L-Transform Practice

MATH 20D, LECTURE D00, FALL 2017

1. Find the unique solution of the following initial value problems by using the Laplace transform

   (a) $y'' + 2y' + 10y = 0, \quad y(0) = 0, y'(0) = 6.$

   (b) $y'' + 4y = 16u_\pi(t), \quad y(0) = 0, y'(0) = 1.$

2. Find the Laplace transform of the following functions.

   (a) $f(t) = \begin{cases} 3t, & \text{if } 0 \le t < 2, \\ 6, & \text{if } t \ge 2. \end{cases}$

   (b) $f(t) = \begin{cases} \sin(2t), & \text{if } 0 \le t < \pi, \\ 0, & \text{if } t \ge \pi. \end{cases}$

   (c) $f(t) = \begin{cases} t^2 - 4t, & \text{if } 0 \le t < 4, \\ 0, & \text{if } t \ge 4. \end{cases}$

   (d) $f(t) = \begin{cases} \cos t, & \text{if } 0 \le t < \pi, \\ -2, & \text{if } t \ge \pi. \end{cases}$

3. Find the inverse Laplace transform of the following functions.

   (a) $F(s) = \frac{s-2}{s^2-4s+13} e^{-\pi s/2}.$

   (b) $F(s) = \frac{s+1}{s^2+2s+10} e^{-\pi s}.$

   (c) $F(s) = \frac{1}{(s+1)^3} e^{-2s} + \frac{2}{s^4} e^{-4s}.$

   (d) $G(s) = \frac{2}{s^2+9} e^{-2\pi s}$

   (e) $F(s) = \frac{1}{(s-2)^3} e^{-3s} + \frac{1}{(s-2)^2} e^{-4s}.$