Mat 102 - Applied Linear Algebra

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  → this course

Contains / will contain:

- Syllabus, Exams schedule, HW.
- Office Hours: 11-12 MWF at AP&M 6341.
- TA information
- Textbook
- Lecture Notes.

Leture 1: Introduction and the geometry of linear equations.

Why study Linear Algebra ?

Each time you are taking a photo by using your cellphone/ camera, have you ever wondered how an image with multiple attributes like color can be stored ?

This is achieved by storing the pixel intensities in a construct called MATRIX. Then this matrix can be processed to ~~identity~~ Identify colors etc.

So any operation which you want to perform on an image would likely use Linear Algebra and matrices at the back end.

* Adding and scaling vectors:

. Example 1. We have already encountered matrices such as

$$\begin{bmatrix} 1 & 4 & 2 & 3 \\ 2 & -1 & 2 & 2 \\ 3 & 2 & -2 & 0 \end{bmatrix}$$

Each column is what we call a column vector.

. Example 2.

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 5 \end{bmatrix}, \quad 7 \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7x_1 \\ 7x_2 \\ 7x_3 \end{bmatrix}.$$

{ Vectors of the same kind can be added. (Matrices)

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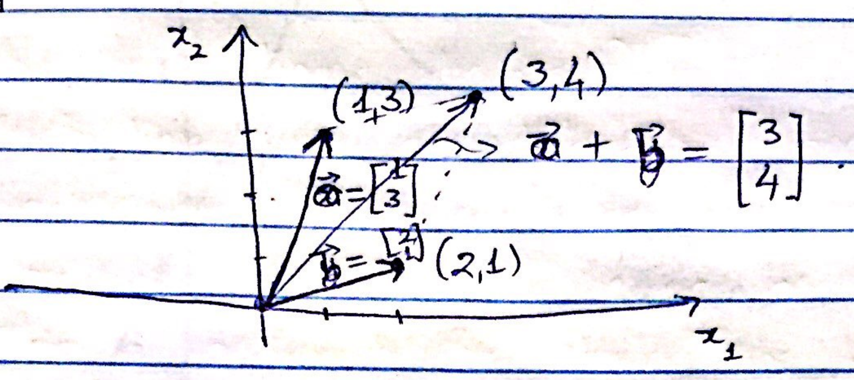
Example 3. (Geometric description of $\mathbb{R}^2$) A vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ represents the point (x_1, x_2) in the plane.



Def: Given vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m \in \mathbb{R}^n$ and scalars $c_1, c_2, \dots, c_m$, the vector

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_m\vec{v}_m$$

is a linear combination of $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$.

Ex. Express $\begin{bmatrix} 1 \\ 5 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

One of the main goals of this class is study how to solve linear systems.

* The row and column picture.

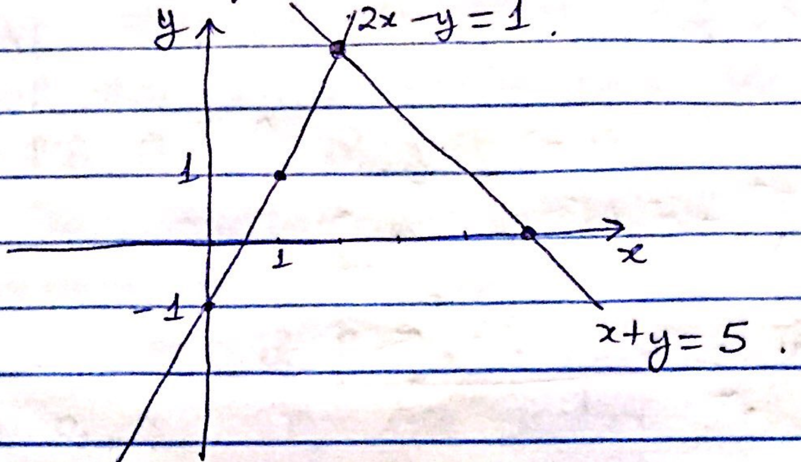
Example 7. We can think of the linear system

$$2x - y = 1$$
$$x + y = 5$$

in two different geometric ways

- Row picture:

Each equation defines a line in $\mathbb{R}^2$



Which points lie on the intersection of these lines?

- Column picture:

The system can be written as $x\begin{bmatrix}2\\1\end{bmatrix} + y\begin{bmatrix}-1\\1\end{bmatrix} = \begin{bmatrix}1\\5\end{bmatrix}$.

