Assignment-2
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J Sai Sri Hari Vamshi
AI21BTECH11014

Problem 1(x):

If events A and B are independent such that $\mathrm{P}(A) = \frac{3}{5}$, $\mathrm{P}(B) = \frac{2}{3}$, find $\mathrm{P}(A+B)$.

Solution:

The given input probabilities and desired values are given in the Table 1,

Event	**Probability**	**Value**
A	$\mathrm{P}(A)$	$\frac{3}{5}$
B	$\mathrm{P}(B)$	$\frac{2}{3}$
$A+B$	$\mathrm{P}(A+B)$	?

Table 1:

It is also given that events A and B are independent which means,

$$\mathrm{P}(AB) = \mathrm{P}(A)\,\mathrm{P}(B) \tag{1}$$

We know, if events E_1 and E_2 were disjoint, we get

$$\mathrm{P}(E_1 + E_2) = \mathrm{P}(E_1) + \mathrm{P}(E_2) \tag{2}$$

Now, for events A and B,

$$A + B = A + (B - A)$$

Here, A and $B - A$ are disjoint. And from equation (2)

$$\begin{aligned} \mathrm{P}(A+B) &= \mathrm{P}(A + (B - A)) \\ &= \mathrm{P}(A) + \mathrm{P}(B - A) \\ &= \mathrm{P}(A) + \mathrm{P}(B - AB) \end{aligned}$$

B and AB are also disjoint, so again using (2)

$$\mathrm{P}(A+B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(AB) \tag{3}$$

This is the General Probabiity Addition Rule.
Using the above two formulas, (1) and (3), we can use the modified probability addition rule for independent sets as,

$$\mathrm{P}(A+B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A)\,\mathrm{P}(B) \tag{4}$$

By substituting the respective values in (4), we get,

$$\begin{aligned} \mathrm{P}(A+B) &= \frac{3}{5} + \frac{2}{3} - \frac{3}{5} \cdot \frac{2}{3} \\ &= \frac{3}{5} + \frac{2}{3} - \frac{2}{5} \\ &= \frac{13}{15} \end{aligned}$$

So, the desired probability $\mathrm{P}(A+B)$ is found to be,

$$\mathrm{P}(A+B) = \frac{13}{15} = 0.8667$$