

# I. Game Theory Basics

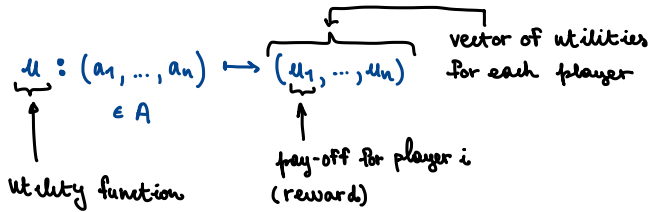
$N = \{1, \dots, n\} \leftarrow$  set of players

$S_i \leftarrow$  set of actions for player  $i$

$A = S_1 \times \dots \times S_n \leftarrow$  action profile  
(set of every possible combinations of actions)

In case of RPS:

$$A = \{(R,R), (R,P), \dots, (S,P), (S,S)\}$$



Pay-off table for RPS:

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

Zero-sum game - if each vector's sum is 0

Ext-sum game - each vector has ext sum

we can reformulate to 0-sum if we add a dummy player with always negative pay-off

pure strategy - if player chooses a single action with probability 1

mixed strategy - at least 2 actions with positive probability  $\rightarrow$  we denote it like  $\sigma$

$\sigma_i(s) \leftarrow$  probability that player  $i$  chooses action  $s$   
( $s \in S_i$ )

$$u_i(\sigma_i, \sigma_{-i}) = \sum_{s \in S_i} \sum_{s' \in S_{-i}} \sigma_i(s) \cdot \sigma_{-i}(s') \cdot u_i(s, s')$$

player 1      player 2

$\rightarrow$  returns a vector

Expected utility for both pl., if player 1 play with strategy  $\sigma_i$  and player 2 plays with strategy  $\sigma_{-i}$   
(In the 2-player case).

The best strategy for every player is to maximize their expected utility

Nash Equilibrium  $\leftarrow$  Both players play best "response" strategies  
No player can improve by changing strategy alone.

Game between Garry and Monica:

		Garry	
		C	M
Monica	C	(2,1)	(0,0)
	M	(0,0)	(1,2)

Labels: "cinema" points to the top row (C), "match" points to the bottom row (M).

$\sigma_{\text{garry}}(C) = x \leftarrow$  probability that Garry chooses a movie

$\Rightarrow \sigma_{\text{garry}}(M) = 1-x$

$\sigma_{\text{monica}}(C) = y$

$\Rightarrow \sigma_{\text{monica}}(M) = (1-y)$

$$u_{\text{monica}}(C) = \underbrace{\sigma_{\text{gary}}(C)}_{\text{probability}} \cdot \underbrace{u_{\text{monica}}(C,C)}_{\text{action utility}} + \underbrace{\sigma_{\text{gary}}(M)}_{\text{probability}} \cdot \underbrace{u_{\text{monica}}(C,M)}_{\substack{\text{monica} \\ \text{gary}}} = x \cdot 2 + (1-x) \cdot 0 = 2x$$

$$u_{\text{monica}}(M) = \sigma_{\text{gary}}(C) \cdot u_{\text{monica}}(M,C) + \sigma_{\text{gary}}(M) \cdot u_{\text{monica}}(M,M) = x \cdot 0 + (1-x) \cdot 1 = 1-x$$

For monica to be indifferent:  $u_{\text{monica}}(C) = u_{\text{monica}}(M)$

$$2x = 1-x$$

$$x = \frac{1}{3} \Rightarrow \sigma_{\text{gary}}(C) = \frac{1}{3}$$

$$\sigma_{\text{gary}}(M) = \frac{2}{3}$$

## II. Regret Matching

1st round

pl	opp	utility for pl	regret for pl	strategy
R	P	-1	0	0
P	P	0	+1	1/3
S	P	+1	+2	2/3

Regret for player  $i$  of not choosing an action  $s_2$  when choosing  $s_1$ :  $r_i(s_1, s_2) = u_i(s_2) - u_i(s_1)$

Regret matching - select actions with probability proportional to positive regrets

Our goal is to minimize expected regrets over time

2nd round

pl	opp	utility for pl	regret for pl	previous regrets	cu	ive regrets	strategy
S	R	-1	0	2		2	1/3
R	R	0	1	0		1	1/6
P	R	+1	2	1		3	1/2

3rd round

pl	opp	utility for pl	regret for pl	previous regrets	cumulative regrets	strategy
R	S	+1	0	2	2	1/2
P	S	-1	-2	1	0	0
S	S	0	-1	3	2	1/2

can there be negative? → should be negative and positive