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Handling subjective information through augmented (fuzzy) computation

Marcelo Loor [a,b,*], Guy De Tré [a]

[a] *Ghent University, Dept. of Telecommunications and Information Processing, Sint-Pietersnieuwstraat 41, B-9000, Ghent, Belgium*
[b] *ESPOL University, Dept. of Electrical and Computer Engineering, Campus Gustavo Galindo V., Km. 30.5 Via Perimetral, Guayaquil, Ecuador*

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Abstract

Since it can result in significant benefits for a company or an individual, the inclusion of information extracted from subjective social media content into a decision making process is becoming a more frequent activity. However, such benefits are usually linked to the usability of the extracted information, which, among other aspects, depends on the reliability of its source. In this regard, people whose understandings of a topic are alike to the understanding possessed by an information seeker can be considered fairly reliable information sources. Hence, we propose a novel technique for detecting social media users with whom an information seeker shares a similar understanding of a given topic. Through this technique, posts on social media are digested to build a kind of database consisting of *augmented Atanassov fuzzy sets*, or AAIFSs for short, each resembling a collection of *experience-based evaluations* given by a particular source with respect to a given topic. Since such AAIFSs can be used in comparisons in which not only the extents but also the contexts of those evaluations are taken into account for computation, extracting more reliable (and usable) information is possible. An illustrative example shows how the proposed technique works and how it can help to detect sources having a common understanding of a topic.

Keywords: Subjective data; Imperfect data; Fuzzy databases; Augmented fuzzy computation

1. Introduction

Experience-based evaluations (XBEs) are appraisals stemmed from the understanding that someone has acquired on a given topic by experience [1]. By way of illustration, one can think of the statement "the countryside of Wales is definitely a place to avoid during summer because of the high levels of pollen," which has been published online by Alice after visiting that place last summer, as an XBE of the countryside of Wales in connection with the topic "places to avoid during summer." Like this statement, XBEs can be very subjective and imprecise. Even so, information

* Corresponding author at: Ghent University, Dept. of Telecommunications and Information Processing, Sint-Pietersnieuwstraat 41, B-9000, Ghent, Belgium.
E-mail addresses: Marcelo.Loor@UGent.be (M. Loor), Guy.DeTre@UGent.be (G. De Tré).

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resulting from them can be useful. If, for instance, a friend of Alice, say Bob, understands that the presence of high levels of pollen is a relevant aspect of a place that should be avoided during summer, Alice's XBE might help him to decide about visiting the countryside of Wales next summer. On the other hand, if Bob understands that the presence of high levels of pollen is not pertinent to that matter, he might disregard Alice's XBE for his decision. A challenge that arises when someone like Bob wants to extract useful information from statements like the one published by Alice is concerned with detecting similar understandings of a topic informally discussed through usually subjective messages that might be posted online by persons considering different contexts.

In a previous work [1], we studied a closely analogous situation. Therein the challenge was to detect similar understandings of the topic behind an evaluation request, in which the XBEs are performed by a heterogeneous group of people. To address that challenge, we proposed the characterization of XBEs by means of *augmented appraisal degrees* (AADs) and *augmented (Atanassov) intuitionistic fuzzy sets* (AAIFSs). It was shown that such concepts along with several operators and functions can deal with (similarity) comparisons of such XBEs. Hence, one might expect that those *augmented computational tools* could help to address the challenge stated above in this paper. However, since posts on social media do not usually result from an explicit evaluation request, a specific problem to solve is *how to extract XBEs regarding a topic from posts that are not necessarily related to that topic.*

Aiming to address that specific problem, we propose in this paper a novel computational intelligence method whereby posts on social media are digested to obtain an AAIFS, which characterizes a collection of XBEs that would have been given by a person regarding a particular topic. The idea behind the proposed *post-digest method* is that the understanding of a topic possessed by a person might be reflected through his/her posts. Hence, the proposed method uses the content of those posts to build a model of such understanding, which is then used for producing *artificial XBEs* that resemble *actual* XBEs performed by that person.

An important aspect of the post-digest method is that the AAIFSs resulting after digesting the messages posted by several users lend themselves to *augmented (fuzzy) computation*, i.e., those AAIFSs can be used in a process in which not only the extents but also the contexts of XBEs are taken into account for computation – herein, by "*context of an XBE*" is meant the conditions that arise when the evaluation is carried out, which mainly depend on the experience of the person who performs the evaluation. Thus, our method can be applied to build a kind of database consisting of AAIFSs that resemble XBEs given by social media users with whom an information seeker shares a similar understanding about one or more topics. A practical motivation here is that, if such a database is available for an information seeker, he/she can use it as a reliable source to obtain information about objects that he/she might have not experienced yet – herein, the word "*object*" refers to a notion or something that exists by itself. For instance, Fig. 1 depicts a situation where messages posted by six social media users, say Pia, Quinn, Rose, Tom, Vera and Warren, have been digested to build a database based on what an information seeker, say Sam, understands about topics A, B and C – for simplicity, only the processes followed to digest the messages posted by Pia and Tom, which yield the AAIFSs $\hat{A}_{@Pia}$ and $\hat{A}_{@Tom}$ respectively, have been illustrated. Assuming in this case that Sam shares a quite similar understanding about topic A with Pia, Sam can find the artificial XBEs digested from the messages posted by Pia (i.e., $\hat{A}_{@Pia}$) to be more reliable when looking for information about objects that are compatible with the way in which he perceives A. Likewise, if Sam shares a very similar understanding about topic B and C with Quinn and Rose respectively, Sam can consider the data digested from the messages posted by Quinn and Rose to be more reliable to get information about objects that are compatible with the way to which he perceives B and C respectively.

Another important and interesting aspect of the post-digest method is that it allows an information seeker to assess the quality of an information source without given details that might compromise his/her privacy. Hence, this method could be applied in situations where someone needs some privacy when looking for pertinent information given by a reliable source. For instance, consider that (topic) A in the previous example refers to "*nice places to stay*." If Sam is interested in a nice place to stay, he can provide the names of some hotels that, according to his experience, are either compatible or noncompatible with his understanding of *nice places to stay*. After using our method to digest the messages posted by him, as well as the messages posted by Pia, Sam will obtain the AAIFSs $\hat{A}_{@Sam}$ and $\hat{A}_{@Pia}$. The elements in $\hat{A}_{@Sam}$ and $\hat{A}_{@Pia}$ will include both the level to which and a collection of features that hint why each hotel is a nice place to stay according to their individual understandings. Thus, by performing an augmented comparison between these AAIFSs, Sam can measure the degree of similarity between his understanding and Pia's about (the topic) "*nice places to stay*." This measure can be used by Sam as an indicator of how reliable Pia could be as an information source on this topic without telling her about his vacation plans.

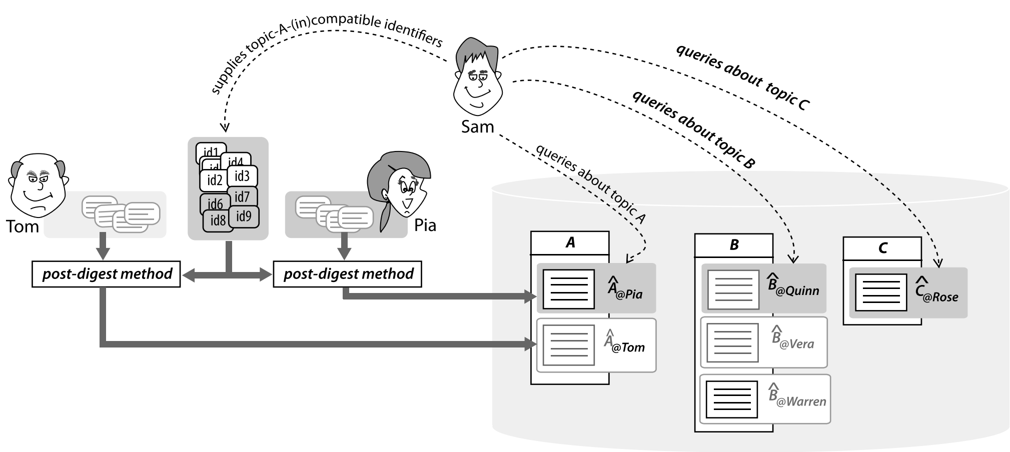

Fig. 1. Practical motivation for this work.

To present the proposed post-digest method, which is the main contribution of this work, we structure the paper as follows. The key ideas and definitions regarding AAIFSs, as well as the formal notation used throughout the paper are presented in the next section. The internal structure and components of the post-digest method are described in Section 3. After that, in Section 4 we present an illustrative example that shows how the proposed method works. Before concluding with some suggestions on future research directions, we present some related work in Section 5.

2. Preliminaries

As was mentioned in the previous section, the aim of the proposed post-digest method is to extract XBEs related to a given topic from posts on social media that might not be connected with that topic. Since a proper mathematical representation is needed for processing such XBEs, in this section we briefly explain how concepts like *augmented appraisal degrees* (AADs) and *augmented Atanassov (intuitionistic) fuzzy sets* (AAIFSs), which have been proposed in [1], can be used for handling XBEs through *augmented (fuzzy) computation*.

2.1. Characterization of XBEs

In the framework of *fuzzy set theory* [2], an XBE can be seen as the result of the evaluation of a proposition p having a canonical form ‘x IS A’, which means “the value of x is compatible with the understanding of A” – this follows from the nontraditional view of fuzzy logic proposed by Zadeh in [3]. Hence, although not being explicitly mentioned as such, concepts like *fuzzy sets* [2], *intuitionistic fuzzy sets* [4,5], *Pythagorean fuzzy sets* [6,7] and *bipolar satisfaction degrees* [8,9], which are all connected to fuzzy set theory, can be applicable for the characterization of XBEs. For instance, consider that Pia is asked to assess the level to which each book in a collection, say $X = \{$‘*BK1*’, ‘*BK2*’, ‘*BK3*’$\}$, is deemed to be (compatible with her understanding of) *a book suitable for teenagers*. Using the fuzzy set concept and denoting her understanding of ‘a book suitable for teenagers’ by A, Pia can express an XBE for each book x in X by means of a *membership grade* [2], which is a real number in the unit interval [0, 1] that indicates the extent to which the proposition ‘x IS A’ is true – herein, the lowest and the highest extents are denoted by 0 and 1 respectively. This means that Pia’s XBEs can be characterized by membership grades like $\mu_{A@Pia}(\text{‘}BK1\text{’}) = 0.3$, $\mu_{A@Pia}(\text{‘}BK2\text{’}) = 0.6$ and $\mu_{A@Pia}(\text{‘}BK3\text{’}) = 0.8$ respectively. These membership grades can be included into a fuzzy set of books suitable for teenagers, say $A_{@Pia}$, which can mathematically be denoted by

$$A_{@Pia} = \{\langle x, \mu_{A@Pia}(x)\rangle \mid (x \in X) \wedge (0 < \mu_{A@Pia}(x) \leq 1)\}.$$

In this case, it is implicitly assumed that the context of an XBE is irrelevant – notice that nothing is said about the features that Pia focused on for the evaluation of each book. Because of this, this kind of characterization of XBEs can be acceptable when a *homogeneous* group of people, i.e., a group consisting of persons having the same (or very similar) understanding of the topic under evaluation, are asked to perform the evaluations.

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In other cases, where XBEs are given by persons with different understandings, it is useful to record hints about the context of their appraisals. For such cases, *augmented appraisal degrees* (AADs) and *augmented Atanassov (intuitionistic) fuzzy sets* (AAIFSs) are proven to be more suitable for representing XBEs [1]. If a membership grade denotes the extent to which a proposition having the canonical form 'x IS A' is true, an AAD additionally hints *why* that proposition is deemed to be true. Therefore, Pia's XBEs can be characterized by AADs such as $\hat{\mu}_{A@Pia}(\text{'}BK1\text{'}) = \langle 0.3, \{\text{'}\textit{friendly writing style}\text{'}\}\rangle$, $\hat{\mu}_{A@Pia}(\text{'}BK2\text{'}) = \langle 0.6, \{\text{'}\textit{simple plot}\text{'}\}\rangle$ and $\hat{\mu}_{A@Pia}(\text{'}BK3\text{'}) = \langle 0.8, \{\text{'}\textit{vivid descriptions}\text{'}, \text{'}\textit{intriguing story}\text{'}\}\rangle$ respectively. In this case, for example, the AAD of *BK1*, namely $\hat{\mu}_{A@Pia}(\text{'}BK1\text{'})$, is a pair $\langle \mu_{A@Pia}(\text{'}BK1\text{'}), F_{\mu_A@Pia}(\text{'}BK1\text{'})\rangle$ denoting the level, namely $\mu_{A@Pia}(\text{'}BK1\text{'}) = 0.3$, to which the proposition '$BK1$ IS A' is true, as well as the particular collection of *BK1*'s features, namely $F_{\mu_A@Pia}(\text{'}BK1\text{'}) = \{\text{'}\textit{friendly writing style}\text{'}\}$, on which Pia focuses her attention to appraise '*BK1* IS A' from her perspective.

