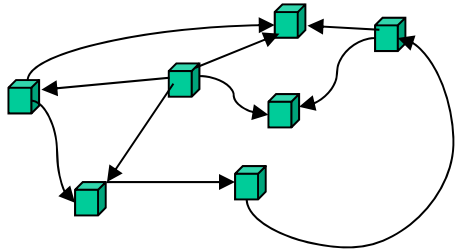
Graph Algorithms

Definitions



- Basic Concepts
  - A ***graph***: $G = (V, E)$

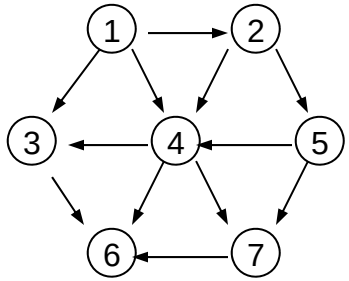
    where V is a set of ***vertices***, E is a set of ***edges***. Each edge is defined as a pair (v, w) with $v, w \in V$.
  - If the pairs that define the edges in E are ordered, G is a ***directed*** graph (or: digraph)
  - If $(v, w) \in E$, w is said to be ***adjacent*** to v. In an undirected graph, w is adjacent to v if and only if v is adjacent to w.
  - A value (***weight***) can be assigned to each edge.
  - A ***path*** in G is a sequence of connected vertices $w_1, w_2, \ldots, w_N \in V$, *i.e.* $(w_i, w_{i+1}) \in E$ $(1 \leq i < N)$. The length of the path is N-1.
  - A zero length path is a path from vertex v to itself (without the edge (v, v), which is called a ***loop***).

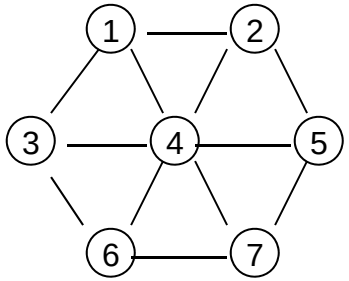
- A simple path is a path in which all vertices are distinct except the *1st* and the last vertices.
- A *cycle* is a path such that $w_1 = w_N$ $(N > 1)$. For an undirected graph, the edges in a cycle are required to be distinct.
- A directed graph is *acyclic* if it has no cycles, and it is called *DAG* (directed acyclic graph).
- A graph is *connected* if there is a path from every vertex to every other vertex.

  If a connected graph is also directed, the graph is said to be *strongly connected*.

  For a directed graph, if the connectivity can only be defined with undirected paths, it is said to be *weakly connected*.
- A complete graph is a graph in which there is an edge between every pair of vertices.
- Example: Airport connections.

– Graph Representations





– Adjacency Matrix

- A 2D $N \times N$ array

$A[i][j]$ = { 1: if there is edge (i, j)

0: otherwise

- Weights of the edge can also be stored in the matrix.
- Memory requirement: $\Theta(|V|^2)$
- A good representation for dense graphs, but inefficient if the graph is sparse.

directed

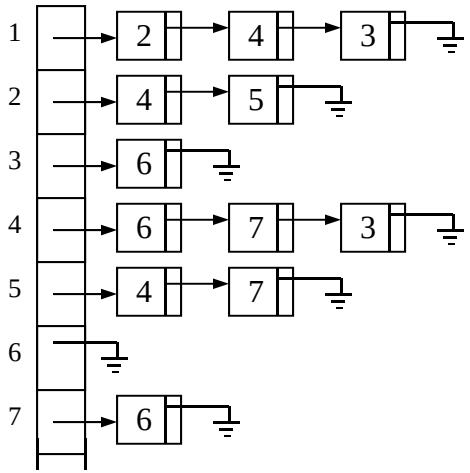
	1	2	3	4	5	6	7
1	0	1	1	1	0	0	0
2	0	0	0	1	1	0	0
3	0	0	0	0	0	1	0
4	0	0	1	0	0	1	1
5	0	0	0	1	0	0	1
6	0	0	0	0	0	0	0
7	0	0	0	0	0	1	0

undirected

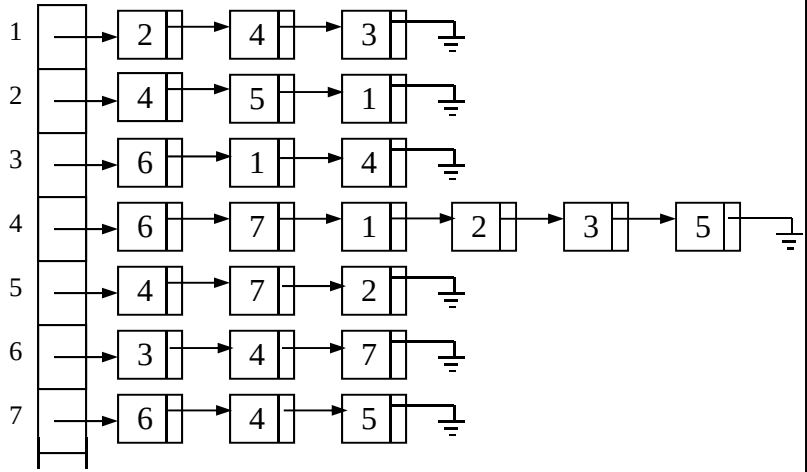
	1	2	3	4	5	6	7
1	0	1	1	1	0	0	0
2	1	0	0	1	1	0	0
3	1	0	0	1	0	1	0
4	1	1	1	0	1	1	1
5	0	1	0	1	0	0	1
6	0	0	1	1	0	0	1
7	0	0	0	1	1	1	0

– Adjacency List
  - A 1D array of linked lists.
    $A[u]$ is a list of all the vertices that are adjacent to u.
  - Memory cost: $\Theta(|V| + |E|)$
  - Weights of the edges can be stored in the nodes of the lists.
  - For undirected graph, each edge appears in two lists (double memory cost)
  - Hashing can be used to map the names of the vertices to the vertex nodes.

directed

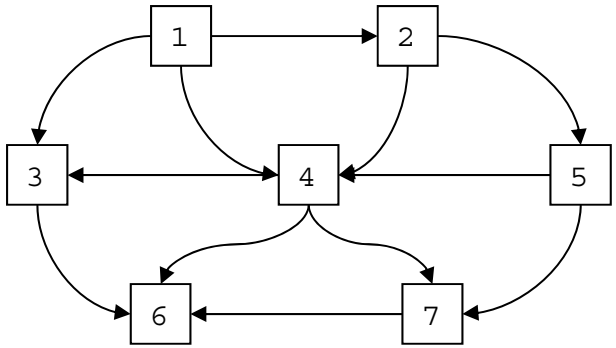


undirected



Topological Sort

- A *topological Sort* is an ordering of vertices in a directed acyclic graph (DAG) such that if there is a path from v_i to v_j, then v_j appears after v_i in the ordering.
- *Topological order* is not possible if there is a cycle in the graph.
- Topological ordering is not unique.
- Example
  - course prerequisite graph



1 2 5 4 3 7 6

or

1 2 5 4 7 3 6

- A simple algorithm
  - *indegree* of vertex v: the number of edges (u, v)
  - topological sorting algorithm:

    a.) compute the indegrees of all vertices

    b.) remove a vertex with *0* indegree and its associated edges

    c.) repeat b.) until all vertices are sorted.

example with directed edge matrix

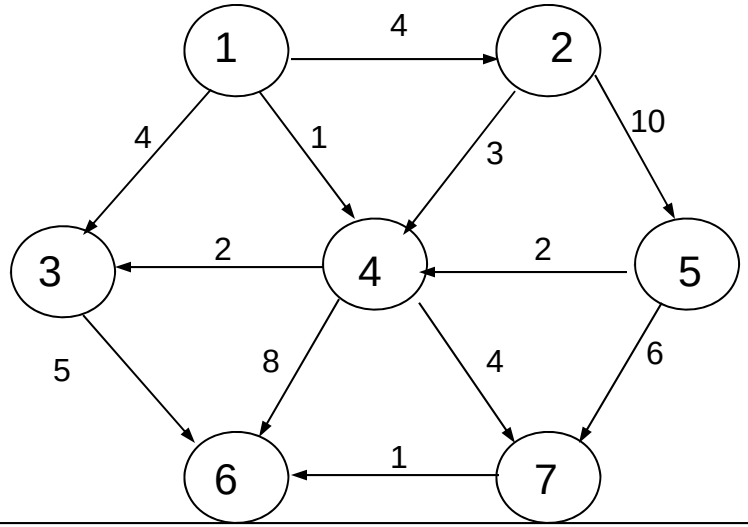
Shortest-Path Algorithms

- The problem
  - Input: a weighted graph

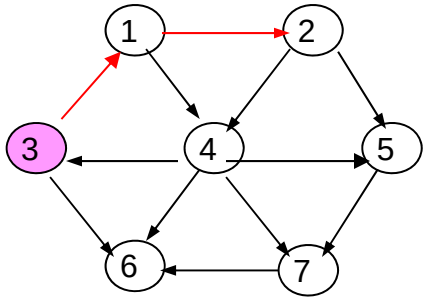
    i.e. each edge (v_i, v_j) has a cost C_{ij}
  - the cost of path $v_1\ v_2 \ldots v_N$ is $\sum_{i=1}^{N-1} C_{i,i+1}$, also called ***weighted path length.***
  - ***unweighted path length***: $N - 1$
  - single-source shortest path problem:

    given as input a weighted graph $G = (V, E)$ and a distinguished vertex, S, find the shortest weighted path from S to every other vertex in G
  - example

    $C(v_1, v_6) = 6$

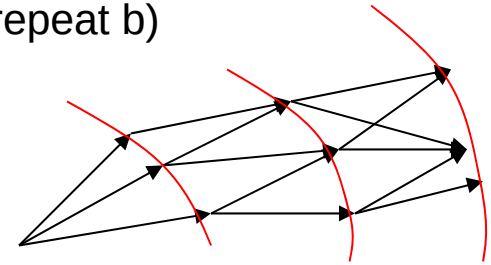


– The single destination problem: find the shortest path from source s to a given destination vertex t.
  - Currently, there are no algorithms that can find the single destination shortest path any faster (by more than a constant factor) than the shortest paths to all vertices.

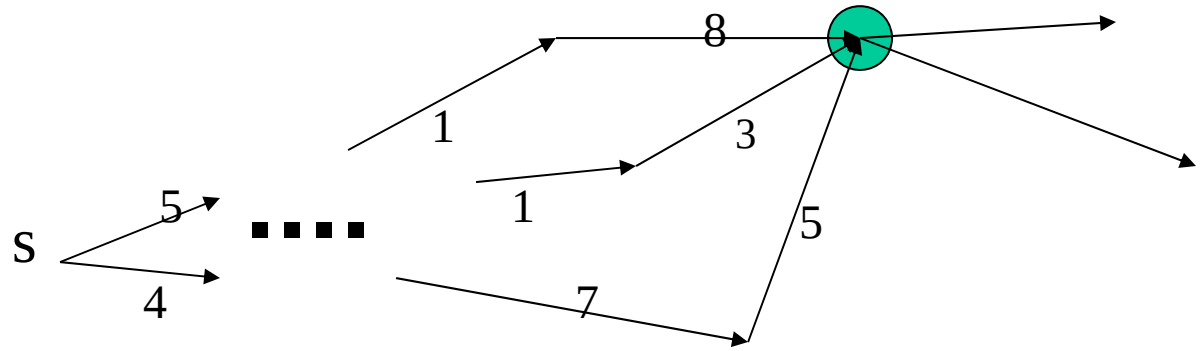


– Unweighted shortest paths
  - A special case of weighted shortest path
  - data structure (table)

  - basic idea
    a) Starting from vertex s, let `s.dis=0` and `x=s`
    b) Mark `dist=x.dist+1` for all vertices that are adjacent to x.
    c) For each of the vertices, x, that are just marked, repeat b)
    d) repeat c) until all vertices are marked.
  - Breadth-first search



– Weighted shortest path
  - Mark vertices as known or unknown



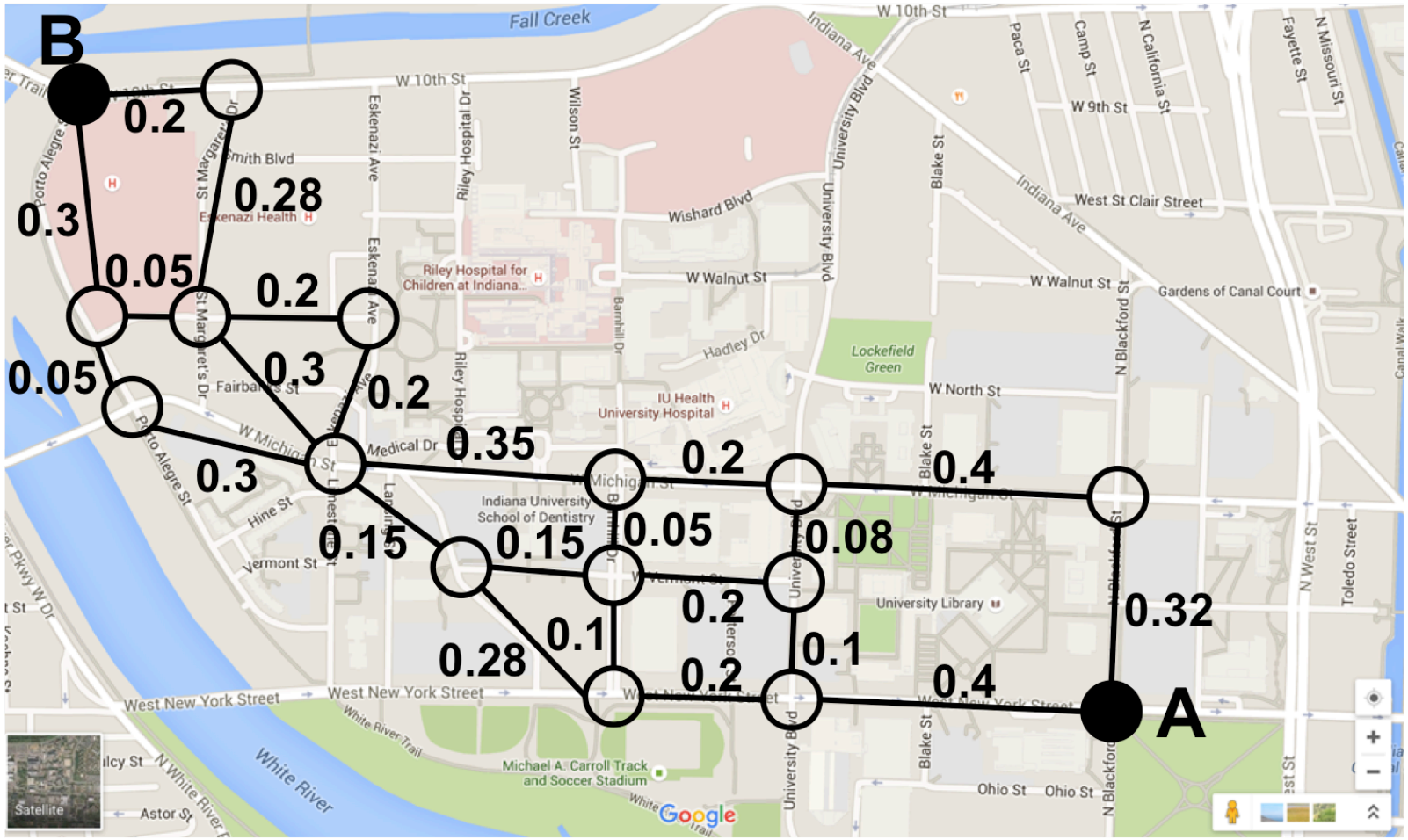
– Greedy algorithm
  - greedy strategy: taking currently best solution at each step.
  - similar idea to the unweighted algorithm.

    At each stage, find the shortest unknown distance vertex v, and update its adjacent vertices: `w.dist=v.dist+c(v,w)`

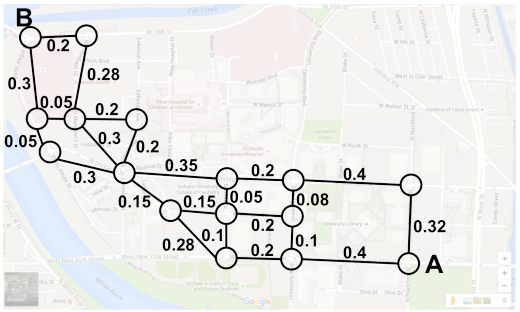
    if it is an improvement over the previous value in w.

Problem?

You want to travel from point A to point B, following the shortest distance.



