^2y + 3yz^2$

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$2\cos^2\theta = \cos 2\theta + 1$

2a)



$z = 5$

$x^2 + y^2 = 9 - 5$

$x^2 + y^2 = 4, \quad z = 5$

let $x = r\cos\theta = 2\cos\theta$

$y = r\sin\theta = 2\sin\theta$

$x = 2\cos\theta, \quad y = 2\sin\theta, \quad z = 5$ where $0 < \theta < 2\pi$

b) $\iint_\Sigma (\nabla \times F) \cdot n \, dS$

$= \int \vec{F} \cdot d\vec{r}$

$= \int 2\sin\theta\, i + 4\cos\theta\, j + 5k \; d(2\cos\theta\, i + 2\sin\theta\, j + 5k)$

$= \int 2\sin\theta \, d(2\cos\theta) + 4\cos\theta \, d(2\sin\theta) + \cancel{5d(5)}$

$= \int -4\sin^2\theta \, d\theta + 8\cos^2\theta \, d\theta$

$= \int_0^{2\pi} 4\cos^2\theta + 4\cos 2\theta \, d\theta$

$= \int_0^{2\pi} 2\cos 2\theta + 2 + 4\cos 2\theta \, d\theta$

$= \int_0^{2\pi} 6\cos 2\theta + 2 \, d\theta$

$= [3\sin 2\theta + 2\theta]_0^{2\pi} = 4\pi$

c) $\iiint_{D} \nabla \cdot (\nabla \times \vec{F})\, dV$

$= \iiint_{D} 0 \, dV$

$= 0$

d) $\iiint_{\Sigma_1} (\nabla \times F) \cdot n \, dS$

$\nabla \times F = \begin{vmatrix} i & j & k \\ \partial_x & \partial_y & \partial_z \\ y & 2x & z \end{vmatrix}$

$= (2-1)k = k$

$\iint k \cdot (-k) \sqrt{0^2 + 0^2 + 1} \, dx\,dy$

$\iint -1 \, dx\,dy$

$\int_0^{2\pi}\int_0^{2} -r \, dr \, d\theta$

$= \int_0^{2\pi} -\frac{1}{2}(2)^2 \, d\theta$

$= -2(2\pi)$

$= -4\pi$