HW06

Math 20D, Lecture D00, Fall 2017

Due Tuesday, Nov 14

Name:

1. Determine whether the members of the given set of vectors are linearly independent for $-\infty < t < \infty$. If they are linearly dependent, find the linear relation among them.

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} e^{-t} \\ 2e^{-t} \end{pmatrix}, \mathbf{x}^{(2)}(t) = \begin{pmatrix} e^{-t} \\ e^{-t} \end{pmatrix}, \text{ and } \mathbf{x}^{(3)}(t) = \begin{pmatrix} 3e^{-t} \\ 0 \end{pmatrix}.$$

2. Let

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} e^{t} \\ te^{t} \end{pmatrix}, \quad \mathbf{x}^{(2)}(t) = \begin{pmatrix} 1 \\ t \end{pmatrix}.$$

Show that $\mathbf{x}^{(1)}(t)$ and $\mathbf{x}^{(2)}(t)$ are linearly dependent at each point in the interval $0 \leq t \leq 1$. Nevertheless, show that $\mathbf{x}^{(1)}(t)$ and $\mathbf{x}^{(2)}(t)$ are linearly independent on $0 \leq t \leq 1$.

3. Consider the vectors $\mathbf{x}^{(1)}(t) = \begin{pmatrix} t \\ 1 \end{pmatrix}$ and $\mathbf{x}^{(2)}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$.

   (a) Compute the Wronskian of $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$.

   (b) In what intervals are $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$ linearly independent?

   (c) What conclusion can be drawn about the coefficients in the system of homogeneous differentials satisfied by $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$?

4. Consider the vectors $\mathbf{x}^{(1)}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix}$ and $\mathbf{x}^{(2)}(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix}$. Answer the same questions as in the above problem.