R.2 Exercise Set

Write an equivalent expression without negative exponents.

1. 3^{-7}
2. $\dfrac{1}{(5.9)^{-4}}$
3. $\dfrac{x^{-5}}{y^{-4}}$
4. $\dfrac{a^{-2}}{b^{-8}}$
5. $\dfrac{m^{-1}n^{-12}}{t^{-6}}$
6. $\dfrac{x^{-9}y^{-17}}{z^{-11}}$

Simplify.

7. 23^0
8. $\left(-\dfrac{2}{5}\right)^0$
9. $z^0 \cdot z^7$
10. $x^{10} \cdot x^0$
11. $5^8 \cdot 5^{-6}$
12. $6^2 \cdot 6^{-7}$
13. $m^{-5} \cdot m^5$
14. $n^9 \cdot n^{-9}$
15. $y^3 \cdot y^{-7}$
16. $b^{-4} \cdot b^{12}$
17. $(x + 3)^4(x + 3)^{-2}$
18. $(y - 1)^{-1}(y - 1)^5$
19. $3^{-3} \cdot 3^8 \cdot 3$
20. $6^7 \cdot 6^{-10} \cdot 6^2$
21. $2x^3 \cdot 3x^2$
22. $3y^4 \cdot 4y^3$
23. $(-3a^{-5})(5a^{-7})$
24. $(-6b^{-4})(2b^{-7})$
25. $(6x^{-3}y^5)(-7x^2y^{-9})$
26. $(8ab^7)(-7a^{-5}b^2)$
27. $(2x)^4(3x)^3$
28. $(4y)^2(3y)^3$
29. $(-2n)^3(5n)^2$
30. $(2x)^5(3x)^2$
31. $\dfrac{y^{35}}{y^{31}}$
32. $\dfrac{x^{26}}{x^{13}}$
33. $\dfrac{b^{-7}}{b^{12}}$
34. $\dfrac{a^{-18}}{a^{-13}}$
35. $\dfrac{x^2y^{-2}}{x^{-1}y}$
36. $\dfrac{x^3y^{-3}}{x^{-1}y^2}$
37. $\dfrac{32x^{-4}y^3}{4x^{-5}y^8}$
38. $\dfrac{20a^5b^{-2}}{5a^7b^{-3}}$
39. $(2x^2y)^4$
40. $(3ab^5)^3$
41. $(-2x^3)^5$
42. $(-3x^2)^4$
43. $(-5c^{-1}d^{-2})^{-2}$
44. $(-4x^{-5}z^{-2})^{-3}$
45. $(3m^4)^3(2m^{-5})^4$
46. $(4n^{-1})^2(2n^3)^3$
47. $\left(\dfrac{2x^{-3}y^7}{z^{-1}}\right)^3$
48. $\left(\dfrac{3x^5y^{-8}}{z^{-2}}\right)^4$
49. $\left(\dfrac{24a^{10}b^{-8}c^7}{12a^6b^{-3}c^5}\right)^{-5}$
50. $\left(\dfrac{125p^{12}q^{-14}r^{22}}{25p^8q^6r^{-15}}\right)^{-4}$

Convert to scientific notation.

51. 16,500,000
52. 359,000
53. 0.000000437
54. 0.0056
55. 234,600,000,000
56. 8,904,000,000
57. 0.00104
58. 0.00000000514
59. *Mass of a Neutron.* The mass of a neutron is about 0.00000000000000000000000000167 kg.
60. *Tech Security Spending.* It is estimated that \$37,800,000,000 will be spent on information technology security systems in 2013 (*Source*: IDC).

Convert to decimal notation.

61. 7.6×10^5
62. 3.4×10^{-6}
63. 1.09×10^{-7}
64. 5.87×10^8
65. 3.496×10^{10}
66. 8.409×10^{11}
67. 5.41×10^{-8}
68. 6.27×10^{-10}
69. The amount of solid waste generated in the United States in a recent year was 2.319×10^8 tons (*Source*: Franklin Associates, Ltd.).
70. The mass of a proton is about 1.67×10^{-24} g.

Compute. Write the answer using scientific notation.

71. $(4.2 \times 10^{7})(3.2 \times 10^{-2})$

72. $(8.3 \times 10^{-15})(7.7 \times 10^{4})$

73. $(2.6 \times 10^{-18})(8.5 \times 10^{7})$

74. $(6.4 \times 10^{12})(3.7 \times 10^{-5})$

75. $\dfrac{6.4 \times 10^{-7}}{8.0 \times 10^{6}}$

76. $\dfrac{1.1 \times 10^{-40}}{2.0 \times 10^{-71}}$

77. $\dfrac{1.8 \times 10^{-3}}{7.2 \times 10^{-9}}$

78. $\dfrac{1.3 \times 10^{4}}{5.2 \times 10^{10}}$

Solve. Write the answer using scientific notation.

79. *Trash on U.S. Roadways.* It is estimated that there were 51.2 billion pieces of trash on 76 million mi of U.S. roadways in a recent year (*Source*: Keep America Beautiful). On average, how many pieces of trash were on each mile of roadway?

80. *Nanowires.* A **nanometer** is 0.000000001 m. Scientists have developed optical nanowires to transmit light waves short distances. A nanowire with a diameter of 360 nanometers has been used in experiments on the transmission of light (*Source: The New York Times*, January 29, 2004). Find the diameter of such a wire in meters.

81. *Population Density.* The tiny country of Monaco has an area of 0.75 mi^2. It is estimated that the population of Monaco will be 38,000 in 2050. Find the number of people per square mile in 2050.

82. *Chesapeake Bay Bridge-Tunnel.* The 17.6-mi long Chesapeake Bay Bridge-Tunnel was completed in 1964. Construction costs were $210 million. Find the average cost per mile.

83. *Distance to a Star.* The nearest star, Alpha Centauri C, is about 4.22 light-years from Earth. One **light-year** is the distance that light travels in one year and is about 5.88×10^{12} mi. How many miles is it from Earth to Alpha Centauri C?

84. *Parsecs.* One **parsec** is about 3.26 light-years and 1 light-year is about 5.88×10^{12} mi. Find the number of miles in 1 parsec.

85. *Nuclear Disintegration.* One gram of radium produces 37 billion disintegrations per second. How many disintegrations are produced in 1 hr?

86. *Length of Earth's Orbit.* The average distance from Earth to the sun is 93 million mi. About how far does Earth travel in a yearly orbit? (Assume a circular orbit.)

Calculate.

87. $5 \cdot 3 + 8 \cdot 3^{2} + 4(6 - 2)$

88. $5[3 - 8 \cdot 3^{2} + 4 \cdot 6 - 2]$

89. $16 \div 4 \cdot 4 \div 2 \cdot 256$

90. $2^{6} \cdot 2^{-3} \div 2^{10} \div 2^{-8}$

91. $\dfrac{4(8 - 6)^{2} - 4 \cdot 3 + 2 \cdot 8}{3^{1} + 19^{0}}$

92. $\dfrac{[4(8-6)^2+4](3-2\cdot 8)}{2^2(2^3+5)}$

Compound Interest. *Use the compound interest formula from Example 10 for Exercises 93–96. Round to the nearest cent.*

93. Suppose that \$3225 is invested at 3.1%, compounded semiannually. How much is in the account at the end of 4 years?

94. Suppose that \$7550 is invested at 2.8%, compounded semiannually. How much is in the account at the end of 5 years?

95. Suppose that \$4100 is invested at 2.3%, compounded quarterly. How much is in the account at the end of 6 years?

96. Suppose that \$4875 is invested at 1.8%, compounded quarterly. How much is in the account at the end of 9 years?

Synthesis

Savings Plan. *The formula*

$$S = P\left[\frac{\left(1+\dfrac{r}{12}\right)^{12\cdot t}-1}{\dfrac{r}{12}}\right]$$

gives the amount S accumulated in a savings plan when a deposit of P dollars is made each month for t years in an account with interest rate r, compounded monthly. Use this formula for Exercises 97–100.

97. James deposits \$250 in a retirement account each month beginning at age 40. If the investment earns 5% interest, compounded monthly, how much will have accumulated in the account when he retires 27 years later?

98. Kayla deposits \$100 in a retirement account each month beginning at age 25. If the investment earns 4% interest, compounded monthly, how much will have accumulated in the account when she retires at age 65?

99. Sue and Richard want to establish a college fund for their daughter that will have accumulated \$120,000 at the end of 18 years. If they can count on an interest rate of 3%, compounded monthly, how much should they deposit each month to accomplish this?

100. Lamont wants to have \$200,000 accumulated in a retirement account by age 70. If he starts making monthly deposits to the plan at age 30 and can count on an interest rate of 4.5%, compounded monthly, how much should he deposit each month in order to accomplish this?

Simplify. Assume that all exponents are integers, all denominators are nonzero, and zero is not raised to a nonpositive power.

101. $(x^t \cdot x^{3t})^2$

102. $(x^y \cdot x^{-y})^3$

103. $(t^{a+x} \cdot t^{x-a})^4$

104. $(m^{x-b} \cdot n^{x+b})^x (m^b n^{-b})^x$

105. $\left[\dfrac{(3x^a y^b)^3}{(-3x^a y^b)^2}\right]^2$

106. $\left[\left(\dfrac{x^r}{y^t}\right)^2\left(\dfrac{x^{2r}}{y^{4t}}\right)^{-2}\right]^{-3}$

EXAMPLE 8 Multiply each of the following.

a) $(4x + 1)^2$ **b)** $(3y^2 - 2)^2$ **c)** $(x^2 + 3y)(x^2 - 3y)$

Solution

a) $(4x + 1)^2 = (4x)^2 + 2 \cdot 4x \cdot 1 + 1^2 = 16x^2 + 8x + 1$

b) $(3y^2 - 2)^2 = (3y^2)^2 - 2 \cdot 3y^2 \cdot 2 + 2^2 = 9y^4 - 12y^2 + 4$

c) $(x^2 + 3y)(x^2 - 3y) = (x^2)^2 - (3y)^2 = x^4 - 9y^2$

Now Try Exercises 27 and 37.

Division of polynomials is discussed in Section 4.3.

R.3 Exercise Set

Determine the terms and the degree of the polynomial.

1. $7x^3 - 4x^2 + 8x + 5$

2. $-3n^4 - 6n^3 + n^2 + 2n - 1$

3. $3a^4b - 7a^3b^3 + 5ab - 2$

4. $6p^3q^2 - p^2q^4 - 3pq^2 + 5$

Perform the indicated operations.

5. $(3ab^2 - 4a^2b - 2ab + 6) + (-ab^2 - 5a^2b + 8ab + 4)$

6. $(-6m^2n + 3mn^2 - 5mn + 2) + (4m^2n + 2mn^2 - 6mn - 9)$

7. $(2x + 3y + z - 7) + (4x - 2y - z + 8) + (-3x + y - 2z - 4)$

8. $(2x^2 + 12xy - 11) + (6x^2 - 2x + 4) + (-x^2 - y - 2)$

9. $(3x^2 - 2x - x^3 + 2) - (5x^2 - 8x - x^3 + 4)$

10. $(5x^2 + 4xy - 3y^2 + 2) - (9x^2 - 4xy + 2y^2 - 1)$

11. $(x^4 - 3x^2 + 4x) - (3x^3 + x^2 - 5x + 3)$

12. $(2x^4 - 3x^2 + 7x) - (5x^3 + 2x^2 - 3x + 5)$

13. $(3a^2)(-7a^4)$

14. $(8y^5)(9y)$

15. $(6xy^3)(9x^4y^2)$

16. $(-5m^4n^2)(6m^2n^3)$

17. $(a - b)(2a^3 - ab + 3b^2)$

18. $(n + 1)(n^2 - 6n - 4)$

19. $(y - 3)(y + 5)$

20. $(z + 4)(z - 2)$

21. $(x + 6)(x + 3)$

22. $(a - 8)(a - 1)$

23. $(2a + 3)(a + 5)$

24. $(3b + 1)(b - 2)$

25. $(2x + 3y)(2x + y)$

26. $(2a - 3b)(2a - b)$

27. $(x + 3)^2$

28. $(z + 6)^2$

29. $(y - 5)^2$

30. $(x - 4)^2$

31. $(5x - 3)^2$

32. $(3x - 2)^2$

33. $(2x + 3y)^2$

34. $(5x + 2y)^2$

35. $(2x^2 - 3y)^2$

36. $(4x^2 - 5y)^2$

37. $(n + 6)(n - 6)$

38. $(m + 1)(m - 1)$

39. $(3y + 4)(3y - 4)$

40. $(2x - 7)(2x + 7)$

41. $(3x - 2y)(3x + 2y)$

42. $(3x + 5y)(3x - 5y)$

43. $(2x + 3y + 4)(2x + 3y - 4)$

44. $(5x + 2y + 3)(5x + 2y - 3)$

45. $(x + 1)(x - 1)(x^2 + 1)$

46. $(y - 2)(y + 2)(y^2 + 4)$

Synthesis

Multiply. Assume that all exponents are natural numbers.

47. $(a^n + b^n)(a^n - b^n)$

48. $(t^a + 4)(t^a - 7)$

49. $(a^n + b^n)^2$

50. $(x^{3m} - t^{5n})^2$

51. $(x - 1)(x^2 + x + 1)(x^3 + 1)$

52. $[(2x - 1)^2 - 1]^2$

53. $(x^{a-b})^{a+b}$

54. $(t^{m+n})^{m+n} \cdot (t^{m-n})^{m-n}$

55. $(a + b + c)^2$

R.4 Factoring

- Factor polynomials by removing a common factor.
- Factor polynomials by grouping.
- Factor trinomials of the type $x^2 + bx + c$.
- Factor trinomials of the type $ax^2 + bx + c, a \neq 1$, using the FOIL method and the grouping method.
- Factor special products of polynomials.

To factor a polynomial, we do the reverse of multiplying; that is, we find an equivalent expression that is written as a product.

Terms with Common Factors

When a polynomial is to be factored, we should always look first to factor out a factor that is common to all the terms using the distributive property. We generally look for the constant common factor with the largest absolute value and for variables with the largest exponent common to all the terms. In this sense, we factor out the "largest" common factor.

EXAMPLE 1 Factor each of the following.

a) $15 + 10x - 5x^2$ b) $12x^2y^2 - 20x^3y$

Solution

a) $15 + 10x - 5x^2 = 5 \cdot 3 + 5 \cdot 2x - 5 \cdot x^2 = 5(3 + 2x - x^2)$

We can always check a factorization by multiplying:

$$5(3 + 2x - x^2) = 15 + 10x - 5x^2.$$

b) There are several factors common to the terms of $12x^2y^2 - 20x^3y$, but $4x^2y$ is the "largest" of these.

$$\begin{aligned} 12x^2y^2 - 20x^3y &= 4x^2y \cdot 3y - 4x^2y \cdot 5x \\ &= 4x^2y(3y - 5x) \end{aligned}$$

Now Try Exercise 3.

b) $16y^3 - 250 = 2(8y^3 - 125)$
$= 2[(2y)^3 - 5^3]$
$= 2(2y - 5)(4y^2 + 10y + 25)$

Now Try Exercises 67 and 69.

Not all polynomials can be factored into polynomials with integer coefficients. An example is $x^2 - x + 7$. There are no real factors of 7 whose sum is -1. In such a case, we say that the polynomial is "not factorable," or **prime**.

CONNECTING THE CONCEPTS

A Strategy for Factoring

A. Always factor out the largest common factor first.

B. Look at the number of terms.

Two terms: Try factoring as a difference of squares first. Next, try factoring as a sum or a difference of cubes. There is no rule for factoring a *sum* of squares.

Three terms: Try factoring as the square of a binomial. Next, try using the FOIL method or the grouping method for factoring a trinomial.

Four or more terms: Try factoring by grouping and factoring out a common binomial factor.

C. Always *factor completely*. If a factor with more than one term can itself be factored further, do so.

R.4 Exercise Set

Factor out the largest common factor.

1. $3x + 18$
2. $5y - 20$
3. $2z^3 - 8z^2$
4. $12m^2 + 3m^6$
5. $4a^2 - 12a + 16$
6. $6n^2 + 24n - 18$
7. $a(b - 2) + c(b - 2)$
8. $a(x^2 - 3) - 2(x^2 - 3)$

Factor by grouping.

9. $3x^3 - x^2 + 18x - 6$
10. $x^3 + 3x^2 + 6x + 18$
11. $y^3 - y^2 + 2y - 2$
12. $y^3 - y^2 + 3y - 3$
13. $24x^3 - 36x^2 + 72x - 108$
14. $5a^3 - 10a^2 + 25a - 50$
15. $x^3 - x^2 - 5x + 5$
16. $t^3 + 6t^2 - 2t - 12$
17. $a^3 - 3a^2 - 2a + 6$
18. $x^3 - x^2 - 6x + 6$

Factor the trinomial.

19. $w^2 - 7w + 10$
20. $p^2 + 6p + 8$
21. $x^2 + 6x + 5$
22. $x^2 - 8x + 12$

23. $t^2 + 8t + 15$
24. $y^2 + 12y + 27$
25. $x^2 - 6xy - 27y^2$
26. $t^2 - 2t - 15$
27. $2n^2 - 20n - 48$
28. $2a^2 - 2ab - 24b^2$
29. $y^2 - 4y - 21$
30. $m^2 - m - 90$
31. $y^4 - 9y^3 + 14y^2$
32. $3z^3 - 21z^2 + 18z$
33. $2x^3 - 2x^2y - 24xy^2$
34. $a^3b - 9a^2b^2 + 20ab^3$
35. $2n^2 + 9n - 56$
36. $3y^2 + 7y - 20$
37. $12x^2 + 11x + 2$
38. $6x^2 - 7x - 20$
39. $4x^2 + 15x + 9$
40. $2y^2 + 7y + 6$
41. $2y^2 + y - 6$
42. $20p^2 - 23p + 6$
43. $6a^2 - 29ab + 28b^2$
44. $10m^2 + 7mn - 12n^2$
45. $12a^2 - 4a - 16$
46. $12a^2 - 14a - 20$

Factor the difference of squares.

47. $z^2 - 81$
48. $m^2 - 4$
49. $16x^2 - 9$
50. $4z^2 - 81$
51. $6x^2 - 6y^2$
52. $8a^2 - 8b^2$
53. $4xy^4 - 4xz^2$
54. $5x^2y - 5yz^4$
55. $7pq^4 - 7py^4$
56. $25ab^4 - 25az^4$

Factor the square of a binomial.

57. $x^2 + 12x + 36$
58. $y^2 - 6y + 9$
59. $9z^2 - 12z + 4$
60. $4z^2 + 12z + 9$
61. $1 - 8x + 16x^2$
62. $1 + 10x + 25x^2$
63. $a^3 + 24a^2 + 144a$
64. $y^3 - 18y^2 + 81y$
65. $4p^2 - 8pq + 4q^2$
66. $5a^2 - 10ab + 5b^2$

Factor the sum or difference of cubes.

67. $x^3 + 64$
68. $y^3 - 8$
69. $m^3 - 216$
70. $n^3 + 1$
71. $8t^3 + 8$
72. $2y^3 - 128$
73. $3a^5 - 24a^2$
74. $250z^4 - 2z$
75. $t^6 + 1$
76. $27x^6 - 8$

Factor completely.

77. $18a^2b - 15ab^2$
78. $4x^2y + 12xy^2$
79. $x^3 - 4x^2 + 5x - 20$
80. $z^3 + 3z^2 - 3z - 9$
81. $8x^2 - 32$
82. $6y^2 - 6$
83. $4y^2 - 5$
84. $16x^2 - 7$
85. $m^2 - 9n^2$
86. $25t^2 - 16$
87. $x^2 + 9x + 20$
88. $y^2 + y - 6$
89. $y^2 - 6y + 5$
90. $x^2 - 4x - 21$
91. $2a^2 + 9a + 4$
92. $3b^2 - b - 2$
93. $6x^2 + 7x - 3$
94. $8x^2 + 2x - 15$
95. $y^2 - 18y + 81$
96. $n^2 + 2n + 1$
97. $9z^2 - 24z + 16$
98. $4z^2 + 20z + 25$
99. $x^2y^2 - 14xy + 49$
100. $x^2y^2 - 16xy + 64$
101. $4ax^2 + 20ax - 56a$
102. $21x^2y + 2xy - 8y$
103. $3z^3 - 24$
104. $4t^3 + 108$
105. $16a^7b + 54ab^7$
106. $24a^2x^4 - 375a^8x$
107. $y^3 - 3y^2 - 4y + 12$
108. $p^3 - 2p^2 - 9p + 18$
109. $x^3 - x^2 + x - 1$
110. $x^3 - x^2 - x + 1$
111. $5m^4 - 20$
112. $2x^2 - 288$
113. $2x^3 + 6x^2 - 8x - 24$
114. $3x^3 + 6x^2 - 27x - 54$
115. $4c^2 - 4cd + d^2$
116. $9a^2 - 6ab + b^2$
117. $m^6 + 8m^3 - 20$
118. $x^4 - 37x^2 + 36$
119. $p - 64p^4$
120. $125a - 8a^4$

Synthesis

Factor.

121. $y^4 - 84 + 5y^2$
122. $11x^2 + x^4 - 80$
123. $y^2 - \frac{8}{49} + \frac{2}{7}y$
124. $t^2 - \frac{27}{100} + \frac{3}{5}t$
125. $x^2 + 3x + \frac{9}{4}$
126. $x^2 - 5x + \frac{25}{4}$
127. $x^2 - x + \frac{1}{4}$
128. $x^2 - \frac{2}{3}x + \frac{1}{9}$
129. $(x + h)^3 - x^3$
130. $(x + 0.01)^2 - x^2$
131. $(y - 4)^2 + 5(y - 4) - 24$
132. $6(2p + q)^2 - 5(2p + q) - 25$

Factor. Assume that variables in exponents represent natural numbers.

133. $x^{2n} + 5x^n - 24$

134. $4x^{2n} - 4x^n - 3$

135. $x^2 + ax + bx + ab$

136. $bdy^2 + ady + bcy + ac$

137. $25y^{2m} - (x^{2n} - 2x^n + 1)$

138. $x^{6a} - t^{3b}$

139. $(y - 1)^4 - (y - 1)^2$

140. $x^6 - 2x^5 + x^4 - x^2 + 2x - 1$

The Basics of Equation Solving

R.5

- Solve linear equations.
- Solve quadratic equations.
- Solve a formula for a given letter.

An **equation** is a statement that two expressions are equal. To **solve** an equation in one variable is to find all the values of the variable that make the equation true. Each of these numbers is a **solution** of the equation. The set of all solutions of an equation is its **solution set**. Equations that have the same solution set are called **equivalent equations**.

Linear Equations and Quadratic Equations

A **linear equation in one variable** is an equation that is equivalent to one of the form $ax + b = 0$, where a and b are real numbers and $a \neq 0$.

A **quadratic equation** is an equation that is equivalent to one of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$.

The following principles allow us to solve many linear equations and quadratic equations.

Equation-Solving Principles

For any real numbers a, b, and c,

The Addition Principle: If $a = b$ is true, then $a + c = b + c$ is true.

The Multiplication Principle: If $a = b$ is true, then $ac = bc$ is true.

The Principle of Zero Products: If $ab = 0$ is true, then $a = 0$ *or* $b = 0$, and if $a = 0$ *or* $b = 0$, then $ab = 0$.

The Principle of Square Roots: If $x^2 = k$, then $x = \sqrt{k}$ *or* $x = -\sqrt{k}$.

EXAMPLE 5 The formula $A = P + Prt$ gives the amount A to which a principal of P dollars will grow when invested at simple interest rate r for t years. Solve the formula for P.

Solution We have

$$A = P + Prt \qquad \text{We want to isolate } P.$$
$$A = P(1 + rt) \qquad \text{Factoring}$$
$$\frac{A}{1 + rt} = \frac{P(1 + rt)}{1 + rt} \qquad \text{Dividing by } 1 + rt \text{ on both sides}$$
$$\frac{A}{1 + rt} = P.$$

Now Try Exercise 77.

The formula $P = \dfrac{A}{1 + rt}$ can be used to determine how much should be invested at simple interest rate r in order to have A dollars t years later.

EXAMPLE 6 Solve $A = \frac{1}{2}h(b_1 + b_2)$ for b_1.

Solution We have

$$A = \frac{1}{2}h(b_1 + b_2) \qquad \text{Formula for the area of a trapezoid}$$
$$2A = h(b_1 + b_2) \qquad \text{Multiplying by 2}$$
$$2A = hb_1 + hb_2 \qquad \text{Removing parentheses}$$
$$2A - hb_2 = hb_1 \qquad \text{Subtracting } hb_2$$
$$\frac{2A - hb_2}{h} = b_1. \qquad \text{Dividing by } h$$

Now Try Exercise 65.

R.5 Exercise Set

Solve.

1. $x - 5 = 7$
2. $y + 3 = 4$
3. $3x + 4 = -8$
4. $5x - 7 = 23$
5. $5y - 12 = 3$
6. $6x + 23 = 5$
7. $6x - 15 = 45$
8. $4x - 7 = 81$
9. $5x - 10 = 45$
10. $6x - 7 = 11$
11. $9t + 4 = -5$
12. $5x + 7 = -13$
13. $8x + 48 = 3x - 12$
14. $15x + 40 = 8x - 9$
15. $7y - 1 = 23 - 5y$
16. $3x - 15 = 15 - 3x$
17. $3x - 4 = 5 + 12x$
18. $9t - 4 = 14 + 15t$
19. $5 - 4a = a - 13$
20. $6 - 7x = x - 14$
21. $3m - 7 = -13 + m$
22. $5x - 8 = 2x - 8$
23. $11 - 3x = 5x + 3$
24. $20 - 4y = 10 - 6y$
25. $2(x + 7) = 5x + 14$
26. $3(y + 4) = 8y$
27. $24 = 5(2t + 5)$
28. $9 = 4(3y - 2)$
29. $5y - (2y - 10) = 25$
30. $8x - (3x - 5) = 40$

31. $7(3x + 6) = 11 - (x + 2)$

32. $9(2x + 8) = 20 - (x + 5)$

33. $4(3y - 1) - 6 = 5(y + 2)$

34. $3(2n - 5) - 7 = 4(n - 9)$

35. $x^2 + 3x - 28 = 0$

36. $y^2 - 4y - 45 = 0$

37. $x^2 + 5x = 0$

38. $t^2 + 6t = 0$

39. $y^2 + 6y + 9 = 0$

40. $n^2 + 4n + 4 = 0$

41. $x^2 + 100 = 20x$

42. $y^2 + 25 = 10y$

43. $x^2 - 4x - 32 = 0$

44. $t^2 + 12t + 27 = 0$

45. $3y^2 + 8y + 4 = 0$

46. $9y^2 + 15y + 4 = 0$

47. $12z^2 + z = 6$

48. $6x^2 - 7x = 10$

49. $12a^2 - 28 = 5a$

50. $21n^2 - 10 = n$

51. $14 = x(x - 5)$

52. $24 = x(x - 2)$

53. $x^2 - 36 = 0$

54. $y^2 - 81 = 0$

55. $z^2 = 144$

56. $t^2 = 25$

57. $2x^2 - 20 = 0$

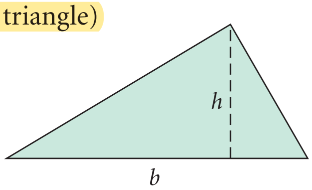
58. $3y^2 - 15 = 0$

59. $6z^2 - 18 = 0$

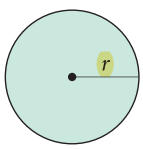
60. $5x^2 - 75 = 0$

Solve.

61. $A = \frac{1}{2}bh$, for b
(Area of a triangle)



62. $A = \pi r^2$, for π
(Area of a circle)



63. $P = 2l + 2w$, for w
(Perimeter of a rectangle)

64. $A = P + Prt$, for r
(Simple interest)

65. $A = \frac{1}{2}h(b_1 + b_2)$, for h
(Area of a trapezoid)

66. $A = \frac{1}{2}h(b_1 + b_2)$, for b_2

67. $V = \frac{4}{3}\pi r^3$, for π
(Volume of a sphere)

68. $V = \frac{4}{3}\pi r^3$, for r^3

69. $F = \frac{9}{5}C + 32$, for C
(Temperature conversion)

70. $Ax + By = C$, for y
(Standard linear equation)

71. $Ax + By = C$, for A

72. $2w + 2h + l = p$, for w

73. $2w + 2h + l = p$, for h

74. $3x + 4y = 12$, for y

75. $2x - 3y = 6$, for y

76. $T = \frac{3}{10}(I - 12{,}000)$, for I

77. $a = b + bcd$, for b

78. $q = p - np$, for p

79. $z = xy - xy^2$, for x

80. $st = t - 4$, for t

Synthesis

Solve.

81. $3[5 - 3(4 - t)] - 2 = 5[3(5t - 4) + 8] - 26$

82. $6[4(8 - y) - 5(9 + 3y)] - 21 = -7[3(7 + 4y) - 4]$

83. $x - \{3x - [2x - (5x - (7x - 1))]\} = x + 7$

84. $23 - 2[4 + 3(x - 1)] + 5[x - 2(x + 3)] = 7\{x - 2[5 - (2x + 3)]\}$

85. $(5x^2 + 6x)(12x^2 - 5x - 2) = 0$

86. $(3x^2 + 7x - 20)(x^2 - 4x) = 0$

87. $3x^3 + 6x^2 - 27x - 54 = 0$

88. $2x^3 + 6x^2 = 8x + 24$

R.6 Exercise Set

Find the domain of the rational expression.

1. $-\frac{5}{3}$

2. $\frac{4}{7-x}$

3. $\frac{3x-3}{x(x-1)}$

4. $\frac{15x-10}{2x(3x-2)}$

5. $\frac{x+5}{x^2+4x-5}$

6. $\frac{(x^2-4)(x+1)}{(x+2)(x^2-1)}$

7. $\frac{7x^2-28x+28}{(x^2-4)(x^2+3x-10)}$

8. $\frac{7x^2+11x-6}{x(x^2-x-6)}$

Simplify.

9. $\frac{x^2-4}{x^2-4x+4}$

10. $\frac{x^2+2x-3}{x^2-9}$

11. $\frac{x^3-6x^2+9x}{x^3-3x^2}$

12. $\frac{y^5-5y^4+4y^3}{y^3-6y^2+8y}$

13. $\frac{6y^2+12y-48}{3y^2-9y+6}$

14. $\frac{2x^2-20x+50}{10x^2-30x-100}$

15. $\frac{4-x}{x^2+4x-32}$

16. $\frac{6-x}{x^2-36}$

Multiply or divide and, if possible, simplify.

17. $\frac{r-s}{r+s} \cdot \frac{r^2-s^2}{(r-s)^2}$

18. $\frac{x^2-y^2}{(x-y)^2} \cdot \frac{1}{x+y}$

19. $\frac{x^2+2x-35}{3x^3-2x^2} \cdot \frac{9x^3-4x}{7x+49}$

20. $\frac{x^2-2x-35}{2x^3-3x^2} \cdot \frac{4x^3-9x}{7x-49}$

21. $\frac{a^2-a-6}{a^2-7a+12} \cdot \frac{a^2-2a-8}{a^2-3a-10}$

22. $\frac{a^2-a-12}{a^2-6a+8} \cdot \frac{a^2+a-6}{a^2-2a-24}$

23. $\frac{m^2-n^2}{r+s} \div \frac{m-n}{r+s}$

24. $\frac{a^2-b^2}{x-y} \div \frac{a+b}{x-y}$

25. $\frac{3x+12}{2x-8} \div \frac{(x+4)^2}{(x-4)^2}$

26. $\frac{a^2-a-2}{a^2-a-6} \div \frac{a^2-2a}{2a+a^2}$

27. $\frac{x^2-y^2}{x^3-y^3} \cdot \frac{x^2+xy+y^2}{x^2+2xy+y^2}$

28. $\frac{c^3+8}{c^2-4} \div \frac{c^2-2c+4}{c^2-4c+4}$

29. $\frac{(x-y)^2-z^2}{(x+y)^2-z^2} \div \frac{x-y+z}{x+y-z}$

30. $\frac{(a+b)^2-9}{(a-b)^2-9} \cdot \frac{a-b-3}{a+b+3}$

Add or subtract and, if possible, simplify.

31. $\frac{7}{5x} + \frac{3}{5x}$

32. $\frac{7}{12y} - \frac{1}{12y}$

33. $\frac{4}{3a+4} + \frac{3a}{3a+4}$

34. $\frac{a-3b}{a+b} + \frac{a+5b}{a+b}$

35. $\frac{5}{4z} - \frac{3}{8z}$

36. $\frac{12}{x^2y} + \frac{5}{xy^2}$

37. $\frac{3}{x+2} + \frac{2}{x^2-4}$

38. $\frac{5}{a-3} - \frac{2}{a^2-9}$

39. $\frac{y}{y^2-y-20} - \frac{2}{y+4}$

40. $\frac{6}{y^2+6y+9} - \frac{5}{y+3}$

41. $\frac{3}{x+y} + \frac{x-5y}{x^2-y^2}$

42. $\frac{a^2+1}{a^2-1} - \frac{a-1}{a+1}$

43. $\frac{y}{y-1} + \frac{2}{1-y}$
(*Note*: $1-y=-1(y-1)$.)

44. $\frac{a}{a-b} + \frac{b}{b-a}$
(*Note*: $b-a=-1(a-b)$.)

45. $\dfrac{x}{2x-3y} - \dfrac{y}{3y-2x}$

46. $\dfrac{3a}{3a-2b} - \dfrac{2a}{2b-3a}$

47. $\dfrac{9x+2}{3x^2-2x-8} + \dfrac{7}{3x^2+x-4}$

48. $\dfrac{3y}{y^2-7y+10} - \dfrac{2y}{y^2-8y+15}$

49. $\dfrac{5a}{a-b} + \dfrac{ab}{a^2-b^2} + \dfrac{4b}{a+b}$

50. $\dfrac{6a}{a-b} - \dfrac{3b}{b-a} + \dfrac{5}{a^2-b^2}$

51. $\dfrac{7}{x+2} - \dfrac{x+8}{4-x^2} + \dfrac{3x-2}{4-4x+x^2}$

52. $\dfrac{6}{x+3} - \dfrac{x+4}{9-x^2} + \dfrac{2x-3}{9-6x+x^2}$

53. $\dfrac{1}{x+1} + \dfrac{x}{2-x} + \dfrac{x^2+2}{x^2-x-2}$

54. $\dfrac{x-1}{x-2} - \dfrac{x+1}{x+2} - \dfrac{x-6}{4-x^2}$

Simplify.

55. $\dfrac{\dfrac{a-b}{b}}{\dfrac{a^2-b^2}{ab}}$

56. $\dfrac{\dfrac{x^2-y^2}{xy}}{\dfrac{x-y}{y}}$

57. $\dfrac{\dfrac{x}{y}-\dfrac{y}{x}}{\dfrac{1}{y}+\dfrac{1}{x}}$

58. $\dfrac{\dfrac{a}{b}-\dfrac{b}{a}}{\dfrac{1}{a}-\dfrac{1}{b}}$

59. $\dfrac{c+\dfrac{8}{c^2}}{1+\dfrac{2}{c}}$

60. $\dfrac{a-\dfrac{a}{b}}{b-\dfrac{b}{a}}$

61. $\dfrac{x^2+xy+y^2}{\dfrac{x^2}{y}-\dfrac{y^2}{x}}$

62. $\dfrac{\dfrac{a^2}{b}+\dfrac{b^2}{a}}{a^2-ab+b^2}$

63. $\dfrac{a-a^{-1}}{a+a^{-1}}$

64. $\dfrac{x^{-1}+y^{-1}}{x^{-3}+y^{-3}}$

65. $\dfrac{\dfrac{1}{x-3}+\dfrac{2}{x+3}}{\dfrac{3}{x-1}-\dfrac{4}{x+2}}$

66. $\dfrac{\dfrac{5}{x+1}-\dfrac{3}{x-2}}{\dfrac{1}{x-5}+\dfrac{2}{x+2}}$

67. $\dfrac{\dfrac{a}{1-a}+\dfrac{1+a}{a}}{\dfrac{1-a}{a}+\dfrac{a}{1+a}}$

68. $\dfrac{\dfrac{1-x}{x}+\dfrac{x}{1+x}}{\dfrac{1+x}{x}+\dfrac{x}{1-x}}$

69. $\dfrac{\dfrac{1}{a^2}+\dfrac{2}{ab}+\dfrac{1}{b^2}}{\dfrac{1}{a^2}-\dfrac{1}{b^2}}$

70. $\dfrac{\dfrac{1}{x^2}-\dfrac{1}{y^2}}{\dfrac{1}{x^2}-\dfrac{2}{xy}+\dfrac{1}{y^2}}$

Synthesis

Simplify.

71. $\dfrac{(x+h)^2-x^2}{h}$

72. $\dfrac{\dfrac{1}{x+h}-\dfrac{1}{x}}{h}$

73. $\dfrac{(x+h)^3-x^3}{h}$

74. $\dfrac{\dfrac{1}{(x+h)^2}-\dfrac{1}{x^2}}{h}$

75. $\left[\dfrac{\dfrac{x+1}{x-1}+1}{\dfrac{x+1}{x-1}-1}\right]^5$

76. $1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{x}}}}$

Perform the indicated operations and, if possible, simplify.

77. $\dfrac{n(n+1)(n+2)}{2\cdot 3} + \dfrac{(n+1)(n+2)}{2}$

78. $\dfrac{n(n+1)(n+2)(n+3)}{2\cdot 3\cdot 4} + \dfrac{(n+1)(n+2)(n+3)}{2\cdot 3}$

79. $\dfrac{x^2-9}{x^3+27}\cdot\dfrac{5x^2-15x+45}{x^2-2x-3} + \dfrac{x^2+x}{4+2x}$

80. $\dfrac{x^2+2x-3}{x^2-x-12} \div \dfrac{x^2-1}{x^2-16} - \dfrac{2x+1}{x^2+2x+1}$

EXAMPLE 7 Convert to radical notation and, if possible, simplify each of the following.

a) $7^{3/4}$ b) $8^{-5/3}$ c) $m^{1/6}$ d) $(-32)^{2/5}$

Solution

a) $7^{3/4} = \sqrt[4]{7^3}$, or $\left(\sqrt[4]{7}\right)^3$

b) $8^{-5/3} = \dfrac{1}{8^{5/3}} = \dfrac{1}{\left(\sqrt[3]{8}\right)^5} = \dfrac{1}{2^5} = \dfrac{1}{32}$

c) $m^{1/6} = \sqrt[6]{m}$

d) $(-32)^{2/5} = \sqrt[5]{(-32)^2} = \sqrt[5]{1024} = 4$, or
$(-32)^{2/5} = \left(\sqrt[5]{-32}\right)^2 = (-2)^2 = 4$

Now Try Exercise 87.

EXAMPLE 8 Convert each of the following to exponential notation.

a) $\left(\sqrt[4]{7xy}\right)^5$ b) $\sqrt[6]{x^3}$

Solution

a) $\left(\sqrt[4]{7xy}\right)^5 = (7xy)^{5/4}$

b) $\sqrt[6]{x^3} = x^{3/6} = x^{1/2}$

Now Try Exercise 97.

We can use the laws of exponents to simplify exponential expressions and radical expressions.

EXAMPLE 9 Simplify and then, if appropriate, write radical notation for each of the following.

a) $x^{5/6} \cdot x^{2/3}$ b) $(x+3)^{5/2}(x+3)^{-1/2}$ c) $\sqrt[3]{\sqrt{7}}$

Solution

a) $x^{5/6} \cdot x^{2/3} = x^{5/6+2/3} = x^{9/6} = x^{3/2} = \sqrt{x^3} = \sqrt{x^2}\sqrt{x} = x\sqrt{x}$

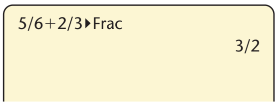
b) $(x+3)^{5/2}(x+3)^{-1/2} = (x+3)^{5/2-1/2} = (x+3)^2$

c) $\sqrt[3]{\sqrt{7}} = \sqrt[3]{7^{1/2}} = (7^{1/2})^{1/3} = 7^{1/6} = \sqrt[6]{7}$

Now Try Exercise 107.

TECHNOLOGY CONNECTION

We can add and subtract rational exponents on a graphing calculator. The FRAC feature from the MATH menu allows us to express the result as a fraction. The addition of the exponents in Example 9(a) is shown here.



EXAMPLE 10 Write an expression containing a single radical: $\sqrt{a}\sqrt[6]{b^5}$.

Solution $\sqrt{a}\sqrt[6]{b^5} = a^{1/2}b^{5/6} = a^{3/6}b^{5/6} = (a^3b^5)^{1/6} = \sqrt[6]{a^3b^5}$

Now Try Exercise 117.

R.7 Exercise Set

Simplify. Assume that variables can represent any real number.

1. $\sqrt{(-21)^2}$
2. $\sqrt{(-7)^2}$
3. $\sqrt{9y^2}$
4. $\sqrt{64t^2}$
5. $\sqrt{(a-2)^2}$
6. $\sqrt{(2b+5)^2}$
7. $\sqrt[3]{-27x^3}$
8. $\sqrt[3]{-8y^3}$
9. $\sqrt[4]{81x^8}$
10. $\sqrt[4]{16z^{12}}$

11. $\sqrt[5]{32}$

12. $\sqrt[5]{-32}$

13. $\sqrt{180}$

14. $\sqrt{48}$

15. $\sqrt{72}$

16. $\sqrt{250}$

17. $\sqrt[3]{54}$

18. $\sqrt[3]{135}$

19. $\sqrt{128c^2d^4}$

20. $\sqrt{162c^4d^6}$

21. $\sqrt[4]{48x^6y^4}$

22. $\sqrt[4]{243m^5n^{10}}$

23. $\sqrt{x^2 - 4x + 4}$

24. $\sqrt{x^2 + 16x + 64}$

Simplify. Assume that no radicands were formed by raising negative quantities to even powers.

25. $\sqrt{15}\sqrt{35}$

26. $\sqrt{21}\sqrt{6}$

27. $\sqrt{8}\sqrt{10}$

28. $\sqrt{12}\sqrt{15}$

29. $\sqrt{2x^3y}\sqrt{12xy}$

30. $\sqrt{3y^4z}\sqrt{20z}$

31. $\sqrt[3]{3x^2y}\sqrt[3]{36x}$

32. $\sqrt[5]{8x^3y^4}\sqrt[5]{4x^4y}$

33. $\sqrt[3]{2(x+4)}\sqrt[3]{4(x+4)^4}$

34. $\sqrt[3]{4(x+1)^2}\sqrt[3]{18(x+1)^2}$

35. $\sqrt[8]{\dfrac{m^{16}n^{24}}{2^8}}$

36. $\sqrt[6]{\dfrac{m^{12}n^{24}}{64}}$

37. $\dfrac{\sqrt{40xy}}{\sqrt{8x}}$

38. $\dfrac{\sqrt[3]{40m}}{\sqrt[3]{5m}}$

39. $\dfrac{\sqrt[3]{3x^2}}{\sqrt[3]{24x^5}}$

40. $\dfrac{\sqrt{128a^2b^4}}{\sqrt{16ab}}$

41. $\sqrt[3]{\dfrac{64a^4}{27b^3}}$

42. $\sqrt{\dfrac{9x^7}{16y^8}}$

43. $\sqrt{\dfrac{7x^3}{36y^6}}$

44. $\sqrt[3]{\dfrac{2yz}{250z^4}}$

45. $5\sqrt{2} + 3\sqrt{32}$

46. $7\sqrt{12} - 2\sqrt{3}$

47. $6\sqrt{20} - 4\sqrt{45} + \sqrt{80}$

48. $2\sqrt{32} + 3\sqrt{8} - 4\sqrt{18}$

49. $8\sqrt{2x^2} - 6\sqrt{20x} - 5\sqrt{8x^2}$

50. $2\sqrt[3]{8x^2} + 5\sqrt[3]{27x^2} - 3\sqrt{x^3}$

51. $(\sqrt{8} + 2\sqrt{5})(\sqrt{8} - 2\sqrt{5})$

52. $(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})$

53. $(2\sqrt{3} + \sqrt{5})(\sqrt{3} - 3\sqrt{5})$

54. $(\sqrt{6} - 4\sqrt{7})(3\sqrt{6} + 2\sqrt{7})$

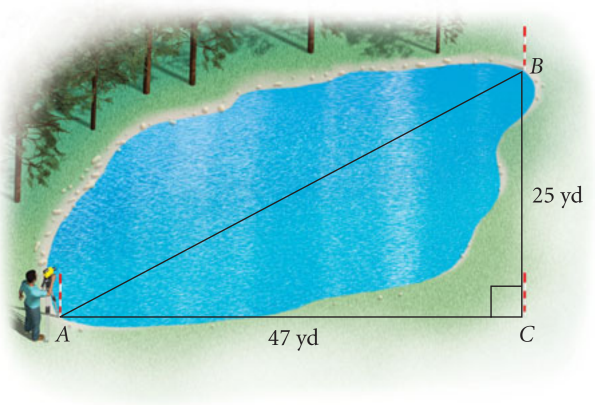
55. $\left(\sqrt{2} - 5\right)^2$

56. $\left(1 + \sqrt{3}\right)^2$

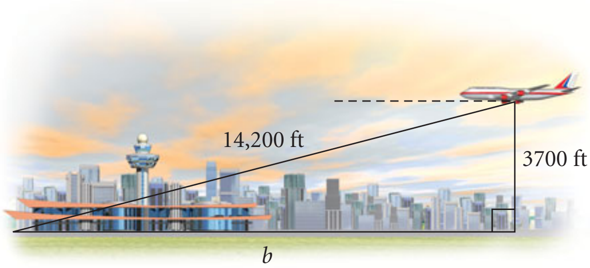
57. $\left(\sqrt{5} - \sqrt{6}\right)^2$

58. $\left(\sqrt{3} + \sqrt{2}\right)^2$

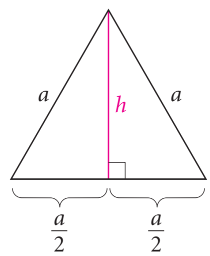
59. *Surveying.* A surveyor places poles at points A, B, and C in order to measure the distance across a pond. The distances AC and BC are measured as shown. Find the distance AB across the pond.



60. *Distance from Airport.* An airplane is flying at an altitude of 3700 ft. The slanted distance directly to the airport is 14,200 ft. How far horizontally is the airplane from the airport?



61. An *equilateral triangle* is shown below.

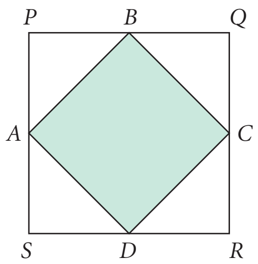


a) Find an expression for its height h in terms of a.
b) Find an expression for its area A in terms of a.

62. An isosceles right triangle has legs of length s. Find an expression for the length of the hypotenuse in terms of s.

63. The diagonal of a square has length $8\sqrt{2}$. Find the length of a side of the square.

64. The area of square $PQRS$ is 100 ft^2, and A, B, C, and D are the midpoints of the sides. Find the area of square $ABCD$.



Rationalize the denominator.

65. $\sqrt{\frac{3}{7}}$

66. $\sqrt{\frac{2}{3}}$

67. $\frac{\sqrt[3]{7}}{\sqrt[3]{25}}$

68. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$

69. $\sqrt[3]{\frac{16}{9}}$

70. $\sqrt[3]{\frac{3}{5}}$

71. $\frac{2}{\sqrt{3} - 1}$

72. $\frac{6}{3 + \sqrt{5}}$

73. $\frac{1 - \sqrt{2}}{2\sqrt{3} - \sqrt{6}}$

74. $\frac{\sqrt{5} + 4}{\sqrt{2} + 3\sqrt{7}}$

75. $\frac{6}{\sqrt{m} - \sqrt{n}}$

76. $\frac{3}{\sqrt{v} + \sqrt{w}}$

Rationalize the numerator.

77. $\frac{\sqrt{50}}{3}$

78. $\frac{\sqrt{12}}{5}$

79. $\sqrt[3]{\frac{2}{5}}$

80. $\sqrt[3]{\frac{7}{2}}$

81. $\frac{\sqrt{11}}{\sqrt{3}}$

82. $\frac{\sqrt{5}}{\sqrt{2}}$

83. $\frac{9 - \sqrt{5}}{3 - \sqrt{3}}$

84. $\frac{8 - \sqrt{6}}{5 - \sqrt{2}}$

85. $\frac{\sqrt{a} + \sqrt{b}}{3a}$

86. $\frac{\sqrt{p} - \sqrt{q}}{1 + \sqrt{q}}$

Convert to radical notation and, if possible, simplify.

87. $y^{5/6}$

88. $x^{2/3}$

89. $16^{3/4}$

90. $4^{7/2}$

91. $125^{-1/3}$

92. $32^{-4/5}$

93. $a^{5/4}b^{-3/4}$

94. $x^{2/5}y^{-1/5}$

95. $m^{5/3}n^{7/3}$

96. $p^{7/6}q^{11/6}$

Convert to exponential notation.

97. $\sqrt[5]{17^3}$

98. $\left(\sqrt[4]{13}\right)^5$

99. $\left(\sqrt[5]{12}\right)^4$

100. $\sqrt[3]{20^2}$

101. $\sqrt[3]{\sqrt{11}}$

102. $\sqrt[3]{\sqrt[4]{7}}$

103. $\sqrt{5}\sqrt[3]{5}$

104. $\sqrt[3]{2}\sqrt{2}$

105. $\sqrt[5]{32^2}$

106. $\sqrt[3]{64^2}$

Simplify and then, if appropriate, write radical notation.

107. $(2a^{3/2})(4a^{1/2})$

108. $(3a^{5/6})(8a^{2/3})$

109. $\left(\frac{x^6}{9b^{-4}}\right)^{1/2}$

110. $\left(\frac{x^{2/3}}{4y^{-2}}\right)^{1/2}$

111. $\frac{x^{2/3}y^{5/6}}{x^{-1/3}y^{1/2}}$

112. $\frac{a^{1/2}b^{5/8}}{a^{1/4}b^{3/8}}$

113. $(m^{1/2}n^{5/2})^{2/3}$

114. $(x^{5/3}y^{1/3}z^{2/3})^{3/5}$

115. $a^{3/4}(a^{2/3} + a^{4/3})$

116. $m^{2/3}(m^{7/4} - m^{5/4})$

Write an expression containing a single radical and simplify.

117. $\sqrt[3]{6}\sqrt{2}$

118. $\sqrt{2}\sqrt[4]{8}$

119. $\sqrt[4]{xy}\sqrt[3]{x^2y}$

120. $\sqrt[3]{ab^2}\sqrt{ab}$

121. $\sqrt[3]{a^4\sqrt{a^3}}$

122. $\sqrt{a^3\sqrt[3]{a^2}}$

123. $\frac{\sqrt{(a + x)^3}\sqrt[3]{(a + x)^2}}{\sqrt[4]{a + x}}$

124. $\frac{\sqrt[4]{(x + y)^2}\sqrt[3]{x + y}}{\sqrt{(x + y)^3}}$

Synthesis

Simplify.

125. $\sqrt{1 + x^2} + \frac{1}{\sqrt{1 + x^2}}$

126. $\sqrt{1 - x^2} - \frac{x^2}{2\sqrt{1 - x^2}}$

127. $\left(\sqrt{a^{\sqrt{a}}}\right)^{\sqrt{a}}$

128. $(2a^3b^{5/4}c^{1/7})^4 \div (54a^{-2}b^{2/3}c^{6/5})^{-1/3}$

REVIEW EXERCISES

Answers for all the review exercises appear in the answer section at the back of the book. If your answer is incorrect, restudy the section indicated in red next to the exercise or the direction line that precedes it.

Determine whether the statement is true or false.

1. If $a < 0$, then $|a| = -a$. [R.1]
2. For any real number a, $a \neq 0$, and any integers m and n, $a^m \cdot a^n = a^{mn}$. [R.2]
3. If $a = b$ is true, then $a + c = b + c$ is true. [R.5]
4. The domain of an algebraic expression is the set of all real numbers for which the expression is defined. [R.6]

In Exercises 5–10, consider the following numbers

$$-7,\ 43,\ -\frac{4}{9},\ \sqrt{17},\ 0,\ 2.191191119\ldots,\ \sqrt[3]{64},$$
$$-\sqrt{2},\ 4\frac{3}{4},\ \frac{12}{7},\ 102,\ \sqrt[5]{5}.$$

5. Which are rational numbers? [R.1]
6. Which are whole numbers? [R.1]
7. Which are integers? [R.1]
8. Which are real numbers? [R.1]
9. Which are natural numbers? [R.1]
10. Which are irrational numbers? [R.1]
11. Write interval notation for $\{x \mid -4 < x \leq 7\}$. [R.1]

Simplify. [R.1]

12. $|24|$
13. $\left|-\frac{7}{8}\right|$
14. Find the distance between -5 and 5 on the number line. [R.1]

Calculate. [R.2]

15. $3 \cdot 2 - 4 \cdot 2^2 + 6(3 - 1)$
16. $\dfrac{3^4 - (6 - 7)^4}{2^3 - 2^4}$

Convert to decimal notation. [R.2]

17. 8.3×10^{-5}
18. 2.07×10^7

Convert to scientific notation. [R.2]

19. 405,000
20. 0.00000039

Compute. Write the answer using scientific notation. [R.2]

21. $(3.1 \times 10^5)(4.5 \times 10^{-3})$
22. $\dfrac{2.5 \times 10^{-8}}{3.2 \times 10^{13}}$

Simplify.

23. $(-3x^4y^{-5})(4x^{-2}y)$ [R.2]
24. $\dfrac{48a^{-3}b^2c^5}{6a^3b^{-1}c^4}$ [R.2]
25. $\sqrt[4]{81}$ [R.7]
26. $\sqrt[5]{-32}$ [R.7]
27. $\dfrac{b - a^{-1}}{a - b^{-1}}$ [R.6]
28. $\dfrac{\dfrac{x^2}{y} + \dfrac{y^2}{x}}{y^2 - xy + x^2}$ [R.6]
29. $(\sqrt{3} - \sqrt{7})(\sqrt{3} + \sqrt{7})$ [R.7]
30. $(5 - \sqrt{2})^2$ [R.7]
31. $8\sqrt{5} + \dfrac{25}{\sqrt{5}}$ [R.7]
32. $(x + t)(x^2 - xt + t^2)$ [R.3]
33. $(5a + 4b)(2a - 3b)$ [R.3]
34. $(6x^2y - 3xy^2 + 5xy - 3) - (-4x^2y - 4xy^2 + 3xy + 8)$ [R.3]

Factor. [R.4]

35. $32x^4 - 40xy^3$

36. $y^3 + 3y^2 - 2y - 6$

37. $24x + 144 + x^2$

38. $9x^3 + 35x^2 - 4x$

39. $9x^2 - 30x + 25$

40. $8x^3 - 1$

41. $18x^2 - 3x + 6$

42. $4x^3 - 4x^2 - 9x + 9$

43. $6x^3 + 48$

44. $a^2b^2 - ab - 6$

45. $2x^2 + 5x - 3$

Solve. [R.5]

46. $2x - 7 = 7$

47. $5x - 7 = 3x - 9$

48. $8 - 3x = -7 + 2x$

49. $6(2x - 1) = 3 - (x + 10)$

50. $y^2 + 16y + 64 = 0$

51. $x^2 - x = 20$

52. $2x^2 + 11x - 6 = 0$

53. $x(x - 2) = 3$

54. $y^2 - 16 = 0$

55. $n^2 - 7 = 0$

56. Divide and simplify:

$$\frac{3x^2 - 12}{x^2 + 4x + 4} \div \frac{x - 2}{x + 2}.$$ [R.6]

57. Subtract and simplify:

$$\frac{x}{x^2 + 9x + 20} - \frac{4}{x^2 + 7x + 12}.$$ [R.6]

Write an expression containing a single radical. [R.7]

58. $\sqrt{y^5}\sqrt[3]{y^2}$

59. $\dfrac{\sqrt{(a + b)^3}\sqrt[3]{a + b}}{\sqrt[6]{(a + b)^7}}$

60. Convert to radical notation: $b^{7/5}$. [R.7]

61. Convert to exponential notation:

$$\sqrt[8]{\frac{m^{32}n^{16}}{3^8}}.$$ [R.7]

62. Rationalize the denominator:

$$\frac{4 - \sqrt{3}}{5 + \sqrt{3}}.$$ [R.7]

63. How long is a guy wire that reaches from the top of a 17-ft pole to a point on the ground 8 ft from the bottom of the pole? [R.7]

64. Calculate: $128 \div (-2)^3 \div (-2) \cdot 3$. [R.2]

A. $\dfrac{8}{3}$ **B.** 24 **C.** 96 **D.** $\dfrac{512}{3}$

65. Factor completely: $9x^2 - 36y^2$. [R.4]

A. $(3x + 6y)(3x - 6y)$
B. $3(x + 2y)(x - 2y)$
C. $9(x + 2y)(x - 2y)$
D. $9(x - 2y)^2$

Synthesis

Mortgage Payments. *The formula*

$$M = P\left[\frac{\frac{r}{12}\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}\right]$$

gives the monthly mortgage payment M on a home loan of P dollars at interest rate r, where n is the total number of payments (12 times the number of years). Use this formula in Exercises 66–69. [R.2]

66. The cost of a house is \$98,000. The down payment is \$16,000, the interest rate is $6\frac{1}{2}\%$, and the loan period is 25 years. What is the monthly mortgage payment?

67. The cost of a house is \$124,000. The down payment is \$20,000, the interest rate is $5\frac{3}{4}\%$, and the loan period is 30 years. What is the monthly mortgage payment?

68. The cost of a house is \$135,000. The down payment is \$18,000, the interest rate is $7\frac{1}{2}\%$, and the loan period is 20 years. What is the monthly mortgage payment?

69. The cost of a house is \$151,000. The down payment is \$21,000, the interest rate is $6\frac{1}{4}\%$, and the loan period is 25 years. What is the monthly mortgage payment?

Multiply. Assume that all exponents are integers. [R.3]

70. $(x^n + 10)(x^n - 4)$

71. $(t^a + t^{-a})^2$

72. $(y^b - z^c)(y^b + z^c)$

73. $(a^n - b^n)^3$

Factor. [R.4]

74. $y^{2n} + 16y^n + 64$

75. $x^{2t} - 3x^t - 28$

76. $m^{6n} - m^{3n}$

Collaborative Discussion and Writing

To the student and the instructor: The Collaborative Discussion and Writing exercises are meant to be answered with one or more sentences. These exercises can also be discussed and answered collaboratively by the entire class or by small groups. Answers to these exercises appear at the back of the book.

