Quiz 2, Section ~~D01~~ D04 MATH 20D, LECTURE D00, FALL 2017

NAME: UID:

There are *two problems*. Write your name on the back too. No CHEATSHEETS or any electronic devices are allowed. Write your answer as clearly as possible to receive full credits. You have 20 mins to finish this quiz.

Problem 1.(5 points.) Find the general solution of

$$y'' + y = 2te^t.$$

Homogenous solution

$$\left.\begin{aligned} &y'' + y = 0 \\ &y_c = c_1 \cos t + c_2 \sin t \end{aligned}\right\} \quad \text{1 pt for } y_c$$

Let $y_p = Ae^t + Bte^t$ } 1pt for correct guess for y_p

$y_p' = Ae^t + Be^t + Bte^t$

$y_p'' = Ae^t + Be^t + Be^t + Bte^t$

Since y_p is a solution the given non homogenous eqn

$$(Ae^t + Be^t + Be^t + Bte^t) + (Ae^t + \cancel{Be^t} + Bte^t) = 2te^t$$

$$(2A + 2B)e^t + 2Bte^t = 2te^t$$

$$\left.\begin{aligned} &2A + 2B = 0 \qquad 2B = 2 \\ &B = -A \qquad\quad \Rightarrow B = 1 \\ &\Rightarrow A = -B = -1 \end{aligned}\right\} \quad \text{1 pt each for } A, B$$

$y_p = -e^t + te^t$

$$\boxed{y(t) = y_c + y_p = c_1 \cos t + c_2 \sin t - e^t + te^t} \quad \text{1 pt for final soln}$$

D05 - Answer

$$\boxed{c_1 \cos t + c_2 \sin t - \frac{5}{2}e^t + \frac{5t}{2}e^t}$$

D0[illegible]- Answer

$$\boxed{c_1 \cos t + c_2 \sin t - 2e^t + 2te^t}$$

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Problem 2.(5 points.) Given $y_1(x) = x$ and $y_2(x) = x^{-1}$ are two solutions of the differential equation

$$x^2y'' + xy' - y = 0,$$

find the general solution of

$$x^2y'' + xy' - y = x^2e^{-x}, \quad x > 0.$$

$$y'' + \frac{y'}{x} - \frac{y}{x^2} = e^{-x}$$ (rewrite the equation)

Let $y_p(x) = u_1(x)\,y_1(x) + u_2(x)\,y_2(x)$ — 1pt

$y_1(x) = x \qquad y_2(x) = \frac{1}{x}$

$$W[y_1, y_2] = y_1y_2' - y_2y_1' = x \times \frac{-1}{x^2} - \frac{1}{x}(1) = \frac{-2}{x}$$ — 1pt for W

$$u_1(x) = -\int \frac{y_2(x)\,g(x)}{W}\,dx = -\int \frac{\frac{1}{x}e^{-x}}{\frac{-2}{x}}\,dx = \frac{1}{2}\int e^{-x}\,dx = -\frac{e^{-x}}{2}$$ — 1pt for $u_1(x)$

$$u_2(x) = \int \frac{y_1(x)\,g(x)}{W}\,dx$$ — 1pt for $u_2(x)$

$$= \int \frac{x e^{-x}}{\frac{-2}{x}}\,dx = \frac{-1}{2}\int x^2e^{-x}\,dx$$

$$= \frac{-1}{2}\left[x^2\frac{e^{-x}}{-1} - \int 2x\left(\frac{e^{-x}}{-1}\right)dx\right] = \frac{1}{2}\left[x^2e^{-x} - 2\int xe^{-x}\,dx\right]$$

$$= \frac{1}{2}\left(x^2e^{-x} - 2\left[x(-e^{-x}) - \int(-e^{-x})\,dx\right]\right)$$

$$= \frac{1}{2}\left(x^2e^{-x} + 2xe^{-x} - 2\int e^{-x}\,dx\right) = \frac{1}{2}\left(x^2e^{-x} + 2xe^{-x} + 2e^{-x}\right)$$

$$= \frac{x^2}{2}e^{-x} + xe^{-x} + e^{-x}$$

$$y_p(x) = \left(-\frac{e^{-x}}{2}\right)x + \left(\frac{x^2}{2}e^{-x} + xe^{-x} + e^{-x}\right)\frac{1}{x} = e^{-x} + \frac{1}{x}e^{-x}$$

$$\boxed{y(x) = c_1x + c_2\frac{1}{x} + e^{-x} + \frac{1}{x}e^{-x}}$$ — 1pt for final Answers