Final, Dec 12, 2012

1) $F(x) = \int_A^x f(t)\,dt \Rightarrow F(A) = 0.$

$F'(x) = f(x).$ for all $x > A.$

a) critical points ~~at~~ occurs when $F'(x) = 0. \Rightarrow f(x) = 0.$

$\Rightarrow$ at $x = C, E, G, \cancel{I}.$

b)

x	A		B		C		D		E		F		G		H		I
$F'(x)$		.	–	–	0	+		+	0	–		–	0	–		–	

local max: at $x = E.$

local min at $x = C.$

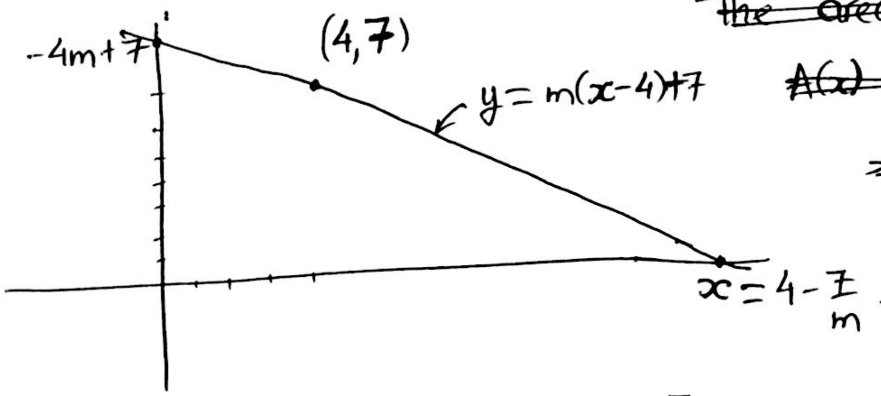
d) Its absolute maximum is at $x = E.$

e) $F''(x) = f'(x) = 0$ at $x = B, D, F, G, H.$

x	A		B		C		D		E		F		G		H		I
$F''(x)$		–	0	+		+	0	–		–	0	+	0	–	0	+	

$\Rightarrow$ inflection points at $x = B, D, F, G, H.$

2)



~~The area of is.~~

~~$A(x) = \frac{1}{2} y \cdot x$~~

$=$

The x-intercept is $x = 4 - \frac{7}{m}.$

y-intercept is $y = -4m + 7.$

$\Rightarrow$ The area function is $A(m) = \frac{1}{2}(-4m+7)\left(4 - \frac{7}{m}\right)$

$$= \frac{1}{2}\left(-16m + 28 + 28 - \frac{49}{m}\right)$$

$$A(m) = \frac{1}{2}\left(-16m + 56 - \frac{49}{m}\right).$$

$$A'(m) = \frac{1}{2}\left(-16 + \frac{49}{m^2}\right) = 0.$$

$$\Rightarrow -16 + \frac{49}{m^2} = 0.$$

$$m^2 = \frac{49}{16} \Rightarrow m = \pm\frac{7}{4}.$$

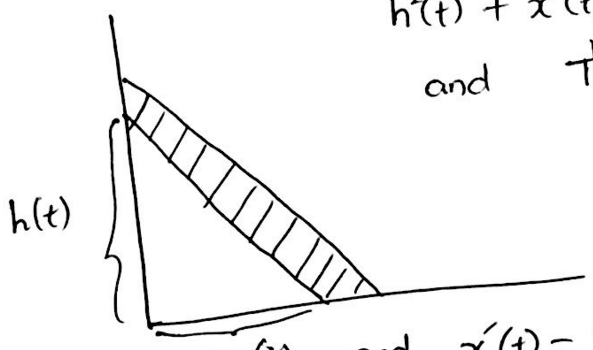
but we only take $m = -\frac{7}{4}$. since $m < 0$.

$$A''(m) = \frac{1}{2}\left(-\frac{98}{m^3}\right) > 0 \quad \text{for } m < 0.$$

$\Rightarrow$ min at $m = -\frac{7}{4}$.

$\Rightarrow$ The eq. of the line is $y = -\frac{7}{4}(x-4) + 7$.

3)



$$h^2(t) + x^2(t) = 25^2$$

and the area $\div$

$$A(t) = \frac{1}{2} h(t)\, x(t).$$

$$A'(t) = \frac{1}{2} h'(t)\, x(t) + \frac{1}{2} h(t)\, x'(t).$$

$$= \frac{1}{2} h'(t)\, x(t) + \frac{3}{2} h(t)$$

$x(t)$ and $x'(t) = 3$ ft/s

when $x(t) = 20$, $A'(t) = \frac{1}{2} h'(t) \cdot 20 + \frac{3}{2} h(t)$.

