Evaluation of a Few Interpolation Techniques of Gravity Values in the Border Region of Brazil and Argentina

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Abstract

Least Squares Collocation (LSC) and kriging are the most used techniques to predict gravity values as well as gravity anomalies. The limitations of LSC technique are mainly related in obtaining an adequate co-variance function. Moreover, LSC and kriging predictions depend strongly on known data distribution. Artificial Neural Network (ANN) is a promising tool to be applied in the interpolation problems. Even though, far from the deterministic ones, these techniques are presented as alternatives for interpolating due their good adaptation to several data distribution and easy implementation for fusion of different kinds of data basis. To test the performance of ANN in face of interpolation problems with respect to LSC and kriging, an experiment was developed in a region in the Brazil–Argentina border. Interpolated gravity values were obtained by LSC and kriging and compared with values obtained by ANN considering different data distributions and by using the same test points where gravity values are known. Considering the need of consistency of datum for predicting gravity related values, only a Brazilian data set was used in the present analysis. The smallest number of reference data for training and the low dispersion reveals the ANN as an alternative for LSC and kriging techniques for the usual poor gravity data distribution in South America.

114.1 Introduction

Due to the impossibility of observing gravity values all over the Earth's surface, in some cases it is necessary to do a prediction of these values from two or more known data. Prediction of gravity values is very useful in Geodesy, when we use techniques such as those related to the solution of the Geodetic Boundary Value Problems (GBVP) where gridded data are needed; or for geopotential numbers computation along with leveling lines. This problem is present in the South American territory where most of vertical networks have only normal-orthometric corrections

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and the vertical datums are only local ones because the poor distribution of gravity data necessary to an adequate GBVP solution.

Interpolation of gravity values is affected by several effects. Most of them are very difficult to model because the need of considering different kinds of data and the fusion of local gravity data with different origin like Digital Topography Models and Global Geopotential Models. Interpolated values from a data set by deterministic or statistical methods in the space domain have some limitations in computations related with inadequate data distribution. The classic technique usually used when predicting the gravity values is the LSC which highly depends on the chosen covariance function and of data distribution. The problem is emphasized when we need to consider different kind of data source. However, with the development of the computing methods, other methodologies can be applied, in some cases, with advantages. We have the Artificial Neural Networks (ANNs) as an example of that.

The used network for testing interpolation techniques has 34 benchmarks in the Brazilian side and other additional reference points. The PREDGRAV tool provided by SIRGAS/WG III (Drewes et al. 2002) based on LSC was used. The kriging technique was tested with basis in the SURFER™ package. ANNs were constructed with a radial basis function with distributed training points in the region. Several tests were realized. The best results with LSC points out a RMS of 1.57 mGal but most cases presented limitations regarding the data distribution. The ANN presented in the better case a RMS of 2.39 mGal in similar situation. But the ANNs have less limitation for data distribution, still working with poor data distribution. Also, the ANN allowed incorporating EGM2008 information which improved the prediction capability reducing the RMS to half. Kriging presented, in general, worse results even for the best data distribution.

114.1.1 Data Set

There is a Bilateral Project (involving Brazilian and Argentine Institutes and Universities) for the connection between Brazilian and Argentine Fundamental Vertical Networks. This project has main purpose to build a vertical net based on geopotential numbers in the Brazil–Argentina border. However, there are several problems, not discussed here, to put the Brazilian and Argentine data sets in a common basis.

Aiming to generate a consistent data basis in the Brazilian–Argentine border region some bi-lateral campaigns were organized in the region of *Corrientes* and *Rio Grande do Sul* states in August and December, 2008. Each country has its data basis referred to different datums. In this sense we are considering only the Brazilian data set in this manuscript. The Brazilian gravity reference in the region is the São Borja gravity station which is a point of the Brazilian Fundamental Gravity Network (ON 1986). Several gravity observations associated with GPS/RTK positioning were realized in the region, most of them over existing benchmarks. The gravity observations were performed with the LaCoste & Romberg G-372 gravimeter calibrated on the Brazilian Absolute Gravity Network (RENEGA) in 2007 and the SCINTREX CG3 with factory original calibration and which was submitted to static drift determination before each campaign. The GPS positioning was performed with a pair of Leica Geosystems 1,200 dual frequency GPS receivers equipped with RTK system. GPS/RTK positioning was used to improve the velocity of position determination for points until distances of 15 km of reference station. However most of points were processed with basis in static relative technique by using some local reference stations realized with reference in GNSS Continuous Monitoring Brazilian Network (RBMC) part of the SIRGAS network. The obtained mean precisions in position for static-relative positioning were: Horizontal: 3 mm; and Vertical 6 mm. In the RTK positioning the mean precisions were: Horizontal 20 mm; and Vertical 45 mm.

