a) A ball of mass M1 and velocity v1 collides with another ball of mass M2 and velocity v2. If the collision is elastic, what are the final velocities of the two balls?

b) A billiard ball of mass m sits on a billiards table near the edge. Another ball of mass M larger than or equal to m (assume both are the same size) is shot towards the first ball with speed v. The first ball will bounce back and forth between the heavier ball and the wall a number of times before the larger ball eventually moves off with a larger speed than the smaller one. If all collisions are elastic, determine the total number of collisions (ball-ball and ball-wall) if M = m, M=100m, and M=10000m (you will probably need to use a computer program e.g. Scratch). Can you guess the result for M = 100^n. Hint: the answer is amazing.

Question 1

How many collisions will there be if M=m?

Question 2

How many collisions will there be if M=100m?

Question 3

How many collisions will there be if M=10000m?

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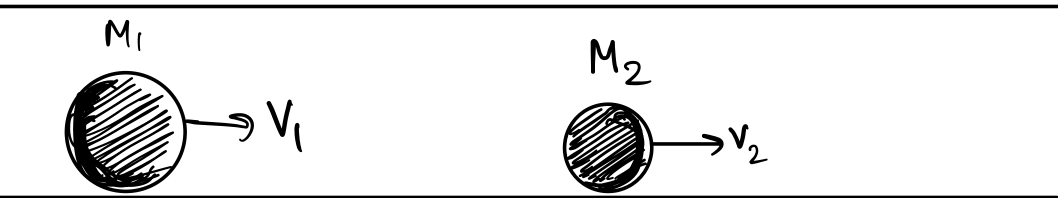
a)



Let velocities after collision be V_1 for M_1

and V_2 for M_2.

ELASTIC COLLISION: so momentum and KE is conserved.

MOMENTUM CONSERVATION:

$$M_1 v_1 + M_2 v_2 = M_1 V_1 + M_2 V_2 \qquad (p_i = p_f)$$

$$M_1(v_1 - V_1) = M_2(V_2 - v_2) \text{ --- } ①$$

ENERGY CONSERVATION:

$$KE_i = KE_f$$

$$\frac{1}{2}M_1(v_1)^2 + \frac{1}{2}M_2(v_2)^2 = \frac{1}{2}M_1(V_1)^2 + \frac{1}{2}M_2(V_2)^2$$

$$M_1(v_1)^2 + M_2(v_2)^2 = M_1(V_1)^2 + M_2(V_2)^2$$

$$M_1(v_1^2 - V_1^2) = M_2(V_2^2 - v_2^2)$$

$$M_1(v_1 - V_1)(v_1 + V_1) = M_2(V_2 - v_2)(V_2 + v_2) \text{ --- } ②$$

② ÷ ①

$$\frac{\cancel{M_1(v_1 - V_1)}(v_1 + V_1)}{\cancel{M_1(v_1 - V_1)}} = \frac{\cancel{M_2(V_2 - v_2)}(V_2 + v_2)}{\cancel{M_2(V_2 - v_2)}}$$

$$M_1(\cancel{v_1 - V_2}) \qquad M_2(\cancel{V_2 - v_2})$$

SOLUTION $v_1 = V_1$ and $V_2 = v_2$ is not valid because this means velocities won't change.

$$v_1 + V_1 = v_2 + V_2$$

$$V_2 = v_1 + V_1 - v_2 \quad \text{———} \quad (3)$$

put (3) in (1)

$$M_1(v_1 - V_1) = M_2((v_1 + V_1 - v_2) - v_2)$$

$$M_1 v_1 - M_1 V_1 = M_2 v_1 + M_2 V_1 - 2M_2 v_2$$

$$(M_1 + M_2)V_1 = M_1 v_1 + 2M_2 v_2 - M_2 v_1$$

$$V_1 = \frac{M_1 v_1 + 2M_2 v_2 - M_2 v_1}{M_1 + M_2}$$

$$V_2 = v_1 + V_1 - v_2$$

$$V_2 = (v_1 - v_2) + \frac{(M_1 v_1 + 2M_2 v_2 - M_2 v_1)}{M_1 + M_2}$$

$$V_2 = M_1 v_1 - M_1 v_2 + \cancel{M_2 v_1} - M_2 v_2 + M_1 v_1 + 2M_2 v_2 - \cancel{M_2 v_1}$$

$$M_1 + M_2$$

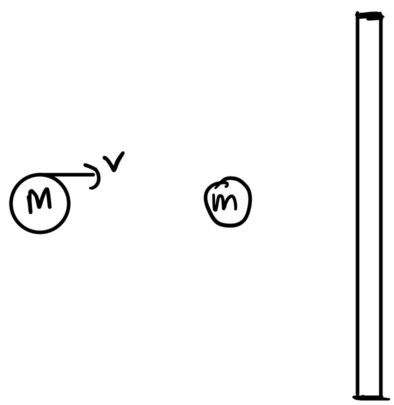
$$V_2 = \frac{2M_1v_1 - M_1v_2 + M_2v_2}{M_1 + M_2}$$

