1) Solve $\frac{dy}{dx} + 2xy^2 = 0$, $y(2) = \frac{1}{5}$.

$$\frac{dy}{dx} = -2xy^2.$$

$$\cancel{\frac{dy}{dx}} \quad \frac{dy}{y^2} = -2x\,dx.$$

$$\int \frac{dy}{y^2} = -\int 2x\,dx.$$

$$-\frac{1}{y} = -x^2 + C.$$

$$y = \frac{-1}{-x^2 + C}.$$

Since $y(2) = \frac{1}{5}$,

$$\frac{1}{5} = \frac{-1}{-\left(\frac{1}{5}\right)^2 + C}.$$

$$-\frac{1}{25} + C = -5.$$

$$C = \frac{1}{25} - 5$$

$$C = -\frac{124}{25}.$$

$\Rightarrow$ Sol. $y(x) = \frac{-1}{-x^2 - 124/25}$

$$\frac{1}{5} = \frac{-1}{-2^2 + C}$$

$$-4 + C = -5$$

$$C = -1$$

$$\boxed{\text{Sol. } y(x) = \frac{1}{x^2 + 1}}$$

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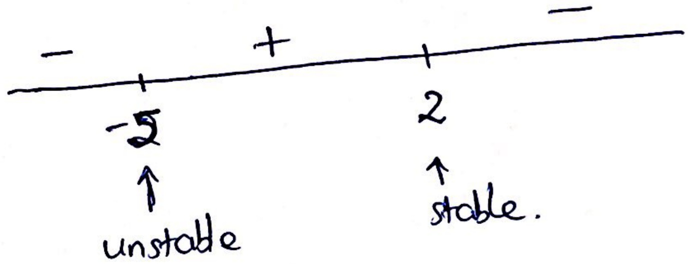
2) $\frac{dy}{dt} = e^y(10 - 3y - y^2)$.

a) Find equilibrium points: set $e^y(10 - 3y - y^2) = 0$

$$\Rightarrow \quad 10 - 3y - y^2 = 0$$
$$y^2 + 3y - 10 = 0$$
$$(y-2)(y+5) = 0$$
$$y = 2 \quad \text{or} \quad y = -5.$$

phase line



b) Since $-5 < -4 = y(0) < 2$, $\phi(t)$ is an increasing function

$$\lim_{t \to \infty} \phi(t) = 2.$$

3) Solve $t^2 \frac{dy}{dt} + t(t+2)y = e^t$, $t > 0$

$$\frac{dy}{dt} + \frac{t+2}{t} y = \frac{e^t}{t^2}.$$

integrating factor: $u(t) = e^{\int \frac{t+2}{t} dt} = e^{\int 1 + \frac{2}{t} dt} = e^{t + 2\ln t} = t^2 e^t$.

$$\Rightarrow \quad y(t) = \frac{\int \left(\frac{e^t}{t^2}\right)(t^2 e^t)\, dt + C}{t^2 e^t} = \frac{\int e^{2t} dt + C}{t^2 e^t} = \frac{\frac{1}{2} e^{2t} + C}{t^2 e^t}.$$

4) a) $x - y^3 + y^2 \sin x = (3xy^2 + 2y\cos x)y'$

$$\underbrace{(x - y^3 + y^2\sin x)}_{M(x,y)} + \underbrace{(-3xy^2 - 2y\cos x)}_{N(x,y).} \frac{dy}{dx} = 0.$$

$M(x,y) = x - y^3 + y^2\sin x$

$N(x,y) = -3xy^2 - 2y\cos x.$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= -3y^2 + 2y\sin x. \\ \frac{\partial N}{\partial x} &= -3y^2 + 2y\sin x \end{aligned}\right\} \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{exact.}$$

b) $\left(x^2y^3 - \frac{1}{1+x^2}\right)dx + (x^3y^2 + \sin y)\,dy = 0.$

$M(x,y) = x^2y^3 - \frac{1}{1+x^2}$

$N(x,y) = x^3y^2 + \sin y.$

$$\begin{aligned} \psi &= \int M\,dx + h(y) \\ &= \int\left(x^2y^3 - \frac{1}{1+x^2}\right)dx + h(y). \\ &= \frac{1}{3}x^3y^3 - \tan^{-1}(x) + h(y). \end{aligned}$$

$$\frac{\partial \psi}{\partial y} = x^3y^2 - 0 + h'(y). = N(x,y).$$

$\Rightarrow \quad x^3y^2 + h'(y) = x^3y^2 + \sin y.$

$h'(y) = \sin y.$

$h(y) = -\cos y.$

Sol. $\boxed{\frac{1}{3}x^3y^3 - \tan^{-1}(x) - \cos y = C.}$