Dijkstra's algorithm



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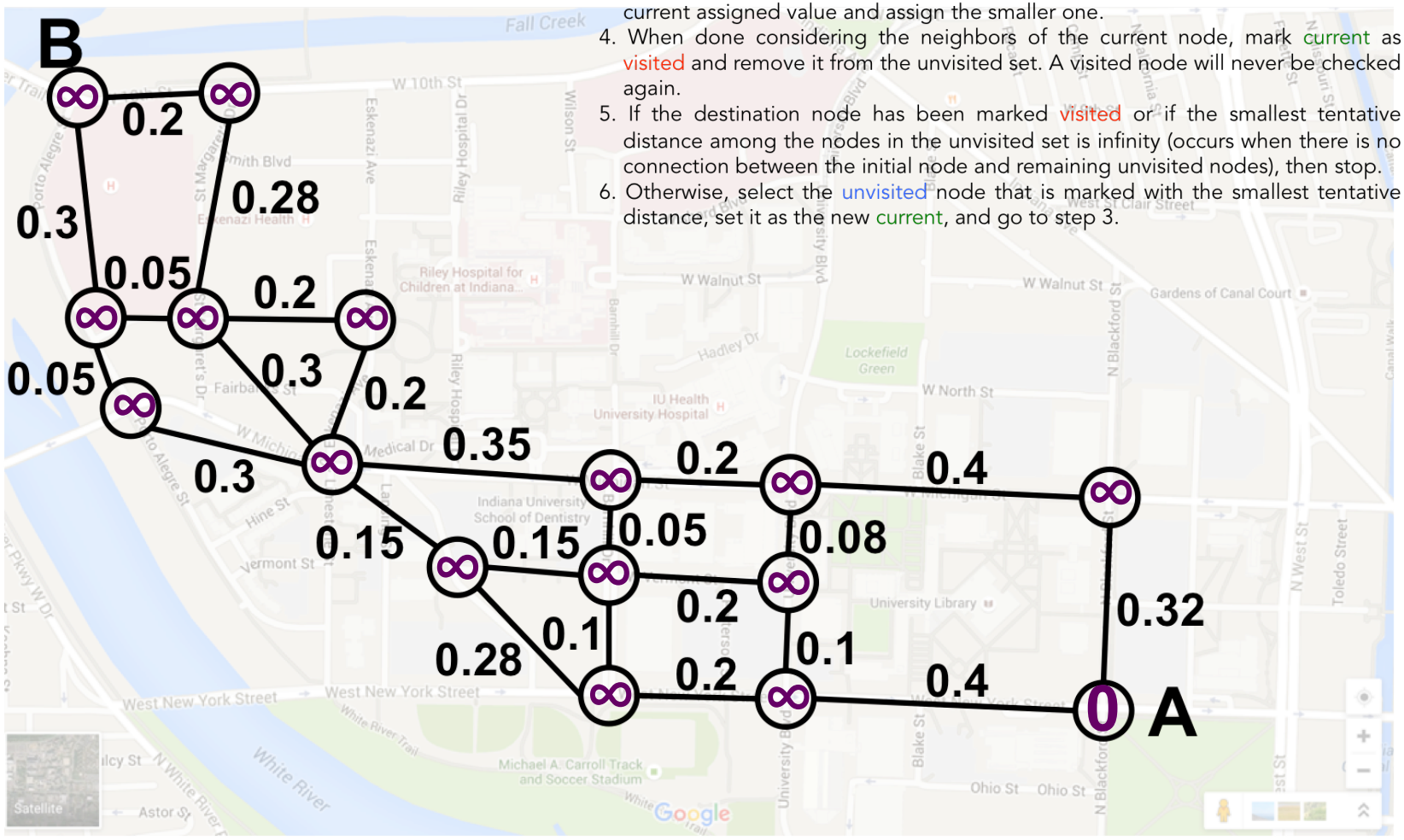

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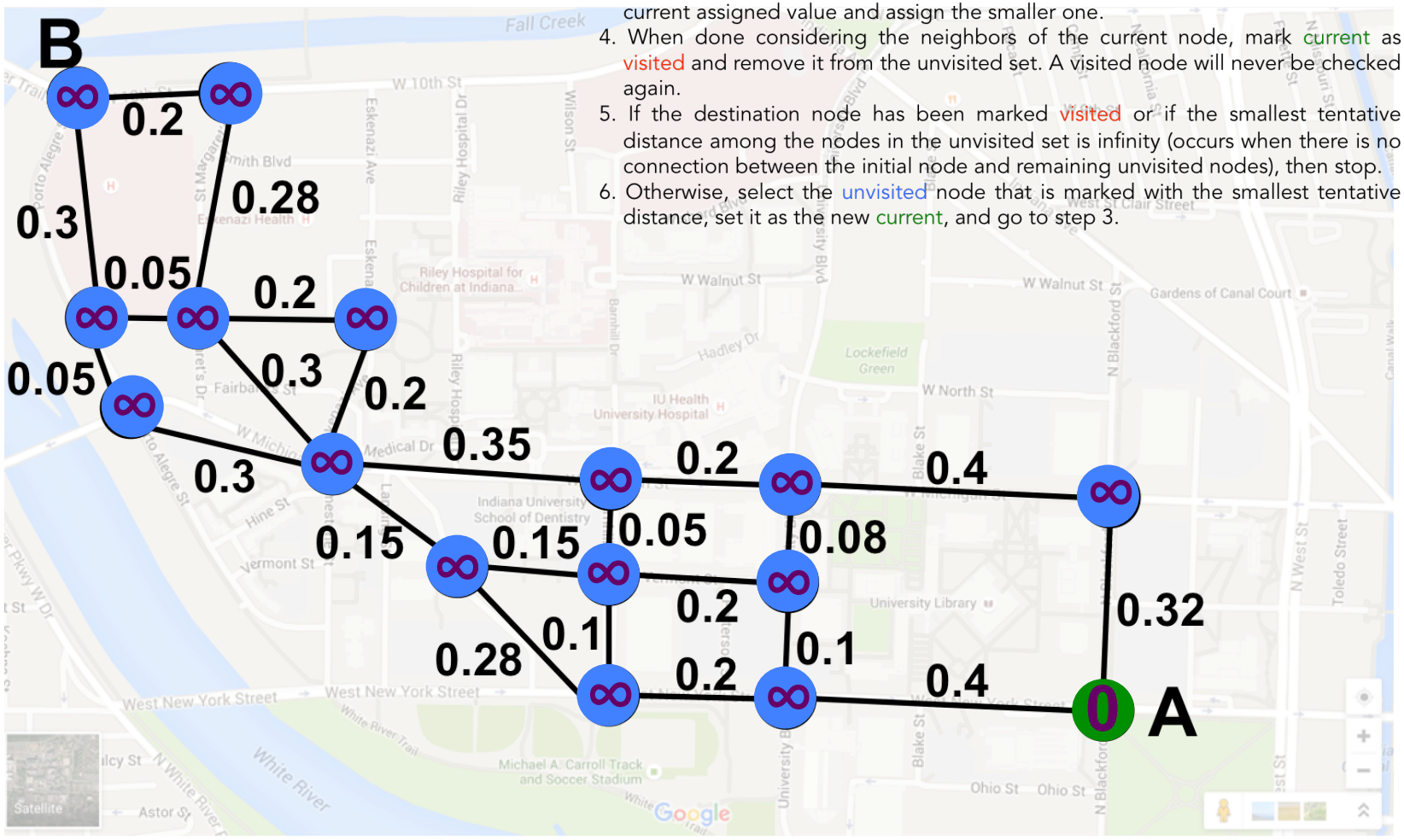

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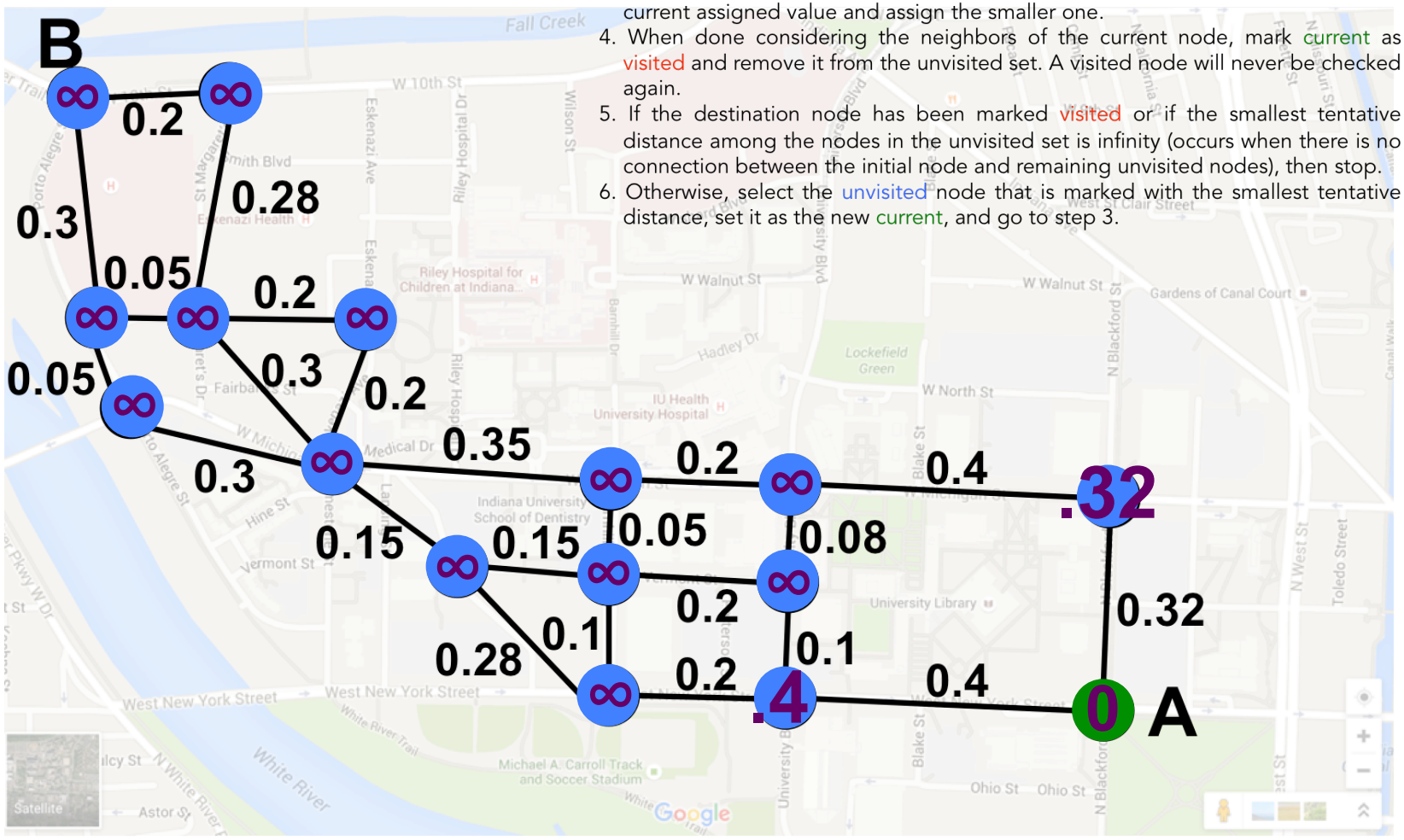

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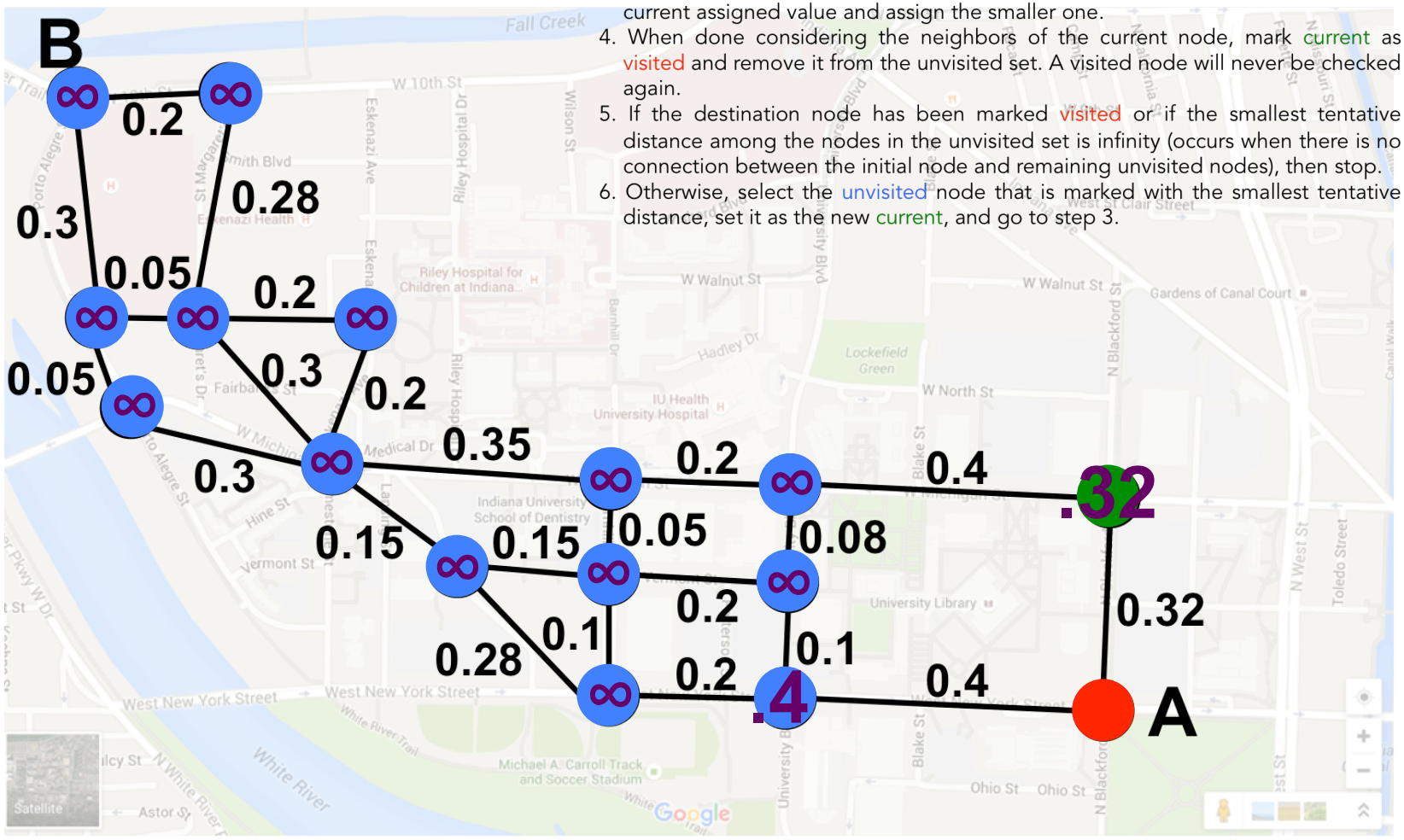

STOP & THINK

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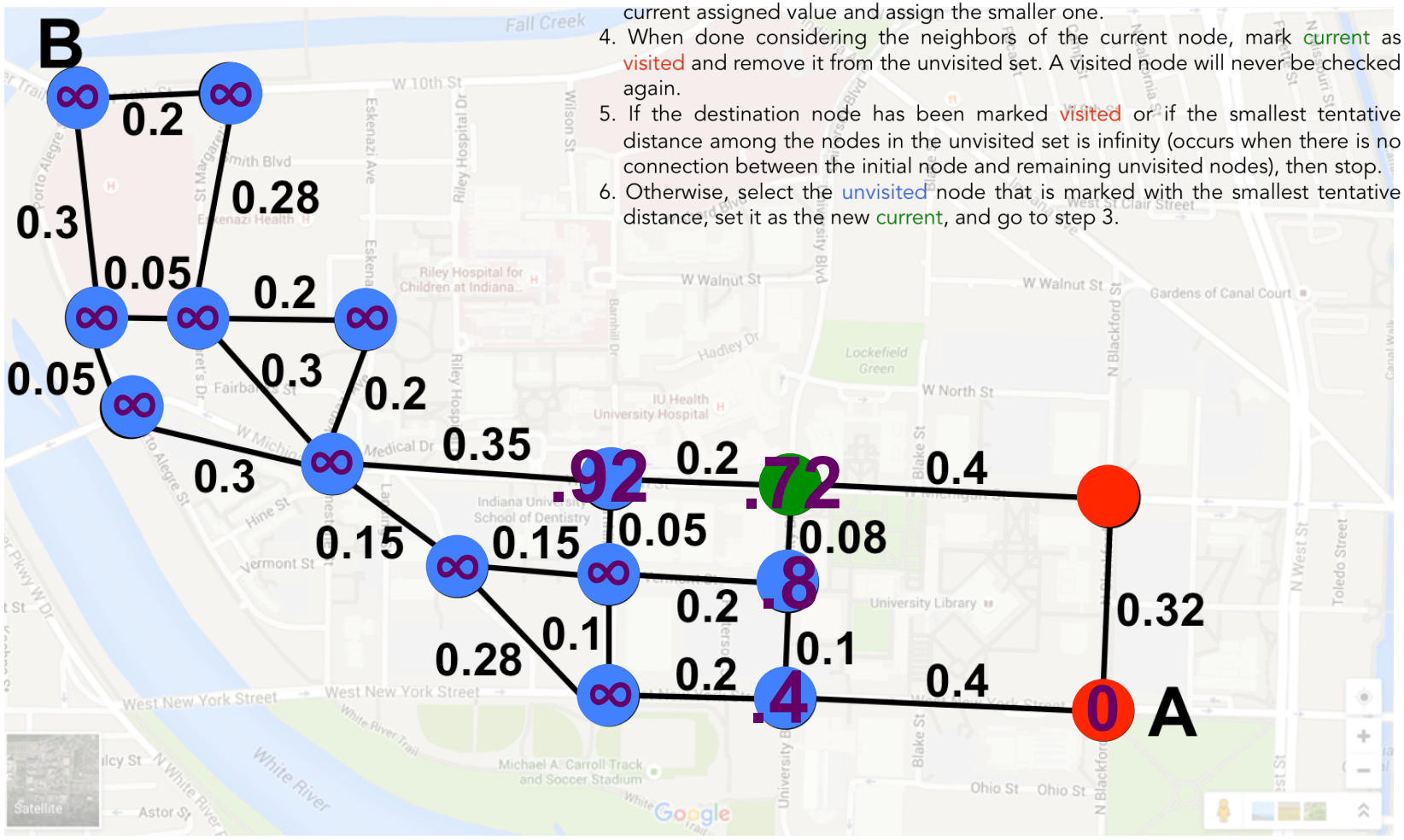

