Chapter 2

Fundamental principles and historical developments of inertial navigation

2.1 Basic concepts

The basic concepts of inertial navigation are outlined here with the aid of some simple examples.

A simple one-dimensional example of navigation involves the determination of the position of a train which is moving along a track between two locations on a perfectly flat plane. It is possible to determine the instantaneous speed of the train and the distance it has travelled from a known starting point by using measurements of its acceleration along the track. Sensors called accelerometers provide such information about their own movement. If an accelerometer is fixed in the train, it will provide information about the acceleration of the train. The time integral of the acceleration measurement provides a continuous estimate of the instantaneous speed of the train, provided its initial speed was known. A second integration yields the distance travelled with respect to a known starting point. The accelerometer together with a computer, or other suitable device capable of integration, therefore constitutes a simple one-dimensional navigation system.

In general, a navigation system is required to provide an indication of the position of a vehicle with respect to a known grid system or reference frame. For instance, it may be required to determine the location of a vehicle in terms of x and y coordinates in a Cartesian reference frame. Considering again the example of a train moving along a track, as depicted in Figure 2.1, it is now necessary to determine the position of the train with respect to the coordinate reference frame shown in the figure.

Given the knowledge of the train's acceleration along the track, and the angle which the track makes with the reference frame, the x and y coordinate positions may be determined. This may be accomplished by resolving the measured acceleration in the reference frame to give x and y components, and by suitable integration of the resolved signals to yield the velocity and position of the train in reference axes.

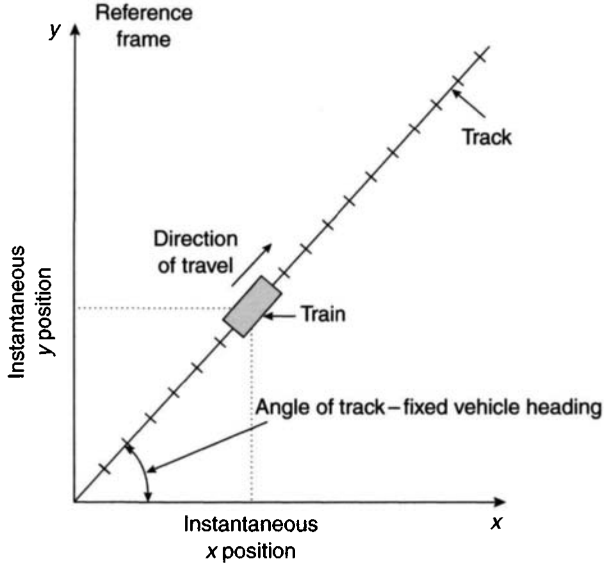

Figure 2.1 One-dimensional navigation

In this simple case, the angle of the track defines the heading of the train with respect to the reference frame.

With the more general situation illustrated in Figure 2.2, where the track curves, it is necessary to detect continuously the translational motion of the train in two directions and changes in its direction of travel, that is, to detect the rotations of the train about the perpendicular to the plane of motion as the train moves along the track.

Two accelerometers are now required to detect the translational motion in perpendicular directions along and perpendicular to the track. One sensor suitable for the measurement of the rotational motion is a gyroscope. Depending on the form of construction of this sensor, it may be used to provide either a direct measure of the train's heading with respect to the reference frame, or a measurement of the turn rate of the train. In the latter case, the angular orientation of the train may be calculated by the integration of this measurement, provided the angle is known at the start of navigation. Given such information, it is possible to relate the measurements of acceleration, which are obtained in an axis set which is fixed in the train, to the reference frame. The instantaneous measurements of acceleration may therefore be resolved in the reference frame and integrated with respect to time to determine the instantaneous velocity and position of the vehicle with respect to that frame.

Clearly then, it is possible to construct a simple, two-dimensional, navigation system using a gyroscope, two accelerometers and a computer. In practice, the inertial sensors may be mounted on a platform which is stabilised in space, and hence isolated from the rotation of the vehicle, or mounted directly on to the vehicle to form a strapdown system. The measurements are processed in the computer to provide continuous estimates of the position, speed and the direction of travel or heading of