we need to find $h(t)$ and $h'(t)$:

$$h(t) = \sqrt{25^2 - 20^2} = \sqrt{15^2} = 15.$$

$$2\, h(t)\, h'(t) + 2\, x(t)\, x'(t) = 0.$$

$$\Rightarrow h'(t) = -\frac{x(t)\, x'(t)}{h(t)} = -\frac{20 \cdot 3}{15} = -4.$$

$\therefore \quad A'(t) = -\frac{4}{2} \cdot 20 \quad + \frac{3}{2}(15) = \frac{45}{2} - 40 = \frac{-35}{2}.$

4) a) $\int \left(\frac{w^2+1}{w} + \frac{1}{w^2+1}\right) dw = \int \left(w + w^{-1} + \frac{1}{w^2+1}\right) dw.$

$$= \frac{w^2}{2} + \ln|w| + \tan^{-1}(w) + C.$$

b) $\int_{-e}^{-1} \frac{2012}{x} dx = 2012 \ln|x| \Big|_{x=-1} - 2012 \ln|x| \Big|_{x=-e}$

$$= 2012 \ln|-1| - 2012 \ln|-e|.$$

$$= 2012(0) - 2012.$$

$$= -2012.$$

c) $\frac{d}{dx} \cos(\sin(x^2)) = -\sin(\sin(x^2)) \cdot \cos(x^2) \cdot 2x.$

5) $v(t) = t^2 + t - 6.$

a) the displacement of the object over the interval $[0, 3]$:

$$\int_0^3 v(t)\,dt = \int_0^3 t^2 + t - 6\,dt = \frac{t^3}{3} + \frac{t^2}{2} - 6t \Big|_{t=0}^{3}$$

$$= \frac{3^3}{3} + \frac{3^2}{2} - 6(3) - 0.$$

$$= 9 + \frac{9}{2} - 18. = -\frac{9}{2}.$$

b) the distance that the object travels over $[0, 3]$:

$$\int_0^3 |v(t)|\,dt = \int_0^3 |t^2 + t - 6|\,dt. \quad = \int_0^2 -(t^2 + t - 6)\,dt + \int_2^3 (t^2 + t - 6)\,dt$$

$$|t^2 + t - 6| = \begin{cases} t^2 + t - 6 & \text{if } t \leq -3 \text{ or } t \geq 2 \\ -(t^2 + t - 6) & \text{if } -3 < t < 2 \end{cases}$$

$$= -\frac{t^3}{3} - \frac{t^2}{2} + 6t \Big|_{t=0}^{2} + \left(\frac{t^3}{3} + \frac{t^2}{2} - 6t\right)\Big|_{t=2}^{3}$$

$$= \left(-\frac{8}{3} - \frac{4}{2} + 12 - 0\right) + \left(\frac{3^3}{3} + \frac{9}{2} - 18 - \frac{8}{3} \cancel{+} -\frac{4}{2} + 12\right)$$

$= \ldots$

6) $f(x) = \tan\left(\frac{\pi}{4} + x\right) - 1.$

$f'(x) = \sec^2\left(\frac{\pi}{4} + x\right).$

At $x = 0$,

$$\begin{aligned} L(x) &= f(0) + f'(0)(x-0) \\ &= \tan\left(\frac{\pi}{4}\right) - 1 + \sec^2\left(\frac{\pi}{4}\right) \cdot x \\ &= 1 - 1 + 2x. \\ &= 2x. \end{aligned}$$

$\therefore f(0.0002) \approx L(0.0002) = 0.0004.$

7) $$\begin{aligned} \lim_{x \to 0^+} x^{\sin(2x)} &= \lim_{x \to 0^+} e^{\ln x^{\sin(2x)}} \\ &= \lim_{x \to 0^+} e^{\sin(2x)\ln x} \\ &= e^{\lim_{x \to 0^+} \sin(2x) \ln x} = e^0 = 1. \end{aligned}$$

Now, compute

$$\begin{aligned} \lim_{x \to 0^+} \sin(2x)\ln x &= \lim_{x \to 0^+} \frac{\ln x}{\csc(2x)} \quad \frac{-\infty}{\infty} \\ &\overset{\text{L'Hôpital}}{=} \lim_{x \to 0^+} \frac{1/x}{-\csc(2x)\cot(2x)\cdot 2} \\ &= \lim_{x \to 0^+} -\frac{\sin(2x)\tan(2x)}{2x} \\ &= -\lim_{x \to 0^+} \frac{\sin(2x)}{2x} \cdot \tan(2x) = 0. \end{aligned}$$

8) a) $F(x) = \int_{x}^{\pi} \sqrt{1 + \sec(2t)}\, dt = -\int_{\pi}^{x} \sqrt{1 + \sec(2t)}\, dt$

$F'(x) = -\sqrt{1 + \sec(2x)}$.

b) $h(x) = \int_{0}^{1/x} \arctan(2t)\, dt$.

$h'(x) = \arctan\left(\frac{2}{x}\right) \cdot \left(-\frac{1}{x^2}\right)$.