77. Anya says that $15 - 6 \div 3 \cdot 4$ is 12. What mistake is she probably making? [R.2]

78. When adding or subtracting rational expressions, we can always find a common denominator by forming the product of all the denominators. Explain why it is usually preferable to find the least common denominator. [R.6]

79. Explain how the rule for factoring a sum of cubes can be used to factor a difference of cubes. [R.4]

80. Explain how you would determine whether $10\sqrt{26} - 50$ is positive or negative without carrying out the actual computation. [R.7]

Chapter R Test

1. Consider the numbers

$$6\frac{6}{7},\ \sqrt{12},\ 0,\ -\frac{13}{4},\ \sqrt[3]{8},\ -1.2,\ 29,\ -5.$$

a) Which are whole numbers?
b) Which are irrational numbers?
c) Which are integers but not natural numbers?
d) Which are rational numbers but not integers?

Simplify.

2. $|-17.6|$ **3.** $\left|\frac{15}{11}\right|$ **4.** $|0|$

5. Write interval notation for $\{x \mid -3 < x \le 6\}$. Then graph the interval.

6. Find the distance between -9 and 6 on the number line.

7. Calculate: $32 \div 2^3 - 12 \div 4 \cdot 3$.

8. Convert to scientific notation: 4,509,000.

9. Convert to decimal notation: 8.6×10^{-5}.

10. Compute and write scientific notation for the answer:

$$\frac{2.7 \times 10^4}{3.6 \times 10^{-3}}.$$

Simplify.

11. $x^{-8} \cdot x^5$

12. $(2y^2)^3(3y^4)^2$

13. $(-3a^5b^{-4})(5a^{-1}b^3)$

14. $(5xy^4 - 7xy^2 + 4x^2 - 3) - (-3xy^4 + 2xy^2 - 2y + 4)$

15. $(y - 2)(3y + 4)$

16. $(4x - 3)^2$

17. $\dfrac{\dfrac{x}{y} - \dfrac{y}{x}}{x + y}$

18. $\sqrt{45}$

19. $\sqrt[3]{56}$

20. $3\sqrt{75} + 2\sqrt{27}$

21. $\sqrt{18}\,\sqrt{10}$

22. $(2 + \sqrt{3})(5 - 2\sqrt{3})$

Factor.

23. $8x^2 - 18$

24. $y^2 - 3y - 18$

25. $2n^2 + 5n - 12$

26. $x^3 + 10x^2 + 25x$

27. $m^3 - 8$

Solve.

28. $7x - 4 = 24$

29. $3(y - 5) + 6 = 8 - (y + 2)$

30. $2x^2 + 5x + 3 = 0$

31. $z^2 - 11 = 0$

32. Multiply and simplify:
$$\frac{x^2 + x - 6}{x^2 + 8x + 15} \cdot \frac{x^2 - 25}{x^2 - 4x + 4}.$$

33. Subtract and simplify:
$$\frac{x}{x^2 - 1} - \frac{3}{x^2 + 4x - 5}.$$

34. Rationalize the denominator:
$$\frac{5}{7 - \sqrt{3}}.$$

35. Convert to radical notation: $m^{3/8}$.

36. Convert to exponential notation: $\sqrt[6]{3^5}$.

37. How long is a guy wire that reaches from the top of a 12-ft pole to a point on the ground 5 ft from the bottom of the pole?

Synthesis

38. Multiply: $(x - y - 1)^2$.

2.3 Exercise Set

Given that $f(x) = 3x + 1$, $g(x) = x^2 - 2x - 6$, *and* $h(x) = x^3$, *find each of the following.*

1. $(f \circ g)(-1)$

2. $(g \circ f)(-2)$

3. $(h \circ f)(1)$

4. $(g \circ h)\left(\frac{1}{2}\right)$

5. $(g \circ f)(5)$

6. $(f \circ g)\left(\frac{1}{3}\right)$

7. $(f \circ h)(-3)$

8. $(h \circ g)(3)$

9. $(g \circ g)(-2)$

10. $(g \circ g)(3)$

11. $(h \circ h)(2)$

12. $(h \circ h)(-1)$

13. $(f \circ f)(-4)$

14. $(f \circ f)(1)$

15. $(h \circ h)(x)$

16. $(f \circ f)(x)$

Find $(f \circ g)(x)$ *and* $(g \circ f)(x)$ *and the domain of each.*

17. $f(x) = x + 3$, $g(x) = x - 3$

18. $f(x) = \frac{4}{5}x$, $g(x) = \frac{5}{4}x$

19. $f(x) = x + 1$, $g(x) = 3x^2 - 2x - 1$

20. $f(x) = 3x - 2$, $g(x) = x^2 + 5$

21. $f(x) = x^2 - 3$, $g(x) = 4x - 3$

22. $f(x) = 4x^2 - x + 10$, $g(x) = 2x - 7$

23. $f(x) = \dfrac{4}{1 - 5x}$, $g(x) = \dfrac{1}{x}$

24. $f(x) = \dfrac{6}{x}$, $g(x) = \dfrac{1}{2x + 1}$

25. $f(x) = 3x - 7$, $g(x) = \dfrac{x + 7}{3}$

26. $f(x) = \frac{2}{3}x - \frac{4}{5}$, $g(x) = 1.5x + 1.2$

27. $f(x) = 2x + 1$, $g(x) = \sqrt{x}$

28. $f(x) = \sqrt{x}$, $g(x) = 2 - 3x$

29. $f(x) = 20$, $g(x) = 0.05$

30. $f(x) = x^4$, $g(x) = \sqrt[4]{x}$

31. $f(x) = \sqrt{x + 5}$, $g(x) = x^2 - 5$

32. $f(x) = x^5 - 2$, $g(x) = \sqrt[5]{x + 2}$

33. $f(x) = x^2 + 2$, $g(x) = \sqrt{3 - x}$

34. $f(x) = 1 - x^2$, $g(x) = \sqrt{x^2 - 25}$

35. $f(x) = \dfrac{1 - x}{x}$, $g(x) = \dfrac{1}{1 + x}$

36. $f(x) = \dfrac{1}{x - 2}$, $g(x) = \dfrac{x + 2}{x}$

37. $f(x) = x^3 - 5x^2 + 3x + 7$, $g(x) = x + 1$

38. $f(x) = x - 1$, $g(x) = x^3 + 2x^2 - 3x - 9$

Find $f(x)$ *and* $g(x)$ *such that* $h(x) = (f \circ g)(x)$. *Answers may vary.*

39. $h(x) = (4 + 3x)^5$

40. $h(x) = \sqrt[3]{x^2 - 8}$

41. $h(x) = \dfrac{1}{(x - 2)^4}$

42. $h(x) = \dfrac{1}{\sqrt{3x + 7}}$

43. $h(x) = \dfrac{x^3 - 1}{x^3 + 1}$

44. $h(x) = |9x^2 - 4|$

45. $h(x) = \left(\dfrac{2 + x^3}{2 - x^3}\right)^6$

46. $h(x) = (\sqrt{x} - 3)^4$

47. $h(x) = \sqrt{\dfrac{x - 5}{x + 2}}$

48. $h(x) = \sqrt{1 + \sqrt{1 + x}}$

49. $h(x) = (x + 2)^3 - 5(x + 2)^2 + 3(x + 2) - 1$

50. $h(x) = 2(x - 1)^{5/3} + 5(x - 1)^{2/3}$

EXAMPLE 6 Divide $2 - 5i$ by $1 - 6i$.

Solution We write fraction notation and then multiply by 1, using the conjugate of the denominator to form the symbol for 1.

$$\frac{2-5i}{1-6i} = \frac{2-5i}{1-6i} \cdot \frac{1+6i}{1+6i}$$ Note that $1 + 6i$ is the conjugate of the divisor, $1 - 6i$.

$$= \frac{(2-5i)(1+6i)}{(1-6i)(1+6i)}$$

$$= \frac{2+12i-5i-30i^2}{1-36i^2}$$

$$= \frac{2+7i+30}{1+36}$$ $i^2 = -1$

$$= \frac{32+7i}{37}$$

$$= \frac{32}{37} + \frac{7}{37}i.$$ Writing the quotient in the form $a + bi$

Now Try Exercise 69.

TECHNOLOGY CONNECTION

With a graphing calculator set in $a + bi$ mode, we can divide complex numbers and express the real and imaginary parts in fraction form, just as we did in Example 6.

```
(2−5i)/(1−6i)▶Frac
              32/37+7/37i
```

3.1 Exercise Set

Express the number in terms of i.

1. $\sqrt{-3}$
2. $\sqrt{-21}$
3. $\sqrt{-25}$
4. $\sqrt{-100}$
5. $-\sqrt{-33}$
6. $-\sqrt{-59}$
7. $-\sqrt{-81}$
8. $-\sqrt{-9}$
9. $\sqrt{-98}$
10. $\sqrt{-28}$

Simplify. Write answers in the form $a + bi$, where a and b are real numbers.

11. $(-5 + 3i) + (7 + 8i)$
12. $(-6 - 5i) + (9 + 2i)$
13. $(4 - 9i) + (1 - 3i)$
14. $(7 - 2i) + (4 - 5i)$
15. $(12 + 3i) + (-8 + 5i)$
16. $(-11 + 4i) + (6 + 8i)$
17. $(-1 - i) + (-3 - i)$
18. $(-5 - i) + (6 + 2i)$
19. $(3 + \sqrt{-16}) + (2 + \sqrt{-25})$
20. $(7 - \sqrt{-36}) + (2 + \sqrt{-9})$
21. $(10 + 7i) - (5 + 3i)$
22. $(-3 - 4i) - (8 - i)$
23. $(13 + 9i) - (8 + 2i)$
24. $(-7 + 12i) - (3 - 6i)$
25. $(6 - 4i) - (-5 + i)$
26. $(8 - 3i) - (9 - i)$
27. $(-5 + 2i) - (-4 - 3i)$
28. $(-6 + 7i) - (-5 - 2i)$
29. $(4 - 9i) - (2 + 3i)$
30. $(10 - 4i) - (8 + 2i)$
31. $\sqrt{-4} \cdot \sqrt{-36}$
32. $\sqrt{-49} \cdot \sqrt{-9}$
33. $\sqrt{-81} \cdot \sqrt{-25}$
34. $\sqrt{-16} \cdot \sqrt{-100}$
35. $7i(2 - 5i)$
36. $3i(6 + 4i)$
37. $-2i(-8 + 3i)$

38. $-6i(-5 + i)$

39. $(1 + 3i)(1 - 4i)$

40. $(1 - 2i)(1 + 3i)$

41. $(2 + 3i)(2 + 5i)$

42. $(3 - 5i)(8 - 2i)$

43. $(-4 + i)(3 - 2i)$

44. $(5 - 2i)(-1 + i)$

45. $(8 - 3i)(-2 - 5i)$

46. $(7 - 4i)(-3 - 3i)$

47. $(3 + \sqrt{-16})(2 + \sqrt{-25})$

48. $(7 - \sqrt{-16})(2 + \sqrt{-9})$

49. $(5 - 4i)(5 + 4i)$

50. $(5 + 9i)(5 - 9i)$

51. $(3 + 2i)(3 - 2i)$

52. $(8 + i)(8 - i)$

53. $(7 - 5i)(7 + 5i)$

54. $(6 - 8i)(6 + 8i)$

55. $(4 + 2i)^2$

56. $(5 - 4i)^2$

57. $(-2 + 7i)^2$

58. $(-3 + 2i)^2$

59. $(1 - 3i)^2$

60. $(2 - 5i)^2$

61. $(-1 - i)^2$

62. $(-4 - 2i)^2$

63. $(3 + 4i)^2$

64. $(6 + 5i)^2$

65. $\dfrac{3}{5 - 11i}$

66. $\dfrac{i}{2 + i}$

67. $\dfrac{5}{2 + 3i}$

68. $\dfrac{-3}{4 - 5i}$

69. $\dfrac{4 + i}{-3 - 2i}$

70. $\dfrac{5 - i}{-7 + 2i}$

71. $\dfrac{5 - 3i}{4 + 3i}$

72. $\dfrac{6 + 5i}{3 - 4i}$

73. $\dfrac{2 + \sqrt{3}i}{5 - 4i}$

74. $\dfrac{\sqrt{5} + 3i}{1 - i}$

75. $\dfrac{1 + i}{(1 - i)^2}$

76. $\dfrac{1 - i}{(1 + i)^2}$

77. $\dfrac{4 - 2i}{1 + i} + \dfrac{2 - 5i}{1 + i}$

78. $\dfrac{3 + 2i}{1 - i} + \dfrac{6 + 2i}{1 - i}$

Simplify.

79. i^{11}

80. i^7

81. i^{35}

82. i^{24}

83. i^{64}

84. i^{42}

85. $(-i)^{71}$

86. $(-i)^6$

87. $(5i)^4$

88. $(2i)^5$

Skill Maintenance

89. Write a slope–intercept equation for the line containing the point $(3, -5)$ and perpendicular to the line $3x - 6y = 7$.

Given that $f(x) = x^2 + 4$ *and* $g(x) = 3x + 5$, *find each of the following.*

90. The domain of $f - g$

91. The domain of f/g

92. $(f - g)(x)$

93. $(f/g)(2)$

94. For the function $f(x) = x^2 - 3x + 4$, construct and simplify the difference quotient

$$\frac{f(x + h) - f(x)}{h}.$$

Synthesis

Determine whether each of the following is true or false.

95. The sum of two numbers that are conjugates of each other is always a real number.

96. The conjugate of a sum is the sum of the conjugates of the individual complex numbers.

97. The conjugate of a product is the product of the conjugates of the individual complex numbers.

Let $z = a + bi$ *and* $\bar{z} = a - bi$.

98. Find a general expression for $1/z$.

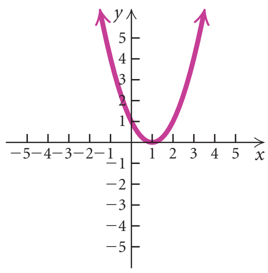
99. Find a general expression for $z\bar{z}$.

100. Solve $z + 6\bar{z} = 7$ for z.

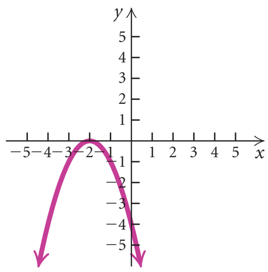
101. Multiply and simplify:

$$[x - (3 + 4i)][x - (3 - 4i)].$$

27.



28.



Solve by completing the square to obtain exact solutions.

29. $x^2 + 6x = 7$

30. $x^2 + 8x = -15$

31. $x^2 = 8x - 9$

32. $x^2 = 22 + 10x$

33. $x^2 + 8x + 25 = 0$

34. $x^2 + 6x + 13 = 0$

35. $3x^2 + 5x - 2 = 0$

36. $2x^2 - 5x - 3 = 0$

Use the quadratic formula to find exact solutions.

37. $x^2 - 2x = 15$

38. $x^2 + 4x = 5$

39. $5m^2 + 3m = 2$

40. $2y^2 - 3y - 2 = 0$

41. $3x^2 + 6 = 10x$

42. $3t^2 + 8t + 3 = 0$

43. $x^2 + x + 2 = 0$

44. $x^2 + 1 = x$

45. $5t^2 - 8t = 3$

46. $5x^2 + 2 = x$

47. $3x^2 + 4 = 5x$

48. $2t^2 - 5t = 1$

49. $x^2 - 8x + 5 = 0$

50. $x^2 - 6x + 3 = 0$

51. $3x^2 + x = 5$

52. $5x^2 + 3x = 1$

53. $2x^2 + 1 = 5x$

54. $4x^2 + 3 = x$

55. $5x^2 + 2x = -2$

56. $3x^2 + 3x = -4$

For each of the following, find the discriminant, $b^2 - 4ac$, and then determine whether one real-number solution, two different real-number solutions, or two different imaginary-number solutions exist.

57. $4x^2 = 8x + 5$

58. $4x^2 - 12x + 9 = 0$

59. $x^2 + 3x + 4 = 0$

60. $x^2 - 2x + 4 = 0$

61. $9x^2 + 6x + 1 = 0$

62. $5t^2 - 4t = 11$

Find the zeros of the function. Give exact answers and approximate solutions rounded to three decimal places when possible.

63. $f(x) = x^2 + 6x + 5$

64. $f(x) = x^2 - x - 2$

65. $f(x) = x^2 - 3x - 3$

66. $f(x) = 3x^2 + 8x + 2$

67. $f(x) = x^2 - 5x + 1$

68. $f(x) = x^2 - 3x - 7$

69. $f(x) = x^2 + 2x - 5$

70. $f(x) = x^2 - x - 4$

71. $f(x) = 2x^2 - x + 4$

72. $f(x) = 2x^2 + 3x + 2$

73. $f(x) = 3x^2 - x - 1$

74. $f(x) = 3x^2 + 5x + 1$

75. $f(x) = 5x^2 - 2x - 1$

76. $f(x) = 4x^2 - 4x - 5$

77. $f(x) = 4x^2 + 3x - 3$

78. $f(x) = x^2 + 6x - 3$

Solve.

79. $x^4 - 3x^2 + 2 = 0$

80. $x^4 + 3 = 4x^2$

81. $x^4 + 3x^2 = 10$

82. $x^4 - 8x^2 = 9$

83. $y^4 + 4y^2 - 5 = 0$

84. $y^4 - 15y^2 - 16 = 0$

85. $x - 3\sqrt{x} - 4 = 0$
($Hint$: Let $u = \sqrt{x}$.)

86. $2x - 9\sqrt{x} + 4 = 0$

87. $m^{2/3} - 2m^{1/3} - 8 = 0$
($Hint$: Let $u = m^{1/3}$.)

88. $t^{2/3} + t^{1/3} - 6 = 0$

89. $x^{1/2} - 3x^{1/4} + 2 = 0$

90. $x^{1/2} - 4x^{1/4} = -3$

91. $(2x - 3)^2 - 5(2x - 3) + 6 = 0$
($Hint$: Let $u = 2x - 3$.)

92. $(3x + 2)^2 + 7(3x + 2) - 8 = 0$

93. $(2t^2 + t)^2 - 4(2t^2 + t) + 3 = 0$

94. $12 = (m^2 - 5m)^2 + (m^2 - 5m)$

Multigenerational Households. *After declining between 1940 and 1980, the number of multigenerational American households has been increasing since 1980. The function* $h(x) = 0.012x^2 - 0.583x + 35.727$ *can be used to estimate the number of multigenerational households in the United States, in millions, x years after 1940 (Source: Pew Research Center). Use this function for Exercises 95 and 96.*

95. In what year were there 40 million multigenerational households?

96. In what year were there 55 million multigenerational households?

TV Channels. *The number of TV channels that the average U.S. home receives has been soaring in recent years. The function* $t(x) = 0.16x^2 + 0.46x + 21.36$ *can be used to estimate this number, where x is the number of years after 1985 (Source: Nielsen Media Research, National People Meter Sample). Use this function for Exercises 97 and 98.*

97. In what year did the average U.S. household receive 50 channels?

98. In what year did the average U.S. household receive 88 channels?

Time of a Free Fall. *The formula* $s = 16t^2$ *is used to approximate the distance s, in feet, that an object falls freely from rest in t seconds. Use this formula for Exercises 99 and 100.*

99. The Taipei 101 Tower, also known as the Taipei Financial Center, in Taipei, Taiwan, is 1670 ft tall. How long would it take an object dropped from the top to reach the ground?

100. At 630 ft, the Gateway Arch in St. Louis is the tallest man-made monument in the United States. How long would it take an object dropped from the top to reach the ground?

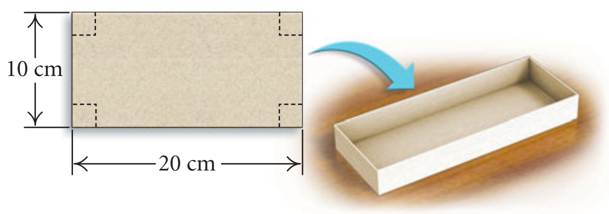
101. The length of a rectangular poster is 1 ft more than the width, and a diagonal of the poster is 5 ft. Find the length and the width.

102. One leg of a right triangle is 7 cm less than the length of the other leg. The length of the hypotenuse is 13 cm. Find the lengths of the legs.

103. One number is 5 greater than another. The product of the numbers is 36. Find the numbers.

104. One number is 6 less than another. The product of the numbers is 72. Find the numbers.

105. *Box Construction.* An open box is made from a 10-cm by 20-cm piece of tin by cutting a square from each corner and folding up the edges. The area of the resulting base is 96 cm^2. What is the length of the sides of the squares?



106. *Petting Zoo Dimensions.* At the Glen Island Zoo, 170 m of fencing was used to enclose a petting area of 1750 m^2. Find the dimensions of the petting area.

107. *Dimensions of a Rug.* Find the dimensions of a Persian rug whose perimeter is 28 ft and whose area is 48 ft^2.

108. *Picture Frame Dimensions.* The frame on a picture is 8 in. by 10 in. outside and is of uniform width. What is the width of the frame if 48 in^2 of the picture shows?

8 in.

10 in.

State whether the function is linear or quadratic.

109. $f(x) = 4 - 5x$

110. $f(x) = 4 - 5x^2$

111. $f(x) = 7x^2$

112. $f(x) = 23x + 6$

113. $f(x) = 1.2x - (3.6)^2$

114. $f(x) = 2 - x - x^2$

Skill Maintenance

Spending on Antipsychotic Drugs. The amount of spending on antipsychotic drugs, used to treat schizophrenia and other conditions, recently edged out cholesterol medications at the top of U.S. sales charts. The function $a(x) = 1.24x + 9.24$ can be used to estimate the amount of spending per year on antipsychotic drugs in the United States, in billions of dollars, x years after 2004 (Source: IMS Health). Use this function for Exercises 115 and 116.

115. Estimate the amount spent on antipsychotic drugs in the United States in 2010.

116. When will the amount of spending on antipsychotic drugs reach \$24 billion?

Determine whether the graph is symmetric with respect to the x-axis, the y-axis, and the origin.

117. $3x^2 + 4y^2 = 5$

118. $y^3 = 6x^2$

Determine whether the function is even, odd, or neither even nor odd.

119. $f(x) = 2x^3 - x$

120. $f(x) = 4x^2 + 2x - 3$

Synthesis

For each equation in Exercises 121–124, under the given condition: **(a)** *Find k and* **(b)** *find a second solution.*

121. $kx^2 - 17x + 33 = 0$; one solution is 3

122. $kx^2 - 2x + k = 0$; one solution is -3

123. $x^2 - kx + 2 = 0$; one solution is $1 + i$

124. $x^2 - (6 + 3i)x + k = 0$; one solution is 3

Solve.

125. $(x - 2)^3 = x^3 - 2$

126. $(x + 1)^3 = (x - 1)^3 + 26$

127. $(6x^3 + 7x^2 - 3x)(x^2 - 7) = 0$

128. $\left(x - \frac{1}{5}\right)\left(x^2 - \frac{1}{4}\right) + \left(x - \frac{1}{5}\right)\left(x^2 + \frac{1}{8}\right) = 0$

129. $x^2 + x - \sqrt{2} = 0$

130. $x^2 + \sqrt{5}x - \sqrt{3} = 0$

131. $2t^2 + (t - 4)^2 = 5t(t - 4) + 24$

132. $9t(t + 2) - 3t(t - 2) = 2(t + 4)(t + 6)$

133. $\sqrt{x - 3} - \sqrt[4]{x - 3} = 2$

134. $x^2 + 3x + 1 - \sqrt{x^2 + 3x + 1} = 8$

135. $\left(y + \frac{2}{y}\right)^2 + 3y + \frac{6}{y} = 4$

136. Solve $\frac{1}{2}at^2 + v_0t + x_0 = 0$ for t.

Simplify. Write answers in the form $a + bi$, where a and b are real numbers. [3.1]

9. $(3 - 2i) + (-4 + 3i)$

10. $(-5 + i) - (2 - 4i)$

11. $(2 + 3i)(4 - 5i)$

12. $\dfrac{3 + i}{-2 + 5i}$

Simplify. [3.1]

13. i^{13}

14. i^{44}

15. $(-i)^5$

16. $(2i)^6$

Solve. [3.2]

17. $x^2 + 3x - 4 = 0$

18. $2x^2 + 6 = -7x$

19. $4x^2 = 24$

20. $x^2 + 100 = 0$

21. Find the zeros of $f(x) = 4x^2 - 8x - 3$ by completing the square. Show your work. [3.2]

In Exercises 22–24, (**a**) *find the discriminant $b^2 - 4ac$, and then determine whether one real-number solution, two different real-number solutions, or two different imaginary-number solutions exist; and* (**b**) *solve the equation, finding exact solutions and approximate solutions rounded to three decimal places, where appropriate.* [3.2]

22. $x^2 - 3x - 5 = 0$

23. $4x^2 - 12x + 9 = 0$

24. $3x^2 + 2x = -1$

Solve. [3.2]

25. $x^4 + 5x^2 - 6 = 0$

26. $2x - 5\sqrt{x} + 2 = 0$

27. One number is 2 more than another. The product of the numbers is 35. Find the numbers. [3.2]

In Exercises 28 and 29:

a) *Find the vertex.* [3.3]
b) *Find the axis of symmetry.* [3.3]
c) *Determine whether there is a maximum or minimum value, and find that value.* [3.3]
d) *Find the range.* [3.3]
e) *Find the intervals on which the function is increasing or decreasing.* [3.3]
f) *Graph the function.* [3.3]

28. $f(x) = x^2 - 6x + 7$

29. $f(x) = -2x^2 - 4x - 5$

30. The sum of the base and the height of a triangle is 16 in. Find the dimensions for which the area is a maximum. [3.3]

Collaborative Discussion and Writing

31. Is the sum of two imaginary numbers always an imaginary number? Explain your answer. [3.1]

32. The graph of a quadratic function can have 0, 1, or 2 x-intercepts. How can you predict the number of x-intercepts without drawing the graph or (completely) solving an equation? [3.2]

33. Discuss two ways in which we used completing the square in this chapter. [3.2], [3.3]

34. Suppose that the graph of $f(x) = ax^2 + bx + c$ has x-intercepts $(x_1, 0)$ and $(x_2, 0)$. What are the x-intercepts of $g(x) = -ax^2 - bx - c$? Explain. [3.3]

3.4 Exercise Set

Solve.

1. $\frac{1}{4} + \frac{1}{5} = \frac{1}{t}$

2. $\frac{1}{3} - \frac{5}{6} = \frac{1}{x}$

3. $\frac{x+2}{4} - \frac{x-1}{5} = 15$

4. $\frac{t+1}{3} - \frac{t-1}{2} = 1$

5. $\frac{1}{2} + \frac{2}{x} = \frac{1}{3} + \frac{3}{x}$

6. $\frac{1}{t} + \frac{1}{2t} + \frac{1}{3t} = 5$

7. $\frac{5}{3x+2} = \frac{3}{2x}$

8. $\frac{2}{x-1} = \frac{3}{x+2}$

9. $x + \frac{6}{x} = 5$

10. $x - \frac{12}{x} = 1$

11. $\frac{6}{y+3} + \frac{2}{y} = \frac{5y-3}{y^2-9}$

12. $\frac{3}{m+2} + \frac{2}{m} = \frac{4m-4}{m^2-4}$

13. $\frac{2x}{x-1} = \frac{5}{x-3}$

14. $\frac{2x}{x+7} = \frac{5}{x+1}$

15. $\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2-25}$

16. $\frac{2}{x^2-9} + \frac{5}{x-3} = \frac{3}{x+3}$

17. $\frac{3x}{x+2} + \frac{6}{x} = \frac{12}{x^2+2x}$

18. $\frac{3y+5}{y^2+5y} + \frac{y+4}{y+5} = \frac{y+1}{y}$

19. $\frac{1}{5x+20} - \frac{1}{x^2-16} = \frac{3}{x-4}$

20. $\frac{1}{4x+12} - \frac{1}{x^2-9} = \frac{5}{x-3}$

21. $\frac{2}{5x+5} - \frac{3}{x^2-1} = \frac{4}{x-1}$

22. $\frac{1}{3x+6} - \frac{1}{x^2-4} = \frac{3}{x-2}$

23. $\frac{8}{x^2-2x+4} = \frac{x}{x+2} + \frac{24}{x^3+8}$

24. $\frac{18}{x^2-3x+9} - \frac{x}{x+3} = \frac{81}{x^3+27}$

25. $\frac{x}{x-4} - \frac{4}{x+4} = \frac{32}{x^2-16}$

26. $\frac{x}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$

27. $\frac{1}{x-6} - \frac{1}{x} = \frac{6}{x^2-6x}$

28. $\frac{1}{x-15} - \frac{1}{x} = \frac{15}{x^2-15x}$

29. $\sqrt{3x-4} = 1$

30. $\sqrt{4x+1} = 3$

31. $\sqrt{2x-5} = 2$

32. $\sqrt{3x+2} = 6$

33. $\sqrt{7-x} = 2$

34. $\sqrt{5-x} = 1$

35. $\sqrt{1-2x} = 3$

36. $\sqrt{2-7x} = 2$

37. $\sqrt[3]{5x-2} = -3$

38. $\sqrt[3]{2x+1} = -5$

39. $\sqrt[4]{x^2-1} = 1$

40. $\sqrt[5]{3x+4} = 2$

41. $\sqrt{y-1} + 4 = 0$

42. $\sqrt{m+1} - 5 = 8$

43. $\sqrt{b+3} - 2 = 1$

44. $\sqrt{x-4} + 1 = 5$

45. $\sqrt{z+2} + 3 = 4$

46. $\sqrt{y-5} - 2 = 3$

47. $\sqrt{2x+1} - 3 = 3$

48. $\sqrt{3x-1} + 2 = 7$

49. $\sqrt{2-x} - 4 = 6$

50. $\sqrt{5-x} + 2 = 8$

51. $\sqrt[3]{6x+9} + 8 = 5$

52. $\sqrt[5]{2x-3} - 1 = 1$

53. $\sqrt{x+4} + 2 = x$

54. $\sqrt{x+1} + 1 = x$

55. $\sqrt{x-3} + 5 = x$

56. $\sqrt{x+3} - 1 = x$

57. $\sqrt{x+7} = x + 1$

58. $\sqrt{6x+7} = x + 2$

59. $\sqrt{3x+3} = x + 1$

60. $\sqrt{2x+5} = x - 5$

61. $\sqrt{5x+1} = x - 1$

62. $\sqrt{7x+4} = x + 2$

63. $\sqrt{x-3} + \sqrt{x+2} = 5$

64. $\sqrt{x} - \sqrt{x-5} = 1$

65. $\sqrt{3x-5} + \sqrt{2x+3} + 1 = 0$

66. $\sqrt{2m-3} = \sqrt{m+7} - 2$

67. $\sqrt{x} - \sqrt{3x-3} = 1$

68. $\sqrt{2x+1} - \sqrt{x} = 1$

69. $\sqrt{2y-5} - \sqrt{y-3} = 1$

70. $\sqrt{4p+5} + \sqrt{p+5} = 3$

71. $\sqrt{y+4} - \sqrt{y-1} = 1$

72. $\sqrt{y+7} + \sqrt{y+16} = 9$

73. $\sqrt{x+5} + \sqrt{x+2} = 3$

74. $\sqrt{6x+6} = 5 + \sqrt{21-4x}$

75. $x^{1/3} = -2$

76. $t^{1/5} = 2$

77. $t^{1/4} = 3$

78. $m^{1/2} = -7$

Solve.

79. $\dfrac{P_1V_1}{T_1} = \dfrac{P_2V_2}{T_2}$, for T_1
(A chemistry formula for gases)

80. $\dfrac{1}{F} = \dfrac{1}{m} + \dfrac{1}{p}$, for F
(A formula from optics)

81. $W = \sqrt{\dfrac{1}{LC}}$, for C
(An electricity formula)

82. $s = \sqrt{\dfrac{A}{6}}$, for A
(A geometry formula)

83. $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$, for R_2
(A formula for resistance)

84. $\dfrac{1}{t} = \dfrac{1}{a} + \dfrac{1}{b}$, for t
(A formula for work rate)

85. $I = \sqrt{\dfrac{A}{P}} - 1$, for P
(A compound-interest formula)

86. $T = 2\pi\sqrt{\dfrac{1}{g}}$, for g
(A pendulum formula)

87. $\dfrac{1}{F} = \dfrac{1}{m} + \dfrac{1}{p}$, for p
(A formula from optics)

88. $\dfrac{V^2}{R^2} = \dfrac{2g}{R+h}$, for h
(A formula for escape velocity)

Skill Maintenance

Find the zero of the function.

89. $f(x) = 15 - 2x$

90. $f(x) = -3x + 9$

91. *Deadly Distractions.* Drivers who were distracted by such things as text-messaging, talking on cell phones, conversing with passengers, and eating were involved in 5870 highway fatalities in 2008. This was an increase of about 18% over the number of distracted-driving fatalities in 2004 and is attributed largely to the increased number of drivers who texted in 2008. (*Source:* NHTSA's National Center for Statistics and Analysis) How many highway fatalities involved distracted driving in 2004?

92. *Big Sites.* Together, the Mall of America in Minnesota and the Disneyland theme park in California occupy 181 acres of land. The Mall of America occupies 11 acres more than Disneyland. (*Sources:* Mall of America; Disneyland) How much land does each occupy?

Synthesis

Solve.

93. $(x - 3)^{2/3} = 2$

94. $\frac{x+3}{x+2} - \frac{x+4}{x+3} = \frac{x+5}{x+4} - \frac{x+6}{x+5}$

95. $\sqrt{x+5} + 1 = \frac{6}{\sqrt{x+5}}$

96. $\sqrt{15 + \sqrt{2x + 80}} = 5$

97. $x^{2/3} = x$

3.5 Solving Equations and Inequalities with Absolute Value

- Solve equations with absolute value.
- Solve inequalities with absolute value.

Equations with Absolute Value

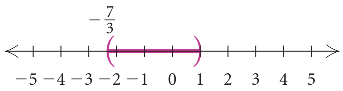
ABSOLUTE VALUE
REVIEW SECTION R.1

Recall that the absolute value of a number is its distance from 0 on the number line. We use this concept to solve equations with absolute value.

For $a > 0$ and an algebraic expression X:

$|X| = a$ is equivalent to $X = -a$ *or* $X = a$.

The solution set is $\left\{x \middle| -\frac{7}{3} < x < 1\right\}$, or $\left(-\frac{7}{3}, 1\right)$. The graph of the solution set is shown below.



Now Try Exercise 45.

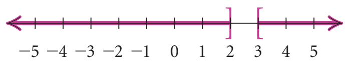
EXAMPLE 4 Solve and graph the solution set: $|5 - 2x| \geq 1$.

Solution We have

$$|5 - 2x| \geq 1$$

$5 - 2x \leq -1$	*or*	$5 - 2x \geq 1$	**Writing an equivalent inequality**
$-2x \leq -6$	*or*	$-2x \geq -4$	**Subtracting 5**
$x \geq 3$	*or*	$x \leq 2.$	**Dividing by -2 and reversing the inequality signs**

The solution set is $\{x \mid x \leq 2 \text{ or } x \geq 3\}$, or $(-\infty, 2] \cup [3, \infty)$. The graph of the solution set is shown below.



Now Try Exercise 47.

3.5 Exercise Set

Solve.

1. $|x| = 7$
2. $|x| = 4.5$
3. $|x| = 0$
4. $|x| = \frac{3}{2}$
5. $|x| = \frac{5}{6}$
6. $|x| = -\frac{3}{5}$
7. $|x| = -10.7$
8. $|x| = 12$
9. $|3x| = 1$
10. $|5x| = 4$
11. $|8x| = 24$
12. $|6x| = 0$
13. $|x - 1| = 4$
14. $|x - 7| = 5$
15. $|x + 2| = 6$
16. $|x + 5| = 1$
17. $|3x + 2| = 1$
18. $|7x - 4| = 8$
19. $\left|\frac{1}{2}x - 5\right| = 17$
20. $\left|\frac{1}{3}x - 4\right| = 13$
21. $|x - 1| + 3 = 6$
22. $|x + 2| - 5 = 9$
23. $|x + 3| - 2 = 8$
24. $|x - 4| + 3 = 9$
25. $|3x + 1| - 4 = -1$
26. $|2x - 1| - 5 = -3$
27. $|4x - 3| + 1 = 7$
28. $|5x + 4| + 2 = 5$
29. $12 - |x + 6| = 5$
30. $9 - |x - 2| = 7$
31. $7 - |2x - 1| = 6$
32. $5 - |4x + 3| = 2$

Solve and write interval notation for the solution set. Then graph the solution set.

33. $|x| < 7$
34. $|x| \leq 4.5$
35. $|x| \leq 2$
36. $|x| < 3$
37. $|x| \geq 4.5$
38. $|x| > 7$
39. $|x| > 3$
40. $|x| \geq 2$
41. $|3x| < 1$
42. $|5x| \leq 4$

43. $|2x| \geq 6$

44. $|4x| > 20$

45. $|x + 8| < 9$

46. $|x + 6| \leq 10$

47. $|x + 8| \geq 9$

48. $|x + 6| > 10$

49. $\left|x - \frac{1}{4}\right| < \frac{1}{2}$

50. $|x - 0.5| \leq 0.2$

51. $|2x + 3| \leq 9$

52. $|3x + 4| < 13$

53. $|x - 5| > 0.1$

54. $|x - 7| \geq 0.4$

55. $|6 - 4x| \geq 8$

56. $|5 - 2x| > 10$

57. $\left|x + \frac{2}{3}\right| \leq \frac{5}{3}$

58. $\left|x + \frac{3}{4}\right| < \frac{1}{4}$

59. $\left|\frac{2x + 1}{3}\right| > 5$

60. $\left|\frac{2x - 1}{3}\right| \geq \frac{5}{6}$

61. $|2x - 4| < -5$

62. $|3x + 5| < 0$

Skill Maintenance

In each of Exercises 63–70, fill in the blank with the correct term. Some of the given choices will not be used.

distance formula	symmetric with respect to the x-axis
midpoint formula	symmetric with respect to the y-axis
function	symmetric with respect to the origin
relation	increasing
x-intercept	decreasing
y-intercept	constant
perpendicular	
parallel	
horizontal lines	
vertical lines	

63. A(n) ________ is a point $(0, b)$.

64. The ________ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

65. A(n) ________ is a correspondence such that each member of the domain corresponds to at least one member of the range.

66. A(n) ________ is a correspondence such that each member of the domain corresponds to exactly one member of the range.

67. ________ are given by equations of the type $y = b$, or $f(x) = b$.

68. Nonvertical lines are ________ if and only if they have the same slope and different y-intercepts.

69. A function f is said to be ________ on an open interval I if, for all a and b in that interval, $a < b$ implies $f(a) > f(b)$.

70. For an equation $y = f(x)$, if replacing x with $-x$ produces an equivalent equation, then the graph is ________.

Synthesis

Solve.

71. $|3x - 1| > 5x - 2$

72. $|x + 2| \leq |x - 5|$

73. $|p - 4| + |p + 4| < 8$

74. $|x| + |x + 1| < 10$

75. $|x - 3| + |2x + 5| > 6$

Solve. [3.2]

5. $(2y + 5)(3y - 1) = 0$

6. $x^2 + 4x - 5 = 0$

7. $3x^2 + 2x = 8$

8. $5x^2 = 15$

9. $x^2 + 10 = 0$

Find the zero(s) of the function. [3.2]

10. $f(x) = x^2 - 2x + 1$

11. $f(x) = x^2 + 2x - 15$

12. $f(x) = 2x^2 - x - 5$

13. $f(x) = 3x^2 + 2x + 3$

Solve.

14. $\dfrac{5}{2x + 3} + \dfrac{1}{x - 6} = 0$ [3.4]

15. $\dfrac{3}{8x + 1} + \dfrac{8}{2x + 5} = 1$ [3.4]

16. $\sqrt{5x + 1} - 1 = \sqrt{3x}$ [3.4]

17. $\sqrt{x - 1} - \sqrt{x - 4} = 1$ [3.4]

18. $|x - 4| = 3$ [3.5]

19. $|2y + 7| = 9$ [3.5]

Solve and write interval notation for the solution set. Then graph the solution set. [3.5]

20. $|5x| \geq 15$

21. $|3x + 4| < 10$

22. $|1 - 6x| < 5$

23. $|x + 4| \geq 2$

24. Solve $\dfrac{1}{M} + \dfrac{1}{N} = \dfrac{1}{P}$ for P. [3.4]

Express in terms of i. [3.1]

25. $-\sqrt{-40}$

26. $\sqrt{-12} \cdot \sqrt{-20}$

27. $\dfrac{\sqrt{-49}}{-\sqrt{-64}}$

Simplify each of the following. Write the answer in the form $a + bi$, where a and b are real numbers. [3.1]

28. $(6 + 2i) + (-4 - 3i)$

29. $(3 - 5i) - (2 - i)$

30. $(6 + 2i)(-4 - 3i)$

31. $\dfrac{2 - 3i}{1 - 3i}$

32. i^{23}

Solve by completing the square to obtain exact solutions. Show your work. [3.2]

33. $x^2 - 3x = 18$

34. $3x^2 - 12x - 6 = 0$

Solve. Give exact solutions. [3.2]

35. $3x^2 + 10x = 8$

36. $r^2 - 2r + 10 = 0$

37. $x^2 = 10 + 3x$

38. $x = 2\sqrt{x} - 1$

39. $y^4 - 3y^2 + 1 = 0$

40. $(x^2 - 1)^2 - (x^2 - 1) - 2 = 0$

41. $(p - 3)(3p + 2)(p + 2) = 0$

42. $x^3 + 5x^2 - 4x - 20 = 0$

In Exercises 43 and 44, complete the square to:

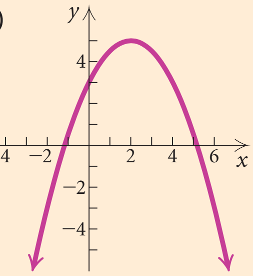
a) *find the vertex;*
b) *find the axis of symmetry;*
c) *determine whether there is a maximum or minimum value and find that value;*
d) *find the range; and*
e) *graph the function.* [3.3]

43. $f(x) = -4x^2 + 3x - 1$

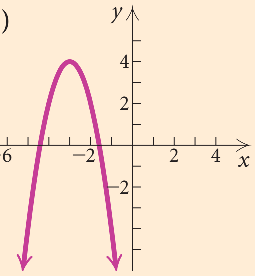
44. $f(x) = 5x^2 - 10x + 3$

In Exercises 45–48, match the equation with one of the figures (a)–(d), which follow. [3.3]

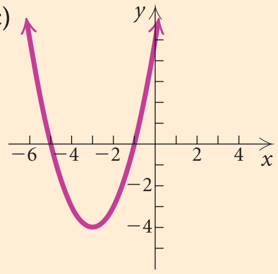
a)



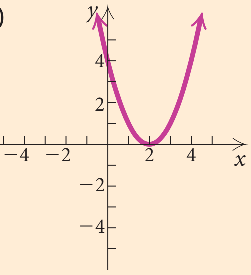
b)



c)



d)



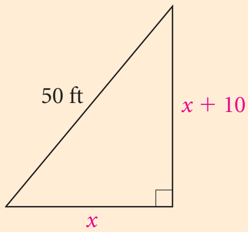
45. $y = (x - 2)^2$

46. $y = (x + 3)^2 - 4$

47. $y = -2(x + 3)^2 + 4$

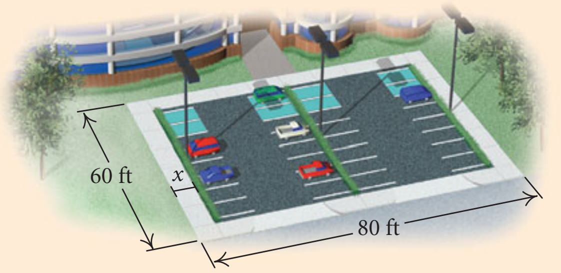
48. $y = -\frac{1}{2}(x - 2)^2 + 5$

49. *Legs of a Right Triangle.* The hypotenuse of a right triangle is 50 ft. One leg is 10 ft longer than the other. What are the lengths of the legs? [3.2]



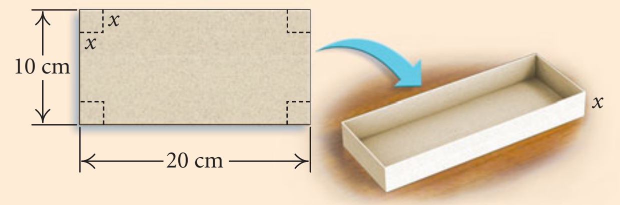
50. *Bicycling Speed.* Logan and Cassidy leave a campsite, Logan biking due north and Cassidy biking due east. Logan bikes 7 km/h slower than Cassidy. After 4 hr, they are 68 km apart. Find the speed of each bicyclist. [3.2]

51. *Sidewalk Width.* A 60-ft by 80-ft parking lot is torn up to install a sidewalk of uniform width around its perimeter. The new area of the parking lot is two-thirds of the old area. How wide is the sidewalk? [3.2]



52. *Maximizing Volume.* The Berniers have 24 ft of flexible fencing with which to build a rectangular "toy corral." If the fencing is 2 ft high, what dimensions should the corral have in order to maximize its volume? [3.3]

53. *Dimensions of a Box.* An open box is made from a 10-cm by 20-cm piece of aluminum by cutting a square from each corner and folding up the edges. The area of the resulting base is 90 cm^2. What is the length of the sides of the squares? [3.2]



54. Find the zeros of $f(x) = 2x^2 - 5x + 1$. [3.2]

A. $\dfrac{5 \pm \sqrt{17}}{2}$ **B.** $\dfrac{5 \pm \sqrt{17}}{4}$

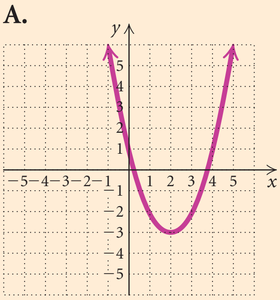
C. $\dfrac{5 \pm \sqrt{33}}{4}$ **D.** $\dfrac{-5 \pm \sqrt{17}}{4}$

55. Solve: $\sqrt{4x + 1} + \sqrt{2x} = 1$. [3.4]

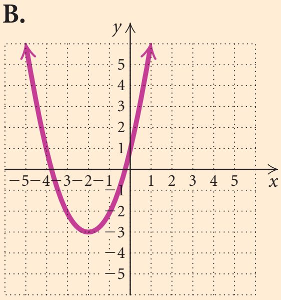
A. There are two solutions.
B. There is only one solution. It is less than 1.
C. There is only one solution. It is greater than 1.
D. There is no solution.

56. The graph of $f(x) = (x - 2)^2 - 3$ is which of the following? [3.3]

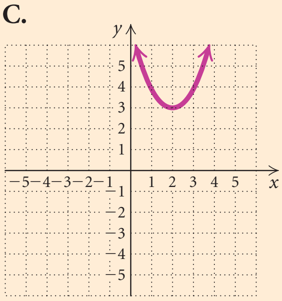
A.



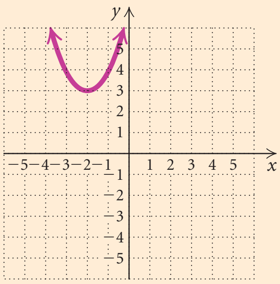
B.



C.



D.



Synthesis

Solve.

57. $\sqrt{\sqrt{\sqrt{x}}} = 2$ [3.4]

58. $(t - 4)^{4/5} = 3$ [3.4]

59. $(x - 1)^{2/3} = 4$ [3.4]

60. $(2y - 2)^2 + y - 1 = 5$ [3.2]

61. $\sqrt{x + 2} + \sqrt[4]{x + 2} - 2 = 0$ [3.2]

62. At the beginning of the year, \$3500 was deposited in a savings account. One year later, \$4000 was deposited in another account. The interest rate was the same for both accounts. At the end of the second year, there was a total of \$8518.35 in the accounts. What was the annual interest rate? [3.2]

63. Find b such that $f(x) = -3x^2 + bx - 1$ has a maximum value of 2. [3.3]

Collaborative Discussion and Writing

64. Is the product of two imaginary numbers always an imaginary number? Explain your answer. [3.1]

65. Is it possible for a quadratic function to have one real zero and one imaginary zero? Why or why not? [3.2]

66. If the graphs of

$$f(x) = a_1(x - h_1)^2 + k_1$$

and

$$g(x) = a_2(x - h_2)^2 + k_2$$

have the same shape, what, if anything, can you conclude about the a's, the h's, and the k's? Explain your answer. [3.3]

67. Explain why it is necessary to check the possible solutions of a rational equation. [3.4]

68. Explain in your own words why it is necessary to check the possible solutions when the principle of powers is used to solve an equation. [3.4]

69. Explain why $|x| < p$ has no solution for $p \leq 0$. [3.5]

70. Explain why all real numbers are solutions of $|x| > p$, for $p < 0$. [3.5]

Chapter 3 Test

Solve. Find exact solutions.

1. $(2x - 1)(x + 5) = 0$

2. $6x^2 - 36 = 0$

3. $x^2 + 4 = 0$

4. $x^2 - 2x - 3 = 0$

5. $x^2 - 5x + 3 = 0$

6. $2t^2 - 3t + 4 = 0$

7. $x + 5\sqrt{x} - 36 = 0$

8. $\frac{3}{3x + 4} + \frac{2}{x - 1} = 2$

9. $\sqrt{x + 4} - 2 = 1$

10. $\sqrt{x + 4} - \sqrt{x - 4} = 2$

11. $|x + 4| = 7$

12. $|4y - 3| = 5$

Solve and write interval notation for the solution set. Then graph the solution set.

13. $|x + 3| \leq 4$

14. $|2x - 1| < 5$

15. $|x + 5| > 2$

16. $|3 - 2x| \geq 7$

17. Solve $\frac{1}{A} + \frac{1}{B} = \frac{1}{C}$ for B.

18. Solve $R = \sqrt{3np}$ for n.

19. Solve $x^2 + 4x = 1$ by completing the square. Find the exact solutions. Show your work.

20. The tallest structure in the United States, at 2063 ft, is the KTHI-TV tower in North Dakota (*Source: The Cambridge Fact Finder*). How long would it take an object falling freely from the top to reach the ground? (Use the formula $s = 16t^2$.)

Express in terms of i.

21. $\sqrt{-43}$

22. $-\sqrt{-25}$

Simplify.

23. $(5 - 2i) - (2 + 3i)$

24. $(3 + 4i)(2 - i)$

25. $\frac{1 - i}{6 + 2i}$

26. i^{33}

Find the zeros of each function.

27. $f(x) = 4x^2 - 11x - 3$

28. $f(x) = 2x^2 - x - 7$

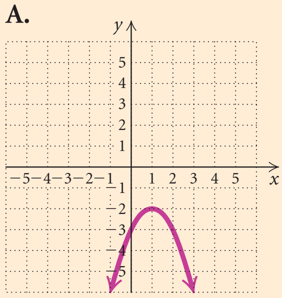
29. For the graph of the function $f(x) = -x^2 + 2x + 8$:

a) Find the vertex.
b) Find the axis of symmetry.
c) State whether there is a maximum or minimum value and find that value.
d) Find the range.
e) Graph the function.

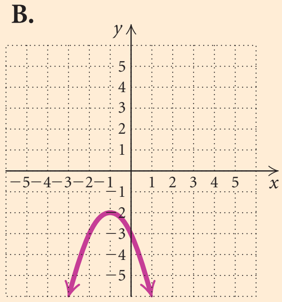
30. *Maximizing Area.* A homeowner wants to fence a rectangular play yard using 80 ft of fencing. The side of the house will be used as one side of the rectangle. Find the dimensions for which the area is a maximum.

31. The graph of $f(x) = x^2 - 2x - 1$ is which of the following?

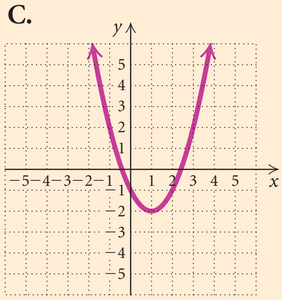
A.



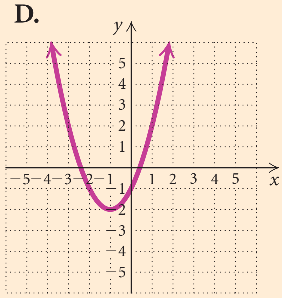
B.



C.



D.



Synthesis

32. Find a such that $f(x) = ax^2 - 4x + 3$ has a maximum value of 12.

4.1 Exercise Set

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial function as constant, linear, quadratic, cubic, or quartic.

1. $g(x) = \frac{1}{2}x^3 - 10x + 8$

2. $f(x) = 15x^2 - 10 + 0.11x^4 - 7x^3$

3. $h(x) = 0.9x - 0.13$

4. $f(x) = -6$

5. $g(x) = 305x^4 + 4021$

6. $h(x) = 2.4x^3 + 5x^2 - x + \frac{7}{8}$

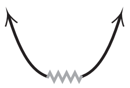
7. $h(x) = -5x^2 + 7x^3 + x^4$

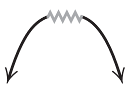
8. $f(x) = 2 - x^2$

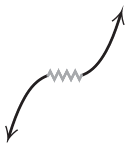
9. $g(x) = 4x^3 - \frac{1}{2}x^2 + 8$

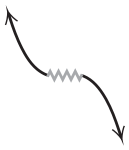
10. $f(x) = 12 + x$

In Exercises 11–18, select one of the following four sketches to describe the end behavior of the graph of the function.

a)


b)


c)


d)


11. $f(x) = -3x^3 - x + 4$

12. $f(x) = \frac{1}{4}x^4 + \frac{1}{2}x^3 - 6x^2 + x - 5$

13. $f(x) = -x^6 + \frac{3}{4}x^4$

14. $f(x) = \frac{2}{5}x^5 - 2x^4 + x^3 - \frac{1}{2}x + 3$

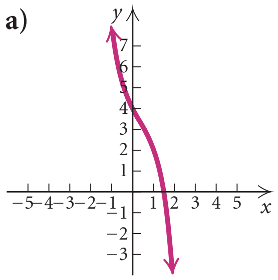
15. $f(x) = -3.5x^4 + x^6 + 0.1x^7$

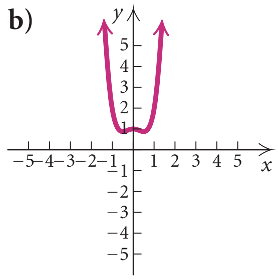
16. $f(x) = -x^3 + x^5 - 0.5x^6$

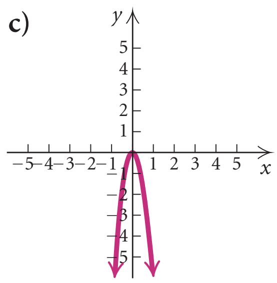
17. $f(x) = 10 + \frac{1}{10}x^4 - \frac{2}{5}x^3$

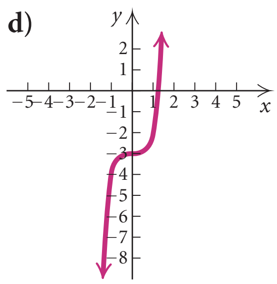
18. $f(x) = 2x + x^3 - x^5$

In Exercises 19–22, use the leading-term test to match the function with one of the graphs (a)–(d), which follow.

a)


b)


c)


d)


19. $f(x) = -x^6 + 2x^5 - 7x^2$

20. $f(x) = 2x^4 - x^2 + 1$

21. $f(x) = x^5 + \frac{1}{10}x - 3$

22. $f(x) = -x^3 + x^2 - 2x + 4$

23. Use substitution to determine whether 4, 5, and -2 are zeros of
$$f(x) = x^3 - 9x^2 + 14x + 24.$$

24. Use substitution to determine whether 2, 3, and -1 are zeros of
$$f(x) = 2x^3 - 3x^2 + x + 6.$$

25. Use substitution to determine whether 2, 3, and -1 are zeros of
$$g(x) = x^4 - 6x^3 + 8x^2 + 6x - 9.$$

26. Use substitution to determine whether 1, -2, and 3 are zeros of
$$g(x) = x^4 - x^3 - 3x^2 + 5x - 2.$$

Find the zeros of the polynomial function and state the multiplicity of each.

27. $f(x) = (x + 3)^2(x - 1)$

28. $f(x) = (x + 5)^3(x - 4)(x + 1)^2$

29. $f(x) = -2(x - 4)(x - 4)(x - 4)(x + 6)$

30. $f(x) = \left(x + \frac{1}{2}\right)(x + 7)(x + 7)(x + 5)$

31. $f(x) = (x^2 - 9)^3$

32. $f(x) = (x^2 - 4)^2$

33. $f(x) = x^3(x - 1)^2(x + 4)$

34. $f(x) = x^2(x + 3)^2(x - 4)(x + 1)^4$

35. $f(x) = -8(x - 3)^2(x + 4)^3x^4$

36. $f(x) = (x^2 - 5x + 6)^2$

37. $f(x) = x^4 - 4x^2 + 3$

38. $f(x) = x^4 - 10x^2 + 9$

39. $f(x) = x^3 + 3x^2 - x - 3$

40. $f(x) = x^3 - x^2 - 2x + 2$

41. $f(x) = 2x^3 - x^2 - 8x + 4$

42. $f(x) = 3x^3 + x^2 - 48x - 16$

Determine whether the statement is true or false.

43. If $P(x) = (x - 3)^4(x + 1)^3$, then the graph of the polynomial function $y = P(x)$ crosses the x-axis at $(3, 0)$.

44. If $P(x) = (x + 2)^2\left(x - \frac{1}{4}\right)^5$, then the graph of the polynomial function $y = P(x)$ crosses the x-axis at $\left(\frac{1}{4}, 0\right)$.

45. If $P(x) = (x - 2)^3(x + 5)^6$, then the graph of $y = P(x)$ is tangent to the x-axis at $(-5, 0)$.

46. If $P(x) = (x + 4)^2(x - 1)^2$, then the graph of $y = P(x)$ is tangent to the x-axis at $(4, 0)$.

47. *Twin Births.* As a result of a greater number of births to older women and the increased use of fertility drugs, the number of twin births in the United States increased approximately 42% from 1990 to 2005 (*Source*: National Center for Health Statistics, U.S. Department of Health and Human Services). The quartic function

$$f(x) = -0.056316x^4 - 19.500154x^3 + 584.892054x^2 - 1518.5717x + 94{,}299.1990,$$

where x is the number of years since 1990, can be used to estimate the number of twin births from 1990 to 2006. Estimate the number of twin births in 1995 and in 2005.

48. *Railroad Miles.* The greatest combined length of U.S.-owned operating railroad track existed in 1916, when industrial activity increased during World War I. The total length has decreased ever since. The data over the years 1900 to 2008 are modeled by the quartic function

$$f(x) = -0.004091x^4 + 1.275179x^3 - 142.589291x^2 + 5069.1067x + 197{,}909.1675,$$

where x is the number of years since 1900 and $f(x)$ is in miles (*Source*: Association of American Railroads). Find the number of miles of operating railroad track in the United States in 1916, in 1960, and in 1985, and estimate the number in 2010. (*Note:* The lengths exclude yard tracks, sidings, and parallel tracks.)

