Math 170A Introduction to Numerical Analysis – Fall 2016

Homework 4 – DUE TUESDAY, NOV 22

1. Section 8.3: Do the following two exercises.

   (a) Consider Jacobi's Method for solving the system $Ax = b$ where

   $$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

   i. Verify that the iteration matrix G corresponding to the Jacobi Method for this system is

   $$G = \begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix},$$

   and verify that the eigenvalues for this matrix are $\lambda_1 = 0.5$ and $\lambda_2 = -0.5$ with corresponding eigenvectors $v_1 = (1, -1)$ and $v_2 = (1, 1)$.

   ii. Suppose that $b = (0, 0)$, so that the true solution is $x = (0, 0)$. If we choose $x^{(0)} = (5, -1)$, then $e^{(0)} = x - x^{(0)} = (-5, 1)$. Verify that $e^{(0)} = c_1 v_1 + c_2 v_2$, where $c_1 = -3$ and $c_2 = -2$.

   iii. Use the fact that $e^{(k)} = c_1 \lambda_1^k v_1 + c_2 \lambda_2^k v_2$ to show that

   $$e^{(k)} = \begin{cases} \left(-\frac{5}{2^k}, \frac{1}{2^k}\right) & \text{for } k \text{ even,} \\ \left(-\frac{1}{2^k}, \frac{5}{2^k}\right) & \text{for } k \text{ odd.} \end{cases}$$

   Compute $\|e^{(3)}\|_2$, $\|e^{(4)}\|_2$, and $\|e^{(5)}\|_2$.

   iv. For the same coefficient matrix A, suppose that any b and $x^{(0)}$ are chosen so that the initial error is $e^{(0)} = x - x^{(0)} = (-5, 1)$. What would you predict that the errors $e^{(1)}, e^{(2)}, e^{(3)}, e^{(4)}$, and $e^{(5)}$ would be?

   (b) Do the above exercise with the Gauss-Seidel Method instead of the Jacobi Method. You will need to find G and its eigenvalues and eigenvectors, as well as c_1 and c_2. (DO NOT TURN IN This exercise.)

2. Section 8.4: 7a, 12.

3. Section 1.7: 10bc, 18, 34, 37.

4. Section 1.8: 4, 9, 10.

Programming

1. Write a function in Matlab that takes as input a symmetric positive definite matrix A, the right hand side vector b, initial guess $x^{(0)}$, and number of iterations, and returns as output the solution of $Ax = b$ as found by performing the *steepest descent* method. Use basic programming, along with Matlab's built in basic matrix computations, such as addition, subtraction, "division," and multiplication, as needed.

(a) Write out or print out your function.

(b) Run the case with A=[9 1; 1 2], b=[1 1] and initial guess [1 1] for 10 iterations. Print out your solution. What is the residual of this result?

2. Write a function in Matlab that takes as input a tridiagonal matrix given as three vectors: $n \times 1$ vector v representing the main diagonal, $(n-1) \times 1$ vector w representing the upper diagonal, and $(n-1) \times 1$ vector z representing the lower diagonal. Have this function output the LU factorization with the U as two vectors and the L as one vector representing the diagonals. Also output the number of flops used. Use only basic programming.

   (a) Write out or print out your function.

   (b) Run the case with n = 10, v the vector of 2's, w and z the vector of -1's. Write down your results for the diagonals of L and U.

   (c) Run the case with n = 50 and n = 100 with v the vector of 2's, w and z the vector of -1's. Write down your results for the number of flops used.