Digital Image Processing

Machine Vision & Image Processing Lab.

Computer Homework 1

Due date: 1404/01/13

※ Happy New Year ※

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Application of **Discrete Fourier Transform** to Image Processing

1. Instructions

a) Submit the assignment as a **Jupyter Notebook** (.ipynb) containing:
   i. Executed results of the code (no empty cells).
   ii. Explanations in Markdown cells for each step of the assignment.

b) Important Notes:
   i. Attach all necessary files (including sample images) to ensure the notebook runs directly.
   ii. Keep each part of the code in separate cells for clarity.
   iii. Add proper documentation and comments in both Markdown and code cells for a better score.

2. Preliminaries

Fourier Transform is used to analyze the frequency characteristics of various filters. For images, **2D Discrete Fourier Transform** (DFT) is used to find the frequency domain. A fast algorithm called **Fast Fourier Transform (FFT)** is used for calculation of DFT. [1]
Let f(x,y) denote an M × N image, for x = 0, 1, 2,…, M − 1 and y = 0, 1, 2,…, N − 1 .The 2D **Discrete Fourier Transform (DFT)** of f, denoted by F(m,n), is given by:

$$F(m.n) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) \exp\left(-2\pi i\left(\frac{x}{M}m + \frac{y}{N}n\right)\right)$$

for m = 0, 1, 2, …, M−1 and n = 0, 1, 2, …, N−1. The M×N rectangular region defined for (m,n) is called the **frequency domain**, and the values of F(m,n) are called the **Fourier coefficients**. The **inverse discrete Fourier transform** is given by [2]:

$$f(x,y) = \frac{1}{\sqrt{MN}} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} F(M,N) \exp\left(2\pi i\left(\frac{x}{M}m + \frac{y}{N}n\right)\right)$$

for x = 0, 1, 2,…, M−1 and y = 0, 1, 2,…, N−1

Before starting the assignment, install and import the required libraries.

```
import numpy as np
import matplotlib.pyplot as plt
import cv2
```

3. Tasks & Implementation

Task 1: Applying Fourier Transform

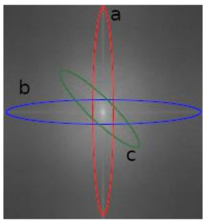
➢ Step 1: Fourier Transform and Visualization

1. Load an image (x-ray-human-skeleton-bones.jpg [3])
2. Convert it to grayscale (if not already).

3. Apply the FFT using
4. Compute and visualize:
   ❖ The magnitude spectrum (using log(1 + |F(m,n)|)).

➢ Step 2: Centering the Frequency Domain and Interpenetrate

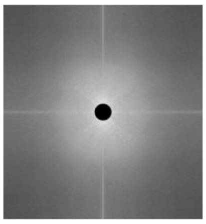
1. Shift the origin of F(m,n) from corner to center. This results in such a way that the center of the spectrum contains the low frequency components whereas other parts contain high frequencies.

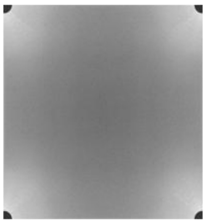


2. Interpret the bright regions (a, b, c) present in the image spectrum of shifted transform and elaborate on the features within the original image that could lead to these bright regions.

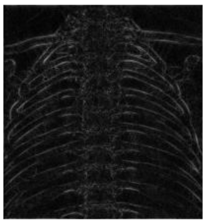
➢ Step 3: High-Pass Filtering (Removing Low Frequencies)

1. Create a high-pass filter mask with a radius of 20 pixels. (circular).
2. Multiply it with the frequency-domain representation.



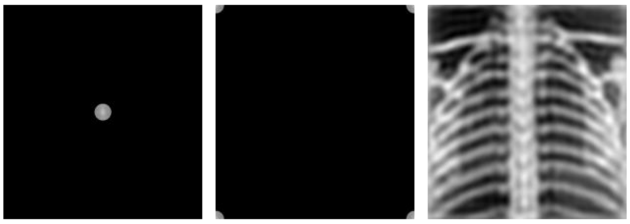


3. Decentralize the origin back to where it was.



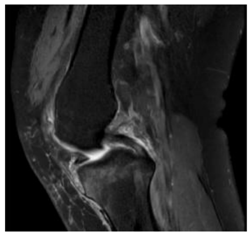
4. Transform back to the spatial domain by using inverse Fourier transform.
5. Show the result and interpret it.

