

1. Define the  $n$ -based integer rounding of an integer  $k$  to be the nearest multiple of  $n$  to  $k$ . If two multiples of  $n$  are equidistant use the greater one. For example the 4-based rounding of 5 is 4 because 5 is closer to 4 than it is to 8, the 5-based rounding of 5 is 5 because 5 is closer to 5 than it is to 10, the 4-based rounding of 6 is 8 because 6 is equidistant from 4 and 8, so the greater one is used, the 13-based rounding of 9 is 13, because 9 is closer to 13 than it is to 0,

Write a function named **doIntegerBasedRounding** that takes an integer array and rounds all its positive elements using  $n$ -based integer rounding. A negative element of the array is not modified and if  $n \leq 0$ , no elements of the array are modified. Finally you may assume that the array has at least two elements. Hint: In integer arithmetic,  $(6/4) * 4 = 4$ . The function signature is `void doIntegerBasedRounding(int[ ] a, int n)` where  $n$  is used to do the rounding

| if a is              | and n is | then a becomes       | reason  |
|----------------------|----------|----------------------|---|
| {1, 2, 3, 4, 5}      | 2        | {2, 2, 4, 4, 6}      | because the 2-based rounding of 1 is 2, the 2-based rounding of 2 is 2, the 2-based rounding of 3 is 4, the 2-based rounding of 4 is 4, and the 2-based rounding of 5 is 6. |
| {1, 2, 3, 4, 5}      | 3        | {0, 3, 3, 3, 6}      | because the 3-based rounding of 1 is 0, the 3-based roundings of 2, 3, 4 are all 3, and the 3-based rounding of 5 is 6.   |
| {1, 2, 3, 4, 5}      | -3       | {1, 2, 3, 4, 5}      | because the array is not changed if $n \leq 0$ .  |
| {-1, -2, -3, -4, -5} | 3        | {-1, -2, -3, -4, -5} | because negative numbers are not rounded  |

2. A number  $n > 0$  is called cube-powerful if it is equal to the sum of the cubes of its digits. Write a function named `isCubePowerful` that returns 1 if its argument is cube-powerful; otherwise it returns 0. The function prototype is `int isCubePowerful(int n);`  
Hint: use modulo 10 arithmetic to get the digits of the number.

Examples:

| if n is | return | because                           |
|---------|--------|-----------------------------------|
| 153     | 1      | because $153 = 1^3 + 5^3 + 3^3$   |
| 370     | 1      | because $370 = 3^3 + 7^3 + 0^3$   |
| 371     | 1      | because $371 = 3^3 + 7^3 + 1^3$   |
| 407     | 1      | because $407 = 4^3 + 0^3 + 7^3$   |
| 87      | 0      | because $87 \neq 8^3 + 7^3$       |
| 0       | 0      | because n must be greater than 0. |
| -81     | 0      | because n must be greater than 0. |

3. An array is zero-plentiful if it contains at least one 0 and every sequence of 0s is of length at least 4. Write a method named **`isZeroPlentiful`** which returns the number of zero sequences if its array argument is zero-plentiful, otherwise it returns 0. If you are programming in Java or C#, the function signature is **`int isZeroPlentiful(int[] a)`**. The function signature is `int isZeroPlentiful(int a[], int len)` where `len` is the number of elements in the array `a`.

Example

| a is  | then function returns | reason  |
|---|-----------------------|---|
| {0, 0, 0, 0, 0} 1                                     | 1                     | because there is one sequence of 0s and its length $\geq 4$ .           |
| {1, 2, 0, 0, 0, 0, 2, -18, 0, 0, 0, 0, 12} 1          | 2                     | because there are two sequences of 0s and both have lengths $\geq 4$ .  |
| {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 8, 0, 0, 0, 0, 0, 0} 1 | 3                     | because there are three sequences of zeros and all have length $\geq 4$ |
| {1, 2, 3, 4} 1  | 0                     | because there must be at least one 0.                                   |
| {1, 0, 0, 0, 2, 0, 0, 0, 0}                           | 0                     | because there is a sequence of zeros whose length is less $< 4$ .       |
| {0}   | 0                     | because there is a sequence of zeroes whose length is $< 4$ .           |

4. An array is called zero-balanced if its elements sum to 0 and for each positive element  $n$ , there exists another element that is the negative of  $n$ . Write a function named **isZeroBalanced** that returns 1 if its argument is a zero-balanced array. The function signature is **int isZeroBalanced(int[ ] a)**

Examples:

| if a is               | return  |
|-----------------------|---|
| {1, 2, -2, -1}        | 1 because elements sum to 0 and each positive element has a corresponding negative element. |
| {-3, 1, 2, -2, -1, 3} | 1 because elements sum to 0 and each positive element has a corresponding negative element. |
| {3, 4, -2, -3, -2}    | 0 because even though this sums to 0, there is no element whose value is -4                 |
| {0, 0, 0, 0, 0, 0}    | 1 this is true vacuously; 0 is not a positive number  |
| {3, -3, -3}           | 0 because it doesn't sum to 0. (Be sure your function handles this array correctly)         |
| {3}                   | 0 because this doesn't sum to 0   |
| {}                    | 0 because it doesn't sum to 0   |