Considering the *degree-of-similarity* semantic interpretation of a membership grade [10], we can also say that Pia evaluates the aforementioned proposition for each book according to which of its features are similar to the features in a *prototype* of a book that represents her understanding of *a book suitable for teenagers*. Hence, we can represent Pia's XBEs by means of an *augmented appraisal function* (AAF) [1], which, in this case, is a mapping $\hat{\mu}_{A@Pia} : X \to \langle I, \mathcal{F}\rangle$ that denotes the correspondence between each book x_i in $X = \{x_1, \dots, x_n\}$ and its AAD $\hat{\mu}_{A@Pia}(x_i) = \langle \mu_{A@Pia}(x_i), F_{\mu_A@Pia}(x_i)\rangle$. In this mapping, I represents the unit interval [0, 1] and it is assumed that $\mathcal{F} = \mathcal{F}_1 \cup \dots \cup \mathcal{F}_n$, where $\mathcal{F}_i$ denotes the collection of x_i's features – this means that $F_{\mu_A@Pia}(x_i) \subseteq \mathcal{F}_i$ holds. As can be noticed, while an AAD is a generalization of a membership grade, an AAF is a generalization of a *membership function* [2]. Thus, the correspondence between each x_i in X and $\hat{\mu}_{A@Pia}(x_i)$ can also be represented by

$$\hat{A}_{@Pia} = \{\langle x_i, \hat{\mu}_{A@Pia}(x_i)\rangle \mid (x_i \in X) \wedge (\hat{\mu}_{A@Pia}(x_i) \in \langle I, \mathcal{F}\rangle)\}.$$

The above-mentioned idea, as well as the fact the evaluation of a proposition like '$BK1$ IS A' can simultaneously include positive and negative aspects detected in $BK1$, have been considered in [1] to incorporate AADs into the *intuitionistic fuzzy set concept* [4,5] as follows:

Definition 1 *(Augmented Atanassov (Intuitionistic) Fuzzy Set [1]).* Consider a concept A and a person P. Consider also a collection $X = \{x_1, \dots, x_n\}$, where each $x_i \in X$ has collection of features $\mathcal{F}_i$. Additionally, consider two propositions, say p and $\bar{p}$, which have the canonical forms 'x_i IS A' (meaning "x_i is *compatible* with the understanding of A") and 'x_i IS NOT A' (meaning "x_i is *incompatible* with the understanding of A") respectively. Assume $I = [0, 1]$ and $\mathcal{F} = \mathcal{F}_1 \cup \dots \cup \mathcal{F}_n$. Let $\hat{\mu}_{A@P}(x_i) = \langle \mu_{A@P}(x_i), F_{\mu_A@P}(x_i)\rangle$ and $\hat{\nu}_{A@P}(x_i) = \langle \nu_{A@P}(x_i), F_{\nu_A@P}(x_i)\rangle$ in $\langle I, \mathcal{F}\rangle$ be two AADs characterizing the evaluations done by P of p and $\bar{p}$ respectively. In this context, a collection $\hat{A}_{@P}$ that denotes the correspondence between each $x_i \in X$ and both $\hat{\mu}_{A@P}(x_i)$ and $\hat{\nu}_{A@P}(x_i)$ such that

$$\hat{A}_{@P} = \left\{\langle x_i, \hat{\mu}_{A@P}(x_i), \hat{\nu}_{A@P}(x_i)\rangle \mid (x_i \in X) \wedge (0 \le \mu_{A@P}(x_i) + \nu_{A@P}(x_i) \le 1)\right\}, \tag{1}$$

is called an *augmented Atanassov fuzzy set*, AAIFS for short.

Along with this definition, an *augmented hesitation margin* expressed by

$$\hat{h}_{A@P}(x_i) = \langle h_{A@P}(x_i), F_{h_A@P}(x_i)\rangle \tag{2}$$

has been proposed to represent the lack of knowledge or hesitation that P might have during the evaluation of 'x_i IS A' and/or 'x_i IS NOT A'. Here, $F_{h_A@P}(x_i)$ is a collection that might contain features in x_i causing such hesitation during the judgment, and $h_{A@P}(x_i)$ is given by

$$h_{A@P}(x_i) = 1 - \mu_{A@P}(x_i) - \nu_{A@P}(x_i). \tag{3}$$

By way of illustration, consider that Pia has expressed by means of a form like the one depicted in Fig. 2 the following judgments – the interested reader is referred to [11] for a description of a pilot test in which a similar form has been used for getting XBEs from a heterogeneous group of people:

- *BK1* has a 0.3-level of compatibility with *a book suitable for teenagers* because of the friendly writing style; but, due to *BK1* has a boring plot, it has also a 0.5-level of incompatibility with that concept.

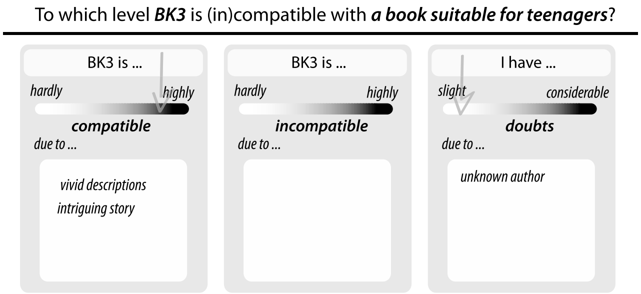

Fig. 2. A form for filling out an XBE.

- *BK2* has a 0.6-level of compatibility (with *a book suitable for teenagers*) because it has a simple plot; however, due to *BK2* has mathematical expressions, it has also a 0.1-level of incompatibility.
- *BK3* has a 0.8-level of compatibility because of the vivid descriptions and the intriguing story; however, the author of *BK3* is unknown.

In this case, these judgments can be characterized as an AAIFS, say $\hat{A}_{@Pia}$, such that

$$\begin{aligned}\hat{A}_{@Pia} = \Big\{ & \Big\langle \text{‘}BK1\text{’}, \langle 0.3, \{\text{‘}\textit{friendly writing style}\text{’}\}\rangle\ \langle 0.5, \{\text{‘}\textit{boring plot}\text{’}\}\rangle \Big\rangle, \\ & \Big\langle \text{‘}BK2\text{’}, \langle 0.6, \{\text{‘}\textit{single plot}\text{’}\}\rangle\ \langle 0.1, \{\text{‘}\textit{mathematical expressions}\text{’}\}\rangle \Big\rangle, \\ & \Big\langle \text{‘}BK3\text{’}, \langle 0.8, \{\text{‘}\textit{vivid descriptions}\text{’}, \text{‘}\textit{intriguing story}\text{’}\}\rangle, \langle 0, \{\}\rangle \Big\rangle \Big\}.\end{aligned}$$

Here, the hesitation from Pia to judge, e.g., ‘*BK3*’ as ‘*a book suitable for teenagers*’ could be

$$\hat{h}_{A@Pia}(\text{‘}BK3\text{’}) = \langle 0.2, \{\text{‘}\textit{unknown author}\text{’}\}\rangle.$$

Notice that, by means of $\hat{A}_{@Pia}$, one can simultaneously characterize all the XBEs performed by Pia about the compatibility and incompatibility of ‘*BK1*’, ‘*BK2*’ and ‘*BK3*’ with the idea of *a book suitable for teenagers*.

2.2. Comparison of XBEs

A comparison between two XBEs can be influenced by how these XBEs are characterized. As an example, consider that another person, say Rod, is asked to assess the level to which each book in $X = \{\text{‘}BK1\text{’}, \text{‘}BK2\text{’}, \text{‘}BK3\text{’}\}$ is deemed to be (compatible with his understanding of) *a book suitable for teenagers*. Consider also that Rod has answered the following:

- *BK1* has a 0.3-level of compatibility with *a book suitable for teenagers* due to its motivational plot; but, since *BK1* contains crayon drawings, it has also a 0.5-level of incompatibility with that concept.
- *BK2* has a 0.6-level of compatibility because it contains mathematical riddles; however, due to *BK2* contains colorful drawings, it has also a 0.3-level of incompatibility.
- *BK3* has a 0.8-level of compatibility because of the intriguing story; however, it has as 0.2-level of incompatibility due to it contains street slang.

Assuming that Pia and Rod share a similar understanding of a book suitable for teenagers, someone might consider the context of each XBE to be irrelevant and, thus, he/she might represent Rod's XBEs by membership grades like $\mu_{A@Rod}(\text{‘}BK1\text{’}) = 0.3$, $\mu_{A@Rod}(\text{‘}BK2\text{’}) = 0.6$ and $\mu_{A@Rod}(\text{‘}BK3\text{’}) = 0.8$ respectively – recall that, by "context of an XBE" we mean the conditions that arise when the XBE is performed, which, in this case, mainly depend on the understanding of a book suitable for teenagers possessed by Rod. If this is the case, Pia's and Rod's XBEs will match – notice that the expressions $\mu_{A@Pia}(\text{‘}BK1\text{’}) = \mu_{A@Rod}(\text{‘}BK1\text{’})$, $\mu_{A@Pia}(\text{‘}BK2\text{’}) = \mu_{A@Rod}(\text{‘}BK2\text{’})$ and $\mu_{A@Pia}(\text{‘}BK3\text{’}) = \mu_{A@Rod}(\text{‘}BK3\text{’})$ hold. However, if it is assumed that Pia and Rod may have different understandings of a book suitable for teenagers, then the context of an XBE will be relevant. Hence, AADs like $\hat{\mu}_{A@Rod}$('*BK1*') $= \langle 0.3, \{$'*motivational plot*'$\}\rangle$, $\hat{\mu}_{A@Rod}$('*BK2*') $= \langle 0.6, \{$ '*mathematical riddles*' $\}\rangle$ and $\hat{\mu}_{A@Rod}$('*BK3*') $= \langle 0.8, \{$'*intriguing story*' $\}\rangle$ can be used and, thus, Pia's and Rod's XBEs will look dissimilar to each other. Such dissimilarity will be more evident if Rod's XBEs are characterized as an AAIFS, say $\hat{A}_{@Rod}$, such that

$$\begin{aligned}\hat{A}_{@Rod} = \Big\{ &\Big\langle \text{'}\mathit{BK1}\text{'}, \langle 0.3, \{\text{'}\mathit{motivational\ plot}\text{'}\}\rangle\ \langle 0.5, \{\text{'}\mathit{crayon\ drawings}\text{'}\}\rangle \Big\rangle,\\ &\Big\langle \text{'}\mathit{BK2}\text{'}, \langle 0.6, \{\text{'}\mathit{mathematical\ riddles}\text{'}\}\rangle\ \langle 0.3, \{\text{'}\mathit{colorful\ drawings}\text{'}\}\rangle \Big\rangle,\\ &\Big\langle \text{'}\mathit{BK3}\text{'}, \langle 0.8, \{\text{'}\mathit{intriguing\ story}\text{'}\}\rangle\ , \langle 0.2, \{\text{'}\mathit{street\ slang}\text{'}\}\rangle \Big\rangle \Big\}.\end{aligned}$$

As suggested above, XBEs given by a heterogeneous group of people can be better compared if some hints of their contexts are included into their characterization. This idea has been considered in [1] to propose an *augmented framework* consisting of several concepts and tools designed to process XBEs characterized as AADs or AAIFSs. In that framework the context of an XBE, which is shaped by a collection of features that hint the reasons of the appraisal, is taken into account for computation. Hence, it is deemed to be a framework that enables *augmented (fuzzy) computation* on XBEs.

Among other concepts included in that augmented framework, one can find the definition of a *connotation alikeness factor* (CAF), which, in its simplest form, is an indicator of the perceived similarity between the contexts of two AADs. For instance, given that the contexts of $\hat{\mu}_{A@Pia}$('*BK3*') and $\hat{\mu}_{A@Rod}$('*BK3*') are shaped by $F_{\mu_A@Pia}$('*BK3*') and $F_{\mu_A@Rod}$('*BK3*') respectively, a CAF between $\hat{\mu}_{A@Pia}$('*BK3*') and $\hat{\mu}_{A@Rod}$('*BK3*') can be established according to level to which $F_{\mu_A@Pia}$('*BK3*') $= \{$'*vivid descriptions*', 'intriguing story'$\}$ and $F_{\mu_A@Rod}$('*BK3*') $= \{$'*intriguing story*'$\}$ are perceived as similar. Such a level can be quantified by means of a number $\Delta_{\mu_A} \in [0, 1]$, where 0 and 1 denote the lowest and the highest levels of similarity respectively. Taken the perspective of Pia as a reference, someone can establish $\Delta_{\mu_A:(Pia,Rod)@Pia} = 0.5$ as a CAF between $\hat{\mu}_{A@Pia}$('*BK3*') and $\hat{\mu}_{A@Rod}$('*BK3*') because Rod has only focused on one out of two *BK3*'s features focused by Pia. Likewise, someone can take the perspective of Rod as reference and establish $\Delta_{\mu_A:(Pia,Rod)@Rod} = 1$ as a CAF between $\hat{\mu}_{A@Pia}$('*BK3*') and $\hat{\mu}_{A@Rod}$('*BK3*') because the only feature that Rod focused on is included into the *BK3*'s features that Pia focused on. Notice that the value of a CAF will depend on the perspective that is taken as a reference, i.e., a CAF is *directional*.

In a more complex form, a CAF can be seen as an indicator of the perceived similarity between the contexts of two AAIFSs. In this case, the value of a CAF can be established according to the perceived level of similarity between collections containing *all* the features recorded in each AAIFS. To illustrate this, consider that $\mathbf{F}_{\mu_A@P}(X)$ and $\mathbf{F}_{\mu_A@Q}(X)$ contain the features recorded in the AADs of the proposition 'x_i IS A' for each object $x_i \in X$ included into the AAIFSs $\hat{A}_{@P}$ and $\hat{A}_{@Q}$ respectively, i.e., $\mathbf{F}_{\mu_A@P}(X) = F_{\mu_A@P}(x_1) \cup \cdots \cup F_{\mu_A@P}(x_n)$ and $\mathbf{F}_{\mu_A@Q}(X) = F_{\mu_A@Q}(x_1) \cup \cdots \cup F_{\mu_A@Q}(x_n)$. Under this consideration, a *membership CAF* between $\hat{A}_{@P}$ and $\hat{A}_{@Q}$, say $\Delta_{\mu_A:(P,Q)@P}$, will indicate from the perspective of P how similar the collections $\mathbf{F}_{\mu_A@P}(X)$ and $\mathbf{F}_{\mu_A@Q}(X)$ are. Likewise, a *nonmembership CAF*, say $\Delta_{\nu_A:(P,Q)@P}$, will indicate from P's perspective how similar $\mathbf{F}_{\nu_A@P}(X)$ and $\mathbf{F}_{\nu_A@Q}$(X) are, where $\mathbf{F}_{\nu_A@P}(X) = F_{\nu_A@P}(x_1) \cup \cdots \cup F_{\nu_A@P}(x_n)$ and $\mathbf{F}_{\nu_A@Q}(X) = F_{\nu_A@Q}(x_1) \cup \cdots \cup F_{\nu_A@Q}(x_n)$.