➢ Step 4: Low-Pass Filtering (Removing High Frequencies).

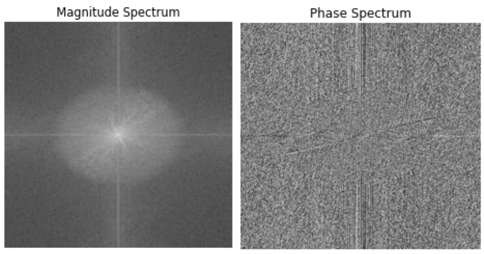


Task 2: Role of Spectrum and Phase in Image Formation

➢ Step 1: Fourier Transform and Reconstruction of MRI image

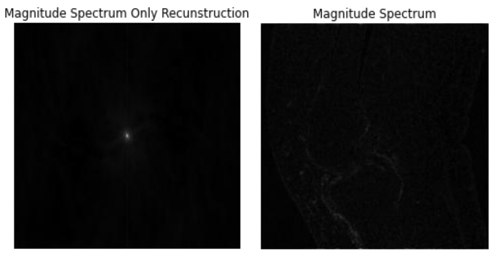


1. Load an image (MRI-Knee_Joint.png [4])
2. Apply the FFT using (Same as Task1. Step1)
3. Compute and visualize:
   - ❖ The magnitude spectrum.
   - ❖ The phase spectrum.



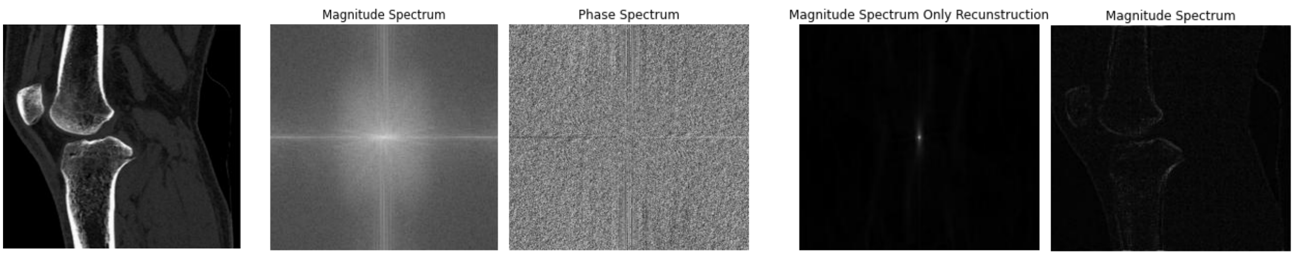
Although there is no detail in this phase spectrum that would lead us by visual analysis to associate it with the structure of its corresponding image, the information in this array is crucial in determining shape features of the image. [5]

4. Reconstruct the Image:
   - ❖ Using only the magnitude spectrum.
   - ❖ Using only the phase spectrum.



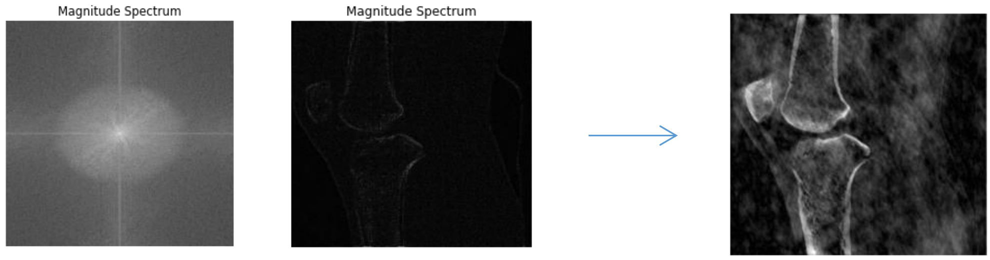
➢ Step 2: Fourier Transform and Reconstruction of CT scan image