FINAL EQUATIONS:

$$V_1 = \frac{M_1 - M_2}{M_1 + M_2} \cdot V_1 + \frac{2M_2}{M_1 + M_2} \cdot V_2$$

$$V_2 = \frac{M_2 - M_1}{M_1 + M_2} \cdot V_2 + \frac{2M_1}{M_1 + M_2} \cdot V_1$$

Q1]



1st case: $M = m$

$m_1 = m_2 = m$

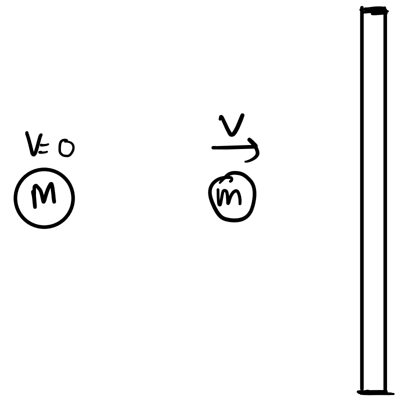
$v_1 = V$

$v_2 = 0$

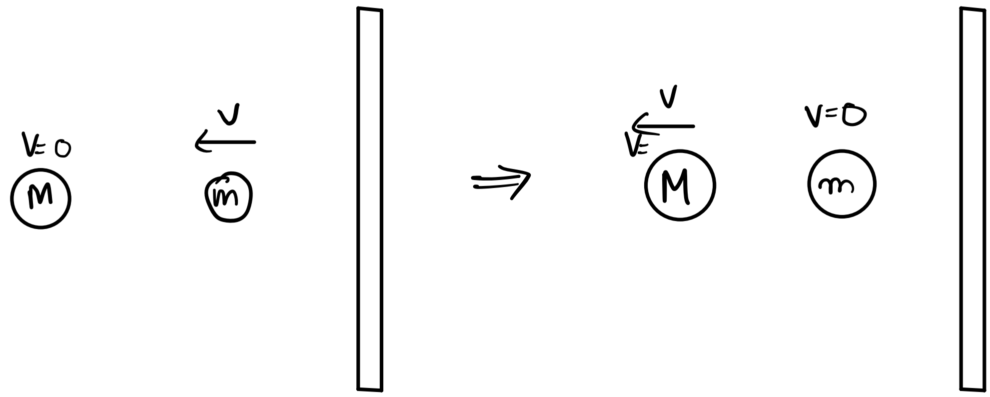
AFTER COLLISION:

$V_1 = 0$

$V_2 = V$



AFTER COLLISION WITH WALL:



∴ 3 collisions

Q2] 31 collisions

Q3] 314 collisions

PICTURE OF CODE:

C HW5_bonus_prblm.c > V2_after(double, double, double, double)

```c

#include <stdio.h>
#include <stdlib.h>
#include <math.h>

/* SIGN CONVENTION */
/* - <--------> + */

double V1_after(double m1, double m2, double v1, double v2);
double V2_after(double m1, double m2, double v1, double v2);

int main(void) {
    double m1; /* mass of big ball */
    double m2; /* mass of small ball */
    printf("ENTER MASS OF LARGE BALL:\n");
    scanf("%lf",&m1);
    printf("ENTER MASS OF SMALL BALL:\n");
    scanf("%lf",&m2);
    double v1 = 10; /* vel of big ball */
    double v2 = 0; /* vel of small ball */
    int num_collisions = 0;

        /* collision with ball */
        double v_f_1 = V1_after(m1,m2,v1,v2);
        double v_f_2 = V2_after(m1,m2,v1,v2);
        v1 = v_f_1;
        v2 = v_f_2;
        num_collisions++;
        printf("%lf\n",v1);
        printf("%lf\n",v2);

    while (fabs(v1) < fabs(v2) /*|| v1 > 0*/) {

        /* collision of small ball with with wall */
        v2 = -v2;
        num_collisions++;
        printf("%lf\n",v1);
        printf("%lf\n",v2);

        /* collison with ball again */
        double v_f_1 = V1_after(m1,m2,v1,v2);
        double v_f_2 = V2_after(m1,m2,v1,v2);
        v1 = v_f_1;
        v2 = v_f_2;
        printf("%lf\n",v1);
        printf("%lf\n",v2);
        num_collisions++;
    }
    printf("Number of collisions: %d\n", num_collisions);
    return 0;
}

double V1_after(double m1, double m2, double v1, double v2) {
    double v_after = (((m1 - m2) / (m1 + m2)) * (v1)) + (((2 * m2) / (m1 + m2)) * (v2));
    return v_after;
}

double V2_after(double m1, double m2, double v1, double v2) {
    double v_after = (((m2 - m1) / (m1 + m2)) * (v2)) + (((2.0 * m1) / (m1 + m2)) * (v1));
    return v_after;
}