Something starts going wrong

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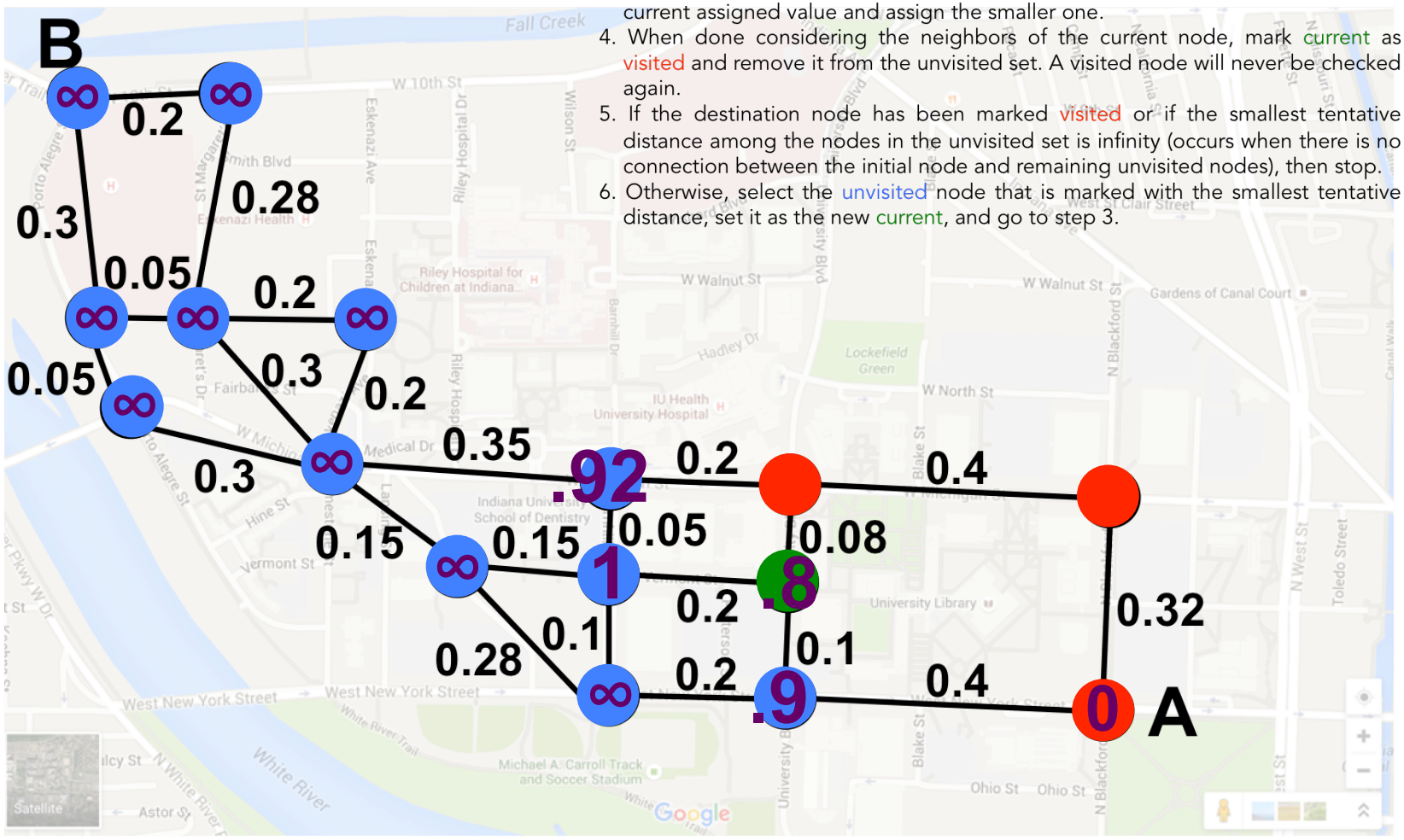

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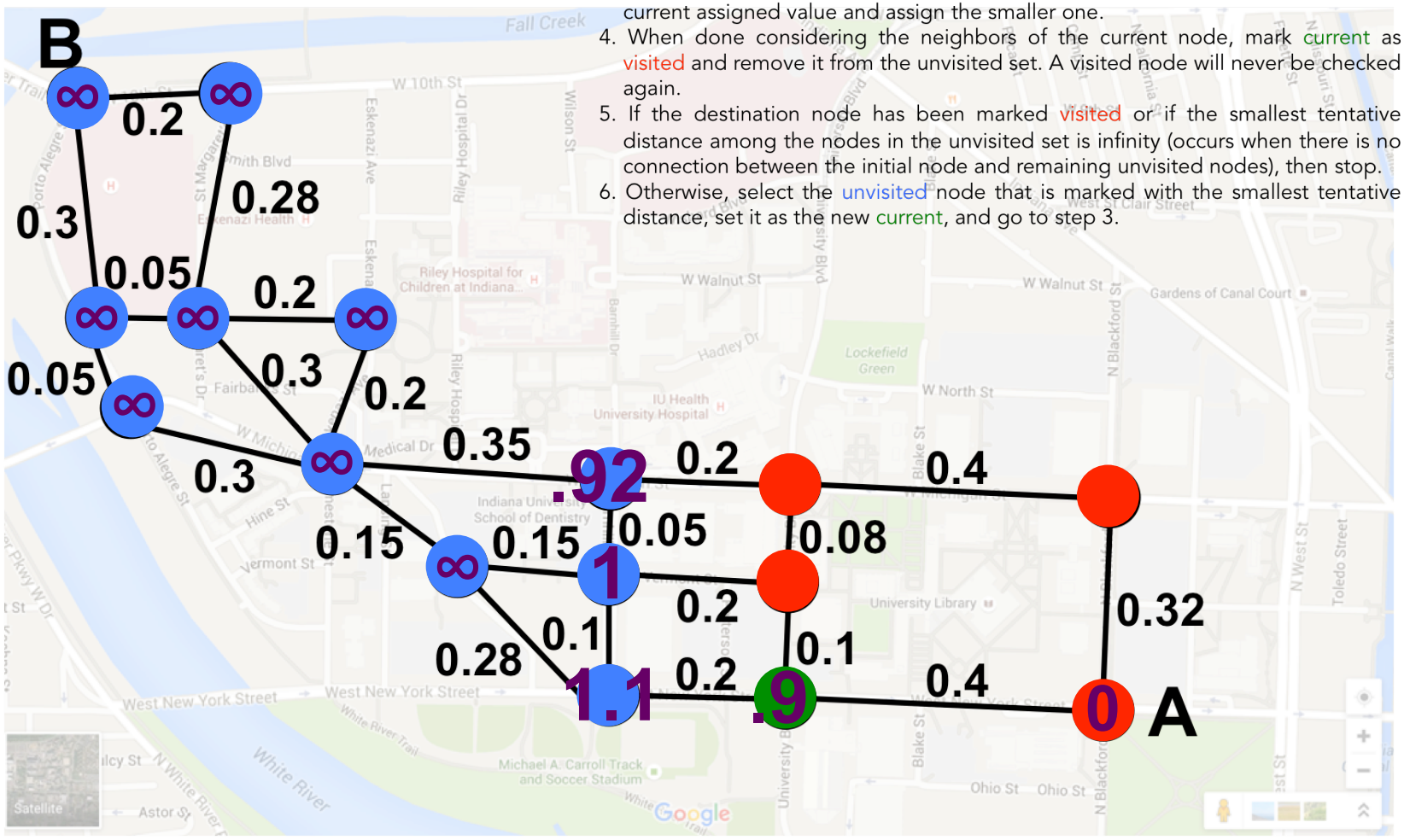

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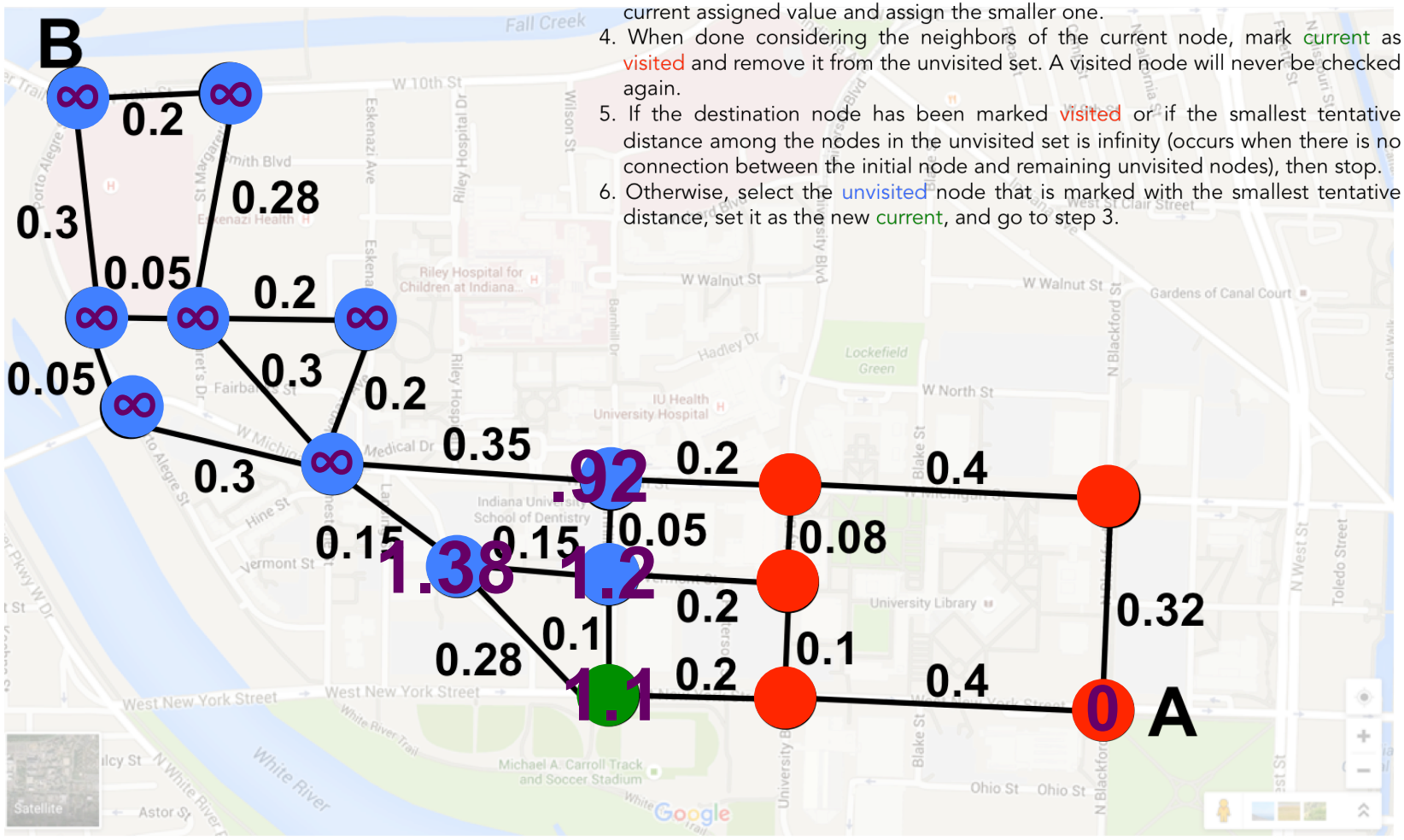

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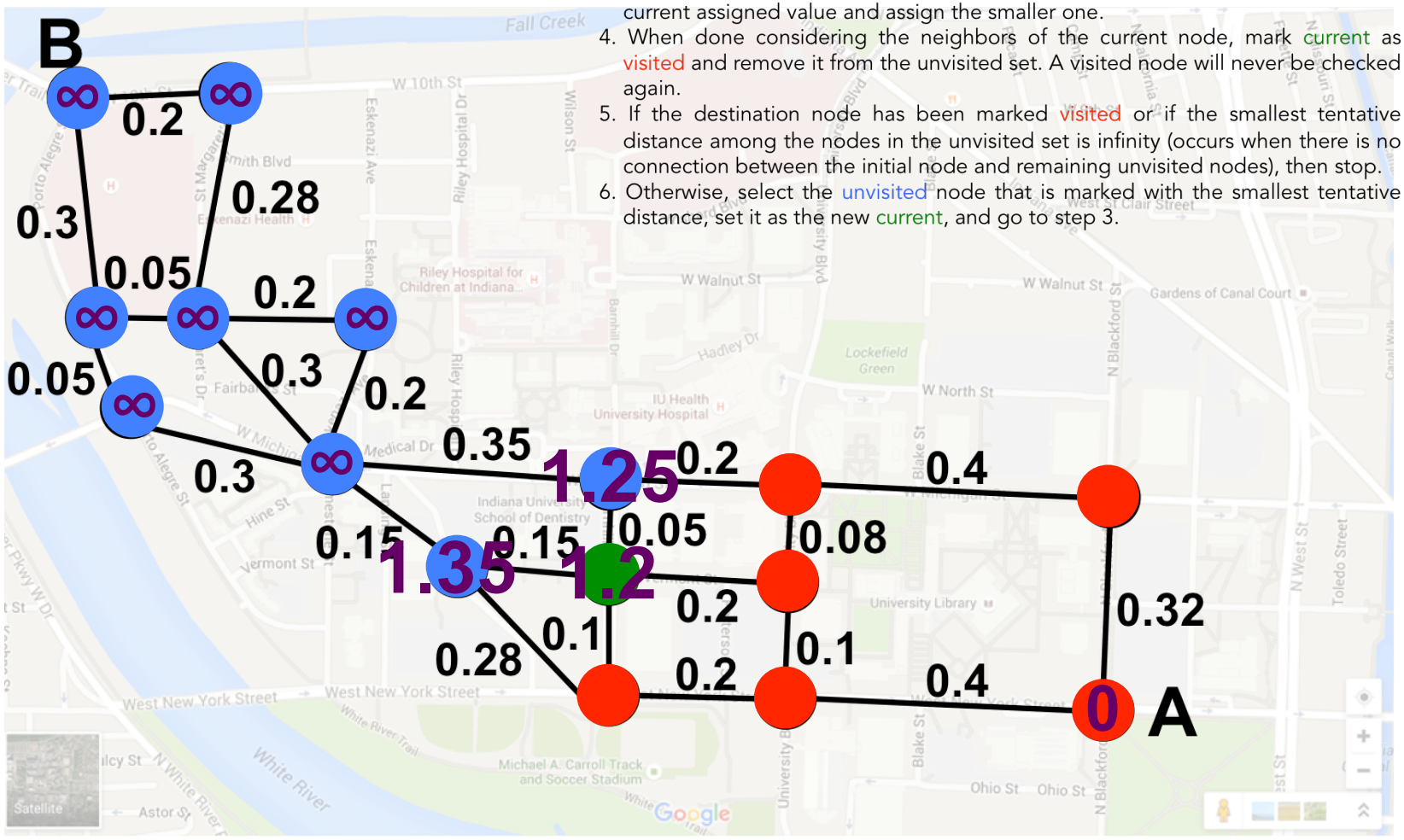