1. Load an image (CT-Knee_Joint.png [6])
2. Step. 1. (1 - 4)

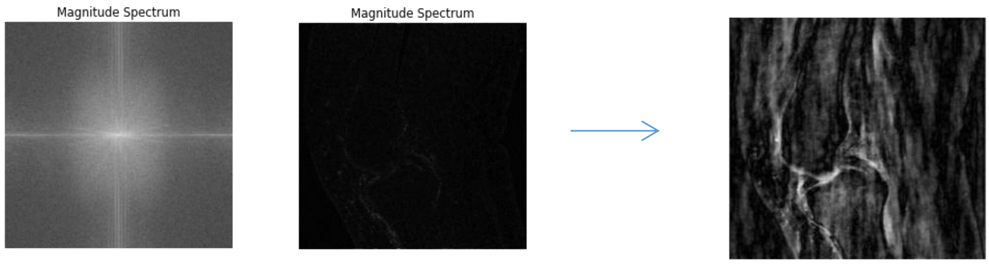


➢ Step 3: Swap the Magnitude and Phase of MRI and CT Scans and Reconstruct

1. Magnitude of the MRI image + Phase of the CT image



2. Magnitude of the CT image + Phase of the MRI image



Task 3: Phase Shift and Image Transformation in Frequency Domain

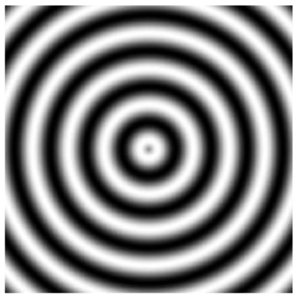
1. Create a periodic 2D sine wave pattern on a grid, as shown below. You will use a frequency of your choice for radial ripples.

$$Z = \sin(2\pi \times f \times \sqrt{X^2 + Y^2})$$

Where f controls the frequency of radial ripples.

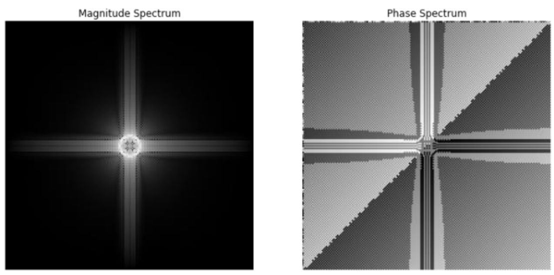
```
x = np.linspace(-2, 2, 200)
y = np.linspace(-2, 2, 200)
X, Y = np.meshgrid(x, y)

f = 2
Z = np.sin(2 * np.pi * f * np.sqrt(X**2 + Y**2))
```



2. Compute the 2D Fourier Transform.

3. Plot the magnitude and phase of the Fourier Transform.

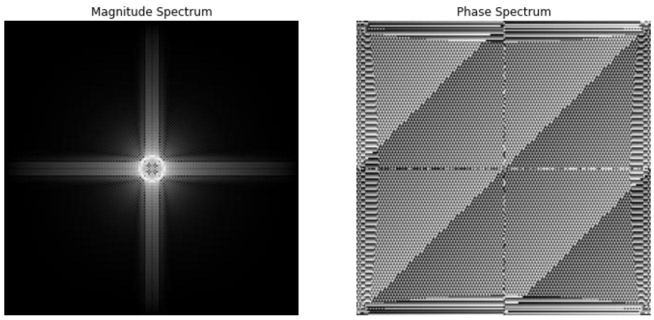


4. Apply a horizontal phase shift to the frequency domain.
   Modifying the phase component.

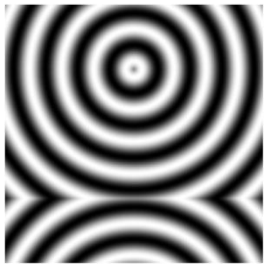
$$F'(U,V) = F(U,V) \cdot e^{j 2 \pi \cdot c}$$

   Where $c = 100$ controls amount of shifting.

