Math 102 - Winter 2013 - Midterm II

Name: ______________________________

Student ID: ______________________________

Section time: ______________________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones.

Read each question carefully, and show all your work.

There are 5 questions which are worth 50 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		11
2		15
3		8
4		8
5		8
Total		50

Problem 1. [*11 points.*]

Consider the subspace V of $\mathbb{R}^3$ spanned by the **orthonormal** vectors

$$\vec{v}_1 = \begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix}.$$

(You do not need to verify that the two vectors are orthonormal.)

(i) [4] Find the projection of the vector $\begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$ onto V.

(ii) [4] Find a basis for the orthogonal complement $V^{\perp}$.

(iii) [3] Find the matrix of the projection onto $V^{\perp}$.

Problem 2. [*15 points.*]

Consider the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ -1 & -2 \\ -1 & -1 \end{bmatrix} \text{ and the vector } b = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

(i) [4] Find the left inverse of A.

(ii) [4] Find the matrix of the orthogonal projection onto the column space of A.

(iii) [4] Find the QR decomposition of the matrix A.

(iv) [3] Find the least squares solution of the system $Ax = b$ using any method you like.

Problem 3. [*8 points.*]

Consider the two matrices

$$A = \begin{bmatrix} 0 & 0 & a \\ 1 & 0 & 3 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

(i) [3] Find the value of a for which the matrices A and B have the same determinant.

(ii) [5] Are there any values of a for which the matrices A and B are similar?

Problem 4. [*8 points.*]

Let $\mathcal{P}$ be the vector space of polynomials of degree less or equal to 2, endowed with the inner product

$$\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)x^2 \, dx.$$

Find an orthogonal basis for $\mathcal{P}$ using Gram-Schmidt on the standard basis $\{1, x, x^2\}$.

Problem 5. [*8 points.*]

Let V be subspace of $\mathbb{R}^n$ of dimension k for $0 < k < n$, and let P denote the matrix of the orthogonal projection Proj: $\mathbb{R}^n \to \mathbb{R}^n$ onto V.

(i) [1] What is the dimension of $V^{\perp}$?

(ii) [2] Explain that the eigenvalues of P equal either 0 and 1, and write down the two corresponding eigenspaces.

(iii) [3] Is P diagonalizable? Why or why not?

(iv) [2] What is the trace of P?