

Math 170A Introduction to Numerical Analysis – Fall 2016

Homework 4 – DUE TUESDAY, NOV 22

1. Section 8.3: Do the following two exercises.

(a) Consider Jacobi's Method for solving the system $Ax = b$ where

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

i. Verify that the iteration matrix G corresponding to the Jacobi Method for this system is

$$G = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix},$$

and verify that the eigenvalues for this matrix are $\lambda_1 = 0.5$ and $\lambda_2 = -0.5$ with corresponding eigenvectors $v_1 = (1, -1)$ and $v_2 = (1, 1)$.

ii. Suppose that $b = (0, 0)$, so that the true solution is $x = (0, 0)$. If we choose $x^{(0)} = (5, -1)$, then $e^{(0)} = x - x^{(0)} = (-5, 1)$. Verify that $e^{(0)} = c_1 v_1 + c_2 v_2$, where $c_1 = -3$ and $c_2 = -2$.

iii. Use the fact that $e^{(k)} = c_1 \lambda_1^k v_1 + c_2 \lambda_2^k v_2$ to show that

$$e^{(k)} = \begin{cases} \begin{pmatrix} -\frac{5}{2^k} & \frac{1}{2^k} \end{pmatrix} & \text{for } k \text{ even,} \\ \begin{pmatrix} -\frac{1}{2^k} & \frac{5}{2^k} \end{pmatrix} & \text{for } k \text{ odd.} \end{cases}$$

Compute $\|e^{(3)}\|_2$, $\|e^{(4)}\|_2$, and $\|e^{(5)}\|_2$.

iv. For the same coefficient matrix A , suppose that any b and $x^{(0)}$ are chosen so that the initial error is $e^{(0)} = x - x^{(0)} = (-5, 1)$. What would you predict that the errors $e^{(1)}, e^{(2)}, e^{(3)}, e^{(4)}$, and $e^{(5)}$ would be?

(b) Do the above exercise with the Gauss-Seidel Method instead of the Jacobi Method. You will need to find G and its eigenvalues and eigenvectors, as well as c_1 and c_2 . (DO NOT TURN IN This exercise.)

2. Section 8.4: 7a, 12.

3. Section 1.7: 10bc, 18, 34, 37.

4. Section 1.8: 4, 9, 10.

Programming

1. Write a function in Matlab that takes as input a symmetric positive definite matrix A , the right hand side vector b , initial guess $x^{(0)}$, and number of iterations, and returns as output the solution of $Ax = b$ as found by performing the *steepest descent* method. Use basic programming, along with Matlab's built in basic matrix computations, such as addition, subtraction, "division," and multiplication, as needed.

- (a) Write out or print out your function.
 - (b) Run the case with $A = \begin{bmatrix} 9 & 1 \\ 1 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and initial guess $\begin{bmatrix} 1 & 1 \end{bmatrix}$ for 10 iterations. Print out your solution. What is the residual of this result?
2. Write a function in Matlab that takes as input a tridiagonal matrix given as three vectors: $n \times 1$ vector v representing the main diagonal, $(n - 1) \times 1$ vector w representing the upper diagonal, and $(n - 1) \times 1$ vector z representing the lower diagonal. Have this function output the LU factorization with the U as two vectors and the L as one vector representing the diagonals. Also output the number of flops used. Use only basic programming.
- (a) Write out or print out your function.
 - (b) Run the case with $n = 10$, v the vector of 2's, w and z the vector of -1's. Write down your results for the diagonals of L and U.
 - (c) Run the case with $n = 50$ and $n = 100$ with v the vector of 2's, w and z the vector of -1's. Write down your results for the number of flops used.