Assignment-7
(Papoulis Chapter 8 Example 8.27)

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Contents

Question

Suppose that $f(x,\theta)\ \theta e^{-\theta x}U(x)$. Then test the likelihood ratio hypothesis

$$H_0 : 0 < \theta \leq \theta_0 \quad against \quad H_1 : \theta > \theta_0$$

Solution

In this problem, θ_0 is the segment $0 < \theta \leq \theta_0$ of the real line and θ is the half-line $\theta < 0$. Thus both hypotheses are composite. The likelihood function

$$f(X, \theta) = \theta^n e^{-n\bar{x}\theta} \tag{1}$$

is shown for $\bar{x} > 1/\theta_0$ and $\bar{x} < 1/\theta_0$. In the half-line $\theta_0 > 0$ this function is maximum for $\theta = 1/\bar{x}$. In the interval $0 < \theta \leq \theta_0$ it is maximum for $\theta = 1/\bar{x}$ if $\bar{x} > 1/\theta_0$ and for $\theta = \theta_0$ if $\bar{x} < 1/\theta_0$. Hence

$$\theta_m = \frac{1}{x} \qquad \theta_{m0} = \begin{cases} 1/\bar{x} & \textit{for} \quad \bar{x} > 1/\theta_0 \\ \theta_0 & \textit{for} \quad \bar{x} < 1/\theta_0 \end{cases} \tag{2}$$

Solution

The likelihood ratio equals

$$\lambda = \begin{cases} 1 \quad \mathit{for} \quad \bar{x} > 1/\theta_0 \\ (\bar{x}\theta_0)^n e^{-n\theta_0\bar{x}+n\theta_0} \quad \mathit{for} \quad \bar{x} < 1/\theta_0 \end{cases} \tag{3}$$

We reject H_o if $\lambda < c$ or, equivalently, if $\bar{x} < c_1$, where c_1 equals the α percentile of the random variable $\bar{x}$.