In Fig. 114.1 is possible to see the range of heights and gravity anomalies variation at the studied region (Fig. 114.2). The apparent low correlation among free air gravity anomalies and heights points out that the relief information is not fundamental for gravity interpolation. It must be emphasized that the Bouguer anomalies seems to have a distribution of values as rough as the free air anomalies.

In general, the gravity data on the whole South America is not adequate because the poor distribution of data. Therefore, the gravity prediction based on all possible related information is still a necessity, mainly because the lack of resources to cover the entire region with gravimetry in a terrestrial conventional form.

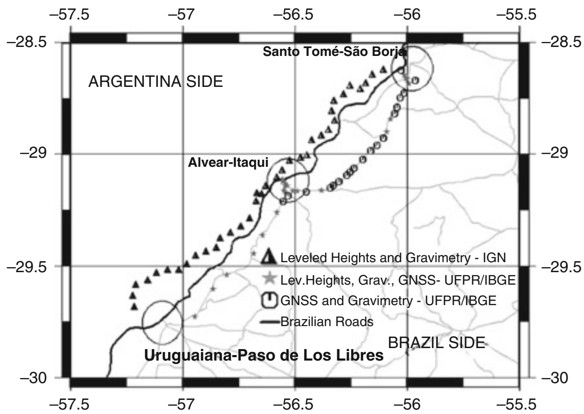


Fig. 114.1 Gravity information after two observation campaigns in the studied area

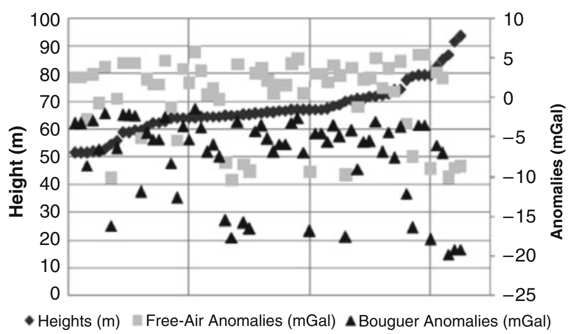


Fig. 114.2 Heights, free-air anomalies and Bouguer anomalies for the data set presented in Fig. 114.2

114.2 Interpolation Techniques

114.2.1 Artificial Neural Networks

ANNs are computing instructions used mainly for the classification of groups. The basic idea came up from the development of the perceptron algorithms by McCulloch & Pits (Negnevitsky 2002). The perceptron allows, using only one function, to distinguish two groups which are linearly separable. However, when the groups are not linearly separable, different functions are used to allow such classification. The result of that classification, known as learning, has, as a final result, a matrix which can be used to make a prediction of different data which belong to the same initial group. An ANN (Fig. 114.3) can therefore, be used with latitude and longitude values to obtain a specific result, such as gravity value (Tierra Criollo and de Freitas 2005).

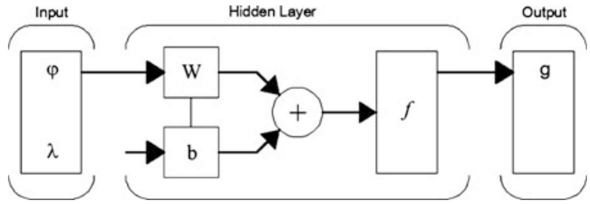


Fig. 114.3 Artificial neural network

In Fig. 114.3, W is an initial vector of weight which is adjusted through interactions with values obtained from the transference function *f*. *b* is a unit vector which is called bias and which increases the weight vector. The iteration process continues until the weights are adjusted so that we can get the desired values in a certain level of confidence. Several kinds of ANNs can be built by using different instruction routines.

