

## 1 Mushu

1. Consider the Tree class below. Suppose we would like to write a method for this Tree class, `getAncestor(int k, Node target)`. This method takes in an integer  $k$  and a Node `target`, and returns the  $k$ 'th ancestor of `target` in our tree (you may assume such an ancestor exists). You may also assume that  $k \geq 0$ , that `target != null`, and that there are no cycles in our tree before we call this method.

```

1  public class Tree<T> {
2      private Node root;
3
4      private class Node{
5          public T item;
6          public ArrayList<Node> children;
7      }
8
9      public Node getAncestor(int k, Node target) {
10         List<Node> list = new LinkedList<>();
11         ancestorHelper(root, target, list);
12         return list.get(list.size() - 1 - k);
13     }
14     private boolean ancestorHelper(Node x, Node target, List<Node> L) {
15         L.add(x);
16         if (x == target) {
17             return true;
18         }
19         for (Node n : x.children) {
20             if (ancestorHelper(n, target, L))
21                 return true;
22         }
23         L.remove(x); // or removeLast
24         return false;
25     }
26 }
27

```



2. Give a bound on the runtime of `getAncestor(int k, Node target)` in the best and worst cases in  $\Theta(\cdot)$  notation in terms of  $N$  and  $k$ , for a tree with  $N$  nodes. How does our choice of list implementation on line 10 affect our runtime?

## 2 Kontakte

We're going to make our own Contacts application! The application must perform two operations: `addName(String name)`, which stores a new contact, and `countPartial(String partial)`, which returns the number of contacts whose names begin with `partial`. Implement both of these methods in the `Contacts` class below. You may find the work already done in the private `Node` class, as well as the method `String::charAt(int index)` useful.

```

1      public class Contacts {
2
3          private class Node {
4              public int numWords;
5              public Map<Character, Node> children;
6
7              public Node() {
8                  numWords = 0;
9                  children = new HashMap<Character, Node>();
10             }
11             // no ending flag,
12             // because we don't need to get strings
13             // here
14             Node root;
15
16             public Contacts() {root = new Node();}
17
18             public void addName(String name) {
19                 Node current = root;
20                 for (int i = 0; i < name.length; i++) {
21                     if (!current.children.containsKey(name.charAt(i))) {
22                         Node n = new Node();
23                         current.children.put(name.charAt(i), n);
24                     }
25                     current = current.children.get(name.charAt(i));
26                     current.numWords += 1;
27                 }
28             }
29
30             public int countPartial(String partial) {
31                 Node current = root;
32                 for (int i = 0; i < partial.length(); i++) {
33                     if (current.children.containsKey(partial.charAt(i))) {
34                         current = current.children.get(partial.charAt(i));
35                     }
36                     else {return 0;}
37                 }
38                 return current.numWords;
39             }
40         }
41     }

```

// finally, current points to  
the last char of partial

### 3 KND Trees

A  $k$ -d tree is a binary tree where each node contains a point of dimension  $k$ . Our goal is to create a tree of points which, when given a  $k$ -dimensional coordinate, can find the point closest to that coordinate (i.e. "what is the closest point to  $(a, b)$ ?" ).

Each node also has a splitting plane, which is one of these  $k$  dimensions. Say a node  $n$  has splitting plane  $x$ . Then everything to the left of  $n$  will have an  $x$ -coordinate less than or equal to  $n$ 's. Similarly, everything to the right of  $n$  will have an equal or greater  $x$ -coordinate. If  $n$  instead split on  $y$ , then the above holds for  $y$ -coordinates.

From the Wikipedia page for  $k$ -d trees, "As one moves down the tree, one cycles through the  $k$  axes used to select the splitting planes."

This means in a 3-dimensional tree:

- the root would have an x-aligned plane
- the roots children would both have y-aligned planes
- the roots grandchildren would all have z-aligned planes
- the roots great-grandchildren would all have x-aligned planes, etc.

1. Consider a 2-d tree in which the root splits on  $x$ . Normally, we want to turn a fixed set of points into a  $k$ -d Tree, and we don't have to worry about later additions. This makes it easier to make our Tree bushy. Discuss how you may do this efficiently, and draw a balanced  $k$ -d Tree of the points  $(2, 3), (5, 4), (9, 6), (4, 7), (8, 1), (7, 2), (10, 10)$

2. What is the closest point in our tree to the coordinate  $(3, 6)$ ? What about  $(2, 5)$ ? What can you conclude about the worst-case runtime for closest point (otherwise known as nearest neighbor) search in a reasonably bushy  $k$ -d tree?