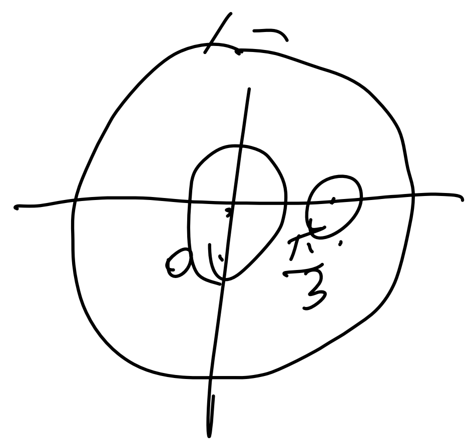
$$z^2 = (z-2)^2.$$



$0, \frac{\pi}{3}$

$$\oint_C = \oint_{|z|=1} + \oint_{|z-\frac{\pi}{3}|=\frac{1}{10}}$$

$$\oint_{|z|=1} \frac{\frac{\sin z}{z-\frac{\pi}{3}}}{z^2}\, dz$$

$$\oint_{|z-\frac{\pi}{3}|=\frac{1}{10}} \frac{\frac{\sin z}{z^2}}{z-\frac{\pi}{3}}\, dz$$

$$= \underbrace{\frac{2\pi i}{1!} \cdot f'(0)} + 2\pi i\, g\left(\frac{\pi}{3}\right)$$

$$\oint_C \frac{f(z)}{(z-z_0)^{n+1}}\, dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$$

$u_x = v_y \quad = -v_y$

$u_y = -v_x$

$v_y = 2x \qquad\qquad v = 2xy + C$

$v_x = 2y$

$\left(e^{i\frac{\pi}{3}}\right)^{2020}$

$= e^{i\frac{2020}{3}\pi}$

$= e^{i\left(673\pi + \frac{\pi}{3}\right)}$

$$= e^{i\left(\pi + \frac{\pi}{3}\right)}$$

$$= e^{i\frac{4\pi}{3}}$$

$$= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

$$= \boxed{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}$$

$$i^{1+i} = e^{(1+i)\,\mathrm{Ln}\, i}$$

$$= e^{(1+i)\left(\ln 1 + i\left(2k\pi + \frac{\pi}{2}\right)\right)}$$

$$= e^{(1+i)\, i\left(2k\pi + \frac{\pi}{2}\right)}$$

$$= e^{(-1+i)(2k\pi + \frac{\pi}{2})}$$

$$= e^{-2k\pi - \frac{\pi}{2}} \cdot e^{i(2k\pi + \frac{\pi}{2})}$$

$$= e^{-2k\pi - \frac{\pi}{2}} e^{i\frac{\pi}{2}}$$

$$= \left(e^{-2k\pi - \frac{\pi}{2}}\right) i$$

$$e^{-2k\pi - \frac{\pi}{2}}, \qquad k \in \mathbb{Z}.$$

$$re^{i\theta}.$$

3. $e^{4i-3} = \underbrace{e^{-3} \cdot e^{4i}}$

$\theta = 4$.

$4 - 2\pi$.

4. $\underbrace{\oint_{|z|=1} e^{z}\,dz}_{=0} + \oint_{|z|=1} |z|\,\bar{z}\,dz$.

$z = e^{i\theta}$.

$$\oint_{|z|=1} 1 \cdot \bar{z} \, dz.$$

$$= \int_0^{2\pi} \cancel{e^{-i\theta}} \cdot i \cancel{e^{i\theta}} \, d\theta$$

$$= \int_0^{2\pi} i \, d\theta = 2\pi i$$

$$F\left[f'(t-2)\right]$$

$$f(t) \rightarrow F(w)$$

$$f(t-2) \rightarrow e^{-jw2} F(w)$$

$$f'(t-2) \Rightarrow jw e^{-2jw} F(w)$$

$$jw(\cos 2w - j\sin 2w) F(w)$$

$$(w\sin 2w + jw\cos 2w) F(w)$$

$$z = x + iy$$

$$z + 1 = x + 1 + iy$$

$$|z+1|^2 = (x+1)^2 + y^2.$$

$$\operatorname{Im}(z) = y.$$

$$f(z) = \left[(x+1)^2 + y^2\right] y.$$

$$\begin{cases} u = [(x+1)^2 + y^2]\, y \\ v = 0 \end{cases}$$

u, v 在 R^2 上可微

$$\begin{cases} u_x = 2(x+1)y = 0 \\ u_y = (x+1)^2 + 3y^2 = 0 \\ v_x = v_y = 0 \end{cases}$$

$$\Rightarrow \begin{cases} u_x = v_y \\ u_y = -v_x \end{cases} \Rightarrow (-1, 0)$$

$x = -1, \ y = 0$

$\Rightarrow$ $(-1,0)$处

可导点为

$$z = -1 + 0i = -1$$

$\Rightarrow$ 在平面不解析

$$f'(-1) = u_x + i v_x$$
$$= 0$$

三.

$$f(z_0) = \frac{1}{2\pi i}\oint_{|z-1|=2} \frac{z^3}{(z-z_0)^2}\,dz.$$

$z_0 \in |z-1|=2$ 内部.

$$f(z_0)=\frac{1}{2\pi i}\cdot\left((z^3)'\Big|_{z=z_0}\right)\cdot\frac{2\pi i}{1!}$$

$$=\frac{1}{2\pi i}\,3z_0^2\cdot 2\pi i$$

$$=3z_0^2.$$

z_0 在 $|z-1|=2$ 外部.

