

CS 323 Computer Problem: Polynomial Interpolation - due 4/15/15

Write a Matlab function

$$\text{function } fz = \text{interp}(x, f, z)$$

which takes vectors x , f of data points:

$$(x_1, f_1), (x_2, f_2), \dots, (x_{n+1}, f_{n+1}),$$

determines the corresponding interpolating polynomial $p_n(x)$ in Newton form, and returns in 'fz' the values of $p_n(x)$ at an array of points z . (Note: We do not use (x_0, f_0) as a sample point because Matlab does not allow zero as a subscript.)

Apply your code to the interpolation problems below, in each of which a given function $f(x)$ is to be interpolated over a prescribed interval $[a, b]$ using $n + 1$ equispaced points

$$a = x_1 < x_2 < x_3 < \dots < x_{n+1} = b.$$

In each case, graph $f(x)$ and the corresponding interpolating polynomials $p_n(x)$ on the same plot. Do this by sampling these functions at a “dense” set of points

$$a = z_1 < z_2 < z_3 < \dots < z_m = b,$$

where, say, $m = 100$, and then plotting these points. Programming notes: The simplest way to initialize z is via the Matlab function 'linspace'. To superimpose several graphs on the same plot, you can use the Matlab commands 'hold on' (puts next graph on same plot) and 'hold off'. Use the Matlab command *legend* to label your plots.

In addition, evaluate and print the maximum error

$$\max_{1 \leq i \leq m} |f(z_i) - p_n(z_i)|$$

for each polynomial $p_n(x)$.

Here are the functions to be approximated:

1. $f(x) = x^6$, $[a, b] = [0, 1]$, $n = 4, 8, 12$,
2. $f(x) = \sin x$, $[a, b] = [0, 2\pi]$, $n = 4, 8, 12$, (in Matlab, π gives the value of π)
3. $f(x) = \begin{cases} .45, & x < 1, \\ .50, & x = 1, \\ .55, & x > 1 \end{cases}$, $[a, b] = [0, 2]$, $n = 4, 8, 12$.
4. $f(x) = \frac{1}{1 + 10x^2}$, $[a, b] = [-1, 1]$, $n = 4, 8, 12$.

Interpret your results.

Note: The error term for the interpolating polynomial implies that

$$\max_{x \in [a, b]} |f(x) - p_n(x)| \leq \frac{(b-a)^{n+1}}{(n+1)!} M_{n+1}; \quad M_{n+1} \equiv \max_{x \in [a, b]} |f^{(n+1)}(x)|.$$

If M_{n+1} remains bounded as $n \rightarrow \infty$, then $\max_{x \in [a, b]} |f(x) - p_n(x)| \rightarrow 0$ as $n \rightarrow \infty$ (because $(b-a)^{n+1}/(n+1)! \rightarrow 0$ as $n \rightarrow \infty$).