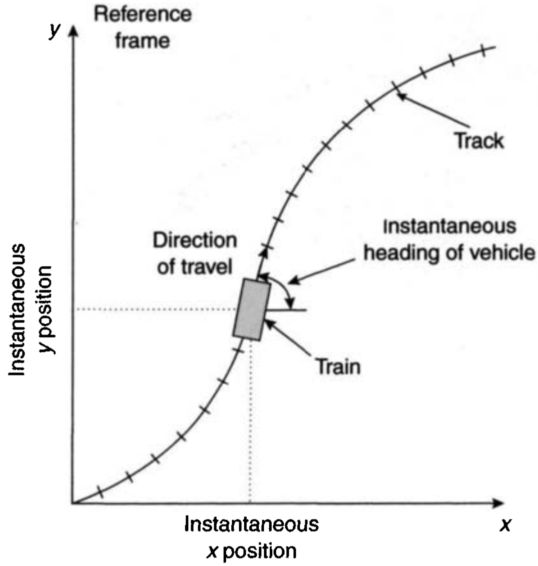

Figure 2.2 *Two-dimensional navigation*

the train. It must be stressed that inertial navigation is fundamentally dependent on an accurate knowledge of position, speed and heading being available prior to the start of navigation. This is because it uses dead reckoning which relies for its operation on the updating of the system's previous estimates of these navigational quantities, commencing with the initial values input to the system at the start of navigation.

It will be apparent from the preceding discussion that successful navigation of a vehicle can be achieved by using the properties of suitable sensors mounted in the vehicle. In general, it is required to determine a vehicle's position with respect to a three-dimensional reference frame. Consequently, if single-axis sensors are used, three gyroscopes will be required to provide measurements of vehicle turn rates about three separate axes, whilst three accelerometers provide the components of acceleration which the vehicle experiences along these axes. For convenience and accuracy, the three axes are usually chosen to be mutually perpendicular.

In most applications, the axis set defined by the sensitive axes of the inertial sensors is made coincident with the axes of the vehicle, or body, in which the sensors are mounted, usually referred to as the body axis set. The measurements provided by the gyroscopes are used to determine the attitude and heading of the body with respect to the reference frame in which it is required to navigate. The attitude and heading information is then used to resolve the accelerometer measurements into the reference frame. The resolved accelerations may then be integrated twice to obtain vehicle velocity and position in the reference frame.

Gyroscopes provide measurements of changes in vehicle attitude or its turn rate with respect to inertial space. Accelerometers, however, are unable to separate the total acceleration of the vehicle, the acceleration with respect to inertial space, from

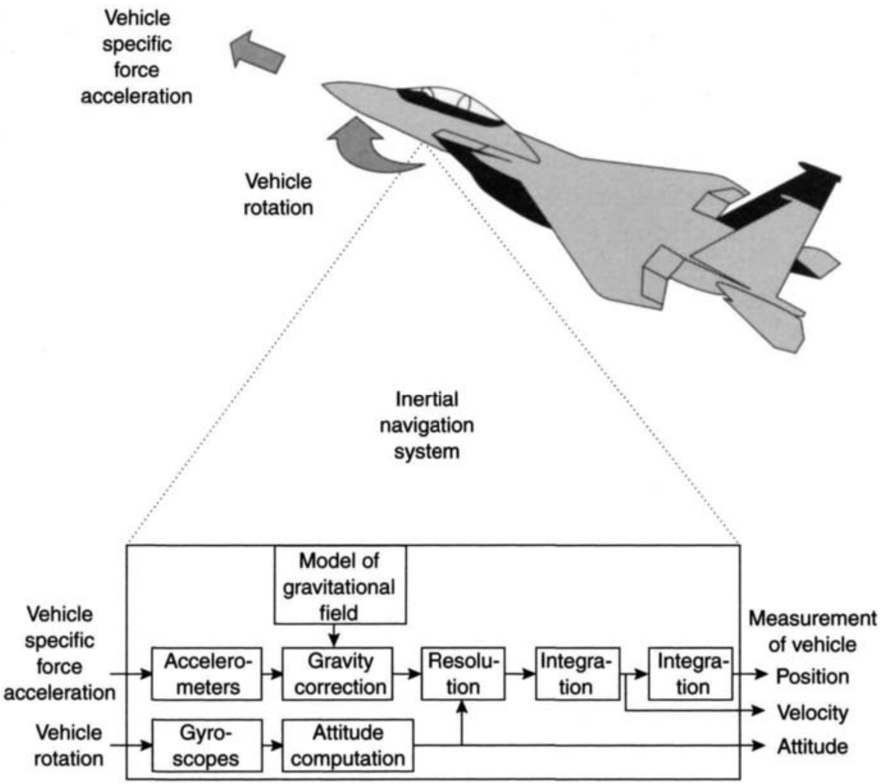

Figure 2.3 Functional components of an inertial navigation system

that caused by the presence of a gravitational field. These sensors do in fact provide measurements of the difference between the true acceleration in space and the acceleration due to gravity.[1] This quantity is the non-gravitational force per unit mass exerted on the instrument, referred to in this text for brevity as 'specific force' [1].