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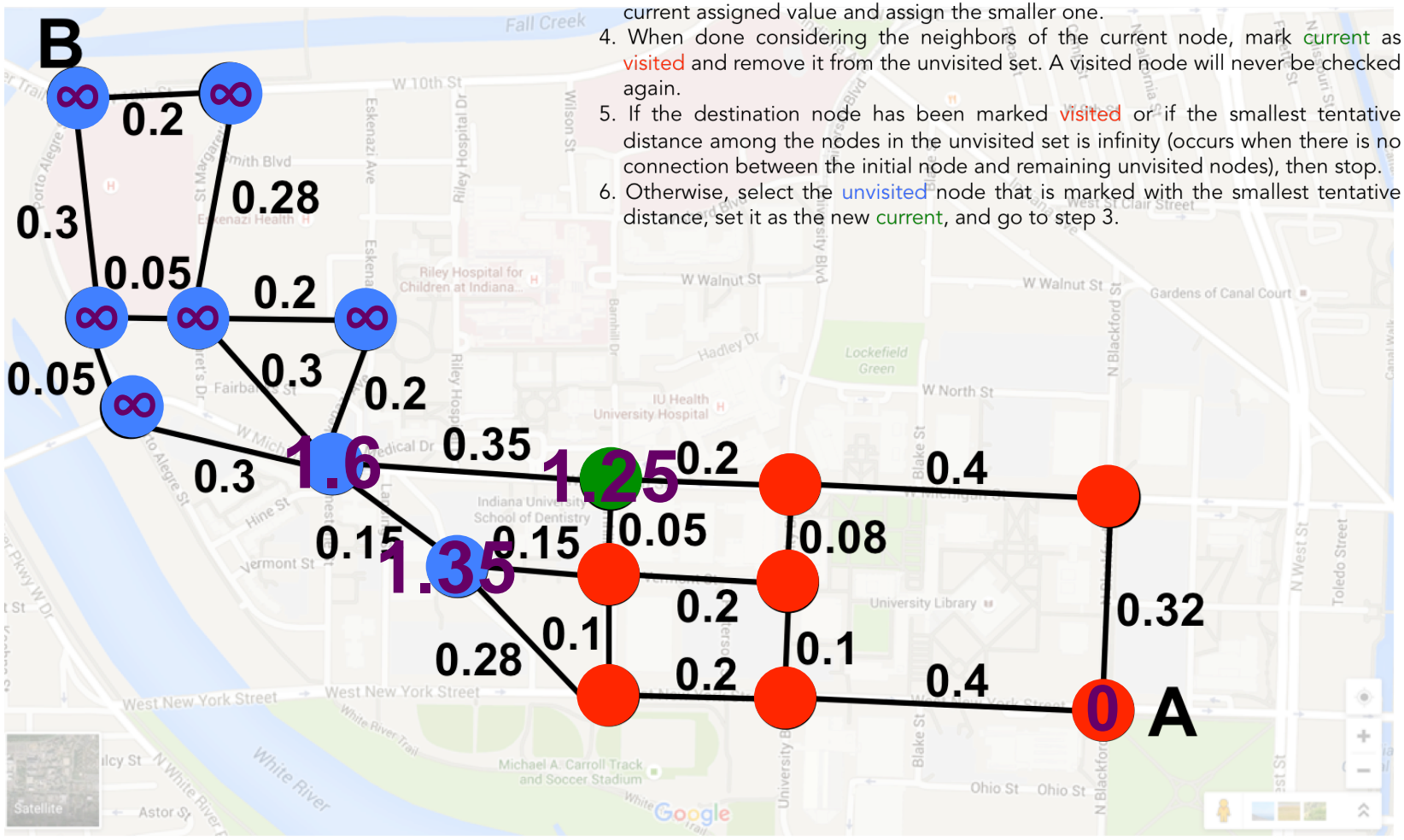

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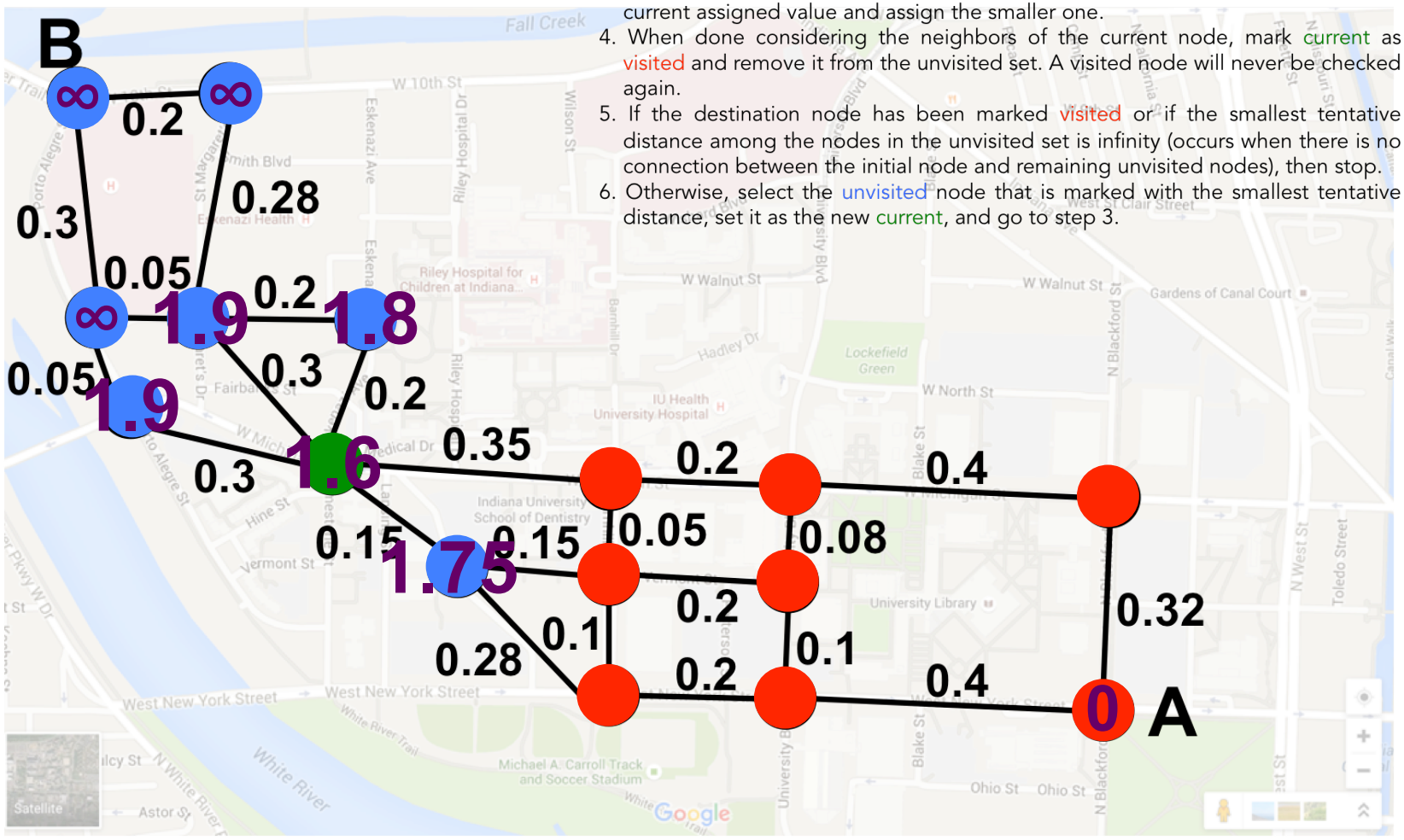

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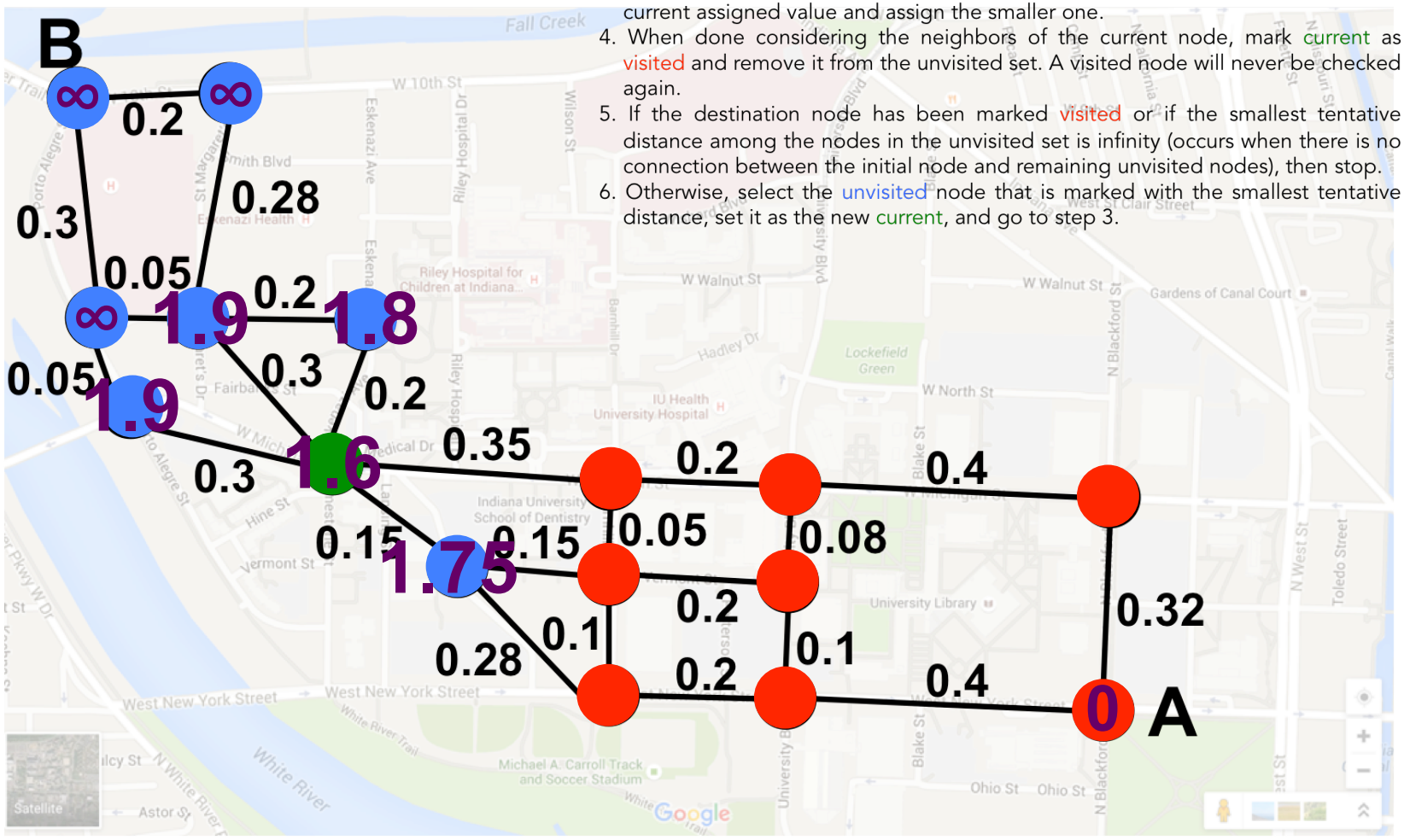

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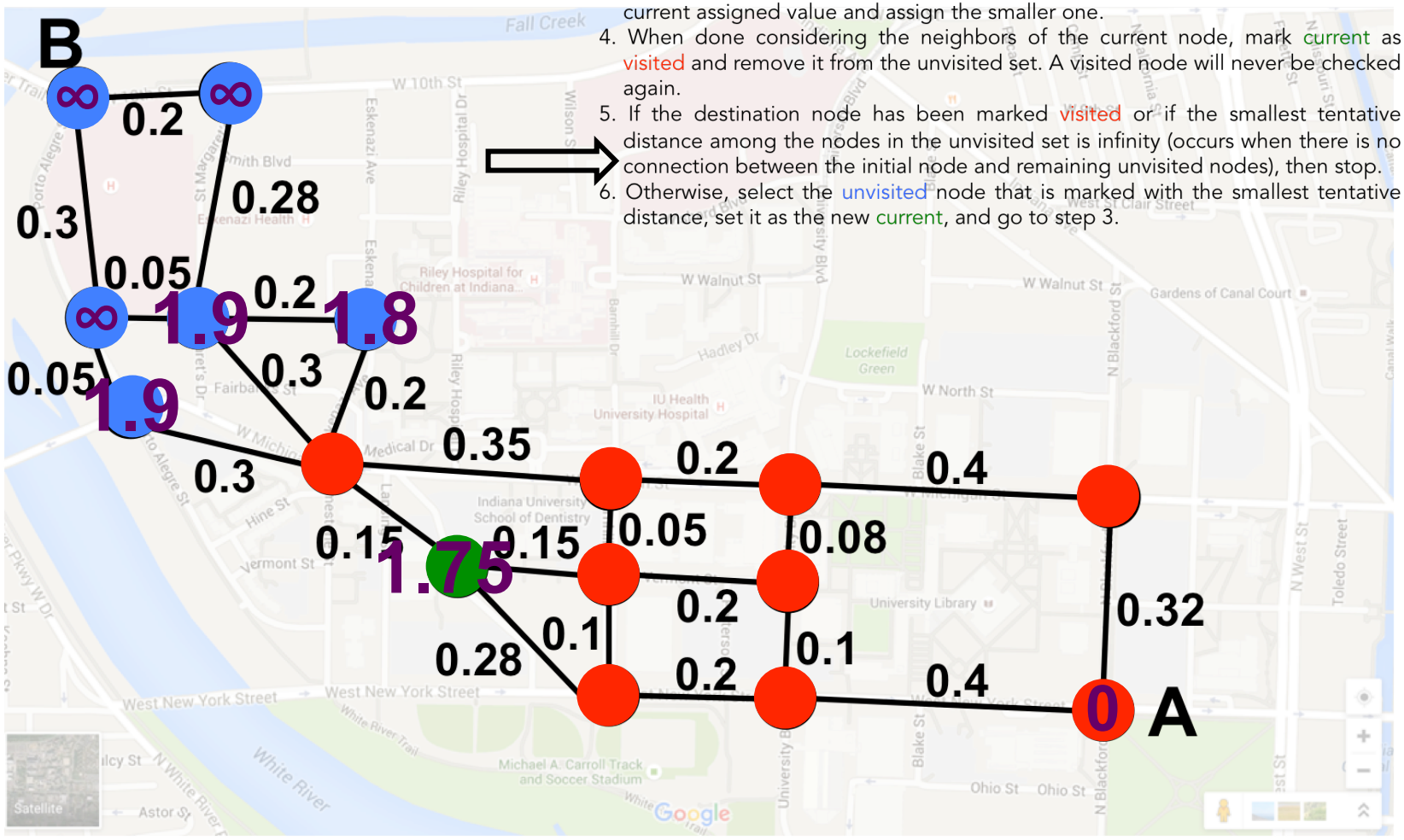

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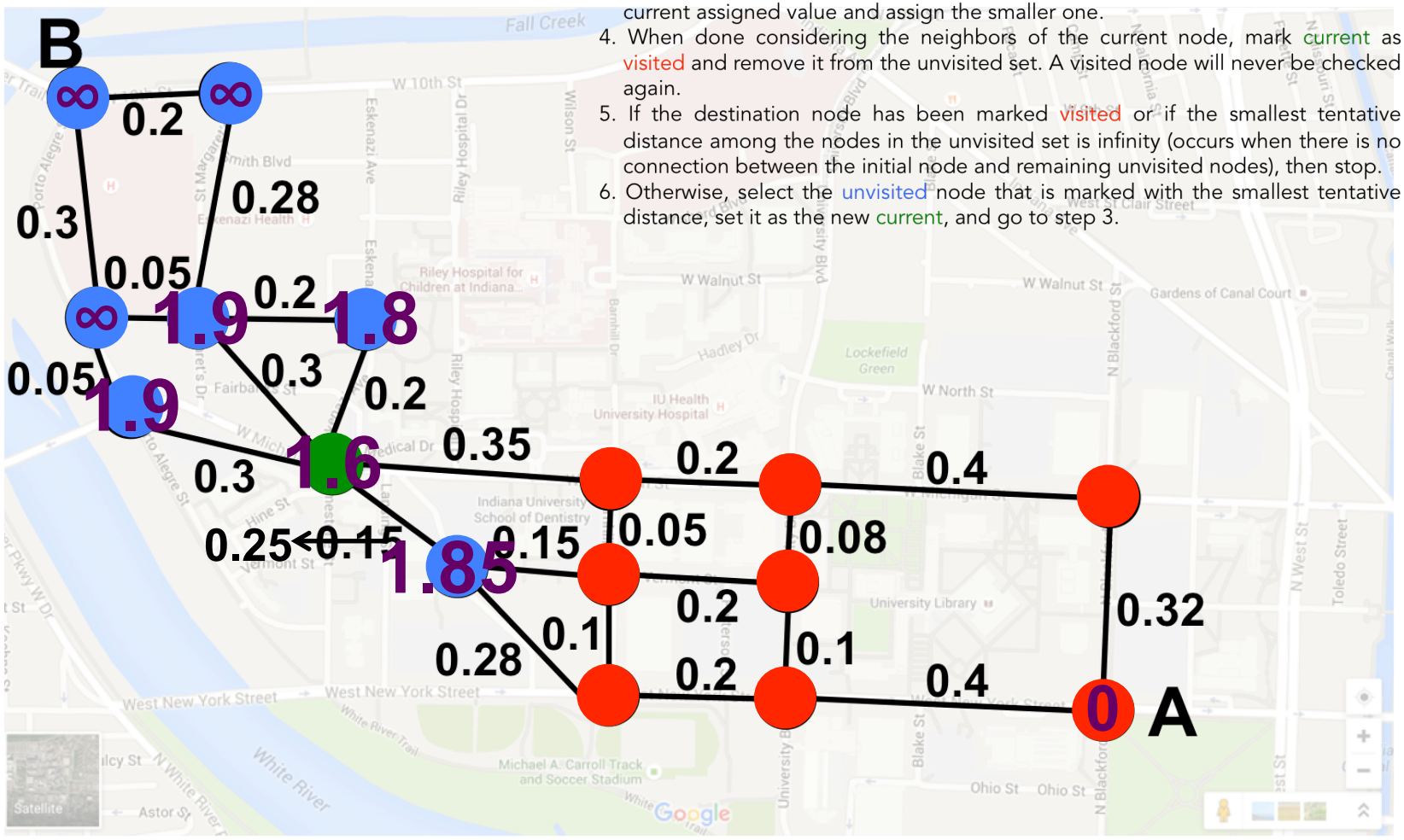

