Assignment-3
(CBSE 11th Ex 16.3)

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Problem 13:

Fill in the blanks in the following Table 1:

$\mathbf{P}(A)$	$\mathbf{P}(B)$	$\mathbf{P}(A \cap B)$	$\mathbf{P}(A \cup B)$
$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	...
0.35	...	0.25	0.6
0.5	0.35	...	0.7

Table 1:

Solution:

The given inputs of probabilities involving events A and B vary among $\mathrm{P}(A)$, $\mathrm{P}(B)$, $\mathrm{P}(AB)$ and $\mathrm{P}(A+B)$ with one of them being an unknown. We know, if events E_1 and E_2 were disjoint, we get

$$\mathrm{P}(E_1 + E_2) = \mathrm{P}(E_1) + \mathrm{P}(E_2) \tag{1}$$
$$\mathrm{P}(E_1 E_2) = 0 \tag{2}$$

Now, for events A and B,

$$A + B = A + (B - A)$$
$$B - A = B - AB$$

Here, A and $B - A$ are disjoint. And from equation (1)

$$\begin{aligned}\mathrm{P}(A+B) &= \mathrm{P}(A + (B - A)) \\ &= \mathrm{P}(A) + \mathrm{P}(B - A) \\ &= \mathrm{P}(A) + \mathrm{P}(B - AB)\end{aligned}$$

B and AB are also disjoint, so again using equation (1)

$$\mathrm{P}(A+B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(AB) \tag{3}$$

This is the General Probabiity Addition Rule.
Using the above formulae (3) we can find the unknown probabilities in the question,

(a) Given values from the question are in Table 2:

Event	Probability	Value
A	$\mathrm{P}(A)$	$\frac{1}{3}$
B	$\mathrm{P}(B)$	$\frac{1}{5}$
AB	$\mathrm{P}(AB)$	$\frac{1}{15}$
$A+B$	$\mathrm{P}(A+B)$	?

Table 2:

Substituting the values above into the formulae (3),

$$\begin{aligned}\mathrm{P}(A+B) &= \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(AB) \\ &= \frac{1}{3} + \frac{1}{5} - \frac{1}{15} \\ &= \frac{7}{15}\end{aligned}$$

Therefore from above, we get the desired probability,

$$\mathrm{P}(A+B)=\frac{7}{15}$$

(b) Given values from the question are in Table 3:

Event	Probability	Value
A	$\mathrm{P}(A)$	0.35
B	$\mathrm{P}(B)$	?
AB	$\mathrm{P}(AB)$	0.25
$A+B$	$\mathrm{P}(A+B)$	0.6

Table 3:

Substituting the values above into the formulae (3),

$$\begin{aligned}\mathrm{P}(A+B) &= \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(AB)\\ \mathrm{P}(B) &= \mathrm{P}(A+B)+\mathrm{P}(AB)-\mathrm{P}(B)\\ &= 0.6+0.25-0.35\\ &= 0.5\end{aligned}$$

Therefore from above, we get the desired probability,

$$\mathrm{P}(B)=0.5$$

(c) Given values from the question are in Table 4:

Event	Probability	Value
A	$\mathrm{P}(A)$	0.5
B	$\mathrm{P}(B)$	0.35
AB	$\mathrm{P}(AB)$	?
$A+B$	$\mathrm{P}(A+B)$	0.7

Table 4:

Substituting the values above into the formulae (3),

$$\begin{aligned}\mathrm{P}(A+B) &= \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(AB)\\ \mathrm{P}(AB) &= \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A+B)\\ &= 0.5+0.35-0.7\\ &= 0.15\end{aligned}$$

Therefore from above, we get the desired probability,

$$\mathrm{P}(AB)=0.15$$