49. *Dog Years.* A dog's life span is typically much shorter than that of a human. The cubic function

$$d(x) = 0.010255x^3 - 0.340119x^2 + 7.397499x + 6.618361,$$

where x is the dog's age, in years, approximates the equivalent human age in years. Estimate the equivalent human age for dogs that are 3, 12, and 16 years old.

50. *Threshold Weight.* In a study performed by Alvin Shemesh, it was found that the **threshold weight** W, defined as the weight above which the risk of death rises dramatically, is given by

$$W(h) = \left(\frac{h}{12.3}\right)^3,$$

where W is in pounds and h is a person's height, in inches. Find the threshold weight of a person who is 5 ft 7 in. tall.

51. *Projectile Motion.* A stone thrown downward with an initial velocity of 34.3 m/sec will travel a distance of s meters, where

$$s(t) = 4.9t^2 + 34.3t$$

and t is in seconds. If a stone is thrown downward at 34.3 m/sec from a height of 294 m, how long will it take the stone to hit the ground?

52. *Games in a Sports League.* If there are x teams in a sports league and all the teams play each other twice, a total of $N(x)$ games are played, where

$$N(x) = x^2 - x.$$

A softball league has 9 teams, each of which plays the others twice. If the league pays $110 per game for the field and the umpires, how much will it cost to play the entire schedule?

53. *Median Home Prices.* The median price for an existing home in the United States peaked at $221,900 in 2006 (*Source:* National Association of REALTORS®). The quartic function

$$h(x) = 56.8328x^4 - 1554.7494x^3 + 10{,}451.8211x^2 - 5655.7692x + 140{,}589.1608,$$

where x is the number of years since 2000, can be used to estimate the median existing-home price from 2000 to 2009. Estimate the median existing-home price in 2002, in 2005, in 2008, and in 2009.

54. *Circulation of Daily Newspapers.* In 1985, the circulation of daily newspapers reached its highest level (*Source*: Newspaper Association of America). The quartic function

$$f(x) = -0.006093x^4 + 0.849362x^3 - 51.892087x^2 + 1627.3581x + 41{,}334.7289,$$

where x is the number of years since 1940, can be used to estimate the circulation of daily newspapers, in thousands, from 1940 to 2008. Using this function, estimate the circulation of daily newspapers in 1945, in 1985, and in 2008.

55. *Interest Compounded Annually.* When P dollars is invested at interest rate i, compounded annually, for t years, the investment grows to A dollars, where

$$A = P(1 + i)^t.$$

Trevor's parents deposit $8000 in a savings account when Trevor is 16 years old. The principal plus interest is to be used for a truck when Trevor is 18 years old. Find the interest rate i if the $8000 grows to $9039.75 in 2 years.

56. *Interest Compounded Annually.* When P dollars is invested at interest rate i, compounded annually, for t years, the investment grows to A dollars, where

$$A = P(1 + i)^t.$$

When Sara enters the 11th grade, her grandparents deposit $10,000 in a college savings account. Find the interest rate i if the $10,000 grows to $11,193.64 in 2 years.

Skill Maintenance

Find the distance between the pair of points.

57. $(3, -5)$ and $(0, -1)$

58. $(4, 2)$ and $(-2, -4)$

59. Find the center and the radius of the circle

$$(x - 3)^2 + (y + 5)^2 = 49.$$

60. The diameter of a circle connects the points $(-6, 5)$ and $(-2, 1)$ on the circle. Find the coordinates of the center of the circle and the length of the radius.

Solve.

61. $2y - 3 \geq 1 - y + 5$

62. $(x - 2)(x + 5) > x(x - 3)$

63. $|x + 6| \geq 7$

64. $\left|x + \frac{1}{4}\right| \leq \frac{2}{3}$

Synthesis

Determine the degree and the leading term of the polynomial function.

65. $f(x) = (x^5 - 1)^2(x^2 + 2)^3$

66. $f(x) = (10 - 3x^5)^2(5 - x^4)^3(x + 4)$

Graphing Polynomial Functions

4.2

- Graph polynomial functions.
- Use the intermediate value theorem to determine whether a function has a real zero between two given real numbers.

Graphing Polynomial Functions

In addition to using the leading-term test and finding the zeros of the function, it is helpful to consider the following facts when graphing a polynomial function.

Graph of a Polynomial Function

If $P(x)$ is a polynomial function of degree n, the graph of the function has:

- at most n real zeros, and thus at most n x-intercepts;
- at most $n - 1$ turning points.

(Turning points on a graph, also called relative maxima and minima, occur when the function changes from decreasing to increasing or from increasing to decreasing.)

4.3 Exercise Set

1. For the function
$$f(x) = x^4 - 6x^3 + x^2 + 24x - 20,$$
use long division to determine whether each of the following is a factor of $f(x)$.

a) $x + 1$ **b)** $x - 2$ **c)** $x + 5$

2. For the function
$$h(x) = x^3 - x^2 - 17x - 15,$$
use long division to determine whether each of the following is a factor of $h(x)$.

a) $x + 5$ **b)** $x + 1$ **c)** $x + 3$

3. For the function
$$g(x) = x^3 - 2x^2 - 11x + 12,$$
use long division to determine whether each of the following is a factor of $g(x)$.

a) $x - 4$ **b)** $x - 3$ **c)** $x - 1$

4. For the function
$$f(x) = x^4 + 8x^3 + 5x^2 - 38x + 24,$$
use long division to determine whether each of the following is a factor of $f(x)$.

a) $x + 6$ **b)** $x + 1$ **c)** $x - 4$

In each of the following, a polynomial $P(x)$ and a divisor $d(x)$ are given. Use long division to find the quotient $Q(x)$ and the remainder $R(x)$ when $P(x)$ is divided by $d(x)$. Express $P(x)$ in the form $d(x) \cdot Q(x) + R(x)$.

5. $P(x) = x^3 - 8$,
$d(x) = x + 2$

6. $P(x) = 2x^3 - 3x^2 + x - 1$,
$d(x) = x - 3$

7. $P(x) = x^3 + 6x^2 - 25x + 18$,
$d(x) = x + 9$

8. $P(x) = x^3 - 9x^2 + 15x + 25$,
$d(x) = x - 5$

9. $P(x) = x^4 - 2x^2 + 3$,
$d(x) = x + 2$

10. $P(x) = x^4 + 6x^3$,
$d(x) = x - 1$

Use synthetic division to find the quotient and the remainder.

11. $(2x^4 + 7x^3 + x - 12) \div (x + 3)$

12. $(x^3 - 7x^2 + 13x + 3) \div (x - 2)$

13. $(x^3 - 2x^2 - 8) \div (x + 2)$

14. $(x^3 - 3x + 10) \div (x - 2)$

15. $(3x^3 - x^2 + 4x - 10) \div (x + 1)$

16. $(4x^4 - 2x + 5) \div (x + 3)$

17. $(x^5 + x^3 - x) \div (x - 3)$

18. $(x^7 - x^6 + x^5 - x^4 + 2) \div (x + 1)$

19. $(x^4 - 1) \div (x - 1)$

20. $(x^5 + 32) \div (x + 2)$

21. $(2x^4 + 3x^2 - 1) \div \left(x - \frac{1}{2}\right)$

22. $(3x^4 - 2x^2 + 2) \div \left(x - \frac{1}{4}\right)$

Use synthetic division to find the function values. Then check your work using a graphing calculator.

23. $f(x) = x^3 - 6x^2 + 11x - 6$; find $f(1)$, $f(-2)$, and $f(3)$.

24. $f(x) = x^3 + 7x^2 - 12x - 3$; find $f(-3)$, $f(-2)$, and $f(1)$.

25. $f(x) = x^4 - 3x^3 + 2x + 8$; find $f(-1)$, $f(4)$, and $f(-5)$.

26. $f(x) = 2x^4 + x^2 - 10x + 1$; find $f(-10)$, $f(2)$, and $f(3)$.

27. $f(x) = 2x^5 - 3x^4 + 2x^3 - x + 8$; find $f(20)$ and $f(-3)$.

28. $f(x) = x^5 - 10x^4 + 20x^3 - 5x - 100$; find $f(-10)$ and $f(5)$.

29. $f(x) = x^4 - 16$; find $f(2)$, $f(-2)$, $f(3)$, and $f(1 - \sqrt{2})$.

30. $f(x) = x^5 + 32$; find $f(2)$, $f(-2)$, $f(3)$, and $f(2 + 3i)$.

Using synthetic division, determine whether the numbers are zeros of the polynomial function.

31. $-3, 2;\ f(x) = 3x^3 + 5x^2 - 6x + 18$

32. $-4, 2;\ f(x) = 3x^3 + 11x^2 - 2x + 8$

33. $-3, 1;\ h(x) = x^4 + 4x^3 + 2x^2 - 4x - 3$

34. $2, -1;\ g(x) = x^4 - 6x^3 + x^2 + 24x - 20$

35. $i, -2i;\ g(x) = x^3 - 4x^2 + 4x - 16$

36. $\frac{1}{3}, 2;\ h(x) = x^3 - x^2 - \frac{1}{9}x + \frac{1}{9}$

37. $-3, \frac{1}{2};\ f(x) = x^3 - \frac{7}{2}x^2 + x - \frac{3}{2}$

38. $i, -i, -2;\ f(x) = x^3 + 2x^2 + x + 2$

Factor the polynomial function $f(x)$. Then solve the equation $f(x) = 0$.

39. $f(x) = x^3 + 4x^2 + x - 6$

40. $f(x) = x^3 + 5x^2 - 2x - 24$

41. $f(x) = x^3 - 6x^2 + 3x + 10$

42. $f(x) = x^3 + 2x^2 - 13x + 10$

43. $f(x) = x^3 - x^2 - 14x + 24$

44. $f(x) = x^3 - 3x^2 - 10x + 24$

45. $f(x) = x^4 - 7x^3 + 9x^2 + 27x - 54$

46. $f(x) = x^4 - 4x^3 - 7x^2 + 34x - 24$

47. $f(x) = x^4 - x^3 - 19x^2 + 49x - 30$

48. $f(x) = x^4 + 11x^3 + 41x^2 + 61x + 30$

Sketch the graph of the polynomial function. Follow the procedure outlined on p. 311. Use synthetic division and the remainder theorem to find the zeros.

49. $f(x) = x^4 - x^3 - 7x^2 + x + 6$

50. $f(x) = x^4 + x^3 - 3x^2 - 5x - 2$

51. $f(x) = x^3 - 7x + 6$

52. $f(x) = x^3 - 12x + 16$

53. $f(x) = -x^3 + 3x^2 + 6x - 8$

54. $f(x) = -x^4 + 2x^3 + 3x^2 - 4x - 4$

Skill Maintenance

Solve. Find exact solutions.

55. $2x^2 + 12 = 5x$

56. $7x^2 + 4x = 3$

Consider the function

$$g(x) = x^2 + 5x - 14$$

in Exercises 57–59.

57. What are the inputs if the output is -14?

58. What is the output if the input is 3?

59. Given an output of -20, find the corresponding inputs.

60. *Movie Ticket Price.* The average price of a movie ticket has increased linearly over the years, rising from \$2.69 in 1980 to \$7.18 in 2008 (*Source*: Motion Picture Association of America). Using these two data points, find a linear function, $f(x) = mx + b$, that models the data. Let x represent the number of years since 1980. Then use this function to estimate the average price of a movie ticket in 1998 and in 2012.

61. The sum of the base and the height of a triangle is 30 in. Find the dimensions for which the area is a maximum.

Synthesis

In Exercises 62 and 63, a graph of a polynomial function is given. On the basis of the graph:

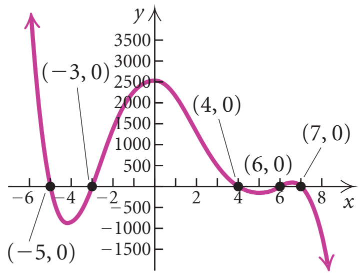
a) *Find as many factors of the polynomial as you can.*

b) *Construct a polynomial function with the zeros shown in the graph.*

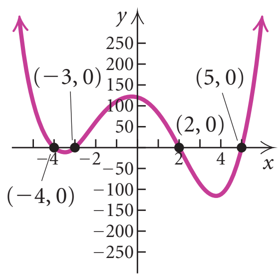
c) *Can you find any other polynomial functions with the given zeros?*

d) *Can you find any other polynomial functions with the given zeros and the same graph?*

62.



63.



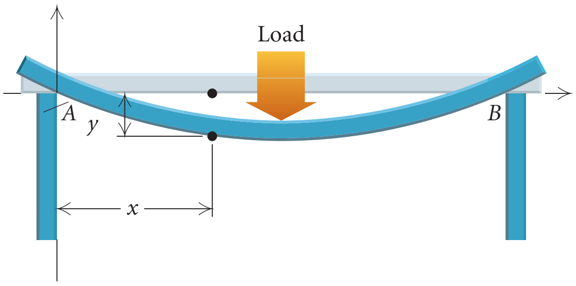
64. For what values of k will the remainder be the same when $x^2 + kx + 4$ is divided by $x - 1$ and by $x + 1$?

65. Find k such that $x + 2$ is a factor of $x^3 - kx^2 + 3x + 7k$.

66. *Beam Deflection.* A beam rests at two points A and B and has a concentrated load applied to its center, as shown below. Let $y =$ the deflection, in feet, of the beam at a distance of x feet from A. Under certain conditions, this deflection is given by

$$y = \frac{1}{13}x^3 - \frac{1}{14}x.$$

Find the zeros of the polynomial in the interval $[0, 2]$.



Solve.

67. $\dfrac{2x^2}{x^2 - 1} + \dfrac{4}{x + 3} = \dfrac{12x - 4}{x^3 + 3x^2 - x - 3}$

68. $\dfrac{6x^2}{x^2 + 11} + \dfrac{60}{x^3 - 7x^2 + 11x - 77} = \dfrac{1}{x - 7}$

69. Find a 15th-degree polynomial for which $x - 1$ is a factor. Answers may vary.

Use synthetic division to divide.

70. $(x^4 - y^4) \div (x - y)$

71. $(x^3 + 3ix^2 - 4ix - 2) \div (x + i)$

72. $(x^2 - 4x - 2) \div [x - (3 + 2i)]$

73. $(x^2 - 3x + 7) \div (x - i)$

Mid-Chapter Mixed Review

Determine whether the statement is true or false.

1. The y-intercept of the graph of the function $P(x) = 5 - 2x^3$ is $(5, 0)$. [4.2]

2. The degree of the polynomial $x - \frac{1}{2}x^4 - 3x^6 + x^5$ is 6. [4.1]

3. If $f(x) = (x + 7)(x - 8)$, then $f(8) = 0$. [4.3]

4. If $f(12) = 0$, then $x + 12$ is a factor of $f(x)$. [4.3]

Find the zeros of the polynomial function and state the multiplicity of each. [4.1]

5. $f(x) = (x^2 - 10x + 25)^3$

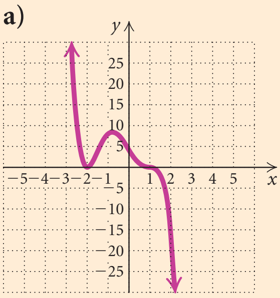
6. $h(x) = 2x^3 + x^2 - 50x - 25$

7. $g(x) = x^4 - 3x^2 + 2$

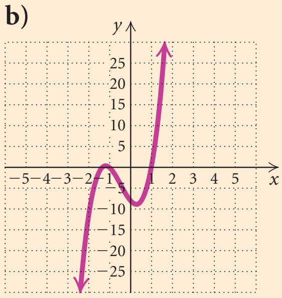
8. $f(x) = -6(x - 3)^2(x + 4)$

In Exercises 9–12, match the function with one of the graphs (a)–(d), which follow. [4.2]

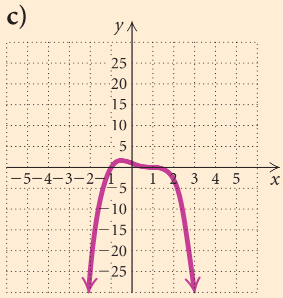
a)



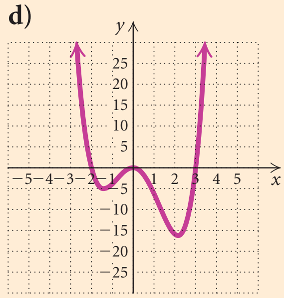
b)



c)



d)



9. $f(x) = x^4 - x^3 - 6x^2$

10. $f(x) = -(x - 1)^3(x + 2)^2$

11. $f(x) = 6x^3 + 8x^2 - 6x - 8$

12. $f(x) = -(x - 1)^3(x + 1)$

Using the intermediate value theorem, determine, if possible, whether the function has at least one real zero between a and b. [4.2]

13. $f(x) = x^3 - 2x^2 + 3;\ a = -2, b = 0$

14. $f(x) = x^3 - 2x^2 + 3;\ a = -\frac{1}{2}, b = 1$

15. For the polynomial $P(x) = x^4 - 6x^3 + x - 2$ and the divisor $d(x) = x - 1$, use long division to find the quotient $Q(x)$ and the remainder $R(x)$ when $P(x)$ is divided by $d(x)$. Express $P(x)$ in the form $d(x) \cdot Q(x) + R(x)$. [4.3]

Use synthetic division to find the quotient and the remainder. [4.3]

16. $(3x^4 - x^3 + 2x^2 - 6x + 6) \div (x - 2)$

17. $(x^5 - 5) \div (x + 1)$

Use synthetic division to find the function values. [4.3]

18. $g(x) = x^3 - 9x^2 + 4x - 10;$ find $g(-5)$

19. $f(x) = 20x^2 - 40x;$ find $f\left(\frac{1}{2}\right)$

20. $f(x) = 5x^4 + x^3 - x;$ find $f\left(-\sqrt{2}\right)$

Using synthetic division, determine whether the numbers are zeros of the polynomial function. [4.3]

21. $-3i, 3;\ f(x) = x^3 - 4x^2 + 9x - 36$

22. $-1, 5;\ f(x) = x^6 - 35x^4 + 259x^2 - 225$

Factor the polynomial function $f(x)$. Then solve the equation $f(x) = 0$. [4.3]

23. $h(x) = x^3 - 2x^2 - 55x + 56$

24. $g(x) = x^4 - 2x^3 - 13x^2 + 14x + 24$

Collaborative Discussion and Writing

25. How is the range of a polynomial function related to the degree of the polynomial? [4.1]

26. Is it possible for the graph of a polynomial function to have no y-intercept? no x-intercepts? Explain your answer. [4.2]

27. Explain why values of a function must be all positive or all negative between consecutive zeros. [4.2]

28. In synthetic division, why is the degree of the quotient 1 less than that of the dividend? [4.3]

Total Number of Zeros	5	
Positive Real	2	0
Negative Real	1	1
Nonreal	2	4

The number of variations of sign in $P(-x)$ is 1. Thus there is exactly 1 negative real zero. Since nonreal, complex conjugates occur in pairs, we also know the possible ways in which nonreal zeros might occur. The table shown at left summarizes all the possibilities for real zeros and nonreal zeros of $P(x)$.

Now Try Exercise 93.

STUDY TIP

It is never too soon to begin reviewing for the final examination. The Skill Maintenance exercises found in each exercise set review and reinforce skills taught in earlier sections. Be sure to do these exercises as you do the homework assignment in each section. Answers to all of the skill maintenance exercises, along with section references, appear at the back of the book.

EXAMPLE 9 $P(x) = 5x^4 - 3x^3 + 7x^2 - 12x + 4$

Solution There are 4 variations of sign. Thus the number of positive real zeros is either

$$4 \quad \text{or} \quad 4 - 2 \quad \text{or} \quad 4 - 4.$$

That is, the number of positive real zeros is 4, 2, or 0.

$$P(-x) = 5x^4 + 3x^3 + 7x^2 + 12x + 4$$

There are 0 changes in sign, so there are no negative real zeros.

Now Try Exercise 81.

EXAMPLE 10 $P(x) = 6x^6 - 2x^2 - 5x$

Solution As written, the polynomial does not satisfy the conditions of Descartes' rule of signs because the constant term is 0. But because x is a factor of every term, we know that the polynomial has 0 as a zero. We can then factor as follows:

$$P(x) = x(6x^5 - 2x - 5).$$

Now we analyze $Q(x) = 6x^5 - 2x - 5$ and $Q(-x) = -6x^5 + 2x - 5$. The number of variations of sign in $Q(x)$ is 1. Therefore, there is exactly 1 positive real zero. The number of variations of sign in $Q(-x)$ is 2. Thus the number of negative real zeros is 2 or 0. The same results apply to $P(x)$. Since nonreal, complex conjugates occur in pairs, we know the possible ways in which nonreal zeros might occur. The table at left summarizes all the possibilities for real zeros and nonreal zeros of $P(x)$.

Total Number of Zeros	6	
0 as a Zero	1	1
Positive Real	1	1
Negative Real	2	0
Nonreal	2	4

Now Try Exercise 95.

4.4 Exercise Set

Find a polynomial function of degree 3 with the given numbers as zeros.

1. $-2, 3, 5$

2. $-1, 0, 4$

3. $-3, 2i, -2i$

4. $2, i, -i$

5. $\sqrt{2}, -\sqrt{2}, 3$

6. $-5, \sqrt{3}, -\sqrt{3}$

7. $1 - \sqrt{3}, 1 + \sqrt{3}, -2$

8. $-4, 1 - \sqrt{5}, 1 + \sqrt{5}$

9. $1 + 6i, 1 - 6i, -4$

10. $1 + 4i, 1 - 4i, -1$

11. $-\frac{1}{3}, 0, 2$

12. $-3, 0, \frac{1}{2}$

13. Find a polynomial function of degree 5 with -1 as a zero of multiplicity 3, 0 as a zero of multiplicity 1, and 1 as a zero of multiplicity 1.

14. Find a polynomial function of degree 4 with -2 as a zero of multiplicity 1, 3 as a zero of multiplicity 2, and -1 as a zero of multiplicity 1.

15. Find a polynomial function of degree 4 with $a_4 = 1$ and with -1 as a zero of multiplicity 3 and 0 as a zero of multiplicity 1.

16. Find a polynomial function of degree 5 with $a_5 = 1$ and with $-\frac{1}{2}$ as a zero of multiplicity 2, 0 as a zero of multiplicity 1, and 1 as a zero of multiplicity 2.

Suppose that a polynomial function of degree 4 with rational coefficients has the given numbers as zeros. Find the other zero(s).

17. $-1, \sqrt{3}, \frac{11}{3}$

18. $-\sqrt{2}, -1, \frac{4}{5}$

19. $-i, 2 - \sqrt{5}$

20. $i, -3 + \sqrt{3}$

21. $3i, 0, -5$

22. $3, 0, -2i$

23. $-4 - 3i, 2 - \sqrt{3}$

24. $6 - 5i, -1 + \sqrt{7}$

Suppose that a polynomial function of degree 5 with rational coefficients has the given numbers as zeros. Find the other zero(s).

25. $-\frac{1}{2}, \sqrt{5}, -4i$

26. $\frac{3}{4}, -\sqrt{3}, 2i$

27. $-5, 0, 2 - i, 4$

28. $-2, 3, 4, 1 - i$

29. $6, -3 + 4i, 4 - \sqrt{5}$

30. $-3 - 3i, 2 + \sqrt{13}, 6$

31. $-\frac{3}{4}, \frac{3}{4}, 0, 4 - i$

32. $-0.6, 0, 0.6, -3 + \sqrt{2}$

Find a polynomial function of lowest degree with rational coefficients that has the given numbers as some of its zeros.

33. $1 + i, 2$

34. $2 - i, -1$

35. $4i$

36. $-5i$

37. $-4i, 5$

38. $3, -i$

39. $1 - i, -\sqrt{5}$

40. $2 - \sqrt{3}, 1 + i$

41. $\sqrt{5}, -3i$

42. $-\sqrt{2}, 4i$

Given that the polynomial function has the given zero, find the other zeros.

43. $f(x) = x^3 + 5x^2 - 2x - 10;\ -5$

44. $f(x) = x^3 - x^2 + x - 1;\ 1$

45. $f(x) = x^4 - 5x^3 + 7x^2 - 5x + 6;\ -i$

46. $f(x) = x^4 - 16;\ 2i$

47. $f(x) = x^3 - 6x^2 + 13x - 20;\ 4$

48. $f(x) = x^3 - 8;\ 2$

List all possible rational zeros of the function.

49. $f(x) = x^5 - 3x^2 + 1$

50. $f(x) = x^7 + 37x^5 - 6x^2 + 12$

51. $f(x) = 2x^4 - 3x^3 - x + 8$

52. $f(x) = 3x^3 - x^2 + 6x - 9$

53. $f(x) = 15x^6 + 47x^2 + 2$

54. $f(x) = 10x^{25} + 3x^{17} - 35x + 6$

For each polynomial function:

a) *Find the rational zeros and then the other zeros; that is, solve $f(x) = 0$.*

b) *Factor $f(x)$ into linear factors.*

55. $f(x) = x^3 + 3x^2 - 2x - 6$

56. $f(x) = x^3 - x^2 - 3x + 3$

57. $f(x) = 3x^3 - x^2 - 15x + 5$

58. $f(x) = 4x^3 - 4x^2 - 3x + 3$

59. $f(x) = x^3 - 3x + 2$

60. $f(x) = x^3 - 2x + 4$

61. $f(x) = 2x^3 + 3x^2 + 18x + 27$

62. $f(x) = 2x^3 + 7x^2 + 2x - 8$

63. $f(x) = 5x^4 - 4x^3 + 19x^2 - 16x - 4$

64. $f(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$

65. $f(x) = x^4 - 3x^3 - 20x^2 - 24x - 8$

66. $f(x) = x^4 + 5x^3 - 27x^2 + 31x - 10$

67. $f(x) = x^3 - 4x^2 + 2x + 4$

68. $f(x) = x^3 - 8x^2 + 17x - 4$

69. $f(x) = x^3 + 8$

70. $f(x) = x^3 - 8$

71. $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - \frac{1}{6}x + \frac{1}{6}$

72. $f(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 + \frac{2}{3}x - \frac{1}{2}$

Find only the rational zeros of the function.

73. $f(x) = x^4 + 2x^3 - 5x^2 - 4x + 6$

74. $f(x) = x^4 - 3x^3 - 9x^2 - 3x - 10$

75. $f(x) = x^3 - x^2 - 4x + 3$

76. $f(x) = 2x^3 + 3x^2 + 2x + 3$

77. $f(x) = x^4 + 2x^3 + 2x^2 - 4x - 8$

78. $f(x) = x^4 + 6x^3 + 17x^2 + 36x + 66$

79. $f(x) = x^5 - 5x^4 + 5x^3 + 15x^2 - 36x + 20$

80. $f(x) = x^5 - 3x^4 - 3x^3 + 9x^2 - 4x + 12$

What does Descartes' rule of signs tell you about the number of positive real zeros and the number of negative real zeros of the function?

81. $f(x) = 3x^5 - 2x^2 + x - 1$

82. $g(x) = 5x^6 - 3x^3 + x^2 - x$

83. $h(x) = 6x^7 + 2x^2 + 5x + 4$

84. $P(x) = -3x^5 - 7x^3 - 4x - 5$

85. $F(p) = 3p^{18} + 2p^4 - 5p^2 + p + 3$

86. $H(t) = 5t^{12} - 7t^4 + 3t^2 + t + 1$

87. $C(x) = 7x^6 + 3x^4 - x - 10$

88. $g(z) = -z^{10} + 8z^7 + z^3 + 6z - 1$

89. $h(t) = -4t^5 - t^3 + 2t^2 + 1$

90. $P(x) = x^6 + 2x^4 - 9x^3 - 4$

91. $f(y) = y^4 + 13y^3 - y + 5$

92. $Q(x) = x^4 - 2x^2 + 12x - 8$

93. $r(x) = x^4 - 6x^2 + 20x - 24$

94. $f(x) = x^5 - 2x^3 - 8x$

95. $R(x) = 3x^5 - 5x^3 - 4x$

96. $f(x) = x^4 - 9x^2 - 6x + 4$

Sketch the graph of the polynomial function. Follow the procedure outlined on p. 311. Use the rational zeros theorem when finding the zeros.

97. $f(x) = 4x^3 + x^2 - 8x - 2$

98. $f(x) = 3x^3 - 4x^2 - 5x + 2$

99. $f(x) = 2x^4 - 3x^3 - 2x^2 + 3x$

100. $f(x) = 4x^4 - 37x^2 + 9$

Skill Maintenance

For Exercises 101 and 102, complete the square to:

a) *find the vertex;*

b) *find the axis of symmetry; and*

c) *determine whether there is a maximum or minimum function value and find that value.*

101. $f(x) = x^2 - 8x + 10$

102. $f(x) = 3x^2 - 6x - 1$

Find the zeros of the function.

103. $f(x) = -\frac{4}{5}x + 8$

104. $g(x) = x^2 - 8x - 33$

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then describe the end behavior of the function's graph and classify the polynomial function as constant, linear, quadratic, cubic, or quartic.

105. $g(x) = -x^3 - 2x^2$

106. $f(x) = -x^2 - 3x + 6$

107. $f(x) = -\frac{4}{9}$

108. $h(x) = x - 2$

109. $g(x) = x^4 - 2x^3 + x^2 - x + 2$

110. $h(x) = x^3 + \frac{1}{2}x^2 - 4x - 3$

Synthesis

111. Consider $f(x) = 2x^3 - 5x^2 - 4x + 3$. Find the solutions of each equation.

a) $f(x) = 0$ **b)** $f(x - 1) = 0$

c) $f(x + 2) = 0$ **d)** $f(2x) = 0$

112. Use the rational zeros theorem and the equation $x^4 - 12 = 0$ to show that $\sqrt[4]{12}$ is irrational.

Find the rational zeros of the function.

113. $P(x) = 2x^5 - 33x^4 - 84x^3 + 2203x^2 - 3348x - 10{,}080$

114. $P(x) = x^6 - 6x^5 - 72x^4 - 81x^2 + 486x + 5832$

The following is a method for solving rational inequalities.

To solve a rational inequality:

1. Find an equivalent inequality with 0 on one side.
2. Change the inequality symbol to an equals sign and solve the related equation.
3. Find values of the variable for which the related rational function is not defined.
4. The numbers found in steps (2) and (3) are called critical values. Use the critical values to divide the x-axis into intervals. Then determine the function's sign in each interval using an x-value from the interval or the graph of the equation.
5. Select the intervals for which the inequality is satisfied and write interval notation or set-builder notation for the solution set. If the inequality symbol is $\leq$ or $\geq$, then the solutions to step (2) should be included in the solution set. The x-values found in step (3) are never included in the solution set.

It works well to use a combination of algebraic and graphical methods to solve polynomial inequalities and rational inequalities. The algebraic methods give exact numbers for the critical values, and the graphical methods usually allow us to see easily what intervals satisfy the inequality.

4.6 Exercise Set

For the function $f(x) = x^2 + 2x - 15$, *solve each of the following.*

1. $f(x) = 0$

2. $f(x) < 0$

3. $f(x) \leq 0$

4. $f(x) > 0$

5. $f(x) \geq 0$

For the function $g(x) = \dfrac{x-2}{x+4}$, *solve each of the following.*

6. $g(x) = 0$

7. $g(x) > 0$

8. $g(x) \leq 0$

9. $g(x) \geq 0$

10. $g(x) < 0$

For the function

$$h(x) = \frac{7x}{(x-1)(x+5)},$$

solve each of the following.

11. $h(x) = 0$

12. $h(x) \leq 0$

13. $h(x) \geq 0$

14. $h(x) > 0$

15. $h(x) < 0$

For the function $g(x) = x^5 - 9x^3$, *solve each of the following.*

16. $g(x) = 0$

17. $g(x) < 0$

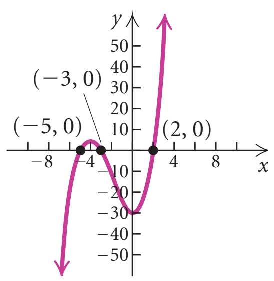
18. $g(x) \leq 0$

19. $g(x) > 0$

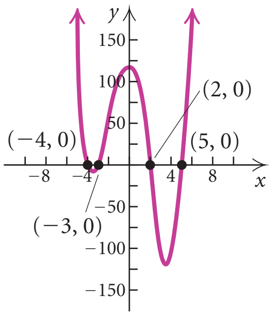
20. $g(x) \geq 0$

In Exercises 21–24, a related function is graphed. Solve the given inequality.

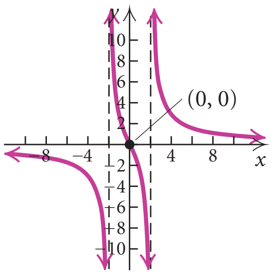
21. $x^3 + 6x^2 < x + 30$



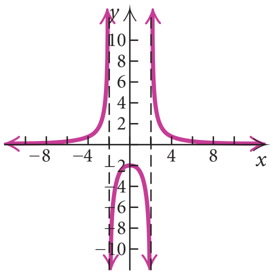
22. $x^4 - 27x^2 - 14x + 120 \geq 0$



23. $\dfrac{8x}{x^2 - 4} \geq 0$



24. $\dfrac{8}{x^2 - 4} < 0$



Solve.

25. $(x - 1)(x + 4) < 0$

26. $(x + 3)(x - 5) < 0$

27. $x^2 + x - 2 > 0$

28. $x^2 - x - 6 > 0$

29. $x^2 - x - 5 \geq x - 2$

30. $x^2 + 4x + 7 \geq 5x + 9$

31. $x^2 > 25$

32. $x^2 \leq 1$

33. $4 - x^2 \leq 0$

34. $11 - x^2 \geq 0$

35. $6x - 9 - x^2 < 0$

36. $x^2 + 2x + 1 \leq 0$

37. $x^2 + 12 < 4x$

38. $x^2 - 8 > 6x$

39. $4x^3 - 7x^2 \leq 15x$

40. $2x^3 - x^2 < 5x$

41. $x^3 + 3x^2 - x - 3 \geq 0$

42. $x^3 + x^2 - 4x - 4 \geq 0$

43. $x^3 - 2x^2 < 5x - 6$

44. $x^3 + x \leq 6 - 4x^2$

45. $x^5 + x^2 \geq 2x^3 + 2$

46. $x^5 + 24 > 3x^3 + 8x^2$

47. $2x^3 + 6 \leq 5x^2 + x$

48. $2x^3 + x^2 < 10 + 11x$

49. $x^3 + 5x^2 - 25x \leq 125$

50. $x^3 - 9x + 27 \geq 3x^2$

51. $0.1x^3 - 0.6x^2 - 0.1x + 2 < 0$

52. $19.2x^3 + 12.8x^2 + 144 \geq 172.8x + 3.2x^4$

List the critical values of the related function. Then solve the inequality.

53. $\dfrac{1}{x + 4} > 0$

54. $\dfrac{1}{x - 3} \leq 0$

55. $\dfrac{-4}{2x + 5} < 0$

56. $\dfrac{-2}{5 - x} \geq 0$

57. $\dfrac{2x}{x - 4} \geq 0$

58. $\dfrac{5x}{x + 1} < 0$

59. $\dfrac{x - 4}{x + 3} - \dfrac{x + 2}{x - 1} \leq 0$

60. $\dfrac{x + 1}{x - 2} - \dfrac{x - 3}{x - 1} < 0$

61. $\dfrac{x + 6}{x - 2} > \dfrac{x - 8}{x - 5}$

62. $\dfrac{x - 7}{x + 2} \geq \dfrac{x - 9}{x + 3}$

63. $\dfrac{x + 1}{x - 2} \geq 3$

64. $\dfrac{x}{x - 5} < 2$

65. $x - 2 > \dfrac{1}{x}$

66. $4 \geq \dfrac{4}{x} + x$

67. $\dfrac{2}{x^2 - 4x + 3} \le \dfrac{5}{x^2 - 9}$

68. $\dfrac{3}{x^2 - 4} \le \dfrac{5}{x^2 + 7x + 10}$

69. $\dfrac{3}{x^2 + 1} \ge \dfrac{6}{5x^2 + 2}$

70. $\dfrac{4}{x^2 - 9} < \dfrac{3}{x^2 - 25}$

71. $\dfrac{5}{x^2 + 3x} < \dfrac{3}{2x + 1}$

72. $\dfrac{2}{x^2 + 3} > \dfrac{3}{5 + 4x^2}$

73. $\dfrac{5x}{7x - 2} > \dfrac{x}{x + 1}$

74. $\dfrac{x^2 - x - 2}{x^2 + 5x + 6} < 0$

75. $\dfrac{x}{x^2 + 4x - 5} + \dfrac{3}{x^2 - 25} \le \dfrac{2x}{x^2 - 6x + 5}$

76. $\dfrac{2x}{x^2 - 9} + \dfrac{x}{x^2 + x - 12} \ge \dfrac{3x}{x^2 + 7x + 12}$

77. *Temperature During an Illness.* A person's temperature T, in degrees Fahrenheit, during an illness is given by the function

$$T(t) = \frac{4t}{t^2 + 1} + 98.6,$$

where t is the time since the onset of the illness, in hours. Find the interval on which the temperature was over 100°F. (See Example 12 in Section 4.5.)

78. *Population Growth.* The population P, in thousands, of a resort community is given by

$$P(t) = \frac{500t}{2t^2 + 9},$$

where t is the time, in months. Find the interval on which the population was 40,000 or greater. (See Exercise 85 in Exercise Set 4.5.)

79. *Total Profit.* Flexl, Inc., determines that its total profit is given by the function

$$P(x) = -3x^2 + 630x - 6000.$$

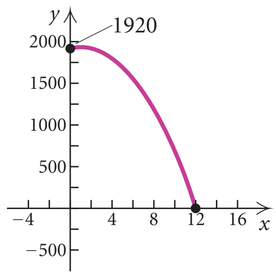
a) Flexl makes a profit for those nonnegative values of x for which $P(x) > 0$. Find the values of x for which Flexl makes a profit.

b) Flexl loses money for those nonnegative values of x for which $P(x) < 0$. Find the values of x for which Flexl loses money.

80. *Height of a Thrown Object.* The function

$$S(t) = -16t^2 + 32t + 1920$$

gives the height S, in feet, of an object thrown upward from a cliff that is 1920 ft high. Here t is the time, in seconds, that the object is in the air.



a) For what times is the height greater than 1920 ft?

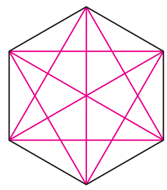
b) For what times is the height less than 640 ft?

81. *Number of Diagonals.* A polygon with n sides has D diagonals, where D is given by the function

$$D(n) = \frac{n(n - 3)}{2}.$$

Find the number of sides n if

$$27 \le D \le 230.$$



82. *Number of Handshakes.* If there are n people in a room, the number N of possible handshakes by all the people in the room is given by the function

$$N(n) = \frac{n(n - 1)}{2}.$$

For what number n of people is

$$66 \le N \le 300?$$

Skill Maintenance

Find an equation for a circle satisfying the given conditions.

83. Center: $(-2, 4)$; radius of length 3

84. Center: $(0, -3)$; diameter of length $\frac{7}{2}$

In Exercises 85 and 86:

a) *Find the vertex.*

b) *Determine whether there is a maximum or minimum value and find that value.*

c) *Find the range.*

85. $h(x) = -2x^2 + 3x - 8$

86. $g(x) = x^2 - 10x + 2$

Synthesis

Solve.

87. $|x^2 - 5| = 5 - x^2$

88. $x^4 - 6x^2 + 5 > 0$

89. $2|x|^2 - |x| + 2 \le 5$

90. $(7 - x)^{-2} < 0$

91. $\left|1 + \frac{1}{x}\right| < 3$

92. $\left|2 - \frac{1}{x}\right| \le 2 + \left|\frac{1}{x}\right|$

93. Write a quadratic inequality for which the solution set is $(-4, 3)$.

94. Write a polynomial inequality for which the solution set is $[-4, 3] \cup [7, \infty)$.

Find the domain of the function.

95. $f(x) = \sqrt{\frac{72}{x^2 - 4x - 21}}$

96. $f(x) = \sqrt{x^2 - 4x - 21}$

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial function as constant, linear, quadratic, cubic, or quartic. [4.1]

6. $f(x) = 7x^2 - 5 + 0.45x^4 - 3x^3$

7. $h(x) = -25$

8. $g(x) = 6 - 0.5x$

9. $f(x) = \frac{1}{3}x^3 - 2x + 3$

Use the leading-term test to describe the end behavior of the graph of the function. [4.1]

10. $f(x) = -\frac{1}{2}x^4 + 3x^2 + x - 6$

11. $f(x) = x^5 + 2x^3 - x^2 + 5x + 4$

Find the zeros of the polynomial function and state the multiplicity of each. [4.1]

12. $g(x) = \left(x - \frac{2}{3}\right)(x + 2)^3(x - 5)^2$

13. $f(x) = x^4 - 26x^2 + 25$

14. $h(x) = x^3 + 4x^2 - 9x - 36$

15. *Interest Compounded Annually.* When P dollars is invested at interest rate i, compounded annually, for t years, the investment grows to A dollars, where

$$A = P(1 + i)^t.$$

a) Find the interest rate i if \$6250 grows to \$6760 in 2 years. [4.1]

b) Find the interest rate i if \$1,000,000 grows to \$1,215,506.25 in 4 years. [4.1]

Sketch the graph of the polynomial function.

16. $f(x) = -x^4 + 2x^3$ [4.2]

17. $g(x) = (x - 1)^3(x + 2)^2$ [4.2]

18. $h(x) = x^3 + 3x^2 - x - 3$ [4.2]

19. $f(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$ [4.2], [4.3], [4.4]

20. $g(x) = 2x^3 + 7x^2 - 14x + 5$ [4.2], [4.4]

Using the intermediate value theorem, determine, if possible, whether the function f has a zero between a and b. [4.2]

21. $f(x) = 4x^2 - 5x - 3;\ a = 1, b = 2$

22. $f(x) = x^3 - 4x^2 + \frac{1}{2}x + 2;\ a = -1, b = 1$

In each of the following, a polynomial $P(x)$ and a divisor $d(x)$ are given. Use long division to find the quotient $Q(x)$ and the remainder $R(x)$ when $P(x)$ is divided by $d(x)$. Express $P(x)$ in the form $d(x) \cdot Q(x) + R(x)$. [4.3]

23. $P(x) = 6x^3 - 2x^2 + 4x - 1,\ d(x) = x - 3$

24. $P(x) = x^4 - 2x^3 + x + 5,\ d(x) = x + 1$

Use synthetic division to find the quotient and the remainder. [4.3]

25. $(x^3 + 2x^2 - 13x + 10) \div (x - 5)$

26. $(x^4 + 3x^3 + 3x^2 + 3x + 2) \div (x + 2)$

27. $(x^5 - 2x) \div (x + 1)$

Use synthetic division to find the indicated function value. [4.3]

28. $f(x) = x^3 + 2x^2 - 13x + 10;\ f(-2)$

29. $f(x) = x^4 - 16;\ f(-2)$

30. $f(x) = x^5 - 4x^4 + x^3 - x^2 + 2x - 100;\ f(-10)$

Using synthetic division, determine whether the given numbers are zeros of the polynomial function. [4.3]

31. $-i, -5;\ f(x) = x^3 - 5x^2 + x - 5$

32. $-1, -2;\ f(x) = x^4 - 4x^3 - 3x^2 + 14x - 8$

33. $\frac{1}{3}, 1;\ f(x) = x^3 - \frac{4}{3}x^2 - \frac{5}{3}x + \frac{2}{3}$

34. $2, -\sqrt{3};\ f(x) = x^4 - 5x^2 + 6$

Factor the polynomial $f(x)$. Then solve the equation $f(x) = 0$. [4.3], [4.4]

35. $f(x) = x^3 + 2x^2 - 7x + 4$

36. $f(x) = x^3 + 4x^2 - 3x - 18$

37. $f(x) = x^4 - 4x^3 - 21x^2 + 100x - 100$

38. $f(x) = x^4 - 3x^2 + 2$

Find a polynomial function of degree 3 with the given numbers as zeros. [4.4]

39. $-4, -1, 2$

40. $-3, 1 - i, 1 + i$

41. $\frac{1}{2}, 1 - \sqrt{2}, 1 + \sqrt{2}$

42. Find a polynomial function of degree 4 with -5 as a zero of multiplicity 3 and $\frac{1}{2}$ as a zero of multiplicity 1. [4.4]

43. Find a polynomial function of degree 5 with -3 as a zero of multiplicity 2, 2 as a zero of multiplicity 1, and 0 as a zero of multiplicity 2. [4.4]

Suppose that a polynomial function of degree 5 with rational coefficients has the given zeros. Find the other zero(s). [4.4]

44. $-\frac{2}{3}, \sqrt{5}, i$

45. $0, 1 + \sqrt{3}, -\sqrt{3}$

46. $-\sqrt{2}, \frac{1}{2}, 1, 2$

Find a polynomial function of lowest degree with rational coefficients and the following as some of its zeros. [4.4]

47. $\sqrt{11}$

48. $-i, 6$

49. $-1, 4, 1 + i$

50. $\sqrt{5}, -2i$

51. $\frac{1}{3}, 0, -3$

List all possible rational zeros. [4.4]

52. $h(x) = 4x^5 - 2x^3 + 6x - 12$

53. $g(x) = 3x^4 - x^3 + 5x^2 - x + 1$

54. $f(x) = x^3 - 2x^2 + x - 24$

For each polynomial function:

a) *Find the rational zeros and then the other zeros; that is, solve $f(x) = 0$.* [4.4]

b) *Factor $f(x)$ into linear factors.* [4.4]

55. $f(x) = 3x^5 + 2x^4 - 25x^3 - 28x^2 + 12x$

56. $f(x) = x^3 - 2x^2 - 3x + 6$

57. $f(x) = x^4 - 6x^3 + 9x^2 + 6x - 10$

58. $f(x) = x^3 + 3x^2 - 11x - 5$

59. $f(x) = 3x^3 - 8x^2 + 7x - 2$

60. $f(x) = x^5 - 8x^4 + 20x^3 - 8x^2 - 32x + 32$

61. $f(x) = x^6 + x^5 - 28x^4 - 16x^3 + 192x^2$

62. $f(x) = 2x^5 - 13x^4 + 32x^3 - 38x^2 + 22x - 5$

What does Descartes' rule of signs tell you about the number of positive real zeros and the number of negative real zeros of each of the following polynomial functions? [4.4]

63. $f(x) = 2x^6 - 7x^3 + x^2 - x$

64. $h(x) = -x^8 + 6x^5 - x^3 + 2x - 2$

65. $g(x) = 5x^5 - 4x^2 + x - 1$

Graph the function. Be sure to label all the asymptotes. List the domain and the x- and y-intercepts. [4.5]

66. $f(x) = \dfrac{x^2 - 5}{x + 2}$

67. $f(x) = \dfrac{5}{(x - 2)^2}$

68. $f(x) = \dfrac{x^2 + x - 6}{x^2 - x - 20}$

69. $f(x) = \dfrac{x - 2}{x^2 - 2x - 15}$

In Exercises 70 and 71, find a rational function that satisfies the given conditions. Answers may vary, but try to give the simplest answer possible. [4.5]

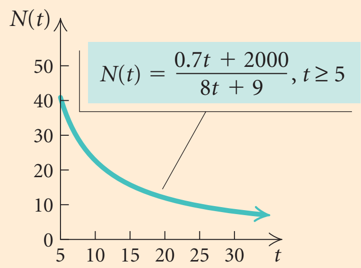
70. Vertical asymptotes $x = -2, x = 3$

71. Vertical asymptotes $x = -2, x = 3$; horizontal asymptote $y = 4$; x-intercept $(-3, 0)$

72. *Medical Dosage.* The function

$$N(t) = \frac{0.7t + 2000}{8t + 9}, \quad t \geq 5$$

gives the body concentration $N(t)$, in parts per million, of a certain dosage of medication after time t, in hours.



a) Find the horizontal asymptote of the graph and complete the following:

$N(t) \to \boxed{}$ as $t \to \infty$. [4.5]

b) Explain the meaning of the answer to part (a) in terms of the application. [4.5]

Solve. [4.6]

73. $x^2 - 9 < 0$

74. $2x^2 > 3x + 2$

75. $(1 - x)(x + 4)(x - 2) \leq 0$

76. $\dfrac{x - 2}{x + 3} < 4$

77. *Height of a Rocket.* The function

$$S(t) = -16t^2 + 80t + 224$$

gives the height S, in feet, of a model rocket launched with a velocity of 80 ft/sec from a hill that is 224 ft high, where t is the time, in seconds.

a) Determine when the rocket reaches the ground. [4.1]

b) On what interval is the height greater than 320 ft? [4.1], [4.6]

78. *Population Growth.* The population P, in thousands, of Novi is given by

$$P(t) = \frac{8000t}{4t^2 + 10},$$

where t is the time, in months. Find the interval on which the population was 400,000 or greater. [4.6]

79. Determine the domain of the function

$$g(x) = \frac{x^2 + 2x - 3}{x^2 - 5x + 6}. \quad [4.5]$$

A. $(-\infty, 2) \cup (2, 3) \cup (3, \infty)$
B. $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$
C. $(-\infty, 2) \cup (3, \infty)$
D. $(-\infty, -3) \cup (1, \infty)$

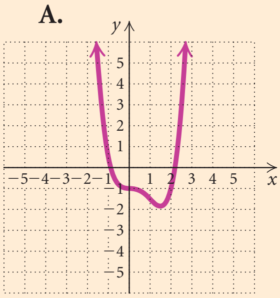
80. Determine the vertical asymptotes of the function

$$f(x) = \frac{x - 4}{(x + 1)(x - 2)(x + 4)}. \quad [4.5]$$

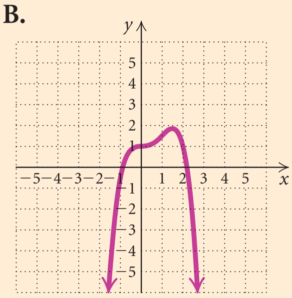
A. $x = 1, x = -2$, and $x = 4$
B. $x = -1, x = 2, x = -4$, and $x = 4$
C. $x = -1, x = 2$, and $x = -4$
D. $x = 4$

81. The graph of $f(x) = -\frac{1}{2}x^4 + x^3 + 1$ is which of the following? [4.2]

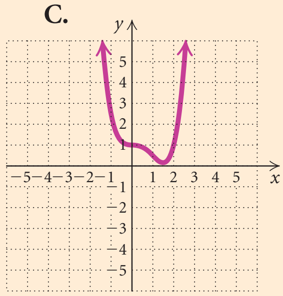
A.



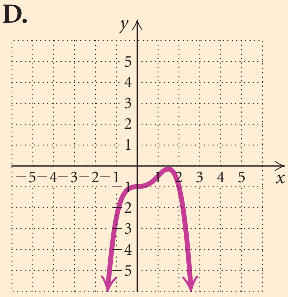
B.



C.



D.



Synthesis

Solve.

82. $x^2 \geq 5 - 2x$ [4.6]

83. $\left|1 - \frac{1}{x^2}\right| < 3$ [4.6]

84. $x^4 - 2x^3 + 3x^2 - 2x + 2 = 0$ [4.4], [4.5]

85. $(x - 2)^{-3} < 0$ [4.6]

86. Express $x^3 - 1$ as a product of linear factors. [4.4]

87. Find k such that $x + 3$ is a factor of $x^3 + kx^2 + kx - 15$. [4.3]

88. When $x^2 - 4x + 3k$ is divided by $x + 5$, the remainder is 33. Find the value of k. [4.3]

Find the domain of the function. [4.5]

89. $f(x) = \sqrt{x^2 + 3x - 10}$

90. $f(x) = \sqrt{x^2 - 3.1x + 2.2} + 1.75$

91. $f(x) = \dfrac{1}{\sqrt{5 - |7x + 2|}}$

Collaborative Discussion and Writing

92. Explain the difference between a polynomial function and a rational function. [4.1], [4.5]

93. Is it possible for a third-degree polynomial with rational coefficients to have no real zeros? Why or why not? [4.4]

94. Explain and contrast the three types of asymptotes considered for rational functions. [4.5]

95. If $P(x)$ is an even function, and by Descartes' rule of signs, $P(x)$ has one positive real zero, how many negative real zeros does $P(x)$ have? Explain. [4.4]

96. Explain why the graph of a rational function cannot have both a horizontal asymptote and an oblique asymptote. [4.5]

97. Under what circumstances would a quadratic inequality have a solution set that is a closed interval? [4.6]

Chapter 4 Test

Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial as constant, linear, quadratic, cubic, or quartic.

1. $f(x) = 2x^3 + 6x^2 - x^4 + 11$

2. $h(x) = -4.7x + 29$

3. Find the zeros of the polynomial function and state the multiplicity of each:
$$f(x) = x(3x - 5)(x - 3)^2(x + 1)^3.$$

4. *Foreign-Born Population.* In 1970, only 4.7% of the U.S. population was foreign-born, while in 2007, 12.6% of the population was foreign-born (*Sources*: Annual Social and Economic Supplements, Current Population Surveys, U.S. Census Bureau, U.S. Department of Commerce). The quartic function
$$f(x) = -0.0000007623221x^4 + 0.00021189064x^3 - 0.016314058x^2 + 0.2440779643x + 13.59260684,$$
where x is the number of years since 1900, can be used to estimate the percent of the U.S. population for years 1900 to 2007 that was foreign-born. Using this function, estimate the percent of the population that was foreign-born in 1930, in 1990, and in 2000.

Sketch the graph of the polynomial function.

5. $f(x) = x^3 - 5x^2 + 2x + 8$

6. $f(x) = -2x^4 + x^3 + 11x^2 - 4x - 12$

Using the intermediate value theorem, determine, if possible, whether the function has a zero between a and b.

7. $f(x) = -5x^2 + 3;\ a = 0, b = 2$

8. $g(x) = 2x^3 + 6x^2 - 3;\ a = -2, b = -1$

9. Use long division to find the quotient $Q(x)$ and the remainder $R(x)$ when $P(x)$ is divided by $d(x)$. Express $P(x)$ in the form $d(x) \cdot Q(x) + R(x)$. Show your work.
$$P(x) = x^4 + 3x^3 + 2x - 5,$$
$$d(x) = x - 1$$

10. Use synthetic division to find the quotient and the remainder. Show your work.
$$(3x^3 - 12x + 7) \div (x - 5)$$

11. Use synthetic division to find $P(-3)$ for $P(x) = 2x^3 - 6x^2 + x - 4$. Show your work.

12. Use synthetic division to determine whether -2 is a zero of $f(x) = x^3 + 4x^2 + x - 6$. Answer yes or no. Show your work.

13. Find a polynomial of degree 4 with -3 as a zero of multiplicity 2 and 0 and 6 as zeros of multiplicity 1.

14. Suppose that a polynomial function of degree 5 with rational coefficients has 1, $\sqrt{3}$, and $2 - i$ as zeros. Find the other zeros.

Find a polynomial function of lowest degree with rational coefficients and the following as some of its zeros.

15. $-10, 3i$

16. $0, -\sqrt{3}, 1 - i$

List all possible rational zeros.

17. $f(x) = 2x^3 + x^2 - 2x + 12$

18. $h(x) = 10x^4 - x^3 + 2x - 5$

For each polynomial function:

a) *Find the rational zeros and then the other zeros; that is, solve* $f(x) = 0$.
b) *Factor* $f(x)$ *into linear factors.*

19. $f(x) = x^3 + x^2 - 5x - 5$

20. $f(x) = 2x^4 - 11x^3 + 16x^2 - x - 6$

21. $f(x) = x^3 + 4x^2 + 4x + 16$

22. $f(x) = 3x^4 - 11x^3 + 15x^2 - 9x + 2$

23. What does Descartes' rule of signs tell you about the number of positive real zeros and the number of negative real zeros of the following function?

$$g(x) = -x^8 + 2x^6 - 4x^3 - 1$$

Graph the function. Be sure to label all the asymptotes. List the domain and the x- and y-intercepts.

24. $f(x) = \dfrac{2}{(x-3)^2}$

25. $f(x) = \dfrac{x+3}{x^2 - 3x - 4}$

26. Find a rational function that has vertical asymptotes $x = -1$ and $x = 2$ and x-intercept $(-4, 0)$.

Solve.

27. $2x^2 > 5x + 3$

28. $\dfrac{x+1}{x-4} \le 3$

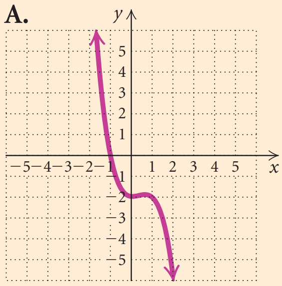
29. The function $S(t) = -16t^2 + 64t + 192$ gives the height S, in feet, of a model rocket launched with a velocity of 64 ft/sec from a hill that is 192 ft high.

a) Determine how long it will take the rocket to reach the ground.

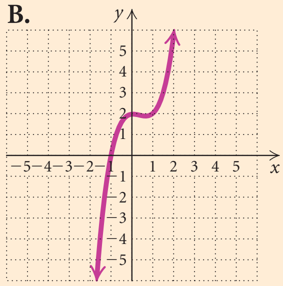
b) Find the interval on which the height of the rocket is greater than 240 ft.

30. The graph of $f(x) = x^3 - x^2 - 2$ is which of the following?

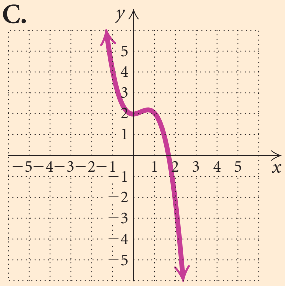
A.



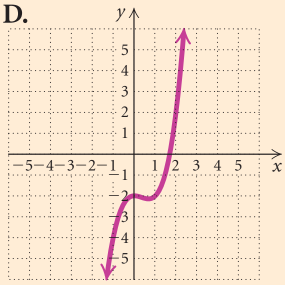
B.



C.



D.



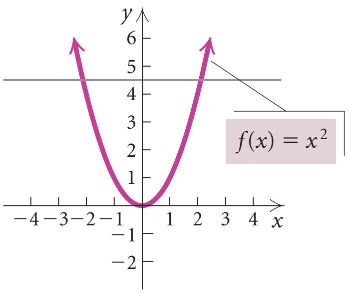
Synthesis

31. Find the domain of $f(x) = \sqrt{x^2 + x - 12}$.

$$(f \circ f^{-1})(x) = f(f^{-1}(x))$$
$$= f\left(\frac{x-8}{5}\right) = 5\left(\frac{x-8}{5}\right) + 8 = x - 8 + 8 = x.$$

Now Try Exercise 77.

