

1) Solve $\frac{dy}{dx} + 2xy^2 = 0$, $y(2) = \frac{1}{5}$.

$$\frac{dy}{dx} = -2xy^2.$$

$$\frac{dy}{y^2} = -2x dx.$$

$$\int \frac{dy}{y^2} = -\int 2x dx.$$

$$-\frac{1}{y} = -x^2 + C.$$

$$y = \frac{-1}{-x^2 + C}.$$

Since $y(2) = \frac{1}{5}$,

~~$$\frac{1}{5} = \frac{-1}{-(\frac{1}{5})^2 + C}.$$~~

~~$$-\frac{1}{25} + C = -5.$$~~

~~$$C = \frac{1}{25} - 5$$~~

~~$$C = -\frac{124}{25}.$$~~

~~$$\Rightarrow \text{Sol. } y(x) = \frac{-1}{-x^2 - \frac{124}{25}}.$$~~

$$\frac{1}{5} = \frac{-1}{-2^2 + C}$$

$$-4 + C = -5$$

$$C = -1$$

$$\text{Sol. } y(x) = \frac{1}{x^2 + 1}$$

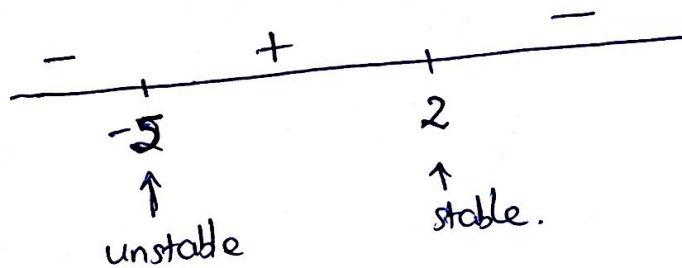
2) $\frac{dy}{dt} = e^y(10 - 3y - y^2)$.

a) Find equilibrium points: set $e^y(10 - 3y - y^2) = 0$

$$\begin{aligned} \Rightarrow 10 - 3y - y^2 &= 0 \\ y^2 + 3y - 10 &= 0 \\ (y - 2)(y + 5) &= 0 \\ y = 2 \quad \text{or} \quad y = -5. \end{aligned}$$



phase line



b) Since

$-5 < -4 = y(0) < 2$, $\phi(t)$ is an increasing function
 $\lim_{t \rightarrow \infty} \phi(t) = 2$.

3) Solve

$$t^2 \frac{dy}{dt} + t(t+2)y = e^t, \quad t > 0$$

$$\frac{dy}{dt} + \frac{t+2}{t} y = \frac{e^t}{t^2}$$

integrating factor: $u(t) = e^{\int \frac{t+2}{t} dt} = e^{\int 1 + \frac{2}{t} dt} = e^{t+2\ln t} = t^2 e^t$.

$$\Rightarrow y(t) = \frac{\int \left(\frac{e^t}{t^2}\right)(t^2 e^t) dt + C}{t^2 e^t} = \frac{\int e^{2t} dt + C}{t^2 e^t} = \frac{\frac{1}{2} e^{2t} + C}{t^2 e^t}$$

$$4) a) \quad x - y^3 + y^2 \sin x = (3xy^2 + 2y \cos x) y'$$

$$\underbrace{(x - y^3 + y^2 \sin x)}_{M(x,y)} + \underbrace{(-3xy^2 - 2y \cos x)}_{N(x,y)} \frac{dy}{dx} = 0.$$

$$M(x,y) = x - y^3 + y^2 \sin x$$

$$N(x,y) = -3xy^2 - 2y \cos x.$$

$$\left. \begin{aligned} \frac{\partial M}{\partial y} &= -3y^2 + 2y \sin x \\ \frac{\partial N}{\partial x} &= -3y^2 + 2y \sin x \end{aligned} \right\} \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{exact.}$$

$$b) \quad (x^2 y^3 - \frac{1}{1+x^2}) dx + (x^3 y^2 + \sin y) dy = 0.$$

$$M(x,y) = x^2 y^3 - \frac{1}{1+x^2}$$

$$N(x,y) = x^3 y^2 + \sin y.$$

$$\psi = \int M dx + h(y)$$

$$= \int (x^2 y^3 - \frac{1}{1+x^2}) dx + h(y).$$

$$= \frac{1}{3} x^3 y^3 - \tan^{-1}(x) + h(y).$$

$$\frac{\partial \psi}{\partial y} = x^3 y^2 - 0 + h'(y) = N(x,y).$$

$$\Rightarrow x^3 y^2 + h'(y) = x^3 y^2 + \sin y.$$

$$h'(y) = \sin y.$$

$$h(y) = -\cos y.$$

$$\text{Sol. } \boxed{\frac{1}{3} x^3 y^3 - \tan^{-1}(x) - \cos y = C.}$$