HW07

MATH 20D, LECTURE D00, FALL 2017

DUE TUESDAY, NOV 28 NAME:

1. Given
$$\vec{x}' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{x}.$$
   (a) Draw a direction field.
   (b) Find the general solution of the given system of equations and describe the behavior of the solution as $t \to \infty$.
   (c) Plot a few trajectories of the system.

2. Solve the given initial value problem. Describe the behavior of the solution as $t \to \infty$.
$$\vec{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

3. Assume that $t > 0$. Solve the given system of equations
$$t\vec{x}' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} \vec{x}.$$

4. Solve
$$\vec{x}' = \begin{pmatrix} -1 & -1 \\ -\frac{1}{2} & -1 \end{pmatrix} \vec{x}.$$
   What are the eigenvalues of the coefficient matrix? Classify the equilibrium point at the origin as to type.

5. Given
$$\vec{x}' = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix} \vec{x}.$$
   (a) Draw a direction field and sketch a few trajectories.
   (b) Express the general solution of the given system of equations in terms of real-valued functions.
   (c) Describe the behavior of the solutions as $t \to \infty$.

6. Given
$$\vec{x}' = \begin{pmatrix} 1 & 2 \\ -5 & -1 \end{pmatrix} \vec{x}.$$
   (a) Draw a direction field and sketch a few trajectories.
   (b) Express the general solution of the given system of equations in terms of real-valued functions.
   (c) Describe the behavior of the solutions as $t \to \infty$.

7. Given
$$\vec{x}' = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \vec{x}.$$

(a) Draw a direction field and sketch a few trajectories.

(b) Describe how the solutions behave as $t \to \infty$.

(c) Find the general solution of the system of equations.

8. Consider again the electric circuit which is described by the system of differential equations

$$\frac{d}{dt}\begin{pmatrix} I \\ V \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{pmatrix}\begin{pmatrix} I \\ V \end{pmatrix}.$$

Suppose that $R = 1\Omega, C = 1F$, and $L = 4H$. Suppose also that $I(0) = 1A$ and $V(0) = 2V$. Find $I(t)$ and $V(t)$.

9. Determine the Taylor series about the point x_0 for the given function. Also determine the radius of convergence of the series.

$$x^2, \quad x_0 = -1.$$

10. Given

$$2y'' + (x+1)y' + 3y = 0, \quad x_0 = 2.$$

(a) Seek power series solutions of the given differential equation about the given point x_0; find the recurrence relation that the coefficients must satisfy.

(b) Find the first four nonzero terms in each of two solutions y_1 and y_2 (unless the series terminates sooner).

11. Given

$$y'' + xy' + 2y = 0, \quad y(0) = 4, y'(0) = -1.$$

Find the first five nonzero terms in the solution of the given initial-value problem.