A comparison between two XBEs also involves a comparison of their appraisal levels. Hence, together with the definition of a CAF, additional tools have been proposed in the aforementioned augmented framework for comparing the appraisal levels recorded in AADs or AAIFSs. One of those tools is the '*as seen from*' operator, $\lfloor\cdot\rfloor$, which is needed to determine how an AAD looks like when it is seen from a particular perspective. For instance, one of the AADs given by Pia, say $\hat{\mu}_{A@Pia}$(*'BK1'*), can be seen from Rod's perspective as $\lfloor\hat{\mu}_{A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod}$. Here, $\lfloor\hat{\mu}_{A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod}$ has the form $\langle\lfloor\mu_{A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod}, \lfloor F_{\mu_A@Pia}\rfloor_{@Rod}\rangle$, where $\lfloor\mu_{A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod}$ and $\lfloor F_{\mu_A@Pia}\rfloor_{@Rod}$ correspond to $\mu_{A@Pia}$('*BK1*') and $F_{\mu_A@Pia}$ respectively as both are seen from the perspective of Rod. Moreover, the equation

$$\lfloor\hat{\mu}_{A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod} = \langle\Delta_{\mu_A:(Pia,Rod)@Rod} \cdot \mu_{A@Pia}(\text{'}\mathit{BK1}\text{'}), F_{\mu_A@Rod}(\text{'}\mathit{BK1}\text{'})\rangle$$

can be used for computing the value of $\lfloor\hat{\mu}_{A@Pia}(x)\rfloor_{@Rod}$. The idea behind this equation is that, when $\mu_{A@Pia}$('*BK1*') is seen from the perspective of Rod, the collection of features detected by him should be taken into account for the appraisal (i.e., $\lfloor F_{\mu_A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod} = F_{\mu_A@Rod}(\text{'}\mathit{BK1}\text{'})$). Thus, $\lfloor\mu_{A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod}$ will depend not only on the value of $\mu_{A@Pia}$('*BK1*') but also on how similar $F_{\mu_A@Pia}$('*BK1*') and $F_{\mu_A@Rod}$('*BK1*') are, i.e., $\lfloor F_{\mu_A@Pia}(\text{'}\mathit{BK1}\text{'})\rfloor_{@Rod}$ will also depend on $\Delta_{\mu_A:(Pia,Rod)@Rod}$.

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Regarding the comparison of the appraisal levels in two AAIFSs, an ℓ-*measure*, defined by

$$\mathrm{sim}^{\alpha}_{\ell @ P}(\hat{A}_{@P}, \hat{A}_{@Q}) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left|\mathrm{dif}^{\alpha}_{\ell @ P}(\mathbf{p_i}, \mathbf{q_i})\right|, \tag{4}$$

has been proposed in [1] as an option to determine the *level* to which $\hat{A}_{@P}$ and $\hat{A}_{@Q}$ are similar as seen from the perspective of P. In this equation, $\mathrm{dif}^{\alpha}_{\ell @ P}(\mathbf{p_i}, \mathbf{q_i})$ represents the *spot-difference* [12,13] between $\mathbf{p_i}$ and $\mathbf{q_i}$ as seen from the perspective of P, which is given by

$$\mathrm{dif}^{\alpha}_{\ell @ P}(\mathbf{p_i}, \mathbf{q_i}) = \left(\mu_{A@P}(x_i) - \lfloor \mu_{A@Q}(x_i)\rfloor_{@P}\right) + \alpha\left(h_{A@P}(x_i) - \lfloor h_{A@Q}(x_i)\rfloor_{@P}\right), \tag{5}$$

where

$$\mathbf{p_i} = \begin{pmatrix} \mu_{A@P}(x_i) + \alpha h_{A@P}(x_i) \\ \nu_{A@P}(x_i) + (1-\alpha) h_{A@P}(x_i) \end{pmatrix} \tag{6}$$

and

$$\mathbf{q_i} = \begin{pmatrix} \mu_{A@Q}(x_i) + \alpha h_{A@Q}(x_i) \\ \nu_{A@Q}(x_i) + (1-\alpha) h_{A@Q}(x_i) \end{pmatrix} \tag{7}$$

are vector interpretations of the AAIFS-elements in $\hat{A}_{@P}$ and $\hat{A}_{@Q}$ respectively [13], $\alpha \in [0, 1]$ is a *hesitation splitter* [12,13], and

$$\lfloor h_{A@Q}(x_i)\rfloor_{@P} = 1 - \lfloor \mu_{A@Q}(x_i)\rfloor_{@P} - \lfloor \nu_{A@Q}(x_i)\rfloor_{@P} \tag{8}$$

is the hesitation margin from Q as seen from P. In this case, α splits any hesitation about the compatibility (or incompatibility) of x_i with the understanding of A: while the α por of the hesitation is added to the membership component, the $(1-\alpha)$ part is added to the nonmembership component. Since α semantically indicates in which proportion any hesitation will favor (or disfavor) the compatibility of x_i with A, the value of α can be set to reflect a particular comparison strategy: while a value close to 1 is assigned in a "pro membership strategy," a value close to 0 is assigned in a "pro nonmembership strategy" and a value close to 0.5 is assigned in a "neutral strategy."

Using $\Delta_{\mu_A:(P,Q)@P}$, $\Delta_{\nu_A:(P,Q)@P}$ and the above ℓ-measure, the comparison between $\hat{A}_{@P}$ and $\hat{A}_{@Q}$ can be denoted by a triplet $\langle \mathrm{sim}^{\alpha}_{\ell @ P}, \Delta_{\mu_A:(P,Q)@P}, \Delta_{\nu_A:(P,Q)@P}\rangle$ or by a fusion of its components such as the given by the equation

$$sim(\hat{A}_{@P}, \hat{A}_{@Q}) = (\lambda_\mu \Delta_{\mu_A:(P,Q)@P} + \lambda_\nu \Delta_{\nu_A:(P,Q)@P})\,\mathrm{sim}^{\alpha}_{\ell @ P}, \tag{9}$$

where $\lambda_\mu, \lambda_\nu \in [0, 1]$ and $\lambda_\mu + \lambda_\nu \leq 1$. As will be shown in Section 4, even though both representations can be used, the triplet $\langle \mathrm{sim}^{\alpha}_{\ell @ P}, \Delta_{\mu_A:(P,Q)@P}, \Delta_{\nu_A:(P,Q)@P}\rangle$ is preferred for comparisons between XBEs where not only the levels but also the contexts of such XBEs are needed.

It is worth mentioning that equations (4) and (9) follow a *directional* approach in which the similarity between, say, a and b is not necessarily equal to the similarity between b and a [14]. This means that $sim(\hat{A}_{@P}, \hat{A}_{@Q})$ is not necessarily equal to $sim(\hat{A}_{@Q}, \hat{A}_{@P})$. Nevertheless, in [1] it has been argued that $sim^{\alpha}_{\ell @ P}(\hat{A}_{@P}, \hat{A}_{@Q}) = sim^{\alpha}_{\ell @ P}(\hat{A}_{@Q}, \hat{A}_{@P})$ only holds when P and Q share the same of very similar understanding of A – the interested reader is referred to [15] for the description of an empirical study in which a directional similarity measure designed to compare two intuitionistic fuzzy sets has been proven to be suitable for comparing two collections of XBEs.

In the next section, we describe how the proposed *post-digest method* can be used for extracting artificial XBEs, which can be processed through the concepts and tools introduced in this section.

3. Post-digest method

As was mentioned in the introduction, we are interested in extracting experience-based evaluations (XBEs) regarding a given topic from posts on social media that are not necessarily related to that topic. Hence, in this section we describe the novel *post-digest method*, which digests messages posted by a person on social media to obtain a collection of XBEs as if they were given by this person regarding the topic (or concept) under consideration.

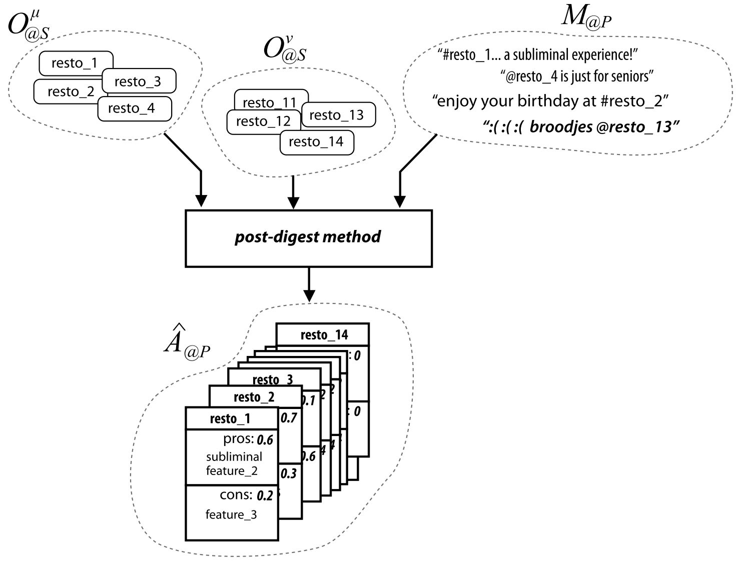


Fig. 3. A general view of the proposed post-digest method.

In a broad sense, the post-digest method uses the notion that the XBE of an object x with respect to a topic A can be seen inside the AAIFS framework as the evaluation of the truth value a proposition p having a canonical form 'x IS A' – recall from the previous section that the meaning of this proposition is "the value x is compatible with the understanding of A." Therefore, along with the collection of messages $M_{@P}$ posted by a person P, the post-digest method takes as input the collections $O^{\mu}_{@S}$ and $O^{\nu}_{@S}$, which contain identifiers of objects that make the proposition p true and false respectively according to an information seeker S. The method uses these inputs to obtain an AAIFS, say $\hat{A}_{@P}$, which characterizes a collection of artificial *performed-by-P* XBEs of the objects whose identifiers are in $O^{\mu}_{@S} \cup O^{\nu}_{@S}$. For instance, Fig. 3 illustrates a situation in which the post-digest method is used for the extraction of XBEs regarding *must-visit restaurants*. In this example, while the collection $O^{\mu}_{@S}$ contains names of restaurants like 'resto_1' or 'resto_2' that are compatible with the understanding of *must-visit restaurants* possessed by an information seeker S, the collection $O^{\nu}_{@S}$ contains names like 'resto_11' or 'resto_13' that are incompatible with such understanding. The names of these restaurants along with the collection $M_{@P}$, which contains messages such as "@resto_4 is just for seniors" posted by a person P, are the inputs of the post-digest method. In this case, an AAIFS $\hat{A}_{@P}$ containing artificial performed-by-P XBEs of the restaurants whose names are in $O^{\mu}_{@S} \cup O^{\nu}_{@S}$ is obtained as a result.

3.1. Idea behind the method

Since an XBE is deemed to be a judgment resulting from what one has learned or understood about a given concept by experience, it is necessary to draw an idea on *how a person could experience (or learn about) a concept*, and *how the experience (or knowledge) acquired by this person could then be reflected in his/her XBEs*.

To address the first question, we consider an intuition in which, *to learn about a given concept, one can study some objects that satisfy or dissatisfy an evaluation criterion related to that concept*. This means that one can look into the features of some objects that favor or disfavor the fulfillment of a criterion related to a concept to learn about the concept. For example, if one wants to learn about *green buildings* (i.e., buildings having features that are good for the environment), one can explore the features (e.g., building material, energy efficiency, water efficiency, etc.) of all the buildings in a particular region (i.e., buildings with a *green* label, and buildings without that label) to determine which of the features are in favor of a *green* label, and which are not.

To answer the second question, we consider a consequence of the previous intuition: *after experiencing with objects that satisfy or dissatisfy an evaluation criterion related to a concept, one obtains a particular knowledge that could be used for appraising the level to which other (new) objects satisfy or dissatisfy the specified criterion.* Hence, after learning which features favor or disfavor the fulfillment of an evaluation criterion, one can take into account those features for the evaluation of the level to which other objects fulfill that criterion.

3.1.1. A representational model

Relying on the above idea, when the fulfillment of a criterion like “be compatible with the understanding of a concept” is appraised on an object, some of its features will be more influential (favoring or disfavoring the fulfillment of this criterion) than others. Thus, we shall assume that the influence of a feature on the appraisal of a criterion depends on both its *weight* (or *relative importance*) and its *direction*: while the weight denotes *how influential (or important) a feature is in relation to the others*, the direction denotes *whether the feature’s influence is ‘in favor of’ or ‘in opposition to’ the criterion*. For instance, a person can consider the feature ‘A+ energy efficiency’ to be twice more influential than the feature ‘D- water efficiency’ to label a building as a *green building*. If so, this person can assign 2 and 1 as the weights of ‘A+ energy efficiency’ and ‘D- water efficiency’ respectively to denote their relative importance. Furthermore, this person can consider that the feature ‘A+ energy efficiency’ is in favor of such a label and, thus, he/she can say that influence of this feature is ‘in favor of’ putting the green label to buildings having this feature. Accordingly, to denote the influence of a feature on the appraisal of a criterion regarding the compatibility of an object with the understanding of a concept, a *feature-influence* representational model has been introduced in [16]. The characteristics and usage of this model are shown in the next example.

