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Linear Algebra - Exam #1 - A. Terras, April 27, 2007

The exam is closed book, no calculators, no computers, no notes, no headphones .... Each problem is worth the same number of points.

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1) **Define** the following and give an example.
   a) linearly independent vectors in a vector space V
   c) linear transformation $T:V\to W$ where V and W are vector spaces

2) Given the matrix A below, find the reduced echelon form of A and then find a basis for the column space ColA and a basis for the null space NulA.

$$A=\begin{bmatrix} -3 & 9 & -2 & -6 \\ 6 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix}.$$

3) **True-False.** Tell whether the following statements are true or false. If true, give a brief explanation and if false, give a counterexample.
   a) The plane consisting of vectors (x,y,z) such that x+y+z = 1 is a subspace of $\mathbb{R}^3$.

   b) Suppose W is a subspace of V and $W^\perp$ denotes the orthogonal complement of W. Then $\dim(W) + \dim(W^\perp) = \dim(V)$.

   c) Suppose A and B are nxn matrices. Then AB=BA.

4) Let $\mathbb{P}_n$ be the vector space of polynomials of degree less than or equal to n.
   a) What is the standard basis B_n for $\mathbb{P}_n$?

   b) Let $L:\mathbb{P}_2\to\mathbb{P}_3$ be the function defined by $Lp(x)=\int_0^x p(t)dt$.

   Find the matrix of L using the basis B_2 for $\mathbb{P}_2$ and B_3 for $\mathbb{P}_3$.

5) Suppose that A is an mxm real matrix. Show that the following statements are equivalent.

   i) The columns of A span $\mathbb{R}^m$.

   ii) The equation $A\vec{x}=\vec{0}$ has a unique solution $\vec{x}=\vec{0}$.

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