Math 170A Introduction to Numerical Analysis – Fall 2016

Practice Problem Midterm 1

1. Let
$$A = \begin{bmatrix} 16 & 4 & -8 & 4 \\ 4 & 5 & 2 & -5 \\ -8 & 2 & 17 & 1 \\ 4 & -5 & 1 & 20 \end{bmatrix}.$$
Matrix A is symmetric positive definite. Compute the Cholesky factor of A.

2. Let A be an $n \times n$ matrix with $A = R^T R$ where R is an upper triangular matrix with $r_{ii} > 0$ for $i = 1, \ldots, n$. Prove $x^T A x > 0$ for all n length vectors $x \neq 0$. Thus A is s.p.d.

3. Let A be s.p.d. with Cholesky factor $R = \begin{bmatrix} 2 & -1 & 2 & 1 \\ 0 & 3 & 0 & -2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Let $b = \begin{bmatrix} 8 \\ 2 \\ 16 \\ 6 \end{bmatrix}$. Solve $Ax = b$ for x using forward substitution and backward substitution. (*Hint:* Notice that $R^T R x = b$ and let $Rx = y$. Solve the first equation for y then solve the second equation for x.)

4. Consider the differential equation $-u''(x) + 10u'(x) + u(x) = 2$ for $0 < x < 1$ with boundary conditions $u(0) = 1$ and $u(1) = 1$. We wish to solve it approximately by the finite difference method. We subdivide the interval $[0, 1]$ into 10 equal subintervals of length $h = \frac{1}{10}$. The subdivision points of the intervals are $x_i = \frac{1}{10} i$ for $i = 0, \ldots, 10$. Write the system of equations in the form $Au = b$ that we would use to solve for approximations $u(x_1), \ldots, u(x_9)$. Don't solve the system. (*Hint:* $u''(x_i) \approx \frac{u(x_{i+1}) - 2u(x_i) + u(x_{i-1})}{h^2}$ and $u'(x_i) \approx \frac{u(x_{i+1}) - u(x_{i-1})}{2h}$.)