114.2.2 Least Squares Collocation

Least Squares Collocation (LSC) is a technique which serves both a prediction and a filtering of data. For example, a formula that uses the least squares collocation concept for prediction gravity anomalies and its details can be found in Tscherning (1974):

$$\tilde{\Delta}\mathrm{g}_\mathrm{P} = \sum_{i=1}^{n} \mathrm{a_i}\Delta\mathrm{g_i}, \tag{114.1}$$

where $\Delta\mathrm{g_i}$ are gravity anomalies observed and $\tilde{\Delta}\mathrm{g}_\mathrm{P}$ is the predicted gravity anomaly at point P. In Hofmann-Wellenhof and Moritz (2005) the treatment of (1) can be found in a matrix form:

$$\tilde{\Delta}\mathrm{g}_\mathrm{P} = \begin{bmatrix} C_{P1} & C_{P2} & \cdots & C_{Pn} \end{bmatrix} \times \cdots$$
$$\begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & C_{nn} \end{bmatrix}^{-1} \times \begin{bmatrix} \Delta\mathrm{g}_1 \\ \Delta\mathrm{g}_2 \\ \vdots \\ \Delta\mathrm{g}_\mathrm{n} \end{bmatrix}. \tag{114.2}$$

Hofmann-Wellenhof and Moritz (2005) consider that "*for optimal prediction, we must know the statistical behavior of the gravity anomalies through the covariance function C*". The C functions are obtained, in general, from terms that depend of positions of points, from spherical distance between those points and other operators. For each different quantity, there is a different covariance function associated, which makes the LSC a not easy problem to solve. Because of many types of covariance functions developed for different situations, this paper used the PREDGRAV, a LSC tool provided by SIRGAS WGIII.

114.2.3 Kriging

Kriging is a prediction process which uses the principle that closer points must presents more similar characteristics than farther away ones. There are many kinds of kriging. This work uses ordinary kriging.

According to Trauth (2007), ordinary point kriging uses a weighted average of the neighboring points to estimate the value of an unobserved point:

$$\tilde{g}_P = \sum_i^n \lambda_i \cdot g_i, \qquad (114.3)$$

where λ_i are the weights which have to be estimated. The sum of the weights should be one to guarantee that the estimates are unbiased:

$$\sum_i^n \lambda_i = 1. \qquad (114.4)$$

The expected error of the estimation has to be zero, in this way we have:

$$E(\tilde{g}_P - g_P) = 0, \qquad (114.5)$$

where g_P is the expected true, but unknown value.

114.3 Prediction of Gravity Values

For training the ANNs, it was used the data configuration shown in Fig. 114.4. These points were the same generated database for kriging and PREDGRAV prediction. Four kinds of ANNs were tested (Cases number 01, 02, 03 and 06). Several other tests can be found in Pereira (2009).

In Case 01, latitude, longitude and height were used in the training to obtain gravity values. The architecture is presented in Fig. 114.5, with 3, 4, 3 and 1 neurons in a hyperbolic -tangent sigmoid, in a radial basis function, in a hyperbolic tangent sigmoid and in a linear transfer function, respectively.

In Case 02 (Figs. 114.6 and 114.7), latitude and longitude were used. However, heights were not

