一、选择题

1. B 2. D 3. C 4. C 5. D

二、填空题

① $z = \cancel{e^{i(\frac{\pi}{2}-\theta)}} = e^{-i(\theta-\frac{\pi}{2})}$

② $e^{i\,\mathrm{Ln}(-1+i)} = e^{i(\ln\sqrt{2}+i(\frac{3}{4}\pi+2k\pi))} = e^{-(\frac{3}{4}\pi+2k\pi)}e^{i\frac{\ln 2}{2}}$

③ $4\pi i$

④ $-\frac{d}{ds}\left(\frac{s}{s^2+4}\right) = \frac{s^2-4}{(s^2+4)^2}$

⑤ $\sin(2t_0)$

三、 $u_x = 2x,\ u_{xx} = 2,\ u_y = -2y\quad u_{yy} = -2$

$u_{xx} + u_{yy} = 0$

故 u 为调和函数

$f'(z) = u_x - i u_y = 2x + i2y = 2z$

故 $f(z) = \int 2z\,dz = z^2 + C$

代入 $f(0)=0$. 得 $f(z) = z^2$.

四、证明：因 $f(z) = u + iv$ 解析. 故 $u_x = v_y,\ u_y = -v_x$ ①

又因 $\overline{f(z)} = u - iv$ 解析. 故 $u_x = -v_y,\ u_y = v_x$ ②

将两式①、②联立，知 $u_x = u_y = v_x = v_y = 0$.

因此 $u \equiv C_1,\ v \equiv C_2$. 故 $f(z)$ 为常数.

五. $\oint_C \frac{\sin z}{z^2(z-\frac{\pi}{3})}dz$ 以0为中心画圆C_0，以$\frac{\pi}{3}$为中心画圆C_1，互不相交。

由复合闭路定理知

$$\oint_C \frac{\sin z}{z^2(z-\frac{\pi}{3})}dz = \oint_{C_0} \frac{\sin z}{z^2(z-\frac{\pi}{3})}dz + \oint_{C_1} \frac{\sin z}{z^2(z-\frac{\pi}{3})}dz$$

再根据柯西积分公式与柯西积分高阶导数公式知，上式为

$$= \left(\frac{\sin z}{z-\frac{\pi}{3}}\right)'\Big|_{z=0} \times 2\pi i + 2\pi i \times \frac{\sin z}{z^2}\Big|_{z=\frac{\pi}{3}}$$

$$= \frac{\cos z(z-\frac{\pi}{3})-\sin z}{(z-\frac{\pi}{3})^2}\Big|_{z=0} \times 2\pi i + 2\pi i \times \frac{\sin\frac{\pi}{3}}{\frac{\pi^2}{9}}$$

$$= 2\pi i \times \frac{-\frac{\pi}{3}}{\frac{\pi^2}{9}} + 2\pi i \times \frac{9\times\frac{\sqrt{3}}{2}}{\pi^2}$$

$$= -6i + \frac{9\sqrt{3}}{2}i$$

六 $\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt = \int_{-\infty}^{+\infty} u(t)e^{-2t}e^{-j\omega t}dt = \int_0^{+\infty} e^{-(2+j\omega)t}dt$

$$= \frac{e^{-(2+j\omega)t}}{-(2+j\omega)}\Big|_0^{+\infty} = \frac{1}{2+j\omega}$$

再由微分性质知

$$F(tf(t)) = j\frac{d}{d\omega}\left(\frac{1}{2+j\omega}\right) = \frac{j(-j)}{(2+j\omega)^2} = \frac{1}{(2+j\omega)^2}.$$

再由位移性质知

$$F(e^{j3t}tf(t)) = \frac{1}{(2+j(\omega-3))^2} = \frac{1}{4+4j(\omega-3)-(\omega-3)^2}$$

七. $L(y''-y) = L\delta(t) \Rightarrow s^2Y(s) - sy(0) - y'(0) - \cancel{sY(s)+y(0)} = 1$ $Y(s)$

$$\Rightarrow (s^2-1)Y(s) = 1$$

$$\Rightarrow Y(s) = \frac{1}{s(s-1)} = \frac{1}{s-1} - \frac{1}{s}$$

故 $L^{-1}(Y(s)) = L^{-1}\left(\frac{1}{s-1}\right) - L^{-1}\left(\frac{1}{s}\right) = e^t - 1$