Consider a concept denoted by A. Let $X = \{x_1, \cdots, x_n\}$ be a collection of objects, where each $x_i \in X$ has a collection of features $\mathcal{F}_i$ – recall from Section 1 that by “*object*” is meant a person, a notion, or something that exists by itself. Also, let p be a proposition having the form ‘x_i IS A’ meaning “x_i is compatible with the understanding of A.” Assume $\mathcal{F} = \mathcal{F}_1 \cup \cdots \cup \mathcal{F}_n$ and that each f_j in $\mathcal{F}$ is related to a dimension in a m-dimension feature space. In this context, the influence of, e.g., $f_1, f_2 \in \mathcal{F}$ on the evaluation of (the truth value of) p can be represented as follows:

- The *resulting overall influence* of the features of an object x_i on the appraisal of p is represented by the vector $\mathbf{x}_i = \beta_{i,1}\hat{\mathbf{f}}_1 + \beta_{i,2}\hat{\mathbf{f}}_2$, where $\beta_{i,1}\hat{\mathbf{f}}_1$ and $\beta_{i,2}\hat{\mathbf{f}}_2$ represent the *overall influence* of the features f_1 and f_2 respectively – here, $\hat{\mathbf{f}}_1$ and $\hat{\mathbf{f}}_2$ are unit vectors that characterize the dimensions corresponding to f_1 and f_2, while $\beta_{i,1}$ and $\beta_{i,2}$ represent the *overall weights* of these features in x_i (see Fig. 4a).
- A particular understanding (or knowledge) of A, say K_A, which is characterized by $\hat{\mathbf{u}}_A$ and t_A, is depicted as a line. While the directional vector $\hat{\mathbf{u}}_A$ points towards a place where the compatibility with the understanding of A is favored, the threshold point t_A denotes a location where such compatibility is neither favored nor disfavored (see Fig. 4b).
- The *specific influence* of a feature, say f_1, on the appraisal of p is depicted as a *vector projection* of the component $\beta_{i,1}\hat{\mathbf{f}}_1$ in $\mathbf{x}_i$ on the line K_A. That is, the specific influence of f_1 corresponds to $\mathbf{f}_{i,1_A} = \beta_{i,1_A}\hat{\mathbf{u}}_A$, where $\beta_{i,1_A}$ represents the *specific weight* of this feature on such an appraisal. By way of illustration, Fig. 4c shows the specific influence of f_1, as well as the specific influence of f_2 on the line K_A. Notice in this figure that, while f_1 is “*in favor of*” the compatibility with the understanding of A because $\mathbf{f}_{i,1_A}$ and $\hat{\mathbf{u}}_A$ have the same direction, f_2 is “*in opposition to*” such compatibility because the directions of $\mathbf{f}_{i,2_A}$ and $\hat{\mathbf{u}}_A$ are opposite to each other.
- The *resulting specific influence of the features* f_1 and f_2 of the object x_i on the appraisal of p is represented as a vector $\mathbf{x}_{i_A}$ (see Fig. 4d). The level to which x_i is compatible (or incompatible) with the understanding of A is given by the magnitude of vector

  $$\mathbf{l}_{i_A} = \mathbf{x}_{i_A} - t_A\hat{\mathbf{u}}_A. \tag{10}$$

  If the direction of $\mathbf{l}_{i_A}$ is opposite to the direction of $\hat{\mathbf{u}}_A$, x_i will be *incompatible* at a level given by $||\mathbf{l}_{i_A}||$, where $||\mathbf{l}_{i_A}||$ is the magnitude of $\mathbf{l}_{i_A}$, i.e., $\sqrt{\mathbf{l}_{i_A} \cdot \mathbf{l}_{i_A}}$. On the other hand, if the directions of $\mathbf{l}_{i_A}$ and $\hat{\mathbf{u}}_A$ are the same, x_i will be *compatible* at a level given by $||\mathbf{l}_{i_A}||$. Modeling the resulting specific influence of the x_i’s features this way allows for the characterization of an XBE in a situation where positive and negative aspects of x_i are simultaneously taken into account (see Section 2).

As mentioned above, the feature-influence representational model is based on the idea of classifying an object according to the fulfillment of a criterion related to the compatibility of this object with the understanding of a concept.

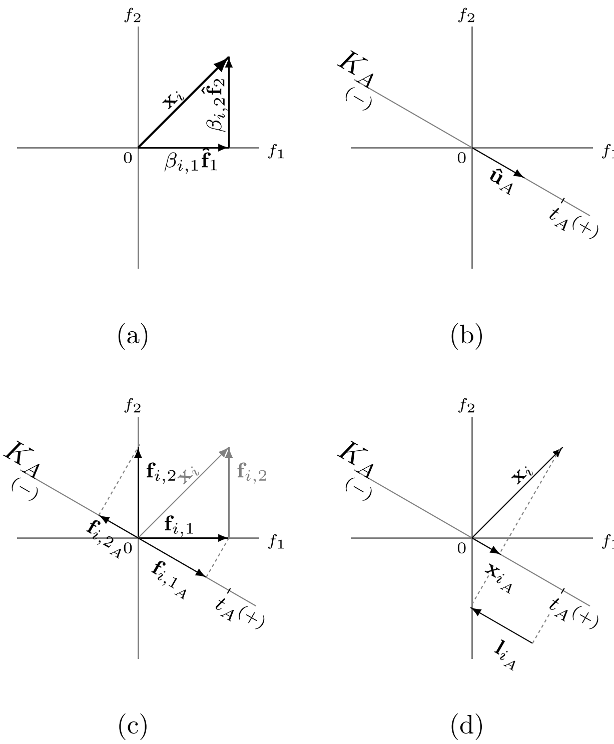


Fig. 4. Representational model of the influence of a feature on the appraisal of a criterion.

In this regard, this model is analogous to the *vector space models* that use the notion of classification. As an example, to classify documents in *information retrieval* [17], documents are represented as vectors in a common vector space where the words in a dictionary are the dimensions. As will be shown later in this section, this analogy allows us to use techniques connected with, e.g., *text categorization* [18,19] into the internal processes of the post-digest method. However, it is worth mentioning that some of those techniques might not be applicable for the comparison of XBEs due to their assumptions. For instance, according to the empirical study presented in [15], the assumption of symmetry makes a cosine similarity measure unsuitable for a comparison between two XBEs characterized as IFSs.

In the next part, the feature-influence representational model is used for describing how to extract XBEs with the proposed method.

3.2. Extracting experience-based evaluations

A message posted by a person may include some hints about his/her understanding of a particular concept. Among such hints, we could find the features that are relevant during the appraisal of the compatibility (or incompatibility) of one or more objects with such understanding. Hence, we could use those features to learn what would be this person's understanding of the concept. After learning so, we could use a model of that understanding (or knowledge) to evaluate the level to which other objects are compatible (or incompatible) with the actual understanding of the concept possessed by this person – i.e., we could obtain XBEs of those objects as if these XBEs had been performed by this person. We shall use this idea throughout the description of the post-digest method. Let us start this description with a formal statement of the problem regarding the extraction of XBEs from subjective social media content:

> Consider a concept A. Consider also two persons: an information seeker, say S, and an information source, say P. In addition, consider a collection $O_{@S} = \{oid_1, \cdots, oid_n\}$ that includes identifiers (e.g., names or codes) of objects that are known by S; consider that, while $O_{@S} = O^{\mu}_{@S} \cup O^{\nu}_{@S}$ holds, $O^{\mu}_{@S}$ and $O^{\nu}_{@S}$ contain, in that order, identifiers of objects that are *compatible* and *incompatible* with the understanding of A possessed by S. Consider finally a collection $M_{@P}$ consisting of messages posted by P. Let $\hat{A}_{@P}$ be an AAIFS that resembles

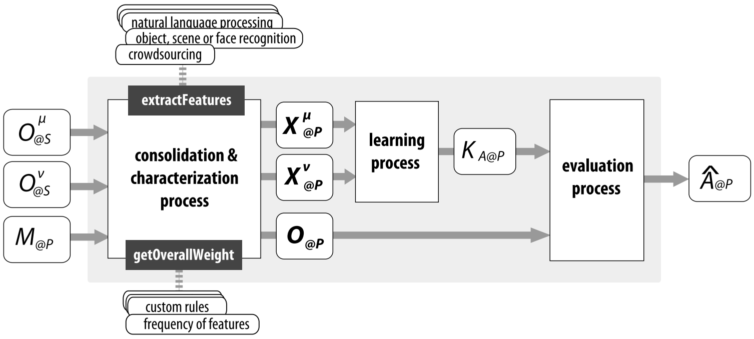
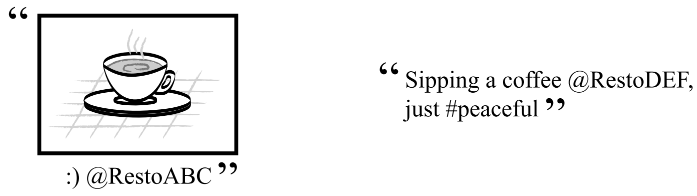


Fig. 5. Internal structure of the post-digest method.



(a) (b)

Fig. 6. Examples of posts on social media.

XBEs done by P of the objects whose identifiers are included in $O_{@S}$. Under these considerations, find $\hat{A}_{@P}$ through the messages in $M_{@P}$ that are related to the identifiers in $O_{@S}$.

To address this problem, we translate the idea described in Section 3.1 into the design of the post-digest method, whose internal structure is depicted in Fig. 5. As noticed, the post-digest method consists of three (sub) processes: (i) a consolidation and characterization process, or CC process for short; (ii) a learning process; and (iii) an evaluation process. In what follows, we describe each of them.

3.2.1. *Consolidation and characterization process*

The purpose of this process is to consolidate the features extracted from each *pertinent message* in $M_{@P}$ and characterize the objects whose identifiers are in $O^{\mu}_{@S} \cup O^{\nu}_{@S}$ – herein, by ‘*pertinent message*’ is meant a message related to any identifier in $O^{\mu}_{@S} \cup O^{\nu}_{@S}$. To that end, the CC process uses the identifiers in $O^{\mu}_{@S} \cup O^{\nu}_{@S}$ and the messages in $M_{@P}$ as input to obtain the collections $\mathbf{X}^{\mu}_{@P}$, $\mathbf{X}^{\nu}_{@P}$, $\mathbf{O}_{@P}$ as output (see Fig. 5). The collections $\mathbf{X}^{\mu}_{@P}$ and $\mathbf{X}^{\nu}_{@P}$ will contain the *overall influence vectors* that, according to the feature-influence representational model (see Section 3.1.1), characterize the overall influence of the features extracted for each pertinent message that is found in $M_{@P}$. With respect to $\mathbf{O}_{@P}$, this collection will contain overall influence vectors that characterize consolidated objects having the features extracted from each pertinent message. The steps of the CC process are implemented in Algorithm 1.

As can be noticed, the extraction of the features of each message is performed by the method *extractFeatures* (see Line 4). Since the content of a message posted on social media can be fairly diverse, the method *extractFeatures* can involve several techniques for the extraction. For instance, to extract the emoticon, the identifiers and the other terms included in the posts depicted in Figs. 6a and 6b, techniques connected with *natural language processing* [20] or *text categorization* [18,19] such as *tokenization*, *stemming*, *named entity extraction* or *stopword removal* [21,22] can be used; analogously, to extract the features of the picture included in the post depicted in Fig. 6a, techniques for object, scene or face recognition such as [23–25] can be applied. As will be shown in Section 4, the extraction of the features can also be performed by a group of (anonymous) persons using a *crowdsourcing* approach [26].

Algorithm 1: Consolidation and characterization process.

