

HW09

MATH 102, WINTER 2018

DUE WEDNESDAY, MARCH 14

NAME:

Problem 1 Diagonalize the 2×2 skew-Hermitian matrix $K = \begin{bmatrix} i & i \\ i & i \end{bmatrix}$. Compute e^{Kt} , and verify that e^{Kt} is unitary. What is the derivative of e^{Kt} at $t = 0$?

Problem 2 Find the unit eigenvectors of the following matrix

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix},$$

and put them into the columns of a unitary matrix U . What property of P makes these eigenvectors orthogonal?

Problem 3 Which pairs are similar? Choose a, b, c, d to prove that the other pairs aren't:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \begin{bmatrix} b & a \\ d & c \end{bmatrix} \quad \begin{bmatrix} c & d \\ a & b \end{bmatrix} \quad \begin{bmatrix} d & c \\ b & a \end{bmatrix}.$$

Problem 4 These Jordan matrices have eigenvalues $0, 0, 0, 0$. They have two eigenvectors (find them). But the block sizes don't match and J is not similar to K :

$$J = \left[\begin{array}{cc|cc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array} \right] \quad \text{and} \quad K = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array} \right]$$