# HW02

MATH 102, WINTER 2018

DUE WEDNESDAY, JAN 24

NAME:

1. What three elimination matrices $E_{21}, E_{31}, E_{32}$ put $A$ into upper triangular form $E_{32}E_{31}E_{21}A = U$? Multiply by $E_{32}^{-1}, E_{31}^{-1}$, and $E_{21}^{-1}$ to factor $A$ into $LU$ where $L = E_{21}^{-1}E_{31}^{-1}E_{32}^{-1}$. Find $L$ and $U$:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}.$$

2. Invert these matrices $A$ by the Gauss-Jordan method starting with $[\ A \mid I\ ]$.

   a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}.$

   b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$

3. For which right-hand sides (find a condition on $b_1, b_2, b_3$) are these systems solvable?

   a) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$

   b) $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$

4. Consider an $n \times n$ matrix $A$. Show that $A^2 = 0$ is possible but $A^T A = 0$ is not possible (unless $A$ is a zero matrix).