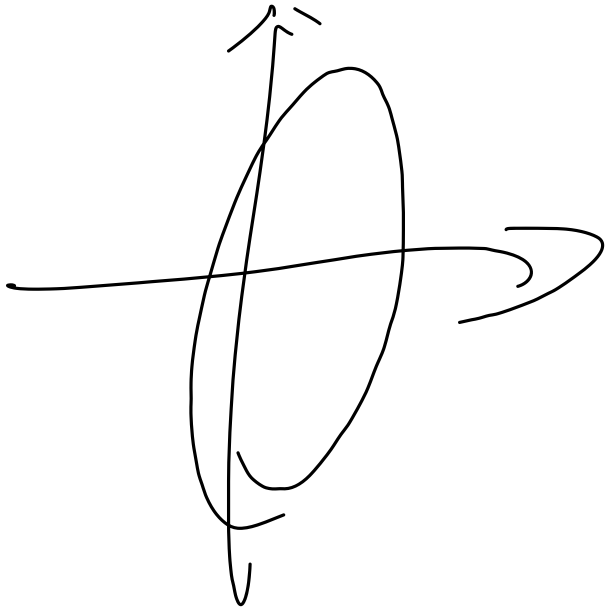
$f(z_0)=0$

$$f(z_0)=\begin{cases} 3z_0^2 & 内 \\ 0 & 外 \end{cases}$$

$$f'(z_0)=\begin{cases} 6z_0 & 内 \\ 0 & 外 \end{cases}$$

$\int f'(1) = 6$

$$f'(2i) = 0$$



III.

$$
\begin{cases}
F(w) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt. \\
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(w)\, e^{jwt}\, dw.
\end{cases}
$$

$$F(w) = \int_{-\infty}^{\infty} f(t) e^{-jwt} \, dt.$$

$$= \int_{-\delta}^{\delta} 1 \cdot e^{-jwt} \, dt$$

$$= \left. \frac{e^{-jwt}}{-jw} \right|_{-\delta}^{\delta}$$

$$= \frac{e^{-jw\delta} - e^{jw\delta}}{-jw}$$

$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

$f(t)$ 在 $t=0$ 连续 + 有界

$$\int_{-\infty}^{\infty} \delta(t) f(t)\, dt = f(0).$$

$$\int_{-\infty}^{\infty} \delta(t-t_0) f(t)\, dt = f(t_0)$$

δ 偶.

$$F[\delta(t)] = 1 \Rightarrow$$

$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$

$$\int_{-\infty}^{t} \delta(t)\,dt = u(t).$$

$$u'(t) = \delta(t).$$

$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} 1\, e^{j\omega t}\, d\omega$$

$$F[1] = 2\pi\,\delta(\omega)$$

$$F[u(t)] = \frac{1}{j\omega} + \pi\,\delta(\omega)$$

$$F[\cos at] = \pi[\delta(\omega-a)+\delta(\omega+a)]$$

$$F[\sin at] = j\pi[\delta(\omega+a)-\delta(\omega-a)]$$

设：$f(t) \to F(w)$

$$\begin{cases} \mathcal{F}[f(t-t_0)] = e^{-jwt_0} F(w) \\ \mathcal{F}^{-1}[F(w-w_0)] = e^{jw_0 t} f(t) \end{cases}$$

$$\boxed{\mathcal{F}[f(at)] = \frac{1}{|a|} F\left(\frac{w}{a}\right)}$$

$$F[f'(t)] = j\omega F(\omega).$$

$$F^{(n)}(\omega) = (-j)^n \cdot F[t^n f(t)]$$

$$\Downarrow$$

$$t^n f(t) \rightarrow . \frac{F^{(n)}(\omega)}{(-j)^n}$$

$f_1 * f_2$

$$= \int_{-\infty}^{\infty} f_1(x)\, f_2(t-x)\, dx :$$

$$F[f_1 * f_2] = F_1(w) \cdot F_2(w)$$

$$F[f_1 \cdot f_2] = \frac{1}{2\pi} F_1(w) * F_2(w).$$

$$\boxed{\int_0^{+\infty} f(t)\, e^{-st}\, dt = F(s).}$$

$$\begin{cases} u(t) \to \dfrac{1}{s} \\ 1 \to \dfrac{1}{s} \\ e^{at} \Rightarrow \dfrac{1}{s-a} \end{cases}$$

$$\cos at \Rightarrow \frac{s}{s^2+a^2}$$

$$\sin at \Rightarrow \frac{a}{s^2+a^2}$$

$$\boxed{\begin{gathered} a>0 \\ f(at) \rightarrow \frac{1}{a} F\left(\frac{s}{a}\right) \end{gathered}}$$

$$f(t) \rightarrow F(s)$$

$$f'(t) \rightarrow sF(s) - f(0)$$

$$f''(t) \rightarrow s^2 F(s) - sf(0) - f'(0)$$

$$f'''(t) \rightarrow s^3 F(s) - s^2 f(0) - s f'(0) - f''(0)$$

$$t^m \Rightarrow \frac{m!}{s^{m+1}}$$

$$t^n f(t) \rightarrow \frac{F^{(n)}(s)}{(-1)^n}$$

$$\frac{f(t)}{t} \rightarrow \int_s^{\infty} F(s)\, ds$$

$$\frac{f(t)}{t^2} \rightarrow \int_s^{\infty} ds \int_s^{\infty} F(s)\, ds.$$

$$f(t - t_0) \rightarrow e^{-st_0} F(s)$$

$$e^{at} f(t) \rightarrow F(s-a)$$

$$L[f_1 * f_2] = F_1(s) F_2(s)$$