Data: $O^{\mu}_{@S}$, $O^{\nu}_{@S}$, $M_{@P}$
Result: $\mathbf{O}_{@P}$, $\mathbf{X}^{\mu}_{@P}$, $\mathbf{X}^{\nu}_{@P}$
1 $\mathbf{O}_{@P}, \mathbf{X}^{\mu}_{@P}, \mathbf{X}^{\nu}_{@P} \leftarrow \{\}$
2 $O_{@P}, X^{\mu}_{@P}, X^{\nu}_{@P} \leftarrow \{\}$
3 **foreach** $m \in M_{@P}$ **do**
4     $F \leftarrow$ `extractFeatures(`m`)`
5   **foreach** $oid \in (O^{\mu}_{@S} \cup O^{\nu}_{@S})$ **do**
6   **if** $oid \in F$ **then**
7             $o_k \leftarrow$ `getObject(`$oid, O_{@P}$`)`
8   **if** $o_k =$ NIL **then** $O_{@P} \leftarrow \{x : oid, \mathcal{F} : F\}$
9   **else** $o_k.\mathcal{F} \leftarrow o_k.\mathcal{F} \cup F$
10             $x_i \leftarrow \{x : i, \mathcal{F} : F\}$ `/* a pertinent message` m `is represented as an object` x_i `*/`
11   **if** $oid \in O^{\mu}_{@S}$ **then** $X^{\mu}_{@P} \leftarrow X^{\mu}_{@P} \cup \{x_i\}$
12   **if** $oid \in O^{\nu}_{@S}$ **then** $X^{\nu}_{@P} \leftarrow X^{\nu}_{@P} \cup \{x_i\}$
13 **foreach** $x_i \in (X^{\mu}_{@P} \cup X^{\nu}_{@P})$ **do**
14   **foreach** $f_j \in x_i.\mathcal{F}$ **do**
15         $\beta_{i,j} \leftarrow$ `getOverallWeight(`f_j, x_i`)`
16         $\mathbf{x}_i \leftarrow \mathbf{x}_i + \beta_{i,j}\hat{\mathbf{f}}_j$
17   **if** $x_i \in X^{\mu}$ **then** $\mathbf{X}^{\mu}_{@P} \leftarrow \mathbf{X}^{\mu}_{@P} \cup \{\mathbf{x}_i\}$
18   **if** $x_i \in X^{\nu}$ **then** $\mathbf{X}^{\nu}_{@P} \leftarrow \mathbf{X}^{\nu}_{@P} \cup \{\mathbf{x}_i\}$
19 **foreach** $o_k \in O_{@P}$ **do**
20   **foreach** $f_j \in o_k.\mathcal{F}$ **do**
21         $\beta_{k,j} \leftarrow$ `getOverallWeight(`f_j, o_k`)`
22         $\mathbf{o}_k \leftarrow \mathbf{o}_k + \beta_{k,j}\hat{\mathbf{f}}_j$
23     $\mathbf{O}_{@P} \leftarrow \mathbf{O}_{@P} \cup \{\mathbf{o}_k\}$
24 **return** $\mathbf{O}_{@P}$, $\mathbf{X}^{\mu}_{@P}$, $\mathbf{X}^{\nu}_{@P}$

Previous to the consolidation of the extracted features, the relevancy of a message is verified (see Line 6). If the identifier of an object in $O^{\mu}_{@S} \cup O^{\nu}_{@S}$ is part of the extracted features, the message will be considered related to this object and, thus, the extracted features will be considered (and consolidated) as features of this object (see Lines 8-9). These features will also be used to represent this (pertinent) message m as an object x_i (see Line 10). After that, x_i is put into either $X^{\mu}_{@P}$ or $X^{\nu}_{@P}$ depending on which collection the identifier of o_k is part of.

To characterize an object x_i in $X_{@P} = (X^{\mu}_{@P} \cup X^{\nu}_{@P})$ as a vector according to the feature-influence representational model, the overall weight of each feature f_j in $x_i.\mathcal{F}$ is computed by means of the method *getOverallWeight* (see Line 15) and, then, the corresponding overall influence vector is aggregated to the resulting overall influence vector $\mathbf{x}_i$ (see Line 16). The method *getOverallWeight* can be implemented following rules given by the information seeker or according to the frequency of the extracted features. In the former case, e.g., an information seeker can consider that the overall weight of a *hashtag*, i.e., a word preceded by '#', should be twice the overall weight of a (plain) word. In the latter case, the overall weight can be computed by, e.g., the equation

$$weight(f_j, x_i) = \begin{cases} \big(1 + \ln n(f_j, x_i)\big) \ln\big(|X_{@P}|/n(f_j, X_{@P})\big) & : \text{if } \big(n(f_j, x_i) > 0\big) \wedge \\ & \quad \big(n(f_j, X_{@P}) > 0\big); \\ 0 & : \text{otherwise}; \end{cases} \tag{11}$$

proposed in [27] where $n(f_j, x_i)$ is the number of occurrences of f_j in x_i, $n(f_j, X_{@P})$ is the number of objects in $X_{@P}$ that contain f_j, and $|X_{@P}|$ is the number of objects in $X_{@P}$. Similar steps are followed to characterize all the objects in $O_{@P}$ as vectors (see Lines 19-23). Finally, all the vector collections are returned in Line 24.

As might be noticed, an important assumption of this method is that a pertinent message is posted by a person, say P, as an "answer" to an evaluation request, which is "submitted" by an information seeker S. Hence, the content of this message would reflect (some of) the features that support the "answer" and, thus, these features could be used to learn what would be P's understanding of the concept behind the evaluation request – this is one of the reasons why in Line 10 a pertinent message is represented as an object that will be part of the next learning process.

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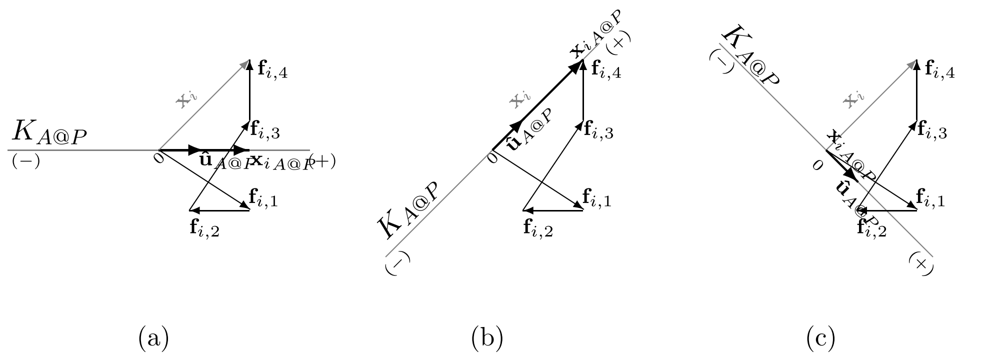

Fig. 7. Varying the resulting specific influence of the features f_1, f_2, f_3 and f_4 of an object x_i on the appraisal of a proposition, say p, with the form 'x_i IS A' meaning "x_i is compatible with the understanding of A."

3.2.2. Learning process

The aim of the learning process is to obtain a model, say $K_{A@P}$, that represents the understanding of (concept) A possessed by (person) P. To do so, the method uses the collections $\mathbf{X}^{\mu}_{@P}$ and $\mathbf{X}^{\nu}_{@P}$, which contain the vectors that represent the pertinent messages posted by P, as *training collections* (see Fig. 5).

Following the idea presented in Section 3.1, this method tries to mimic a learning behavior where one can learn about A by studying (the features of) the objects in $\mathbf{X}^{\mu}_{@P}$ and $\mathbf{X}^{\nu}_{@P}$ which are respectively compatible and incompatible with the understanding of A. Thus, the main step of this method can be stated as follows:

> Compute the constituents of $K_{A@P}$, i.e., $\hat{\mathbf{u}}_{A@P} = \omega_1\hat{\mathbf{f}}_1 + \cdots + \omega_m\hat{\mathbf{f}}_m$ and $t_{A@P}$, in such a way that (i) the correspondence between each $\mathbf{x}_i \in \mathbf{X}^{\mu}_{@P}$ (or $\mathbf{x}_i \in \mathbf{X}^{\nu}_{@P}$), as well as the resulting specific influence of its features are preserved, and (ii) both the vector sum of the specific influences of the features of objects in $\mathbf{X}^{\mu}_{@P}$ and the vector sum of the specific influences of the features of objects in $\mathbf{X}^{\nu}_{@P}$ are maximized.

To illustrate how this step works, let us visualize it through the example presented in Fig. 7 where the following have been depicted:

- a line $K_{A@P}$ that represents the understanding of A possessed by P and is characterized by a directional vector $\hat{\mathbf{u}}_{A@P}$ and a threshold $t_{A@P}$ – for readability, the threshold $t_{A@P}$ has not been depicted;
- a vector $\mathbf{x}_i = \mathbf{f}_{i,1} + \mathbf{f}_{i,2} + \mathbf{f}_{i,3} + \mathbf{f}_{i,4}$ that represents the *resulting overall influence* of the features of x_i, namely f_1, f_2, f_3 and f_4, on the appraisal of a proposition p having the form 'x_i IS A' meaning "x_i is compatible with the understanding of A," and
- a vector $\mathbf{x}_{i\,A@P} = \mathbf{f}_{i,1\,A@P} + \mathbf{f}_{i,2\,A@P} + \mathbf{f}_{i,3\,A@P} + \mathbf{f}_{i,4\,A@P}$ that represents the *resulting specific influence* of the features of x_i on the appraisal of p – for readability, the vectors $\mathbf{f}_{i,1\,A@P}$, $\mathbf{f}_{i,2\,A@P}$, $\mathbf{f}_{i,3\,A@P}$ and $\mathbf{f}_{i,4\,A@P}$ have not been depicted.

During a learning process, $\hat{\mathbf{u}}_{A@P}$ and $t_{A@P}$ can be varied in order to increase (or decrease) the resulting specific influence of the features f_1, f_2, f_3 and f_4 on the appraisal of p. For instance, turning $\hat{\mathbf{u}}_{A@P}$ counterclockwise as shown in Fig. 7b yields an increment of the resulting specific influence of these features, i.e., the magnitude of $\mathbf{x}_{i\,A@P}$ increases. In contrast, turning $\hat{\mathbf{u}}_{A@P}$ clockwise as shown in Fig. 7c makes the resulting influence of these features disappears, i.e., the magnitude of $\mathbf{x}_{i\,A@P}$ becomes 0. The idea here is to find potential *suitable* directional vectors and threshold points for a given concept, where by '*suitable*' is meant that the resulting specific influence of the features of each x_i in a training collection must correspond to the appraisal given for p. Thus, e.g., if x_i is an object that makes (the value of) p true, i.e., $\mathbf{x}_i \in \mathbf{X}^{\mu}_{@P}$, the resulting specific influence of its features must be in favor of the compatibility of x_i with the understanding of A in such a way that the threshold $t_{A@P}$ is exceeded. After the potential suitable directional vectors and threshold points have been found, the *optimal couple*, say $\langle\hat{\mathbf{u}}_{A@P}, t_{A@P}\rangle$, which maximizes both the aggregate of the specific influences of the features in favor of the criterion and the aggregate of the specific influences of the features in opposition to this criterion, must be chosen.

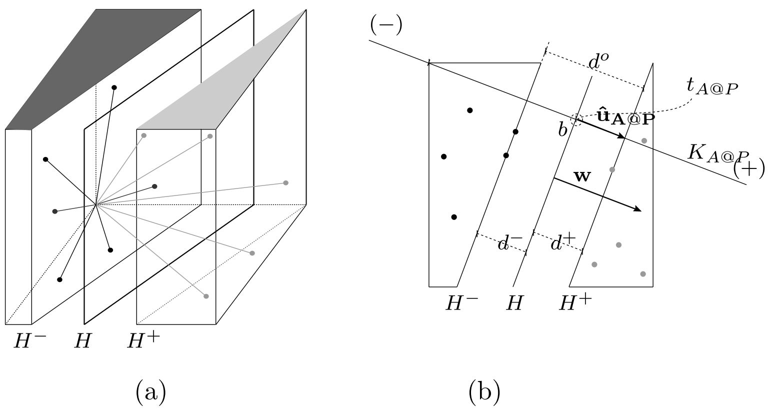

Fig. 8. *Support vector machines* can be used to compute the optimal couple $\langle \hat{\mathbf{u}}_{A@P}, t_{A@P} \rangle$.

To perform the computation of the optimal couple $\langle \hat{\mathbf{u}}_{A@P}, t_{A@P} \rangle$, one can use *support vector machines*, SVM for short [28,29], which has been successfully used for pattern recognition problems in statistical learning theory – see, e.g., the applications for optical character recognition [30], face detection [31], and text categorization [32].

A SVM is a classifier based on the idea that a *separating hyperplane*, i.e., a surface with m-1 dimensions that separates a m-dimensional space into two parts, can be used to categorize objects. Hence, to compute $\langle \hat{\mathbf{u}}_{A@P}, t_{A@P} \rangle$, we use a SVM as follows:

Let p be the aforementioned proposition and let $X_{@P} = X^{\mu}_{@P} \cup X^{\nu}_{@P}$ be a training collection where each $x_i \in X_{@P}$ is an object having a collection of features $\mathcal{F}_i$. Assume $\mathcal{F} = \mathcal{F}_1 \cup \cdots \cup \mathcal{F}_n$. Then, assume that each $x_i \in X_{@P}$ has a label $y_i \in \{-1, 1\}$ such that $y_i = 1$ indicates that x_i belongs to $X^{\mu}_{@P}$ (positive example) and $y_i = -1$ means that x_i belongs to $X^{\nu}_{@P}$ (negative example). Finally, assume that each $x_i \in X_{@P}$ is denoted by a vector $\mathbf{x}_i$ according to the feature-influence representational model. In this context, $\hat{\mathbf{u}}_{A@P}$ and $t_{A@P}$ are given by the equations

$$\hat{\mathbf{u}}_{A@P} = \frac{\mathbf{w}}{\|\mathbf{w}\|} \tag{12}$$

and

$$t_{A@P} = -\frac{b}{\|\mathbf{w}\|} \tag{13}$$

respectively, where both $\mathbf{w}$ and a term b define the hyperplane $H : \mathbf{w} \cdot \mathbf{x_i} + b$ that separates vectors $\mathbf{x}_i$ corresponding to positive examples and negative ones according to the SVM model.

The rationale behind the above equations is depicted in Fig. 8. In this figure, vectors corresponding to objects $x_i \in X^{\mu}_{@P}$ (positive examples depicted as gray circles) and objects $x_i \in X^{\nu}_{@P}$ (negative examples depicted as black circles) are presented in a 3D-view (a) and in a 2D-view (b). The hyperplane $H : \mathbf{w} \cdot \mathbf{x_i} + b$ separates the positive from the negative examples. The hyperplanes H^+ and H^- are parallel to H: while H^+ contains the closest positive example(s) to H, H^- contains the closest negative example(s) to H. The distance $d^o = d^+ + d^-$ between H^+ and H^- is the largest. The *support vectors* are the vectors whose tips lie either on H^- or H^+.