Restricting a Domain

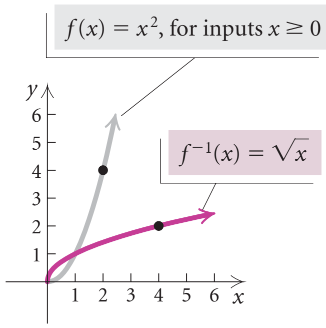


In the case in which the inverse of a function is not a function, the domain of the function can be restricted to allow the inverse to be a function. We saw in Examples 4 and 5(b) that $f(x) = x^2$ is not one-to-one. The graph is shown at left.

Suppose that we had tried to find a formula for the inverse as follows:

$$y = x^2 \qquad \text{Replacing } f(x) \text{ with } y$$
$$x = y^2 \qquad \text{Interchanging } x \text{ and } y$$
$$\pm\sqrt{x} = y. \qquad \text{Solving for } y$$

This is not the equation of a function. An input of, say, 4 would yield two outputs, -2 and 2. In such cases, it is convenient to consider "part" of the function by restricting the domain of $f(x)$. For example, if we restrict the domain of $f(x) = x^2$ to nonnegative numbers, then its inverse is a function, as shown with the graphs of $f(x) = x^2, x \geq 0$, and $f^{-1}(x) = \sqrt{x}$ below.



5.1 Exercise Set

Find the inverse of the relation.

1. $\{(7, 8), (-2, 8), (3, -4), (8, -8)\}$
2. $\{(0, 1), (5, 6), (-2, -4)\}$
3. $\{(-1, -1), (-3, 4)\}$
4. $\{(-1, 3), (2, 5), (-3, 5), (2, 0)\}$

Find an equation of the inverse relation.

5. $y = 4x - 5$
6. $2x^2 + 5y^2 = 4$
7. $x^3y = -5$
8. $y = 3x^2 - 5x + 9$

9. $x = y^2 - 2y$

10. $x = \frac{1}{2}y + 4$

Graph the equation by substituting and plotting points. Then reflect the graph across the line $y = x$ to obtain the graph of its inverse.

11. $x = y^2 - 3$

12. $y = x^2 + 1$

13. $y = 3x - 2$

14. $x = -y + 4$

15. $y = |x|$

16. $x + 2 = |y|$

Given the function f, prove that f is one-to-one using the definition of a one-to-one function on p. 390.

17. $f(x) = \frac{1}{3}x - 6$

18. $f(x) = 4 - 2x$

19. $f(x) = x^3 + \frac{1}{2}$

20. $f(x) = \sqrt[3]{x}$

Given the function g, prove that g is not one-to-one using the definition of a one-to-one function on p. 390.

21. $g(x) = 1 - x^2$

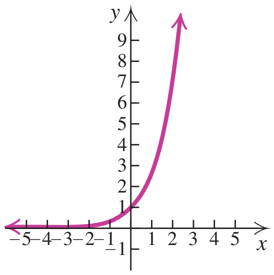
22. $g(x) = 3x^2 + 1$

23. $g(x) = x^4 - x^2$

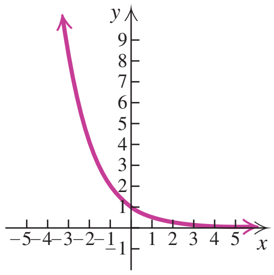
24. $g(x) = \dfrac{1}{x^6}$

Using the horizontal-line test, determine whether the function is one-to-one.

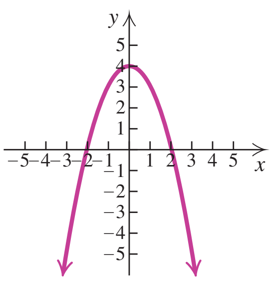
25. $f(x) = 2.7^x$



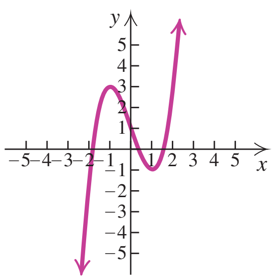
26. $f(x) = 2^{-x}$



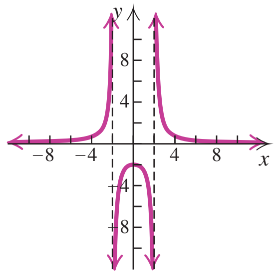
27. $f(x) = 4 - x^2$



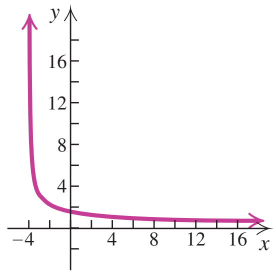
28. $f(x) = x^3 - 3x + 1$



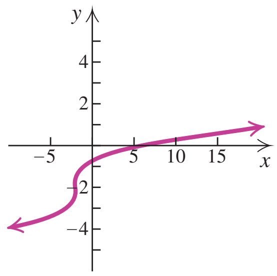
29. $f(x) = \dfrac{8}{x^2 - 4}$



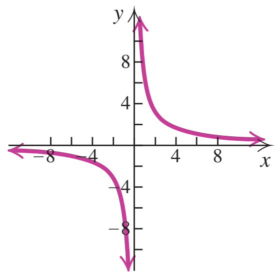
30. $f(x) = \sqrt{\dfrac{10}{4 + x}}$



31. $f(x) = \sqrt[3]{x + 2} - 2$



32. $f(x) = \dfrac{8}{x}$



Graph the function and determine whether the function is one-to-one using the horizontal-line test.

33. $f(x) = 5x - 8$

34. $f(x) = 3 + 4x$

35. $f(x) = 1 - x^2$

36. $f(x) = |x| - 2$

37. $f(x) = |x + 2|$

38. $f(x) = -0.8$

39. $f(x) = -\dfrac{4}{x}$

40. $f(x) = \dfrac{2}{x + 3}$

41. $f(x) = \frac{2}{3}$

42. $f(x) = \frac{1}{2}x^2 + 3$

43. $f(x) = \sqrt{25 - x^2}$

44. $f(x) = -x^3 + 2$

In Exercises 45–60, for each function:

a) *Determine whether it is one-to-one.*

b) *If the function is one-to-one, find a formula for the inverse.*

45. $f(x) = x + 4$

46. $f(x) = 7 - x$

47. $f(x) = 2x - 1$

48. $f(x) = 5x + 8$

49. $f(x) = \dfrac{4}{x + 7}$

50. $f(x) = -\dfrac{3}{x}$

51. $f(x) = \dfrac{x+4}{x-3}$

52. $f(x) = \dfrac{5x-3}{2x+1}$

53. $f(x) = x^3 - 1$

54. $f(x) = (x+5)^3$

55. $f(x) = x\sqrt{4-x^2}$

56. $f(x) = 2x^2 - x - 1$

57. $f(x) = 5x^2 - 2,\ x \geq 0$

58. $f(x) = 4x^2 + 3,\ x \geq 0$

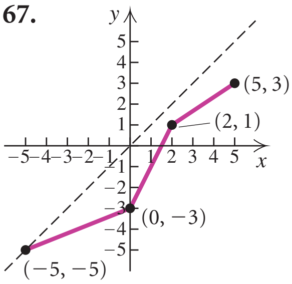
59. $f(x) = \sqrt{x+1}$

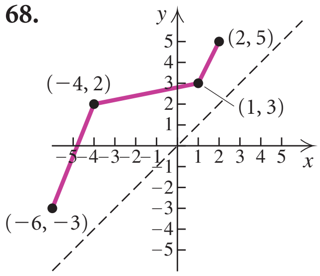
60. $f(x) = \sqrt[3]{x-8}$

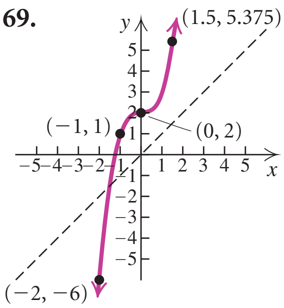
Find the inverse by thinking about the operations of the function and then reversing, or undoing, them. Check your work algebraically.

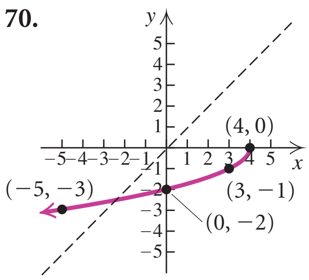
	FUNCTION	INVERSE
61.	$f(x) = 3x$	$f^{-1}(x) =$
62.	$f(x) = \frac{1}{4}x + 7$	$f^{-1}(x) =$
63.	$f(x) = -x$	$f^{-1}(x) =$
64.	$f(x) = \sqrt[3]{x} - 5$	$f^{-1}(x) =$
65.	$f(x) = \sqrt[3]{x-5}$	$f^{-1}(x) =$
66.	$f(x) = x^{-1}$	$f^{-1}(x) =$

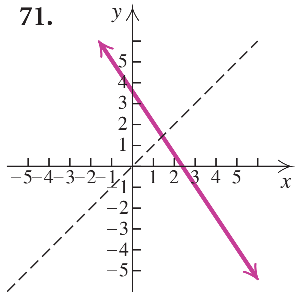
Each graph in Exercises 67–72 is the graph of a one-to-one function f. Sketch the graph of the inverse function f^{-1}.

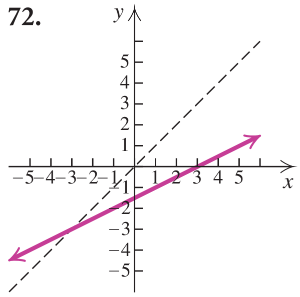
67.


68.


69.


70.


71.


72.


For the function f, use composition of functions to show that f^{-1} is as given.

73. $f(x) = \frac{7}{8}x,\ f^{-1}(x) = \frac{8}{7}x$

74. $f(x) = \dfrac{x+5}{4},\ f^{-1}(x) = 4x - 5$

75. $f(x) = \dfrac{1-x}{x},\ f^{-1}(x) = \dfrac{1}{x+1}$

76. $f(x) = \sqrt[3]{x+4},\ f^{-1}(x) = x^3 - 4$

77. $f(x) = \dfrac{2}{5}x + 1,\ f^{-1}(x) = \dfrac{5x-5}{2}$

78. $f(x) = \dfrac{x+6}{3x-4},\ f^{-1}(x) = \dfrac{4x+6}{3x-1}$

Find the inverse of the given one-to-one function f. Give the domain and the range of f and of f^{-1}, and then graph both f and f^{-1} on the same set of axes.

79. $f(x) = 5x - 3$

80. $f(x) = 2 - x$

81. $f(x) = \dfrac{2}{x}$

82. $f(x) = -\dfrac{3}{x+1}$

83. $f(x) = \dfrac{1}{3}x^3 - 2$

84. $f(x) = \sqrt[3]{x} - 1$

85. $f(x) = \dfrac{x+1}{x-3}$

86. $f(x) = \dfrac{x-1}{x+2}$

87. Find $f(f^{-1}(5))$ and $f^{-1}(f(a))$:
$$f(x) = x^3 - 4.$$

88. Find $(f^{-1}(f(p))$ and $f(f^{-1}(1253))$:
$$f(x) = \sqrt[5]{\frac{2x-7}{3x+4}}.$$

89. *Women's Shoe Sizes.* A function that will convert women's shoe sizes in the United States to those in Australia is

$$s(x) = \frac{2x - 3}{2}$$

(*Source*: OnlineConversion.com).

a) Determine the women's shoe sizes in Australia that correspond to sizes 5, $7\frac{1}{2}$, and 8 in the United States.

b) Find a formula for the inverse of the function.

c) Use the inverse function to determine the women's shoe sizes in the United States that correspond to sizes 3, $5\frac{1}{2}$, and 7 in Australia.

90. *Swimming Lessons.* A city swimming league determines that the cost per person of a group swim lesson is given by the formula

$$C(x) = \frac{60 + 2x}{x},$$

where x is the number of people in the group and $C(x)$ is in dollars. Find $C^{-1}(x)$ and explain what it represents.

91. *Spending on Pets.* The total amount of spending per year, in billions of dollars, on pets in the United States x years after 2000 is given by the function

$$P(x) = 2.1782x + 25.3$$

(*Source*: Animal Pet Products Manufacturing Association).

a) Determine the total amount of spending per year on pets in 2005 and in 2010.

b) Find $P^{-1}(x)$ and explain what it represents.

92. *Converting Temperatures.* The following formula can be used to convert Fahrenheit temperatures x to Celsius temperatures $T(x)$:

$$T(x) = \frac{5}{9}(x - 32).$$

a) Find $T(-13°)$ and $T(86°)$.

b) Find $T^{-1}(x)$ and explain what it represents.

Skill Maintenance

Consider the following quadratic functions. Without graphing them, answer the questions below.

a) $f(x) = 2x^2$
b) $f(x) = -x^2$
c) $f(x) = \frac{1}{4}x^2$
d) $f(x) = -5x^2 + 3$
e) $f(x) = \frac{2}{3}(x - 1)^2 - 3$
f) $f(x) = -2(x + 3)^2 + 1$
g) $f(x) = (x - 3)^2 + 1$
h) $f(x) = -4(x + 1)^2 - 3$

93. Which functions have a maximum value?

94. Which graphs open up?

95. Consider (a) and (c). Which graph is narrower?

96. Consider (d) and (e). Which graph is narrower?

5.4 Exercise Set

Express as a sum of logarithms.

1. $\log_3(81 \cdot 27)$
2. $\log_2(8 \cdot 64)$
3. $\log_5(5 \cdot 125)$
4. $\log_4(64 \cdot 4)$
5. $\log_t 8Y$
6. $\log 0.2x$
7. $\ln xy$
8. $\ln ab$

Express as a product.

9. $\log_b t^3$
10. $\log_a x^4$
11. $\log y^8$
12. $\ln y^5$
13. $\log_c K^{-6}$
14. $\log_b Q^{-8}$
15. $\ln \sqrt[3]{4}$
16. $\ln \sqrt{a}$

Express as a difference of logarithms.

17. $\log_t \frac{M}{8}$
18. $\log_a \frac{76}{13}$
19. $\log \frac{x}{y}$
20. $\ln \frac{a}{b}$
21. $\ln \frac{r}{s}$
22. $\log_b \frac{3}{w}$

Express in terms of sums and differences of logarithms.

23. $\log_a 6xy^5z^4$
24. $\log_a x^3y^2z$
25. $\log_b \frac{p^2q^5}{m^4b^9}$
26. $\log_b \frac{x^2y}{b^3}$
27. $\ln \frac{2}{3x^3y}$
28. $\log \frac{5a}{4b^2}$
29. $\log \sqrt{r^3t}$
30. $\ln \sqrt[3]{5x^5}$
31. $\log_a \sqrt{\frac{x^6}{p^5q^8}}$
32. $\log_c \sqrt[3]{\frac{y^3z^2}{x^4}}$
33. $\log_a \sqrt[4]{\frac{m^8n^{12}}{a^3b^5}}$
34. $\log_a \sqrt{\frac{a^6b^8}{a^2b^5}}$

Express as a single logarithm and, if possible, simplify.

35. $\log_a 75 + \log_a 2$
36. $\log 0.01 + \log 1000$
37. $\log 10{,}000 - \log 100$
38. $\ln 54 - \ln 6$
39. $\frac{1}{2}\log n + 3\log m$
40. $\frac{1}{2}\log a - \log 2$
41. $\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x$
42. $\frac{2}{5}\log_a x - \frac{1}{3}\log_a y$
43. $\ln x^2 - 2\ln\sqrt{x}$
44. $\ln 2x + 3(\ln x - \ln y)$
45. $\ln(x^2 - 4) - \ln(x + 2)$
46. $\log(x^3 - 8) - \log(x - 2)$
47. $\log(x^2 - 5x - 14) - \log(x^2 - 4)$
48. $\log_a \frac{a}{\sqrt{x}} - \log_a \sqrt{ax}$
49. $\ln x - 3[\ln(x - 5) + \ln(x + 5)]$
50. $\frac{2}{3}\left[\ln(x^2 - 9) - \ln(x + 3)\right] + \ln(x + y)$
51. $\frac{3}{2}\ln 4x^6 - \frac{4}{5}\ln 2y^{10}$
52. $120\left(\ln\sqrt[5]{x^3} + \ln\sqrt[3]{y^2} - \ln\sqrt[4]{16z^5}\right)$

Given that $\log_a 2 \approx 0.301$, $\log_a 7 \approx 0.845$, and $\log_a 11 \approx 1.041$, find each of the following, if possible. Round the answer to the nearest thousandth.

53. $\log_a \frac{2}{11}$
54. $\log_a 14$
55. $\log_a 98$
56. $\log_a \frac{1}{7}$
57. $\frac{\log_a 2}{\log_a 7}$
58. $\log_a 9$

Given that $\log_b 2 \approx 0.693$, $\log_b 3 \approx 1.099$, and $\log_b 5 \approx 1.609$, find each of the following, if possible. Round the answer to the nearest thousandth.

59. $\log_b 125$
60. $\log_b \frac{5}{3}$
61. $\log_b \frac{1}{6}$
62. $\log_b 30$
63. $\log_b \frac{3}{b}$
64. $\log_b 15b$

Simplify.

65. $\log_p p^3$
66. $\log_t t^{2713}$
67. $\log_e e^{|x-4|}$
68. $\log_q q^{\sqrt{3}}$
69. $3^{\log_3 4x}$
70. $5^{\log_5 (4x-3)}$

71. $10^{\log w}$

72. $e^{\ln x^3}$

73. $\ln e^{8t}$

74. $\log 10^{-k}$

75. $\log_b \sqrt{b}$

76. $\log_b \sqrt{b^3}$

Skill Maintenance

In each of Exercises 77–86, classify the function as linear, quadratic, cubic, quartic, rational, exponential, or logarithmic.

77. $f(x) = 5 - x^2 + x^4$

78. $f(x) = 2^x$

79. $f(x) = -\frac{3}{4}$

80. $f(x) = 4^x - 8$

81. $f(x) = -\dfrac{3}{x}$

82. $f(x) = \log x + 6$

83. $f(x) = -\frac{1}{3}x^3 - 4x^2 + 6x + 42$

84. $f(x) = \dfrac{x^2 - 1}{x^2 + x - 6}$

85. $f(x) = \frac{1}{2}x + 3$

86. $f(x) = 2x^2 - 6x + 3$

Synthesis

Solve for x.

87. $5^{\log_5 8} = 2x$

88. $\ln e^{3x-5} = -8$

Express as a single logarithm and, if possible, simplify.

89. $\log_a (x^2 + xy + y^2) + \log_a (x - y)$

90. $\log_a (a^{10} - b^{10}) - \log_a (a + b)$

Express as a sum or a difference of logarithms.

91. $\log_a \dfrac{x - y}{\sqrt{x^2 - y^2}}$

92. $\log_a \sqrt{9 - x^2}$

93. Given that $\log_a x = 2$, $\log_a y = 3$, and $\log_a z = 4$, find

$$\log_a \frac{\sqrt[4]{y^2 z^5}}{\sqrt[4]{x^3 z^{-2}}}.$$

Determine whether each of the following is true. Assume that a, x, M, and N are positive.

94. $\log_a M + \log_a N = \log_a (M + N)$

95. $\log_a M - \log_a N = \log_a \dfrac{M}{N}$

96. $\dfrac{\log_a M}{\log_a N} = \log_a M - \log_a N$

97. $\dfrac{\log_a M}{x} = \log_a M^{1/x}$

98. $\log_a x^3 = 3 \log_a x$

99. $\log_a 8x = \log_a x + \log_a 8$

100. $\log_N (MN)^x = x \log_N M + x$

Suppose that $\log_a x = 2$. Find each of the following.

101. $\log_a \left(\dfrac{1}{x}\right)$

102. $\log_{1/a} x$

103. Simplify:

$\log_{10} 11 \cdot \log_{11} 12 \cdot \log_{12} 13 \cdots \log_{998} 999 \cdot \log_{999} 1000.$

Write each of the following without using logarithms.

104. $\log_a x + \log_a y - mz = 0$

105. $\ln a - \ln b + xy = 0$

Prove each of the following for any base a and any positive number x.

106. $\log_a \left(\dfrac{1}{x}\right) = -\log_a x = \log_{1/a} x$

107. $\log_a \left(\dfrac{x + \sqrt{x^2 - 5}}{5}\right) = -\log_a \left(x - \sqrt{x^2 - 5}\right)$

EXAMPLE 9 Solve: $\ln(4x + 6) - \ln(x + 5) = \ln x$.

Algebraic Solution

We have

$$\ln(4x + 6) - \ln(x + 5) = \ln x$$

$$\ln \frac{4x + 6}{x + 5} = \ln x \qquad \text{Using the quotient rule}$$

$$\frac{4x + 6}{x + 5} = x \qquad \text{Using the property of logarithmic equality}$$

$$(x + 5) \cdot \frac{4x + 6}{x + 5} = x(x + 5) \qquad \text{Multiplying by } x + 5$$

$$4x + 6 = x^2 + 5x$$

$$0 = x^2 + x - 6$$

$$0 = (x + 3)(x - 2) \qquad \text{Factoring}$$

$$x + 3 = 0 \quad \textit{or} \quad x - 2 = 0$$

$$x = -3 \quad \textit{or} \quad x = 2.$$

The number -3 is not a solution because $4(-3) + 6 = -6$ and $\ln(-6)$ is not a real number. The value 2 checks and is the solution.

Visualizing the Solution

The solution of the equation

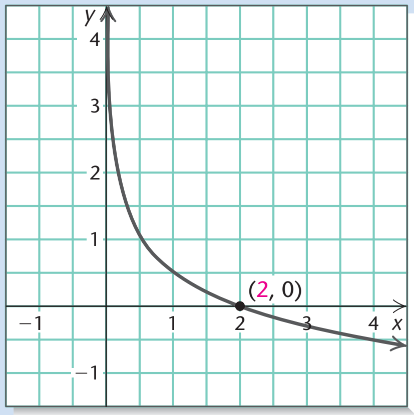
$$\ln(4x + 6) - \ln(x + 5) = \ln x$$

is the zero of the function

$$f(x) = \ln(4x + 6) - \ln(x + 5) - \ln x.$$

The solution is also the first coordinate of the x-intercept of the graph of the function.

$f(x) = \ln(4x + 6) - \ln(x + 5) - \ln x$



The solution of the equation is 2. From the graph, we can easily see that there is only one solution.

Now Try Exercise 43.

5.5 Exercise Set

Solve the exponential equation.

1. $3^x = 81$
2. $2^x = 32$
3. $2^{2x} = 8$
4. $3^{7x} = 27$
5. $2^x = 33$
6. $2^x = 40$
7. $5^{4x-7} = 125$
8. $4^{3x-5} = 16$
9. $27 = 3^{5x} \cdot 9^{x^2}$
10. $3^{x^2+4x} = \frac{1}{27}$
11. $84^x = 70$
12. $28^x = 10^{-3x}$
13. $10^{-x} = 5^{2x}$
14. $15^x = 30$
15. $e^{-c} = 5^{2c}$
16. $e^{4t} = 200$
17. $e^t = 1000$
18. $e^{-t} = 0.04$
19. $e^{-0.03t} = 0.08$
20. $1000e^{0.09t} = 5000$
21. $3^x = 2^{x-1}$
22. $5^{x+2} = 4^{1-x}$

23. $(3.9)^x = 48$

24. $250 - (1.87)^x = 0$

25. $e^x + e^{-x} = 5$

26. $e^x - 6e^{-x} = 1$

27. $3^{2x-1} = 5^x$

28. $2^{x+1} = 5^{2x}$

29. $2e^x = 5 - e^{-x}$

30. $e^x + e^{-x} = 4$

Solve the logarithmic equation.

31. $\log_5 x = 4$

32. $\log_2 x = -3$

33. $\log x = -4$

34. $\log x = 1$

35. $\ln x = 1$

36. $\ln x = -2$

37. $\log_{64} \frac{1}{4} = x$

38. $\log_{125} \frac{1}{25} = x$

39. $\log_2 (10 + 3x) = 5$

40. $\log_5 (8 - 7x) = 3$

41. $\log x + \log (x - 9) = 1$

42. $\log_2 (x + 1) + \log_2 (x - 1) = 3$

43. $\log_2 (x + 20) - \log_2 (x + 2) = \log_2 x$

44. $\log (x + 5) - \log (x - 3) = \log 2$

45. $\log_8 (x + 1) - \log_8 x = 2$

46. $\log x - \log (x + 3) = -1$

47. $\log x + \log (x + 4) = \log 12$

48. $\log_3 (x + 14) - \log_3 (x + 6) = \log_3 x$

49. $\log (x + 8) - \log (x + 1) = \log 6$

50. $\ln x - \ln (x - 4) = \ln 3$

51. $\log_4 (x + 3) + \log_4 (x - 3) = 2$

52. $\ln (x + 1) - \ln x = \ln 4$

53. $\log (2x + 1) - \log (x - 2) = 1$

54. $\log_5 (x + 4) + \log_5 (x - 4) = 2$

55. $\ln (x + 8) + \ln (x - 1) = 2 \ln x$

56. $\log_3 x + \log_3 (x + 1) = \log_3 2 + \log_3 (x + 3)$

Solve.

57. $\log_6 x = 1 - \log_6 (x - 5)$

58. $2^{x^2-9x} = \dfrac{1}{256}$

59. $9^{x-1} = 100(3^x)$

60. $2 \ln x - \ln 5 = \ln (x + 10)$

61. $e^x - 2 = -e^{-x}$

62. $2 \log 50 = 3 \log 25 + \log (x - 2)$

Skill Maintenance

In Exercises 63–66:

a) *Find the vertex.*

b) *Find the axis of symmetry.*

c) *Determine whether there is a maximum or minimum value and find that value.*

63. $g(x) = x^2 - 6$

64. $f(x) = -x^2 + 6x - 8$

65. $G(x) = -2x^2 - 4x - 7$

66. $H(x) = 3x^2 - 12x + 16$

Synthesis

Solve using any method.

67. $\dfrac{e^x + e^{-x}}{e^x - e^{-x}} = 3$

68. $\dfrac{5^x - 5^{-x}}{5^x + 5^{-x}} = 8$

69. $\ln (\log x) = 0$

70. $\ln (\ln x) = 2$

71. $\sqrt{\ln x} = \ln \sqrt{x}$

72. $\ln \sqrt[4]{x} = \sqrt{\ln x}$

73. $(\log_3 x)^2 - \log_3 x^2 = 3$

74. $\log_3 (\log_4 x) = 0$

75. $\ln x^2 = (\ln x)^2$

76. $(\log x)^2 - \log x^2 = 3$

77. $5^{2x} - 3 \cdot 5^x + 2 = 0$

78. $e^{2x} - 9 \cdot e^x + 14 = 0$

79. $\log_3 |x| = 2$

80. $x \left(\ln \frac{1}{6}\right) = \ln 6$

81. $\ln x^{\ln x} = 4$

82. $x^{\log x} = \dfrac{x^3}{100}$

83. $\dfrac{\sqrt{(e^{2x} \cdot e^{-5x})^{-4}}}{e^x \div e^{-x}} = e^7$

84. $\dfrac{(e^{3x+1})^2}{e^4} = e^{10x}$

85. $|\log_5 x| + 3 \log_5 |x| = 4$

86. $e^x < \frac{4}{5}$

87. $|2^{x^2} - 8| = 3$

88. Given that $a = \log_8 225$ and $b = \log_2 15$, express a as a function of b.

89. Given that

$$\begin{aligned}\log_2[\log_3(\log_4 x)] &= \log_3[\log_2(\log_4 y)]\\ &= \log_4[\log_3(\log_2 z)]\\ &= 0,\end{aligned}$$

find $x + y + z$.

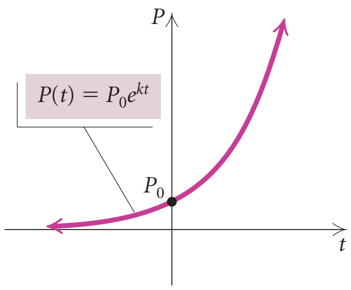
90. Given that $a = (\log_{125} 5)^{\log_5 125}$, find the value of $\log_3 a$.

91. Given that $f(x) = e^x - e^{-x}$, find $f^{-1}(x)$ if it exists.

5.6 Applications and Models: Growth and Decay; Compound Interest

- Solve applied problems involving exponential growth and decay.
- Solve applied problems involving compound interest.

Exponential functions and logarithmic functions with base e are rich in applications to many fields such as business, science, psychology, and sociology.



Population Growth

The function

$$P(t) = P_0 e^{kt}, \quad k > 0$$

is a model of many kinds of population growth, whether it be a population of people, bacteria, cell phones, or money. In this function, P_0 is the population at time 0, P is the population after time t, and k is called the **exponential growth rate**. The graph of such an equation is shown at left.

EXAMPLE 1 *Population Growth of Mexico.* In 2009, the population of Mexico was about 111.2 million, and the exponential growth rate was 1.13% per year (*Source*: *CIA World Factbook*, 2009).

a) Find the exponential growth function.
b) Estimate the population in 2014.
c) After how long will the population be double what it was in 2009?

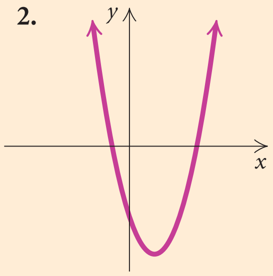
Chapter 5 Test

1. Find the inverse of the relation

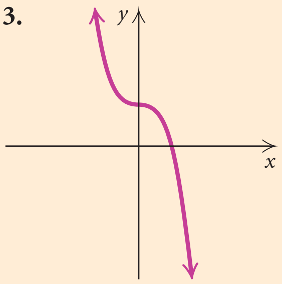
$$\{(-2, 5), (4, 3), (0, -1), (-6, -3)\}.$$

Determine whether the function is one-to-one. Answer yes or no.

2.



3.



In Exercises 4–7, given the function:

a) *Sketch the graph and determine whether the function is one-to-one.*

b) *If it is one-to-one, find a formula for the inverse.*

4. $f(x) = x^3 + 1$

5. $f(x) = 1 - x$

6. $f(x) = \dfrac{x}{2 - x}$

7. $f(x) = x^2 + x - 3$

8. Use composition of functions to show that f^{-1} is as given:

$$f(x) = -4x + 3, \qquad f^{-1}(x) = \frac{3 - x}{4}.$$

9. Find the inverse of the one-to-one function

$$f(x) = \frac{1}{x - 4}.$$

Give the domain and the range of f and of f^{-1} and then graph both f and f^{-1} on the same set of axes.

Graph the function.

10. $f(x) = 4^{-x}$

11. $f(x) = \log x$

12. $f(x) = e^x - 3$

13. $f(x) = \ln(x + 2)$

Find each of the following. Do not use a calculator.

14. $\log 0.00001$

15. $\ln e$

16. $\ln 1$

17. $\log_4 \sqrt[5]{4}$

18. Convert to an exponential equation: $\ln x = 4$.

19. Convert to a logarithmic equation: $3^x = 5.4$.

Find each of the following using a calculator. Round to four decimal places.

20. $\ln 16$

21. $\log 0.293$

22. Find $\log_6 10$ using the change-of-base formula.

23. Express as a single logarithm:

$$2 \log_a x - \log_a y + \tfrac{1}{2}\log_a z.$$

24. Express $\ln \sqrt[5]{x^2 y}$ in terms of sums and differences of logarithms.

25. Given that $\log_a 2 = 0.328$ and $\log_a 8 = 0.984$, find $\log_a 4$.

26. Simplify: $\ln e^{-4t}$.

Solve.

27. $\log_{25} 5 = x$

28. $\log_3 x + \log_3 (x + 8) = 2$

29. $3^{4-x} = 27^x$

30. $e^x = 65$

31. *Earthquake Magnitude.* The earthquake in Bam, in southeast Iran, on December 26, 2003, had an intensity of $10^{6.6} \cdot I_0$ (*Source*: U.S. Geological Survey). What was its magnitude on the Richter scale?

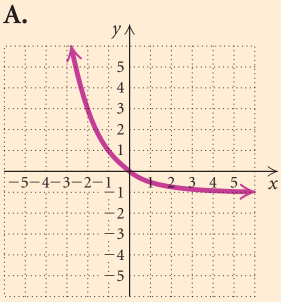
32. *Growth Rate.* A country's population doubled in 45 years. What was the exponential growth rate?

33. *Compound Interest.* Suppose $1000 is invested at interest rate k, compounded continuously, and grows to $1144.54 in 3 years.

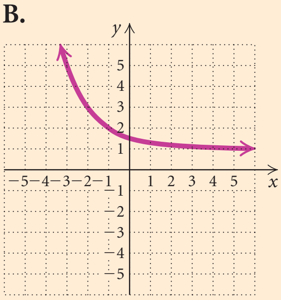
a) Find the interest rate.
b) Find the exponential growth function.
c) Find the balance after 8 years.
d) Find the doubling time.

34. The graph of $f(x) = 2^{x-1} + 1$ is which of the following?

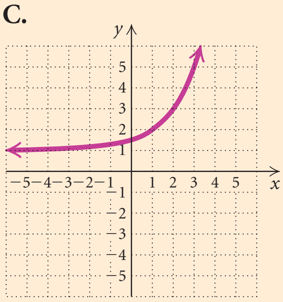
A.



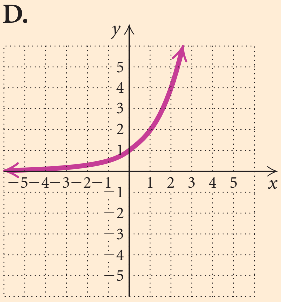
B.



C.



D.



Synthesis

35. Solve: $4^{\sqrt[3]{x}} = 8$.

7. Given that $\sin \alpha = \dfrac{\sqrt{5}}{3}$, $\cos \alpha = \dfrac{2}{3}$, and $\tan \alpha = \dfrac{\sqrt{5}}{2}$, find $\csc \alpha$, $\sec \alpha$, and $\cot \alpha$.

8. Given that $\sin \beta = \dfrac{2\sqrt{2}}{3}$, $\cos \beta = \dfrac{1}{3}$, and $\tan \beta = 2\sqrt{2}$, find $\csc \beta$, $\sec \beta$, and $\cot \beta$.

Given a function value of an acute angle, find the other five trigonometric function values.

9. $\sin \theta = \frac{24}{25}$ **10.** $\cos \sigma = 0.7$

11. $\tan \phi = 2$ **12.** $\cot \theta = \frac{1}{3}$

13. $\csc \theta = 1.5$ **14.** $\sec \beta = \sqrt{17}$

15. $\cos \beta = \dfrac{\sqrt{5}}{5}$ **16.** $\sin \sigma = \frac{10}{11}$

Find the exact function value.

17. $\cos 45°$ **18.** $\tan 30°$

19. $\sec 60°$ **20.** $\sin 45°$

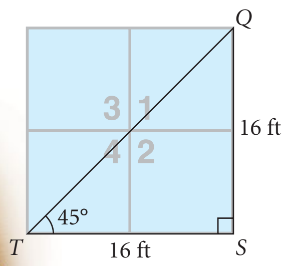
21. $\cot 60°$ **22.** $\csc 45°$

23. $\sin 30°$ **24.** $\cos 60°$

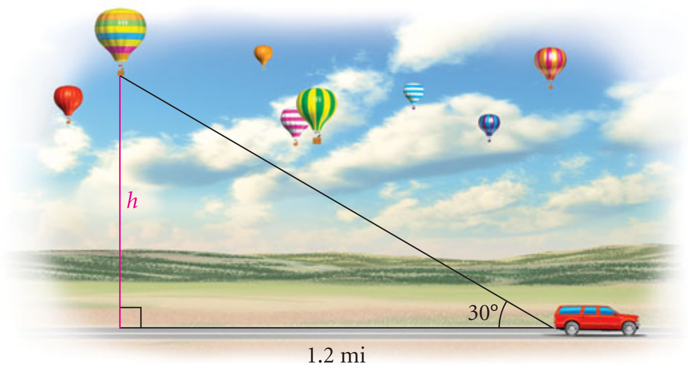
25. $\tan 45°$ **26.** $\sec 30°$

27. $\csc 30°$ **28.** $\tan 60°$

29. *Four Square.* The game Four Square is making a comeback on college campuses. The game is played on a 16-ft square court divided into four smaller squares that meet in the center (*Source*: www.squarefour.org/rules). If a line is drawn diagonally from one corner to another corner, then a right triangle QTS is formed, where $\angle QTS$ is 45°. Using a trigonometric function, find the length of the diagonal. Round the answer to the nearest tenth of a foot.



30. *Height of a Hot-Air Balloon.* As a hot-air balloon began to rise, the ground crew drove 1.2 mi to an observation station. The initial observation from the station estimated the angle between the ground and the line of sight to the balloon to be 30°. Approximately how high was the balloon at that point? (We are assuming that the wind velocity was low and that the balloon rose vertically for the first few minutes.)



Convert to decimal degree notation. Round to two decimal places.

31. $9°43'$ **32.** $52°15'$

33. $35°50''$ **34.** $64°53'$

35. $3°2'$ **36.** $19°47'23''$

37. $49°38'46''$ **38.** $76°11'34''$

39. $15'5''$ **40.** $68°2''$

41. $5°53''$ **42.** $44'10''$

Convert to degrees, minutes, and seconds. Round to the nearest second.

43. $17.6°$ **44.** $20.14°$

45. $83.025°$ **46.** $67.84°$

47. $11.75°$ **48.** $29.8°$

49. $47.8268°$ **50.** $0.253°$

51. $0.9°$ **52.** $30.2505°$

53. $39.45°$ **54.** $2.4°$

Find the function value. Round to four decimal places.

55. $\cos 51°$ **56.** $\cot 17°$

57. $\tan 4°13'$ **58.** $\sin 26.1°$

59. $\sec 38.43°$ **60.** $\cos 74°10'40''$

61. $\cos 40.35°$

62. $\csc 45.2°$

63. $\sin 69°$

64. $\tan 63°48'$

65. $\tan 85.4°$

66. $\cos 4°$

67. $\csc 89.5°$

68. $\sec 35.28°$

69. $\cot 30°25'6''$

70. $\sin 59.2°$

Find the acute angle θ, to the nearest tenth of a degree, for the given function value.

71. $\sin \theta = 0.5125$

72. $\tan \theta = 2.032$

73. $\tan \theta = 0.2226$

74. $\cos \theta = 0.3842$

75. $\sin \theta = 0.9022$

76. $\tan \theta = 3.056$

77. $\cos \theta = 0.6879$

78. $\sin \theta = 0.4005$

79. $\cot \theta = 2.127$
$\left(\textit{Hint}: \tan \theta = \dfrac{1}{\cot \theta}.\right)$

80. $\csc \theta = 1.147$

81. $\sec \theta = 1.279$

82. $\cot \theta = 1.351$

Find the exact acute angle θ for the given function value.

83. $\sin \theta = \dfrac{\sqrt{2}}{2}$

84. $\cot \theta = \dfrac{\sqrt{3}}{3}$

85. $\cos \theta = \dfrac{1}{2}$

86. $\sin \theta = \dfrac{1}{2}$

87. $\tan \theta = 1$

88. $\cos \theta = \dfrac{\sqrt{3}}{2}$

89. $\csc \theta = \dfrac{2\sqrt{3}}{3}$

90. $\tan \theta = \sqrt{3}$

91. $\cot \theta = \sqrt{3}$

92. $\sec \theta = \sqrt{2}$

Use the cofunction and reciprocal identities to complete each of the following.

93. $\cos 20° = \_\_\_\_\_\ 70° = \dfrac{1}{\_\_\_\_\_\ 20°}$

94. $\sin 64° = \_\_\_\_\_\ 26° = \dfrac{1}{\_\_\_\_\_\ 64°}$

95. $\tan 52° = \cot \_\_\_\_\_ = \dfrac{1}{\_\_\_\_\_\ 52°}$

96. $\sec 13° = \csc \_\_\_\_\_ = \dfrac{1}{\_\_\_\_\_\ 13°}$

97. Given that
$\sin 65° \approx 0.9063$, $\cos 65° \approx 0.4226$,
$\tan 65° \approx 2.1445$, $\cot 65° \approx 0.4663$,
$\sec 65° \approx 2.3662$, $\csc 65° \approx 1.1034$,
find the six function values of 25°.

98. Given that
$\sin 8° \approx 0.1392$, $\cos 8° \approx 0.9903$,
$\tan 8° \approx 0.1405$, $\cot 8° \approx 7.1154$,
$\sec 8° \approx 1.0098$, $\csc 8° \approx 7.1853$,
find the six function values of 82°.

99. Given that $\sin 71°10'5'' \approx 0.9465$, $\cos 71°10'5'' \approx 0.3228$, and $\tan 71°10'5'' \approx 2.9321$, find the six function values of $18°49'55''$.

100. Given that $\sin 38.7° \approx 0.6252$, $\cos 38.7° \approx 0.7804$, and $\tan 38.7° \approx 0.8012$, find the six function values of 51.3°.

101. Given that $\sin 82° = p$, $\cos 82° = q$, and $\tan 82° = r$, find the six function values of 8° in terms of p, q, and r.

Skill Maintenance

Graph the function.

102. $f(x) = 2^{-x}$

103. $f(x) = e^{x/2}$

104. $g(x) = \log_2 x$

105. $h(x) = \ln x$

Solve.

106. $e^t = 10{,}000$

107. $5^x = 625$

108. $\log (3x + 1) - \log (x - 1) = 2$

109. $\log_7 x = 3$

Synthesis

110. Given that $\cos \theta = 0.9651$, find $\csc (90° - \theta)$.

111. Given that $\sec \beta = 1.5304$, find $\sin (90° - \beta)$.

Mid-Chapter Mixed Review

Determine whether the statement is true or false.

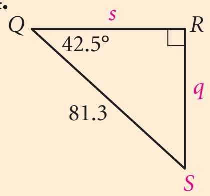
1. If $\sin \alpha > 0$ and $\cot \alpha > 0$, then α is in the first quadrant. [6.3]

2. The lengths of corresponding sides in similar triangles are in the same ratio. [6.1]

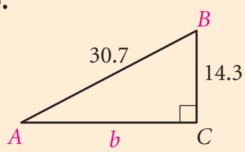
3. If θ is an acute angle and $\csc \theta \approx 1.5539$, then $\cos(90° - \theta) \approx 0.6435$. [6.1]

Solve the right triangle. [6.2]

4.



5.



Find two positive angles and two negative angles that are coterminal with the given angle. Answers may vary. [6.3]

6. $-75°$

7. $214°30'$

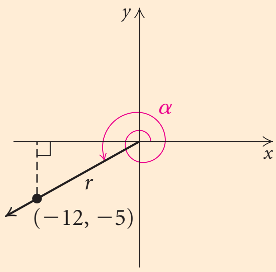
Find the complement and the supplement of the given angle. [6.3]

8. $18.2°$

9. $87°15'10''$

10. Given that $\sin 25° = 0.4226$, $\cos 25° = 0.9063$, and $\tan 25° = 0.4663$, find the six trigonometric function values for 155°. Use a calculator, but do not use the trigonometric function keys. [6.3]

11. Find the six trigonometric function values for the angle shown. [6.3]



12. Given $\cot \theta = 2$ and θ in quadrant III, find the other five function values. [6.3]

13. Given $\cos \alpha = \frac{2}{9}$ and $0° < \alpha < 90°$, find the other five trigonometric function values. [6.1]

14. Convert $42°08'50''$ to decimal degree notation. Round to four decimal places. [6.1]

15. Convert 51.18° to degrees, minutes, and seconds. [6.1]

16. Given that $\sin 9° \approx 0.1564$, $\cos 9° \approx 0.9877$, and $\tan 9° \approx 0.1584$, find the six function values of 81°. [6.1]

17. If $\tan \theta = 2.412$ and θ is acute, find the angle to the nearest tenth of a degree. [6.1]

18. *Aerial Navigation.* An airplane travels at 200 mph for $1\frac{1}{2}$ hr in a direction of 285° from Atlanta. At the end of this time, how far west of Atlanta is the plane? [6.3]

Without a calculator, find the exact function value. [6.1], [6.3]

19. $\tan 210°$ **20.** $\sin 45°$ **21.** $\cot 30°$ **22.** $\sec 135°$

23. $\cos 45°$ **24.** $\csc(-30°)$ **25.** $\sin 90°$ **26.** $\cos 270°$

27. $\sin 120°$ **28.** $\sec 180°$ **29.** $\tan(-240°)$ **30.** $\cot(-315°)$

31. $\sin 750°$ **32.** $\csc 45°$ **33.** $\cos 210°$ **34.** $\cot 0°$

35. $\csc 150°$ **36.** $\tan 90°$ **37.** $\sec 3600°$ **38.** $\cos 495°$

Find the function value. Round the answer to four decimal places. [6.1], [6.3]

39. $\cos 39.8°$ **40.** $\sec 50°$ **41.** $\tan 2183°$ **42.** $\sin 10°28'03''$

43. $\csc(-74°)$ **44.** $\cot 142.7°$ **45.** $\sin(-40.1°)$ **46.** $\cos 87°15'$

Collaborative Discussion and Writing

47. Why do the function values of θ depend only on the angle and not on the choice of a point on the terminal side? [6.3]

48. Explain the difference between reciprocal functions and cofunctions. [6.1]

49. In Section 6.1, the trigonometric functions are defined as functions of acute angles. What appear to be the ranges for the sine, cosine, and tangent functions given the restricted domain as the set of angles whose measures are greater than 0° and less than 90°? [6.1]

50. Why is the domain of the tangent function different from the domains of the sine function and the cosine function? [6.3]

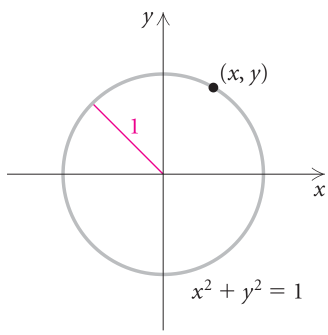
6.4 Radians, Arc Length, and Angular Speed

- Find points on the unit circle determined by real numbers.
- Convert between radian measure and degree measure; find coterminal, complementary, and supplementary angles.
- Find the length of an arc of a circle; find the measure of a central angle of a circle.
- Convert between linear speed and angular speed.

CIRCLES

REVIEW SECTION 1.1

Another useful unit of angle measure is called a *radian*. To introduce radian measure, we use a circle centered at the origin with a radius of length 1. Such a circle is called a **unit circle**. Its equation is $x^2 + y^2 = 1$.



6.4 Exercise Set

For each of Exercises 1–4, sketch a unit circle and mark the points determined by the given real numbers.

1. a) $\frac{\pi}{4}$ **b)** $\frac{3\pi}{2}$ **c)** $\frac{3\pi}{4}$

d) π **e)** $\frac{11\pi}{4}$ **f)** $\frac{17\pi}{4}$

2. a) $\frac{\pi}{2}$ **b)** $\frac{5\pi}{4}$ **c)** 2π

d) $\frac{9\pi}{4}$ **e)** $\frac{13\pi}{4}$ **f)** $\frac{23\pi}{4}$

3. a) $\frac{\pi}{6}$ **b)** $\frac{2\pi}{3}$ **c)** $\frac{7\pi}{6}$

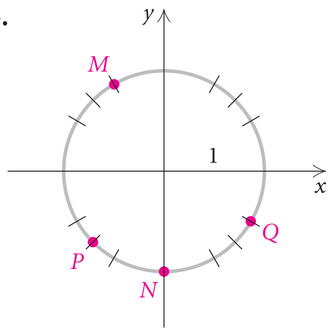
d) $\frac{10\pi}{6}$ **e)** $\frac{14\pi}{6}$ **f)** $\frac{23\pi}{4}$

4. a) $-\frac{\pi}{2}$ **b)** $-\frac{3\pi}{4}$ **c)** $-\frac{5\pi}{6}$

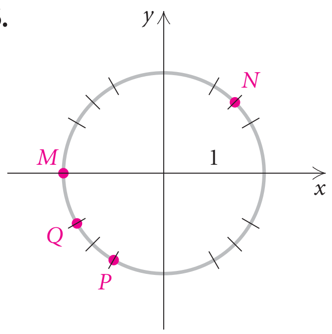
d) $-\frac{5\pi}{2}$ **e)** $-\frac{17\pi}{6}$ **f)** $-\frac{9\pi}{4}$

Find two real numbers between -2π and 2π that determine each of the points on the unit circle.

5.



6.



For Exercises 7 and 8, sketch a unit circle and mark the approximate location of the point determined by the given real number.

7. a) 2.4 **b)** 7.5
c) 32 **d)** 320

8. a) 0.25 **b)** 1.8
c) 47 **d)** 500

Find a positive angle and a negative angle that are coterminal with the given angle. Answers may vary.

9. $\frac{\pi}{4}$ **10.** $\frac{5\pi}{3}$

11. $\frac{7\pi}{6}$ **12.** π

13. $-\frac{2\pi}{3}$ **14.** $-\frac{3\pi}{4}$

Find the complement and the supplement.

15. $\frac{\pi}{3}$ **16.** $\frac{5\pi}{12}$ **17.** $\frac{3\pi}{8}$

18. $\frac{\pi}{4}$ **19.** $\frac{\pi}{12}$ **20.** $\frac{\pi}{6}$

Convert to radian measure. Leave the answer in terms of π.

21. 75° **22.** 30°

23. 200° **24.** $-135°$

25. $-214.6°$ **26.** 37.71°

27. $-180°$ **28.** 90°

29. 12.5° **30.** 6.3°

31. $-340°$ **32.** $-60°$

Convert to radian measure. Round the answer to two decimal places.

33. 240° **34.** 15°

35. $-60°$ **36.** 145°

37. 117.8° **38.** $-231.2°$

39. 1.354° **40.** 584°

41. 345° **42.** −75°

43. 95° **44.** 24.8°

Convert to degree measure. Round the answer to two decimal places.

45. $-\frac{3\pi}{4}$ **46.** $\frac{7\pi}{6}$

47. 8π **48.** $-\frac{\pi}{3}$

49. 1 **50.** −17.6

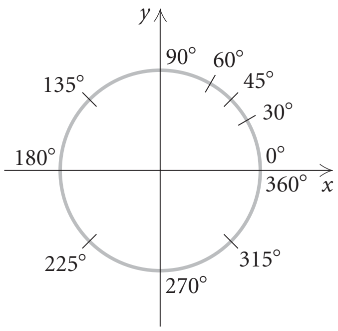
51. 2.347 **52.** 25

53. $\frac{5\pi}{4}$ **54.** -6π

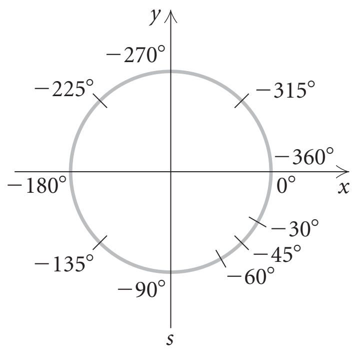
55. −90 **56.** 37.12

57. $\frac{2\pi}{7}$ **58.** $\frac{\pi}{9}$

59. Certain positive angles are marked here in degrees. Find the corresponding radian measures.



60. Certain negative angles are marked here in degrees. Find the corresponding radian measures.



Arc Length and Central Angles. *Complete the table below. Round the answers to two decimal places.*

	Distance, *s* (arc length)	Radius, *r*	Angle, θ
61.	8 ft	$3\frac{1}{2}$ ft	______
62.	200 cm	______	45°
63.	______	4.2 in.	$\frac{5\pi}{12}$
64.	16 yd	______	5

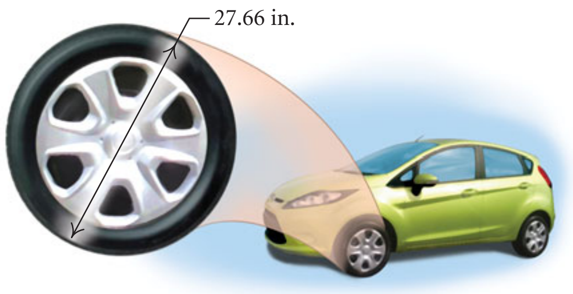
65. In a circle with a 120-cm radius, an arc 132 cm long subtends an angle of how many radians? how many degrees, to the nearest degree?

66. In a circle with a 10-ft diameter, an arc 20 ft long subtends an angle of how many radians? how many degrees, to the nearest degree?

67. In a circle with a 2-yd radius, how long is an arc associated with an angle of 1.6 radians?

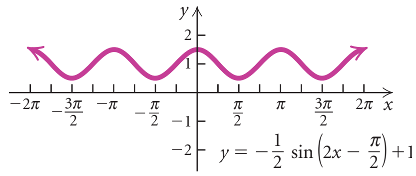
68. In a circle with a 5-m radius, how long is an arc associated with an angle of 2.1 radians?

69. *Angle of Revolution.* A tire on a 2011 Ford Fiesta has an outside diameter of 27.66 in. Through what angle (in radians) does the tire turn while traveling 1 mi?



70. *Angle of Revolution.* Through how many radians does the minute hand of a wristwatch rotate from 12:40 P.M. to 1:30 P.M.?

2. Stretch or shrink the graph vertically according to A. Reflect across the x-axis if $A < 0$. (Amplitude $= |A|$)
3. Translate the graph horizontally according to C/B. $\left(\text{Phase shift} = \dfrac{C}{B}\right)$
4. Translate the graph vertically according to D.



REVIEW EXERCISES

Determine whether the statement is true or false.

1. Given that $(-a, b)$ is a point on the unit circle and θ is in the second quadrant, then $\cos\theta$ is a. [6.4]

2. Given that $(-c, -d)$ is a point on the unit circle and θ is in the second quadrant, then $\tan\theta = -\dfrac{c}{d}$. [6.4]

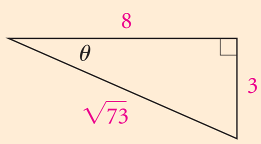
3. The measure 300° is greater than the measure 5 radians. [6.4]

4. If $\sec\theta > 0$ and $\cot\theta < 0$, then θ is in the fourth quadrant. [6.3]

5. The amplitude of $y = \frac{1}{2}\sin x$ is twice as large as the amplitude of $y = \sin\frac{1}{2}x$. [6.6]

6. The supplement of $\dfrac{9}{13}\pi$ is greater than the complement of $\dfrac{\pi}{6}$. [6.4]

7. Find the six trigonometric function values of θ. [6.1]



8. Given that β is acute and $\beta = \dfrac{\sqrt{91}}{10}$, find the other five trigonometric function values. [6.1]

Find the exact function value, if it exists.

9. $\cos 45°$ [6.1]
10. $\cot 60°$ [6.1]
11. $\cos 495°$ [6.3]
12. $\sin 150°$ [6.3]
13. $\sec(-270°)$ [6.3]
14. $\tan(-600°)$ [6.3]
15. $\csc 60°$ [6.1]
16. $\cot(-45°)$ [6.3]

17. Convert 22.27° to degrees, minutes, and seconds. Round the answer to the nearest second. [6.1]

18. Convert $47°33'27''$ to decimal degree notation. Round the answer to two decimal places. [6.1]

Find the function value. Round the answer to four decimal places. [6.3]

19. $\tan 2184°$
20. $\sec 27.9°$
21. $\cos 18°13'42''$
22. $\sin 245°24'$
23. $\cot(-33.2°)$
24. $\sin 556.13°$

Find θ in the interval indicated. Round the answer to the nearest tenth of a degree. [6.3]

25. $\cos\theta = -0.9041,\ (180°, 270°)$

26. $\tan\theta = 1.0799,\ (0°, 90°)$

Find the exact acute angle θ, in degrees, given the function value. [6.1]

27. $\sin\theta = \dfrac{\sqrt{3}}{2}$
28. $\tan\theta = \sqrt{3}$
29. $\cos\theta = \dfrac{\sqrt{2}}{2}$
30. $\sec\theta = \dfrac{2\sqrt{3}}{3}$

31. Given that $\sin 59.1° \approx 0.8581$, $\cos 59.1° \approx 0.5135$, and $\tan 59.1° \approx 1.6709$, find the six function values for 30.9°. [6.1]

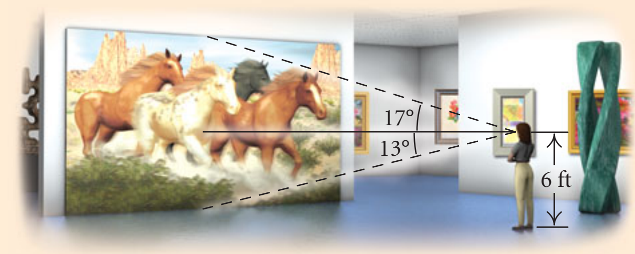
Solve each of the following right triangles. Standard lettering has been used. [6.2]

32. $a = 7.3$, $c = 8.6$

33. $a = 30.5$, $B = 51.17°$

34. One leg of a right triangle bears east. The hypotenuse is 734 m long and bears N57°23′E. Find the perimeter of the triangle.

35. An observer's eye is 6 ft above the floor. A mural is being viewed. The bottom of the mural is at floor level. The observer looks down 13° to see the bottom and up 17° to see the top. How tall is the mural?



For angles of the following measures, state in which quadrant the terminal side lies. [6.3]

36. 142°11′5″ **37.** $-635.2°$ **38.** $-392°$

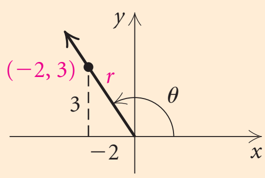
Find a positive angle and a negative angle that are coterminal with the given angle. Answers may vary.

39. 65° [6.3] **40.** $\dfrac{7\pi}{3}$ [6.4]

Find the complement and the supplement.