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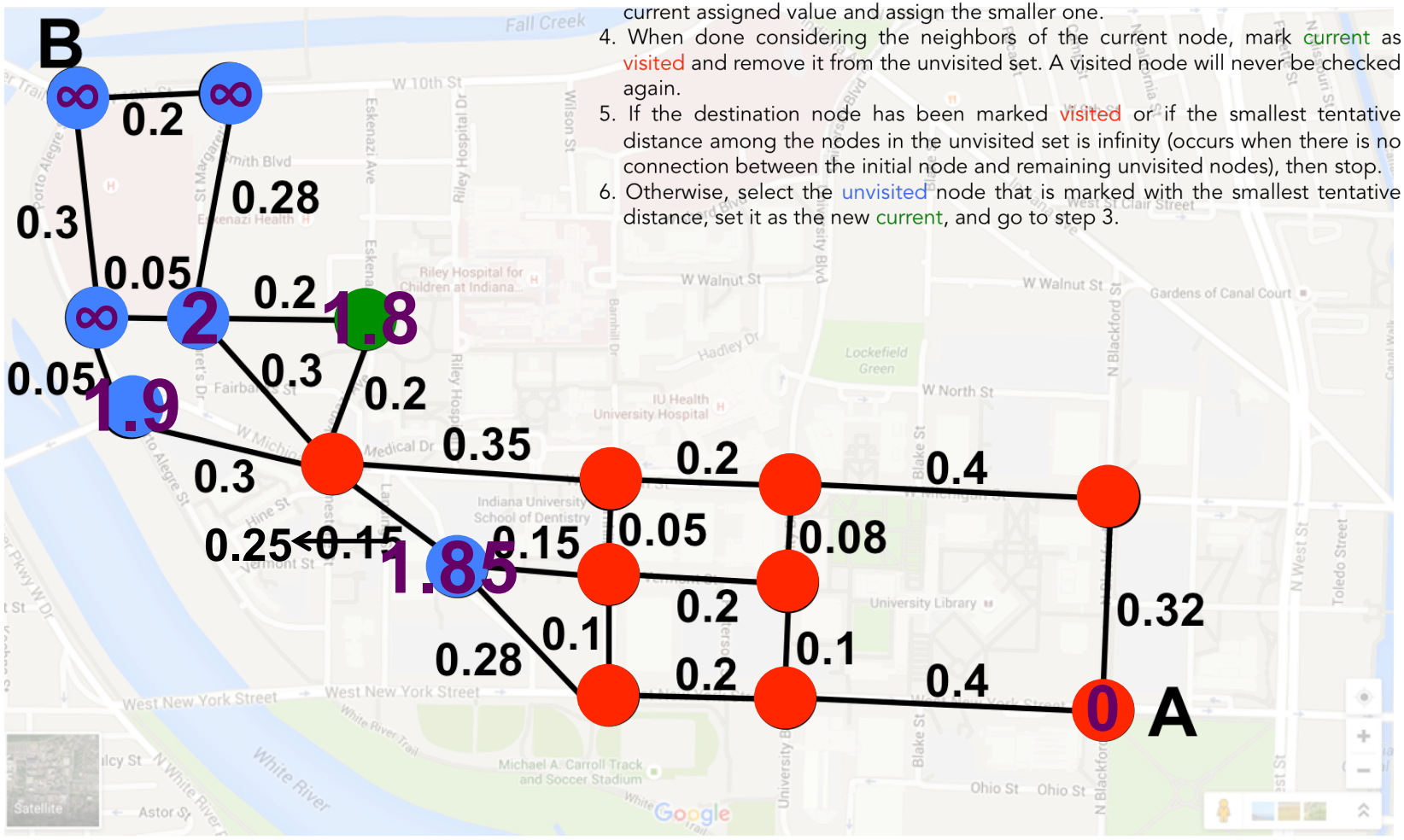

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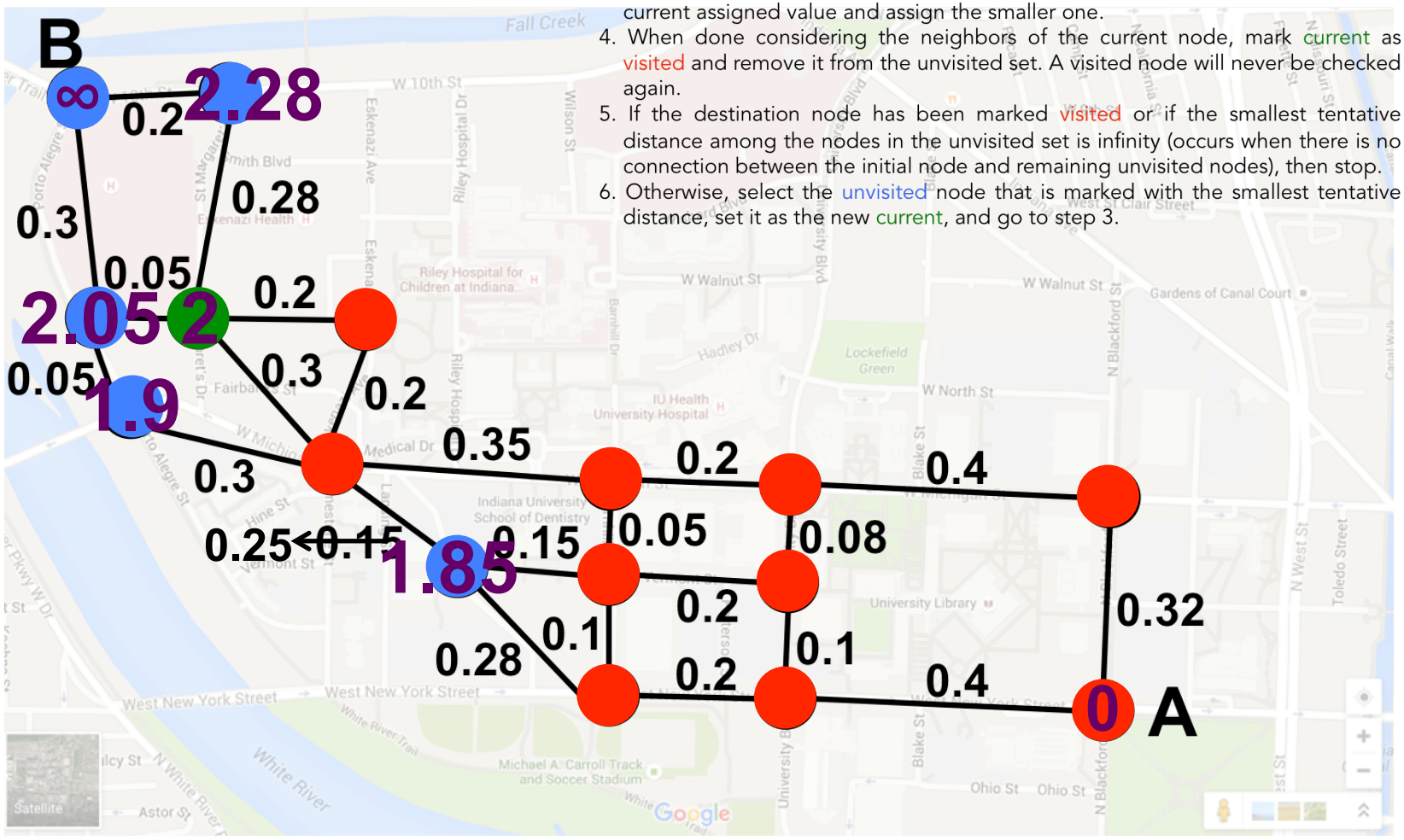

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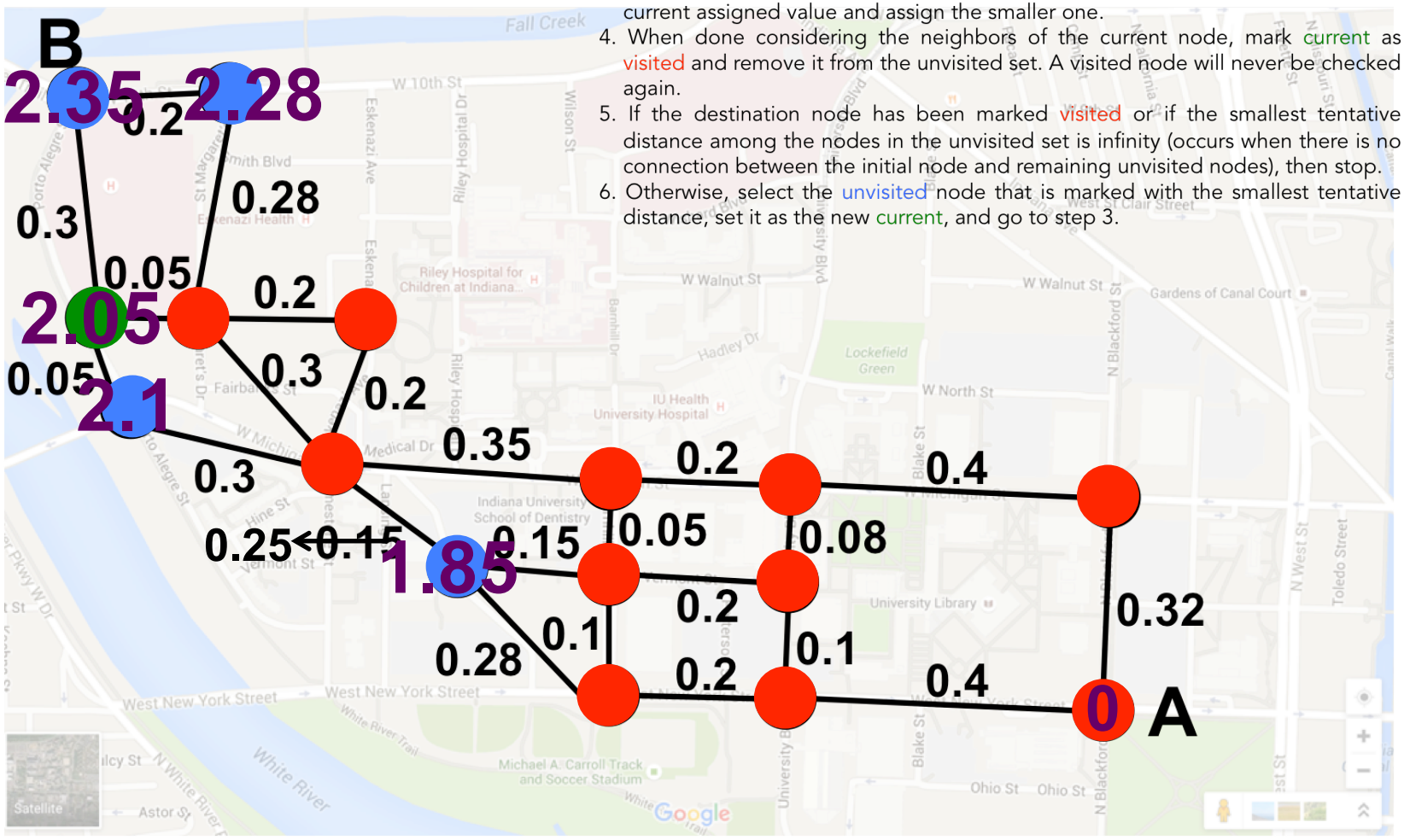

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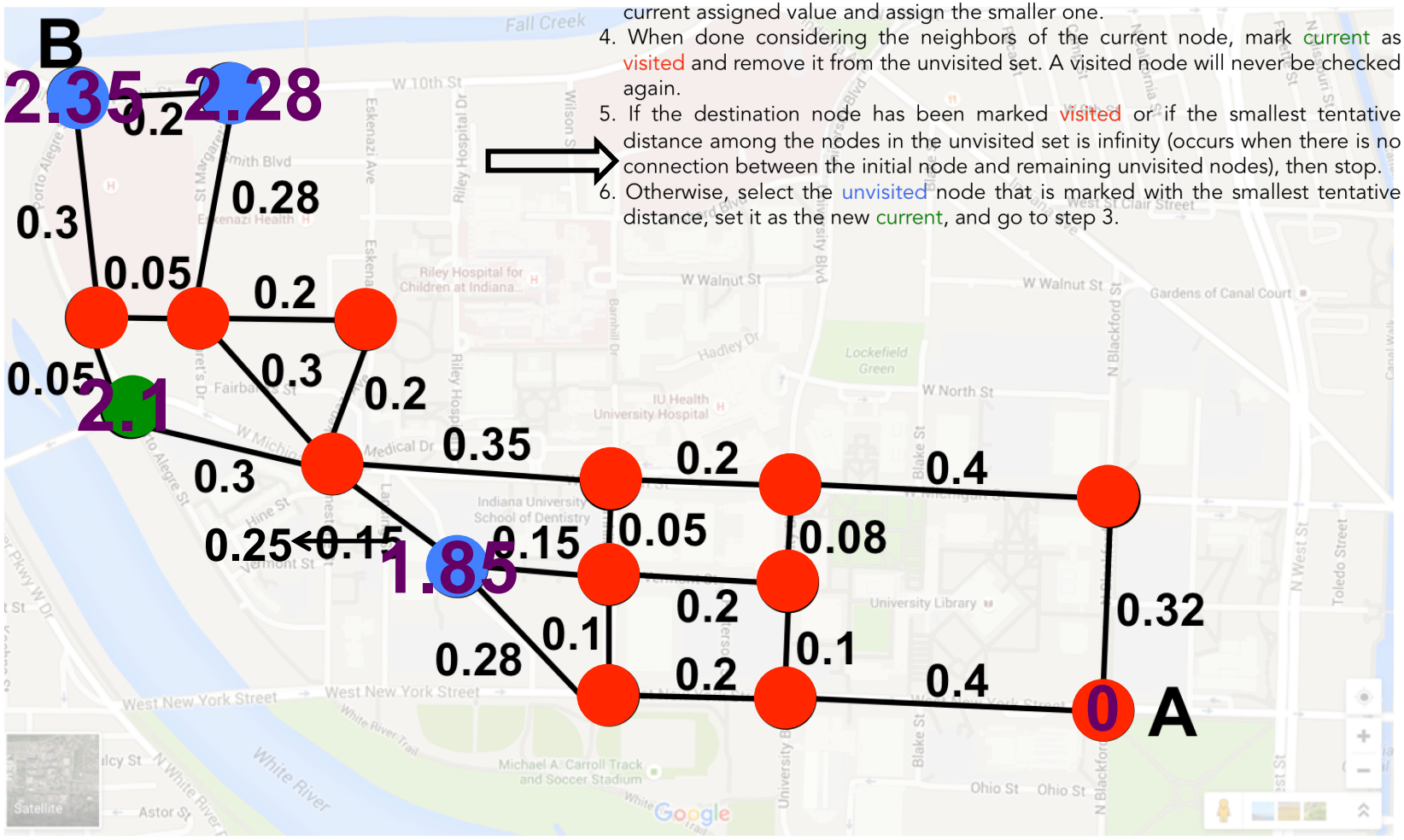

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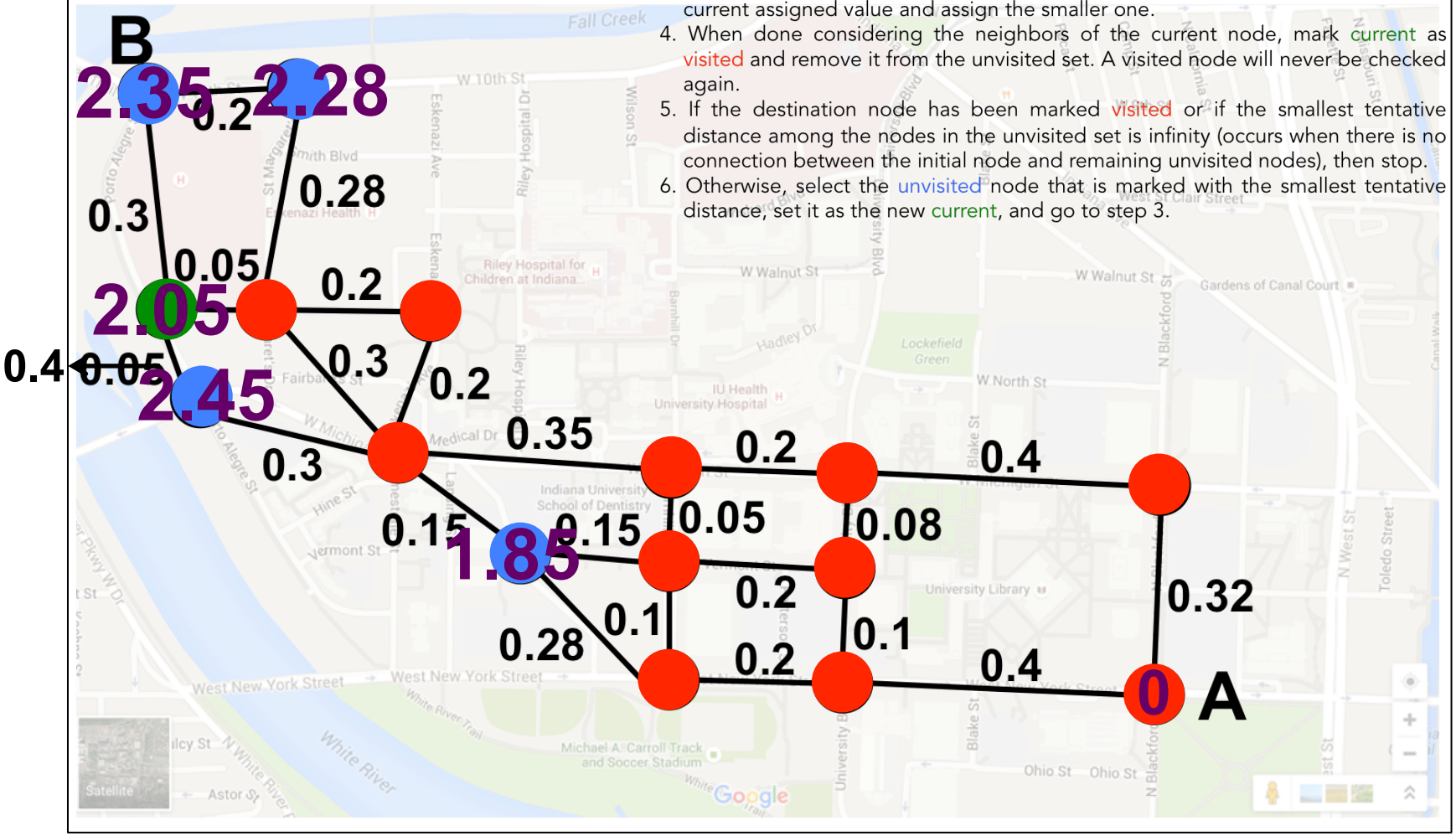