In that regard, finding an optimal hyperplane that separates objects belonging to a given category from others can be applied to maximize both the aggregate of the specific influences of the features in favor of the compatibility with the understanding of A and the aggregate of the specific influences of the features in opposition to such compatibility.

To find an optimal separating hyperplane, one can assume that $d^+ = d^- = 1$ and consider all the vectors $\mathbf{x_i} \in \mathbf{X}^{\mu}_{\mathbf{@P}} \cup \mathbf{X}^{\nu}_{\mathbf{@P}}$ satisfy the following constraints:

- if $\mathbf{x_i}$ belongs to $\mathbf{X}^{\mu}_{\mathbf{@P}}$ (i.e., $y_i = 1$), then

$$\mathbf{w} \cdot \mathbf{x_i} + b \geq 1; \tag{14}$$

- if $\mathbf{x_i}$ belongs to $\mathbf{X}^{\nu}_{\mathbf{@P}}$ (i.e., $y_i = -1$), then

$$\mathbf{w} \cdot \mathbf{x_i} + b \leq -1. \tag{15}$$

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Thus, one can maximize $d^o = \frac{2}{||\mathbf{w}||}$. However, for simplicity, instead of maximizing $\frac{2}{||\mathbf{w}||}$ minimizing $\frac{1}{2}\|\mathbf{w}\|^2$ is preferred:

Find $\mathbf{w}$ and b such that $\frac{1}{2}\|\mathbf{w}\|^2$ is minimized and the constraints (14) and (15) hold for each vector $\mathbf{x_i}$.

This is a problem in which a quadratic function is optimized subject to linear constraints. The solution involves first switching to the following Lagrangian formulation of the problem [33]:

Let $\lambda_1, \cdots, \lambda_n$ be positive Lagrange multipliers, where a multiplier λ_i is associated to each $y_i(\mathbf{w} \cdot \mathbf{x_i} + b) - 1 \geq 0$, which is a "compact" version of the aforementioned constraints; and let Λ be a Lagrangian defined by

$$\Lambda = \frac{1}{2}\|\mathbf{w}\|^2 - \sum_{i=1}^{n} \lambda_i \left(y_i(\mathbf{w} \cdot \mathbf{x}_i + b) - 1\right). \tag{16}$$

Find $\mathbf{w}$, b and all the λ_i such that Λ is minimized.

This problem is then reformulated to the following equivalent dual problem [33]:

Find $\lambda_1, \cdots, \lambda_n$ such that the gradient of Λ with respect to $\mathbf{w}$ and b yields zero, and Λ is maximized.

The condition for the gradient of Λ results in

$$\mathbf{w} = \sum_{i=1}^{n} \lambda_i y_i \mathbf{x}_i \tag{17}$$

and

$$\sum_{i=1}^{n} \lambda_i y_i = 0, \tag{18}$$

which are replaced in (16) to obtain

$$\Lambda = \sum_{i=1}^{n} \lambda_i - \frac{1}{2} \sum_{i=1,k=1}^{n} \lambda_i \lambda_k y_i y_k (\mathbf{x}_i \cdot \mathbf{x}_k). \tag{19}$$

Notice that in this formulation Λ is maximized with respect to the multipliers λ_i subject to (18) and $\lambda_i \geq 0$. The solution here is given by both Equation (17) and

$$b = y_i - \mathbf{w} \cdot \mathbf{x}_i, \tag{20}$$

for any $\mathbf{x}_i$ such that $\lambda_i > 0$. Notice also that there is a multiplier λ_i for each $\mathbf{x}_i$. In the solution, the vectors $\mathbf{x}_i$ having multipliers $\lambda_i > 0$ are the *support vectors*support vectors – in [34], a software tool called *SVMLight* has been provided to perform the computation of both Equation (17) and Equation (20).

It is worth mentioning that, when the separating hyperplane H is not linear, the Equation (19) can be rewritten as

$$\Lambda = \sum_{i=1}^{n} \lambda_i - \frac{1}{2} \sum_{i=1,k=1}^{n} \lambda_i \lambda_k y_i y_k (K(\mathbf{x}_i, \mathbf{x}_k)). \tag{21}$$

In this equation, K is a *kernel function* [28,29] that computes the inner product (or an interpretation of similarity) between $\mathbf{x}_i$ and $\mathbf{x}_k$ in a higher dimensional space in which H is linear. For instance, K can be a *linear kernel* defined by $K(\mathbf{x}_i \cdot \mathbf{x}_k) = \mathbf{x}_i \cdot \mathbf{x}_k$ or a *polynomial kernel* of degree d defined by $K(\mathbf{x}_i \cdot \mathbf{x}_k) = (\mathbf{x}_i \cdot \mathbf{x}_k + 1)^d$.

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3.2.3. Evaluation process

The purpose of the evaluation process is to determine the level to which each object x_i in $\mathbf{O}_{@P} = \mathbf{O}^{\mu}_{@P} \cup \mathbf{O}^{\nu}_{@P}$ is compatible (or incompatible) with the understanding of A possessed by P, i.e., $K_{A@P}$ (see Fig. 5). As was mentioned in Section 3.1.1, such level of compatibility (or incompatibility) corresponds to the level to which the *resulting specific influence* of its features exceeds (or is below) the threshold $t_{A@P}$, which is part of $K_{A@P}$. Thus, according to (10), the evaluation process consists of the computation of $\mathbf{l}_{i\,A@P} = \mathbf{x}_{i\,A@P} - t_A \hat{\mathbf{u}}_{A@P}$ for each $\mathbf{x}_i \in \mathbf{O}_{@P}$, where

$$\mathbf{x}_{i\,A@P} = \left(\mathbf{x}_i \cdot \hat{\mathbf{u}}_{A@P}\right) \hat{\mathbf{u}}_{A@P}. \tag{22}$$

However, since an AAIFS, say $\hat{A}_{@P}$, is expected as result, it is needed to represent each evaluation as an AAIFS element, say $\langle x_i, \hat{\mu}_{A@P}(x_i), \hat{\nu}_{A@P}(x_i)\rangle$ – recall from Definition 1 in Section 2.1 that $\hat{\mu}_{A@P}(x_i)$ and $\hat{\nu}_{A@P}(x_i)$ are two AADs, namely $\langle \mu_{A@P}(x_i), F_{\mu_A@P}(x_i)\rangle$ and $\langle \nu_{A@P}(x_i), F_{\nu_A@P}(x_i)\rangle$ respectively, where $\mu_{A@P}(x_i)$ and $\nu_{A@P}(x_i)$ represent, in that order, the levels of compatibility and incompatibility of x_i with the understanding of A, and $F_{\mu_A@P}(x_i)$ and $F_{\nu_A@P}(x_i)$ represent two collections that include features of x_i that justify such levels. To obtain such AAIFS elements, we use the procedure presented in [16] as follows:

Let $O_{@P} = \{x_1, \cdots, x_n\}$ be the collection of the objects characterized by the vectors in $\mathbf{O}_{@P}$. Let x_i be one of the objects in $O_{@P}$ and let $\mathcal{F}_i$ be a collection of the features of x_i. Assume $\mathcal{F} = \mathcal{F}_1 \cup \cdots \cup \mathcal{F}_n$. In this context, to identify whether $f_j \in \mathcal{F}$ is part of $F_{\mu_A@P}(x_i)$ or $F_{\nu_A@P}(x_i)$, we compute the *specific influence* of f_j, i.e., $\mathbf{f}_{i,j_{A@P}} = \beta_{i,j\,A@P}\hat{\mathbf{u}}_{A@P}$, where $\beta_{i,j\,A@P} = \beta_{i,j}\omega_j$ according to the following three cases: (i) when the direction of $\mathbf{f}_{i,j_{A@P}}$ is the same as the direction of $\hat{\mathbf{u}}_{A@P}$, i.e., $\beta_{i,j\,A@P} > 0$, f_j favors the compatibility and, thus, f_j will be included into $F_{\mu_A@P}(x_i)$; (ii) when the direction of $\mathbf{f}_{i,j_{A@P}}$ is opposite to the direction of $\hat{\mathbf{u}}_{A@P}$, i.e., $\beta_{i,j\,A@P} < 0$, f_j favors the incompatibility and, thus, f_j will be included into $F_{\nu_A@P}(x_i)$; and (iii) when f_j does not reflect any influence, i.e., $\beta_{i,j\,A@P} = 0$, f_j will be excluded from both $F_{\mu_A@P}(x_i)$ and $F_{\nu_A@P}(x_i)$. To compute $\mu_{A@P}(x_i)$ and $\nu_{A@P}(x_i)$, we use the equations

$$\mu_{A@P}(x_i) = \check{\mu}_{A@P}(x_i)/\eta \tag{23}$$

and

$$\nu_{A@P}(x_i) = \check{\nu}_{A@P}(x_i)/\eta \tag{24}$$

respectively (cf. the equations presented in [35] to obtain a *traditional* IFS), where

$$\check{\mu}_{A@P}(x_i) = \begin{cases} \frac{1}{\|\mathbf{x}_i\|}\left(|t_{A@P}| + \sum_{j=1}^{m} \beta_{i,j\,A@P}\right) & : \text{if } \left(\forall(i,j) : \beta_{i,j\,A@P} > 0\right) \wedge (t_{A@P} < 0)\,; \\ \frac{1}{\|\mathbf{x}_i\|}\left(\sum_{j=1}^{m} \beta_{i,j\,A@P}\right) & : \text{if } \left(\forall(i,j) : \beta_{i,j\,A@P} > 0\right) \wedge (t_{A@P} \geq 0)\,; \\ 0 & : \text{otherwise;} \end{cases} \tag{25}$$

$$\check{\nu}_{A@P}(x_i) = \begin{cases} \frac{1}{\|\mathbf{x}_i\|}\left(t_{A@P} + \sum_{j=1}^{m} |\beta_{i,j\,A@P}|\right) & : \text{if } \left(\forall(i,j) : \beta_{i,j\,A@P} < 0\right) \wedge (t_{A@P} > 0) \\ \frac{1}{\|\mathbf{x}_i\|}\left(\sum_{j=1}^{m} |\beta_{i,j\,A@P}|\right) & : \text{if } \left(\forall(i,j) : \beta_{i,j\,A@P} < 0\right) \wedge (t_{A@P} \leq 0)\,; \\ 0 & : \text{otherwise;} \end{cases} \tag{26}$$

and

$$\eta = \max\left(1, \check{\mu}_{A@P}(x_i) + \check{\nu}_{A@P}(x_i)\right), \forall x_i \in O_{@P}. \tag{27}$$

At this point, all the internal processes of the post-digest method have been described. In the next section, we will show how these processes work together to obtain an AAIFS that represents a collection of XBEs.

4. Illustrative example

In this section, we present an example in which music album reviews were digested to detect reviewers who share a similar understanding about *top-rank albums*. To do so, in the first part we describe how the proposed method was configured and used to digest the reviews and, thus, obtain AAIFSs that represent XBEs of music albums. Then, in the second part, we explain how those AAIFSs were used in augmented comparisons aiming to detect the reviewers who share a similar understanding of what a top-rank album is.

In this example, we made use of a dataset containing music album reviews posted on Amazon.com between 1998-04-28 and 2014-07-23, which is part of the Amazon reviews[1] compiled in [36]. Among others, a review within this dataset consists of an identifier of a reviewer, an identifier of an album, a text describing the reviewer's judgment about the album, and an integer value between 1 and 5 that represents the score of the album assigned by the reviewer, where 1 and 5 are the lowest and the highest scores respectively. Accordingly, the following nomenclature is used throughout the example: (a) the collection of all the music album reviews in the dataset is denoted by $\mathcal{M}$; (b) the collection of the reviewers who have posted a review in $\mathcal{M}$ is denoted by $\mathcal{R}$; (c) the collection of the albums reviewed by any of the reviewers in $\mathcal{R}$ is denoted by $\mathcal{O}$; and (d) finally, the concept *top-rank albums* is denoted by A.

4.1. Digesting music album reviews

Consider two reviewers in $\mathcal{R}$, say S and P. Let one of them, say S, be a reference reviewer (or someone who acts as an information seeker). Now, consider that $O^{\mu}_{@S}$ and $O^{\nu}_{@S}$ are two collections containing identifiers[2] of the albums that are respectively compatible and incompatible with the understanding of *top-rank albums* possessed by S. Consider finally a collection in $\mathcal{M}$, say $M_{@P}$, consisting of the reviews posted by P. In this context, we describe below how we configured and used the proposed method to digest $M_{@P}$ and, thus, obtain an AAIFS, say $\hat{A}_{S@P}$, as a result[3] – here, $\hat{A}_{S@P}$ will include evaluations that result after learning what would be P's understanding of A to make the albums in $O_{@S} = O^{\mu}_{@S} \cup O^{\nu}_{@S}$ objects that satisfy or dissatisfy the aforementioned criterion.

To begin with, we configured the methods *extractFeatures* and *getOverallWeight* (see Algorithm 1) as follows. In the case of the method *extractFeatures*, two versions were implemented: one version using techniques connected to *natural language processing* and the other version using *crowdsourcing*. In the former version, the text included in a review is first split into words (or *tokens*) using separators such as commas, semi-colons, colons and blank-spaces. After that, *stop words* (i.e., words such as '*among*', '*where*', '*too*', etc. [21]) are removed from the previous list of words. Then, each word in the reduced list is stemmed using the Porter algorithm [22]. Finally, the resulting stemmed words are considered to be the features of the album evaluated in that review. Fig. 9a shows the features extracted from a review included in the dataset after using this version of the method *extractFeatures*.

