

HW08

MATH 102, WINTER 2018

DUE WEDNESDAY, MARCH 7

NAME:

Problem 1 Find the rank and all four eigenvalues for both the matrix of ones and the checker board matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

Which eigenvectors correspond to nonzero eigenvalues?

Problem 2 Suppose the eigenvector matrix S has $S^T = S^{-1}$. Show that $A = S\Lambda S^{-1}$ is symmetric and has orthogonal eigenvectors.

Problem 3 Diagonalize A and compute $S\Lambda^k S^{-1}$ to prove this formula for A^k :

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad \text{has} \quad A^k = \frac{1}{2} \begin{bmatrix} 3^k + 1 & 3^k - 1 \\ 3^k - 1 & 3^k + 1 \end{bmatrix}.$$

Problem 4 Write $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ in the form $S\Lambda S^{-1}$. Find e^{At} from $S e^{\Lambda t} S^{-1}$.