Hence, the measurements provided by the accelerometers, especially when close to a large body such as the Earth, must be combined with knowledge of the gravitational field of that body in order to determine the acceleration of the vehicle with respect to inertial space. Using this information, vehicle acceleration relative to the body may be derived.

The navigational function is therefore fulfilled by combining the measurements of vehicle rotation and specific force with knowledge of the gravitational field to compute estimates of attitude, velocity and position with respect to a pre-defined reference frame. A schematic representation of such an inertial navigation system is shown in Figure 2.3.

[1] Algebraically, the sum of the acceleration with respect to inertial space and the acceleration due to gravitational attraction.

2.2 Summary

It follows from the introductory discussion that the essential functions which an inertial navigation system must perform may be defined as follows:

- determination of the angular motion of a vehicle using gyroscopic sensors, from which its attitude relative to a reference frame may be derived;
- measure specific force using accelerometers;
- resolve the specific force measurements into the reference frame using the knowledge of attitude derived from the information provided by the gyroscopes;
- evaluate the force resulting from the gravitational field – the gravitational attraction of the Earth in the case of systems operating in the vicinity of the Earth;
- integrate the resolved specific force measurements to obtain estimates of the velocity and position of the vehicle.

The later chapters describe the principles of inertial navigation in some depth and provide detailed information on system mechanisations, inertial sensor technology, computational aspects (including algorithms), design analysis and applications of such systems. However, prior to this, it is instructive to have a brief review of the historical developments which have led to the current state of development of present day inertial navigation systems and their technology.

2.3 Historical developments

From the earliest times, people have moved from one place to another by finding or 'knowing' their way; this skill has required some form of navigation. There is an oblique reference to inertial navigation in the Bible [2]. Generally, as in the case of the biblical reference, the earliest applications were on land. Then as the desire developed to explore farther afield, instruments were developed for marine applications. More recently, there have been significant developments in inertial sensors, and systems for inertial navigation on land, in the air, on or under the oceans as well as in space to the planets and beyond.

Our earliest ancestors travelled in search of food, usually on land. As they developed, they crossed rivers generally using landmarks, that is, navigation by observation. Further development of position fixing techniques saw the Polynesians cross the Pacific Ocean about two millennia ago using their understanding of celestial bodies and landmarks. These techniques can only be used in clear weather conditions. During the thirteenth century, the Chinese discovered the properties of lodestone and applied the principles of magnetism to fabricate a compass. They used this instrument to navigate successfully across the south China Sea. This device could be used irrespective of visibility but was difficult to use in rough weather. The other significant development to help the long distance traveller was the sextant, which enabled position fixes to be made accurately on land.

In the seventeenth century, Sir Isaac Newton defined the laws of mechanics and gravitation, which are the fundamental principles on which inertial navigation is based. Despite this, it was to be about another two centuries before the inertial sensors were developed that would enable the demonstration of inertial navigation techniques. However, in the early eighteenth century, there were several significant developments; Serson demonstrating a stabilised sextant [3] and Harrison devising an accurate chronometer, the former development enabling sightings to be taken of celestial objects without reference to the horizon and the latter enabling an accurate determination of longitude. These instruments, when used with charts and reference tables of location of celestial bodies, enabled accurate navigation to be achieved, provided the objects were visible.

Foucault is generally credited with the discovery of the gyroscopic effect in 1852. He was certainly the first to use the word. There were others, such as Bohneberger, Johnson and Lemarle, developing similar instruments. All of these people were investigating the rotational motion of the Earth and the demonstration of rotational dynamics. They were using the ability of the spin axis of a rotating disc to remain fixed in space. Later in the nineteenth century, many fine gyroscopic instruments were made. In addition, there were various ingenious applications of the gyroscopic principle in heavy equipment such as the grinding mill.

A significant discovery was made in 1890 by Professor G.H. Bryan concerning the ringing of hollow cylinders, a phenomenon later applied to solid-state gyroscopes.