Regarding the crowdsourcing version, we asked a large number of anonymous contributors to extract the features of the albums. To do so, we built a task in which a contributor is presented with a review and asked to perform three steps: (a) read the review, (b) type an album's feature that seems to be important to the reviewer and (c) repeat the step 2 with different features. After collecting the provided features, we performed a de-duplication process of them based on their perceived similarity – e.g., '*fun*', '*fun album*' and '*fun and party feel*' were considered to be the same feature. Fig. 9b shows the features extracted using this version of the method *extractFeatures*.

In the case of the method *getOverallWeight*, we implemented only one version, which is based on the Equation (11). In this example, while x_i in Equation (11) characterizes a *pertinent* album, f_j represents one of its extracted features.

After setting up the methods *extractFeatures* and *getOverallWeight*, we prepared the inputs of the post-digest method, i.e., $M_{@P}$, O^{μ}_{S} and O^{ν}_{S} as described next. While all the reviews posted by P were included into $M_{@P}$, the identifiers of the 10 best-rated and the 10 worst-rated albums reviewed by S were put into $O^{\mu}_{@S}$ and $O^{\nu}_{@S}$ respectively. Here, we assumed that the 10 best-rated and the 10 worst-rated albums reviewed by S are compatible and incompatible respectively with his/her understanding of *top-rank albums*. It is worth mentioning that, to reduce the processing cost (in particular the crowdsourcing cost), we took into account only reviewers who have evaluated at least 6 albums with score greater than 3 and at least 6 albums with score less than 3. After doing so, the number of reviewers was reduced to 55.

Once the inputs were prepared, each version of the post-digest method was executed for each pair of reviewers $\langle S, P\rangle$, where S is the reviewer acting as information seeker. This means that a pair of AAIFSs $\langle \hat{A}_{S@S}, \hat{A}_{S@P}\rangle$ was obtained for each version of the post-digest method. In the next part, we describe how a comparison between each pair of AAIFSs was performed.

[1] https://snap.stanford.edu/data/web-Amazon.html.

[2] "Amazon Standard Identification Numbers" (ASINs) like "B00000016W" or "B0000004X1" were used in this example.

[3] Since the example includes multiple reviewers acting as information seekers, the subscript S was included into the notation to keep track of the reviewer acting as an information seeker in a particular instance of the digest process.

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"For the most part, The Cars were a singles band, but their debut stands as one of the best ever new wave albums. The hit songs were solid, including the amazing threesome, "Let the Good Times Roll," "My Best Friend's Girl" and "Just What I Needed" that kick off the album. But side two is were The Cars demonstrate their artistic complexity, particularly on the lengthy double track "Moving in Stereo/All Mixed Up." "You're All I've Got Tonight" and "Bye Bye Love" make for another couple of great singles as well. This album is so good, it could stand as a greatest hits package all on its own.

part, car, singl, band, debut, stand, on, best, new, wave, album, hit, song, solid, includ, amaz, threesom, good, time, roll, friend, s, girl, need, kick, side, two, demonstr, artist, complex, particularli, lengthi, doubl, track, move, stereo/al, mix, up, re, ve, tonight, bye, love, make, anoth, coupl, great, well, greatest, packag

(a)

- solid hit songs
- artistic complexity
- greatest hits included

(b)

Fig. 9. *Extracting the features of a particular music album that may be contained in the text of a review*. While the list (a) shows the features extracted using techniques connected to natural language processing, the list (b) shows the features extracted using crowdsourcing.

4.2. Finding reviewers with similar understandings

Since the AAIFSs in a resulting couple $\langle \hat{A}_{S@S}, \hat{A}_{S@P}\rangle$ contain the features that reviewers S and P would have focused on while posting their reviews, a comparison between $\hat{A}_{S@S}$ and $\hat{A}_{S@P}$ could determine the level to which S and P share a similar understanding of top-rank albums. As was mentioned in Section 2.2, a (membership or nonmembership) CAF can be used as an indicator of that level. Hence, in this example we computed approximations of both a membership CAF, say $\Delta_{\mu_A:(S,P)@S}$, and a nonmembership CAF, say $\Delta_{\nu_A:(S,P)@S}$, for each couple $\langle \hat{A}_{S@S}, \hat{A}_{S@P}\rangle$ as described below.

Let $X = \{x_1, \cdots, x_n\}$ be a collection of the music albums whose identifiers are included in $O^{\mu}_{@S} \cup O^{\mu}_{@S}$. Let $\mathbf{F}_{\mu_A@S}(X) = F_{\mu_A@S}(x_1) \cup \cdots \cup F_{\mu_A@S}(x_n)$ and $\mathbf{F}_{\nu_A@S}(X) = F_{\nu_A@S}(x_1) \cup \cdots \cup F_{\nu_A@S}(x_n)$ be the collections of the features recorded in $\hat{A}_{S@S}$. Let $\mathbf{F}_{\mu_A@P}(X) = F_{\mu_A@P}(x_1) \cup \cdots \cup F_{\mu_A@P}(x_n)$ and $\mathbf{F}_{\nu_A@P}(X) = F_{\nu_A@P}(x_1) \cup \cdots \cup F_{\nu_P@S}(x_n)$ be the collections of the features recorded in $\hat{A}_{S@P}$. Finally, let $n(C)$ be a function that counts the number of elements in a collection C. In this context, we made use of the *ratio model* presented in [14] for computing $\Delta_{\mu_A:(S,P)@S}$ and $\Delta_{\nu_A:(S,P)@S}$ for each couple $\langle \hat{A}_{S@S}, \hat{A}_{S@P}\rangle$ by means of

$$\begin{aligned}\Delta_{\mu_A:(S,P)@S} = {}& n(\mathbf{F}_{\mu_A@S}(X) \cap \mathbf{F}_{\mu_A@P}(X)) / \big(n(\mathbf{F}_{\mu_A@S}(X) \cap \mathbf{F}_{\mu_A@P}(X)) \\ &+ \lambda_1 n(\mathbf{F}_{\mu_A@S}(X) - \mathbf{F}_{\mu_A@P}(X)) + \lambda_2 n(\mathbf{F}_{\mu_A@P}(X) - \mathbf{F}_{\mu_A@S}(X))\big)\end{aligned} \tag{28}$$

and

$$\begin{aligned}\Delta_{\nu_A:(S,P)@S} = {}& n(\mathbf{F}_{\nu_A@S}(X) \cap \mathbf{F}_{\nu_A@P}(X)) / \big(n(\mathbf{F}_{\nu_A@S}(X) \cap \mathbf{F}_{\nu_A@P}(X)) \\ &+ \lambda_1 n(\mathbf{F}_{\nu_A@S}(X) - \mathbf{F}_{\nu_A@P}(X)) + \lambda_2 n(\mathbf{F}_{\nu_A@P}(X) - \mathbf{F}_{\nu_A@S}(X))\big)\end{aligned} \tag{29}$$

respectively. The parameters λ_1 and λ_2 were set to 1 and 0 respectively in both equations. This was done to exclude the features that have not been considered by S – notice that, e.g., while $n(\mathbf{F}_{\mu_A@S}(X) \cap \mathbf{F}_{\mu_A@P}(X))$ denotes the number of common features, $n(\mathbf{F}_{\mu_A@S}(X) - \mathbf{F}_{\mu_A@P}(X))$ and $n(\mathbf{F}_{\mu_A@P}(X) - \mathbf{F}_{\mu_A@S}(X))$ denote, respectively, the number of features that belong exclusively to $\mathbf{F}_{\mu_A@S}(X)$ and the number of features that belong exclusively to $\mathbf{F}_{\mu_A@P}(X)$.

By way of illustration, the CAFs between the couples $\langle \hat{A}_{R_{35}@R_{35}}, \hat{A}_{R_{35}@P}\rangle$, where R_{35} denotes a reviewer who acts as an information seeker and $P \in \{R_{36}, R_4, R_7, R_{38}, R_9, R_2, R_{26}, R_5, R_6\}$ represents a reviewer who acts as information source, are listed in Tables 1 and 2. While the CAFs in former table correspond to couples of AAIFSs whose features were extracted through techniques connected to natural language processing, the CAFs in the latter table correspond to couples of AAIFSs whose features were extracted through using crowdsourcing techniques.

Notice in Tables 1 and 2 that the approximations of CAFs depend mainly on the features recorded in the AAIFSs (examples of such AAIFSs are listed in Tables A.3, A.4, A.5 and A.6). For instance, in Table 1 the computed CAFs between $\hat{A}_{R_{35}@R_{35}}$ and $\hat{A}_{R_{35}@R_{36}}$ are $\Delta_{\mu_A:(R_{35},R_{36})@R_{35}} = 0.35$ and $\Delta_{\nu_A:(R_{35},R_{36})@R_{35}} = 0.09$ while in Table 2 the

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Table 1
Approximations of CAFs between $\hat{A}_{R_{35}@R_{35}}$ and $\hat{A}_{R_{35}@P}$, where $P \in \{R_{36}, R_4, R_7, R_{38}, R_9, R_2, R_{26}, R_5, R_6\}$ (Features extracted through techniques connected to natural language processing).

	R_{35}								
P	R_{36}	R_4	R_7	R_{38}	R_9	R_2	R_{26}	R_5	R_6
$n(\mathbf{F}_{\mu_A@R_{35}}(X) \cap \mathbf{F}_{\mu_A@P}(X))$	9	2	1	2	2	2	0	0	1
$n(\mathbf{F}_{\mu_A@R_{35}}(X) - \mathbf{F}_{\mu_A@P}(X))$	17	24	25	24	24	24	26	26	25
$n(\mathbf{F}_{\mu_A@P}(X) - \mathbf{F}_{\mu_A@R_{35}}(X))$	8	7	1	2	15	25	0	12	2
$\Delta_{\mu_A:(R_{35},P)@R_{35}}$	0.35	0.08	0.04	0.08	0.08	0.08	0	0	0.04
$n(\mathbf{F}_{\nu_A@R_{35}}(X) \cap \mathbf{F}_{\nu_A@P}(X))$	3	0	2	1	2	6	0	2	2
$n(\mathbf{F}_{\nu_A@R_{35}}(X) - \mathbf{F}_{\nu_A@P}(X))$	31	34	32	33	32	28	34	32	32
$n(\mathbf{F}_{\nu_A@P}(X) - \mathbf{F}_{\nu_A@R_{35}}(X))$	14	9	4	3	25	37	0	16	14
$\Delta_{\nu_A:(R_{35},P)@R_{35}}$	0.09	0	0.06	0.03	0.06	0.18	0	0.06	0.06

Table 2
Approximations of CAFs between $\hat{A}_{R_{35}@R_{35}}$ and $\hat{A}_{R_{35}@P}$, where $P \in \{R_{36}, R_4, R_7, R_{38}, R_9, R_2, R_{26}, R_5, R_6\}$ (Features extracted through crowdsourcing techniques).

	R_{35}								
P	R_{36}	R_4	R_7	R_{38}	R_9	R_2	R_{26}	R_5	R_6
$n(\mathbf{F}_{\mu_A@R_{35}}(X) \cap \mathbf{F}_{\mu_A@P}(X))$	1	0	0	0	0	0	0	0	1
$n(\mathbf{F}_{\mu_A@R_{35}}(X) - \mathbf{F}_{\mu_A@P}(X))$	41	42	42	42	42	42	42	42	41
$n(\mathbf{F}_{\mu_A@P}(X) - \mathbf{F}_{\mu_A@S}(X))$	7	0	0	0	0	7	0	0	4
$\Delta_{\mu_A:(R_{35},P)@R_{35}}$	0.02	0	0	0	0	0	0	0	0.02
$n(\mathbf{F}_{\nu_A@R_{35}}(X) \cap \mathbf{F}_{\nu_A@P}(X))$	4	0	0	0	0	0	0	0	0
$n(\mathbf{F}_{\nu_A@R_{35}}(X) - \mathbf{F}_{\nu_A@P}(X))$	40	44	44	44	44	44	44	44	44
$n(\mathbf{F}_{\nu_A@P}(X) - \mathbf{F}_{\nu_A@R_{35}}(X))$	14	9	4	3	25	37	0	16	14
$\Delta_{\nu_A:(R_{35},P)@R_{35}}$	0.09	0	0	0	0	0	0	0	0

CAFs computed for this couple are $\Delta_{\mu_A:(R_{35},R_{36})@R_{35}} = 0.02$ and $\Delta_{\nu_A:(R_{35},R_{36})@R_{35}} = 0.09$ respectively. Yet, it is possible to detect that the understanding of *top-rank albums* possessed by R_{35} looks more similar to R_{36}'s understanding than R_4's. It is worth mentioning that the identifiers of the albums were not taking into account for counting the number of common features. For this reason, CAFs like the ones computed for a couple like $\langle \hat{A}_{R_{35}@R_{35}}, \hat{A}_{R_{35}@R_{26}} \rangle$ yield 0.

5. Related work

A study about the contributions of fuzzy set theory on machine learning and data mining is presented in [37]. Although that study has mainly focused on fuzzy analysis rather than fuzzy data, the author indicates that contributions such as *interpretability*, *representation of uncertainty* and *incorporation of knowledge* seem to be useful for fuzzy data processing. Such contributions are evident in our work when an augmented (fuzzy) comparison between two AAIFSs resulting after digesting social media posts is performed.

A work about fuzzy computing for data mining [38] shows how information granules can be represented as fuzzy sets and how those granules can be processed using the fuzzy set framework during a data mining process. In a similar way, contextual information granules could be characterized as AAIFSs and processed using the mechanisms included into the augmented framework.

