hw03

December 4, 2024

[5]:
```python
import numpy as np

#
mu = np.array([1, 1])  #
Sigma = np.array([[1, 0.8], [0.8, 1]])  #

#
N = 10

#
samples = np.random.multivariate_normal(mu, Sigma, N)

#
print("Generated samples:")
print(samples)
```

```
Generated samples:
[[-0.32935268  0.02018352]
 [ 1.09153774  0.24522839]
 [ 2.36224352  2.43833085]
 [ 0.95597091  0.63368881]
 [-0.33062434  0.23237103]
 [ 0.75067723  0.31981519]
 [ 2.33137891  3.04416585]
 [ 2.52947733  2.01944625]
 [ 1.6249431   0.12128652]
 [ 1.72495975  2.12623945]]
```

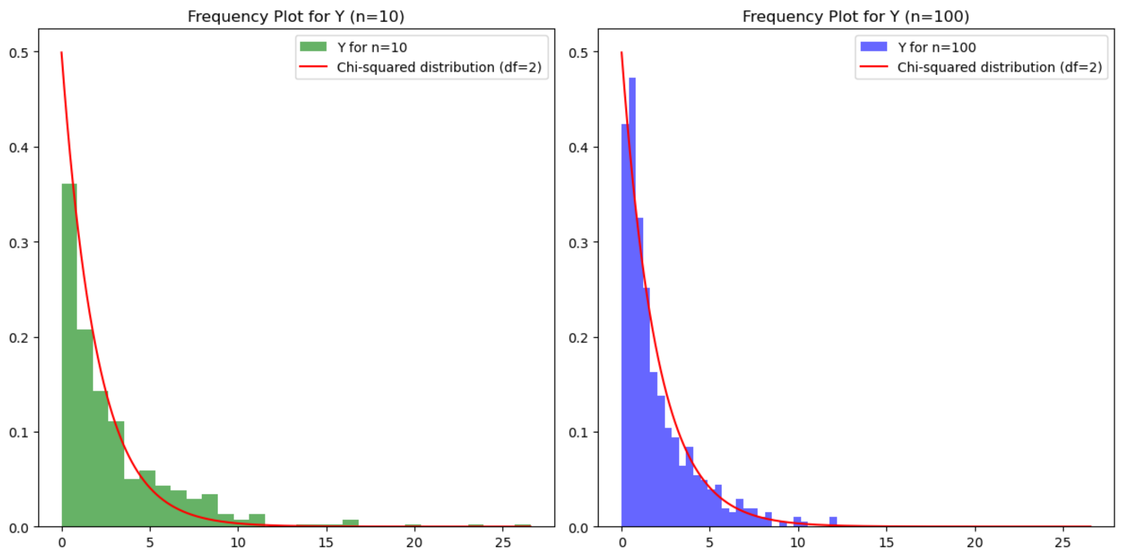
[6]:
```python
import numpy as np

#
mu = np.array([1, 1])  #
Sigma = np.array([[1, 0.8], [0.8, 1]])  #

#
N = 10

#
```

```
samples = np.random.multivariate_normal(mu, Sigma, N)

#       x_bar
x_bar = np.mean(samples, axis=0)

#       S
S = np.cov(samples, rowvar=False)

#
print("Sample mean vector (x_bar):")
print(x_bar)
print("\nSample covariance matrix (S):")
print(S)

#
print("\nTrue mean vector (mu):")
print(mu)
print("\nTrue covariance matrix (Sigma):")
print(Sigma)

#
mean_diff = np.linalg.norm(x_bar - mu)
print(f"\nDifference between sample mean and true mean: {mean_diff}")

#       Frobenius
cov_diff = np.linalg.norm(S - Sigma, 'fro')
print(f"Difference between sample covariance matrix and true covariance matrix:␣
 ↪{cov_diff}")
```

```
Sample mean vector (x_bar):
[1.21134872 1.11962782]

Sample covariance matrix (S):
[[1.05659677 1.1916667 ]
 [1.1916667  1.65175746]]

True mean vector (mu):
[1 1]

True covariance matrix (Sigma):
[[1.  0.8]
 [0.8 1. ]]

Difference between sample mean and true mean: 0.2428561206176943
Difference between sample covariance matrix and true covariance matrix:
0.8572027752035326
```

```
[7]: import numpy as np
     import matplotlib.pyplot as plt
     from scipy.stats import chi2

     #
     mu = np.array([1, 1])  #
     Sigma = np.array([[1, 0.8], [0.8, 1]])  #

     #
     n_values = [10, 100]
     n_repeats = 500

     #
     def generate_Y(mu, Sigma, n, repeats):
         Y_values = []
         for _ in range(repeats):
             #
             samples = np.random.multivariate_normal(mu, Sigma, n)

             #
             x_bar = np.mean(samples, axis=0)
             S = np.cov(samples, rowvar=False)

             #  Y
             diff = x_bar - mu
             S_inv = np.linalg.inv(S)
             Y = n * np.dot(np.dot(diff, S_inv), diff.T)
             Y_values.append(Y)

         return Y_values

     #      Y
     Y_10 = generate_Y(mu, Sigma, 10, n_repeats)
     Y_100 = generate_Y(mu, Sigma, 100, n_repeats)

     #
     fig, ax = plt.subplots(1, 2, figsize=(12, 6))

     # 10
     ax[0].hist(Y_10, bins=30, density=True, alpha=0.6, color='g', label='Y for␣
      ↪n=10')
     x = np.linspace(min(Y_10), max(Y_10), 1000)
     ax[0].plot(x, chi2.pdf(x, df=2), 'r-', label='Chi-squared distribution (df=2)')
     ax[0].set_title('Frequency Plot for Y (n=10)')
     ax[0].legend()

     # 100
```

```
ax[1].hist(Y_100, bins=30, density=True, alpha=0.6, color='b', label='Y for␣
 ↪n=100')
ax[1].plot(x, chi2.pdf(x, df=2), 'r-', label='Chi-squared distribution (df=2)')
ax[1].set_title('Frequency Plot for Y (n=100)')
ax[1].legend()

plt.tight_layout()
plt.show()

#       Y
print(f"Mean of Y for n=10: {np.mean(Y_10)}")
print(f"Mean of Y for n=100: {np.mean(Y_100)}")
```



```
Mean of Y for n=10: 2.8864128867069128
Mean of Y for n=100: 1.9803328276690826
```