HW06

Math 102, Winter 2018

Due Wednesday, Feb 21

Name:

Problem 1 Express the Gram-Schmidt orthogonalization of $\boldsymbol{a}_1, \boldsymbol{a}_2$ as $A = QR$:

$$\boldsymbol{a}_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \quad \boldsymbol{a}_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

Problem 2 a) Find a basis for the subspace S in $\mathbb{R}^4$ spanned by all solutions of

$$x_1 + x_2 + x_3 - x_4 = 0.$$

b) Find a basis for the orthogonal complement $S^\perp$.

c) Find $\boldsymbol{b}_1$ in S and $\boldsymbol{b}_2$ in $S^\perp$ so that $\boldsymbol{b}_1 + \boldsymbol{b}_2 = \boldsymbol{b} = (1, 1, 1, 1)$.

Problem 3 By applying row operations to produce an upper triangular U, compute

$$\det \begin{bmatrix} 1 & 2 & -2 & 0 \\ 2 & 3 & -4 & 1 \\ -1 & -2 & 0 & 2 \\ 0 & 2 & 5 & 3 \end{bmatrix} \quad \text{and} \quad \det \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}.$$

Exchange rows 3 and 4 of the second matrix and recompute the pivots and determinant.

Problem 4 If Q is an orthogonal matrix, so that $Q^T Q = I$, prove that $\det Q$ equals $+1$ and -1. What kind of box is formed from the rows (or columns) of Q?