Partie 0 _ Matrices de passage.

$\boxed{u : E \longrightarrow F}$ $\quad$ $\dim E = p$, $\dim F = n$

$\mathcal{B}_E = (e_1, \dots, e_p)$
$\mathcal{B}_E' = (e_1', \dots, e_p')$
$\mathcal{B}_F = (f_1, \dots, f_n)$
$\mathcal{B}_F' = (f_1', \dots, f_n')$

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\underset{\mathcal{B}_E}{E} \arrow[r, "\underline{\underline{Id_E}}\ P^{-1}"] \arrow[d, "\underline{\underline{u}}"', "A"] & \underset{\mathcal{B}_E'}{E} \arrow[d, "\underline{\underline{u}}\ A'"] \\
\underset{\mathcal{B}_F}{F} & \underset{\mathcal{B}_F'}{F} \arrow[l, "Q\ \underline{\underline{Id_F}}"]
\end{tikzcd}$$

$A = QA'P^{-1}$

$u = Id_F \circ u \circ Id_E$

$A = \underset{\mathcal{B}_E, \mathcal{B}_F}{Mat}(u)$ $\qquad$ $A' = \underset{\mathcal{B}_E', \mathcal{B}_F'}{Mat}(u)$

$$\boxed{\begin{aligned} A &= QA'P^{-1} \\ A' &= Q^{-1}AP \end{aligned}}$$

$$A = \begin{array}{c} \begin{array}{ccc} u(e_1) & \dots & u(e_p) \end{array} \\ \text{base départ} \\ \left(\begin{array}{c} \\ \underset{\mathcal{B}_E, \mathcal{B}_F}{Mat}(u) \\ \\ \end{array} \right) \end{array} \begin{array}{l} f_1 \\ \vdots \\ \text{base arrivée} \\ \vdots \\ f_n \end{array}$$

$\underline{A \in \mathcal{M}_{n,p}(\mathbb{K})}$

$$A' = \begin{array}{c} \begin{array}{ccc} u(e_1') & \dots & u(e_p') \end{array} \\ \left(\begin{array}{c} \\ \underset{\mathcal{B}_E', \mathcal{B}_F'}{Mat}(u) \\ \\ \end{array} \right) \end{array} \begin{array}{l} f_1' \\ \vdots \\ f_n' \end{array}$$

$\underline{A' \in \mathcal{M}_{n,p}(\mathbb{K})}$

$$\boxed{\begin{aligned} &\bullet\ X' = P^{-1}X \\ &\bullet\ Y' = Q^{-1}Y \\ &\bullet\ X = PX' \\ &\bullet\ Y = QY' \end{aligned}}$$

- Matrice de passage de la base $\mathcal{B}_E$ ds la base $\mathcal{B}_E'$

$$P = \begin{array}{c} \begin{array}{ccc} Id_E(e_1') & \dots & Id_E(e_p') \end{array} \\ \text{base de départ} \\ \left(\begin{array}{c} \\ \underset{\mathcal{B}_E', \mathcal{B}_E}{Mat}(Id_E) \\ \\ \text{coordonnées des } (e_1', \dots, e_p') \text{ selon les} \\ (e_1, \dots, e_p) \end{array} \right) \end{array} \begin{array}{l} e_1 \\ \vdots \\ \text{base arrivée} \\ \vdots \\ e_p \end{array}$$

$\underline{P \in \mathcal{M}_{p,p}(\mathbb{K})}$

- Matrice de passage de la base $\mathcal{B}_F$ à la base $\mathcal{B}_F'$

$$Q = \begin{array}{c} \begin{array}{ccc} Id_F(f_1') & \dots & Id_F(f_n') \end{array} \\ \text{base départ} \\ \left(\begin{array}{c} \\ \underset{\mathcal{B}_F', \mathcal{B}_F}{Mat}(Id_F) \\ \text{coordonnées des } (f_1' \dots f_n') \\ \text{selon les } (f_1, \dots f_n) \end{array} \right) \end{array} \begin{array}{l} f_1 \\ \vdots \\ \text{base arrivée} \\ \vdots \\ f_n \end{array}$$

$\underline{Q \in \mathcal{M}_{n,n}(\mathbb{K})}$

- $\boxed{y = u(x) \Leftrightarrow Y = AX}$

avec $\begin{cases} Y = \underset{\mathcal{B}_F}{Mat}(y) & \begin{pmatrix} \vdots \end{pmatrix} n \\ X = \underset{\mathcal{B}_E}{Mat}(x) & \begin{pmatrix} \vdots \end{pmatrix} p \end{cases}$

distinction entre un vecteur du $\mathbb{K}$-EV E et la matrice (colonne) donnant ses coordonnées dans la base E

- $\boxed{y = u(x) \Leftrightarrow Y' = A'X'}$

avec $\begin{cases} Y' = \underset{\mathcal{B}_F}{Mat}(y) & \begin{pmatrix} \vdots \end{pmatrix} n \\ X' = \underset{\mathcal{B}_E}{Mat}(x) & \begin{pmatrix} \vdots \end{pmatrix} p \end{cases}$

$Y = AX$

$$\underset{n \times p}{\begin{pmatrix} A \end{pmatrix}} \underset{p \times 1}{\begin{pmatrix} X \end{pmatrix}} = \underset{n \times 1}{\begin{pmatrix} Y \end{pmatrix}}$$