The early years of the twentieth century saw the development of the gyrocompass for the provision of a directional reference. The basic principle of this instrument is the indication of true north by establishing the equilibrium between the effect of its pendulosity and the angular momentum of the rotating base carrying the compass. Initially, this instrument was sensitive to acceleration. Professor Max Schuler produced an instrument with a vertical erection system enabling an accurate vertical reference to be defined [4]. This instrument was tuned to the undamped natural period defined by $2\pi\sqrt{R/g}$, approximately equal to 84 minutes, where R is the radius of the Earth and g is the acceleration caused by the Earth's gravitational field. Later, this technique became known as 'Schuler tuning' [5], a phrase originated by Dr Walter Wrigley of MIT. This ingenious method produced a directional instrument insensitive to acceleration for use at sea. Elmer and Lawrence Sperry improved the design of the gyrocompass with further refinements by Brown and Perry. These instruments provided the first steps towards all-weather, autonomous navigation. The Sperry brothers were also at the forefront of the application of the gyroscopic effect to control and guidance in the early twentieth century. They produced navigation and autopilot equipment for use in aircraft and gyroscopes for use in torpedoes.

Rate of turn indicators, artificial horizons and directional gyroscopes for aircraft were being produced in the 1920s. At a similar time, side-slip sensors were being developed, early open-loop accelerometers, and Schuler was demonstrating a north-seeking device for land use giving an accuracy of 22 seconds of arc! There was significant progress during the early part of the twentieth century with the development of stable platforms for fire control systems for guns on ships and the identification of the concept for an inertial navigation system. Boykow identified the use of accelerometers and gyroscopes to produce a full inertial navigation system.

However, at this stage, the quality of the inertial sensors was not suitable for the production and demonstration of such a system.

World War II saw the demonstration of the principles of inertial guidance in the V1 and V2 rockets by German scientists, a prime step forward being the use of a system with feedback leading to accurate guidance. At this time there was much activity in various parts of the world devising new types of inertial sensors, improving their accuracy and, in 1949, the first publication suggesting the concept of the strapdown technique for navigation.

The pace of development and innovation quickened in the 1950s with many significant developments for seaborne and airborne applications. More accurate sensors were produced, with the accuracy of the gyroscope being increased substantially. The error in such sensors was reduced from about 15°/hour to about 0.01°/hour, Professor Charles Stark Draper and his coworkers at MIT being largely responsible for many technical advances with the demonstration of the floated rate-integrating gyroscope [6]. It was also during the 1950s that the principle of force-feedback was applied to the proof mass in an accelerometer to produce an accurate acceleration sensing instrument.

The early part of the 1950s saw the fabrication of a stabilised platform inertial navigation system followed by the first crossing of the United States of America by an aircraft using full inertial navigation. Inertial navigation systems became standard equipment in military aircraft, ships and submarines during the 1960s, all of these applications using the so-called stable platform technology. This era saw further significant developments with increases in the accuracy of sensors, the miniaturisation of these devices and the start of ring laser gyroscope development. Major projects of this period in which inertial system technology was applied were the ballistic missile programmes and the exploration of space.

Similar progress has taken place in the last two decades; one major advance being the application of the micro-computer and development of gyroscopes with large dynamic ranges enabling the strapdown principle to be realised. This has enabled the size and complexity of the inertial navigation system to be reduced significantly for very many applications. The use of novel methods has enabled small, reliable, rugged and accurate inertial sensors to be produced that are relatively inexpensive, thus enabling a very wide range of diverse applications as discussed below. This period has also seen significant advances in the development of solid-state sensors such as optical fibre gyroscopes and silicon accelerometers.

The development of inertial navigation systems in recent years has been characterised by the gradual move from stable platform to strapdown technology as indicated in Figure 2.4. The figure gives an indication of the increasing application of strapdown systems which has resulted from advances in gyroscope technology. Milestones in this continuing development have occurred as a result of the development of the miniature rate-integrating gyroscope, the dynamically tuned gyroscope and more recently, ring laser and fibre optic rate sensors and vibratory gyroscopes, all of which are described in Chapters 4 and 5. MEMS sensors have provided an exciting development that should expand the range of applications of inertial navigation.

Strapdown systems are becoming widely used for aircraft and guided missile applications. More recently this technology has been applied to ship and submarine