41. 13.4° [6.3] **42.** $\dfrac{\pi}{6}$ [6.4]

43. Find the six trigonometric function values for the angle θ shown. [6.3]



44. Given that $\tan \theta = 2/\sqrt{5}$ and that the terminal side is in quadrant III, find the other five function values. [6.3]

45. An airplane travels at 530 mph for $3\frac{1}{2}$ hr in a direction of 160° from Minneapolis, Minnesota. At the end of that time, how far south of Minneapolis is the airplane? [6.3]

46. On a unit circle, mark and label the points determined by $7\pi/6$, $-3\pi/4$, $-\pi/3$, and $9\pi/4$. [6.4]

For angles of the following measures, convert to radian measure in terms of π, and convert to radian measure not in terms of π. Round the answer to two decimal places. [6.4]

47. 145.2° **48.** $-30°$

Convert to degree measure. Round the answer to two decimal places. [6.4]

49. $\dfrac{3\pi}{2}$ **50.** 3

51. -4.5 **52.** 11π

53. Find the length of an arc of a circle, given a central angle of $\pi/4$ and a radius of 7 cm. [6.4]

54. An arc 18 m long on a circle of radius 8 m subtends an angle of how many radians? how many degrees, to the nearest degree? [6.4]

55. A waterwheel in a watermill has a radius of 7 ft and makes a complete revolution in 70 sec. What is the linear speed, in feet per minute, of a point on the rim? [6.4]

56. An automobile wheel has a diameter of 14 in. If the car travels at a speed of 55 mph, what is the angular velocity, in radians per hour, of a point on the edge of the wheel? [6.4]

57. The point $\left(\frac{3}{5}, -\frac{4}{5}\right)$ is on a unit circle. Find the coordinates of its reflections across the x-axis, the y-axis, and the origin. [6.5]

Find the exact function value, if it exists. [6.5]

58. $\cos \pi$

59. $\tan \dfrac{5\pi}{4}$

60. $\sin \dfrac{5\pi}{3}$

61. $\sin\left(-\dfrac{7\pi}{6}\right)$

62. $\tan \dfrac{\pi}{6}$

63. $\cos(-13\pi)$

Find the function value. Round the answer to four decimal places. [6.5]

64. $\sin 24$

65. $\cos(-75)$

66. $\cot 16\pi$

67. $\tan \dfrac{3\pi}{7}$

68. $\sec 14.3$

69. $\cos\left(-\dfrac{\pi}{5}\right)$

70. Graph each of the six trigonometric functions from -2π to 2π. [6.5]

71. What is the period of each of the six trigonometric functions? [6.5]

72. Complete the following table. [6.5]

Function	Domain	Range
sine		
cosine		
tangent		

73. Complete the following table with the sign of the specified trigonometric function value in each of the four quadrants. [6.3]

Function	I	II	III	IV
sine				
cosine				
tangent				

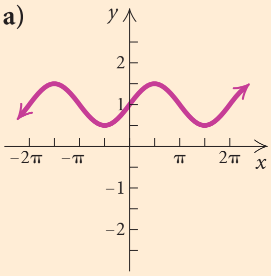
Determine the amplitude, the period, and the phase shift of the function, and sketch the graph of the function. [6.6]

74. $y = \sin\left(x + \dfrac{\pi}{2}\right)$

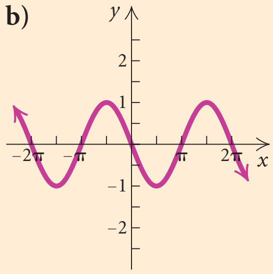
75. $y = 3 + \dfrac{1}{2}\cos\left(2x - \dfrac{\pi}{2}\right)$

In Exercises 76–79, match the function with one of the graphs (a)–(d), which follow. [6.6]

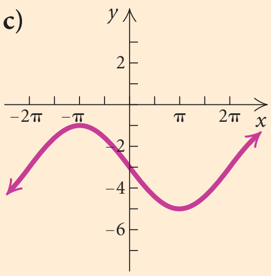
a)



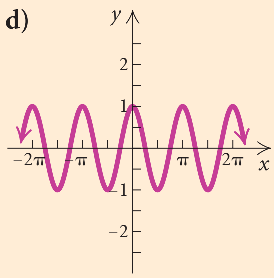
b)



c)



d)



76. $y = \cos 2x$

77. $y = \dfrac{1}{2}\sin x + 1$

78. $y = -2\sin\dfrac{1}{2}x - 3$

79. $y = -\cos\left(x - \dfrac{\pi}{2}\right)$

80. Sketch a graph of $y = 3\cos x + \sin x$ for values of x between 0 and 2π. [6.6]

81. Graph: $f(x) = e^{-0.7x}\cos x$. [6.6]

82. Which of the following is the reflection of $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$ across the y-axis? [6.5]

A. $\left(\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}\right)$

B. $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

C. $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

D. $\left(\dfrac{\sqrt{3}}{2}, -\dfrac{1}{2}\right)$

83. Which of the following is the domain of the cosine function? [6.5]

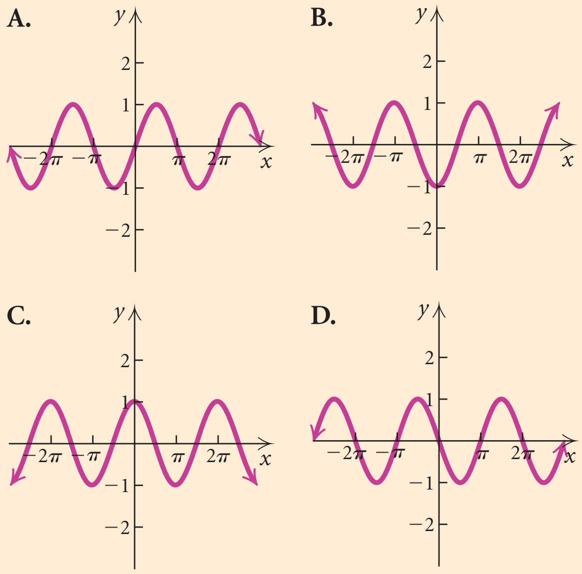
A. $(-1, 1)$

B. $(-\infty, \infty)$

C. $[0, \infty)$

D. $[-1, 1]$

84. The graph of $f(x) = -\cos(-x)$ is which of the following? [6.6]

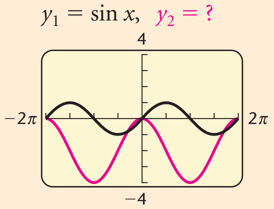


Synthesis

85. Graph $y = 3 \sin(x/2)$, and determine the domain, the range, and the period. [6.6]

86. In the graph below, $y_1 = \sin x$ is shown and y_2 is shown in red. Express y_2 as a transformation of the graph of y_1. [6.6]

$y_1 = \sin x,\ y_2 = ?$



87. Find the domain of $y = \log(\cos x)$. [6.6]

88. Given that $\sin x = 0.6144$ and that the terminal side is in quadrant II, find the other basic circular function values. [6.3]

Collaborative Discussion and Writing

89. Compare the terms radian and degree. [6.1], [6.4]

90. In circular motion with a fixed angular speed, the length of the radius is directly proportional to the linear speed. Explain why with an example. [6.4]

91. Explain why both the sine function and the cosine function are continuous, but the tangent function, defined as sine/cosine, is not continuous. [6.5]

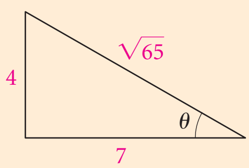
92. In the transformation steps listed in Section 6.6, why must step (1) precede step (3)? Give an example that illustrates this. [6.6]

93. In the equations $y = A \sin(Bx - C) + D$ and $y = A \cos(Bx - C) + D$, which constants translate the graphs and which constants stretch and shrink the graphs? Describe in your own words the effect of each constant. [6.6]

94. Two new cars are each driven at an average speed of 60 mph for an extended highway test drive of 2000 mi. The diameters of the wheels of the two cars are 15 in. and 16 in., respectively. If the cars use tires of equal durability and profile, differing only by the diameter, which car will probably need new tires first? Explain you answer. [6.4]

Chapter 6 Test

1. Find the six trigonometric function values of θ.



Find the exact function value, if it exists.

2. $\sin 120°$

3. $\tan(-45°)$

4. $\cos 3\pi$

5. $\sec \frac{5\pi}{4}$

6. Convert $38°27'56''$ to decimal degree notation. Round the answer to two decimal places.

Find the function values. Round the answers to four decimal places.

7. $\tan 526.4°$

8. $\sin(-12°)$

9. $\sec \dfrac{5\pi}{9}$

10. $\cos 76.07$

11. Find the exact acute angle θ, in degrees, for which $\sin\theta = \frac{1}{2}$.

12. Given that $\sin 28.4° \approx 0.4756$, $\cos 28.4° \approx 0.8796$, and $\tan 28.4° \approx 0.5407$, find the six trigonometric function values for $61.6°$.

13. Solve the right triangle with $b = 45.1$ and $A = 35.9°$. Standard lettering has been used.

14. Find a positive angle and a negative angle coterminal with a $112°$ angle.

15. Find the supplement of $\dfrac{5\pi}{6}$.

16. Given that $\sin\theta = -4/\sqrt{41}$ and that the terminal side is in quadrant IV, find the other five trigonometric function values.

17. Convert $210°$ to radian measure in terms of π.

18. Convert $\dfrac{3\pi}{4}$ to degree measure.

19. Find the length of an arc of a circle given a central angle of $\pi/3$ and a radius of 16 cm.

Consider the function $y = -\sin(x - \pi/2) + 1$ for Exercises 20–23.

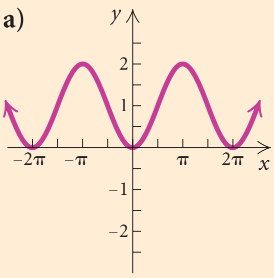
20. Find the amplitude.

21. Find the period.

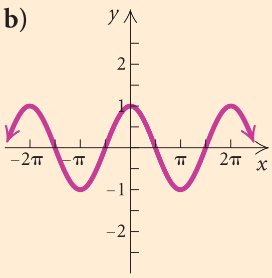
22. Find the phase shift.

23. Which is the graph of the function?

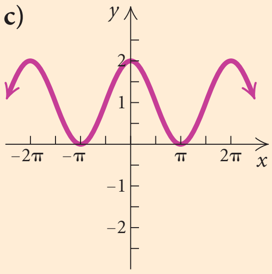
a)



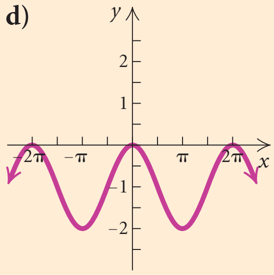
b)



c)



d)



24. *Angle of Elevation.* The longest escalator in the world is in the subway system in St. Petersburg, Russia. The escalator is 330.7 m long and rises a vertical distance of 59.7 m. What is its angle of elevation?

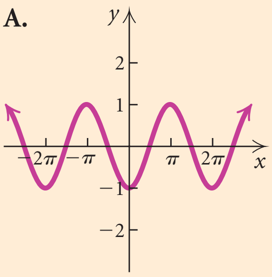
25. *Location.* A pickup-truck camper travels at 50 mph for 6 hr in a direction of 115° from Buffalo, Wyoming. At the end of that time, how far east of Buffalo is the camper?

26. *Linear Speed.* A ferris wheel has a radius of 6 m and revolves at 1.5 rpm. What is the linear speed, in meters per minute?

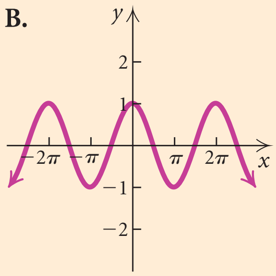
27. Graph: $f(x) = \frac{1}{2}x^2 \sin x$.

28. The graph of $f(x) = -\sin(-x)$ is which of the following?

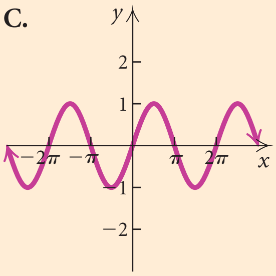
A.



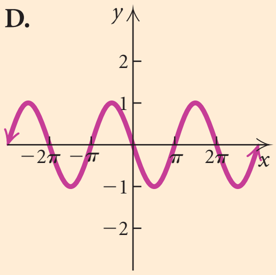
B.



C.



D.



Synthesis

29. Determine the domain of $f(x) = \dfrac{-3}{\sqrt{\cos x}}$.

7.1 Exercise Set

Multiply and simplify.

1. $(\sin x - \cos x)(\sin x + \cos x)$
2. $\tan x (\cos x - \csc x)$
3. $\cos y \sin y (\sec y + \csc y)$
4. $(\sin x + \cos x)(\sec x + \csc x)$
5. $(\sin \phi - \cos \phi)^2$
6. $(1 + \tan x)^2$
7. $(\sin x + \csc x)(\sin^2 x + \csc^2 x - 1)$
8. $(1 - \sin t)(1 + \sin t)$

Factor and simplify.

9. $\sin x \cos x + \cos^2 x$
10. $\tan^2 \theta - \cot^2 \theta$
11. $\sin^4 x - \cos^4 x$
12. $4 \sin^2 y + 8 \sin y + 4$
13. $2 \cos^2 x + \cos x - 3$
14. $3 \cot^2 \beta + 6 \cot \beta + 3$
15. $\sin^3 x + 27$
16. $1 - 125 \tan^3 s$

Simplify.

17. $\dfrac{\sin^2 x \cos x}{\cos^2 x \sin x}$
18. $\dfrac{30 \sin^3 x \cos x}{6 \cos^2 x \sin x}$
19. $\dfrac{\sin^2 x + 2 \sin x + 1}{\sin x + 1}$
20. $\dfrac{\cos^2 \alpha - 1}{\cos \alpha + 1}$
21. $\dfrac{4 \tan t \sec t + 2 \sec t}{6 \tan t \sec t + 2 \sec t}$
22. $\dfrac{\csc(-x)}{\cot(-x)}$
23. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x}$
24. $\dfrac{4 \cos^3 x}{\sin^2 x} \cdot \left(\dfrac{\sin x}{4 \cos x}\right)^2$
25. $\dfrac{5 \cos \phi}{\sin^2 \phi} \cdot \dfrac{\sin^2 \phi - \sin \phi \cos \phi}{\sin^2 \phi - \cos^2 \phi}$
26. $\dfrac{\tan^2 y}{\sec y} \div \dfrac{3 \tan^3 y}{\sec y}$
27. $\dfrac{1}{\sin^2 s - \cos^2 s} - \dfrac{2}{\cos s - \sin s}$
28. $\left(\dfrac{\sin x}{\cos x}\right)^2 - \dfrac{1}{\cos^2 x}$
29. $\dfrac{\sin^2 \theta - 9}{2 \cos \theta + 1} \cdot \dfrac{10 \cos \theta + 5}{3 \sin \theta + 9}$
30. $\dfrac{9 \cos^2 \alpha - 25}{2 \cos \alpha - 2} \cdot \dfrac{\cos^2 \alpha - 1}{6 \cos \alpha - 10}$

Simplify. Assume that all radicands are nonnegative.

31. $\sqrt{\sin^2 x \cos x} \cdot \sqrt{\cos x}$
32. $\sqrt{\cos^2 x \sin x} \cdot \sqrt{\sin x}$
33. $\sqrt{\cos \alpha \sin^2 \alpha} - \sqrt{\cos^3 \alpha}$
34. $\sqrt{\tan^2 x - 2 \tan x \sin x + \sin^2 x}$
35. $\left(1 - \sqrt{\sin y}\right)\left(\sqrt{\sin y} + 1\right)$
36. $\sqrt{\cos \theta}\left(\sqrt{2 \cos \theta} + \sqrt{\sin \theta \cos \theta}\right)$

Rationalize the denominator.

37. $\sqrt{\dfrac{\sin x}{\cos x}}$
38. $\sqrt{\dfrac{\cos x}{\tan x}}$
39. $\sqrt{\dfrac{\cos^2 y}{2 \sin^2 y}}$
40. $\sqrt{\dfrac{1 - \cos \beta}{1 + \cos \beta}}$

Rationalize the numerator.

41. $\sqrt{\dfrac{\cos x}{\sin x}}$
42. $\sqrt{\dfrac{\sin x}{\cot x}}$
43. $\sqrt{\dfrac{1 + \sin y}{1 - \sin y}}$
44. $\sqrt{\dfrac{\cos^2 x}{2 \sin^2 x}}$

Use the given substitution to express the given radical expression as a trigonometric function without radicals. Assume that $a > 0$ and $0 < \theta < \pi/2$. Then find expressions for the indicated trigonometric functions.

45. Let $x = a \sin \theta$ in $\sqrt{a^2 - x^2}$. Then find $\cos \theta$ and $\tan \theta$.

46. Let $x = 2 \tan \theta$ in $\sqrt{4 + x^2}$. Then find $\sin \theta$ and $\cos \theta$.

47. Let $x = 3 \sec \theta$ in $\sqrt{x^2 - 9}$. Then find $\sin \theta$ and $\cos \theta$.

48. Let $x = a \sec \theta$ in $\sqrt{x^2 - a^2}$. Then find $\sin \theta$ and $\cos \theta$.

Use the given substitution to express the given radical expression as a trigonometric function without radicals. Assume that $0 < \theta < \pi/2$.

49. Let $x = \sin \theta$ in $\dfrac{x^2}{\sqrt{1 - x^2}}$.

50. Let $x = 4 \sec \theta$ in $\dfrac{\sqrt{x^2 - 16}}{x^2}$.

Use the sum and difference identities to evaluate exactly.

51. $\sin \dfrac{\pi}{12}$

52. $\cos 75°$

53. $\tan 105°$

54. $\tan \dfrac{5\pi}{12}$

55. $\cos 15°$

56. $\sin \dfrac{7\pi}{12}$

First write each of the following as a trigonometric function of a single angle. Then evaluate.

57. $\sin 37° \cos 22° + \cos 37° \sin 22°$

58. $\cos 83° \cos 53° + \sin 83° \sin 53°$

59. $\cos 19° \cos 5° - \sin 19° \sin 5°$

60. $\sin 40° \cos 15° - \cos 40° \sin 15°$

61. $\dfrac{\tan 20° + \tan 32°}{1 - \tan 20° \tan 32°}$

62. $\dfrac{\tan 35° - \tan 12°}{1 + \tan 35° \tan 12°}$

63. Derive the formula for the tangent of a sum.

64. Derive the formula for the tangent of a difference.

Assuming that $\sin u = \frac{3}{5}$ and $\sin v = \frac{4}{5}$ and that u and v are between 0 and $\pi/2$, evaluate each of the following exactly.

65. $\cos (u + v)$

66. $\tan (u - v)$

67. $\sin (u - v)$

68. $\cos (u - v)$

Assuming that $\sin \theta = 0.6249$ and $\cos \phi = 0.1102$ and that both θ and ϕ are first-quadrant angles, evaluate each of the following.

69. $\tan (\theta + \phi)$

70. $\sin (\theta - \phi)$

71. $\cos (\theta - \phi)$

72. $\cos (\theta + \phi)$

Simplify.

73. $\sin (\alpha + \beta) + \sin (\alpha - \beta)$

74. $\cos (\alpha + \beta) - \cos (\alpha - \beta)$

75. $\cos (u + v) \cos v + \sin (u + v) \sin v$

76. $\sin (u - v) \cos v + \cos (u - v) \sin v$

Skill Maintenance

Solve.

77. $2x - 3 = 2\left(x - \frac{3}{2}\right)$

78. $x - 7 = x + 3.4$

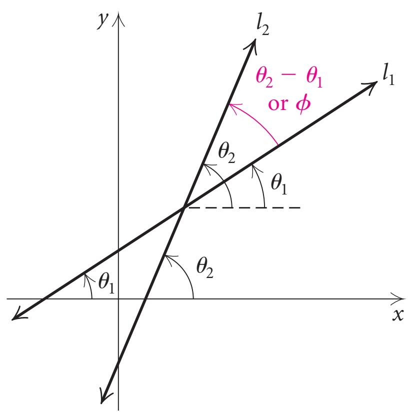
Given that $\sin 31° = 0.5150$ and $\cos 31° = 0.8572$, find the specified function value.

79. $\sec 59°$

80. $\tan 59°$

Synthesis

Angles Between Lines. *One of the identities gives an easy way to find an angle formed by two lines. Consider two lines with equations l_1: $y = m_1x + b_1$ and l_2: $y = m_2x + b_2$.*



The slopes m_1 and m_2 are the tangents of the angles θ_1 and θ_2 that the lines form with the positive direction of the x-axis. Thus we have $m_1 = \tan\theta_1$ and $m_2 = \tan\theta_2$. To find the measure of $\theta_2 - \theta_1$, or ϕ, we proceed as follows:

$$\tan\phi = \tan(\theta_2 - \theta_1) = \frac{\tan\theta_2 - \tan\theta_1}{1 + \tan\theta_2\tan\theta_1} = \frac{m_2 - m_1}{1 + m_2 m_1}.$$

This formula also holds when the lines are taken in the reverse order. When ϕ is acute, $\tan\phi$ will be positive. When ϕ is obtuse, $\tan\phi$ will be negative.

Find the measure of the angle from l_1 to l_2.

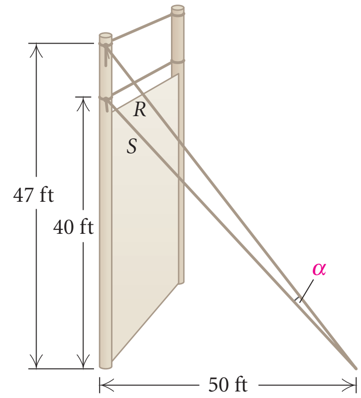
81. l_1: $2x = 3 - 2y$,
l_2: $x + y = 5$

82. l_1: $3y = \sqrt{3}x + 3$,
l_2: $y = \sqrt{3}x + 2$

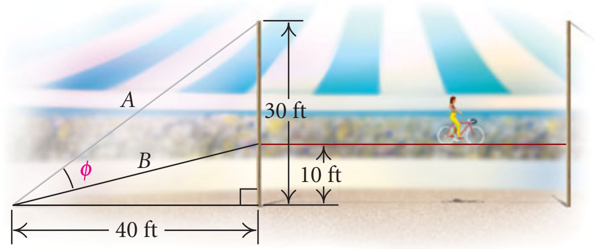
83. l_1: $y = 3$,
l_2: $x + y = 5$

84. l_1: $2x + y - 4 = 0$,
l_2: $y - 2x + 5 = 0$

85. *Rope Course and Climbing Wall.* For a rope course and climbing wall, a guy wire R is attached 47 ft high on a vertical pole. Another guy wire S is attached 40 ft above the ground on the same pole. (*Source*: Experiential Resources, Inc., Todd Domeck, Owner) Find the angle α between the wires if they are attached to the ground 50 ft from the pole.



86. *Circus Guy Wire.* In a circus, a guy wire A is attached to the top of a 30-ft pole. Wire B is used for performers to walk up to the tight wire, 10 ft above the ground. Find the angle ϕ between the wires if they are attached to the ground 40 ft from the pole.



87. Given that $f(x) = \cos x$, show that

$$\frac{f(x+h) - f(x)}{h} = \cos x\left(\frac{\cos h - 1}{h}\right) - \sin x\left(\frac{\sin h}{h}\right).$$

88. Given that $f(x) = \sin x$, show that

$$\frac{f(x+h) - f(x)}{h} = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right).$$

Show that each of the following is not an identity by finding a replacement or replacements for which the sides of the equation do not name the same number.

89. $\dfrac{\sin 5x}{x} = \sin 5$

90. $\sqrt{\sin^2\theta} = \sin\theta$

91. $\cos(2\alpha) = 2\cos\alpha$

92. $\sin(-x) = \sin x$

93. $\dfrac{\cos 6x}{\cos x} = 6$

94. $\tan^2\theta + \cot^2\theta = 1$

Find the slope of line l_1, where m_2 is the slope of line l_2 and ϕ is the smallest positive angle from l_1 to l_2.

95. $m_2 = \frac{2}{3}$, $\phi = 30°$

96. $m_2 = \frac{4}{3}$, $\phi = 45°$

97. Line l_1 contains the points $(-3, 7)$ and $(-3, -2)$. Line l_2 contains $(0, -4)$ and $(2, 6)$. Find the smallest positive angle from l_1 to l_2.

98. Line l_1 contains the points $(-2, 4)$ and $(5, -1)$. Find the slope of line l_2 such that the angle from l_1 to l_2 is 45°.

99. Find an identity for $\cos 2\theta$. (*Hint*: $2\theta = \theta + \theta$.)

100. Find an identity for $\sin 2\theta$. (*Hint*: $2\theta = \theta + \theta$.)

Derive the identity.

101. $\tan\left(x + \frac{\pi}{4}\right) = \frac{1 + \tan x}{1 - \tan x}$

102. $\sin\left(x - \frac{3\pi}{2}\right) = \cos x$

103. $\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin\alpha \cos\beta$

104. $\frac{\sin(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{\tan\alpha + \tan\beta}{1 + \tan\alpha \tan\beta}$

7.2 Identities: Cofunction, Double-Angle, and Half-Angle

- Use cofunction identities to derive other identities.
- Use the double-angle identities to find function values of twice an angle when one function value is known for that angle.
- Use the half-angle identities to find function values of half an angle when one function value is known for that angle.
- Simplify trigonometric expressions using the double-angle identities and the half-angle identities.

Cofunction Identities

Each of the identities listed below yields a conversion to a *cofunction*. For this reason, we call them cofunction identities.

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x, \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x,$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x, \qquad \cot\left(\frac{\pi}{2} - x\right) = \tan x,$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x, \qquad \csc\left(\frac{\pi}{2} - x\right) = \sec x$$

We verified the first two of these identities in Section 7.1. The other four can be proved using the first two and the definitions of the trigonometric functions. These identities hold for all real numbers, and thus, for all angle

7.2 Exercise Set

1. Given that $\sin(3\pi/10) \approx 0.8090$ and $\cos(3\pi/10) \approx 0.5878$, find each of the following.

a) The other four function values for $3\pi/10$

b) The six function values for $\pi/5$

2. Given that

$$\sin\frac{\pi}{12} = \frac{\sqrt{2-\sqrt{3}}}{2} \text{ and } \cos\frac{\pi}{12} = \frac{\sqrt{2+\sqrt{3}}}{2},$$

find exact answers for each of the following.

a) The other four function values for $\pi/12$

b) The six function values for $5\pi/12$

3. Given that $\sin\theta = \frac{1}{3}$ and that the terminal side is in quadrant II, find exact answers for each of the following.

a) The other function values for θ

b) The six function values for $\pi/2 - \theta$

c) The six function values for $\theta - \pi/2$

4. Given that $\cos\phi = \frac{4}{5}$ and that the terminal side is in quadrant IV, find exact answers for each of the following.

a) The other function values for ϕ

b) The six function values for $\pi/2 - \phi$

c) The six function values for $\phi + \pi/2$

Find an equivalent expression for each of the following.

5. $\sec\left(x + \frac{\pi}{2}\right)$

6. $\cot\left(x - \frac{\pi}{2}\right)$

7. $\tan\left(x - \frac{\pi}{2}\right)$

8. $\csc\left(x + \frac{\pi}{2}\right)$

Find the exact value of $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, *and the quadrant in which* 2θ *lies.*

9. $\sin\theta = \frac{4}{5}$, θ in quadrant I

10. $\cos\theta = \frac{5}{13}$, θ in quadrant I

11. $\cos\theta = -\frac{3}{5}$, θ in quadrant III

12. $\tan\theta = -\frac{15}{8}$, θ in quadrant II

13. $\tan\theta = -\frac{5}{12}$, θ in quadrant II

14. $\sin\theta = -\frac{\sqrt{10}}{10}$, θ in quadrant IV

15. Find an equivalent expression for $\cos 4x$ in terms of function values of x.

16. Find an equivalent expression for $\sin^4\theta$ in terms of function values of θ, 2θ, or 4θ, raised only to the first power.

Use the half-angle identities to evaluate exactly.

17. $\cos 15°$

18. $\tan 67.5°$

19. $\sin 112.5°$

20. $\cos\frac{\pi}{8}$

21. $\tan 75°$

22. $\sin\frac{5\pi}{12}$

Given that $\sin\theta = 0.3416$ *and* θ *is in quadrant I, find each of the following using identities.*

23. $\sin 2\theta$

24. $\cos\frac{\theta}{2}$

25. $\sin\frac{\theta}{2}$

26. $\sin 4\theta$

Simplify.

27. $2\cos^2\frac{x}{2} - 1$

28. $\cos^4 x - \sin^4 x$

29. $(\sin x - \cos x)^2 + \sin 2x$

30. $(\sin x + \cos x)^2$

31. $\dfrac{2 - \sec^2 x}{\sec^2 x}$

32. $\dfrac{1 + \sin 2x + \cos 2x}{1 + \sin 2x - \cos 2x}$

33. $(-4\cos x\sin x + 2\cos 2x)^2 + (2\cos 2x + 4\sin x\cos x)^2$

34. $2\sin x\cos^3 x - 2\sin^3 x\cos x$

Skill Maintenance

Complete the identity.

35. $1 - \cos^2 x =$

36. $\sec^2 x - \tan^2 x =$

37. $\sin^2 x - 1 =$

38. $1 + \cot^2 x =$

39. $\csc^2 x - \cot^2 x =$

40. $1 + \tan^2 x =$

41. $1 - \sin^2 x =$

42. $\sec^2 x - 1 =$

Consider the following functions (a)–(f). Without graphing them, answer questions 43–46 below.

a) $f(x) = 2\sin\left(\frac{1}{2}x - \frac{\pi}{2}\right)$

b) $f(x) = \frac{1}{2}\cos\left(2x - \frac{\pi}{4}\right) + 2$

c) $f(x) = -\sin\left[2\left(x - \frac{\pi}{2}\right)\right] + 2$

d) $f(x) = \sin(x + \pi) - \frac{1}{2}$

e) $f(x) = -2\cos(4x - \pi)$

f) $f(x) = -\cos\left[2\left(x - \frac{\pi}{8}\right)\right]$

43. Which functions have a graph with an amplitude of 2?

44. Which functions have a graph with a period of π?

45. Which functions have a graph with a period of 2π?

46. Which functions have a graph with a phase shift of $\frac{\pi}{4}$?

Synthesis

47. Given that $\cos 51° \approx 0.6293$, find the six function values for 141°.

Simplify.

48. $\sin\left(\frac{\pi}{2} - x\right)[\sec x - \cos x]$

49. $\cos(\pi - x) + \cot x \sin\left(x - \frac{\pi}{2}\right)$

50. $\dfrac{\cos x - \sin\left(\frac{\pi}{2} - x\right)\sin x}{\cos x - \cos(\pi - x)\tan x}$

51. $\dfrac{\cos^2 y \sin\left(y + \frac{\pi}{2}\right)}{\sin^2 y \sin\left(\frac{\pi}{2} - y\right)}$

Find $\sin\theta$, $\cos\theta$, *and* $\tan\theta$ *under the given conditions.*

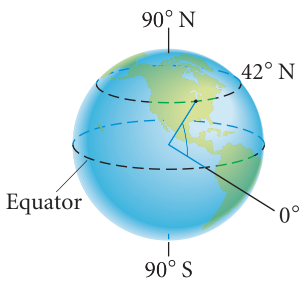
52. $\cos 2\theta = \frac{7}{12},\ \frac{3\pi}{2} \le 2\theta \le 2\pi$

53. $\tan\frac{\theta}{2} = -\frac{5}{3},\ \pi < \theta \le \frac{3\pi}{2}$

54. *Nautical Mile.* *Latitude* is used to measure north–south location on the earth between the equator and the poles. For example, Chicago has latitude 42°N. (See the figure.) In Great Britain, the *nautical mile* is defined as the length of a minute of arc of the earth's radius. Since the earth is flattened slightly at the poles, a British nautical mile varies with latitude. In fact, it is given, in feet, by the function

$$N(\phi) = 6066 - 31\cos 2\phi,$$

where ϕ is the latitude in degrees.



a) What is the length of a British nautical mile at Chicago?

b) What is the length of a British nautical mile at the North Pole?

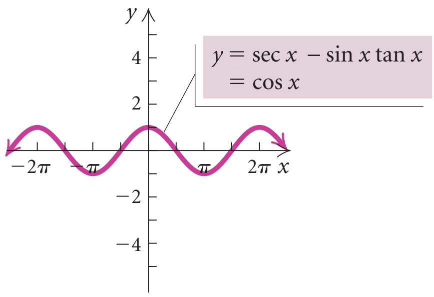
c) Express $N(\phi)$ in terms of $\cos\phi$ only; that is, do not use the double angle.

55. *Acceleration Due to Gravity.* The acceleration due to gravity is often denoted by g in a formula such as

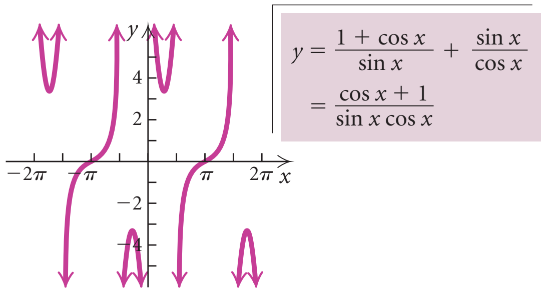
7.3 Exercise Set

Prove the identity.

1. $\sec x - \sin x \tan x = \cos x$



2. $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos\theta + 1}{\sin\theta\cos\theta}$



3. $\dfrac{1-\cos x}{\sin x} = \dfrac{\sin x}{1+\cos x}$

4. $\dfrac{1+\tan y}{1+\cot y} = \dfrac{\sec y}{\csc y}$

5. $\dfrac{1+\tan\theta}{1-\tan\theta} + \dfrac{1+\cot\theta}{1-\cot\theta} = 0$

6. $\dfrac{\sin x + \cos x}{\sec x + \csc x} = \dfrac{\sin x}{\sec x}$

7. $\dfrac{\cos^2\alpha + \cot\alpha}{\cos^2\alpha - \cot\alpha} = \dfrac{\cos^2\alpha\tan\alpha + 1}{\cos^2\alpha\tan\alpha - 1}$

8. $\sec 2\theta = \dfrac{\sec^2\theta}{2-\sec^2\theta}$

9. $\dfrac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$

10. $\dfrac{\cos(u-v)}{\cos u \sin v} = \tan u + \cot v$

11. $1 - \cos 5\theta \cos 3\theta - \sin 5\theta \sin 3\theta = 2\sin^2\theta$

12. $\cos^4 x - \sin^4 x = \cos 2x$

13. $2\sin\theta\cos^3\theta + 2\sin^3\theta\cos\theta = \sin 2\theta$

14. $\dfrac{\tan 3t - \tan t}{1 + \tan 3t \tan t} = \dfrac{2\tan t}{1-\tan^2 t}$

15. $\dfrac{\tan x - \sin x}{2\tan x} = \sin^2\dfrac{x}{2}$

16. $\dfrac{\cos^3\beta - \sin^3\beta}{\cos\beta - \sin\beta} = \dfrac{2+\sin 2\beta}{2}$

17. $\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$

18. $\cos^2 x(1-\sec^2 x) = -\sin^2 x$

19. $\tan\theta(\tan\theta + \cot\theta) = \sec^2\theta$

20. $\dfrac{\cos\theta + \sin\theta}{\cos\theta} = 1 + \tan\theta$

21. $\dfrac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$

22. $\dfrac{\tan y + \cot y}{\csc y} = \sec y$

23. $\dfrac{1+\sin x}{1-\sin x} + \dfrac{\sin x - 1}{1+\sin x} = 4\sec x \tan x$

24. $\tan\theta - \cot\theta = (\sec\theta - \csc\theta)(\sin\theta + \cos\theta)$

25. $\cos^2\alpha\cot^2\alpha = \cot^2\alpha - \cos^2\alpha$

26. $\dfrac{\tan x + \cot x}{\sec x + \csc x} = \dfrac{1}{\cos x + \sin x}$

27. $2\sin^2\theta\cos^2\theta + \cos^4\theta = 1 - \sin^4\theta$

28. $\dfrac{\cot\theta}{\csc\theta - 1} = \dfrac{\csc\theta + 1}{\cot\theta}$

29. $\dfrac{1+\sin x}{1-\sin x} = (\sec x + \tan x)^2$

30. $\sec^4 s - \tan^2 s = \tan^4 s + \sec^2 s$

31. Verify the product-to-sum identities (3) and (4) using the sine sum and difference identities.

32. Verify the sum-to-product identities (5)–(8) using the product-to-sum identities (1)–(4).

Use the product-to-sum identities and the sum-to-product identities to find identities for each of the following.

33. $\sin 3\theta - \sin 5\theta$

34. $\sin 7x - \sin 4x$

35. $\sin 8\theta + \sin 5\theta$

36. $\cos \theta - \cos 7\theta$

37. $\sin 7u \sin 5u$

38. $2 \sin 7\theta \cos 3\theta$

39. $7 \cos \theta \sin 7\theta$

40. $\cos 2t \sin t$

41. $\cos 55° \sin 25°$

42. $7 \cos 5\theta \cos 7\theta$

Use the product-to-sum identities and the sum-to-product identities to prove each of the following.

43. $\sin 4\theta + \sin 6\theta = \cot \theta (\cos 4\theta - \cos 6\theta)$

44. $\tan 2x (\cos x + \cos 3x) = \sin x + \sin 3x$

45. $\cot 4x (\sin x + \sin 4x + \sin 7x)$
$= \cos x + \cos 4x + \cos 7x$

46. $\tan \dfrac{x + y}{2} = \dfrac{\sin x + \sin y}{\cos x + \cos y}$

47. $\cot \dfrac{x + y}{2} = \dfrac{\sin y - \sin x}{\cos x - \cos y}$

48. $\tan \dfrac{\theta + \phi}{2} \tan \dfrac{\phi - \theta}{2} = \dfrac{\cos \theta - \cos \phi}{\cos \theta + \cos \phi}$

49. $\tan \dfrac{\theta + \phi}{2} (\sin \theta - \sin \phi)$
$= \tan \dfrac{\theta - \phi}{2} (\sin \theta + \sin \phi)$

50. $\sin 2\theta + \sin 4\theta + \sin 6\theta = 4 \cos \theta \cos 2\theta \sin 3\theta$

Skill Maintenance

For each function:

a) *Graph the function.*
b) *Determine whether the function is one-to-one.*
c) *If the function is one-to-one, find an equation for its inverse.*
d) *Graph the inverse of the function.*

51. $f(x) = 3x - 2$

52. $f(x) = x^3 + 1$

53. $f(x) = x^2 - 4,\ x \geq 0$

54. $f(x) = \sqrt{x + 2}$

Solve.

55. $2x^2 = 5x$

56. $3x^2 + 5x - 10 = 18$

57. $x^4 + 5x^2 - 36 = 0$

58. $x^2 - 10x + 1 = 0$

59. $\sqrt{x - 2} = 5$

60. $x = \sqrt{x + 7} + 5$

Synthesis

Prove the identity.

61. $\ln |\tan x| = -\ln |\cot x|$

62. $\ln |\sec \theta + \tan \theta| = -\ln |\sec \theta - \tan \theta|$

63. $\log (\cos x - \sin x) + \log (\cos x + \sin x)$
$= \log \cos 2x$

64. *Mechanics.* The following equation occurs in the study of mechanics:

$$\sin \theta = \frac{I_1 \cos \phi}{\sqrt{(I_1 \cos \phi)^2 + (I_2 \sin \phi)^2}}.$$

It can happen that $I_1 = I_2$. Assuming that this happens, simplify the equation.

65. *Alternating Current.* In the theory of alternating current, the following equation occurs:

$$R = \frac{1}{\omega C(\tan \theta + \tan \phi)}.$$

Show that this equation is equivalent to

$$R = \frac{\cos \theta \cos \phi}{\omega C \sin (\theta + \phi)}.$$

66. *Electrical Theory.* In electrical theory, the following equations occur:

$$E_1 = \sqrt{2}E_t \cos \left(\theta + \frac{\pi}{P}\right)$$

and

$$E_2 = \sqrt{2}E_t \cos \left(\theta - \frac{\pi}{P}\right).$$

Assuming that these equations hold, show that

$$\frac{E_1 + E_2}{2} = \sqrt{2}E_t \cos \theta \cos \frac{\pi}{P}$$

and

$$\frac{E_1 - E_2}{2} = -\sqrt{2}E_t \sin \theta \sin \frac{\pi}{P}.$$

Mid-Chapter Mixed Review

Determine whether the statement is true or false.

1. $\sin x\,(\csc x - \cot x) = 1 - \cos x$ [7.1]

2. $\sin 42° = \sqrt{\dfrac{1 + \cos 84°}{2}}$ [7.2]

3. $\sin \dfrac{\pi}{9} = \cos \dfrac{7\pi}{18}$ [7.2]

4. $\cos^2 x \neq \cos x^2$ [7.1]

For Exercises 5–14 choose an expression from expressions A–J to complete the identity. [7.1], [7.2]

5. $\cos(-x) =$
6. $\cos(u + v) =$
7. $\tan 2x =$
8. $\tan\left(\dfrac{\pi}{2} - x\right) =$
9. $1 + \cot^2 x =$
10. $\sin \dfrac{x}{2} =$
11. $\sin 2x =$
12. $\sin(u - v) =$
13. $\csc\left(\dfrac{\pi}{2} - x\right) =$
14. $\cos \dfrac{x}{2} =$

A. $2 \sin x \cos x$
B. $\pm\sqrt{\dfrac{1 + \cos x}{2}}$
C. $\csc^2 x$
D. $\dfrac{2 \tan x}{1 - \tan^2 x}$
E. $\pm\sqrt{\dfrac{1 - \cos x}{2}}$
F. $\sec x$
G. $\sin u \cos v - \cos u \sin v$
H. $\cos u \cos v - \sin u \sin v$
I. $\cot x$
J. $\cos x$

Simplify.

15. $\sqrt{\dfrac{\cot x}{\sin x}}$ [7.1]

16. $\dfrac{1}{\sin^2 x} - \left(\dfrac{\cos x}{\sin x}\right)^2$ [7.1]

17. $\dfrac{2\cos^2 x - 5\cos x - 3}{\cos x - 3}$ [7.1]

18. $\dfrac{\sin x}{\tan(-x)}$ [7.1]

19. $(\cos x - \sin x)^2$ [7.2]

20. $1 - 2\sin^2 \dfrac{x}{2}$ [7.2]

21. Rationalize the denominator:
$$\sqrt{\frac{\sec x}{1 - \cos x}}.$$ [7.1]

22. Write $\cos 41° \cos 29° + \sin 41° \sin 29°$ as a trigonometric function of a single angle and then evaluate. [7.1]

23. Evaluate $\cos \dfrac{3\pi}{8}$ exactly. [7.1]

24. Evaluate $\sin 105°$ exactly. [7.1]

25. Assume that $\sin \alpha = \dfrac{5}{13}$ and $\sin \beta = \dfrac{12}{13}$ and that α and β are between 0 and $\pi/2$, and evaluate $\tan(\alpha - \beta)$. [7.1]

26. Find the exact value of $\sin 2\theta$ and the quadrant in which 2θ lies if $\cos \theta = -\dfrac{4}{5}$, with θ in quadrant II. [7.2]

Prove the identity. [7.3]

27. $\cos^2 \dfrac{x}{2} = \dfrac{\tan x + \sin x}{2 \tan x}$

28. $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$

29. $\dfrac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = \dfrac{2 + \sin 2x}{2}$

30. $\sin 6\theta - \sin 2\theta = \tan 2\theta\,(\cos 2\theta + \cos 6\theta)$

Collaborative Discussion and Writing

31. Explain why $\tan(x + 450°)$ cannot be simplified using the tangent sum formula, but can be simplified using the sine and cosine sum formulas. [7.1]

32. Discuss and compare the graphs of $y = \sin x$, $y = \sin 2x$, and $y = \sin(x/2)$. [7.2]

33. What restrictions must be placed on the variable in each of the following identities? Why? [7.3]

a) $\sin 2x = \dfrac{2\tan x}{1 + \tan^2 x}$

b) $\dfrac{1 - \cos x}{\sin x} = \dfrac{\sin x}{1 + \cos x}$

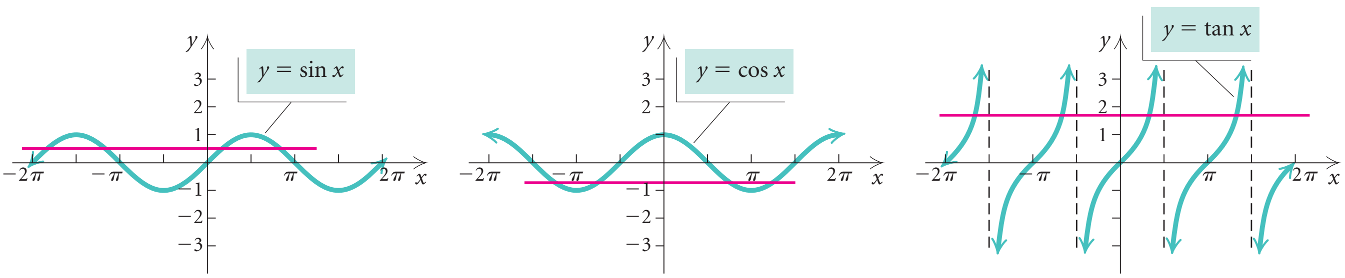
34. Find all errors in the following:

$$\begin{aligned} &2\sin^2 2x + \cos 4x \\ &\quad = 2(2\sin x\cos x)^2 + 2\cos 2x \\ &\quad = 8\sin^2 x\cos^2 x + 2(\cos^2 x + \sin^2 x) \\ &\quad = 8\sin^2 x\cos^2 x + 2. \end{aligned}$$ [7.2]

7.4 Inverses of the Trigonometric Functions

- **Find values of the inverse trigonometric functions.**
- **Simplify expressions such as $\sin(\sin^{-1} x)$ and $\sin^{-1}(\sin x)$.**
- **Simplify expressions involving compositions such as $\sin\left(\cos^{-1}\frac{1}{2}\right)$ without using a calculator.**
- **Simplify expressions such as $\sin\arctan(a/b)$ by making a drawing and reading off appropriate ratios.**

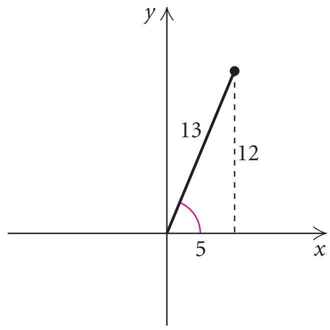
In this section, we develop inverse trigonometric functions. The graphs of the sine, cosine, and tangent functions follow. Do these functions have inverses that are functions? They do have inverses if they are one-to-one, which means that they pass the horizontal-line test.



INVERSE FUNCTIONS
REVIEW SECTION 5.1

Note that for each function, a horizontal line (shown in red) crosses the graph more than once. Therefore, none of them has an inverse that is a function.

To find $\sin\left(\cos^{-1}\frac{5}{13}\right)$, we use a reference triangle in quadrant I and determine that the sine of the angle whose cosine is $\frac{5}{13}$ is $\frac{12}{13}$.



Our expression now simplifies to

$$\frac{1}{2}\cdot\frac{5}{13}+\frac{\sqrt{3}}{2}\cdot\frac{12}{13}, \quad \text{or} \quad \frac{5+12\sqrt{3}}{26}.$$

Thus,

$$\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{5}{13}\right)=\frac{5+12\sqrt{3}}{26}.$$

Now Try Exercise 63.

7.4 Exercise Set

Find each of the following exactly in radians and degrees.

1. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

2. $\cos^{-1}\frac{1}{2}$

3. $\tan^{-1} 1$

4. $\sin^{-1} 0$

5. $\cos^{-1}\frac{\sqrt{2}}{2}$

6. $\sec^{-1}\sqrt{2}$

7. $\tan^{-1} 0$

8. $\tan^{-1}\frac{\sqrt{3}}{3}$

9. $\cos^{-1}\frac{\sqrt{3}}{2}$

10. $\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

11. $\csc^{-1} 2$

12. $\sin^{-1}\frac{1}{2}$

13. $\cot^{-1}(-\sqrt{3})$

14. $\tan^{-1}(-1)$

15. $\sin^{-1}\left(-\frac{1}{2}\right)$

16. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

17. $\cos^{-1} 0$

18. $\sin^{-1}\frac{\sqrt{3}}{2}$

19. $\sec^{-1} 2$

20. $\csc^{-1}(-1)$

Use a calculator to find each of the following in radians, rounded to four decimal places, and in degrees, rounded to the nearest tenth of a degree.

21. $\tan^{-1} 0.3673$

22. $\cos^{-1}(-0.2935)$

23. $\sin^{-1} 0.9613$

24. $\sin^{-1}(-0.6199)$

25. $\cos^{-1}(-0.9810)$

26. $\tan^{-1} 158$

27. $\csc^{-1}(-6.2774)$

28. $\sec^{-1} 1.1677$

29. $\tan^{-1}(1.091)$

30. $\cot^{-1} 1.265$

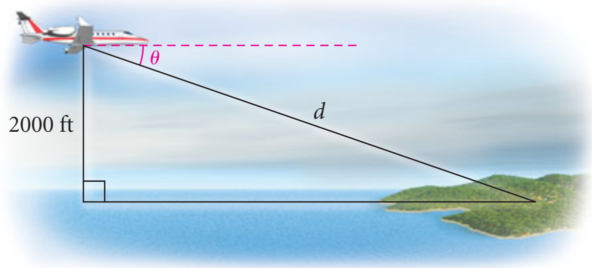
31. $\sin^{-1}(-0.8192)$

32. $\cos^{-1}(-0.2716)$

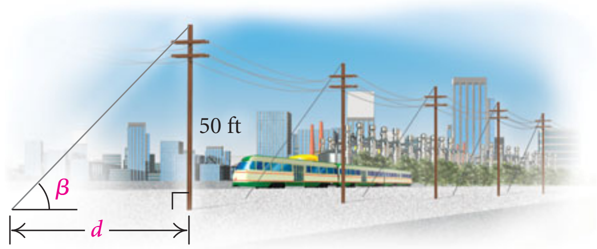
33. State the domains of the inverse sine, inverse cosine, and inverse tangent functions.

34. State the ranges of the inverse sine, inverse cosine, and inverse tangent functions.

35. *Angle of Depression.* An airplane is flying at an altitude of 2000 ft toward an island. The straight-line distance from the airplane to the island is d feet. Express θ, the angle of depression, as a function of d.



36. *Angle of Inclination.* A guy wire is attached to the top of a 50-ft pole and stretched to a point that is d feet from the bottom of the pole. Express β, the angle of inclination, as a function of d.



Evaluate.

37. $\sin(\sin^{-1} 0.3)$

38. $\tan[\tan^{-1}(-4.2)]$

39. $\cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right]$

40. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

41. $\sin^{-1}\left(\sin\frac{\pi}{5}\right)$

42. $\cot^{-1}\left(\cot\frac{2\pi}{3}\right)$

43. $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

44. $\cos^{-1}\left(\cos\frac{\pi}{7}\right)$

45. $\sin\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$

46. $\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

47. $\tan\left(\cos^{-1}\frac{\sqrt{2}}{2}\right)$

48. $\cos^{-1}(\sin\pi)$

49. $\sin^{-1}\left(\cos\frac{\pi}{6}\right)$

50. $\sin^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$

51. $\tan(\sin^{-1} 0.1)$

52. $\cos\left(\tan^{-1}\frac{\sqrt{3}}{4}\right)$

53. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

54. $\tan^{-1}\left[\tan\left(-\frac{3\pi}{4}\right)\right]$

Find each of the following.

55. $\sin\left(\tan^{-1}\frac{a}{3}\right)$

56. $\tan\left(\cos^{-1}\frac{3}{x}\right)$

57. $\cot\left(\sin^{-1}\frac{p}{q}\right)$

58. $\sin(\cos^{-1} x)$

59. $\tan\left(\sin^{-1}\frac{p}{\sqrt{p^2+9}}\right)$

60. $\tan\left(\frac{1}{2}\sin^{-1}\frac{1}{2}\right)$

61. $\cos\left(\frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{2}\right)$

62. $\sin\left(2\cos^{-1}\frac{3}{5}\right)$

Evaluate.

63. $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}+\cos^{-1}\frac{3}{5}\right)$

64. $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}\frac{3}{5}\right)$

65. $\sin(\sin^{-1} x + \cos^{-1} y)$

66. $\cos(\sin^{-1} x - \cos^{-1} y)$

67. $\sin(\sin^{-1} 0.6032 + \cos^{-1} 0.4621)$

68. $\cos(\sin^{-1} 0.7325 - \cos^{-1} 0.4838)$

Skill Maintenance

In each of Exercises 69–76, fill in the blank with the correct term. Some of the given choices will not be used.

linear speed	congruent
angular speed	circular
angle of elevation	periodic
angle of depression	period
complementary	amplitude
supplementary	quadrantal
similar	radian measure

69. A function f is said to be ______________ if there exists a positive constant p such that $f(s+p) = f(s)$ for all s in the domain of f.

70. The ______________ of a rotation is the ratio of the distance s traveled by a point at a radius r from the center of rotation to the length of the radius r.

71. Triangles are ____________ if their corresponding angles have the same measure.

72. The angle between the horizontal and a line of sight below the horizontal is called a(n) ____________.

73. ____________ is the amount of rotation per unit of time.

74. Two positive angles are ____________ if their sum is 180°.

75. The ____________ of a periodic function is one half of the distance between its maximum and minimum function values.

76. Trigonometric functions with domains composed of real numbers are called ____________ functions.

Synthesis

Prove the identity.

77. $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$

78. $\tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2}$

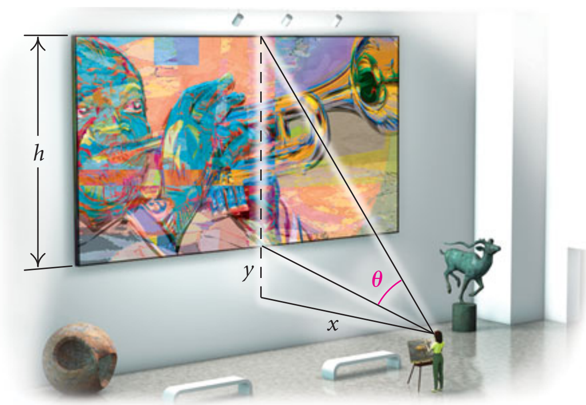
79. $\sin^{-1} x = \tan^{-1} \dfrac{x}{\sqrt{1 - x^2}}$

80. $\tan^{-1} x = \sin^{-1} \dfrac{x}{\sqrt{x^2 + 1}}$

81. $\sin^{-1} x = \cos^{-1} \sqrt{1 - x^2}$, for $x \geq 0$

82. $\cos^{-1} x = \tan^{-1} \dfrac{\sqrt{1 - x^2}}{x}$, for $x > 0$

83. *Height of a Mural.* An art student's eye is at a point A, looking at a mural of height h, with the bottom of the mural y feet above the eye (see the figure). The eye is x feet from the wall. Write an expression for θ in terms of x, y, and h. Then evaluate the expression when $x = 20$ ft, $y = 7$ ft, and $h = 25$ ft.



84. Use a calculator to approximate the following expression:

$$16 \tan^{-1} \frac{1}{5} - 4 \tan^{-1} \frac{1}{239}.$$

What number does this expression seem to approximate?

7.5 Solving Trigonometric Equations

▶ **Solve trigonometric equations.**

When an equation contains a trigonometric expression with a variable, such as $\cos x$, it is called a trigonometric equation. Some trigonometric equations are identities, such as $\sin^2 x + \cos^2 x = 1$. Now we consider equations, such as $2 \cos x = -1$, that are usually not identities. As we have done for other types of equations, we will solve such equations by finding all values for x that make the equation true.

7.5 Exercise Set

Solve, finding all solutions. Express the solutions in both radians and degrees.

1. $\cos x = \frac{\sqrt{3}}{2}$

2. $\sin x = -\frac{\sqrt{2}}{2}$

3. $\tan x = -\sqrt{3}$

4. $\cos x = -\frac{1}{2}$

5. $\sin x = \frac{1}{2}$

6. $\tan x = -1$

7. $\cos x = -\frac{\sqrt{2}}{2}$

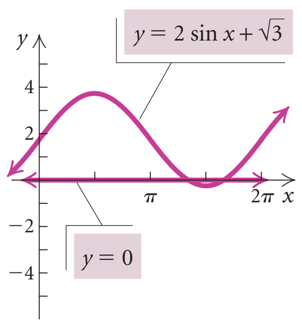
8. $\sin x = \frac{\sqrt{3}}{2}$

Solve, finding all solutions in $[0, 2\pi)$ *or* $[0°, 360°)$.

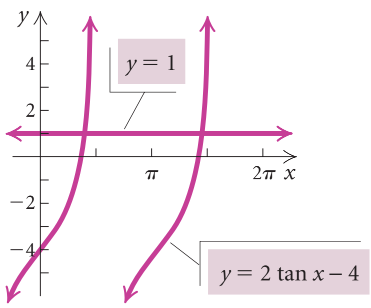
9. $2 \cos x - 1 = -1.2814$

10. $\sin x + 3 = 2.0816$

11. $2 \sin x + \sqrt{3} = 0$



12. $2 \tan x - 4 = 1$



13. $2 \cos^2 x = 1$

14. $\csc^2 x - 4 = 0$

15. $2 \sin^2 x + \sin x = 1$

16. $\cos^2 x + 2 \cos x = 3$

17. $2 \cos^2 x - \sqrt{3} \cos x = 0$

18. $2 \sin^2 \theta + 7 \sin \theta = 4$

19. $6 \cos^2 \phi + 5 \cos \phi + 1 = 0$

20. $2 \sin t \cos t + 2 \sin t - \cos t - 1 = 0$

21. $\sin 2x \cos x - \sin x = 0$

22. $5 \sin^2 x - 8 \sin x = 3$

23. $\cos^2 x + 6 \cos x + 4 = 0$

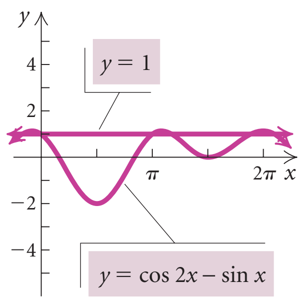
24. $2 \tan^2 x = 3 \tan x + 7$

25. $7 = \cot^2 x + 4 \cot x$

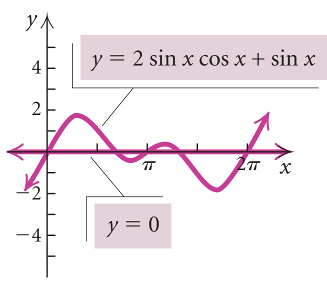
26. $3 \sin^2 x = 3 \sin x + 2$

Solve, finding all solutions in $[0, 2\pi)$.

27. $\cos 2x - \sin x = 1$



28. $2 \sin x \cos x + \sin x = 0$



29. $\tan x \sin x - \tan x = 0$

30. $\sin 4x - 2 \sin 2x = 0$

31. $\sin 2x \cos x + \sin x = 0$

32. $\cos 2x \sin x + \sin x = 0$

33. $2 \sec x \tan x + 2 \sec x + \tan x + 1 = 0$

34. $\sin 2x \sin x - \cos 2x \cos x = -\cos x$

35. $\sin 2x + \sin x + 2 \cos x + 1 = 0$

36. $\tan^2 x + 4 = 2 \sec^2 x + \tan x$

37. $\sec^2 x - 2 \tan^2 x = 0$

38. $\cot x = \tan (2x - 3\pi)$

39. $2 \cos x + 2 \sin x = \sqrt{6}$

40. $\sqrt{3} \cos x - \sin x = 1$

41. $\sec^2 x + 2 \tan x = 6$

42. $5 \cos 2x + \sin x = 4$

43. $\cos (\pi - x) + \sin \left(x - \dfrac{\pi}{2}\right) = 1$

44. $\dfrac{\sin^2 x - 1}{\cos \left(\dfrac{\pi}{2} - x\right) + 1} = \dfrac{\sqrt{2}}{2} - 1$

45. *Daylight Hours.* The number of daylight hours in Kajaani, Finland, varies from approximately 4.3 hr on December 21 to 20.7 on June 11.