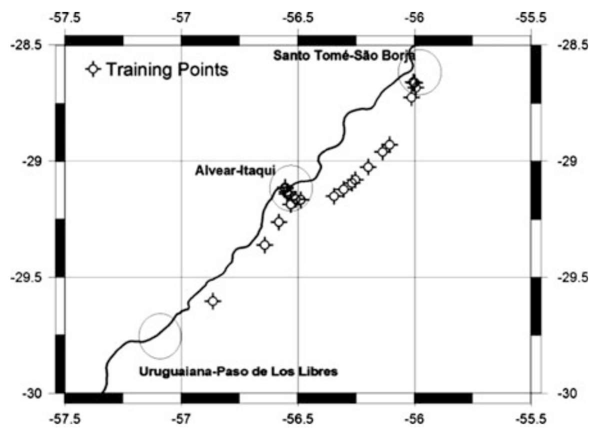


Fig. 114.4 Database for the initial prediction

Fig. 114.5 Case 01 of ANN

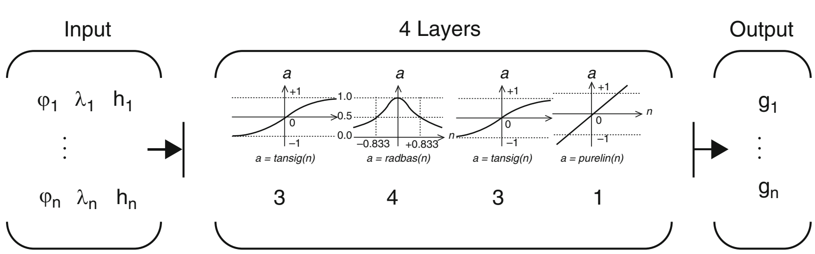
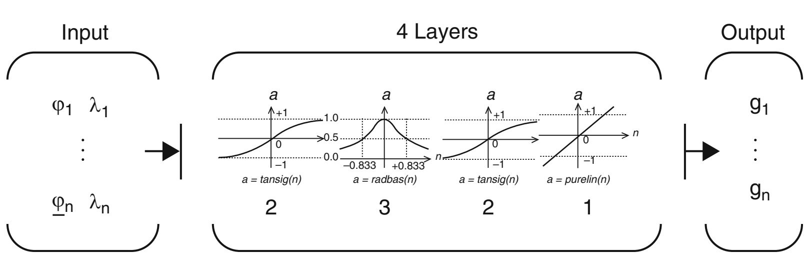


Fig. 114.6 Case 02 of ANN



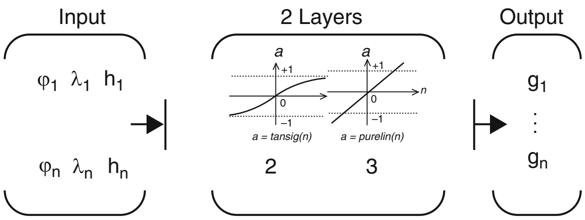


Fig. 114.7 Case 03 of ANN

used. The structure is similar to case 01. However, the numbers of neurons in each layer are not the same.

For Case 04, it was used PREDGRAV, and in the case number 05, kriging.

114.4 Results

As already mentioned, the RMS was calculated considering the local observations built in the campaigns as a reference. The next figures show the predicted grids from the techniques presented. In the all cases of ANN, the trainning's goal was 0.02. The worst case of ANN reached the desired values after 504 epochs. In the following figures the isolines from gridding must be added by 979,000 mGal. Figure 114.8 shows the grid from local observations of g. The next ones (Figs. 114.9–114.13) shows the results reached.

Another possibility offered by ANN is to use quantities derived from global geopotential models without modifications in the original routines. Case 06 related with this approach is showed in Fig. 114.14. Latitude, longitude and heights can be obtained from local frame and the learning can be improved with geoidal heights from EGM 2008 to compute gravity values. The worst Case (03) was used to test this hypothesis becaming Case 06.

With the same points of initial database, the RMS computed for the Case 6 was 3.68 mGal, which confirmed the hipotesis.

Table 114.1 summarizes the main obtained results in this work:

Other interesting situation happen when the geometry and number of database points is changed. It must be emphasized that in PREDGRAV applied for the LSC, it is necessary at least 30 different points to generate the database. For the cases involving ANN it is shown that the number of training points could be a half part for applying the LSC technique. Another

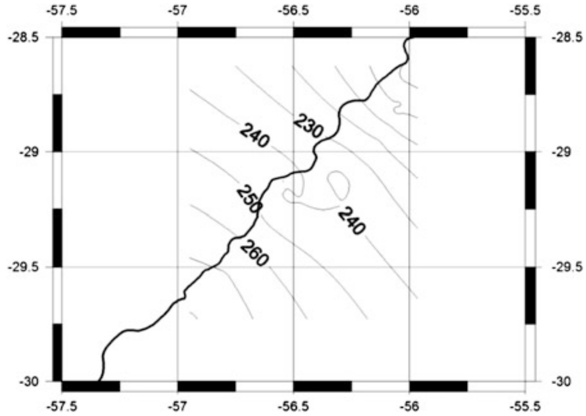


Fig. 114.8 Grid from g observe.