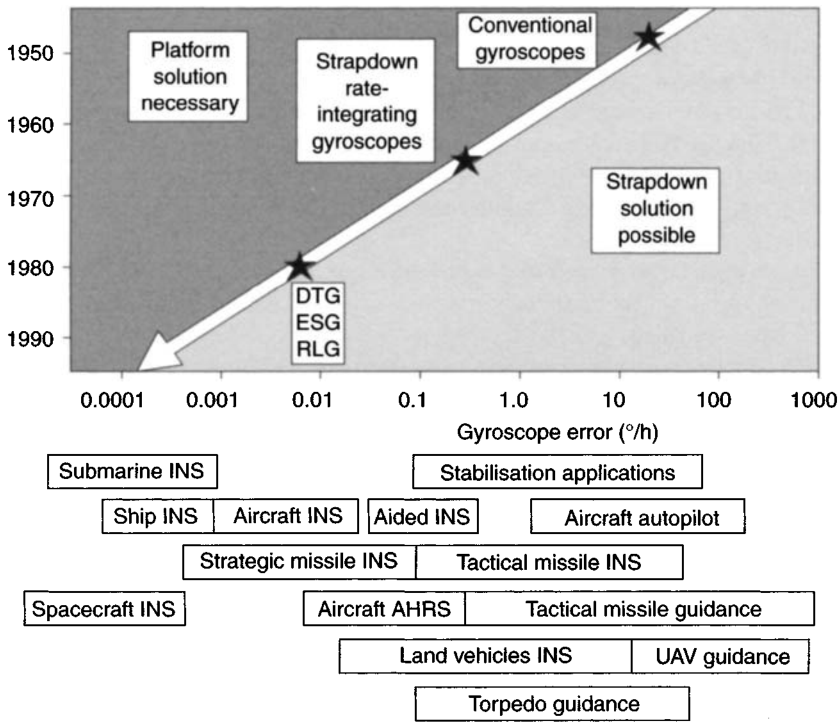

Figure 2.4 Strapdown sensor development and some applications

applications as indicated in Figure 2.4. The diagram shows other applications for strapdown technology, the accuracy required from the gyroscopes being related to the position and size of the box in which it is mentioned.

2.4 The modern-day inertial navigation system

From the preceding section, it is clear that the range of applications in which inertial navigation systems can and are being used is very extensive, covering navigation of ships, aircraft, tactical and strategic missiles and spacecraft. In addition, there are some more novel applications in the field of robotics, active suspension in racing or high performance motor cars and for surveying underground wells and pipelines.

Such diverse applications call for navigation systems having a very broad range of performance capabilities, as well as large differences in the periods of time over which they will be required to provide navigation data. For instance, tactical missile applications may require inertial navigation and guidance to an accuracy of a few hundred metres for periods of minutes or even a few seconds, whilst other airborne systems are required to operate for several hours whilst maintaining knowledge of aircraft position to an accuracy of one or two nautical miles or better. In the cases of marine or space applications, such systems may be required to provide navigation data to similar accuracy over periods of weeks, months or even longer in the case of interplanetary exploration. One extreme example is the Voyager spacecraft which has been navigating through the solar system and beyond for more than 25 years.

Although the basic principles of inertial navigation systems do not change from one application to another, it will come as no surprise to find that the accuracy of the inertial sensors and the precision to which the associated computation must be carried out varies dramatically over the broad range of applications indicated earlier. It follows therefore that the instrument technologies and the techniques used for the implementation of the navigation function in such diverse applications also vary greatly. Part of the function of this text is to provide some insight into the methods and technologies appropriate to some of the different types of inertial system application outlined earlier.

2.5 Trends in inertial sensor development

A number of categories of inertial sensor have made significant progress over the last decade. The development of micro-machined electromechanical systems (MEMS) devices has been spectacular and the performance that can be achieved is approaching inertial grade, so that is likely to be readily achieved in the near term. The development of micro-optical machined electromechanical systems (MOEMS) is expected to provide very high performance sensors in the medium term.

Progress with refinement of the fibre optical gyroscope is likely to continue so it should start to replace the more expensive ring laser gyroscope in the near term.

New techniques, such as the cold atom interferometers, are being researched for highly specialised investigations and very precise measurements, but these approaches are a long way from maturity.

The big drive from the systems applications viewpoint will be further reduction in cost and complexity, with a corresponding leap in reliability.

Inertial sensor performance is discussed in Chapters 4–7.

References

1 BRITTING, K.: 'Inertial navigation system analysis' (Wiley Interscience, New York, 1971)

2 The Bible, Amos 7:7–9

3 SORG, H.W.: 'From Serson to Draper – two centuries of gyroscopic development', *Journal of Institute of Navigation*, 1976–77, **23** (4)

4 SCHULER, M.: 'Die Storung von Pendul-und Kreiselapparaten durch die Beschleunigung der Fahrzeuges', *Physikalische Zeitschrift B*, 1923, **24**

5 WRIGLEY, W., HOLLISTER, W.M., and DENHARD, W.G.: 'Gyroscopic theory, design and instrumentation' (MIT Press, 1969)

6 DRAPER, C., WRIGLEY, W., and HOVORKA, J.: 'Inertial guidance' (Pergamon Press, 1960)