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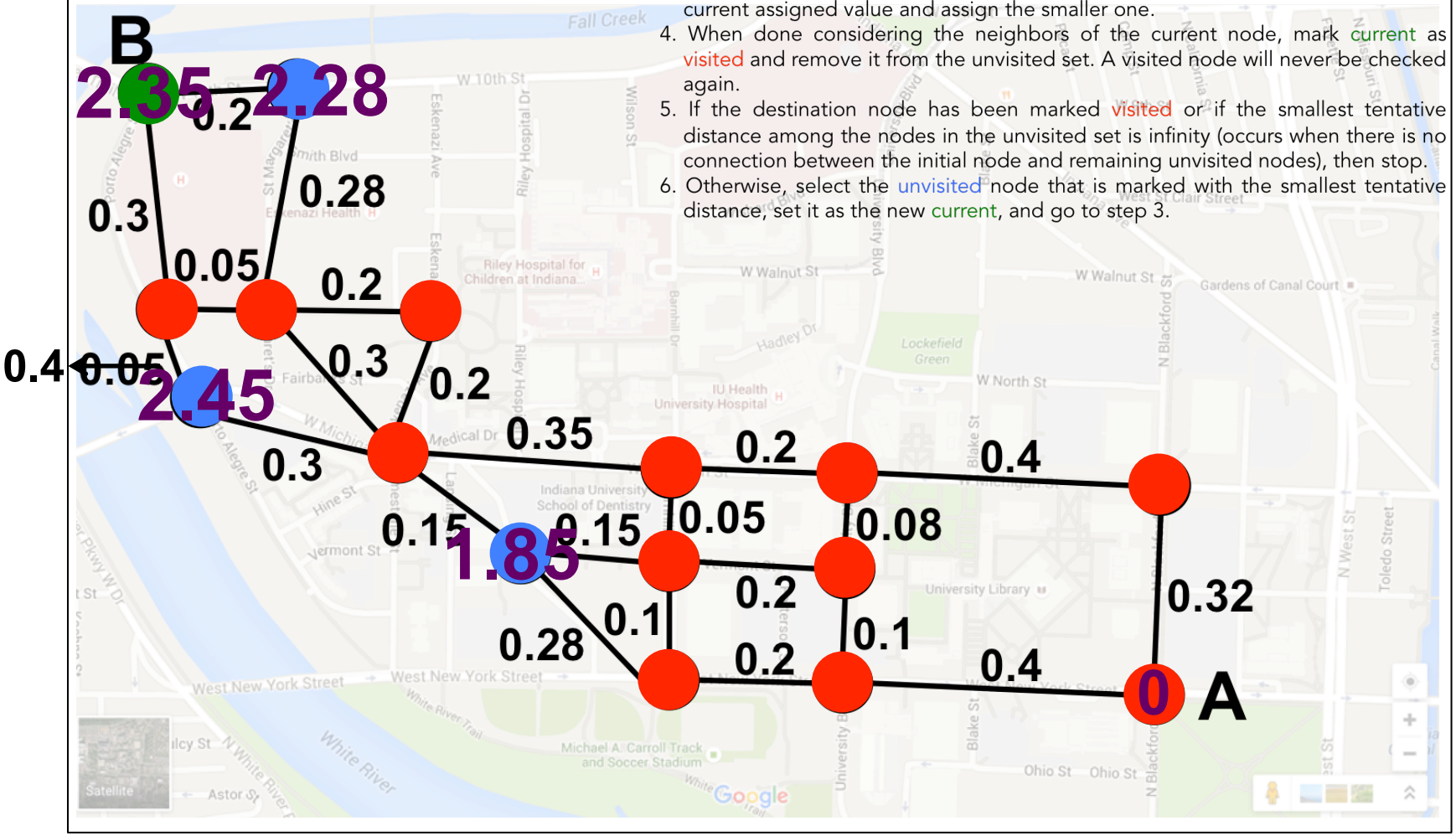

Let's go back.
STOP & THINK

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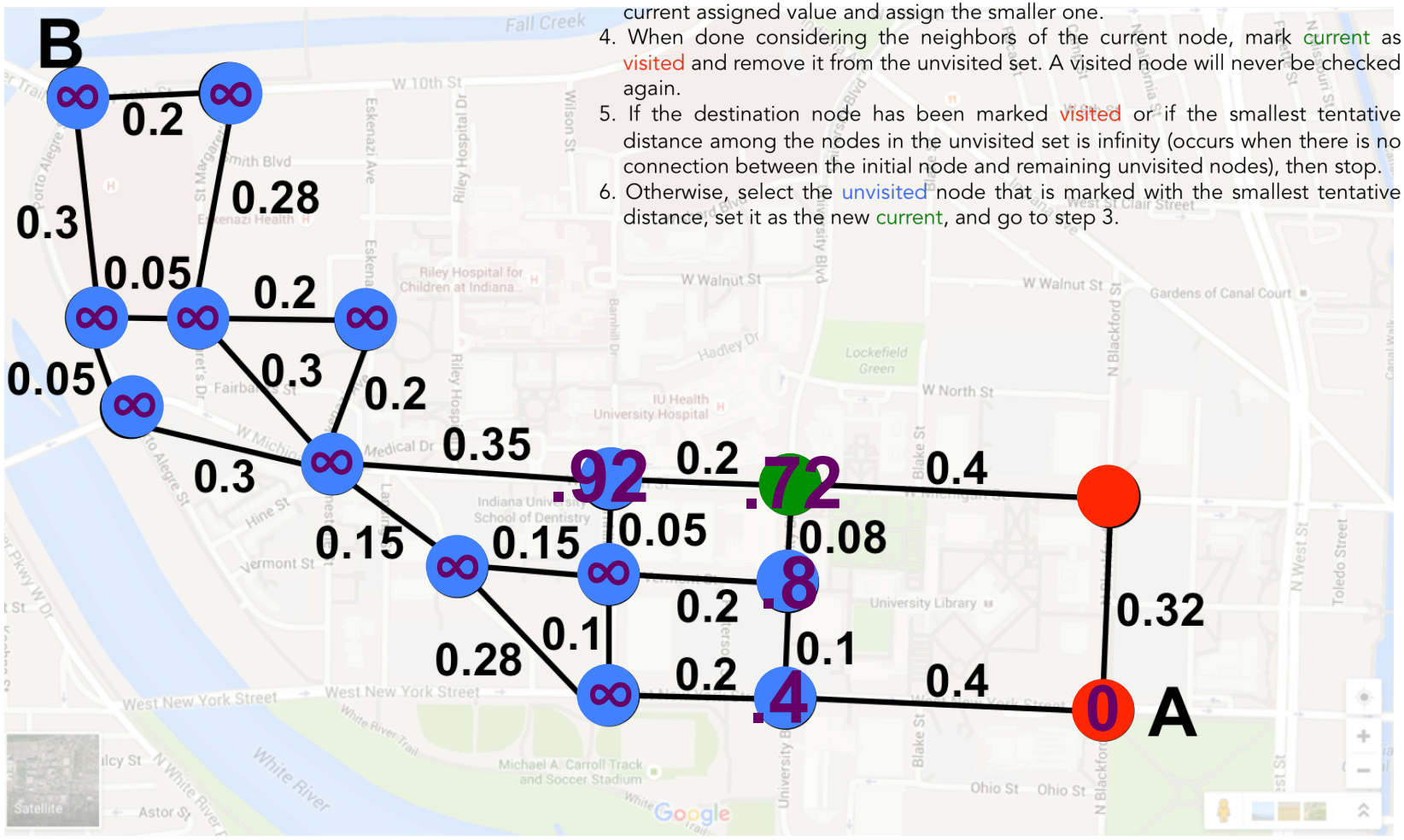

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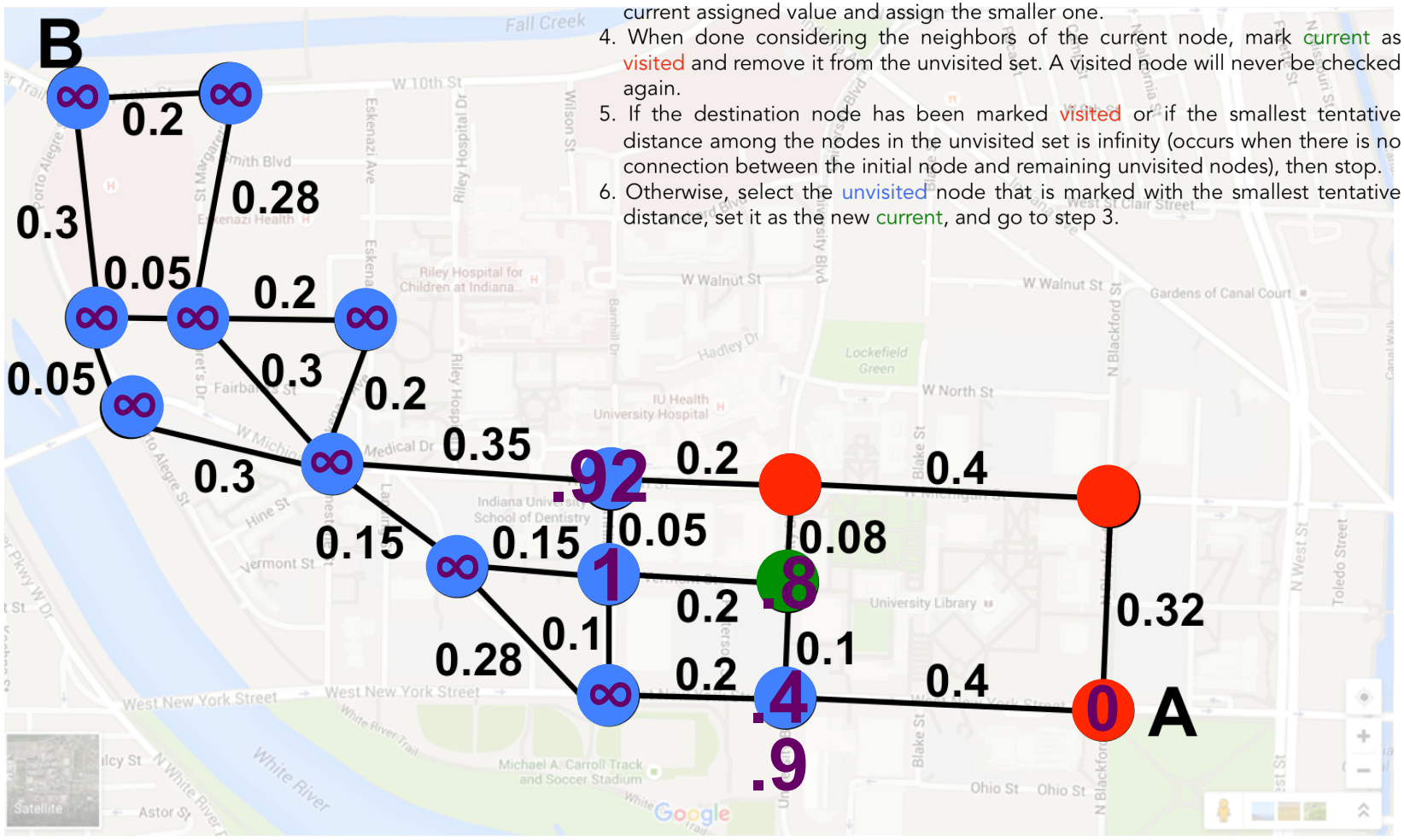

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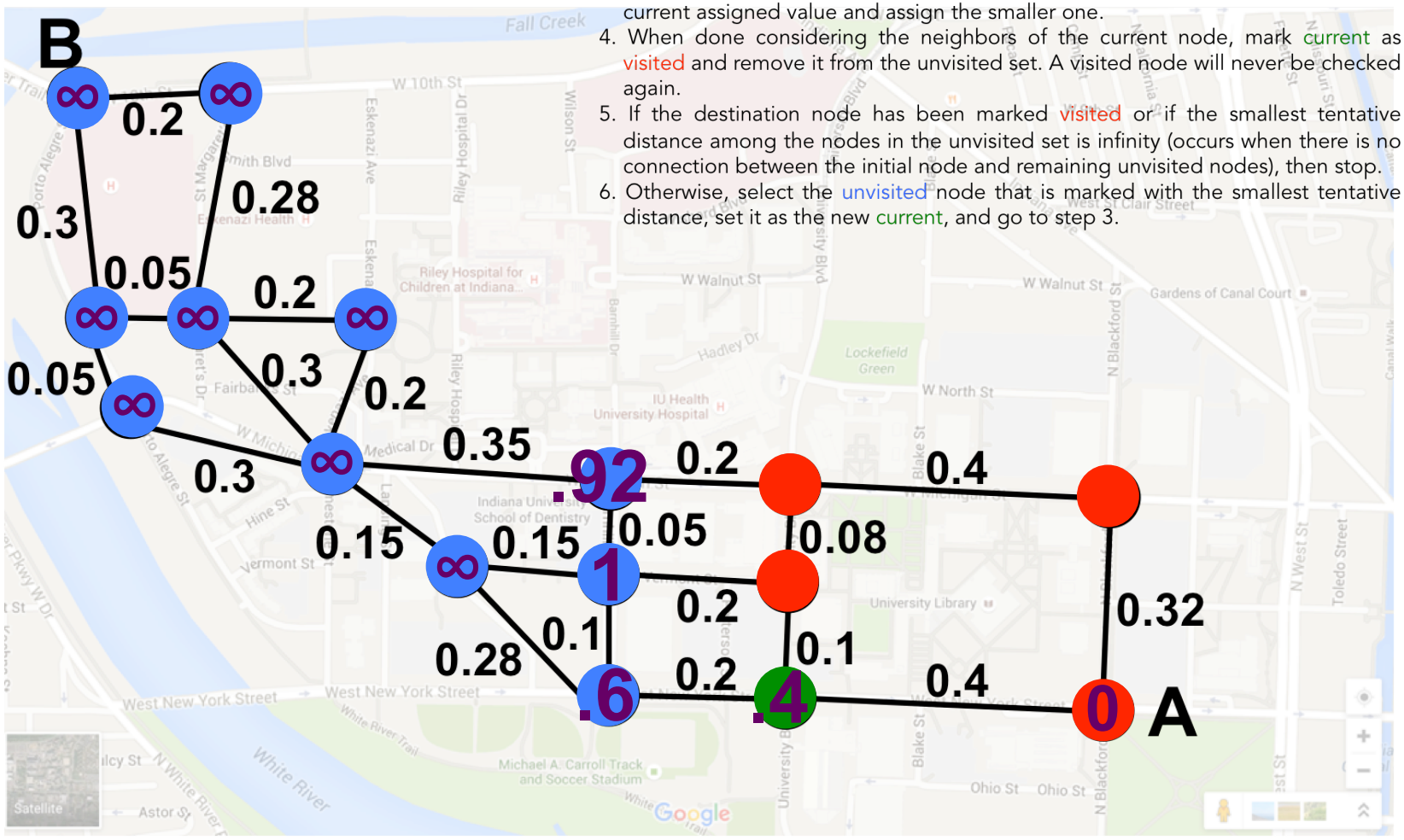