5. Plot the magnitude and phase of the Fourier Transform after Shifting.



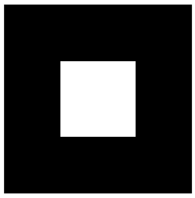
6. Apply the inverse Fourier Transform



7. Visualize the reconstructed image
8. Experiment with different values of c =200, -100

9. Create a simple image with the following specifications and repeat the steps from the previous exercise.

```
width, height = 200, 200
image = np.zeros((height, width))
image[60:140, 60:140] = 1
```



After applying a phase shift in the frequency domain and reconstructing this **non-periodic** image:

a) What does the resulting image look like?
b) Why does the difference occur?
c) Interpret the result both visually and mathematically.

Task 4 (+20 points): Hiding and Revealing a Password in an Image Using Fourier Transform

This task involves hiding an image (a password in this case) within another image using Fourier Transform, a technique known as image-based Steganography [7]. The provided instructions describe a process of combining the 2D Discrete Fourier Transform (DFT) of two images in such a way that the smaller image's information is hidden in the frequency domain of the larger image. The smaller image will be hidden in the higher frequency components of the larger image's DFT.

Let's break down the steps:

1. Load the Two Provided Images:
   - ❖ The two images you'll work with are RGonzalez.jpg and Secrete Message.png.
2. Obtain the 2D DFT of Both Images
3. Bring Low-Frequency Components to the Center.
4. Apply the Weakened Coefficient to the Password Image in the Frequency Domain.

   The password image will be embedded into the higher frequency components of the host image. Weaken the strength of the password image using a coefficient C, where $C \in [0,1]$
   - ❖ If C = 1, no weakening is applied (password image is fully inserted).
   - ❖ If C = 0, the password image is not visible at all.

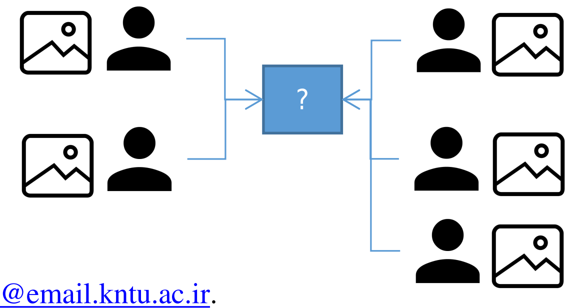
   To combine them:
   Extract the higher frequency components of the host image.
   Insert the password image (scaled by into these higher frequencies).

5. Obtain the Inverse Fourier Transform to Get the Image with Hidden Information.
6. Extract the Hidden Password Image from the Image with Hidden Information
7. Display the Results
   a) Display the following images:
   b) The host image.
   c) The password image.
   d) The hidden image (host with embedded password).
   e) The extracted password image.

8. Evaluate Image Quality Using MSE, SSIM, and VIF
   - ❖ MSE (Mean Squared Error): Measures the average squared difference between the original and the modified image. A lower value indicates less distortion.
   - ❖ SSIM (Structural Similarity Index): Measures the perceptual similarity between two images. Higher values (close to 1) indicate that the images are visually similar.
   - ❖ VIF (Visual Information Fidelity): Measures how well the image preserves visual information based on human perception.

Task 5 (+10 points): Image Secret Sharing Schemes.

In this task, the password image will be securely divided into multiple parts, ensuring that **all parts are required** to reconstruct the original password. Using Fourier Transform, and method mentioned in Task 2 image can be split into multiple parts , which are then distributed among <u>five</u> individuals in a way that no single part reveals meaningful information. Each participant receives a unique share of the data, and only when all shares are combined can the original password image be recovered using the inverse Fourier transform. This approach ensures security, as partial access to the data remains unreadable, making it ideal for secure password sharing and authentication.



Best of Luck
For further information, do not hesitate to contact me at a.najafi@email.kntu.ac.ir.