HW01

MATH 102, WINTER 2018

DUE WEEK 1 AND 2

Reference: 1. L. N. Trefethen and D. Bau: Numerical Linear Algebra, SIAM, 1997

Reading: Review what you learned from your Linear Algebra classes.

1. (a) Let M be the matrix of data points

$$M = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 3 & 9 \\ 4 & 16 \end{bmatrix}.$$

   What are $M^T M$ and MM^T?

   (b) Prove that if A is any matrix, then $A^T A$ and AA^T are *symmetric*. (Recall that a matrix M is symmetric if $M = M^T$.)

2. (1.4-Trefethen & Bau) Let $f_1, \ldots, f_8$ be a set of functions defined on the interval $[1, 8]$ with the property that for any numbers $d_1, \ldots, d_8$, there exists a set of coefficients $c_1, \ldots, c_8$ such that

$$\sum_{j=1}^{8} c_j f_j(i) = d_i, \qquad i = 1, \ldots, 8.$$

   (a) Show that $d_1, \ldots, d_8$ determine $c_1, \ldots, c_8$ uniquely.

   (b) Let A be the 8×8 matrix representing the linear mapping from data $d_1, \ldots, d_8$ to coefficients $c_1, \ldots, c_8$. What is the i, j entry of A^{-1}?

3. (1.3-Trefethen & Bau) We say that a square or rectangular matrix R with entries r_{ij} is *upper-triangular* if $r_{ij} = 0$ for $i > j$. Show that if R is a nonsingular $m \times m$ upper-triangular matrix, then R^{-1} is also upper-triangular. (Note that the analogous result also holds for lower-triangular matrices.)

4. Recall that a matrix $A \in \mathbb{R}^{m\times n}, m \geq n$, is said to have full rank if its columns are linearly independent, i.e., for $\boldsymbol{a}_j$ the jth column of A, $c_1\boldsymbol{a}_1 + \ldots + c_n\boldsymbol{a}_n = 0 \Longrightarrow c_1 = \ldots = c_n = 0$. Show that A has full rank if and only if no two distinct vectors are mapped to the same vector.