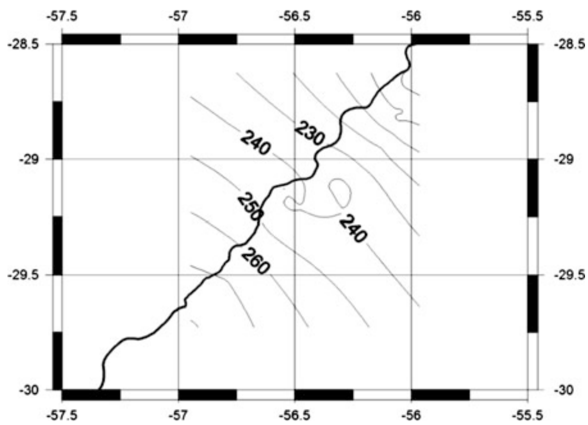


Fig. 114.9 Isolines of predicted grid from ANN Case 01

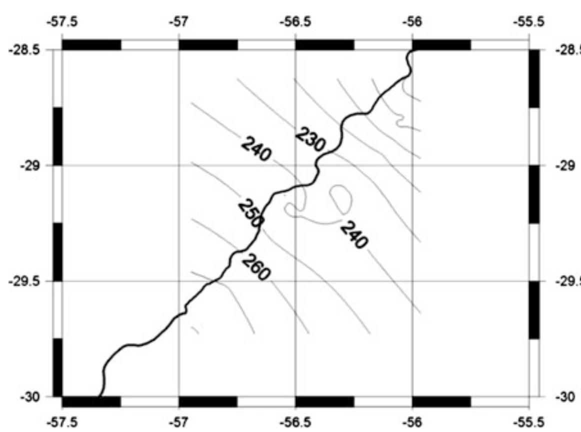


Fig. 114.10 Isolines of predicted grid from ANN Case 02

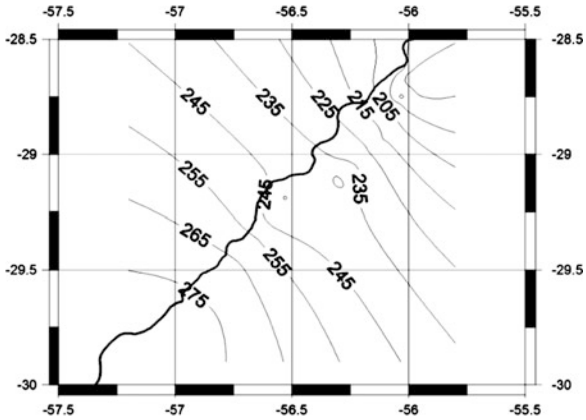


Fig. 114.11 Isolines of predicted grid from ANN Case 03

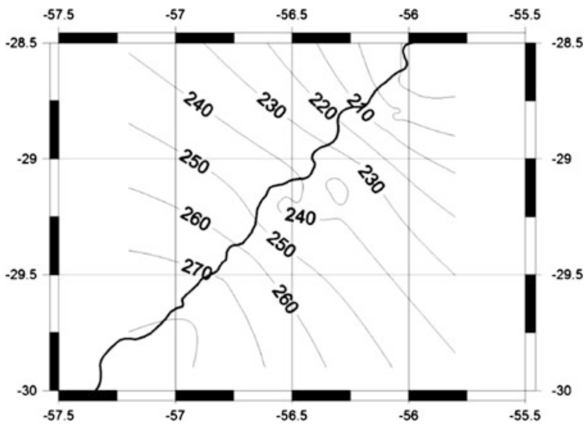


Fig. 114.12 Isolines of predicted grid from PREDGRAV (Case 04)

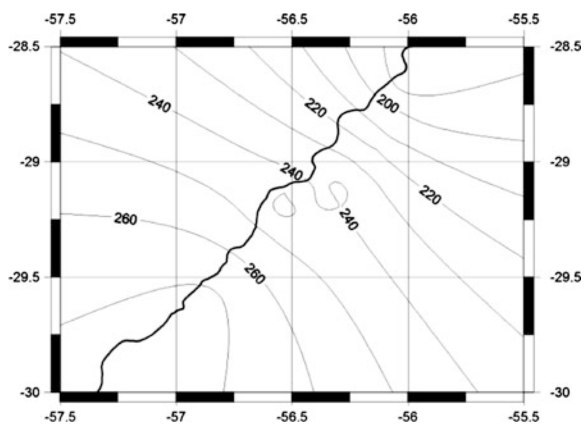


Fig. 114.13 Isolines of predicted grid from kriging (Case 05)