Which linear combinations of $\begin{bmatrix}2\\1\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$ produce $\begin{bmatrix}1\\5\end{bmatrix}$?

This example has unique solution $x = 2$ and $y = 3$.

$\Rightarrow$ • $(2,3)$ is the only intersection of the two lines $2x - y = 1$ and $x + y = 5$.

• $2\begin{bmatrix}2\\1\end{bmatrix} + 3\begin{bmatrix}-1\\1\end{bmatrix}$ is the only linear combination producing $\begin{bmatrix}1\\5\end{bmatrix}$.

Exercise: Consider the vectors

$$\vec{a}_1 = \begin{bmatrix}1\\0\\3\end{bmatrix}, \quad \vec{a}_2 = \begin{bmatrix}4\\2\\14\end{bmatrix}, \quad \vec{a}_3 = \begin{bmatrix}3\\6\\10\end{bmatrix}, \quad \vec{b} = \begin{bmatrix}-1\\8\\-5\end{bmatrix}$$

Determine if $\vec{b}$ is a linear combination of $\vec{a}_1, \vec{a}_2, \vec{a}_3$.

* The span of a set of vectors.

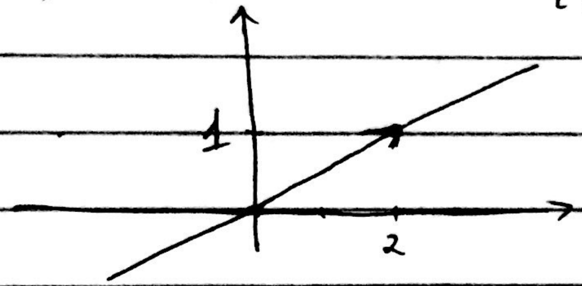
Def. The span of vectors $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m$ is the set of all their linear combinations. We denote it by $\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m\}$.

In other words, $\text{span}\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_m\}$ is the set of all vectors of the form

$$c_1\vec{v}_1 + c_2\vec{v}_2 + \dots + c_m\vec{v}_m,$$

where $c_1, c_2, \dots, c_m$ are scalars

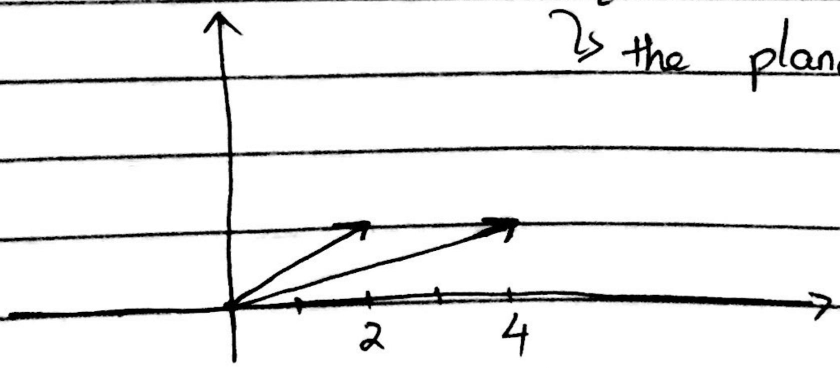
E.g. a) Describe $\text{span}\left\{\begin{bmatrix}2\\1\end{bmatrix}\right\}$ geometrically?



a line with a direction vector $\begin{bmatrix}2\\1\end{bmatrix}$.

i.e., $x - 2y = 0$.

b) Describe $\text{span}\left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}4\\1\end{bmatrix}\right\}$ geometrically?



$\Rightarrow$ the plane $\mathbb{R}^2$

c) Describe $\text{span}\left\{\begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}4\\2\end{bmatrix}\right\}$ geometrically?

(same as part a).

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A single (nonzero) vector ~~all~~ always spans a line, two vectors $\vec{v}_1$, $\vec{v}_2$ usually span a plane but it could also be just a line (if $\vec{v}_1 = \alpha \vec{v}_2$).

<u>Example</u>. Is span$\left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \right\}$ a line or a plane?

<u>Sol.</u> A ~~line~~ plane (why?)

<u>Example</u>. Consider $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 5 \end{bmatrix}$ $\vec{b} = \begin{bmatrix} 8 \\ 3 \\ 17 \end{bmatrix}$.

Is $\vec{b}$ in the plane spanned by the columns of A?

<u>Sol.</u> If yes, there exist scalars x and y such that

$$x \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + y \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ 17 \end{bmatrix}.$$

But we cannot find such x and y.

$\Rightarrow$ No!