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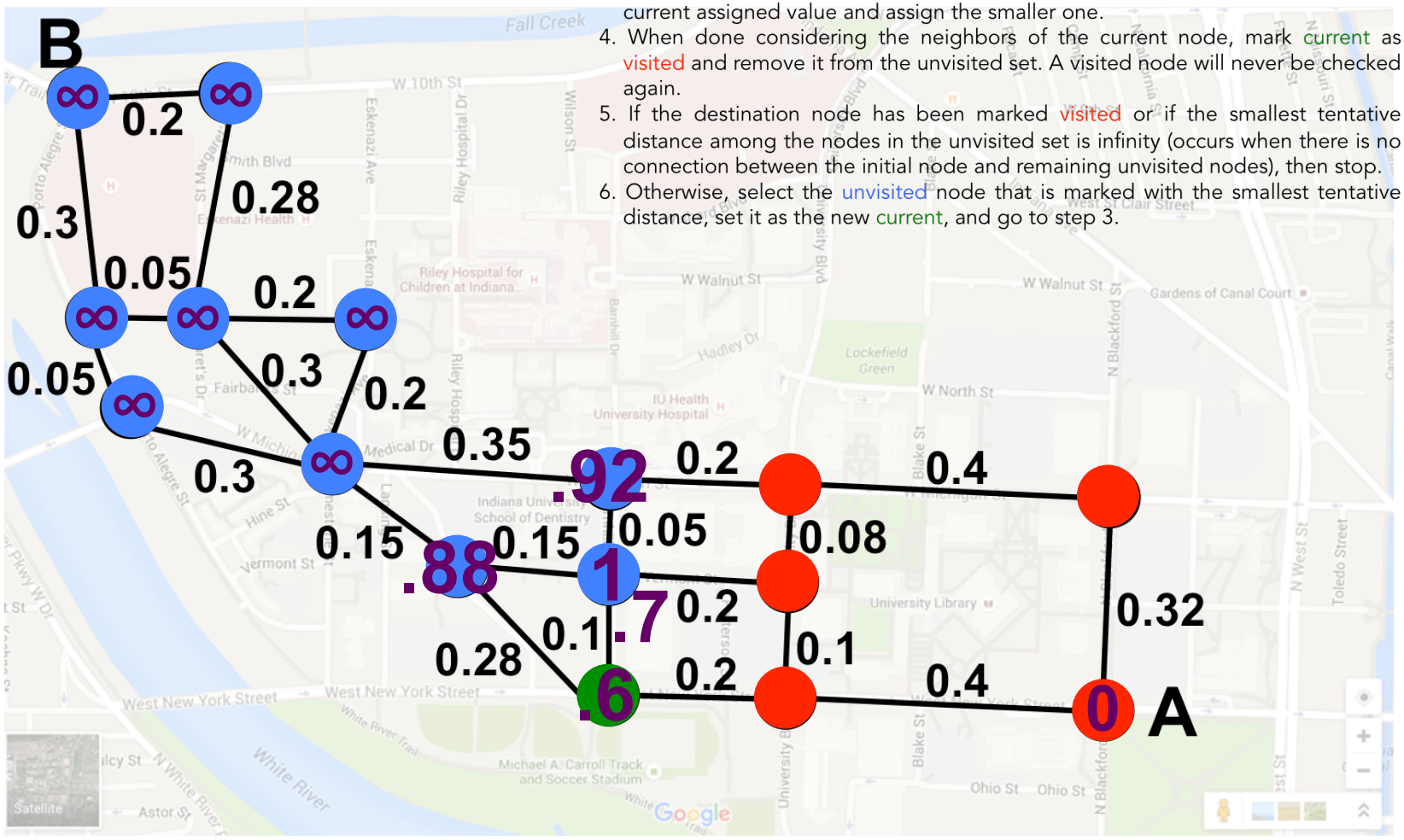

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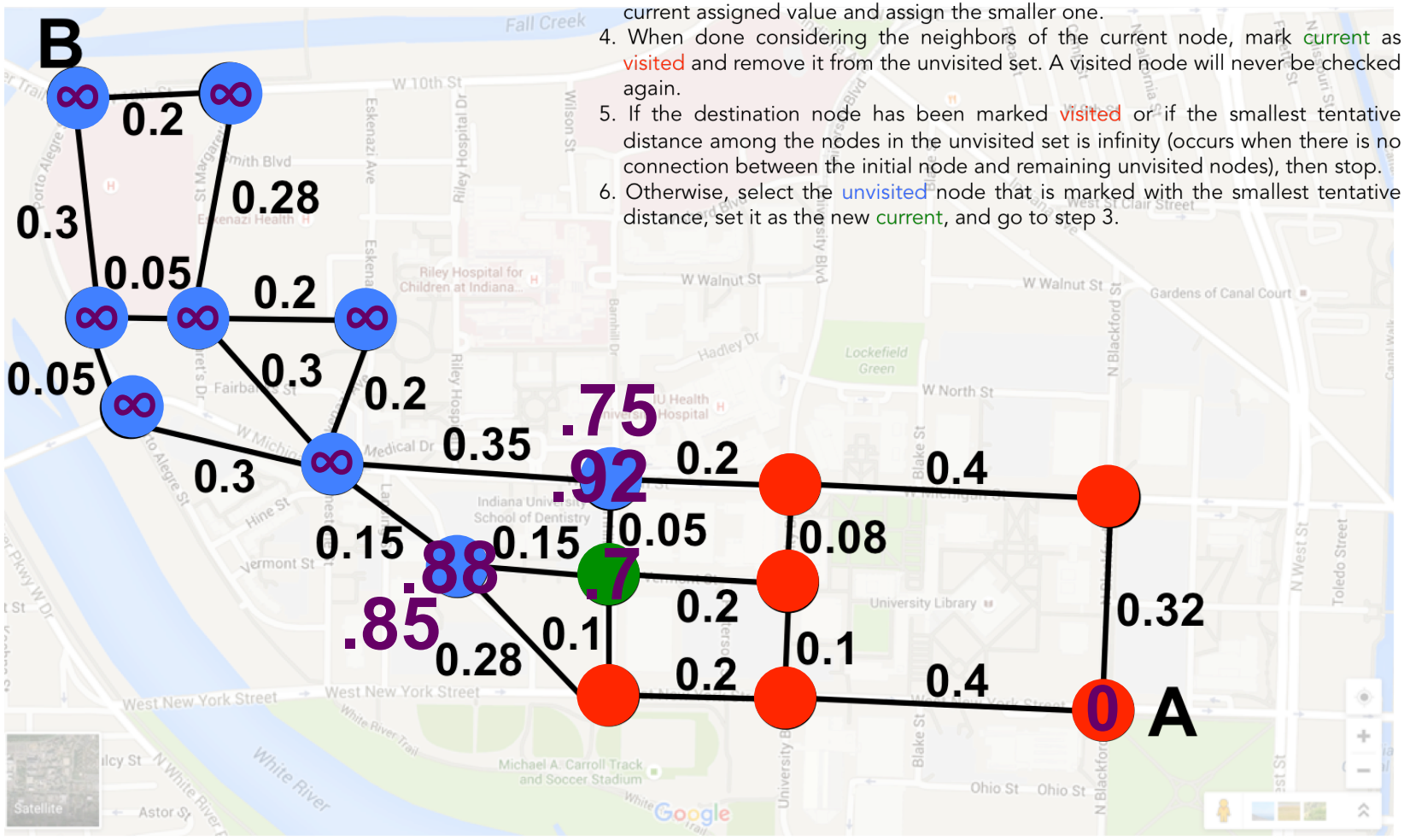

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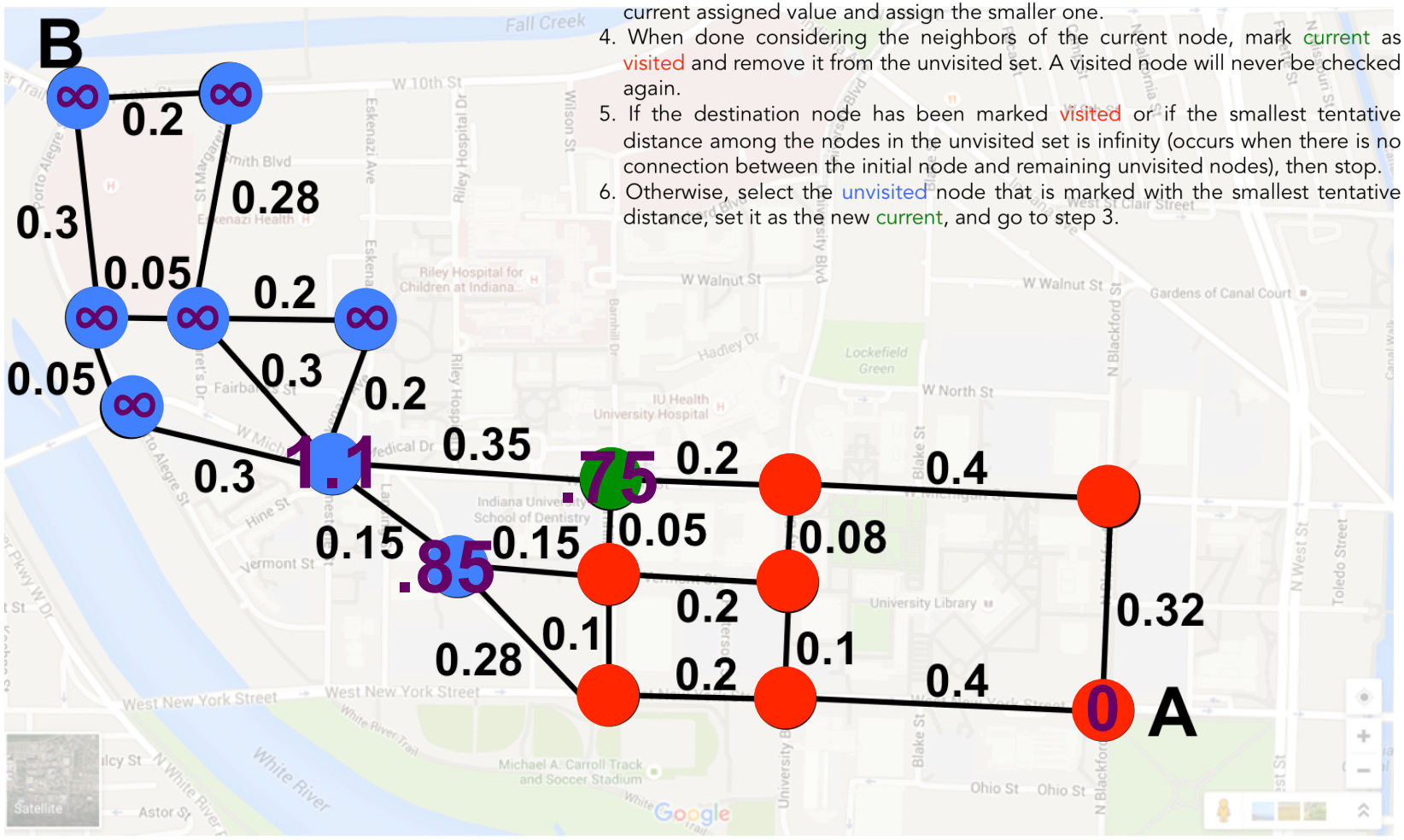

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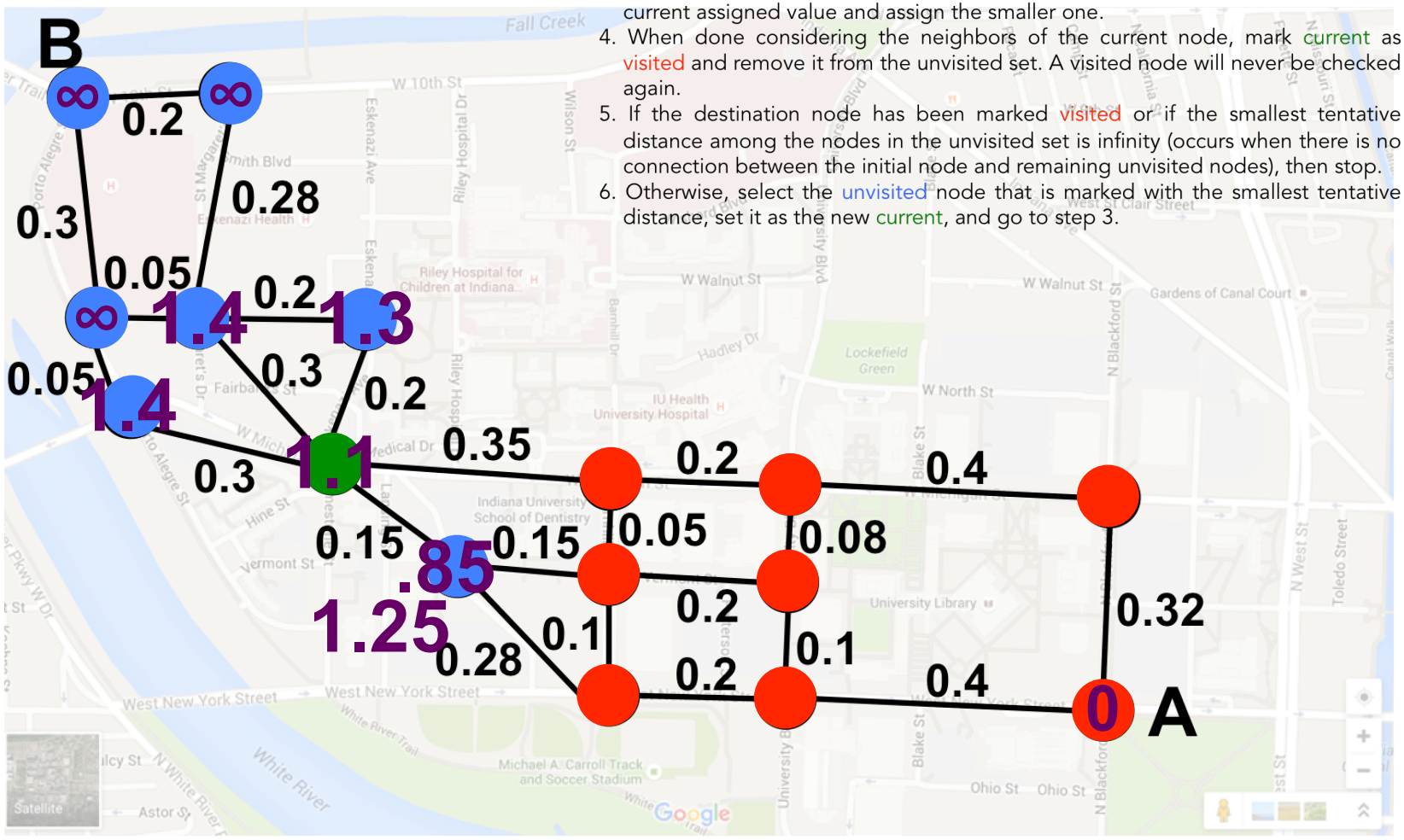

We cannot reach the target--trapped in local minima:

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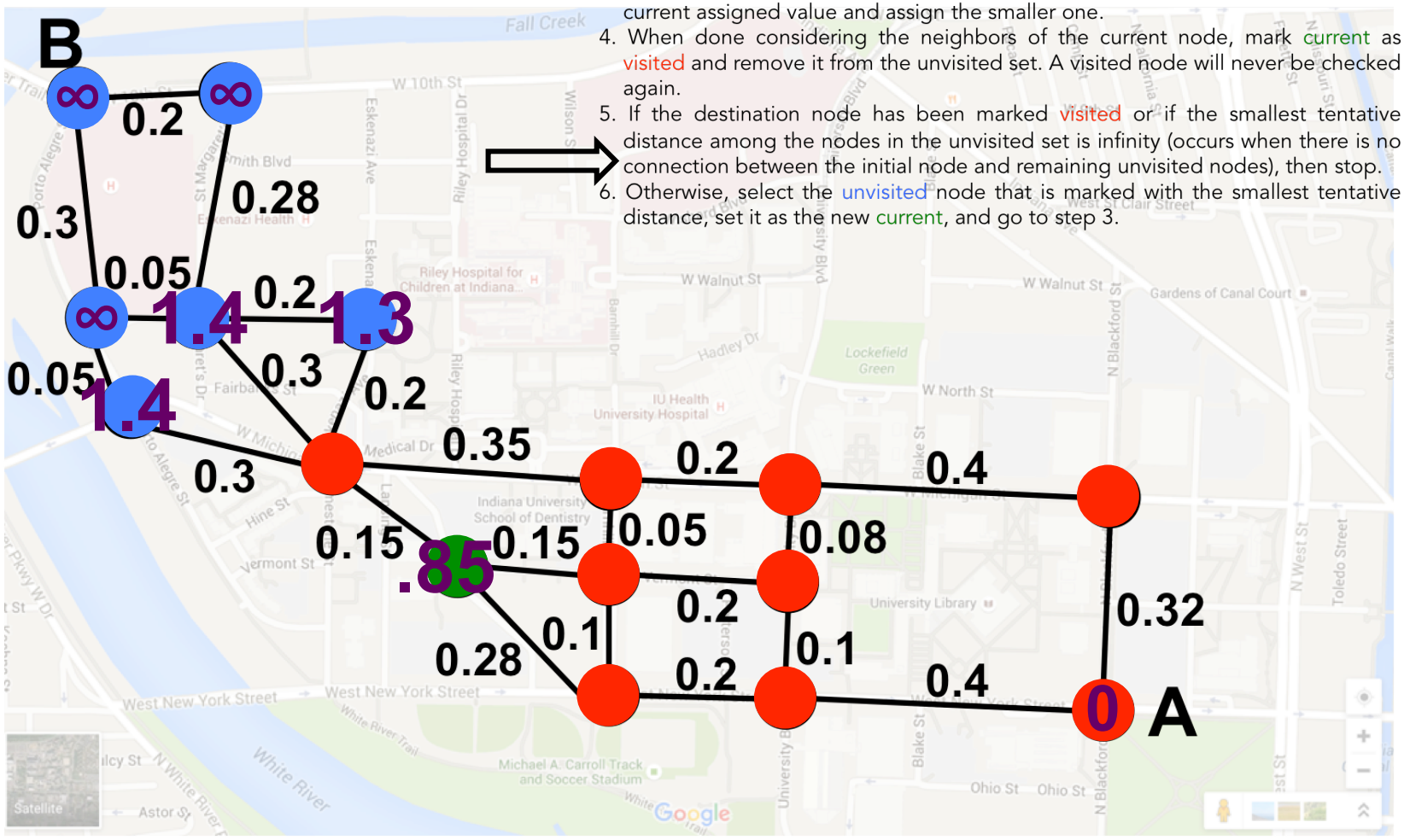
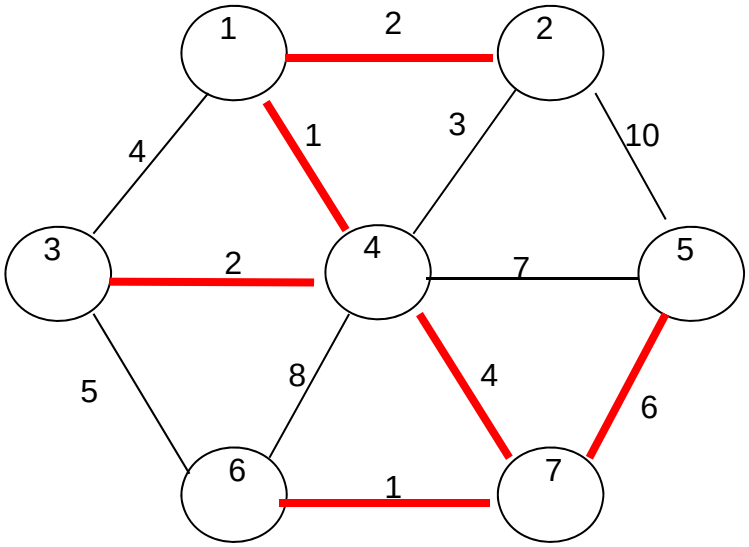


Minimum Spanning Tree

- The problem
  - Consider undirected graph only
  - *Spanning tree*: a tree by graph edges that connect all the vertices of the graph.
  - *Minimum spanning tree*: a spanning tree with the minimum total edge cost.
  - Minimum spanning tree of graph G exists if and only if G is connected.



remember: a graph is connected if there is a path from every vertex to every other vertex.