The following sine function can be used to approximate the number of daylight hours, y, in Kajaani for day x:

$$y = 7.8787 \sin (0.0166x - 1.2723) + 12.1840.$$

a) Approximate the number of daylight hours in Kajaani for April 5 ($x = 95$), for August 18 ($x = 230$), and for November 29 ($x = 333$). (Hint: Set the calculator in radian mode.)

b) Determine on which days of the year there will be about 12 hr of daylight.

46. *Sales of Skis.* Sales of certain products fluctuate in cycles. The following sine function can be used to estimate the total amount of sales of skis, y, in thousands of dollars, in month x, for a business in a northern climate:

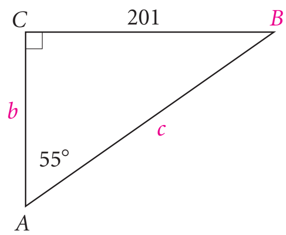
$$y = 9.584 \sin (0.436x + 2.097) + 10.558.$$

Approximate the total amount of sales to the nearest dollar for December and for July. (Hint: Set the calculator in radian mode.)

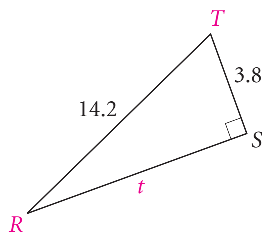
Skill Maintenance

Solve the right triangle.

47.



48.



Solve.

49. $\dfrac{x}{27} = \dfrac{4}{3}$

50. $\dfrac{0.01}{0.7} = \dfrac{0.2}{h}$

Synthesis

Solve in $[0, 2\pi)$.

51. $|\sin x| = \dfrac{\sqrt{3}}{2}$

52. $|\cos x| = \dfrac{1}{2}$

53. $\sqrt{\tan x} = \sqrt[4]{3}$

54. $12 \sin x - 7\sqrt{\sin x} + 1 = 0$

55. $\ln(\cos x) = 0$

56. $e^{\sin x} = 1$

57. $\sin(\ln x) = -1$

58. $e^{\ln(\sin x)} = 1$

59. *Temperature During an Illness.* The temperature T, in degrees Fahrenheit, of a patient t days into a 12-day illness is given by

$$T(t) = 101.6° + 3° \sin\left(\frac{\pi}{8}t\right).$$

Find the times t during the illness at which the patient's temperature was 103°.

60. *Satellite Location.* A satellite circles the earth in such a manner that it is y miles from the equator (north or south, height from the surface not considered) t minutes after its launch, where

$$y = 5000\left[\cos\frac{\pi}{45}(t - 10)\right].$$

At what times t in the interval $[0, 240]$, the first 4 hr, is the satellite 3000 mi north of the equator?

61. *Nautical Mile.* (See Exercise 54 in Exercise Set 7.2.) In Great Britain, the *nautical mile* is defined as the length of a minute of arc of the earth's radius. Since the earth is flattened at the poles, a British nautical mile varies with latitude. In fact, it is given, in feet, by the function

$$N(\phi) = 6066 - 31 \cos 2\phi,$$

where ϕ is the latitude in degrees. At what latitude north is the length of a British nautical mile found to be 6040 ft?

62. *Acceleration Due to Gravity.* (See Exercise 55 in Exercise Set 7.2.) The acceleration due to gravity is often denoted by g in a formula such as $S = \frac{1}{2}gt^2$, where S is the distance that an object falls in t seconds. The number g is generally considered constant, but in fact it varies slightly with latitude. If ϕ stands for latitude, in degrees, an excellent approximation of g is given by the formula

$$g = 9.78049(1 + 0.005288 \sin^2 \phi - 0.000006 \sin^2 2\phi),$$

where g is measured in meters per second per second at sea level. At what latitude north does $g = 9.8$?

Solve.

63. $\cos^{-1} x = \cos^{-1} \frac{3}{5} - \sin^{-1} \frac{4}{5}$

64. $\sin^{-1} x = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2}$

65. Suppose that $\sin x = 5 \cos x$. Find $\sin x \cos x$.

REVIEW EXERCISES

Determine whether the statement is true or false.

1. $\sin^2 s \neq \sin s^2$. [7.1]

2. Given $0 < \alpha < \pi/2$ and $0 < \beta < \pi/2$ and that $\sin(\alpha + \beta) = 1$ and $\sin(\alpha - \beta) = 0$, then $\alpha = \pi/4$. [7.1]

3. If the terminal side of θ is in quadrant IV, then $\tan\theta < \cos\theta$. [7.1]

4. $\cos 5\pi/12 = \cos 7\pi/12$. [7.2]

5. Given that $\sin\theta = -\frac{2}{5}$, $\tan\theta < \cos\theta$. [7.1]

Complete the Pythagorean identity. [7.1]

6. $1 + \cot^2 x =$

7. $\sin^2 x + \cos^2 x =$

Multiply and simplify. [7.1]

8. $(\tan y - \cot y)(\tan y + \cot y)$

9. $(\cos x + \sec x)^2$

Factor and simplify. [7.1]

10. $\sec x \csc x - \csc^2 x$

11. $3\sin^2 y - 7\sin y - 20$

12. $1000 - \cos^3 u$

Simplify. [7.1]

13. $\dfrac{\sec^4 x - \tan^4 x}{\sec^2 x + \tan^2 x}$

14. $\dfrac{2\sin^2 x}{\cos^3 x} \cdot \left(\dfrac{\cos x}{2\sin x}\right)^2$

15. $\dfrac{3\sin x}{\cos^2 x} \cdot \dfrac{\cos^2 x + \cos x \sin x}{\sin^2 x - \cos^2 x}$

16. $\dfrac{3}{\cos y - \sin y} - \dfrac{2}{\sin^2 y - \cos^2 y}$

17. $\left(\dfrac{\cot x}{\csc x}\right)^2 + \dfrac{1}{\csc^2 x}$

18. $\dfrac{4\sin x \cos^2 x}{16\sin^2 x \cos x}$

In Exercises 19–21, assume that all radicands are nonnegative.

19. Simplify:

$$\sqrt{\sin^2 x + 2\cos x \sin x + \cos^2 x}.$$ [7.1]

20. Rationalize the denominator: $\sqrt{\dfrac{1 + \sin x}{1 - \sin x}}$. [7.1]

21. Rationalize the numerator: $\sqrt{\dfrac{\cos x}{\tan x}}$. [7.1]

22. Given that $x = 3\tan\theta$, express $\sqrt{9 + x^2}$ as a trigonometric function without radicals. Assume that $0 < \theta < \pi/2$. [7.1]

Use the sum and difference formulas to write equivalent expressions. You need not simplify. [7.1]

23. $\cos\left(x + \dfrac{3\pi}{2}\right)$

24. $\tan(45° - 30°)$

25. Simplify: $\cos 27° \cos 16° + \sin 27° \sin 16°$. [7.1]

26. Find $\cos 165°$ exactly. [7.1]

27. Given that $\tan\alpha = \sqrt{3}$ and $\sin\beta = \sqrt{2}/2$ and that α and β are between 0 and $\pi/2$, evaluate $\tan(\alpha - \beta)$ exactly. [7.1]

28. Assume that $\sin\theta = 0.5812$ and $\cos\phi = 0.2341$ and that both θ and ϕ are first-quadrant angles. Evaluate $\cos(\theta + \phi)$. [7.1]

Complete the cofunction identity. [7.2]

29. $\cos\left(x + \dfrac{\pi}{2}\right) =$

30. $\cos\left(\dfrac{\pi}{2} - x\right) =$

31. $\sin\left(x - \dfrac{\pi}{2}\right) =$

32. Given that $\cos\alpha = -\frac{3}{5}$ and that the terminal side is in quadrant III:

a) Find the other function values for α. [7.2]

b) Find the six function values for $\pi/2 - \alpha$. [7.2]

c) Find the six function values for $\alpha + \pi/2$. [7.2]

33. Find an equivalent expression for $\csc\left(x - \dfrac{\pi}{2}\right)$. [7.2]

34. Find $\tan 2\theta$, $\cos 2\theta$, and $\sin 2\theta$ and the quadrant in which 2θ lies, where $\cos\theta = -\frac{4}{5}$ and θ is in quadrant III. [7.2]

35. Find $\sin\dfrac{\pi}{8}$ exactly. [7.2]

36. Given that $\sin \beta = 0.2183$ and β is in quadrant I, find $\sin 2\beta$, $\cos \frac{\beta}{2}$, and $\cos 4\beta$. [7.2]

Simplify. [7.2]

37. $1 - 2 \sin^2 \frac{x}{2}$

38. $(\sin x + \cos x)^2 - \sin 2x$

39. $2 \sin x \cos^3 x + 2 \sin^3 x \cos x$

40. $\dfrac{2 \cot x}{\cot^2 x - 1}$

Prove the identity. [7.3]

41. $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$

42. $\dfrac{1 + \cos 2\theta}{\sin 2\theta} = \cot \theta$

43. $\dfrac{\tan y + \sin y}{2 \tan y} = \cos^2 \dfrac{y}{2}$

44. $\dfrac{\sin x - \cos x}{\cos^2 x} = \dfrac{\tan^2 x - 1}{\sin x + \cos x}$

Use the product-to-sum identities and the sum-to-product identities to find identities for each of the following. [7.3]

45. $3 \cos 2\theta \sin \theta$

46. $\sin \theta - \sin 4\theta$

Find each of the following exactly in both radians and degrees. [7.4]

47. $\sin^{-1}\left(-\dfrac{1}{2}\right)$

48. $\cos^{-1} \dfrac{\sqrt{3}}{2}$

49. $\tan^{-1} 1$

50. $\sin^{-1} 0$

Use a calculator to find each of the following in radians, rounded to four decimal places, and in degrees, rounded to the nearest tenth of a degree. [7.4]

51. $\cos^{-1}(-0.2194)$

52. $\cot^{-1} 2.381$

Evaluate. [7.4]

53. $\cos\left(\cos^{-1} \dfrac{1}{2}\right)$

54. $\tan^{-1}\left(\tan \dfrac{\sqrt{3}}{3}\right)$

55. $\sin^{-1}\left(\sin \dfrac{\pi}{7}\right)$

56. $\cos\left(\sin^{-1} \dfrac{\sqrt{2}}{2}\right)$

Find each of the following. [7.4]

57. $\cos\left(\tan^{-1} \dfrac{b}{3}\right)$

58. $\cos\left(2 \sin^{-1} \dfrac{4}{5}\right)$

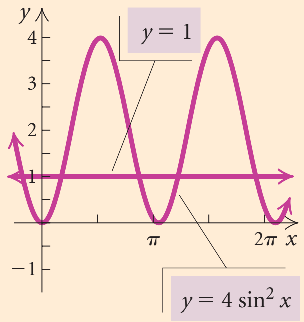
Solve, finding all solutions. Express the solutions in both radians and degrees. [7.5]

59. $\cos x = -\dfrac{\sqrt{2}}{2}$

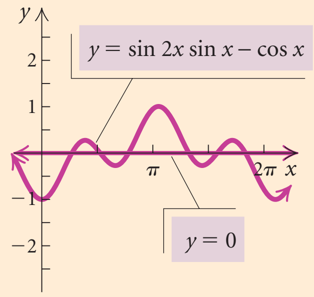
60. $\tan x = \sqrt{3}$

Solve, finding all solutions in $[0, 2\pi)$. [7.5]

61. $4 \sin^2 x = 1$



62. $\sin 2x \sin x - \cos x = 0$



63. $2 \cos^2 x + 3 \cos x = -1$

64. $\sin^2 x - 7 \sin x = 0$

65. $\csc^2 x - 2 \cot^2 x = 0$

66. $\sin 4x + 2 \sin 2x = 0$

67. $2 \cos x + 2 \sin x = \sqrt{2}$

68. $6 \tan^2 x = 5 \tan x + \sec^2 x$

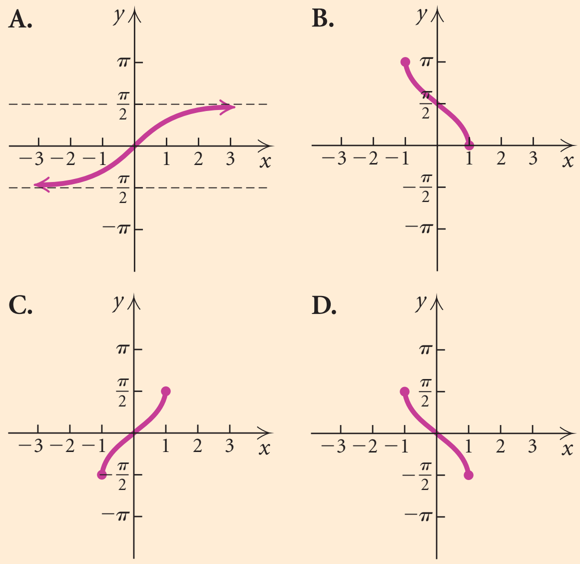
69. Which of the following is the domain of the function $\cos^{-1} x$? [7.4]

A. $(0, \pi)$ **B.** $[-1, 1]$
C. $[-\pi/2, \pi/2]$ **D.** $(-\infty, \infty)$

70. Simplify: $\sin^{-1}\left(\sin \dfrac{7\pi}{6}\right)$. [7.4]

A. $-\pi/6$ **B.** $7\pi/6$
C. $-1/2$ **D.** $11\pi/6$

71. The graph of $f(x) = \sin^{-1} x$ is which of the following? [7.4]



Synthesis

72. Find the measure of the angle from l_1 to l_2:

$$l_1\colon\ x + y = 3 \qquad l_2\colon\ 2x - y = 5. \quad [7.1]$$

73. Find an identity for $\cos(u + v)$ involving only cosines. [7.1], [7.2]

74. Simplify: $\cos\left(\frac{\pi}{2} - x\right)[\csc x - \sin x]$. [7.2]

75. Find $\sin \theta$, $\cos \theta$, and $\tan \theta$ under the given conditions:

$$\sin 2\theta = \frac{1}{5}, \quad \frac{\pi}{2} \le 2\theta < \pi. \quad [7.2]$$

76. Prove the following equation to be an identity:

$$\ln e^{\sin t} = \sin t. \quad [7.3]$$

77. Graph: $y = \sec^{-1} x$. [7.4]

78. Show that

$$\tan^{-1} x = \frac{\sin^{-1} x}{\cos^{-1} x}$$

is *not* an identity. [7.4]

79. Solve $e^{\cos x} = 1$ in $[0, 2\pi)$. [7.5]

Collaborative Discussion and Writing

80. Why are the ranges of the inverse trigonometric functions restricted? [7.4]

81. Jan lists her answer to a problem as $\pi/6 + k\pi$, for any integer k, while Jacob lists his answer as $\pi/6 + 2k\pi$ and $7\pi/6 + 2k\pi$, for any integer k. Are their answers equivalent? Why or why not? [7.5]

82. How does the graph of $y = \sin^{-1} x$ differ from the graph of $y = \sin x$? [7.4]

83. What is the difference between a trigonometric equation that is an identity and a trigonometric equation that is not an identity? Give an example of each. [7.1], [7.5]

84. Why is it that

$$\sin \frac{5\pi}{6} = \frac{1}{2}, \quad \text{but} \quad \sin^{-1}\left(\frac{1}{2}\right) \ne \frac{5\pi}{6}? \quad [7.4]$$

Chapter 7 Test

Simplify.

1. $\dfrac{2\cos^2 x - \cos x - 1}{\cos x - 1}$

2. $\left(\dfrac{\sec x}{\tan x}\right)^2 - \dfrac{1}{\tan^2 x}$

3. Rationalize the denominator:
$$\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}.$$
Assume that the radicand is nonnegative.

4. Given that $x = 2\sin\theta$, express $\sqrt{4 - x^2}$ as a trigonometric function without radicals. Assume $0 < \theta < \pi/2$.

Use the sum or difference identities to evaluate exactly.

5. $\sin 75°$

6. $\tan \dfrac{\pi}{12}$

7. Assuming that $\cos u = \frac{5}{13}$ and $\cos v = \frac{12}{13}$ and that u and v are between 0 and $\pi/2$, evaluate $\cos(u - v)$ exactly.

8. Given that $\cos\theta = -\frac{2}{3}$ and that the terminal side is in quadrant II, find $\cos(\pi/2 - \theta)$.

9. Given that $\sin\theta = -\frac{4}{5}$ and θ is in quadrant III, find $\sin 2\theta$ and the quadrant in which 2θ lies.

10. Use a half-angle identity to evaluate $\cos\dfrac{\pi}{12}$ exactly.

11. Given that $\sin\theta = 0.6820$ and that θ is in quadrant I, find $\cos(\theta/2)$.

12. Simplify: $(\sin x + \cos x)^2 - 1 + 2\sin 2x$.

Prove each of the following identities.

13. $\csc x - \cos x \cot x = \sin x$

14. $(\sin x + \cos x)^2 = 1 + \sin 2x$

15. $(\csc\beta + \cot\beta)^2 = \dfrac{1 + \cos\beta}{1 - \cos\beta}$

16. $\dfrac{1 + \sin\alpha}{1 + \csc\alpha} = \dfrac{\tan\alpha}{\sec\alpha}$

Use the product-to-sum identities and the sum-to-product identities to find identities for each of the following.

17. $\cos 8\alpha - \cos\alpha$

18. $4\sin\beta\cos 3\beta$

19. Find $\sin^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$ exactly in degrees.

20. Find $\tan^{-1}\sqrt{3}$ exactly in radians.

21. Use a calculator to find $\cos^{-1}(-0.6716)$ in radians, rounded to four decimal places.

22. Evaluate $\cos\left(\sin^{-1}\dfrac{1}{2}\right)$.

23. Find $\tan\left(\sin^{-1}\dfrac{5}{x}\right)$.

24. Evaluate $\cos\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right)$.

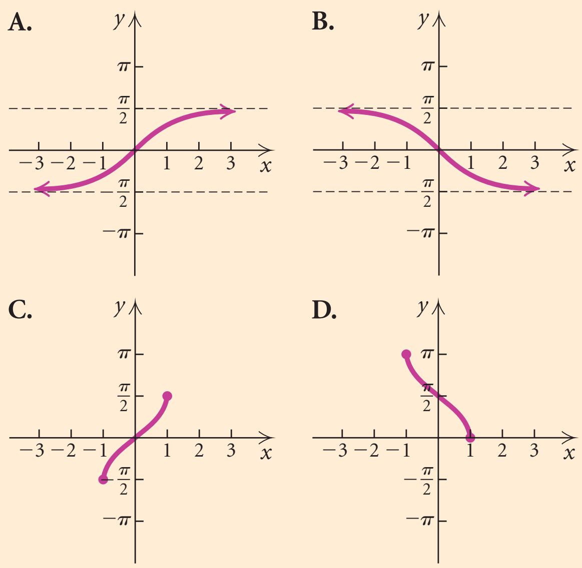
Solve, finding all solutions in $[0, 2\pi)$.

25. $4\cos^2 x = 3$

26. $2\sin^2 x = \sqrt{2}\sin x$

27. $\sqrt{3}\cos x + \sin x = 1$

28. The graph of $f(x) = \cos^{-1} x$ is which of the following?



Synthesis

29. Find $\cos\theta$, given that $\cos 2\theta = \dfrac{5}{6}$, $\dfrac{3\pi}{2} < \theta < 2\pi$.

21. $-3\sqrt{2} - 3\sqrt{2}i$

22. $-\frac{9}{2} - \frac{9\sqrt{3}}{2}i$

Find standard notation, $a + bi$.

23. $3(\cos 30° + i \sin 30°)$

24. $6(\cos 120° + i \sin 120°)$

25. $10(\cos 270° + i \sin 270°)$

26. $3(\cos 0° + i \sin 0°)$

27. $\sqrt{8}\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

28. $5\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$

29. $2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

30. $3\left[\cos\left(-\frac{3\pi}{4}\right) + i \sin\left(-\frac{3\pi}{4}\right)\right]$

31. $\sqrt{2}[\cos(-60°) + i \sin(-60°)]$

32. $4(\cos 135° + i \sin 135°)$

Multiply or divide and leave the answer in trigonometric notation.

33. $\dfrac{12(\cos 48° + i \sin 48°)}{3(\cos 6° + i \sin 6°)}$

34. $5\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right) \cdot 2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)$

35. $2.5(\cos 35° + i \sin 35°) \cdot 4.5(\cos 21° + i \sin 21°)$

36. $\dfrac{\frac{1}{2}\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)}{\frac{3}{8}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)}$

Convert to trigonometric notation and then multiply or divide.

37. $(1 - i)(2 + 2i)$

38. $(1 + i\sqrt{3})(1 + i)$

39. $\dfrac{1 - i}{1 + i}$

40. $\dfrac{1 - i}{\sqrt{3} - i}$

41. $(3\sqrt{3} - 3i)(2i)$

42. $(2\sqrt{3} + 2i)(2i)$

43. $\dfrac{2\sqrt{3} - 2i}{1 + \sqrt{3}i}$

44. $\dfrac{3 - 3\sqrt{3}i}{\sqrt{3} - i}$

Raise the number to the given power and write trigonometric notation for the answer.

45. $\left[2\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]^3$

46. $[2(\cos 120° + i \sin 120°)]^4$

47. $(1 + i)^6$

48. $(-\sqrt{3} + i)^5$

Raise the number to the given power and write standard notation for the answer.

49. $[3(\cos 20° + i \sin 20°)]^3$

50. $[2(\cos 10° + i \sin 10°)]^9$

51. $(1 - i)^5$

52. $(2 + 2i)^4$

53. $\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)^{12}$

54. $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^{10}$

Find the square roots of the number.

55. $-i$

56. $1 + i$

57. $2\sqrt{2} - 2\sqrt{2}i$

58. $-\sqrt{3} - i$

Find the cube roots of the number.

59. i

60. $-64i$

61. $2\sqrt{3} - 2i$

62. $1 - \sqrt{3}i$

63. Find and graph the fourth roots of 16.

64. Find and graph the fourth roots of i.

65. Find and graph the fifth roots of -1.

66. Find and graph the sixth roots of 1.

67. Find the tenth roots of 8.

68. Find the ninth roots of -4.

69. Find the sixth roots of -1.

70. Find the fourth roots of 12.

Find all the complex solutions of the equation.

71. $x^3 = 1$

72. $x^5 - 1 = 0$

73. $x^4 + i = 0$

74. $x^4 + 81 = 0$

75. $x^6 + 64 = 0$

76. $x^5 + \sqrt{3} + i = 0$

Skill Maintenance

Convert to degree measure.

77. $\dfrac{\pi}{12}$

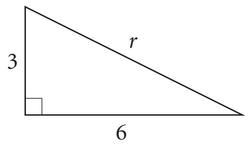
78. 3π

Convert to radian measure.

79. $330°$

80. $-225°$

81. Find r.



82. Graph these points in the rectangular coordinate system: $(2, -1)$, $(0, 3)$, and $\left(-\frac{1}{2}, -4\right)$.

Find the function value using coordinates of points on the unit circle.

83. $\sin \dfrac{2\pi}{3}$

84. $\cos \dfrac{\pi}{6}$

85. $\cos \dfrac{\pi}{4}$

86. $\sin \dfrac{5\pi}{6}$

Synthesis

Solve.

87. $x^2 + (1 - i)x + i = 0$

88. $3x^2 + (1 + 2i)x + 1 - i = 0$

89. Find polar notation for $(\cos \theta + i \sin \theta)^{-1}$.

90. Show that for any complex number z,
$$|z| = |-z|.$$

91. Show that for any complex number z and its conjugate $\bar{z}$,
$$|z| = |\bar{z}|.$$
(*Hint*: Let $z = a + bi$ and $\bar{z} = a - bi$.)

92. Show that for any complex number z and its conjugate $\bar{z}$,
$$|z\bar{z}| = |z^2|.$$
(*Hint*: Let $z = a + bi$ and $\bar{z} = a - bi$.)

93. Show that for any complex number z,
$$|z^2| = |z|^2.$$

94. Show that for any complex numbers z and w,
$$|z \cdot w| = |z| \cdot |w|.$$
(*Hint*: Let $z = r_1(\cos \theta_1 + i \sin \theta_1)$ and $w = r_2(\cos \theta_2 + i \sin \theta_2)$.)

95. Show that for any complex number z and any nonzero, complex number w,
$$\left|\frac{z}{w}\right| = \frac{|z|}{|w|}.$$
(Use the hint for Exercise 94.)

96. On a complex plane, graph $|z| = 1$.

97. On a complex plane, graph $z + \bar{z} = 3$.

98. Solve: $x^6 - 1 = 0$.

Mid-Chapter Mixed Review

Determine whether the statement is true or false.

1. Any triangle, right or oblique, can be solved if at least one side and any other two measures are known. [8.1]
2. The absolute value of $-i$ is 1. [8.3]
3. The law of cosines cannot be used to solve a triangle when all three sides are known. [8.2]
4. Since angle measures determine only the shape of a triangle and not the size, we cannot solve a triangle when only the three angle measures are given. [8.1]

Solve $\triangle ABC$, if possible. [8.1], [8.2]

5. $a = 8.3$ in., $A = 52°$, and $C = 65°$

6. $A = 27.2°$, $c = 33$ m, and $a = 14$ m

7. $a = 17.8$ yd, $b = 13.1$ yd, and $c = 25.6$ yd

8. $a = 29.4$ cm, $b = 40.8$ cm, and $A = 42.7°$

9. $A = 148°$, $b = 200$ yd, and $c = 185$ yd

10. $b = 18$ ft, $c = 27$ ft, and $B = 28°$

11. Find the area of the triangle with $C = 54°$, $a = 38$ in., and $b = 29$ in. [8.2]

Graph the complex number and find its absolute value. [8.1]

12. $-5 + 3i$

13. $-i$

14. 4

15. $1 - 5i$

Find trigonometric notation. [8.3]

16. $\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$

17. $1 - \sqrt{3}i$

18. $5i$

19. $-2 - 2i$

Find standard notation. [8.3]

20. $2\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

21. $12(\cos 30° + i\sin 30°)$

22. $\sqrt{5}(\cos 0° + i\sin 0°)$

23. $4\left[\cos\left(-\frac{3\pi}{2}\right) + i\sin\left(-\frac{3\pi}{2}\right)\right]$

Multiply or divide and leave the answer in trigonometric notation. [8.3]

24. $8(\cos 20° + i\sin 20°) \cdot 2(\cos 25° + i\sin 25°)$

25. $3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) \div \frac{1}{3}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

Convert to trigonometric notation and then multiply or divide. [8.3]

26. $(1 - i)\left(\sqrt{3} - i\right)$

27. $\frac{1 - \sqrt{3}i}{1 + i}$

28. Raise $(1 - i)^7$ to the indicated power and write trigonometric notation for the answer. [8.3]

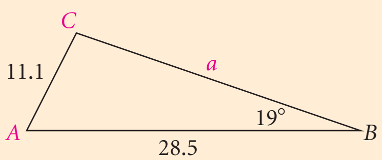
29. Raise $[2(\cos 15° + i\sin 15°)]^4$ to the indicated power and write standard notation for the answer. [8.3]

30. Find the square roots of $-2 - 2\sqrt{3}i$. [8.3]

31. Find the cube roots of -1. [8.3]

Collaborative Discussion and Writing

32. Try to solve this triangle using the law of cosines. Then explain why it is easier to solve it using the law of sines. [8.2]



33. Explain why these statements are not contradictory:

The number 1 has one real cube root.
The number 1 has three complex cube roots.
[8.3]

34. Explain why we cannot solve a triangle given SAS with the law of sines. [8.2]

35. Explain why the law of sines cannot be used to find the first angle when solving a triangle given three sides. [8.1]

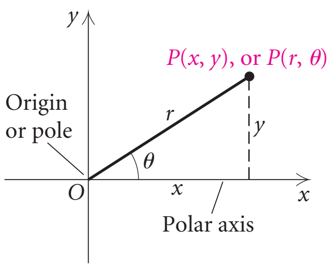
36. Explain why trigonometric notation for a complex number is not unique, but rectangular, or standard, notation is unique. [8.3]

37. Explain why $x^6 - 2x^3 + 1 = 0$ has 3 distinct solutions, $x^6 - 2x^3 = 0$ has 4 distinct solutions, and $x^6 - 2x = 0$ has 6 distinct solutions. [8.3]

Polar Coordinates and Graphs

8.4

- Graph points given their polar coordinates.
- Convert from rectangular coordinates to polar coordinates and from polar coordinates to rectangular coordinates.
- Convert from rectangular equations to polar equations and from polar equations to rectangular equations.
- Graph polar equations.



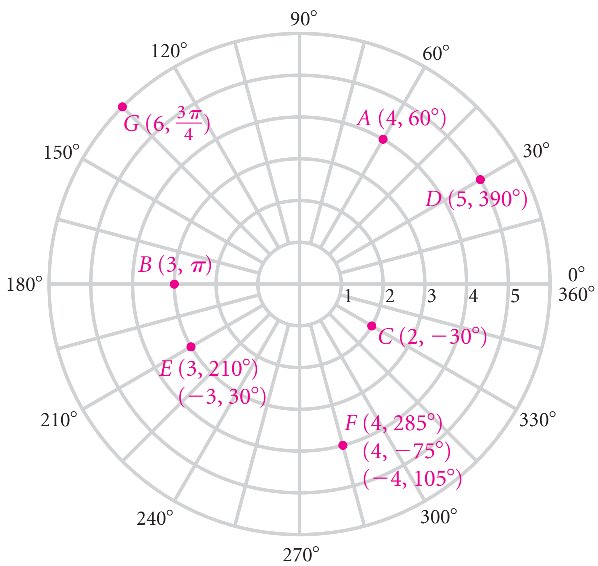
Polar Coordinates

All graphing throughout this text has been done with rectangular coordinates, (x, y), in the Cartesian coordinate system. We now introduce the polar coordinate system. As shown in the diagram at left, any point P has rectangular coordinates (x, y) and polar coordinates (r, θ). On a polar graph, the origin is called the **pole**, and the positive half of the x-axis is called the **polar axis**. The point P can be plotted given the directed angle θ from the polar axis to the ray OP and the directed distance r from the pole to the point. The angle θ can be expressed in degrees or radians.

To plot points on a polar graph:

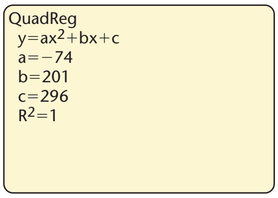
1. Locate the directed angle θ.
2. Move a directed distance r from the pole. If $r > 0$, move along ray OP. If $r < 0$, move in the opposite direction of ray OP.

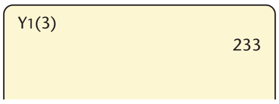
Polar graph paper, shown below, facilitates plotting. Points B and G illustrate that θ may be in radians. Points E and F illustrate that the polar coordinates of a point are not unique.



TECHNOLOGY CONNECTION

The function in Example 4 can also be found using the QUADRATIC REGRESSION feature on a graphing calculator. Note that the method of Example 4 works when we have exactly three data points, whereas the QUADRATIC REGRESSION feature on a graphing calculator can be used for *three or more* points.





Solution We let $x =$ the number of years after 2006 and $s(x) =$ snow-sports sales. Then $x = 0$ corresponds to 2006, $x = 1$ corresponds to 2007, and $x = 2$ corresponds to 2008. We use the three data points $(0, 296)$, $(1, 423)$, and $(2, 402)$ to find a, b, and c in the function $f(x) = ax^2 + bx + c$.

First, we substitute:

$$\begin{aligned} & f(x) = ax^2 + bx + c \\ \text{For } (0, 296)\text{:}\quad & 296 = a \cdot 0^2 + b \cdot 0 + c, \\ \text{For } (1, 423)\text{:}\quad & 423 = a \cdot 1^2 + b \cdot 1 + c, \\ \text{For } (2, 402)\text{:}\quad & 402 = a \cdot 2^2 + b \cdot 2 + c. \end{aligned}$$

We now have a system of equations in the variables a, b, and c:

$$\begin{aligned} c &= 296, \\ a + b + c &= 423, \\ 4a + 2b + c &= 402. \end{aligned}$$

Solving this system, we get $(-74, 201, 296)$.

Thus,

$$f(x) = -74x^2 + 201x + 296.$$

To estimate snow-sports sales in 2009, we find $f(3)$, since 2009 is 3 yr after 2006:

$$\begin{aligned} f(3) &= -74 \cdot 3^2 + 201 \cdot 3 + 296 \\ &= \$233 \text{ million}. \end{aligned}$$

Now Try Exercise 33.

9.2 Exercise Set

Solve the system of equations.

1. $$\begin{aligned} x + y + z &= 2, \\ 6x - 4y + 5z &= 31, \\ 5x + 2y + 2z &= 13 \end{aligned}$$

2. $$\begin{aligned} x + 6y + 3z &= 4, \\ 2x + y + 2z &= 3, \\ 3x - 2y + z &= 0 \end{aligned}$$

3. $$\begin{aligned} x - y + 2z &= -3, \\ x + 2y + 3z &= 4, \\ 2x + y + z &= -3 \end{aligned}$$

4. $$\begin{aligned} x + y + z &= 6, \\ 2x - y - z &= -3, \\ x - 2y + 3z &= 6 \end{aligned}$$

5. $$\begin{aligned} x + 2y - z &= 5, \\ 2x - 4y + z &= 0, \\ 3x + 2y + 2z &= 3 \end{aligned}$$

6. $$\begin{aligned} 2x + 3y - z &= 1, \\ x + 2y + 5z &= 4, \\ 3x - y - 8z &= -7 \end{aligned}$$

7. $$\begin{aligned} x + 2y - z &= -8, \\ 2x - y + z &= 4, \\ 8x + y + z &= 2 \end{aligned}$$

8. $$\begin{aligned} x + 2y - z &= 4, \\ 4x - 3y + z &= 8, \\ 5x - y &= 12 \end{aligned}$$

9. $$\begin{aligned} 2x + y - 3z &= 1, \\ x - 4y + z &= 6, \\ 4x - 7y - z &= 13 \end{aligned}$$

10. $$\begin{aligned} x + 3y + 4z &= 1, \\ 3x + 4y + 5z &= 3, \\ x + 8y + 11z &= 2 \end{aligned}$$

11. $$\begin{aligned} 4a + 9b &= 8, \\ 8a + 6c &= -1, \\ 6b + 6c &= -1 \end{aligned}$$

12. $$\begin{aligned} 3p + 2r &= 11, \\ q - 7r &= 4, \\ p - 6q &= 1 \end{aligned}$$

13. $$\begin{aligned} 2x + z &= 1, \\ 3y - 2z &= 6, \\ x - 2y &= -9 \end{aligned}$$

14. $$\begin{aligned} 3x + 4z &= -11, \\ x - 2y &= 5, \\ 4y - z &= -10 \end{aligned}$$

15. $$\begin{aligned} w + x + y + z &= 2, \\ w + 2x + 2y + 4z &= 1, \\ -w + x - y - z &= -6, \\ -w + 3x + y - z &= -2 \end{aligned}$$

16. $$\begin{aligned} w + x - y + z &= 0, \\ -w + 2x + 2y + z &= 5, \\ -w + 3x + y - z &= -4, \\ -2w + x + y - 3z &= -7 \end{aligned}$$

17. *Winter Olympic Sites.* The Winter Olympics have been held a total of 21 times on the continents of North America, Europe, and Asia. The number of

European sites is 5 more than the total number of sites in North America and Asia. There are 4 more sites in North America than in Asia. (*Source*: *USA Today* research) Find the number of Winter Olympic sites on each continent.

18. *Top Apple Growers.* The top three apple growers in the world—China, the United States, and Turkey—grew a total of about 74 billion lb of apples in a recent year. China produced 44 billion lb more than the combined production of the United States and Turkey. The United States produced twice as many pounds of apples as Turkey. (*Source*: U.S. Apple Association) Find the number of pounds of apples produced by each country.

19. *Restaurant Meals.* The total number of restaurant-purchased meals that the average person will eat in a restaurant, in a car, or at home in a year is 170. The total number of these meals eaten in a car or at home exceeds the number eaten in a restaurant by 14. Twenty more restaurant-purchased meals will be eaten in a restaurant than at home. (*Source*: The NPD Group) Find the number of restaurant-purchased meals eaten in a restaurant, the number eaten in a car, and the number eaten at home.

20. *Adopting Abroad.* The three foreign countries from which the largest number of children were adopted in 2009 were China, Ethiopia, and Russia. A total of 6864 children were adopted from these countries. The number of children adopted from China was 862 fewer than the total number adopted from Ethiopia and Russia. Twice the number adopted from Russia is 171 more than the number adopted from China. (*Source*: U.S. Department of State) Find the number of children adopted from each country.

21. *Jolts of Caffeine.* One 8-oz serving each of brewed coffee, Red Bull energy drink, and Mountain Dew soda contains 197 mg of caffeine. One serving of brewed coffee has 6 mg more caffeine than two servings of Mountain Dew. One serving of Red Bull contains 37 mg less caffeine than one serving each of brewed coffee and Mountain Dew. (*Source*: Australian Institute of Sport) Find the amount of caffeine in one serving of each beverage.

22. *Mother's Day Spending.* The top three Mother's Day gifts are flowers, jewelry, and gift certificates. The total of the average amounts spent on these gifts is $53.42. The average amount spent on jewelry is $4.40 more than the average amount spent on gift certificates. Together, the average amounts spent on flowers and gift certificates is $15.58 more than the average amount spent on jewelry. (*Source*: BIGresearch) What is the average amount spent on each type of gift?

23. *Favorite Pets.* Americans own a total of about 355 million fish, cats, and dogs as pets. The number of fish owned is 11 million more than the total number of cats and dogs owned, and 16 million more cats are owned than dogs. (*Source*: American Pet Products Manufacturers Association) How many of each type of pet do Americans own?

24. *Mail-Order Business.* Natural Fibers Clothing charges $4 for shipping orders of $25 or less, $8 for orders from $25.01 to $75, and $10 for orders over $75. One week shipping charges for 600 orders totaled $4280. Eighty more orders for $25 or less were shipped than orders for more than $75. Find the number of orders shipped at each rate.

25. *Biggest Weekend at the Box Office.* At the time of this writing, the movies *The Dark Knight*, *Spider-Man 3*, and *The Twilight Saga: New Moon* hold the record for having the three highest-grossing weekends at the box office, with a total of $452 million. Together, *Spider-Man 3* and *New Moon* earned $136 million more than *The Dark Knight*. *New Moon* earned $15 million less than *The Dark Knight*. (*Source*: the-numbers.com) Find the amount earned by each movie.

26. *Spring Cleaning.* In a group of 100 adults, 70 say they are most likely to do spring housecleaning in March, April, or May. Of these 70, the number who clean in April is 14 more than the total number who clean in March and May. The total number who clean in April and May is 2 more than three times the number who clean in March. (*Source*: Zoomerang online survey) Find the number who clean in each month.

27. *Nutrition.* A hospital dietician must plan a lunch menu that provides 485 Cal, 41.5 g of carbohydrates, and 35 mg of calcium. A 3-oz serving of broiled ground beef contains 245 Cal, 0 g of carbohydrates, and 9 mg of calcium. One baked potato contains 145 Cal, 34 g of carbohydrates, and 8 mg of calcium. A one-cup serving of strawberries contains 45 Cal, 10 g of carbohydrates, and 21 mg of calcium. (*Source*: *Home and Garden Bulletin No. 72*, U.S. Government Printing Office, Washington, D.C. 20402) How many servings of each are required to provide the desired nutritional values?

28. *Nutrition.* A diabetic patient wishes to prepare a meal consisting of roasted chicken breast, mashed potatoes, and peas. A 3-oz serving of roasted skinless chicken breast contains 140 Cal, 27 g of protein, and 64 mg of sodium. A one-cup serving of mashed potatoes contains 160 Cal, 4 g of protein, and 636 mg of sodium, and a one-cup serving of peas contains 125 Cal, 8 g of protein, and 139 mg of sodium. (*Source*: *Home and Garden Bulletin No. 72*, U.S. Government Printing Office, Washington, D.C. 20402) How many servings of each should be used if the meal is to contain 415 Cal, 50.5 g of protein, and 553 mg of sodium?

29. *Investment.* Jamal earns a year-end bonus of $5000 and puts it in 3 one-year investments that pay $243 in simple interest. Part is invested at 3%, part at 4%, and part at 6%. There is $1500 more invested at 6% than at 3%. Find the amount invested at each rate.

30. *Investment.* Casey receives $126 per year in simple interest from three investments. Part is invested at 2%, part at 3%, and part at 4%. There is $500 more invested at 3% than at 2%. The amount invested at 4% is three times the amount invested at 3%. Find the amount invested at each rate.

31. *Price Increases.* Orange juice, a raisin bagel, and a cup of coffee from Kelly's Koffee Kart cost a total of $5.35. Kelly posts a notice announcing that, effective the following week, the price of orange juice will increase 25% and the price of bagels will increase 20%. After the increase, the same purchase will cost a total of $6.20, and orange juice will cost 50¢ more than coffee. Find the price of each item before the increase.

32. *Cost of Snack Food.* Martin and Eva pool their loose change to buy snacks on their coffee break. One day, they spent \$6.75 on 1 carton of milk, 2 donuts, and 1 cup of coffee. The next day, they spent \$8.50 on 3 donuts and 2 cups of coffee. The third day, they bought 1 carton of milk, 1 donut, and 2 cups of coffee and spent \$7.25. On the fourth day, they have a total of \$6.45 left. Is this enough to buy 2 cartons of milk and 2 donuts?

33. *Job Loss.* The table below lists the percent of American workers who responded that they were likely to be laid off from their jobs in the coming year, represented in terms of the number of years after 1990.

Year, x	Percent Who Say Lay-off Likely
1990, 0	16
1997, 7	9
2010, 20	21

Source: Gallup Poll

a) Fit a quadratic function $f(x) = ax^2 + bx + c$ to the data.

b) Use the function to estimate the percent of workers who responded that they were likely to be laid off in the coming year in 2003.

34. *Student Loans.* The table below lists the volume of nonfederal student loans, in billions of dollars, represented in terms of the number of years after 2004.

Year, x	Volume of Student Loans (in billions)
2004, 0	\$15.1
2006, 2	20.5
2008, 4	11.0

Source: Trends in Student Aid

a) Fit a quadratic function $f(x) = ax^2 + bx + c$ to the data.

b) Use the function to estimate the volume of nonfederal student loans in 2007.

35. *Farm Acreage.* The table below lists the average size of U.S. farms, represented in terms of the number of years since 1997.

Year, x	Average Size of U.S. Farms (in number of acres)
1997, 0	431
2002, 5	441
2007, 10	418

Source: 2007 Census of Agriculture

a) Fit a quadratic function $f(x) = ax^2 + bx + c$ to the data.

b) Use the function to estimate the average size of U.S. farms in 2009.

36. *Book Shipments.* The table below lists the number of books shipped by publishers, in billions, represented in terms of the number of years after 2007.

Year, x	Books Shipped (in billions)
2007, 0	3.127
2008, 1	3.079
2009, 2	3.101

Source: Book Industry Study Group, Inc.

a) Fit a quadratic function $f(x) = ax^2 + bx + c$ to the data.

b) Use the function to estimate the number of books shipped in 2012.

Skill Maintenance

In each of Exercises 37–44, fill in the blank with the correct term. Some of the given choices will not be used.

Descartes' rule of signs
the leading-term test
the intermediate value theorem
the fundamental theorem of algebra
a polynomial function
a rational function
a one-to-one function
a constant function
a horizontal asymptote
a vertical asymptote
an oblique asymptote
direct variation
inverse variation
a horizontal line
a vertical line
parallel
perpendicular

37. Two lines with slopes m_1 and m_2 are ____________ if and only if the product of their slopes is -1.

38. We can use ____________ to determine the behavior of the graph of a polynomial function as $x \to \infty$ or as $x \to -\infty$.

39. If it is possible for ____________ to cross a graph more than once, then the graph is not the graph of a function.

40. A function is ____________ if different inputs have different outputs.

41. ____________ is a function that is a quotient of two polynomials.

42. If a situation gives rise to a function $f(x) = k/x$, or $y = k/x$, where k is a positive constant, we say that we have ____________.

43. ____________ of a rational function $p(x)/q(x)$, where $p(x)$ and $q(x)$ have no common factors other than constants, occurs at an x-value that makes the denominator 0.

44. When the numerator and the denominator of a rational function have the same degree, the graph of the function has ____________.

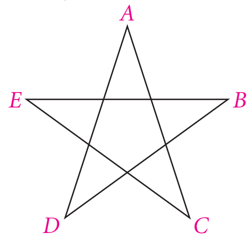
Synthesis

In Exercises 45 and 46, let u represent $1/x$, v represent $1/y$, and w represent $1/z$. Solve first for u, v, and w. Then solve the system of equations.

45. $\dfrac{2}{x} - \dfrac{1}{y} - \dfrac{3}{z} = -1,$
$\dfrac{2}{x} - \dfrac{1}{y} + \dfrac{1}{z} = -9,$
$\dfrac{1}{x} + \dfrac{2}{y} - \dfrac{4}{z} = 17$

46. $\dfrac{2}{x} + \dfrac{2}{y} - \dfrac{3}{z} = 3,$
$\dfrac{1}{x} - \dfrac{2}{y} - \dfrac{3}{z} = 9,$
$\dfrac{7}{x} - \dfrac{2}{y} + \dfrac{9}{z} = -39$

47. Find the sum of the angle measures at the tips of the star.



48. *Transcontinental Railroad.* Use the following facts to find the year in which the first U.S. transcontinental railroad was completed. The sum of the digits in the year is 24. The units digit is 1 more than the hundreds digit. Both the tens and the units digits are multiples of three.

In Exercises 49 and 50, three solutions of an equation are given. Use a system of three equations in three variables to find the constants and write the equation.

49. $Ax + By + Cz = 12;$
$\left(1, \frac{3}{4}, 3\right), \left(\frac{4}{3}, 1, 2\right)$, and $(2, 1, 1)$

50. $y = B - Mx - Nz;$
$(1, 1, 2), (3, 2, -6)$, and $\left(\frac{3}{2}, 1, 1\right)$

In Exercises 51 and 52, four solutions of the equation $y = ax^3 + bx^2 + cx + d$ are given. Use a system of four equations in four variables to find the constants a, b, c, and d and write the equation.

51. $(-2, 59), (-1, 13), (1, -1)$, and $(2, -17)$

52. $(-2, -39), (-1, -12), (1, -6)$, and $(3, 16)$

53. *Theater Attendance.* A performance at the Bingham Performing Arts Center was attended by 100 people. The audience consisted of adults, students, and children. The ticket prices were \$10 each for adults, \$3 each for students, and 50 cents each for children. The total amount of money taken in was \$100. How many adults, students, and children were in attendance? Does there seem to be some information missing? Do some careful reasoning.

9.3 Exercise Set

Determine the order of the matrix.

1. $\begin{bmatrix} 1 & -6 \\ -3 & 2 \\ 0 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 7 \\ -5 \\ -1 \\ 3 \end{bmatrix}$

3. $\begin{bmatrix} 2 & -4 & 0 & 9 \end{bmatrix}$

4. $\begin{bmatrix} -8 \end{bmatrix}$

5. $\begin{bmatrix} 1 & -5 & -8 \\ 6 & 4 & -2 \\ -3 & 0 & 7 \end{bmatrix}$

6. $\begin{bmatrix} 13 & 2 & -6 & 4 \\ -1 & 18 & 5 & -12 \end{bmatrix}$

Write the augmented matrix for the system of equations.

7. $2x - y = 7,$
$x + 4y = -5$

8. $3x + 2y = 8,$
$2x - 3y = 15$

9. $x - 2y + 3z = 12,$
$2x \quad\quad - 4z = 8,$
$3y + z = 7$

10. $x + y - z = 7,$
$3y + 2z = 1,$
$-2x - 5y \quad\quad = 6$

Write the system of equations that corresponds to the augmented matrix.

11. $\left[\begin{array}{cc|c} 3 & -5 & 1 \\ 1 & 4 & -2 \end{array}\right]$

12. $\left[\begin{array}{cc|c} 1 & 2 & -6 \\ 4 & 1 & -3 \end{array}\right]$

13. $\left[\begin{array}{ccc|c} 2 & 1 & -4 & 12 \\ 3 & 0 & 5 & -1 \\ 1 & -1 & 1 & 2 \end{array}\right]$

14. $\left[\begin{array}{ccc|c} -1 & -2 & 3 & 6 \\ 0 & 4 & 1 & 2 \\ 2 & -1 & 0 & 9 \end{array}\right]$

Solve the system of equations using Gaussian elimination or Gauss–Jordan elimination.

15. $4x + 2y = 11,$
$3x - y = 2$

16. $2x + y = 1,$
$3x + 2y = -2$

17. $5x - 2y = -3,$
$2x + 5y = -24$

18. $2x + y = 1,$
$3x - 6y = 4$

19. $3x + 4y = 7,$
$-5x + 2y = 10$

20. $5x - 3y = -2,$
$4x + 2y = 5$

21. $3x + 2y = 6,$
$2x - 3y = -9$

22. $x - 4y = 9,$
$2x + 5y = 5$

23. $x - 3y = 8,$
$-2x + 6y = 3$

24. $4x - 8y = 12,$
$-x + 2y = -3$

25. $-2x + 6y = 4,$
$3x - 9y = -6$

26. $6x + 2y = -10,$
$-3x - y = 6$

27. $x + 2y - 3z = 9,$
$2x - y + 2z = -8,$
$3x - y - 4z = 3$

28. $x - y + 2z = 0,$
$x - 2y + 3z = -1,$
$2x - 2y + z = -3$

29. $4x - y - 3z = 1,$
$8x + y - z = 5,$
$2x + y + 2z = 5$

30. $3x + 2y + 2z = 3,$
$x + 2y - z = 5,$
$2x - 4y + z = 0$

31. $x - 2y + 3z = -4,$
$3x + y - z = 0,$
$2x + 3y - 5z = 1$

32. $2x - 3y + 2z = 2,$
$x + 4y - z = 9,$
$-3x + y - 5z = 5$

33. $2x - 4y - 3z = 3,$
$x + 3y + z = -1,$
$5x + y - 2z = 2$

34. $x + y - 3z = 4,$
$4x + 5y + z = 1,$
$2x + 3y + 7z = -7$

35. $p + q + r = 1,$
$p + 2q + 3r = 4,$
$4p + 5q + 6r = 7$

36. $m + n + t = 9,$
$m - n - t = -15,$
$3m + n + t = 2$

37. $a + b - c = 7,$
$a - b + c = 5,$
$3a + b - c = -1$

38. $a - b + c = 3,$
$2a + b - 3c = 5,$
$4a + b - c = 11$

39. $-2w + 2x + 2y - 2z = -10,$
$w + x + y + z = -5,$
$3w + x - y + 4z = -2,$
$w + 3x - 2y + 2z = -6$

40. $-w + 2x - 3y + z = -8,$
$-w + x + y - z = -4,$
$w + x + y + z = 22,$
$-w + x - y - z = -14$

Use Gaussian elimination or Gauss–Jordan elimination in Exercises 41–44.

41. *Borrowing.* Ishikawa Manufacturing borrowed \$30,000 to buy a new piece of equipment. Part of the money was borrowed at 8%, part at 10%, and part at 12%. The annual interest was \$3040, and the total amount borrowed at 8% and 10% was twice the

amount borrowed at 12%. How much was borrowed at each rate?

42. *Stamp Purchase.* Ricardo spent \$22.35 on 44¢ and 17¢ stamps. He bought a total of 60 stamps. How many of each type did he buy?

43. *Time of Return.* The Houlihans pay their babysitter \$5 per hour before 11 P.M. and \$7.50 per hour after 11 P.M. One evening they went out for 5 hr and paid the sitter \$30. What time did they come home?

44. *Advertising Expense.* eAuction.com spent a total of \$11 million on advertising in fiscal years 2010, 2011, and 2012. The amount spent in 2012 was three times the amount spent in 2010. The amount spent in 2011 was \$3 million less than the amount spent in 2012. How much was spent on advertising each year?

Skill Maintenance

In Exercises 45–52, classify the function as linear, quadratic, cubic, quartic, rational, exponential, or logarithmic.

45. $f(x) = 3^{x-1}$

46. $f(x) = 3x - 1$

47. $f(x) = \dfrac{3x - 1}{x^2 + 4}$

48. $f(x) = -\frac{3}{4}x^4 + \frac{9}{2}x^3 + 2x^2 - 4$

49. $f(x) = \ln(3x - 1)$

50. $f(x) = \frac{3}{4}x^3 - x$

51. $f(x) = 3$

52. $f(x) = 2 - x - x^2$

Synthesis

In Exercises 53 and 54, three solutions of the equation $y = ax^2 + bx + c$ are given. Use a system of three equations in three variables and Gaussian elimination or Gauss–Jordan elimination to find the constants a, b, and c and write the equation.

53. $(-3, 12)$, $(-1, -7)$, and $(1, -2)$

54. $(-1, 0)$, $(1, -3)$, and $(3, -22)$

55. Find two different row-echelon forms of

$$\begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix}.$$

56. Consider the system of equations

$$\begin{aligned} x - y + 3z &= -8, \\ 2x + 3y - z &= 5, \\ 3x + 2y + 2kz &= -3k. \end{aligned}$$

For what value(s) of k, if any, will the system have:

a) no solution?
b) exactly one solution?
c) infinitely many solutions?

Solve using matrices.

57. $$\begin{aligned} y &= x + z, \\ 3y + 5z &= 4, \\ x + 4 &= y + 3z \end{aligned}$$

58. $$\begin{aligned} x + y &= 2z, \\ 2x - 5z &= 4, \\ x - z &= y + 8 \end{aligned}$$

59. $$\begin{aligned} x - 4y + 2z &= 7, \\ 3x + y + 3z &= -5 \end{aligned}$$

60. $$\begin{aligned} x - y - 3z &= 3, \\ -x + 3y + z &= -7 \end{aligned}$$

61. $$\begin{aligned} 4x + 5y &= 3, \\ -2x + y &= 9, \\ 3x - 2y &= -15 \end{aligned}$$

62. $$\begin{aligned} 2x - 3y &= -1, \\ -x + 2y &= -2, \\ 3x - 5y &= 1 \end{aligned}$$

9.4 Exercise Set

Find x and y.

1. $\begin{bmatrix} 5 & x \end{bmatrix} = \begin{bmatrix} y & -3 \end{bmatrix}$

2. $\begin{bmatrix} 6x \\ 25 \end{bmatrix} = \begin{bmatrix} -9 \\ 5y \end{bmatrix}$

3. $\begin{bmatrix} 3 & 2x \\ y & -8 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & -8 \end{bmatrix}$

4. $\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$

For Exercises 5–20, let

$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -3 & 5 \\ 2 & -1 \end{bmatrix}$,

$\mathbf{C} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$,

$\mathbf{E} = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$, $\mathbf{F} = \begin{bmatrix} 3 & 3 \\ -1 & -1 \end{bmatrix}$,

$\mathbf{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Find each of the following.

5. $\mathbf{A} + \mathbf{B}$

6. $\mathbf{B} + \mathbf{A}$

7. $\mathbf{E} + \mathbf{0}$

8. $2\mathbf{A}$

9. $3\mathbf{F}$

10. $(-1)\mathbf{D}$

11. $3\mathbf{F} + 2\mathbf{A}$

12. $\mathbf{A} - \mathbf{B}$

13. $\mathbf{B} - \mathbf{A}$

14. $\mathbf{AB}$

15. $\mathbf{BA}$

16. $\mathbf{0F}$

17. $\mathbf{CD}$

18. $\mathbf{EF}$

19. $\mathbf{AI}$

20. $\mathbf{IA}$

Find the product, if possible.

21. $\begin{bmatrix} -1 & 0 & 7 \\ 3 & -5 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ -4 \\ 1 \end{bmatrix}$

22. $\begin{bmatrix} 6 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -2 & 0 \\ 5 & -3 \end{bmatrix}$

23. $\begin{bmatrix} -2 & 4 \\ 5 & 1 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ -1 & 4 \end{bmatrix}$

24. $\begin{bmatrix} 2 & -1 & 0 \\ 0 & 5 & 4 \end{bmatrix} \begin{bmatrix} -3 & 1 & 0 \\ 0 & 2 & -1 \\ 5 & 0 & 4 \end{bmatrix}$

25. $\begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix} \begin{bmatrix} -6 & 5 & 8 \\ 0 & 4 & -1 \end{bmatrix}$

26. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & -4 & 3 \\ 2 & 1 & 0 \\ -1 & 0 & 6 \end{bmatrix}$

27. $\begin{bmatrix} 1 & -4 & 3 \\ 0 & 8 & 0 \\ -2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

28. $\begin{bmatrix} 4 \\ -5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 6 & -7 \\ 0 & -3 \end{bmatrix}$

29. *Budget.* For the month of June, Nelia budgets \$300 for food, \$80 for clothes, and \$40 for entertainment.

a) Write a 1×3 matrix **B** that represents the amounts budgeted for these items.

b) After receiving a raise, Nelia increases the amount budgeted for each item in July by 5%. Find a matrix **R** that represents the new amounts.

c) Find **B** + **R** and tell what the entries represent.

30. *Produce.* The produce manager at Dugan's Market orders 40 lb of tomatoes, 20 lb of zucchini, and 30 lb of onions from a local farmer one week.

a) Write a 1×3 matrix **A** that represents the amount of each item ordered.

b) The following week the produce manager increases her order by 10%. Find a matrix **B** that represents this order.

c) Find **A** + **B** and tell what the entries represent.

31. *Nutrition.* A 3-oz serving of roasted, skinless chicken breast contains 140 Cal, 27 g of protein, 3 g of fat, 13 mg of calcium, and 64 mg of sodium. One-half cup of potato salad contains 180 Cal, 4 g of protein, 11 g of fat, 24 mg of calcium, and 662 mg of sodium. One broccoli spear contains 50 Cal, 5 g of protein, 1 g of fat, 82 mg of calcium, and 20 mg of sodium. (*Source*: *Home and Garden Bulletin No. 72*, U.S. Government Printing Office, Washington, D.C. 20402)

a) Write 1×5 matrices **C**, **P**, and **B** that represent the nutritional values of each food.

b) Find $\mathbf{C} + 2\mathbf{P} + 3\mathbf{B}$ and tell what the entries represent.

32. *Nutrition.* One slice of cheese pizza contains 290 Cal, 15 g of protein, 9 g of fat, and 39 g of carbohydrates. One-half cup of gelatin dessert contains 70 Cal, 2 g of protein, 0 g of fat, and 17 g of carbohydrates. One cup of whole milk contains 150 Cal, 8 g of protein, 8 g of fat, and 11 g of carbohydrates. (*Source*: *Home and Garden Bulletin No. 72*, U.S. Government Printing Office, Washington, D.C. 20402)

a) Write 1×4 matrices **P**, **G**, and **M** that represent the nutritional values of each food.

b) Find $3\mathbf{P} + 2\mathbf{G} + 2\mathbf{M}$ and tell what the entries represent.

