HW05

Math 20D, Lecture D00, Fall 2017

Due Tuesday, Nov 7

Name:

1. Find the general solution of the given differential equation.

$$y'' + 9y = 9\sec^2(3t), \quad 0 < t < \pi/6.$$

2. Verify that the given functions y_1 and y_2 satisfy the corresponding homogeneous equation; then find a particular solution of the given nonhomogeneous equation.

   (a) $t^2y'' - 2y = 2t^2 - 1, \quad t > 0; \quad y_1(t) = t^2, y_2(t) = t^{-1}$.

   (b) $x^2y'' - 3xy' + 4y = x^2\ln x, \quad x > 0; \quad y_1(x) = x^2, y_2(x) = x^2\ln x$.

3. Use the method of reduction of order to solve the given differential equation.

$$t^2y'' - 2ty' + 2y = 4t^2, \quad t > 0; y_1(t) = t.$$

4. Transform the given equation into a system of first-order equations.

$$u'' + 0.5u' + 2u = 0.$$

5. Given

$$\begin{aligned} x_1' &= 3x_1 - 2x_2, \quad x_1(0) = 3 \\ x_2' &= 2x_1 - 2x_2, \quad x_2(0) = \frac{1}{2}. \end{aligned}$$

   (a) Transform the given system into a single equation of second-order.

   (b) Find x_1 and x_2 that also satisfy the given initial conditions.

6. Given

$$\begin{aligned} x_1' &= -\frac{1}{2}x_1 + 2x_2, \quad x_1(0) = -2 \\ x_2' &= -2x_1 - \frac{1}{2}x_2, \quad x_2(0) = 2. \end{aligned}$$

   (a) Transform the given system into a single equation of second-order.

   (b) Find x_1 and x_2 that also satisfy the given initial conditions.

   (c) Sketch the graph of the solution in the x_1x_2-plane for $t \geq 0$.

7. If $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$ and $B = \begin{pmatrix} i & 3 \\ 2 & -2i \end{pmatrix}$, find AB.

8. If the given matrix is nonsingular, find its inverse. If the matrix is singular, verify that its determinant is zero.

$$\begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}.$$