– Prim's algorithm
  - a greedy algorithm
  - basic steps: starting from an arbitrary vertex as the root. At any point of the algorithm, there is a set of vertices that can be included in the tree, and the rest are not. The algorithm, at each stage, find a new vertex, v, to add to the tree such that the cost of (u, v) is the smallest among all edges where u is in the tree and v is not.
  - The same process as the greedy algorithm in shortest path problem

    a.) the graph is undirected

    b.) the "dist" is defined as the shortest cost of edge (u, v), with u currently in the tree (known) and v currently not (unknown)

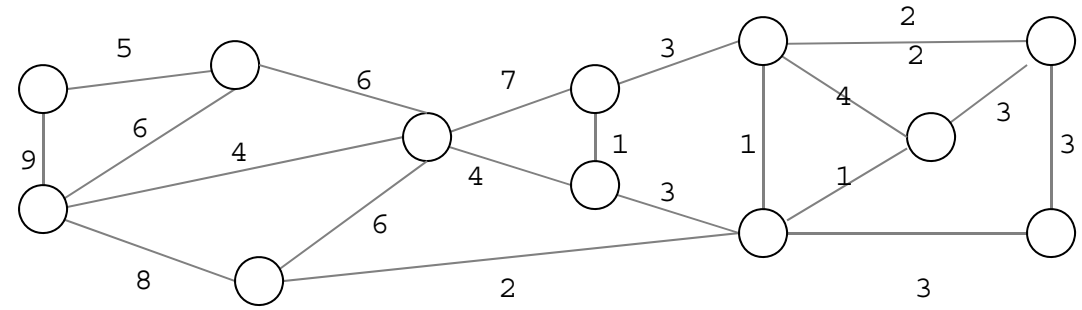
    c.) updating rule:

```
for (each vertex w adjacent to v)
  w.dist = min(w.dist, c(w, v))
```

  - running time

    $O(|V|^2)$: without heap

    $O(|E|\log |V|)$: using a binary heap.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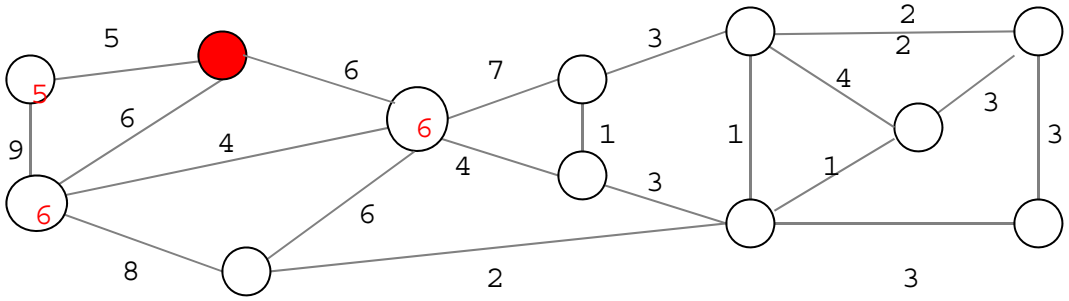
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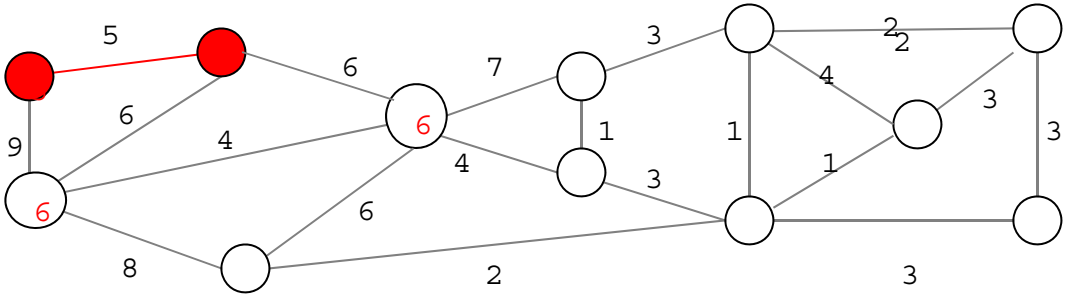
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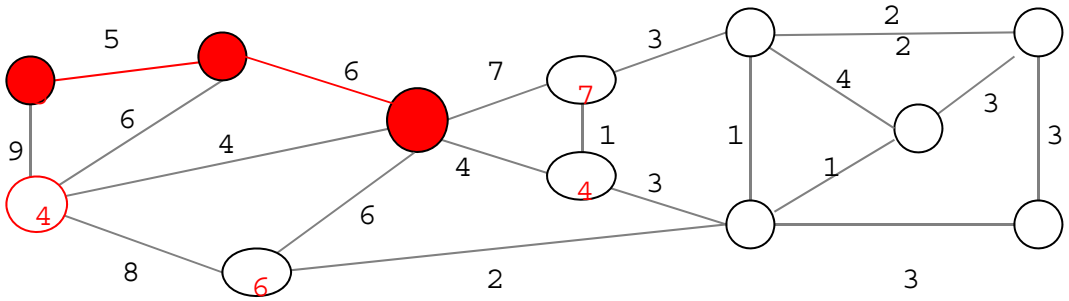
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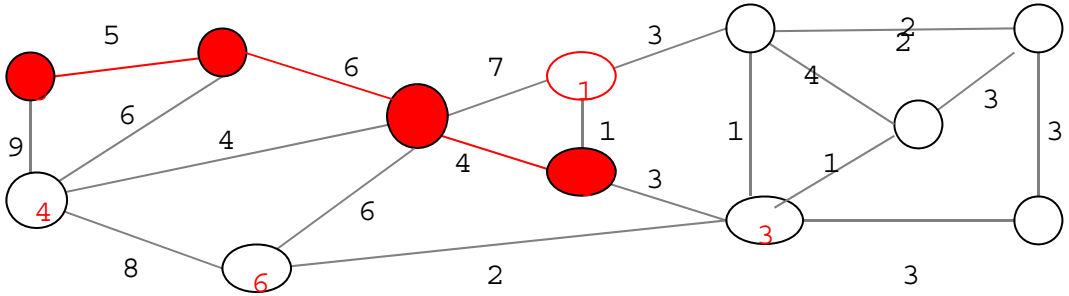
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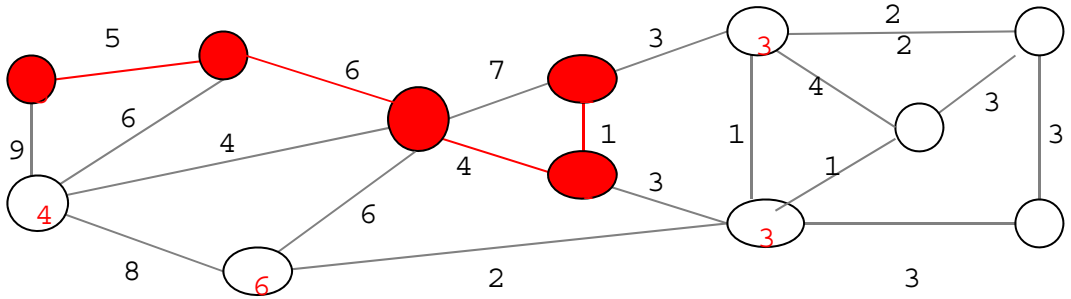
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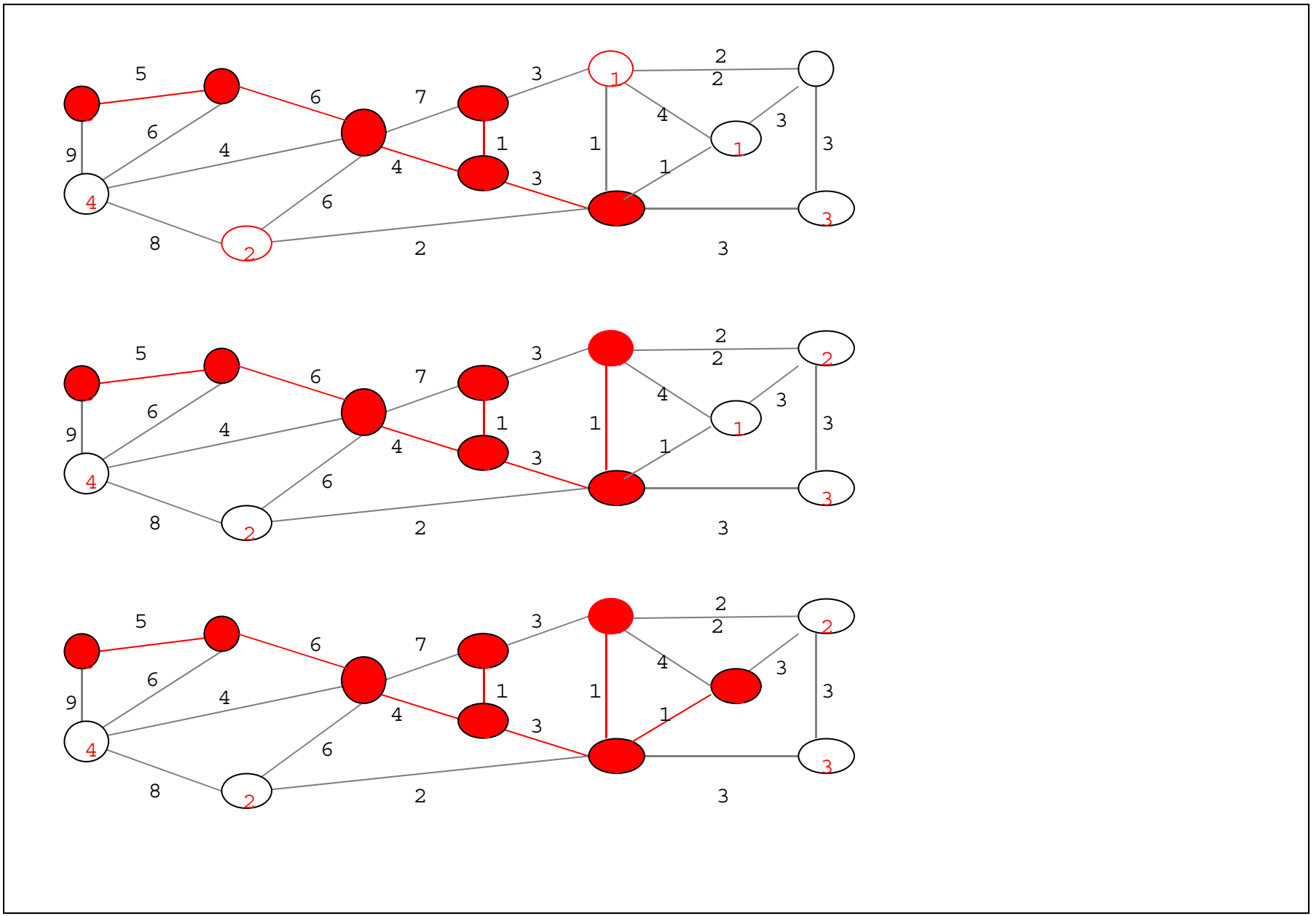
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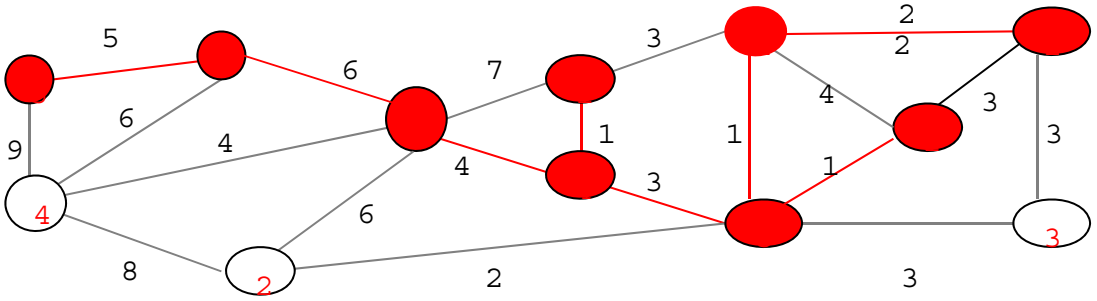
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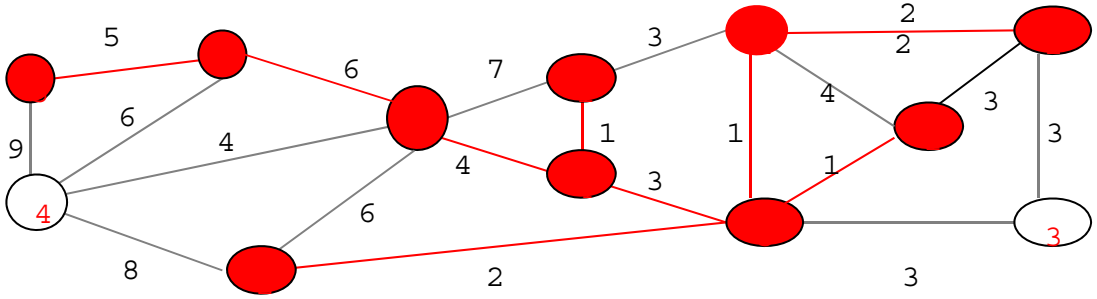
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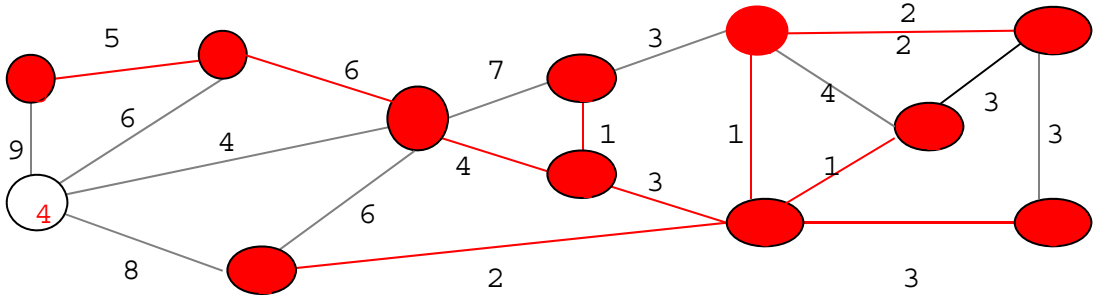
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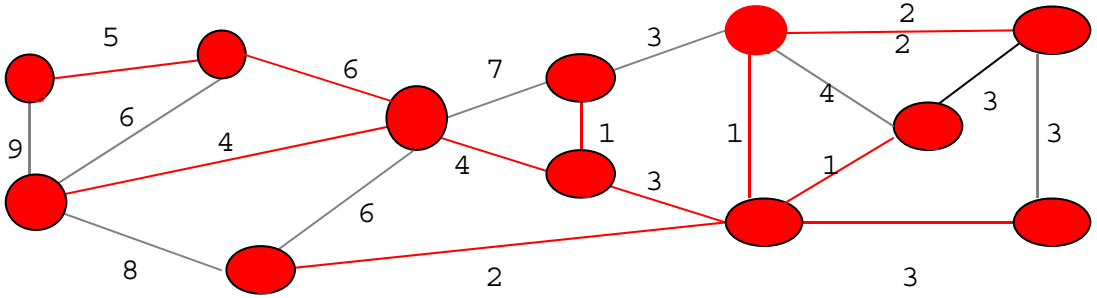
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