33. *Food Service Management.* The food service manager at a large hospital is concerned about maintaining reasonable food costs. The table below lists the cost per serving, in dollars, for items on four menus.

Menu	Meat	Potato	Vegetable	Salad	Dessert
1	1.50	0.15	0.26	0.23	0.64
2	1.55	0.14	0.24	0.21	0.75
3	1.62	0.22	0.31	0.28	0.53
4	1.70	0.20	0.29	0.33	0.68

On a particular day, a dietician orders 65 meals from menu 1, 48 from menu 2, 93 from menu 3, and 57 from menu 4.

a) Write the information in the table as a 4×5 matrix **M**.

b) Write a row matrix **N** that represents the number of each menu ordered.

c) Find the product **NM**.

d) State what the entries of **NM** represent.

34. *Food Service Management.* A college food service manager uses a table like the one below to list the number of units of ingredients, by weight, required for various menu items.

	White Cake	Bread	Coffee Cake	Sugar Cookies
Flour	1	2.5	0.75	0.5
Milk	0	0.5	0.25	0
Eggs	0.75	0.25	0.5	0.5
Butter	0.5	0	0.5	1

The cost per unit of each ingredient is 25 cents for flour, 34 cents for milk, 54 cents for eggs, and 83 cents for butter.

a) Write the information in the table as a 4×4 matrix **M**.

b) Write a row matrix **C** that represents the cost per unit of each ingredient.

c) Find the product **CM**.

d) State what the entries of **CM** represent.

35. *Production Cost.* Karin supplies two small campus coffee shops with homemade chocolate chip cookies, oatmeal cookies, and peanut butter cookies. The table below shows the number of each type of cookie, in dozens, that Karin sold in one week.

	Mugsy's Coffee Shop	The Coffee Club
Chocolate Chip	8	15
Oatmeal	6	10
Peanut Butter	4	3

Karin spends \$4 for the ingredients for one dozen chocolate chip cookies, \$2.50 for the ingredients for one dozen oatmeal cookies, and \$3 for the ingredients for one dozen peanut butter cookies.

a) Write the information in the table as a 3×2 matrix **S**.

b) Write a row matrix **C** that represents the cost, per dozen, of the ingredients for each type of cookie.

c) Find the product **CS**.

d) State what the entries of **CS** represent.

36. *Profit.* A manufacturer produces exterior plywood, interior plywood, and fiberboard, which are shipped to two distributors. The table below shows the number of units of each type of product that are shipped to each warehouse.

	Distributor 1	Distributor 2
Exterior Plywood	900	500
Interior Plywood	450	1000
Fiberboard	600	700

The profits from each unit of exterior plywood, interior plywood, and fiberboard are \$5, \$8, and \$4, respectively.

a) Write the information in the table as a 3×2 matrix **M**.
b) Write a row matrix **P** that represents the profit, per unit, of each type of product.
c) Find the product **PM**.
d) State what the entries of **PM** represent.

37. *Profit.* In Exercise 35, suppose that Karin's profits on one dozen chocolate chip, oatmeal, and peanut butter cookies are \$6, \$4.50, and \$5.20, respectively.

a) Write a row matrix **P** that represents this information.
b) Use the matrices **S** and **P** to find Karin's total profit from each coffee shop.

38. *Production Cost.* In Exercise 36, suppose that the manufacturer's production costs for each unit of exterior plywood, interior plywood, and fiberboard are \$20, \$25, and \$15, respectively.

a) Write a row matrix **C** that represents this information.
b) Use the matrices **M** and **C** to find the total production cost for the products shipped to each distributor.

Write a matrix equation equivalent to the system of equations.

39. $2x - 3y = 7,$
$x + 5y = -6$

40. $-x + y = 3,$
$5x - 4y = 16$

41. $x + y - 2z = 6,$
$3x - y + z = 7,$
$2x + 5y - 3z = 8$

42. $3x - y + z = 1,$
$x + 2y - z = 3,$
$4x + 3y - 2z = 11$

43. $3x - 2y + 4z = 17,$
$2x + y - 5z = 13$

44. $3x + 2y + 5z = 9,$
$4x - 3y + 2z = 10$

45. $-4w + x - y + 2z = 12,$
$w + 2x - y - z = 0,$
$-w + x + 4y - 3z = 1,$
$2w + 3x + 5y - 7z = 9$

46. $12w + 2x + 4y - 5z = 2,$
$-w + 4x - y + 12z = 5,$
$2w - x + 4y = 13,$
$2x + 10y + z = 5$

Skill Maintenance

In Exercises 47–50:

a) *Find the vertex.*
b) *Find the axis of symmetry.*
c) *Determine whether there is a maximum or minimum value and find that value.*
d) *Graph the function.*

47. $f(x) = x^2 - x - 6$

48. $f(x) = 2x^2 - 5x - 3$

49. $f(x) = -x^2 - 3x + 2$

50. $f(x) = -3x^2 + 4x + 4$

Synthesis

For Exercises 51–54, let

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \quad \textit{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}.$$

51. Show that

$$(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B}) \neq \mathbf{A}^2 - \mathbf{B}^2,$$

where

$$\mathbf{A}^2 = \mathbf{AA} \quad \text{and} \quad \mathbf{B}^2 = \mathbf{BB}.$$

52. Show that
$$(\mathbf{A}+\mathbf{B})(\mathbf{A}+\mathbf{B}) \neq \mathbf{A}^2 + 2\mathbf{AB} + \mathbf{B}^2.$$

53. Show that
$$(\mathbf{A}+\mathbf{B})(\mathbf{A}-\mathbf{B}) = \mathbf{A}^2 + \mathbf{BA} - \mathbf{AB} - \mathbf{B}^2.$$

54. Show that
$$(\mathbf{A}+\mathbf{B})(\mathbf{A}+\mathbf{B}) = \mathbf{A}^2 + \mathbf{BA} + \mathbf{AB} + \mathbf{B}^2.$$

In Exercises 55–59, let

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2n} \\ b_{31} & b_{32} & b_{33} & \cdots & b_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & b_{m3} & \cdots & b_{mn} \end{bmatrix},$$

and
$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1n} \\ c_{21} & c_{22} & c_{23} & \cdots & c_{2n} \\ c_{31} & c_{32} & c_{33} & \cdots & c_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ c_{m1} & c_{m2} & c_{m3} & \cdots & c_{mn} \end{bmatrix},$$

and let k and l be any scalars.

55. Prove that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.

56. Prove that $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$.

57. Prove that $(kl)\mathbf{A} = k(l\mathbf{A})$.

58. Prove that $k(\mathbf{A} + \mathbf{B}) = k\mathbf{A} + k\mathbf{B}$.

59. Prove that $(k + l)\mathbf{A} = k\mathbf{A} + l\mathbf{A}$.

Mid-Chapter Mixed Review

Determine whether the statement is true or false.

1. For a system of two linear equations in two variables, if the graphs of the equations are parallel lines, then the system of equations has infinitely many solutions. [9.1]
2. One of the properties of a matrix written in row-echelon form is that all the rows consisting entirely of 0's are at the bottom of the matrix. [9.3]
3. We can multiply two matrices only when the number of columns in the first matrix is equal to the number of rows in the second matrix. [9.4]
4. Addition of matrices is not commutative. [9.4]

Solve. [9.1], [9.2]

5. $2x + y = -4,$
$x = y - 5$

6. $x + y = 4,$
$y = 2 - x$

7. $2x - 3y = 8,$
$3x + 2y = -1$

8. $x - 3y = 1,$
$6y = 2x - 2$

9. $x + 2y + 3z = 4,$
$x - 2y + z = 2,$
$2x - 6y + 4z = 7$

10. *e-Commerce.* computerwarehouse.com charges \$3 for shipping orders up to 10 lb, \$5 for orders from 10 lb up to 15 lb, and \$7.50 for orders of 15 lb or more. One day shipping charges for 150 orders totaled \$680. The number of orders under 10 lb was three times the number of orders weighing 15 lb or more. Find the number of packages shipped at each rate. [9.2]

Solve the system of equations using Gaussian elimination or Gauss–Jordan elimination. [9.3]

11. $2x + y = 5,$
$3x + 2y = 6$

12. $3x + 2y - 3z = -2,$
$2x + 3y + 2z = -2,$
$x + 4y + 4z = 1$

For Exercises 13–20, let

$$\mathbf{A} = \begin{bmatrix} 3 & -1 \\ 5 & 4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -2 & 6 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -4 & 1 & -1 \\ 2 & 3 & -2 \end{bmatrix}, \quad \textit{and} \quad \mathbf{D} = \begin{bmatrix} -2 & 3 & 0 \\ 1 & -1 & 2 \\ -3 & 4 & 1 \end{bmatrix}.$$

Find each of the following. [9.4]

13. $\mathbf{A} + \mathbf{B}$

14. $\mathbf{B} - \mathbf{A}$

15. $4\mathbf{D}$

16. $2\mathbf{A} + 3\mathbf{B}$

17. $\mathbf{AB}$

18. $\mathbf{BA}$

19. $\mathbf{BC}$

20. $\mathbf{DC}$

21. Write a matrix equation equivalent to the following system of equations: [9.4]

$$\begin{aligned} 2x - y + 3z &= 7, \\ x + 2y - z &= 3, \\ 3x - 4y + 2z &= 5. \end{aligned}$$

Collaborative Discussion and Writing

22. Explain in your own words when the elimination method for solving a system of equations is preferable to the substitution method. [9.1]

23. Given two linear equations in three variables, $Ax + By + Cz = D$ and $Ex + Fy + Gz = H$, explain how you could find a third equation such that the system contains dependent equations. [9.2]

24. Explain in your own words why the augmented matrix below represents a system of dependent equations. [9.3]

$$\left[\begin{array}{rrr|r} 1 & -3 & 2 & -5 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

25. Is it true that if $\mathbf{AB} = \mathbf{0}$, for matrices $\mathbf{A}$ and $\mathbf{B}$, then $\mathbf{A} = \mathbf{0}$ or $\mathbf{B} = \mathbf{0}$? Why or why not? [9.4]

9.5 Exercise Set

Determine whether **B** *is the inverse of* **A**.

1. $\mathbf{A} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

2. $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$

3. $\mathbf{A} = \begin{bmatrix} -1 & -1 & 6 \\ 1 & 0 & -2 \\ 1 & 0 & -3 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$

4. $\mathbf{A} = \begin{bmatrix} -2 & 0 & -3 \\ 5 & 1 & 7 \\ -3 & 0 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 4 & 0 & -3 \\ 1 & 1 & 1 \\ -3 & 0 & 2 \end{bmatrix}$

Use the Gauss–Jordan method to find $\mathbf{A}^{-1}$*, if it exists. Check your answers by finding* $\mathbf{A}^{-1}\mathbf{A}$ *and* $\mathbf{A}\mathbf{A}^{-1}$.

5. $\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

6. $\mathbf{A} = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

7. $\mathbf{A} = \begin{bmatrix} 6 & 9 \\ 4 & 6 \end{bmatrix}$

8. $\mathbf{A} = \begin{bmatrix} -4 & -6 \\ 2 & 3 \end{bmatrix}$

9. $\mathbf{A} = \begin{bmatrix} 4 & -3 \\ 1 & -2 \end{bmatrix}$

10. $\mathbf{A} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

11. $\mathbf{A} = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix}$

12. $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

13. $\mathbf{A} = \begin{bmatrix} 1 & -4 & 8 \\ 1 & -3 & 2 \\ 2 & -7 & 10 \end{bmatrix}$

14. $\mathbf{A} = \begin{bmatrix} -2 & 5 & 3 \\ 4 & -1 & 3 \\ 7 & -2 & 5 \end{bmatrix}$

15. $\mathbf{A} = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 3 & 4 \\ -1 & -1 & -1 \end{bmatrix}$

16. $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & -2 \\ -1 & 3 & 3 \end{bmatrix}$

17. $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

18. $\mathbf{A} = \begin{bmatrix} 7 & -1 & -9 \\ 2 & 0 & -4 \\ -4 & 0 & 6 \end{bmatrix}$

19. $\mathbf{A} = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

20. $\mathbf{A} = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

21. $\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 3 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

22. $\mathbf{A} = \begin{bmatrix} -2 & -3 & 4 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & -2 & 5 & 1 \end{bmatrix}$

23. $\mathbf{A} = \begin{bmatrix} 1 & -14 & 7 & 38 \\ -1 & 2 & 1 & -2 \\ 1 & 2 & -1 & -6 \\ 1 & -2 & 3 & 6 \end{bmatrix}$

24. $\mathbf{A} = \begin{bmatrix} 10 & 20 & -30 & 15 \\ 3 & -7 & 14 & -8 \\ -7 & -2 & -1 & 2 \\ 4 & 4 & -3 & 1 \end{bmatrix}$

In Exercises 25–28, a system of equations is given, together with the inverse of the coefficient matrix. Use the inverse of the coefficient matrix to solve the system of equations.

25. $11x + 3y = -4,$ $\quad 7x + 2y = 5;$ $\quad \mathbf{A}^{-1} = \begin{bmatrix} 2 & -3 \\ -7 & 11 \end{bmatrix}$

26. $8x + 5y = -6,$ $\quad 5x + 3y = 2;$ $\quad \mathbf{A}^{-1} = \begin{bmatrix} -3 & 5 \\ 5 & -8 \end{bmatrix}$

27. $\begin{aligned} 3x + y &= 2, \\ 2x - y + 2z &= -5, \\ x + y + z &= 5; \end{aligned}$ $\mathbf{A}^{-1} = \frac{1}{9}\begin{bmatrix} 3 & 1 & -2 \\ 0 & -3 & 6 \\ -3 & 2 & 5 \end{bmatrix}$

28. $\begin{aligned} y - z &= -4, \\ 4x + y &= -3, \\ 3x - y + 3z &= 1; \end{aligned}$ $\mathbf{A}^{-1} = \frac{1}{5}\begin{bmatrix} -3 & 2 & -1 \\ 12 & -3 & 4 \\ 7 & -3 & 4 \end{bmatrix}$

Solve the system of equations using the inverse of the coefficient matrix of the equivalent matrix equation.

29. $\begin{aligned} 4x + 3y &= 2, \\ x - 2y &= 6 \end{aligned}$

30. $\begin{aligned} 2x - 3y &= 7, \\ 4x + y &= -7 \end{aligned}$

31. $\begin{aligned} 5x + y &= 2, \\ 3x - 2y &= -4 \end{aligned}$

32. $\begin{aligned} x - 6y &= 5, \\ -x + 4y &= -5 \end{aligned}$

33. $\begin{aligned} x + z &= 1, \\ 2x + y &= 3, \\ x - y + z &= 4 \end{aligned}$

34. $\begin{aligned} x + 2y + 3z &= -1, \\ 2x - 3y + 4z &= 2, \\ -3x + 5y - 6z &= 4 \end{aligned}$

35. $\begin{aligned} 2x + 3y + 4z &= 2, \\ x - 4y + 3z &= 2, \\ 5x + y + z &= -4 \end{aligned}$

36. $\begin{aligned} x + y &= 2, \\ 3x + 2z &= 5, \\ 2x + 3y - 3z &= 9 \end{aligned}$

37. $\begin{aligned} 2w - 3x + 4y - 5z &= 0, \\ 3w - 2x + 7y - 3z &= 2, \\ w + x - y + z &= 1, \\ -w - 3x - 6y + 4z &= 6 \end{aligned}$

38. $\begin{aligned} 5w - 4x + 3y - 2z &= -6, \\ w + 4x - 2y + 3z &= -5, \\ 2w - 3x + 6y - 9z &= 14, \\ 3w - 5x + 2y - 4z &= -3 \end{aligned}$

39. *Sales.* Kayla sold a total of 145 Italian sausages and hot dogs from her curbside pushcart and collected \$242.05. She sold 45 more hot dogs than sausages. How many of each did she sell?

40. *Price of School Supplies.* Rubio bought 4 lab record books and 3 highlighters for \$17.83. Marcus bought 3 lab record books and 2 highlighters for \$13.05. Find the price of each item.

41. *Cost.* Green-Up Landscaping bought 4 tons of topsoil, 3 tons of mulch, and 6 tons of pea gravel for \$2825. The next week the firm bought 5 tons of topsoil, 2 tons of mulch, and 5 tons of pea gravel for \$2663. Pea gravel costs \$17 less per ton than topsoil. Find the price per ton for each item.

42. *Investment.* Donna receives \$230 per year in simple interest from three investments totaling \$8500. Part is invested at 2.2%, part at 2.65%, and the rest at 3.05%. There is \$1500 more invested at 3.05% than at 2.2%. Find the amount invested at each rate.

Skill Maintenance

Use synthetic division to find the function values.

43. $f(x) = x^3 - 6x^2 + 4x - 8$; find $f(-2)$

44. $f(x) = 2x^4 - x^3 + 5x^2 + 6x - 4$; find $f(3)$

Solve.

45. $2x^2 + x = 7$

46. $\dfrac{1}{x + 1} - \dfrac{6}{x - 1} = 1$

47. $\sqrt{2x + 1} - 1 = \sqrt{2x - 4}$

48. $x - \sqrt{x} - 6 = 0$

Factor the polynomial $f(x)$.

49. $f(x) = x^3 - 3x^2 - 6x + 8$

50. $f(x) = x^4 + 2x^3 - 16x^2 - 2x + 15$

Synthesis

State the conditions under which $\mathbf{A}^{-1}$ exists. Then find a formula for $\mathbf{A}^{-1}$.

51. $\mathbf{A} = [x]$

52. $\mathbf{A} = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$

53. $\mathbf{A} = \begin{bmatrix} 0 & 0 & x \\ 0 & y & 0 \\ z & 0 & 0 \end{bmatrix}$

54. $\mathbf{A} = \begin{bmatrix} x & 1 & 1 & 1 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & w \end{bmatrix}$

In practice, it is not necessary to evaluate D_z. When we have found values for x and y, we can substitute them into one of the equations to find z.

Now Try Exercise 37.

9.6 Exercise Set

Evaluate the determinant.

1. $\begin{vmatrix} 5 & 3 \\ -2 & -4 \end{vmatrix}$

2. $\begin{vmatrix} -8 & 6 \\ -1 & 2 \end{vmatrix}$

3. $\begin{vmatrix} 4 & -7 \\ -2 & 3 \end{vmatrix}$

4. $\begin{vmatrix} -9 & -6 \\ 5 & 4 \end{vmatrix}$

5. $\begin{vmatrix} -2 & -\sqrt{5} \\ -\sqrt{5} & 3 \end{vmatrix}$

6. $\begin{vmatrix} \sqrt{5} & -3 \\ 4 & 2 \end{vmatrix}$

7. $\begin{vmatrix} x & 4 \\ x & x^2 \end{vmatrix}$

8. $\begin{vmatrix} y^2 & -2 \\ y & 3 \end{vmatrix}$

Use the following matrix for Exercises 9–16:

$$\mathbf{A} = \begin{bmatrix} 7 & -4 & -6 \\ 2 & 0 & -3 \\ 1 & 2 & -5 \end{bmatrix}.$$

9. Find M_{11}, M_{32}, and M_{22}.

10. Find M_{13}, M_{31}, and M_{23}.

11. Find A_{11}, A_{32}, and A_{22}.

12. Find A_{13}, A_{31}, and A_{23}.

13. Evaluate $|\mathbf{A}|$ by expanding across the second row.

14. Evaluate $|\mathbf{A}|$ by expanding down the second column.

15. Evaluate $|\mathbf{A}|$ by expanding down the third column.

16. Evaluate $|\mathbf{A}|$ by expanding across the first row.

Use the following matrix for Exercises 17–22:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 4 & 1 & 0 & 0 \\ 5 & 6 & 7 & 8 \\ -2 & -3 & -1 & 0 \end{bmatrix}.$$

17. Find M_{12} and M_{44}.

18. Find M_{41} and M_{33}.

19. Find A_{22} and A_{34}.

20. Find A_{24} and A_{43}.

21. Evaluate $|\mathbf{A}|$ by expanding across the first row.

22. Evaluate $|\mathbf{A}|$ by expanding down the third column.

Evaluate the determinant.

23. $\begin{vmatrix} 3 & 1 & 2 \\ -2 & 3 & 1 \\ 3 & 4 & -6 \end{vmatrix}$

24. $\begin{vmatrix} 3 & -2 & 1 \\ 2 & 4 & 3 \\ -1 & 5 & 1 \end{vmatrix}$

25. $\begin{vmatrix} x & 0 & -1 \\ 2 & x & x^2 \\ -3 & x & 1 \end{vmatrix}$

26. $\begin{vmatrix} x & 1 & -1 \\ x^2 & x & x \\ 0 & x & 1 \end{vmatrix}$

Solve using Cramer's rule.

27. $-2x + 4y = 3,$
$3x - 7y = 1$

28. $5x - 4y = -3,$
$7x + 2y = 6$

29. $2x - y = 5,$
$x - 2y = 1$

30. $3x + 4y = -2,$
$5x - 7y = 1$

31. $2x + 9y = -2,$
$4x - 3y = 3$

32. $2x + 3y = -1,$
$3x + 6y = -0.5$

33. $2x + 5y = 7,$
$3x - 2y = 1$

34. $3x + 2y = 7,$
$2x + 3y = -2$

35. $3x + 2y - z = 4,$
$3x - 2y + z = 5,$
$4x - 5y - z = -1$

36. $3x - y + 2z = 1,$
$x - y + 2z = 3,$
$-2x + 3y + z = 1$

37. $3x + 5y - z = -2,$
$x - 4y + 2z = 13,$
$2x + 4y + 3z = 1$

38. $3x + 2y + 2z = 1,$
$5x - y - 6z = 3,$
$2x + 3y + 3z = 4$

39. $x - 3y - 7z = 6,$
$2x + 3y + z = 9,$
$4x + y = 7$

40. $x - 2y - 3z = 4,$
$3x - 2z = 8,$
$2x + y + 4z = 13$

41. $6y + 6z = -1,$
$8x + 6z = -1,$
$4x + 9y = 8$

42. $3x + 5y = 2,$
$2x - 3z = 7,$
$4y + 2z = -1$

Skill Maintenance

Determine whether the function is one-to-one, and if it is, find a formula for $f^{-1}(x)$.

43. $f(x) = 3x + 2$

44. $f(x) = x^2 - 4$

45. $f(x) = |x| + 3$

46. $f(x) = \sqrt[3]{x} + 1$

Simplify. Write answers in the form $a + bi$, where a and b are real numbers.

47. $(3 - 4i) - (-2 - i)$

48. $(5 + 2i) + (1 - 4i)$

49. $(1 - 2i)(6 + 2i)$

50. $\dfrac{3 + i}{4 - 3i}$

Synthesis

Solve.

51. $\begin{vmatrix} x & 5 \\ -4 & x \end{vmatrix} = 24$

52. $\begin{vmatrix} y & 2 \\ 3 & y \end{vmatrix} = y$

53. $\begin{vmatrix} x & -3 \\ -1 & x \end{vmatrix} \geq 0$

54. $\begin{vmatrix} y & -5 \\ -2 & y \end{vmatrix} < 0$

55. $\begin{vmatrix} x + 3 & 4 \\ x - 3 & 5 \end{vmatrix} = -7$

56. $\begin{vmatrix} m + 2 & -3 \\ m + 5 & -4 \end{vmatrix} = 3m - 5$

57. $\begin{vmatrix} 2 & x & 1 \\ 1 & 2 & -1 \\ 3 & 4 & -2 \end{vmatrix} = -6$

58. $\begin{vmatrix} x & 2 & x \\ 3 & -1 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -10$

Rewrite the expression using a determinant. Answers may vary.

59. $2L + 2W$

60. $\pi r + \pi h$

61. $a^2 + b^2$

62. $\frac{1}{2}h(a + b)$

63. $2\pi r^2 + 2\pi rh$

64. $x^2y^2 - Q^2$

EXAMPLE 5 Decompose into partial fractions:

$$\frac{11x^2 - 8x - 7}{(2x^2 - 1)(x - 3)}.$$

Solution The decomposition has the following form:

$$\frac{11x^2 - 8x - 7}{(2x^2 - 1)(x - 3)} = \frac{Ax + B}{2x^2 - 1} + \frac{C}{x - 3}.$$

Adding and equating numerators, we get

$$\begin{aligned} 11x^2 - 8x - 7 &= (Ax + B)(x - 3) + C(2x^2 - 1) \\ &= Ax^2 - 3Ax + Bx - 3B + 2Cx^2 - C, \end{aligned}$$

or $\quad 11x^2 - 8x - 7 = (A + 2C)x^2 + (-3A + B)x + (-3B - C).$

We then equate corresponding coefficients:

$$\begin{aligned} 11 &= A + 2C, && \text{The coefficients of the } x^2\text{-terms} \\ -8 &= -3A + B, && \text{The coefficients of the } x\text{-terms} \\ -7 &= -3B - C. && \text{The constant terms} \end{aligned}$$

We solve this system of three equations and obtain

$$A = 3, \quad B = 1, \quad \text{and} \quad C = 4.$$

The decomposition is as follows:

$$\frac{11x^2 - 8x - 7}{(2x^2 - 1)(x - 3)} = \frac{3x + 1}{2x^2 - 1} + \frac{4}{x - 3}.$$

Now Try Exercise 13.

9.8 Exercise Set

Decompose into partial fractions.

1. $\dfrac{x + 7}{(x - 3)(x + 2)}$

2. $\dfrac{2x}{(x + 1)(x - 1)}$

3. $\dfrac{7x - 1}{6x^2 - 5x + 1}$

4. $\dfrac{13x + 46}{12x^2 - 11x - 15}$

5. $\dfrac{3x^2 - 11x - 26}{(x^2 - 4)(x + 1)}$

6. $\dfrac{5x^2 + 9x - 56}{(x - 4)(x - 2)(x + 1)}$

7. $\dfrac{9}{(x + 2)^2(x - 1)}$

8. $\dfrac{x^2 - x - 4}{(x - 2)^3}$

9. $\dfrac{2x^2 + 3x + 1}{(x^2 - 1)(2x - 1)}$

10. $\dfrac{x^2 - 10x + 13}{(x^2 - 5x + 6)(x - 1)}$

11. $\dfrac{x^4 - 3x^3 - 3x^2 + 10}{(x + 1)^2(x - 3)}$

12. $\dfrac{10x^3 - 15x^2 - 35x}{x^2 - x - 6}$

13. $\dfrac{-x^2 + 2x - 13}{(x^2 + 2)(x - 1)}$

14. $\dfrac{26x^2 + 208x}{(x^2 + 1)(x + 5)}$

15. $\dfrac{6 + 26x - x^2}{(2x - 1)(x + 2)^2}$

16. $\dfrac{5x^3 + 6x^2 + 5x}{(x^2 - 1)(x + 1)^3}$

17. $\dfrac{6x^3 + 5x^2 + 6x - 2}{2x^2 + x - 1}$

18. $\dfrac{2x^3 + 3x^2 - 11x - 10}{x^2 + 2x - 3}$

19. $\dfrac{2x^2 - 11x + 5}{(x - 3)(x^2 + 2x - 5)}$

20. $\dfrac{3x^2 - 3x - 8}{(x - 5)(x^2 + x - 4)}$

21. $\dfrac{-4x^2 - 2x + 10}{(3x + 5)(x + 1)^2}$

22. $\dfrac{26x^2 - 36x + 22}{(x - 4)(2x - 1)^2}$

23. $\dfrac{36x + 1}{12x^2 - 7x - 10}$

24. $\dfrac{-17x + 61}{6x^2 + 39x - 21}$

25. $\dfrac{-4x^2 - 9x + 8}{(3x^2 + 1)(x - 2)}$

26. $\dfrac{11x^2 - 39x + 16}{(x^2 + 4)(x - 8)}$

Skill Maintenance

Find the zeros of the polynomial function.

27. $f(x) = x^3 + x^2 + 9x + 9$

28. $f(x) = x^3 - 3x^2 + x - 3$

29. $f(x) = x^3 + x^2 - 3x - 2$

30. $f(x) = x^4 - x^3 - 5x^2 - x - 6$

31. $f(x) = x^3 + 5x^2 + 5x - 3$

Synthesis

Decompose into partial fractions.

32. $\dfrac{9x^3 - 24x^2 + 48x}{(x - 2)^4(x + 1)}$
[*Hint*: Let the expression equal
$$\frac{A}{x + 1} + \frac{P(x)}{(x - 2)^4}$$
and find $P(x)$].

33. $\dfrac{x}{x^4 - a^4}$

34. $\dfrac{1}{e^{-x} + 3 + 2e^x}$

35. $\dfrac{1 + \ln x^2}{(\ln x + 2)(\ln x - 3)^2}$

REVIEW EXERCISES

Determine whether the statement is true or false.

1. A system of equations with exactly one solution is consistent and has independent equations. [9.1]

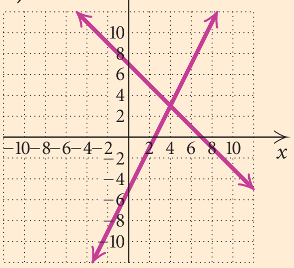
2. A system of two linear equations in two variables can have exactly two solutions. [9.1]

3. For any $m \times n$ matrices **A** and **B**, $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$. [9.4]

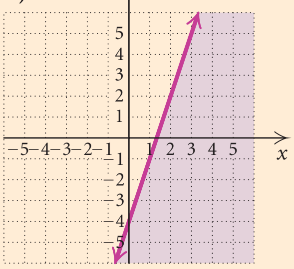
4. In general, matrix multiplication is commutative. [9.4]

In Exercises 5–12, match the equations or inequalities with one of the graphs (a)–(h), which follow.

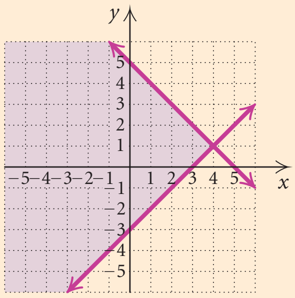
a)



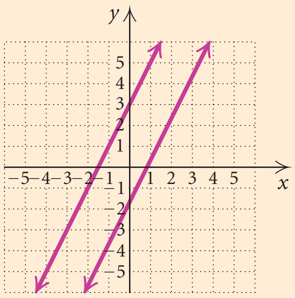
b)



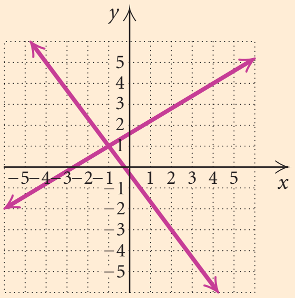
c)



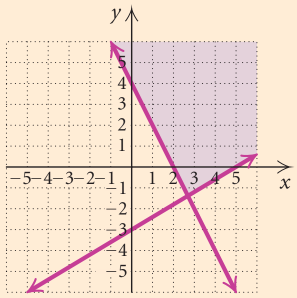
d)



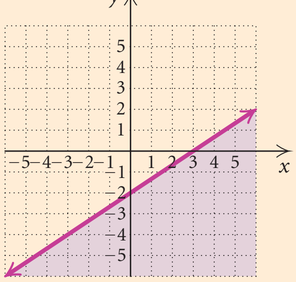
e)



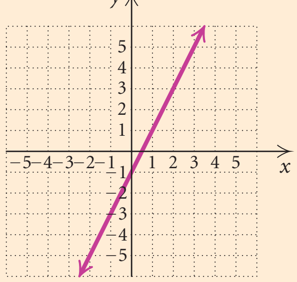
f)



g)



h)



5. $x + y = 7,$
$2x - y = 5$ [9.1]

6. $3x - 5y = -8,$
$4x + 3y = -1$ [9.1]

7. $y = 2x - 1,$
$4x - 2y = 2$ [9.1]

8. $6x - 3y = 5,$
$y = 2x + 3$ [9.1]

9. $y \leq 3x - 4$ [9.7]

10. $2x - 3y \geq 6$ [9.7]

11. $x - y \leq 3,$
$x + y \leq 5$ [9.7]

12. $2x + y \geq 4,$
$3x - 5y \leq 15$ [9.7]

Solve.

13. $5x - 3y = -4,$
$3x - y = -4$ [9.1]

14. $2x + 3y = 2,$
$5x - y = -29$ [9.1]

15. $x + 5y = 12,$
$5x + 25y = 12$ [9.1]

16. $x + y = -2,$
$-3x - 3y = 6$ [9.1]

17. $x + 5y - 3z = 4,$
$3x - 2y + 4z = 3,$
$2x + 3y - z = 5$ [9.2]

18. $2x - 4y + 3z = -3,$
$-5x + 2y - z = 7,$
$3x + 2y - 2z = 4$ [9.2]

19. $x - y = 5,$
$y - z = 6,$
$z - w = 7,$
$x + w = 8$ [9.2]

20. Classify each of the systems in Exercises 13–19 as consistent or inconsistent. [9.1], [9.2]

21. Classify each of the systems in Exercises 13–19 as having dependent equations or independent equations. [9.1], [9.2]

Solve the system of equations using Gaussian elimination or Gauss–Jordan elimination. [9.3]

22. $x + 2y = 5,$
$2x - 5y = -8$

23. $3x + 4y + 2z = 3,$
$5x - 2y - 13z = 3,$
$4x + 3y - 3z = 6$

24. $3x + 5y + z = 0,$
$2x - 4y - 3z = 0,$
$x + 3y + z = 0$

25. $w + x + y + z = -2,$
$-3w - 2x + 3y + 2z = 10,$
$2w + 3x + 2y - z = -12,$
$2w + 4x - y + z = 1$

26. *Coins.* The value of 75 coins, consisting of nickels and dimes, is \$5.95. How many of each kind are there? [9.1]

27. *Investment.* The Mendez family invested \$5000, part at 3% and the remainder at 3.5%. The annual income from both investments is \$167. What is the amount invested at each rate? [9.1]

28. *Nutrition.* A dietician must plan a breakfast menu that provides 460 Cal, 9 g of fat, and 55 mg of calcium. One plain bagel contains 200 Cal, 2 g of fat, and 29 mg of calcium. A one-tablespoon serving of cream cheese contains 100 Cal, 10 g of fat, and 24 mg of calcium. One banana contains 105 Cal, 1 g of fat, and 7 g of calcium. (*Source*: *Home and Garden Bulletin No. 72*, U.S. Government Printing Office, Washington D.C. 20402) How many servings of each are required to provide the desired nutritional values? [9.2]

29. *Test Scores.* A student has a total of 226 on three tests. The sum of the scores on the first and second tests exceeds the score on the third test by 62. The first score exceeds the second by 6. Find the three scores. [9.2]

30. *Trademarks.* The table below lists the number of trademarks renewed, in thousands, in the United States, represented in terms of the number of years since 2006.

Year, *x*	Number of Trademarks Renewed (in thousands)
2006, 0	40
2007, 1	48
2008, 2	40

Source: U.S. Patent and Trademark Office

a) Use a system of equations to fit a quadratic function $f(x) = ax^2 + bx + c$ to the data. [9.2]

b) Use the function to estimate the number of trademarks renewed in 2009. [9.2]

For Exercises 31–38, let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -2 & 0 & 1 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} -1 & 0 & 6 \\ 1 & -2 & 0 \\ 0 & 1 & -3 \end{bmatrix},$$

and

$$\mathbf{C} = \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}.$$

Find each of the following, if possible. [9.4]

31. $\mathbf{A} + \mathbf{B}$

32. $-3\mathbf{A}$

33. $-\mathbf{A}$

34. $\mathbf{AB}$

35. $\mathbf{B} + \mathbf{C}$

36. $\mathbf{A} - \mathbf{B}$

37. $\mathbf{BA}$

38. $\mathbf{A} + 3\mathbf{B}$

39. *Food Service Management.* The table below lists the cost per serving, in dollars, for items on four menus that are served at an elder-care facility.

Menu	Meat	Potato	Vegetable	Salad	Dessert
1	0.98	0.23	0.30	0.28	0.45
2	1.03	0.19	0.27	0.34	0.41
3	1.01	0.21	0.35	0.31	0.39
4	0.99	0.25	0.29	0.33	0.42

On a particular day, a dietician orders 32 meals from menu 1, 19 from menu 2, 43 from menu 3, and 38 from menu 4.

a) Write the information in the table as a 4×5 matrix **M**. [9.4]

b) Write a row matrix **N** that represents the number of each menu ordered. [9.4]

c) Find the product **NM**. [9.4]

d) State what the entries of **NM** represent. [9.4]

Find $\mathbf{A}^{-1}$, *if it exists.* [9.5]

40. $\mathbf{A} = \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$

41. $\mathbf{A} = \begin{bmatrix} 0 & 0 & 3 \\ 0 & -2 & 0 \\ 4 & 0 & 0 \end{bmatrix}$

42. $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & -5 & 0 \\ 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

43. Write a matrix equation equivalent to this system of equations:

$$\begin{aligned} 3x - 2y + 4z &= 13, \\ x + 5y - 3z &= 7, \\ 2x - 3y + 7z &= -8. \end{aligned}$$ [9.4]

Solve the system of equations using the inverse of the coefficient matrix of the equivalent matrix equation. [9.5]

44. $2x + 3y = 5,$
$3x + 5y = 11$

45. $5x - y + 2z = 17,$
$3x + 2y - 3z = -16,$
$4x - 3y - z = 5$

46. $w - x - y + z = -1,$
$2w + 3x - 2y - z = 2,$
$-w + 5x + 4y - 2z = 3,$
$3w - 2x + 5y + 3z = 4$

Evaluate the determinant. [9.6]

47. $\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix}$

48. $\begin{vmatrix} \sqrt{3} & -5 \\ -3 & -\sqrt{3} \end{vmatrix}$

49. $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & 4 & -3 \end{vmatrix}$

50. $\begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 0 \\ -1 & 3 & 1 \end{vmatrix}$

Solve using Cramer's rule. [9.6]

51. $5x - 2y = 19,$
$7x + 3y = 15$

52. $x + y = 4,$
$4x + 3y = 11$

53. $3x - 2y + z = 5,$
$4x - 5y - z = -1,$
$3x + 2y - z = 4$

54. $2x - y - z = 2,$
$3x + 2y + 2z = 10,$
$x - 5y - 3z = -2$

Graph. [9.7]

55. $y \leq 3x + 6$

56. $4x - 3y \geq 12$

57. Graph this system of inequalities and find the coordinates of any vertices formed. [9.7]

$$\begin{aligned} 2x + y &\geq 9, \\ 4x + 3y &\geq 23, \\ x + 3y &\geq 8, \\ x &\geq 0, \\ y &\geq 0 \end{aligned}$$

58. Find the maximum value and the minimum value of $T = 6x + 10y$ subject to

$$\begin{aligned} x + y &\leq 10, \\ 5x + 10y &\geq 50, \\ x &\geq 2, \\ y &\geq 0. \end{aligned}$$ [9.7]

59. *Maximizing a Test Score.* Marita is taking a test that contains questions in group A worth 7 points each and questions in group B worth 12 points each. The total number of questions answered must be at least 8. If Marita knows that group A questions take 8 min each and group B questions take 10 min each and the maximum time for the test is 80 min, how many questions from each group must she answer correctly in order to maximize her score? What is the maximum score? [9.7]

Decompose into partial fractions. [9.8]

60. $\dfrac{5}{(x + 2)^2(x + 1)}$

61. $\dfrac{-8x + 23}{2x^2 + 5x - 12}$

62. Solve: $2x + y = 7,$
$x - 2y = 6.$ [9.1]

A. x and y are both positive numbers.
B. x and y are both negative numbers.
C. x is positive and y is negative.
D. x is negative and y is positive.

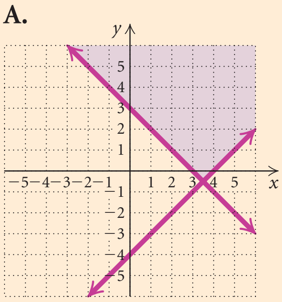
63. Which is *not* a row-equivalent operation on a matrix? [9.3]

A. Interchange any two columns.
B. Interchange any two rows.
C. Add two rows.
D. Multiply each entry in a row by -3.

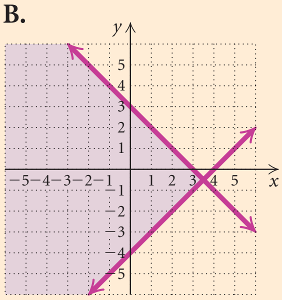
64. The graph of the given system of inequalities is which of the following? [9.7]

$$x + y \leq 3,$$
$$x - y \leq 4$$

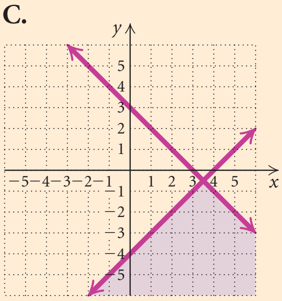
A.



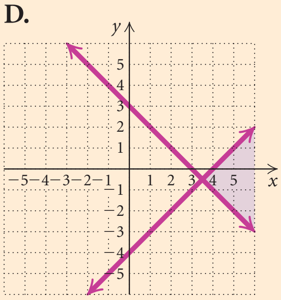
B.



C.



D.



Synthesis

65. One year, Don invested a total of \$40,000, part at 4%, part at 5%, and the rest at $5\frac{1}{2}$%. The total amount of interest received on the investments was \$1990. The interest received on the $5\frac{1}{2}$% investment was \$590 more than the interest received on the 4% investment. How much was invested at each rate? [9.2]

Solve.

66. $\frac{2}{3x} + \frac{4}{5y} = 8,$
$\frac{5}{4x} - \frac{3}{2y} = -6$ [9.1]

67. $\frac{3}{x} - \frac{4}{y} + \frac{1}{z} = -2,$
$\frac{5}{x} + \frac{1}{y} - \frac{2}{z} = 1,$
$\frac{7}{x} + \frac{3}{y} + \frac{2}{z} = 19$ [9.2]

Graph. [9.7]

68. $|x| - |y| \leq 1$

69. $|xy| > 1$

Collaborative Discussion and Writing

70. Cassidy solves the equation $2x + 5 = 3x - 7$ by finding the point of intersection of the graphs of $y_1 = 2x + 5$ and $y_2 = 3x - 7$. She finds the same point when she solves the system of equations

$$y = 2x + 5,$$
$$y = 3x - 7.$$

Explain the difference between the solution of the equation and the solution of the system of equations. [9.1]

71. For square matrices **A** and **B**, is it true, in general, that $(\mathbf{AB})^2 = \mathbf{A}^2\mathbf{B}^2$? Explain. [9.4]

72. Given the system of equations

$$a_1x + b_1y = c_1,$$
$$a_2x + b_2y = c_2,$$

explain why the equations are dependent or the system is inconsistent when

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0. \text{ [9.6]}$$

73. If the lines $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ are parallel, what can you say about the values of

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}, \quad \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad \text{and} \quad \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}? \text{ [9.6]}$$

74. Describe how the graph of a linear inequality differs from the graph of a linear equation. [9.7]

75. What would you say to a classmate who tells you that the partial fraction decomposition of

$$\frac{3x^2 - 8x + 9}{(x + 3)(x^2 - 5x + 6)}$$

is

$$\frac{2}{x + 3} + \frac{x - 1}{x^2 - 5x + 6}?$$

Explain. [9.8]

Chapter 9 Test

Solve. Use any method. Also determine whether the system is consistent or inconsistent and whether the equations are dependent or independent.

1. $3x + 2y = 1,$
$2x - y = -11$

2. $2x - y = 3,$
$2y = 4x - 6$

3. $x - y = 4,$
$3y = 3x - 8$

4. $2x - 3y = 8,$
$5x - 2y = 9$

Solve.

5. $4x + 2y + z = 4,$
$3x - y + 5z = 4,$
$5x + 3y - 3z = -2$

6. *Ticket Sales.* One evening 750 tickets were sold for Shortridge Community College's spring musical. Tickets cost \$3 for students and \$5 for nonstudents. Total receipts were \$3066. How many of each type of ticket were sold?

7. Tricia, Maria, and Antonio can process 352 telephone orders per day. Tricia and Maria together can process 224 orders per day while Tricia and Antonio together can process 248 orders per day. How many orders can each of them process alone?

For Exercises 8–13, let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 5 & 2 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} -5 & 1 \\ -2 & 4 \end{bmatrix},$$

and

$$\mathbf{C} = \begin{bmatrix} 3 & -4 \\ -1 & 0 \end{bmatrix}.$$

Find each of the following, if possible.

8. $\mathbf{B} + \mathbf{C}$

9. $\mathbf{A} - \mathbf{C}$

10. $\mathbf{CB}$

11. $\mathbf{AB}$

12. $2\mathbf{A}$

13. $\mathbf{C}^{-1}$

14. *Food Service Management.* The table below lists the cost per serving, in dollars, for items on three lunch menus served at a senior citizens' center.

Menu	Main Dish	Side Dish	Dessert
1	0.95	0.40	0.39
2	1.10	0.35	0.41
3	1.05	0.39	0.36

On a particular day, 26 Menu 1 meals, 18 Menu 2 meals, and 23 Menu 3 meals are served.

a) Write the information in the table as a 3×3 matrix **M**.
b) Write a row matrix **N** that represents the number of each menu served.
c) Find the product **NM**.
d) State what the entries of **NM** represent.

15. Write a matrix equation equivalent to the system of equations

$$\begin{aligned} 3x - 4y + 2z &= -8, \\ 2x + 3y + z &= 7, \\ x - 5y - 3z &= 3. \end{aligned}$$

16. Solve the system of equations using the inverse of the coefficient matrix of the equivalent matrix equation.

$$\begin{aligned} 3x + 2y + 6z &= 2, \\ x + y + 2z &= 1, \\ 2x + 2y + 5z &= 3 \end{aligned}$$

Evaluate the determinant.

17. $\begin{vmatrix} 3 & -5 \\ 8 & 7 \end{vmatrix}$

18. $\begin{vmatrix} 2 & -1 & 4 \\ -3 & 1 & -2 \\ 5 & 3 & -1 \end{vmatrix}$

19. Solve using Cramer's rule. Show your work.

$$\begin{aligned} 5x + 2y &= -1, \\ 7x + 6y &= 1 \end{aligned}$$

20. Graph: $3x + 4y \leq -12$.

21. Find the maximum value and the minimum value of $Q = 2x + 3y$ subject to

$$\begin{aligned} x + \quad y &\le 6, \\ 2x - 3y &\ge -3, \\ x &\ge 1, \\ y &\ge 0. \end{aligned}$$

22. *Maximizing Profit.* Casey's Cakes prepares pound cakes and carrot cakes. In a given week, at most 100 cakes can be prepared, of which 25 pound cakes and 15 carrot cakes are required by regular customers. The profit from each pound cake is \$3 and the profit from each carrot cake is \$4. How many of each type of cake should be prepared in order to maximize the profit? What is the maximum profit?

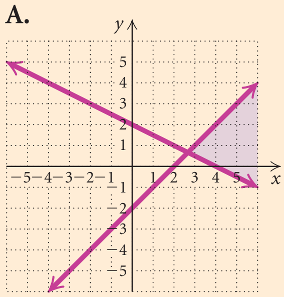
23. Decompose into partial fractions:

$$\frac{3x - 11}{x^2 + 2x - 3}.$$

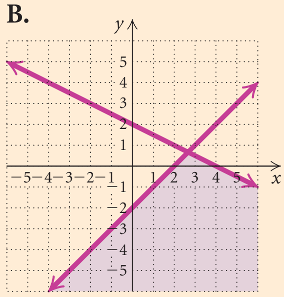
24. The graph of the given system of inequalities is which of the following?

$$\begin{aligned} x + 2y &\ge 4, \\ x - \quad y &\le 2 \end{aligned}$$

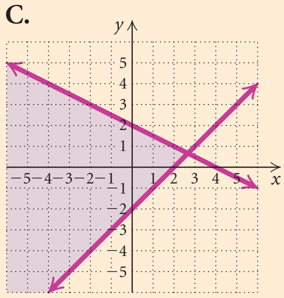
A.



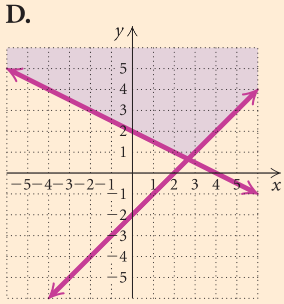
B.



C.



D.



Synthesis

25. Three solutions of the equation $Ax - By = Cz - 8$ are $(2, -2, 2)$, $(-3, -1, 1)$, and $(4, 2, 9)$. Find A, B, and C.

11.1 Exercise Set

In each of the following, the nth term of a sequence is given. Find the first 4 terms, a_{10}, and a_{15}.

1. $a_n = 4n - 1$

2. $a_n = (n-1)(n-2)(n-3)$

3. $a_n = \dfrac{n}{n-1}, \; n \geq 2$

4. $a_n = n^2 - 1, \; n \geq 3$

5. $a_n = \dfrac{n^2 - 1}{n^2 + 1}$

6. $a_n = \left(-\dfrac{1}{2}\right)^{n-1}$

7. $a_n = (-1)^n n^2$

8. $a_n = (-1)^{n-1}(3n - 5)$

9. $a_n = 5 + \dfrac{(-2)^{n+1}}{2^n}$

10. $a_n = \dfrac{2n - 1}{n^2 + 2n}$

Find the indicated term of the given sequence.

11. $a_n = 5n - 6; \; a_8$

12. $a_n = (3n - 4)(2n + 5); \; a_7$

13. $a_n = (2n + 3)^2; \; a_6$

14. $a_n = (-1)^{n-1}(4.6n - 18.3); \; a_{12}$

15. $a_n = 5n^2(4n - 100); \; a_{11}$

16. $a_n = \left(1 + \dfrac{1}{n}\right)^2; \; a_{80}$

17. $a_n = \ln e^n; \; a_{67}$

18. $a_n = 2 - \dfrac{1000}{n}; \; a_{100}$

Predict the general term, or nth term, a_n, of the sequence. Answers may vary.

19. $2, 4, 6, 8, 10, \ldots$

20. $3, 9, 27, 81, 243, \ldots$

21. $-2, 6, -18, 54, \ldots$

22. $-2, 3, 8, 13, 18, \ldots$

23. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \ldots$

24. $\sqrt{2}, 2, \sqrt{6}, 2\sqrt{2}, \sqrt{10}, \ldots$

25. $1 \cdot 2, 2 \cdot 3, 3 \cdot 4, 4 \cdot 5, \ldots$

26. $-1, -4, -7, -10, -13, \ldots$

27. $0, \log 10, \log 100, \log 1000, \ldots$

28. $\ln e^2, \ln e^3, \ln e^4, \ln e^5, \ldots$

Find the indicated partial sums for the sequence.

29. $1, 2, 3, 4, 5, 6, 7, \ldots; \; S_3$ and S_7

30. $1, -3, 5, -7, 9, -11, \ldots; \; S_2$ and S_5

31. $2, 4, 6, 8, \ldots; \; S_4$ and S_5

32. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots; \; S_1$ and S_5

Find and evaluate the sum.

33. $\displaystyle\sum_{k=1}^{5} \frac{1}{2k}$

34. $\displaystyle\sum_{i=1}^{6} \frac{1}{2i + 1}$

35. $\displaystyle\sum_{i=0}^{6} 2^i$

36. $\displaystyle\sum_{k=4}^{7} \sqrt{2k - 1}$

37. $\displaystyle\sum_{k=7}^{10} \ln k$

38. $\displaystyle\sum_{k=1}^{4} \pi k$

39. $\displaystyle\sum_{k=1}^{8} \frac{k}{k + 1}$

40. $\displaystyle\sum_{i=1}^{5} \frac{i - 1}{i + 3}$

41. $\displaystyle\sum_{i=1}^{5} (-1)^i$

42. $\displaystyle\sum_{k=0}^{5} (-1)^{k+1}$

43. $\displaystyle\sum_{k=1}^{8} (-1)^{k+1} 3k$

44. $\displaystyle\sum_{k=0}^{7} (-1)^k 4^{k+1}$

45. $\displaystyle\sum_{k=0}^{6} \frac{2}{k^2 + 1}$

46. $\displaystyle\sum_{i=1}^{10} i(i + 1)$

47. $\displaystyle\sum_{k=0}^{5} (k^2 - 2k + 3)$

48. $\displaystyle\sum_{k=1}^{10} \frac{1}{k(k + 1)}$

49. $\displaystyle\sum_{i=0}^{10} \frac{2^i}{2^i + 1}$

50. $\displaystyle\sum_{k=0}^{3} (-2)^{2k}$

Write sigma notation. Answers may vary.

51. $5 + 10 + 15 + 20 + 25 + \cdots$

52. $7 + 14 + 21 + 28 + 35 + \cdots$

53. $2 - 4 + 8 - 16 + 32 - 64$

54. $3 + 6 + 9 + 12 + 15$

55. $-\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{4}{5} - \frac{5}{6} + \frac{6}{7}$

56. $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2}$

57. $4 - 9 + 16 - 25 + \cdots + (-1)^n n^2$

58. $9 - 16 + 25 + \cdots + (-1)^{n+1} n^2$

59. $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \cdots$

60. $\frac{1}{1 \cdot 2^2} + \frac{1}{2 \cdot 3^2} + \frac{1}{3 \cdot 4^2} + \frac{1}{4 \cdot 5^2} + \cdots$

Find the first 4 terms of the recursively defined sequence.

61. $a_1 = 4,\ a_{n+1} = 1 + \frac{1}{a_n}$

62. $a_1 = 256,\ a_{n+1} = \sqrt{a_n}$

63. $a_1 = 6561,\ a_{n+1} = (-1)^n \sqrt{a_n}$

64. $a_1 = e^Q,\ a_{n+1} = \ln a_n$

65. $a_1 = 2, a_2 = 3,\ a_{n+1} = a_n + a_{n-1}$

66. $a_1 = -10, a_2 = 8,\ a_{n+1} = a_n - a_{n-1}$

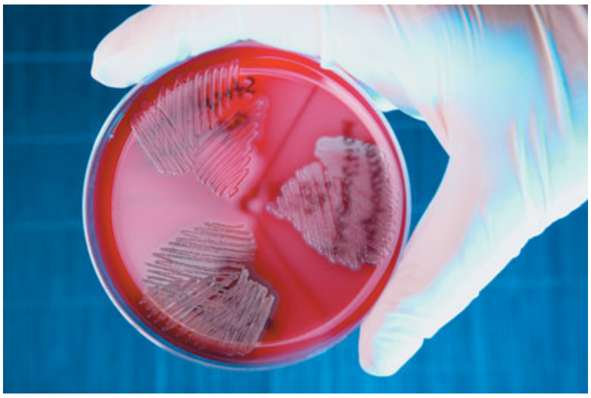
67. *Compound Interest.* Suppose that \$1000 is invested at 6.2%, compounded annually. The value of the investment after n years is given by the sequence model

$$a_n = \$1000(1.062)^n, \quad n = 1, 2, 3, \ldots.$$

a) Find the first 10 terms of the sequence.
b) Find the value of the investment after 20 yr.

68. *Salvage Value.* The value of an office machine is \$5200. Its salvage value each year is 75% of its value the year before. Give a sequence that lists the salvage value of the machine for each year of a 10-yr period.

69. *Bacteria Growth.* Suppose a single cell of bacteria divides into two every 15 min. Suppose that the same rate of division is maintained for 4 hr. Give a sequence that lists the number of cells after successive 15-min periods.



70. *Wage Sequence.* Torrey is paid \$8.30 per hour for working at Red Freight Limited. Each year he receives a \$0.30 hourly raise. Give a sequence that lists Torrey's hourly wage over a 10-yr period.

71. *Fibonacci Sequence: Rabbit Population Growth.* One of the most famous recursively defined sequences is the **Fibonacci sequence**. In 1202, the Italian mathematician Leonardo da Pisa, also called Fibonacci, proposed the following model for rabbit population growth. Suppose that every month each mature pair of rabbits in the population produces a new pair that begins reproducing after two months, and also suppose that no rabbits die. Beginning with one pair of newborn rabbits, the population can be modeled by the following recursively defined sequence:

$$a_1 = 1, \quad a_2 = 1, \quad a_n = a_{n-1} + a_{n-2}, \text{ for } n \geq 3,$$

where a_n is the total number of pairs of rabbits in month n. Find the first 7 terms of the Fibonacci sequence.

Skill Maintenance

Solve.

72. $3x - 2y = 3,$
$2x + 3y = -11$

73. *Tourism.* A total of 18.1 million international visitors traveled to New York City in 2008 and 2009. The number of international visitors in 2009 was 0.9 million fewer than in 2008. (*Source*: NYC & Co.) Find the number of international visitors to New York City in each year.

Find the center and the radius of the circle with the given equation.

74. $x^2 + y^2 - 6x + 4y = 3$

75. $x^2 + y^2 + 5x - 8y = 2$

Synthesis

Find the first 5 terms of the sequence, and then find S_5.

76. $a_n = \frac{1}{2^n} \log 1000^n$

77. $a_n = i^n, i = \sqrt{-1}$

78. $a_n = \ln(1 \cdot 2 \cdot 3 \cdots n)$

For each sequence, find a formula for S_n.

79. $a_n = \ln n$

80. $a_n = \frac{1}{n} - \frac{1}{n+1}$

11.2 Arithmetic Sequences and Series

- For any arithmetic sequence, find the *n*th term when *n* is given and *n* when the *n*th term is given; and given two terms, find the common difference and construct the sequence.
- Find the sum of the first *n* terms of an arithmetic sequence.

A sequence in which each term after the first is found by adding the same number to the preceding term is an **arithmetic sequence**.

Arithmetic Sequences

The sequence 2, 5, 8, 11, 14, 17, . . . is arithmetic because adding 3 to any term produces the next term. In other words, the difference between any term and the preceding one is 3. Arithmetic sequences are also called *arithmetic progressions*.

Arithmetic Sequence

A sequence is **arithmetic** if there exists a number d, called the **common difference**, such that $a_{n+1} = a_n + d$ for any integer $n \geq 1$.

EXAMPLE 1 For each of the following arithmetic sequences, identify the first term, a_1, and the common difference, d.

a) 4, 9, 14, 19, 24, . . .

b) 34, 27, 20, 13, 6, -1, -8, . . .

c) 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, . . .

15. In the sequence of Exercise 8, what term is the number 1.67?

16. In the sequence of Exercise 10, what term is -296?

17. In the sequence of Exercise 11, what term is -27?

18. Find a_{20} when $a_1 = 14$ and $d = -3$.

19. Find a_1 when $d = 4$ and $a_8 = 33$.

20. Find d when $a_1 = 8$ and $a_{11} = 26$.

21. Find n when $a_1 = 25$, $d = -14$, and $a_n = -507$.

22. In an arithmetic sequence, $a_{17} = -40$ and $a_{28} = -73$. Find a_1 and d. Write the first 5 terms of the sequence.

23. In an arithmetic sequence, $a_{17} = \frac{25}{3}$ and $a_{32} = \frac{95}{6}$. Find a_1 and d. Write the first 5 terms of the sequence.