Regarding the importance of enabling augmented computation, the survey conducted in [39] highlights the positive effects of including context within an information fusion process. Some of the surveyed works show how context in *soft data* (or humans judgments) could improve the quality of fused information. Hence, we foresee a potential use of AAIFSs characterizing digested information within this related area. As an example, we found in [40] a more specific area in which our method could be applied to. The authors in that work use data fusion and mining techniques

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to propose a *reputation generation* procedure by which an indicator of the reputation of a product (or entity) is computed. A step within that procedure is the *opinion filtering* process, in which opinions that are not related on the target product are filtered out. Our method could help in this step to filter out opinions given by people having contrasting understandings of the topic under study.

With respect to the interpretability of the post-digest method, a study about the importance of the interpretability of the model, algorithm(s) and the output(s) used during a fuzzy data mining process is presented in [41]. Agreeing with that, we have described in detail the proposed method and emphasized the benefits of getting an AAIFS as a result.

Making an analogy with the *entity-relationship model* proposed in [42], one can consider that a topic, say A, can be connected to an "*entity set*" that has associated the proposition 'x IS A' to test whether an "*entity*" x is one of its members. In this regard, a resulting AAIFS will correspond to a *reduced* subset (or view) of the topic-A-related "entity set," which contains "entities" that make 'x IS A' true enough according to an information seeker. This means that, although digesting the messages posted by social media users can be a time-consuming process, querying data from the topic-A-related subset will be a rather fast process – as was pointed out in Section 3, some steps of the post-digest process can be performed by human contributors using crowdsourcing services [26], which might need considerable time to be completed. An advantage on this regard is that a topic-A-related subset can be used for discovering patterns or summarize content according to the different understandings of the topic A that social media users may have. Hence, techniques used in opinion mining [43] and information fusion [44] would benefit from the application of the proposed method in, e.g., performing sentiment analysis or making personalized summaries according to the individual preferences of an information seeker.

Studies in the area of *probabilistic topic modeling* [45], in which *abstract topics* are inferred from messages (or documents), might be applicable for finding users that share a similar understanding of a topic. The idea in probabilistic topic modeling is that documents might exhibit different topics. Hence, one can say that documents might exhibit different understandings of a topic, which can be evident through their latent topic structures. For instance, in [46] an algorithm based on both collaborative filtering and probabilistic topic modeling has been proposed for recommending scientific articles to researchers who are part of social networks. In that case, researchers build personal libraries with references to relevant articles that share with other researchers. Such libraries are used by the algorithm for recommending other relevant articles. However, it is worth mentioning that such recommendations might also be influenced for the individual experiences that other researchers may have. This means that a researcher might receive recommendations of articles that he/she considers to be incompatible with his particular understanding of a topic because other researchers consider those articles compatible with their own understandings of that topic.

Latent semantic analysis (LSA) [47,48] is another technique proposed for discovering concepts that might be hidden in text documents (or messages). In LSA, (latent) concepts result from the association of terms (or features) with documents, which is known as a *semantic structure*. Since those semantic structures could be used to determine the level of participation of other terms or documents in a particular concept, one might assume that documents given by users having a similar understanding of the concept will have a similar level of participation. In this regard, LSA might also be applicable for detecting users that share a similar understanding of a particular topic.

6. Conclusions

In this paper we have described a computational intelligence method whereby subjective messages posted by a person on social media are digested to obtain an augmented Atanassov fuzzy set (AAIFS), which characterizes a collection of *artificial* experience-based evaluations (XBEs) resembling *actual* XBEs performed by this person in relation to a specific topic (or concept).

Such AAIFSs lend themselves to augmented computation, i.e., they can be used in a process in which not only the extent but also the context of the characterized XBEs are taken into account for computation. Hence, those AAIFSs can be used for measuring the level to which the contexts of the characterized XBEs are alike. In this regard, the proposed *post-digest method* is deemed to be a mechanism for detecting people with whom an information seeker shares a similar understanding about one or more topics.

Since people sharing a similar understanding can be considered fairly reliable sources of subjective information, our method can be used for building a kind of database consisting of XBEs that might help an information seeker to find relevant and useful information according to his/her own understanding of a topic. This is a key aspect of our method because it could be implemented together with techniques used in, say, opinion mining or information fusion to produce personalized summaries according to a very specific understanding that an information seeker may have.

Another important aspect of the post-digest method is that it allows an information seeker to assess the quality of potential information sources without revealing details that might compromise his/her privacy. Therefore, our method could be used in situations where someone needs some privacy when looking for usable subjective information.

The applicability of the proposed method to detect information sources that demonstrate a common understanding of a topic has been illustrated in an example in which music album reviews were digested to detect reviewers having a similar understanding about top-rank albums. The implementation of internal components of the method through techniques connected to natural language processing and crowdsourcing has been also illustrated in the example.

An aspect that will be further investigated is the use of augmented computation in *tailored querying and clustering methods* that accept digested social media content as input. Here, by a *tailored method* is meant a method by which one can obtain results in congruence with one's own understanding of a topic.

Appendix A. Examples of extracted AAIFSs

Examples of the AAIFSs extracted according to the procedure described in Section 4 are presented in this appendix. AAIFSs whose features were extracted through techniques connected to natural language processing (NLP) are listed in Tables A.3 and A.5. AAIFSs whose features were extracted through crowdsourcing techniques are listed in Tables A.4 and A.6 while examples of those features are shown in Table A.7.

Table A.3
Content of the AAIFS $\hat{A}_{R_{35}@R_{35}}$ with features extracted through natural language processing techniques.

x	$\mu_A(x)$	$\nu_A(x)$	$h_A(x)$	$F_{\mu_A}(x)$	$F_{\nu_A}(x)$
B000091R4X	0.27	0	0.73	'album', 'best', 'seen', 'ts', 've',	
B0000CC4VD	0.27	0	0.73	'album', 'best', 'blueprint', 'rocafella', 's',	
B0001BXYRO	0.25	0	0.75	'album', 'great', 'last',	
B0001KL5C6	0.27	0	0.73	'album', 'classic', 'death', 'row',	
B0008ENJ06	0.24	0	0.76	'better', 'even', 'origin',	
B0009XFIZK	0.27	0	0.73	'album', 'best', 'juvenil', 'made',	
B000A3DFYU	0.24	0	0.76	'masterpiec',	
B000AY9OGW	0.27	0	0.73	'album', 'best', 'dirti', 'on', 'south',	
B000E97HB2	0.27	0	0.73	'album', 'best', 'ghostfac', 'made',	'killah',
B000G1QX4A	0.27	0	0.73	'again', 'album', 'dog', 'great', 'year',	
B0009G3BWE	0.01	0.22	0.78	's',	'champ', 'entertain', 'lightli', 'peopl', 'predic',
B000ASTEE0	0	0.21	0.79		'dai', 'ok',
B000BJS4P8	0	0.2	0.79	'on',	'chapter', 'dissappoint', 'final',
B000BY826Y	0.03	0.17	0.79	'great',	'bad',
B000E3K3BY	0.04	0.24	0.73	'album',	'shouldn', 't', 'top',
B000F0UV2M	0	0.21	0.79		'cd', 'good', 'ok', 'usual',
B000F0UV3Q	0	0.2	0.8		'down', 'killah', 'sat', 'season',
B000F4TM6Y	0	0.21	0.79		'citi', 'go', 'joc', 'new', 'nowher',
B000F8DT1O	0	0.21	0.79		'add', 'out', 'run', 'water',
B000FBFT7I	0	0.21	0.79		'good', 'light', 'pine', 'pole', 'tree',

Table A.4
Content of the AAIFS $\hat{A}_{R_{35}@R_{35}}$ with features extracted through crowdsourcing techniques.

x	$\mu_A(x)$	$\nu_A(x)$	$h_A(x)$	$F_{\mu_A}(x)$	$F_{\nu_A}(x)$
B000091R4X	0.25	0	0.75	10135, 10572, 11885, 11886, 11887	
B0000CC4VD	0.25	0.01	0.74	10483, 11878, 11982, 11983, 11984, 11985, 11986	10309
B0001BXYRO	0.24	0	0.76	10779, 12095, 12096, 12097, 12098	
B0001KL5C6	0.26	0	0.74	11986, 12106, 12107, 12108	
B0008ENJ06	0.26	0	0.74	10067, 10202, 10997, 11078, 12368	
B0009XFIZK	0.25	0	0.75	10572, 10582, 10997, 12444	
B000A3DFYU	0.26	0	0.74	10048, 10135, 10638, 11986, 12368, 12449, 12450	
B000AY9OGW	0.26	0	0.74	10135, 10332, 11065, 12108, 12368, 12488	
B000E97HB2	0.25	0	0.75	10524, 10954, 12108, 12589, 12590, 12591	
B000G1QX4A	0.24	0	0.76	10024, 12682, 12683	
B0009G3BWE	0	0.23	0.77		11024, 11363, 12386, 12387
B000ASTEE0	0	0.23	0.77		10080, 11024, 11212, 12409, 12474, 12475, 12476
B000BJS4P8	0	0.23	0.77		10401, 12518, 12520
B000BY826Y	0	0.21	0.79	10572	12562, 12563
B000E3K3BY	0	0.23	0.77	10954	10055, 11641, 11835, 12575, 12576
B000F0UV2M	0	0.21	0.79		10309, 12616, 12617, 12618
B000F0UV3Q	0	0.23	0.77	10572	10055, 12438, 12627, 12628
B000F4TM6Y	0	0.23	0.77		10055, 10156, 10175, 10382, 12388, 12576, 12635, 12636, 12637
B000F8DT1O	0	0.23	0.77		10054, 10055, 10597, 12639, 12640
B000FBFT7I	0	0.23	0.77		10020, 10532, 12390, 12641, 12642, 12643

Table A.5
Content of the AAIFS $\hat{A}_{R_{35}@R_{36}}$ with features extracted through natural language processing techniques.

x	$\mu_A(x)$	$\nu_A(x)$	$h_A(x)$	$F_{\mu_A}(x)$	$F_{\nu_A}(x)$
B000091R4X	0	0	1		
B0000CC4VD	0.3	0	0.7	‘album’, ‘best’, ‘far’, ‘rap’, ‘year’,	
B0001BXYRO	0	0	1		
B0001KL5C6	0	0	1		
B0008ENJ06	0.3	0	0.7	‘album’, ‘hot’,	
B0009XFIZK	0.28	0	0.72	‘album’, ‘finest’, ‘juvenil’, ‘on’, ‘s’,	
B000A3DFYU	0.3	0	0.7	‘slump’, ‘sophomor’,	
B000AY9OGW	0.3	0	0.7	‘best’, ‘dirti’, ‘on’, ‘s’, ‘south’,	
B000E97HB2	0.3	0	0.7	‘album’, ‘best’, ‘debut’,	
B000G1QX4A	0.3	0	0.7	‘back’, ‘dog’,	
B0009G3BWE	0	0.29	0.71		‘2005’, ‘ic’, ‘vanilla’,
B000ASTEE0	0	0.22	0.78	‘finest’, ‘s’,	‘chi’, ‘town’,
B000BJS4P8	0.05	0.28	0.67	‘album’,	‘underr’, ‘veri’,
B000BY826Y	0	0	1		
B000E3K3BY	0	0.29	0.71		‘barrel’, ‘bottom’, ‘more’,
B000F0UV2M	0	0	1		
B000F0UV3Q	0	0.29	0.71		‘masterpiec’,
B000F4TM6Y	0	0.29	0.71	‘s’,	‘career’, ‘down’, ‘go’, ‘joc’, ‘thing’, ‘yung’,
B000F8DT1O	0	0	1		
B000FBFT7I	0	0	1		

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Table A.6
Content of the AAIFS $\hat{A}_{R_{35}@R_{36}}$ with features extracted through crowdsourcing techniques.

x	$\mu_A(x)$	$\nu_A(x)$	$h_A(x)$	$F_{\mu_A}(x)$	$F_{\nu_A}(x)$
B000091R4X	0	0	1		
B0000CC4VD	0.44	0	0.56	10290, 11987	
B0001BXYRO	0	0	1		
B0001KL5C6	0	0	1		
B0008ENJ06	0	0	1		
B0009XFIZK	0	0	1		
B000A3DFYU	0	0	1		
B000AY9OGW	0	0	1		
B000E97HB2	0.46	0	0.54	11851, 12108, 12597, 12598	
B000G1QX4A	0.4	0	0.6	10055, 12108, 12684	
B0009G3BWE	0	0.31	0.69		10081, 10309, 11471, 12388, 12389, 12390, 12391
B000ASTEE0	0	0	1		
B000BJS4P8	0	0	1		
B000BY826Y	0	0	1		
B000E3K3BY	0	0.31	0.69		10037, 11471, 12579, 12580, 12581, 12582
B000F0UV2M	0	0	1		
B000F0UV3Q	0.04	0.35	0.61	12108	10135, 11986, 12632, 12633, 12634
B000F4TM6Y	0.03	0.35	0.62	10055	10650, 10733, 12438, 12638
B000F8DT1O	0	0	1		
B000FBFT7I	0	0	1		

Table A.7
Examples of the features extracted using crowdsourcing techniques.

Code	Description
10572	"solid hit songs"
11885	"perfect single songs"
11886	"the best TS album"
11887	"set the standard for producing"
11982	"Beanie Sigel"
10309	"great songs"
11878	"Grammy nominated performance by Beanie Sigel"
10483	"feelin it in the air was a great video"
12095	"greatest mobb deep ever"
12096	"it was darker grimier and sleeker than their other albums"
12106	"one of the best and greatest comeback"
12107	"better tan xzibit's"
12368	"great rapping"
11078	"awesome album"
10997	"exciting and overall perfect album"
10582	"successful"
12450	"great writing"
10048	"great skills"
10638	"enjoyable album"

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