Quiz 3, Section D05 MATH 20D, LECTURE D00, FALL 2017

NAME: UID:

There are *two problems.* Write your name on the back too. No CHEATSHEETS or any electronic devices are allowed. Write your answer as clearly as possible to receive full credits. You have 20 mins to finish this quiz.

Problem 1.(5 points.) Transform the following second order ODE to a system of ODEs.

$$2y'' + 3y' + 4y = 0.$$

Suppose $y = x_1$, $y' = x_2$ } 1 pt for x_1, x_2

$x_1' = y' = x_2 \Rightarrow x_1' = x_2$ } 1pt for x_1'

$x_2' = y''$

$2x_2' + 3x_2 + 4x_1 = 0$ } 1pt for x_2'

$\Rightarrow x_2' = -2x_1 - \frac{3}{2}x_2$

$$\boxed{\begin{aligned} x_1' &= x_2 \\ x_2' &= -2x_1 - \frac{3}{2}x_2 \end{aligned}}$$

} 1pt each for correct answer

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Problem 2.(5 points.) Given

$$\vec{x}' = A\vec{x}, \quad \text{where } A = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}.$$

Let $\vec{x}^{(1)} = \begin{pmatrix} 5\cos(t) \\ 2\cos(t) + \sin(t) \end{pmatrix}$ and $\vec{x}^{(2)} = \begin{pmatrix} 5\sin(t) \\ 2\sin(t) - \cos(t) \end{pmatrix}$.

a) Show that $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$ are solutions of the system $\vec{x}' = A\vec{x}$.

b) Show that $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$ form a fundamental set of solutions.

c) What is the general solution of $\vec{x}' = A\vec{x}$.

a) $\vec{x}'^{(1)} = \begin{bmatrix} -5\sin t \\ -2\sin t + \cos t \end{bmatrix}$ $\quad A\vec{x}^{(1)} = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 5\cos t \\ 2\cos t + \sin t \end{bmatrix}$

$= \begin{bmatrix} -5\sin t \\ 5\cos t - 4\cos t - 2\sin t \end{bmatrix}$

$\boxed{1pt}$ $\therefore \vec{x}'^{(1)} = A\vec{x}^{(1)}$

$\vec{x}'^{(2)} = \begin{bmatrix} 5\cos t \\ 2\cos t + \sin t \end{bmatrix}$ $\quad A\vec{x}^{(2)} = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 5\sin t \\ 2\sin t - \cos t \end{bmatrix}$

$= \begin{bmatrix} 5\cos t \\ \sin t + 2\cos t \end{bmatrix}$

$\boxed{1pt}$ $\therefore \vec{x}_1'^{(2)} = A\vec{x}^{(2)}$

$\vec{x}^{(1)}, \vec{x}^{(2)}$ are solutions of the system $\vec{x}' = A\vec{x}$

b) $\boxed{1pt}$ $W[\vec{x}^{(1)}, \vec{x}^{(2)}] = \begin{vmatrix} 5\cos t & 5\sin t \\ 2\cos t + \sin t & 2\sin t - \cos t \end{vmatrix} = 10\sin t\cos t - 5\cos^2 t - 10\sin t\cos t - 5\sin^2 t$

$= -5 \neq 0$

$\boxed{1pt}$ $W \neq 0$, $\therefore \vec{x}^{(1)}, \vec{x}^{(2)}$ are independent, hence form a fundamental set of solutions

c) $c_1\vec{x}^{(1)} + c_2\vec{x}^{(2)} = c_1\begin{pmatrix} 5\cos t \\ 2\cos t + \sin t \end{pmatrix} + c_2\begin{pmatrix} 5\sin t \\ 2\sin t - \cos t \end{pmatrix}$

$\boxed{1pt}$