24. Find the sum of the first 14 terms of the series $11 + 7 + 3 + \cdots$.

25. Find the sum of the first 20 terms of the series $5 + 8 + 11 + 14 + \cdots$.

26. Find the sum of the first 300 natural numbers.

27. Find the sum of the first 400 even natural numbers.

28. Find the sum of the odd numbers 1 to 199, inclusive.

29. Find the sum of the multiples of 7 from 7 to 98, inclusive.

30. Find the sum of all multiples of 4 that are between 14 and 523.

31. If an arithmetic series has $a_1 = 2$, $d = 5$, and $n = 20$, what is S_n?

32. If an arithmetic series has $a_1 = 7$, $d = -3$, and $n = 32$, what is S_n?

Find the sum.

33. $\sum_{k=1}^{40} (2k + 3)$

34. $\sum_{k=5}^{20} 8k$

35. $\sum_{k=0}^{19} \frac{k-3}{4}$

36. $\sum_{k=2}^{50} (2000 - 3k)$

37. $\sum_{k=12}^{57} \frac{7-4k}{13}$

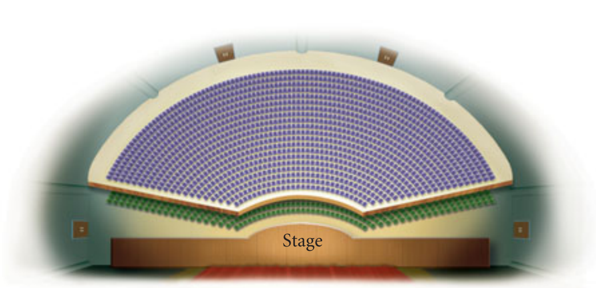
38. $\sum_{k=101}^{200} (1.14k - 2.8) - \sum_{k=1}^{5} \left(\frac{k+4}{10}\right)$

39. *Stacking Poles.* How many poles will be in a stack of telephone poles if there are 50 in the first layer, 49 in the second, and so on, with 6 in the top layer?

40. *Investment Return.* Max sets up an investment situation for a client that will return \$5000 the first year, \$6125 the second year, \$7250 the third year, and so on, for 25 yr. How much is received from the investment altogether?

41. *Total Savings.* If 10¢ is saved on October 1, 20¢ is saved on October 2, 30¢ on October 3, and so on, how much is saved during the 31 days of October?

42. *Theater Seating.* Theaters are often built with more seats per row as the rows move toward the back. Suppose that the first balcony of a theater has 28 seats in the first row, 32 in the second, 36 in the third, and so on, for 20 rows. How many seats are in the first balcony altogether?



43. *Parachutist Free Fall.* When a parachutist jumps from an airplane, the distances, in feet, that the parachutist falls in each successive second before pulling the ripcord to release the parachute are as follows:

$$16, 48, 80, 112, 144, \ldots.$$

Is this sequence arithmetic? What is the common difference? What is the total distance fallen in 10 sec?

44. *Small Group Interaction.* In a social science study, Stephan found the following data regarding an interaction measurement r_n for groups of size n.

n	r_n
3	0.5908
4	0.6080
5	0.6252
6	0.6424
7	0.6596
8	0.6768
9	0.6940
10	0.7112

Source: *American Sociological Review*, **17** (1952)

Is this sequence arithmetic? What is the common difference?

45. *Garden Plantings.* A gardener is making a planting in the shape of a trapezoid. It will have 35 plants in the front row, 31 in the second row, 27 in the third row, and so on. If the pattern is consistent, how many plants will there be in the last row? How many plants are there altogether?

46. *Band Formation.* A formation of a marching band has 10 marchers in the front row, 12 in the second row, 14 in the third row, and so on, for 8 rows. How many marchers are in the last row? How many marchers are there altogether?

47. *Raw Material Production.* In a manufacturing process, it took 3 units of raw materials to produce 1 unit of a product. The raw material needs thus formed the sequence

$$3, 6, 9, \ldots, 3n, \ldots.$$

Is this sequence arithmetic? What is the common difference?

Skill Maintenance

Solve.

48. $7x - 2y = 4,$
$x + 3y = 17$

49. $2x + y + 3z = 12,$
$x - 3y + 2z = 11,$
$5x + 2y - 4z = -4$

50. Find the vertices and the foci of the ellipse with the equation $9x^2 + 16y^2 = 144$.

51. Find an equation of the ellipse with vertices $(0, -5)$ and $(0, 5)$ and minor axis of length 4.

Synthesis

52. Find three numbers in an arithmetic sequence such that the sum of the first and third is 10 and the product of the first and second is 15.

53. Find a formula for the sum of the first n odd natural numbers:

$$1 + 3 + 5 + \cdots + (2n - 1).$$

54. Find the first 10 terms of the arithmetic sequence for which

$$a_1 = \$8760 \quad \text{and} \quad d = -\$798.23.$$

Then find the sum of the first 10 terms.

55. Find the first term and the common difference for the arithmetic sequence for which

$$a_2 = 40 - 3q \quad \text{and} \quad a_4 = 10p + q.$$

56. The zeros of this polynomial function form an arithmetic sequence. Find them.

$$f(x) = x^4 + 4x^3 - 84x^2 - 176x + 640$$

If p, m, and q form an arithmetic sequence, it can be shown that $m = (p + q)/2$. (See Exercise 63.) The number m is the ***arithmetic mean****, or* ***average****, of p and q. Given two numbers p and q, if we find k other numbers $m_1, m_2, \ldots, m_k$ such that*

$$p, m_1, m_2, \ldots, m_k, q$$

forms an arithmetic sequence, we say that we have "inserted k arithmetic means between p and q."

57. Insert three arithmetic means between 4 and 12.

58. Insert three arithmetic means between -3 and 5.

59. Insert four arithmetic means between 4 and 13.

60. Insert ten arithmetic means between 27 and 300.

61. Insert enough arithmetic means between 1 and 50 so that the sum of the resulting series will be 459.

62. *Straight-Line Depreciation.* A company buys an office machine for \$5200 on January 1 of a given year. The machine is expected to last for 8 yr, at the end of which time its **trade-in value**, or **salvage value**, will be \$1100. If the company's accountant figures the decline in value to be the same each year, then its **book values**, or **salvage values**, after t years, $0 \leq t \leq 8$, form an arithmetic sequence given by

$$a_t = C - t\left(\frac{C - S}{N}\right),$$

where C is the original cost of the item ($5200), N is the number of years of expected life (8), and S is the salvage value ($1100).

a) Find the formula for a_t for the straight-line depreciation of the office machine.

b) Find the salvage value after 0 yr, 1 yr, 2 yr, 3 yr, 4 yr, 7 yr, and 8 yr.

63. Prove that if p, m, and q form an arithmetic sequence, then

$$m = \frac{p + q}{2}.$$

11.3 Geometric Sequences and Series

- Identify the common ratio of a geometric sequence, and find a given term and the sum of the first n terms.
- Find the sum of an infinite geometric series, if it exists.

A sequence in which each term after the first is found by multiplying the preceding term by the same number is a **geometric sequence**.

Geometric Sequences

Consider the sequence:

$$2,\ 6,\ 18,\ 54,\ 162, \ldots.$$

Note that multiplying each term by 3 produces the next term. We call the number 3 the **common ratio** because it can be found by dividing any term by the preceding term. A geometric sequence is also called a *geometric progression*.

Geometric Sequence

A sequence is **geometric** if there is a number r, called the **common ratio**, such that

$$\frac{a_{n+1}}{a_n} = r, \quad \text{or} \quad a_{n+1} = a_n r, \quad \text{for any integer } n \geq 1.$$

11.3 Exercise Set

Find the common ratio.

1. $2, 4, 8, 16, \ldots$

2. $18, -6, 2, -\frac{2}{3}, \ldots$

3. $-1, 1, -1, 1, \ldots$

4. $-8, -0.8, -0.08, -0.008, \ldots$

5. $\frac{2}{3}, -\frac{4}{3}, \frac{8}{3}, -\frac{16}{3}, \ldots$

6. $75, 15, 3, \frac{3}{5}, \ldots$

7. $6.275, 0.6275, 0.06275, \ldots$

8. $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \ldots$

9. $5, \frac{5a}{2}, \frac{5a^2}{4}, \frac{5a^3}{8}, \ldots$

10. $\$780, \$858, \$943.80, \$1038.18, \ldots$

Find the indicated term.

11. $2, 4, 8, 16, \ldots;$ the 7th term

12. $2, -10, 50, -250, \ldots;$ the 9th term

13. $2, 2\sqrt{3}, 6, \ldots;$ the 9th term

14. $1, -1, 1, -1, \ldots;$ the 57th term

15. $\frac{7}{625}, -\frac{7}{25}, \ldots;$ the 23rd term

16. $\$1000, \$1060, \$1123.60, \ldots;$ the 5th term

Find the nth, or general, term.

17. $1, 3, 9, \ldots$

18. $25, 5, 1, \ldots$

19. $1, -1, 1, -1, \ldots$

20. $-2, 4, -8, \ldots$

21. $\frac{1}{x}, \frac{1}{x^2}, \frac{1}{x^3}, \ldots$

22. $5, \frac{5a}{2}, \frac{5a^2}{4}, \frac{5a^3}{8}, \ldots$

23. Find the sum of the first 7 terms of the geometric series
$$6 + 12 + 24 + \cdots.$$

24. Find the sum of the first 10 terms of the geometric series
$$16 - 8 + 4 - \cdots.$$

25. Find the sum of the first 9 terms of the geometric series
$$\tfrac{1}{18} - \tfrac{1}{6} + \tfrac{1}{2} - \cdots.$$

26. Find the sum of the geometric series
$$-8 + 4 + (-2) + \cdots + \left(-\tfrac{1}{32}\right).$$

Determine whether the statement is true or false.

27. The sequence $2, -2\sqrt{2}, 4, -4\sqrt{2}, 8, \ldots$ is geometric.

28. The sequence with general term $3n$ is geometric.

29. The sequence with general term 2^n is geometric.

30. Multiplying a term of a geometric sequence by the common ratio produces the next term of the sequence.

31. An infinite geometric series with common ratio -0.75 has a sum.

32. Every infinite geometric series has a limit.

Find the sum, if it exists.

33. $4 + 2 + 1 + \cdots$

34. $7 + 3 + \frac{9}{7} + \cdots$

35. $25 + 20 + 16 + \cdots$

36. $100 - 10 + 1 - \frac{1}{10} + \cdots$

37. $8 + 40 + 200 + \cdots$

38. $-6 + 3 - \frac{3}{2} + \frac{3}{4} - \cdots$

39. $0.6 + 0.06 + 0.006 + \cdots$

40. $\sum_{k=0}^{10} 3^k$

41. $\sum_{k=1}^{11} 15\left(\frac{2}{3}\right)^k$

42. $\sum_{k=0}^{50} 200(1.08)^k$

43. $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-1}$

44. $\sum_{k=1}^{\infty} 2^k$

45. $\sum_{k=1}^{\infty} 12.5^k$

46. $\sum_{k=1}^{\infty} 400(1.0625)^k$

47. $\sum_{k=1}^{\infty} \$500(1.11)^{-k}$

48. $\sum_{k=1}^{\infty} \$1000(1.06)^{-k}$

49. $\sum_{k=1}^{\infty} 16(0.1)^{k-1}$

50. $\sum_{k=1}^{\infty} \frac{8}{3}\left(\frac{1}{2}\right)^{k-1}$

Find fraction notation.

51. $0.131313\ldots$, or $0.\overline{13}$

52. $0.2222\ldots$, or $0.\overline{2}$

53. $8.999\overline{9}$

54. $6.1616\overline{16}$

55. $3.4125\overline{125}$

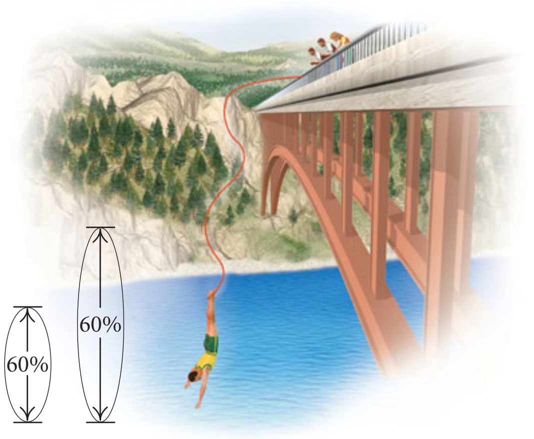
56. $12.7809\overline{809}$

57. *Daily Doubling Salary.* Suppose someone offered you a job for the month of February (28 days) under the following conditions. You will be paid \$0.01 the 1st day, \$0.02 the 2nd, \$0.04 the 3rd, and so on, doubling your previous day's salary each day. How much would you earn altogether?

58. *Bouncing Ping-Pong Ball.* A ping-pong ball is dropped from a height of 16 ft and always rebounds $\frac{1}{4}$ of the distance fallen.

a) How high does it rebound the 6th time?
b) Find the total sum of the rebound heights of the ball.

59. *Bungee Jumping.* A bungee jumper always rebounds 60% of the distance fallen. A bungee jump is made using a cord that stretches to 200 ft.



a) After jumping and then rebounding 9 times, how far has a bungee jumper traveled upward (the total rebound distance)?
b) About how far will a jumper have traveled upward (bounced) before coming to rest?

60. *Population Growth.* Hadleytown has a present population of 100,000, and the population is increasing by 3% each year.

a) What will the population be in 15 yr?
b) How long will it take for the population to double?

61. *Amount of an Annuity.* To create a college fund, a parent makes a sequence of 18 yearly deposits of \$1000 each in a savings account on which interest is compounded annually at 3.2%. Find the amount of the annuity.

62. *Amount of an Annuity.* A sequence of yearly payments of P dollars is invested at the end of each of N years at interest rate i, compounded annually. The total amount in the account, or the amount of the annuity, is V.

a) Show that

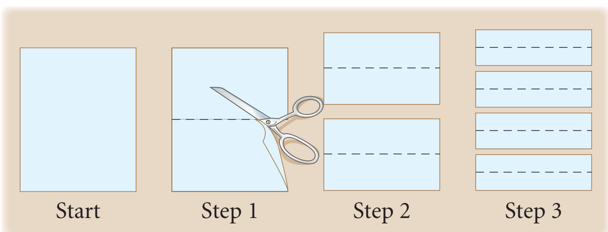
$$V = \frac{P[(1 + i)^N - 1]}{i}.$$

b) Suppose that interest is compounded n times per year and deposits are made every compounding period. Show that the formula for V is then given by

$$V = \frac{P\left[\left(1 + \frac{i}{n}\right)^{nN} - 1\right]}{i/n}.$$

63. *Loan Repayment.* A family borrows \$120,000. The loan is to be repaid in 13 yr at 12% interest, compounded annually. How much will have been repaid at the end of 13 yr?

64. *Doubling the Thickness of Paper.* A piece of paper is 0.01 in. thick. It is cut and stacked repeatedly in such a way that its thickness is doubled each time for 20 times. How thick is the result?



65. *The Economic Multiplier.* Suppose the government is making a \$13,000,000,000 expenditure to stimulate the economy. If 85% of this is spent again, and so on, what is the total effect on the economy?

66. *Advertising Effect.* Gigi's Cupcake Truck is about to open for business in a city of 3,000,000 people, traveling to several curbside locations in the city each day to sell cupcakes. The owners plan an advertising campaign that they think will induce 30% of the people to buy their cupcakes. They estimate that if those people like the product, they will induce 30% · 30% · 3,000,000 more to buy the product, and those will induce 30% · 30% · 30% · 3,000,000 and so on. In all, how many people will buy Gigi's cupcakes as a result of the advertising campaign? What percentage of the population is this?

Skill Maintenance

For each pair of functions, find $(f \circ g)(x)$ *and* $(g \circ f)(x)$.

67. $f(x) = x^2,\ g(x) = 4x + 5$

68. $f(x) = x - 1,\ g(x) = x^2 + x + 3$

Solve.

69. $5^x = 35$

70. $\log_2 x = -4$

Synthesis

71. Prove that

$$\sqrt{3} - \sqrt{2}, \quad 4 - \sqrt{6}, \quad \text{and} \quad 6\sqrt{3} - 2\sqrt{2}$$

form a geometric sequence.

72. Consider the sequence

$$4, \quad 20.4, \quad 104.04, \quad 531.6444, \ldots.$$

What is the error in using $a_{277} = 4(5.1)^{276}$ to find the 277th term?

73. Consider the sequence

$$x + 3, \quad x + 7, \quad 4x - 2, \ldots.$$

a) If the sequence is arithmetic, find x and then determine each of the 3 terms and the 4th term.

b) If the sequence is geometric, find x and then determine each of the 3 terms and the 4th term.

74. Find the sum of the first n terms of

$$1 + x + x^2 + \cdots.$$

75. Find the sum of the first n terms of

$$x^2 - x^3 + x^4 - x^5 + \cdots.$$

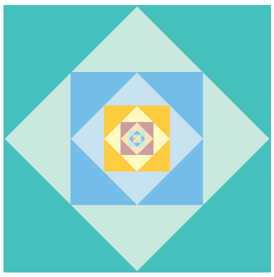
In Exercises 76 and 77, assume that $a_1, a_2, a_3, \ldots$ *is a geometric sequence.*

76. Prove that $a_1^2, a_2^2, a_3^2, \ldots$ is a geometric sequence.

77. Prove that $\ln a_1, \ln a_2, \ln a_3, \ldots$ is an arithmetic sequence.

78. Prove that $5^{a_1}, 5^{a_2}, 5^{a_3}, \ldots$ is a geometric sequence, if $a_1, a_2, a_3, \ldots$ is an arithmetic sequence.

79. The sides of a square are 16 cm long. A second square is inscribed by joining the midpoints of the sides, successively. In the second square, we repeat the process, inscribing a third square. If this process is continued indefinitely, what is the sum of all the areas of all the squares? (*Hint*: Use an infinite geometric series.)



EXAMPLE 3 Prove: For every natural number n, $n < 2^n$.

Proof. We first list S_n, S_1, S_k, and S_{k+1}.

S_n: $n < 2^n$

S_1: $1 < 2^1$

S_k: $k < 2^k$

S_{k+1}: $k + 1 < 2^{k+1}$

(1) *Basis step.* S_1, as listed, is true since $2^1 = 2$ and $1 < 2$.

(2) *Induction step.* We let k be any natural number. We assume S_k to be true and try to show that it implies that S_{k+1} is true. Now

$$k < 2^k \quad \text{This is } S_k.$$
$$2k < 2 \cdot 2^k \quad \text{Multiplying by 2 on both sides}$$
$$2k < 2^{k+1} \quad \text{Adding exponents on the right}$$
$$k + k < 2^{k+1}. \quad \text{Rewriting } 2k \text{ as } k + k$$

Since k is any natural number, we know that $1 \le k$. Thus,

$$k + 1 \le k + k. \quad \text{Adding } k \text{ on both sides of } 1 \le k$$

Putting the results $k + 1 \le k + k$ and $k + k < 2^{k+1}$ together gives us

$$k + 1 < 2^{k+1}. \quad \text{This is } S_{k+1}.$$

We have shown that for all natural numbers k, $S_k \rightarrow S_{k+1}$. This completes the induction step. It and the basis step tell us that the proof is complete.

Now Try Exercise 11.

11.4 Exercise Set

List the first five statements in the sequence that can be obtained from each of the following. Determine whether each of the five statements is true or false.

1. $n^2 < n^3$

2. $n^2 - n + 41$ is prime. Find a value for n for which the statement is false.

3. A polygon of n sides has $[n(n - 3)]/2$ diagonals.

4. The sum of the angles of a polygon of n sides is $(n - 2) \cdot 180°$.

Use mathematical induction to prove each of the following.

5. $2 + 4 + 6 + \cdots + 2n = n(n + 1)$

6. $4 + 8 + 12 + \cdots + 4n = 2n(n + 1)$

7. $1 + 5 + 9 + \cdots + (4n - 3) = n(2n - 1)$

8. $3 + 6 + 9 + \cdots + 3n = \dfrac{3n(n + 1)}{2}$

9. $2 + 4 + 8 + \cdots + 2^n = 2(2^n - 1)$

10. $2 \le 2^n$

11. $n < n + 1$

12. $3^n < 3^{n+1}$

13. $2n \le 2^n$

14. $\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \cdots + \dfrac{1}{n(n + 1)} = \dfrac{n}{n + 1}$

15. $\dfrac{1}{1 \cdot 2 \cdot 3} + \dfrac{1}{2 \cdot 3 \cdot 4} + \dfrac{1}{3 \cdot 4 \cdot 5} + \cdots + \dfrac{1}{n(n + 1)(n + 2)} = \dfrac{n(n + 3)}{4(n + 1)(n + 2)}$

16. If x is any real number greater than 1, then for any natural number n, $x \le x^n$.

The following formulas can be used to find sums of powers of natural numbers. Use mathematical induction to prove each formula.

17. $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$

18. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

19. $1^3 + 2^3 + 3^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$

20. $1^4 + 2^4 + 3^4 + \cdots + n^4$
$= \dfrac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$

21. $1^5 + 2^5 + 3^5 + \cdots + n^5$
$= \dfrac{n^2(n+1)^2(2n^2+2n-1)}{12}$

Use mathematical induction to prove each of the following.

22. $\displaystyle\sum_{i=1}^{n}(3i-1) = \frac{n(3n+1)}{2}$

23. $\displaystyle\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$

24. $\left(1+\dfrac{1}{1}\right)\left(1+\dfrac{1}{2}\right)\left(1+\dfrac{1}{3}\right)\cdots\left(1+\dfrac{1}{n}\right)$
$= n + 1$

25. The sum of n terms of an arithmetic sequence:
$a_1 + (a_1 + d) + (a_1 + 2d) + \cdots + [a_1 + (n-1)d]$
$= \dfrac{n}{2}[2a_1 + (n-1)d]$

Skill Maintenance

Solve.

26. $2x - 3y = 1,$
$3x - 4y = 3$

27. $x + y + z = 3,$
$2x - 3y - 2z = 5,$
$3x + 2y + 2z = 8$

28. *e-Commerce.* ebooks.com ran a one-day promotion offering a hardback title for \$24.95 and a paperback title for \$9.95. A total of 80 books were sold and \$1546 was taken in. How many of each type of book were sold?

29. *Investment.* Martin received \$104 in simple interest one year from three investments. Part is invested at 1.5%, part at 2%, and part at 3%. The amount invested at 2% is twice the amount invested at 1.5%. There is \$400 more invested at 3% than at 2%. Find the amount invested at each rate.

Synthesis

Use mathematical induction to prove each of the following.

30. The sum of n terms of a geometric sequence:
$$a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1} = \frac{a_1 - a_1r^n}{1-r}$$

31. $x + y$ is a factor of $x^{2n} - y^{2n}$.

Prove each of the following using mathematical induction. Do the basis step for $n = 2$.

32. For every natural number $n \ge 2$,
$$2n + 1 < 3^n.$$

33. For every natural number $n \ge 2$,
$\log_a (b_1 b_2 \cdots b_n)$
$= \log_a b_1 + \log_a b_2 + \cdots + \log_a b_n.$

Prove each of the following for any complex numbers z_1, $z_2, \ldots, z_n$, where $i^2 = -1$ and $\bar{z}$ is the conjugate of z. (See Section 3.1.)

34. $\overline{z^n} = \bar{z}^n$

35. $\overline{z_1 + z_2 + \cdots + z_n} = \overline{z_1} + \overline{z_2} + \cdots + \overline{z_n}$

36. $\overline{z_1 z_2 \cdots z_n} = \overline{z_1} \cdot \overline{z_2} \cdot \cdots \cdot \overline{z_n}$

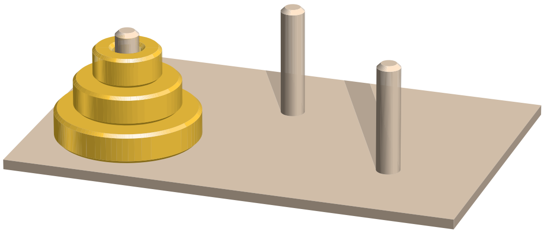
37. i^n is either $1, -1, i$, or $-i$.

For any integers a and b, b is a factor of a if there exists an integer c such that $a = bc$. Prove each of the following for any natural number n.

38. 2 is a factor of $n^2 + n$.

39. 3 is a factor of $n^3 + 2n$.

40. *The Tower of Hanoi Problem.* There are three pegs on a board. On one peg are n disks, each smaller than the one on which it rests. The problem is to move this pile of disks to another peg. The final order must be the same, but you can move only one disk at a time and can never place a larger disk on a smaller one.



a) What is the *smallest* number of moves needed to move 3 disks? 4 disks? 2 disks? 1 disk?

b) Conjecture a formula for the *smallest* number of moves needed to move n disks. Prove it by mathematical induction.

Mid-Chapter Mixed Review

Determine whether the statement is true or false.

1. The general term of the sequence $1, -2, 3, -4, \ldots$ can be expressed as $a_n = n$. [11.1]

2. To find the common difference of an arithmetic sequence, choose any term except the first and then subtract the preceding term from it. [11.2]

3. The sequence $7, 3, -1, -5, \ldots$ is geometric. [11.2], [11.3]

4. If we can show that $S_k \rightarrow S_{k+1}$ for some natural number k, then we know than S_n is true for all natural numbers n. [11.4]

In each of the following, the nth term of a sequence is given. Find the first 4 terms, a_9, and a_{14}.

5. $a_n = 3n + 5$ [11.1]

6. $a_n = (-1)^{n+1}(n - 1)$ [11.1]

Predict the general term, or nth term, a_n, of the sequence. Answers may vary.

7. 3, 6, 9 12, 15, . . . [11.1]

8. $-1, 4, -9, 16, -25, \ldots$ [11.1]

9. Find the partial sum S_4 for the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$. [11.1]

10. Find and evaluate the sum $\sum_{k=1}^{5} k(k + 1)$. [11.1]

11. Write sigma notation for the sum $-4 + 8 - 12 + 16 - 20 + \cdots$. [11.1]

12. Find the first 4 terms of the sequence defined by $a_1 = 2, a_{n+1} = 4a_n - 2$. [11.1]

13. Find the common difference of the arithmetic sequence $12, 7, 2, -3, \ldots$. [11.2]

14. Find the 10th term of the arithmetic sequence 4, 6, 8, 10, ... [11.2]

15. In the sequence in Exercise 14, what term is the number 44? [11.2]

16. Find the sum of the first 16 terms of the arithmetic series $6 + 11 + 16 + 21 + \cdots$. [11.2]

17. Find the common ratio of the geometric sequence $16, -8, 4, -2, 1, \ldots$ [11.3]

18. Find (**a**) the 8th term and (**b**) the sum of the first 10 terms of the geometric sequence $\frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, \ldots$ [11.3]

Find the sum, if it exists.

19. $-8 + 4 - 2 + 1 - \cdots$ [11.3]

20. $\sum_{k=0}^{\infty} 5^k$ [11.3]

21. *Landscaping.* A landscaper is planting a triangular flower bed with 36 plants in the first row, 30 plants in the second row, 24 in the third row, and so on, for a total of 6 rows. How many plants will be planted in all? [11.2]

22. *Amount of an Annuity.* To save money for adding a bedroom to their home, at the end of each of 4 yr the Davidsons deposit \$1500 in an account that pays 4% interest, compounded annually. Find the total amount of the annuity. [11.3]

23. Prove: For every natural number n,
$1 + 4 + 7 + \cdots + (3n - 2) = \frac{1}{2}n(3n - 1)$. [11.4]

Collaborative Discussion and Writing

24. The sum of the first n terms of an arithmetic sequence can be given by

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d].$$

Compare this formula to

$$S_n = \frac{n}{2}(a_1 + a_n).$$

Discuss the reasons for the use of one formula over the other. [11.2]

25. It is said that as a young child, the mathematician Karl F. Gauss (1777–1855) was able to compute the sum $1 + 2 + 3 + \cdots + 100$ very quickly in his head to the amazement of a teacher. Explain how Gauss might have done this had he possessed some knowledge of arithmetic sequences and series. Then give a formula for the sum of the first n natural numbers. [11.2]

26. Write a problem for a classmate to solve. Devise the problem so that a geometric series is involved and the solution is "The total amount in the bank is \$900$(1.08)^{40}$, or about \$19,552." [11.3]

27. Write an explanation of the idea behind mathematical induction for a fellow student. [11.4]

In general:

> For a set of n objects in which n_1 are of one kind, n_2 are of another kind, . . . , and n_k are of a kth kind, the number of distinguishable permutations is
>
> $$\frac{n!}{n_1! \cdot n_2! \cdot \cdots \cdot n_k!}.$$

EXAMPLE 10 In how many distinguishable ways can the letters of the word CINCINNATI be arranged?

Solution There are 2 C's, 3 I's, 3 N's, 1 A, and 1 T for a total of 10 letters. Thus,

$$N = \frac{10!}{2! \cdot 3! \cdot 3! \cdot 1! \cdot 1!}, \quad \text{or} \quad 50{,}400.$$

The letters of the word CINCINNATI can be arranged in 50,400 distinguishable ways.

Now Try Exercise 35.

11.5 Exercise Set

Evaluate.

1. $_6P_6$
2. $_4P_3$
3. $_{10}P_7$
4. $_{10}P_3$
5. $5!$
6. $7!$
7. $0!$
8. $1!$
9. $\dfrac{9!}{5!}$
10. $\dfrac{9!}{4!}$
11. $(8-3)!$
12. $(8-5)!$
13. $\dfrac{10!}{7!\,3!}$
14. $\dfrac{7!}{(7-2)!}$
15. $_8P_0$
16. $_{13}P_1$
17. $_{52}P_4$
18. $_{52}P_5$
19. $_nP_3$
20. $_nP_2$
21. $_nP_1$
22. $_nP_0$

In each of Exercises 23–41, give your answer using permutation notation, factorial notation, or other operations. Then evaluate.

How many permutations are there of the letters in each of the following words, if all the letters are used without repetition?

23. MARVIN
24. JUDY
25. UNDERMOST
26. COMBINES
27. How many permutations are there of the letters of the word UNDERMOST if the letters are taken 4 at a time?
28. How many permutations are there of the letters of the word COMBINES if the letters are taken 5 at a time?
29. How many 5-digit numbers can be formed using the digits 2, 4, 6, 8, and 9 without repetition? with repetition?

30. In how many ways can 7 athletes be arranged in a straight line?

31. *Program Planning.* A program is planned to have 5 musical numbers and 4 speeches. In how many ways can this be done if a musical number and a speech are to alternate and a musical number is to come first?

32. A professor is going to grade her 24 students on a curve. She will give 3 A's, 5 B's, 9 C's, 4 D's, and 3 F's. In how many ways can she do this?

33. *Phone Numbers.* How many 7-digit phone numbers can be formed with the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, assuming that the first number cannot be 0 or 1? Accordingly, how many telephone numbers can there be within a given area code, before the area needs to be split with a new area code?

34. How many distinguishable code symbols can be formed from the letters of the word BUSINESS? BIOLOGY? MATHEMATICS?

35. Suppose the expression $a^2b^3c^4$ is rewritten without exponents. In how many distinguishable ways can this be done?

36. *Coin Arrangements.* A penny, a nickel, a dime, and a quarter are arranged in a straight line.

a) Considering just the coins, in how many ways can they be lined up?
b) Considering the coins and heads and tails, in how many ways can they be lined up?

37. How many code symbols can be formed using 5 out of 6 letters of A, B, C, D, E, F if the letters:

a) are not repeated?
b) can be repeated?
c) are not repeated but must begin with D?
d) are not repeated but must begin with DE?

38. *License Plates.* A state forms its license plates by first listing a number that corresponds to the county in which the owner of the car resides. (The names of the counties are alphabetized and the number is its location in that order.) Then the plate lists a letter of the alphabet, and this is followed by a number from 1 to 9999. How many such plates are possible if there are 80 counties?

39. *Zip Codes.* A U.S. postal zip code is a five-digit number.

a) How many zip codes are possible if any of the digits 0 to 9 can be used?
b) If each post office has its own zip code, how many possible post offices can there be?

40. *Zip-Plus-4 Codes.* A zip-plus-4 postal code uses a 9-digit number like 75247-5456. How many 9-digit zip-plus-4 postal codes are possible?

41. *Social Security Numbers.* A social security number is a 9-digit number like 243-47-0825.

a) How many different social security numbers can there be?
b) There are about 311 million people in the United States. Can each person have a unique social security number?

Skill Maintenance

Find the zero(s) of the function.

42. $f(x) = 4x - 9$

43. $f(x) = x^2 + x - 6$

44. $f(x) = 2x^2 - 3x - 1$

45. $f(x) = x^3 - 4x^2 - 7x + 10$

Synthesis

Solve for n.

46. ${}_nP_5 = 7 \cdot {}_nP_4$

47. ${}_nP_4 = 8 \cdot {}_{n-1}P_3$

48. ${}_nP_5 = 9 \cdot {}_{n-1}P_4$

49. ${}_nP_4 = 8 \cdot {}_nP_3$

50. Show that $n! = n(n-1)(n-2)(n-3)!$.

51. *Single-Elimination Tournaments.* In a single-elimination sports tournament consisting of n teams, a team is eliminated when it loses one game. How many games are required to complete the tournament?

52. *Double-Elimination Tournaments.* In a double-elimination softball tournament consisting of n teams, a team is eliminated when it loses two games. At most, how many games are required to complete the tournament?

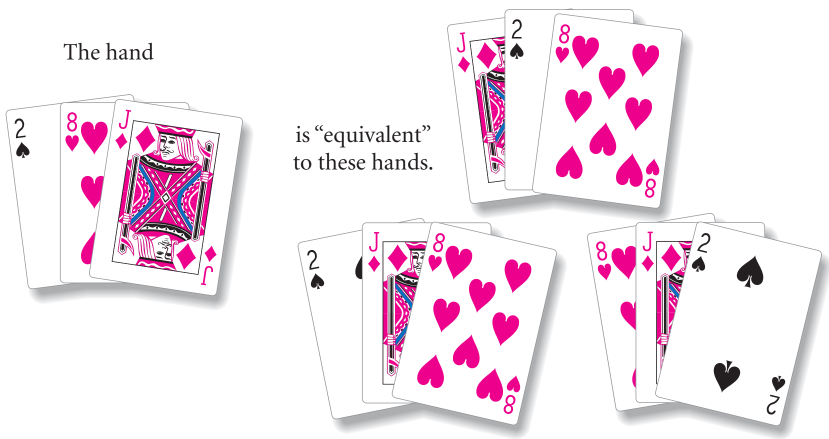
11.6 Combinatorics: Combinations

▶ **Evaluate combination notation and solve related applied problems.**

We now consider counting techniques in which order is not considered.

▶ Combinations

We sometimes make a selection from a set *without regard to order*. Such a selection is called a *combination*. If you play cards, for example, you know that in most situations the *order* in which you hold cards is not important. That is,



Each hand contains the same combination of three cards.

11.6 Exercise Set

Evaluate.

1. ${}_{13}C_2$

2. ${}_{9}C_6$

3. $\binom{13}{11}$

4. $\binom{9}{3}$

5. $\binom{7}{1}$

6. $\binom{8}{8}$

7. $\dfrac{{}_5P_3}{3!}$

8. $\dfrac{{}_{10}P_5}{5!}$

9. $\binom{6}{0}$

10. $\binom{6}{1}$

11. $\binom{6}{2}$

12. $\binom{6}{3}$

13. $\binom{7}{0}+\binom{7}{1}+\binom{7}{2}+\binom{7}{3}+\binom{7}{4}+\binom{7}{5}+\binom{7}{6}+\binom{7}{7}$

14. $\binom{6}{0}+\binom{6}{1}+\binom{6}{2}+\binom{6}{3}+\binom{6}{4}+\binom{6}{5}+\binom{6}{6}$

15. ${}_{52}C_4$

16. ${}_{52}C_5$

17. $\binom{27}{11}$

18. $\binom{37}{8}$

19. $\binom{n}{1}$

20. $\binom{n}{3}$

21. $\binom{m}{m}$

22. $\binom{t}{4}$

In each of the following exercises, give an expression for the answer using permutation notation, combination notation, factorial notation, or other operations. Then evaluate.

23. *Fraternity Officers.* There are 23 students in a fraternity. How many sets of 4 officers can be selected?

24. *League Games.* How many games can be played in a 9-team sports league if each team plays all other teams once? twice?

25. *Test Options.* On a test, a student is to select 10 out of 13 questions. In how many ways can this be done?

26. *Senate Committees.* Suppose the Senate of the United States consists of 58 Democrats and 42 Republicans. How many committees can be formed consisting of 6 Democrats and 4 Republicans?

27. *Test Options.* Of the first 10 questions on a test, a student must answer 7. Of the second 5 questions, the student must answer 3. In how many ways can this be done?

28. *Lines and Triangles from Points.* How many lines are determined by 8 points, no 3 of which are collinear? How many triangles are determined by the same points?

29. *Poker Hands.* How many 5-card poker hands are possible with a 52-card deck?

30. *Bridge Hands.* How many 13-card bridge hands are possible with a 52-card deck?

31. *Baskin-Robbins Ice Cream.* Burt Baskin and Irv Robbins began making ice cream in 1945. Initially they developed 31 flavors—one for each day of the month. (*Source*: Baskin-Robbins)

a) How many 2-dip cones are possible using the 31 original flavors if order of flavors is to be considered and no flavor is repeated?

b) How many 2-dip cones are possible if order is to be considered and a flavor can be repeated?

c) How many 2-dip cones are possible if order is not considered and no flavor is repeated?

Skill Maintenance

Solve.

32. $3x - 7 = 5x + 10$

33. $2x^2 - x = 3$

34. $x^2 + 5x + 1 = 0$

35. $x^3 + 3x^2 - 10x = 24$

Synthesis

36. *Full House.* A full house in poker consists of three of a kind and a pair (two of a kind). How many full houses are there that consist of 3 aces and 2 queens? (See Section 8.8 for a description of a 52-card deck.)

37. *Flush.* A flush in poker consists of a 5-card hand with all cards of the same suit. How many 5-card hands (flushes) are there that consist of all diamonds?

38. There are n points on a circle. How many quadrilaterals can be inscribed with these points as vertices?

39. *League Games.* How many games are played in a league with n teams if each team plays each other team once? twice?

Solve for n.

40. $\binom{n+1}{3} = 2 \cdot \binom{n}{2}$

41. $\binom{n}{n-2} = 6$

42. $\binom{n}{3} = 2 \cdot \binom{n-1}{2}$

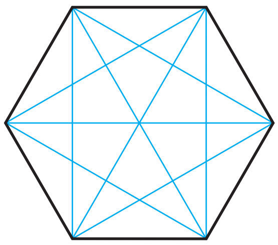
43. $\binom{n+2}{4} = 6 \cdot \binom{n}{2}$

44. Prove that

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

for any natural numbers n and k, $k \leq n$.

45. How many line segments are determined by the n vertices of an n-gon? Of these, how many are diagonals? Use mathematical induction to prove the result for the diagonals.



Solution The toppings on each hamburger are the elements of a subset of the set of all possible toppings, the empty set being a plain hamburger. The total number of possible hamburgers is

$$\binom{7}{0} + \binom{7}{1} + \binom{7}{2} + \cdots + \binom{7}{7} = 2^7 = 128.$$

Thus Wendy's serves hamburgers in 128 different ways.

Now Try Exercise 33.

11.7 Exercise Set

Expand.

1. $(x + 5)^4$
2. $(x - 1)^4$
3. $(x - 3)^5$
4. $(x + 2)^9$
5. $(x - y)^5$
6. $(x + y)^8$
7. $(5x + 4y)^6$
8. $(2x - 3y)^5$
9. $\left(2t + \frac{1}{t}\right)^7$
10. $\left(3y - \frac{1}{y}\right)^4$
11. $(x^2 - 1)^5$
12. $(1 + 2q^3)^8$
13. $(\sqrt{5} + t)^6$
14. $(x - \sqrt{2})^6$
15. $\left(a - \frac{2}{a}\right)^9$
16. $(1 + 3)^n$
17. $(\sqrt{2} + 1)^6 - (\sqrt{2} - 1)^6$
18. $(1 - \sqrt{2})^4 + (1 + \sqrt{2})^4$
19. $(x^{-2} + x^2)^4$
20. $\left(\frac{1}{\sqrt{x}} - \sqrt{x}\right)^6$

Find the indicated term of the binomial expansion.

21. 3rd; $(a + b)^7$
22. 6th; $(x + y)^8$
23. 6th; $(x - y)^{10}$
24. 5th; $(p - 2q)^9$
25. 12th; $(a - 2)^{14}$
26. 11th; $(x - 3)^{12}$
27. 5th; $(2x^3 - \sqrt{y})^8$
28. 4th; $\left(\frac{1}{b^2} + \frac{b}{3}\right)^7$
29. Middle; $(2u - 3v^2)^{10}$
30. Middle two; $(\sqrt{x} + \sqrt{3})^5$

Determine the number of subsets of each of the following.

31. A set of 7 elements
32. A set of 6 members
33. The set of letters of the Greek alphabet, which contains 24 letters
34. The set of letters of the English alphabet, which contains 26 letters
35. What is the degree of $(x^5 + 3)^4$?
36. What is the degree of $(2 - 5x^3)^7$?

Expand each of the following, where $i^2 = -1$.

37. $(3 + i)^5$
38. $(1 + i)^6$
39. $(\sqrt{2} - i)^4$
40. $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^{11}$
41. Find a formula for $(a - b)^n$. Use sigma notation.
42. Expand and simplify:
$$\frac{(x + h)^{13} - x^{13}}{h}.$$
43. Expand and simplify:
$$\frac{(x + h)^n - x^n}{h}.$$
Use sigma notation.

Skill Maintenance

Given that $f(x) = x^2 + 1$ and $g(x) = 2x - 3$, find each of the following.

44. $(f + g)(x)$

45. $(fg)(x)$

46. $(f \circ g)(x)$

47. $(g \circ f)(x)$

Synthesis

Solve for x.

48. $\sum_{k=0}^{8} \binom{8}{k} x^{8-k} 3^k = 0$

49. $\sum_{k=0}^{4} \binom{4}{k} (-1)^k x^{4-k} 6^k = 81$

50. Find the term of
$$\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{12}$$
that does not contain x.

51. Find the middle term of $(x^2 - 6y^{3/2})^6$.

52. Find the ratio of the 4th term of
$$\left(p^2 - \frac{1}{2}p\sqrt[3]{q}\right)^5$$
to the 3rd term.

53. Find the term of
$$\left(\sqrt[3]{x} - \frac{1}{\sqrt{x}}\right)^7$$
containing $1/x^{1/6}$.

54. *Money Combinations.* A money clip contains one each of the following bills: \$1, \$2, \$5, \$10, \$20, \$50, and \$100. How many different sums of money can be formed using the bills?

Find the sum.

55. ${}_{100}C_0 + {}_{100}C_1 + \cdots + {}_{100}C_{100}$

56. ${}_nC_0 + {}_nC_1 + \cdots + {}_nC_n$

Simplify.

57. $\sum_{k=0}^{23} \binom{23}{k} (\log_a x)^{23-k} (\log_a t)^k$

58. $\sum_{k=0}^{15} \binom{15}{k} i^{30-2k}$

59. Use mathematical induction and the property
$$\binom{n}{r-1} + \binom{n}{r} = \binom{n+1}{r}$$
to prove the binomial theorem.

Probability

11.8

► **Compute the probability of a simple event.**

When a coin is tossed, we can reason that the chance, or likelihood, that it will fall heads is 1 out of 2, or the **probability** that it will fall heads is $\frac{1}{2}$. Of course, this does not mean that if a coin is tossed 10 times it will necessarily fall heads 5 times. If the coin is a "fair coin" and it is tossed a great many

REVIEW EXERCISES

Determine whether the statement is true or false.

1. A sequence is a function. [11.1]

2. An infinite geometric series with $r = -1$ has a limit. [11.3]

3. Permutations involve order and arrangements of objects. [11.5]

4. The total number of subsets of a set with n elements is n^2. [11.7]

5. Find the first 4 terms, a_{11}, and a_{23}:
$$a_n = (-1)^n\left(\frac{n^2}{n^4+1}\right).$$ [11.1]

6. Predict the general, or nth, term. Answers may vary.
$$2, -5, 10, -17, 26, \ldots$$ [11.1]

7. Find and evaluate:
$$\sum_{k=1}^{4} \frac{(-1)^{k+1}3^k}{3^k - 1}.$$ [11.1]

8. Write sigma notation. Answers may vary.
$0 + 3 + 8 + 15 + 24 + 35 + 48$ [11.1]

9. Find the 10th term of the arithmetic sequence
$$\frac{3}{4}, \frac{13}{12}, \frac{17}{12}, \ldots.$$ [11.2]

10. Find the 6th term of the arithmetic sequence
$$a - b, a, a + b, \ldots.$$ [11.2]

11. Find the sum of the first 18 terms of the arithmetic sequence
$$4, 7, 10, \ldots.$$ [11.2]

12. Find the sum of the first 200 natural numbers. [11.2]

13. The 1st term in an arithmetic sequence is 5, and the 17th term is 53. Find the 3rd term. [11.2]

14. The common difference in an arithmetic sequence is 3. The 10th term is 23. Find the first term. [11.2]

15. For a geometric sequence, $a_1 = -2$, $r = 2$, and $a_n = -64$. Find n and S_n. [11.3]

16. For a geometric sequence, $r = \frac{1}{2}$ and $S_5 = \frac{31}{2}$. Find a_1 and a_5. [11.3]

Find the sum of each infinite geometric series, if it exists. [11.3]

17. $25 + 27.5 + 30.25 + 33.275 + \cdots$

18. $0.27 + 0.0027 + 0.000027 + \cdots$

19. $\frac{1}{2} - \frac{1}{6} + \frac{1}{18} - \cdots$

20. Find fraction notation for $2.\overline{43}$. [11.3]

21. Insert four arithmetic means between 5 and 9. [11.2]

22. *Bouncing Golfball.* A golfball is dropped to the pavement from a height of 30 ft. It always rebounds three-fourths of the distance that it drops. How far (up and down) will the ball have traveled when it hits the pavement for the 6th time? [11.3]

23. *The Amount of an Annuity.* To create a college fund, a parent makes a sequence of 18 yearly deposits of \$2000 each in a savings account on which interest is compounded annually at 2.8%. Find the amount of the annuity. [11.3]

24. *Total Gift.* Suppose you receive 10¢ on the first day of the year, 12¢ on the 2nd day, 14¢ on the 3rd day, and so on.
    a) How much will you receive on the 365th day? [11.2]
    b) What is the sum of these 365 gifts? [11.2]

25. *The Economic Multiplier.* Suppose the government is making a \$24,000,000,000 expenditure for travel to Mars. If 73% of this amount is spent again, and so on, what is the total effect on the economy? [11.3]

Use mathematical induction to prove each of the following. [11.4]

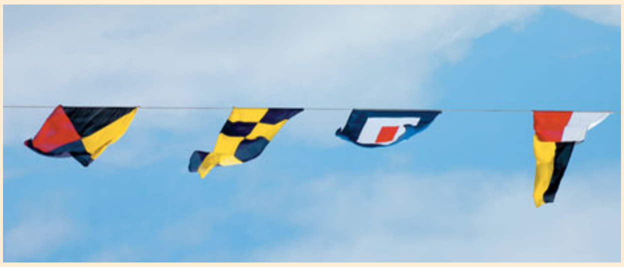
26. For every natural number n,
$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}.$$

27. For every natural number n,
$$1 + 3 + 3^2 + \cdots + 3^{n-1} = \frac{3^n - 1}{2}.$$

28. For every natural number $n \geq 2$,
$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\cdots\left(1 - \frac{1}{n}\right) = \frac{1}{n}.$$

29. *Book Arrangements.* In how many ways can 6 books be arranged on a shelf? [11.5]

30. *Flag Displays.* If 9 different signal flags are available, how many different displays are possible using 4 flags in a row? [11.5]



31. *Prize Choices.* The winner of a contest can choose any 8 of 15 prizes. How many different sets of prizes can be chosen? [11.6]

32. *Fraternity–Sorority Names.* The Greek alphabet contains 24 letters. How many fraternity or sorority names can be formed using 3 different letters? [11.5]

33. *Letter Arrangements.* In how many distinguishable ways can the letters of the word TENNESSEE be arranged? [11.5]

34. *Floor Plans.* A manufacturer of houses has 1 floor plan but achieves variety by having 3 different roofs, 4 different ways of attaching the garage, and 3 different types of entrances. Find the number of different houses that can be produced. [11.5]

35. *Code Symbols.* How many code symbols can be formed using 5 out of 6 of the letters of G, H, I, J, K, L if the letters:

a) cannot be repeated? [11.5]

b) can be repeated? [11.5]

c) cannot be repeated but must begin with K? [11.5]

d) cannot be repeated but must end with IGH? [11.5]

36. Determine the number of subsets of a set containing 8 members. [11.7]

Expand. [11.7]

37. $(m + n)^7$

38. $(x - \sqrt{2})^5$

39. $(x^2 - 3y)^4$

40. $\left(a + \frac{1}{a}\right)^8$

41. $(1 + 5i)^6$, where $i^2 = -1$

42. Find the 4th term of $(a + x)^{12}$. [11.7]

43. Find the 12th term of $(2a - b)^{18}$. Do not multiply out the factorials. [11.7]

44. *Rolling Dice.* What is the probability of getting a 10 on a roll of a pair of dice? on a roll of 1 die? [11.8]

45. *Drawing a Card.* From a deck of 52 cards, 1 card is drawn at random. What is the probability that it is a club? [11.8]

46. *Drawing Three Cards.* From a deck of 52 cards, 3 are drawn at random without replacement. What is the probability that 2 are aces and 1 is a king? [11.8]

47. *Election Poll.* Three people were running for mayor in an election campaign. A poll was conducted to see which candidate was favored. During the polling, 86 favored candidate A, 97 favored B, and 23 favored C. Assuming that the poll is a valid indicator of the election results, what is the probability that the election will be won by A? B? C? [11.8]

48. Which of the following is the 25th term of the arithmetic sequence 12, 10, 8, 6, . . . ? [11.2]

A. -38 **B.** -36

C. 32 **D.** 60

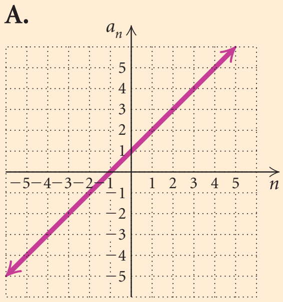
49. What is the probability of getting a total of 4 on a roll of a pair of dice? [11.8]

A. $\frac{1}{12}$ **B.** $\frac{1}{9}$

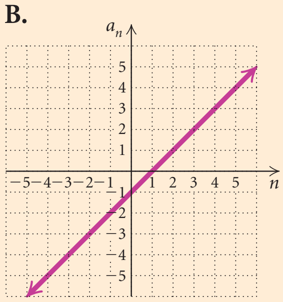
C. $\frac{1}{6}$ **D.** $\frac{5}{36}$

50. The graph of the sequence whose general term is $a_n = n - 1$ is which of the following? [11.1]

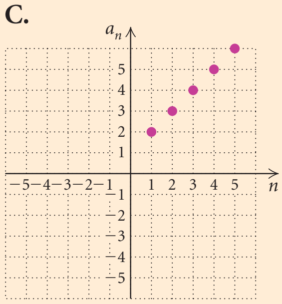
A.



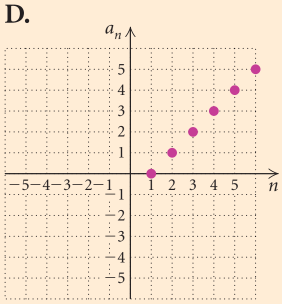
B.



C.



D.



Synthesis

51. Explain why the following cannot be proved by mathematical induction: For every natural number n,

a) $3 + 5 + \cdots + (2n + 1) = (n + 1)^2$. [11.4]
b) $1 + 3 + \cdots + (2n - 1) = n^2 + 3$. [11.4]

52. Suppose that $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ are geometric sequences. Prove that $c_1, c_2, \ldots, c_n$ is a geometric sequence, where $c_n = a_n b_n$. [11.3]

53. Suppose that $a_1, a_2, \ldots, a_n$ is an arithmetic sequence. Is $b_1, b_2, \ldots, b_n$ an arithmetic sequence if:

a) $b_n = |a_n|$? [11.2]
b) $b_n = a_n + 8$? [11.2]
c) $b_n = 7a_n$? [11.2]
d) $b_n = \frac{1}{a_n}$? [11.2]
e) $b_n = \log a_n$? [11.2]
f) $b_n = a_n^3$? [11.2]

54. The zeros of this polynomial function form an arithmetic sequence. Find them. [11.2]

$$f(x) = x^4 - 4x^3 - 4x^2 + 16x$$

55. Write the first 3 terms of the infinite geometric series with $r = -\frac{1}{3}$ and $S_\infty = \frac{3}{8}$. [11.3]

56. Simplify:

$$\sum_{k=0}^{10} (-1)^k \binom{10}{k} (\log x)^{10-k} (\log y)^k. \quad [11.6]$$

Solve for n. [11.6]

57. $\binom{n}{6} = 3 \cdot \binom{n-1}{5}$

58. $\binom{n}{n-1} = 36$

59. Solve for a:

$$\sum_{k=0}^{5} \binom{5}{k} 9^{5-k} a^k = 0. \quad [11.7]$$

Collaborative Discussion and Writing

60. How "long" is 15? Suppose you own 15 books and decide to make up all the possible arrangements of the books on a shelf. About how long, in years, would it take you if you were to make one arrangement per second? Write out the reasoning you used for this problem in the form of a paragraph. [11.5]

61. *Circular Arrangements.* In how many ways can the numbers on a clock face be arranged? See if you can derive a formula for the number of distinct circular arrangements of n objects. Explain your reasoning. [11.5]

62. Give an explanation that you might use with a fellow student to explain that

$$\binom{n}{k} = \binom{n}{n-k}. \quad [11.6]$$

63. Explain why a "combination" lock should really be called a "permutation" lock. [11.6]

64. Discuss the advantages and disadvantages of each method of finding a binomial expansion. Give examples of when you might use one method rather than the other. [11.7]

Chapter 11 Test

1. For the sequence whose nth term is $a_n = (-1)^n(2n + 1)$, find a_{21}.

2. Find the first 5 terms of the sequence with general term

$$a_n = \frac{n+1}{n+2}.$$

3. Find and evaluate:

$$\sum_{k=1}^{4} (k^2 + 1).$$

Write sigma notation. Answers may vary.

4. $4 + 8 + 12 + 16 + 20 + 24$

5. $2 + 4 + 8 + 16 + 32 + \cdots$

6. Find the first 4 terms of the recursively defined sequence

$$a_1 = 3, \quad a_{n+1} = 2 + \frac{1}{a_n}.$$

7. Find the 15th term of the arithmetic sequence $2, 5, 8, \ldots$.

8. The 1st term of an arithmetic sequence is 8 and the 21st term is 108. Find the 7th term.

9. Find the sum of the first 20 terms of the series $17 + 13 + 9 + \cdots$.

10. Find the sum: $\sum_{k=1}^{25} (2k + 1)$.

11. Find the 11th term of the geometric sequence $10, -5, \frac{5}{2}, -\frac{5}{4}, \ldots$.

12. For a geometric sequence, $r = 0.2$ and $S_4 = 1248$. Find a_1.

Find the sum, if it exists.

13. $\sum_{k=1}^{8} 2^k$

14. $18 + 6 + 2 + \cdots$

15. Find fraction notation for $0.\overline{56}$.

16. *Salvage Value.* The value of an office machine is \$10,000. Its salvage value each year is 80% of its value the year before. Give a sequence that lists the salvage value of the machine for each year of a 6-yr period.

17. *Hourly Wage.* Tamika accepts a job, starting with an hourly wage of \$8.50, and is promised a raise of 25¢ per hour every three months for 4 yr. What will Tamika's hourly wage be at the end of the 4-yr period?

18. *Amount of an Annuity.* To create a college fund, a parent makes a sequence of 18 equal yearly deposits of \$2500 in a savings account on which interest is compounded annually at 5.6%. Find the amount of the annuity.

19. Use mathematical induction to prove that, for every natural number n,

$$2 + 5 + 8 + \cdots + (3n - 1) = \frac{n(3n + 1)}{2}.$$

Evaluate.

20. ${}_{15}P_6$ **21.** ${}_{21}C_{10}$ **22.** $\binom{n}{4}$

23. How many 4-digit numbers can be formed using the digits 1, 3, 5, 6, 7, and 9 without repetition?

24. How many code symbols can be formed using 4 of the 6 letters A, B, C, X, Y, Z if the letters:

a) can be repeated?
b) are not repeated and must begin with Z?

25. *Scuba Club Officers.* The Bay Woods Scuba Club has 28 members. How many sets of 4 officers can be selected from this group?

26. *Test Options.* On a test with 20 questions, a student must answer 8 of the first 12 questions and 4 of the last 8. In how many ways can this be done?

27. Expand: $(x + 1)^5$.

28. Find the 5th term of the binomial expansion $(x - y)^7$.

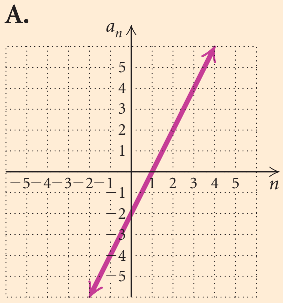
29. Determine the number of subsets of a set containing 9 members.

30. *Marbles.* Suppose that we select, without looking, one marble from a bag containing 6 red marbles and 8 blue marbles. What is the probability of selecting a blue marble?

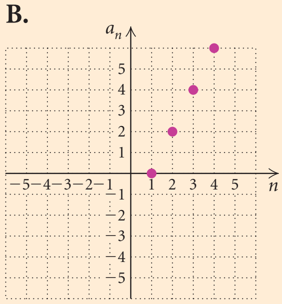
31. *Drawing Coins.* Ethan has 6 pennies, 5 dimes, and 4 quarters in his pocket. Six coins are drawn at random. What is the probability of getting 1 penny, 2 dimes, and 3 quarters?

32. The graph of the sequence whose general term is $a_n = 2n - 2$ is which of the following?

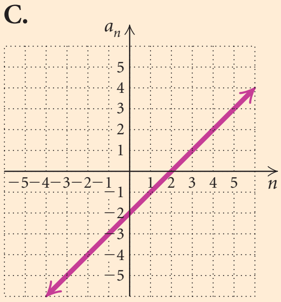
A.



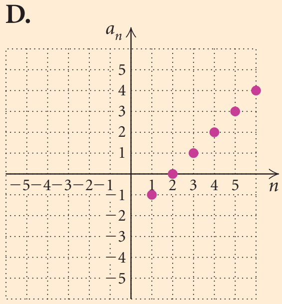
B.



C.



D.



Synthesis

33. Solve for n: ${}_nP_7 = 9 \cdot {}_nP_6$.