Vector Calculus Practice Midterm 2

1. Find all points (u_0, v_0) such that the parametrization $\Phi(u,v) = (u+v, u+v, 2uv)$ is not smooth (regular).

2. Let S be the triangle with vertices $(1,0,0), (0,1,0), (0,0,1)$.

   (a) Find the domain D in $\mathbb{R}^2$ and the linear function $g : D \to \mathbb{R}$ so that S is the graph of $z = g(x,y)$.

   (b) Calculate the surface integral $\int\int_S x^2 z \, dS$.

3. Let D be the region bounded by the lines

$$x + y = 0, \quad x + y = 2, \quad x - y = 0, \quad x - y = 2.$$

   Evaluate

$$\int\int_D (x+y)e^{x^2-y^2} \, dxdy$$

   using the change of variables $u = x + y, v = x - y$.

4. Use a change of variables to calculate

$$\int\int_R e^{(y-3x)} \sin(y-x) \, dxdy$$

   where R is the region bounded by

$$y - x = 0, \quad y - x = 1, \quad y - 3x = 0, \quad y - 3x = 2.$$

5. Compute the integral of $f(x,y,z) = x+y+yz$ along the path $\mathbf{c}(t) = (sint, cost, t), 0 \leq t \leq 2\pi$.

6. Compute the line integral $\int_{\mathbf{C}} yze^{xyz} \, dx + xze^{xyz} \, dy + xye^{xyz} \, dz$, where $\mathbf{C}$ is the path $\mathbf{c}(t) = (t, 2t, 3t), 0 \leq t \leq 2$ (Hint: there are two solutions to this).

7. Compute the line integral $\int_{\mathbf{c}} \mathbf{F} \cdot \; d\mathbf{s}$ where $\mathbf{F}(x,y,z) = (2x + ze^{xz}, 2y, xe^{xz})$ and $\mathbf{c}(t) = \left(cos^3(t), sin^2(t), (\frac{t}{2\pi})^5 \right)$ for $0 \leq t \leq 2\pi$ (Hint: Avoid the direct calculation).

8. Let $\mathbf{F} = x\mathbf{i}+2y\mathbf{j}+3z\mathbf{k}$. Compute the flux of $\mathbf{F}$ through a surface defined by $x = 1, y^2+z^2 \leq 9$.

9. Let S be the part of the cone $z^2 = x^2 + y^2$ with z between 1 and 2 oriented by the normal pointing out of the cone.

   (a) Find the surface area of S.

(b) Compute $\int\int_S \mathbf{F} \cdot \, d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

10. Let S be the portion of the cylinder $x^2 + y^2 = 4$ bounded between the planes $z = 0$ and $x - z + 3 = 0$ oriented with an outward pointing normal (away from the z-axis). Compute the following.

    (a) $A(S)$, the surface area of S.

    (b) $\int\int_S (x, y, xyz) \cdot \, dS$

11. Find the surface area of the unit sphere S that is contained in the cylinder $x^2 + y^2 = \frac{1}{4}$.

Answers

1. Any (u_0, v_0) such that $u_0 = v_0$
2. (a) D is a triangle in $\mathbb{R}^2$ with vertices $(0,0), (1,0)$ and $(0,1)$. (b) $\frac{\sqrt{3}}{60}$
3. $\dfrac{e^4 - 3}{4}$
4. $\frac{1}{2}(e^2 - 1)(1 - \cos 1)$
5. 0
6. $e^{48} - 1$
7. $e - 1$
8. 9π
9. (a) $3\sqrt{2}\pi$ (b) 0
10. (a) 12π (b) 24π
11. $4\pi(1 - \frac{\sqrt{3}}{2})$