Math 102 - Winter 2013 - Midterm I

Name: ____________________

Student ID: ____________________

Section time: ____________________

Instructions:

Please print your name, student ID and section time.

During the test, you may not use books, calculators or telephones.

Read each question carefully, and show all your work. Answers with no explanation will receive no credit, even if they are correct.

There are 4 questions which are worth 50 points. You have 50 minutes to complete the test.

Question	Score	Maximum
1		7
2		12
3		12
4		19
Total		50

Problem 1. [*7 points.*]

Find the LU-decomposition of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & -3 & 1 \\ 3 & 7 & 5 \end{bmatrix}.$$

Problem 2. [*12 points.*]

Are the following vector spaces or not? Simply write "VECTOR SPACE" or "NOT VECTOR SPACE". No justification is necessary.

(i) [2] The set of upper triangular $n \times n$ matrices.

(ii) [2] The set $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \text{ in } \mathbb{R}^3 : x_1 x_2 x_3 \geq 0 \right\}$.

(iii) [2] The set $\left\{ \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \text{ in } \mathbb{R}^4 : 2x - 3y + z - 2w = 0 \text{ and } x - 2y - z + 3w = 0 \right\}$.

(iv) [2] The set of polynomials $P(x, y)$ of degree at most 3 in two variables such that

$$P(0,0) = \frac{\partial P}{\partial x}(0,0) = 0.$$

(v) [2] The set of $n \times n$ matrices A such that $\texttt{rref} A = I_n$.

(vi) [2] The set of $n \times n$ matrices A that commute with a fixed permutation matrix P that is, $PA = AP$.

Problem 3. [*12 points.*]

Consider the vector space $\mathcal{P}$ of polynomials $f(x)$ of degree less or equal to 2. Let

$$T : \mathcal{P} \to \mathcal{P}$$

be the transformation

$$T(f) = f'' + xf'.$$

(i) [4] Show that $\{1, x, x^2 - x - 1\}$ is a basis for $\mathcal{P}$.

(ii) [4] Briefly explain why T is a linear transformation.

(iii) [4] Find the matrix of the transformation T in the basis of part (i).

Problem 4. [*19 points.*]

Consider the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 & 1 \\ -4 & 5 & -9 & 1 \\ 2 & -5 & 12 & 2 \end{bmatrix}.$$

(i) [4] Give a basis for $C(A)$. What is the rank of A?

(ii) [4] Give a basis for the null space of A. What is the nullity of A?

(iii) [3] Show that the columns $\mathbf{c_1}, \mathbf{c_2}, \mathbf{c_3}, \mathbf{c_4}$ of A are linearly dependent by exhibiting explicit relations between them.

(iv) [2] Does A admit a left inverse, a right inverse, neither, or both?

(v) [3] What is the dimension of the left null space of A? What is the dimension of the row space of A?

(vi) [3] Write down the general solution to the following system of equations

$$Ax = \begin{bmatrix} 2 \\ 5 \\ -5 \end{bmatrix}.$$