$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial r} \times \frac{\partial r}{\partial x} + \frac{\partial f}{\partial \theta} \times \frac{\partial \theta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial r} \times \frac{\partial r}{\partial y} + \frac{\partial f}{\partial \theta} \times \frac{\partial \theta}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial r} \times \frac{x}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial \theta} \times \frac{-y}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial r} \times \frac{y}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial \theta} \times \frac{x}{x^2+y^2}\right)$$

$$= \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) \times \frac{\partial r}{\partial x} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right) \times \frac{\partial \theta}{\partial x}\right] \times \frac{x}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial r} \times \frac{\partial}{\partial x}\left(\frac{x}{\sqrt{x^2+y^2}}\right)$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{\partial r}{\partial x} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{\partial \theta}{\partial x}\right] \times \frac{-y}{x^2+y^2} + \frac{\partial f}{\partial \theta} \times \frac{\partial}{\partial x}\left(\frac{-y}{x^2+y^2}\right)$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) \times \frac{\partial r}{\partial y} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right) \times \frac{\partial \theta}{\partial y}\right] \times \frac{y}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial r} \times \frac{\partial}{\partial y}\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{\partial r}{\partial y} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{\partial \theta}{\partial y}\right] \times \frac{x}{x^2+y^2} + \frac{\partial f}{\partial \theta} \times \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)$$

$$= \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) \times \frac{x}{\sqrt{x^2+y^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right) \times \frac{-y}{x^2+y^2}\right] \times \frac{x}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial r} \times \left[\frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}\right]$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{x}{\sqrt{x^2+y^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{-y}{x^2+y^2}\right] \times \frac{-y}{x^2+y^2} + \frac{\partial f}{\partial \theta} \times \left[\frac{2xy}{(x^2+y^2)^2}\right]$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) \times \frac{y}{\sqrt{x^2+y^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial r}\right) \times \frac{x}{x^2+y^2}\right] \times \frac{y}{\sqrt{x^2+y^2}} + \frac{\partial f}{\partial r} \times \left[\frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}\right]$$

$$+ \left[\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{y}{\sqrt{x^2+y^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{x}{x^2+y^2}\right] \times \frac{x}{x^2+y^2} + \frac{\partial f}{\partial \theta} \times \left[\frac{-2xy}{(x^2+y^2)^2}\right]$$

$$= \frac{\partial}{\partial r}\left(\frac{\partial f}{\partial r}\right) + \frac{\partial f}{\partial r} \times \frac{1}{\sqrt{x^2+y^2}} + \frac{\partial}{\partial \theta}\left(\frac{\partial f}{\partial \theta}\right) \times \frac{1}{x^2+y^2}$$

$$= \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2}$$