```

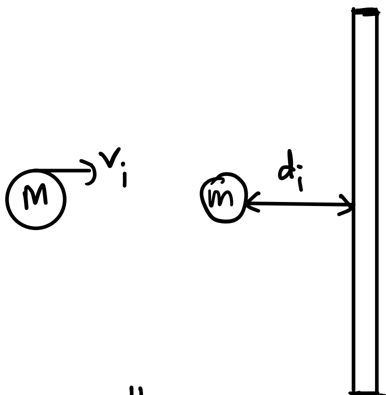
num of collisions btw balls $= \frac{\pi}{2}\sqrt{\frac{M}{m}}$

if $\underline{M} = 100^n$

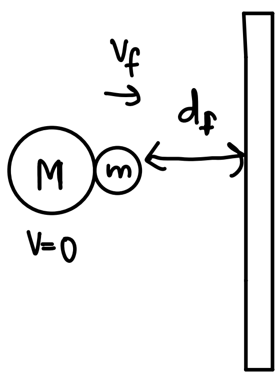
$\overline{m}$

num of collisions (wall + ball) = $\boxed{\pi \cdot 10^{n}}$ digits of pi

* find distance after which big ball changes direction.



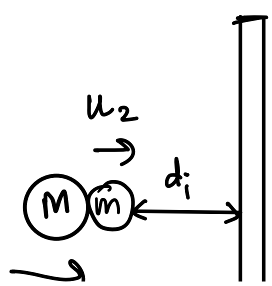
$$E_i = \frac{1}{2} M v_i^2$$



$$E_f = \frac{1}{2} m v_f^2$$

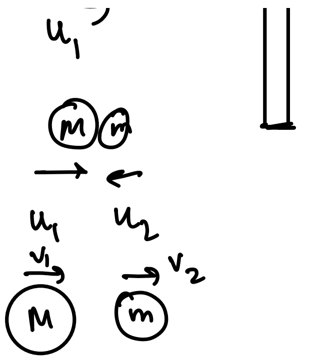
$$E_i = E_f \quad \therefore M v_i^2 = m v_f^2$$

num of collisions = $\boxed{\frac{\pi}{2} \cdot 10^{n}}$



$$d_2 = d_i - u_1 \cdot \Delta t$$

$$\Delta t = d_i + d_i - u_i \cdot \frac{d_i}{v_2}$$



$$\frac{\quad}{u_2} \qquad \frac{\quad}{u_1+u_2}$$

$$\Delta t = d_i\left(\frac{1}{u_2} + \frac{1 - u_1/u_2}{u_1+u_2}\right)$$

mirror image

$$\Delta t = d_i\left(\frac{u_1+u_2+u_2-u_1}{(u_2)(u_1+2)}\right)$$

$\Uparrow$

$$\Delta t = \frac{d_i(2)}{u_1+u_2} = \frac{2d_i}{u_1+u_2}$$

$$\Delta t = \frac{2d_i}{u_1+u_2}$$

$$d_2 = d_i\left(1 - \frac{2u_1}{u_1+u_2}\right)$$

for n^{th} collision $\leftarrow$

$$\boxed{d_n = \frac{d_{n-1}(u_2-u_1)}{(u_1+u_2)}}$$

$$d_n\left(u^2_{n-1} + u^1_{n-1}\right) = d_{n-1}\left(u^2_{n-1} - u^1_{n-1}\right) \rightarrow \text{invariant}$$

velocity of separation = velocity of approach

$$d_n\left(v^2_n - v^1_n\right) = d_{n-1}\left(u^2_{n-1} - u^1_{n-1}\right)$$

$V_1, V_2 \longrightarrow$ velocities after collision

$U_1, U_2 \longrightarrow$ velocities before collision

$$u_1^{n-1} - u_2^{n-1} = V_2^{n} - V_1^{n}$$

$$d_n \left(V_n^2 - V_n^1 \right) = d_{i^\circ} \left(V_{i^\circ}^2 - V_{i^\circ}^1 \right)$$

$(V_n^2)^2 m = (V_{i^\circ}^1)^2 \cdot M$ and $V_n^1 = 0$ and $V_{i^\circ}^2 = 0$

$$d_n \left(V_n^2 \right) = d_{i^\circ} \left(V_{i^\circ}^1 \right)$$

$$\boxed{\frac{d_n}{d_{i^\circ}} = \sqrt{\frac{m}{M}}}$$

$$\boxed{d_n = d_{i^\circ} \cdot 10^{-n}}$$