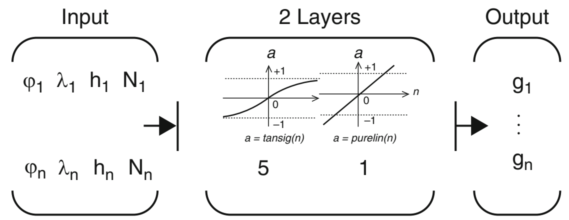


Fig. 114.14 Case 06 of ANN with learning based on geoidal heights

Table 114.1 RMS of different predictions without restriction of data distribution

Case	RMS (mGal)
ANN Case 01	4.08
ANN Case 02	2.39
ANN Case 03	6.64
PREDGRAV (Case 4)	1.57
Kriging (Case 5)	3.10
ANN Case 06	3.68

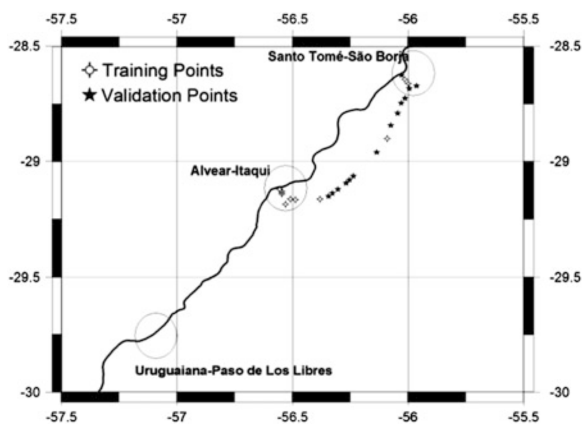


Fig. 114.15 Database of ANN Case 03 where less of 30 points were used in the learning

aspect is that the ANN is less exigent about data distribution.

Figure 114.15 shows a case where the database has less than 30 points and the computed RMS was 1.18 mGal with the ANN Case 03.

Some these effects could be related to terrain corrections. However, if we calculate that quantity from Digital Elevation Model DTM2006 (ICGEM 2010) (Fig. 114.16), the magnitude of results shows that the differences in prediction data not comes from local effects once that the terrain corrections are, at least, two times smaller.

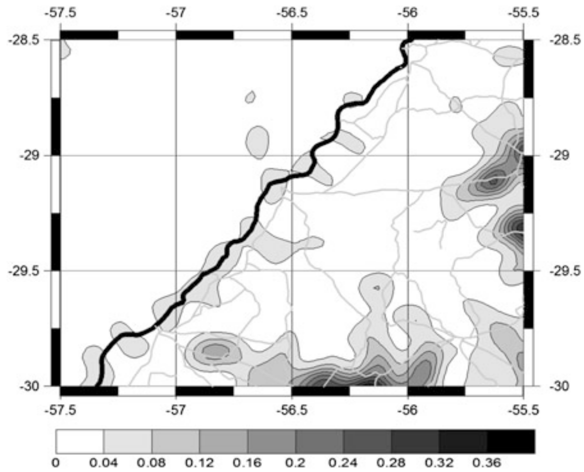


Fig. 114.16 Terrain Corrections for database used (mGal)

In the Brazilian case, because of heterogeneities of height system, again the ANN can be used with advantages, once that gravity anomaly is strongly dependent of point's height. In spite of positive results about of ANN, it must be considered that these results are referred to the studied region and therefore kriging and LSC concepts applied in other regions can furnish different results.

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Conclusions

The ANNs are very easy to apply for gravity prediction, even considering the integration of different data basis. It must be emphasized that the worst case of interpolation with ANN (Case 03) could be improved, reducing the RMS to a half part by integrating EGM2008 geoid heights. This process is not trivial to implement in the LSC because the difficulties to establish the covariance function in this case. For the cases in that it is necessary to integrate many parameters in LSC; the central problem is to obtain the covariance functions. However, for builting an ANN, additional care must be taken into account. The number of layers, usually related to separable groups, is not observed in case of prediction of gravity values. There were examples in which the number of neurons is the same as the number of points used in the learning, but the RMS was one order higher than Case number 01 of ANN.

It must be considered as a special case that the structure of Case 03 applied to few training points still works with good performance while LSC and kriging do not work.

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