Philip Rego P4

Code explanation

In the main function I call interp three times for each n in the four problems. Linspace is used to space out the x's and z's where m=100. A hold is put around each of the problems so that each graph shows the f(z) and pn(z) functions in the same graph. The instructions where ambiguous saying “Do this by sampling these functions at a “dense” set of points” I wasn't sure if “these” refers to only the pn(x) functions or the pn(x) functions and the f(x) function. I chose the later, plotting f(z) and pn(z). The first thing the interp function calls is the DivDif function. The DivDif function calculates the divided difference. It basically calculates the divided difference the same way it is done by hand but uses a 2d array. It's not very elegant but it's very efficient. The interp function goes on to add the x's to the coefficients that divided difference calculated. At this point pn(x) is formed and fz if calculated for each of the 100 z values. To calculate the values I use the matlab function polyval() which solves the polynomial with nested multiplication. Now the f(z) and pn(z) values are graphed. Finally the max error is calculated by scanning all the error values for each z and the error is displayed, but since the fz results are displayed in the main function the max error is displayed before the fz values.

Analysis

For the first problem when n=4 we see that the error is high at 0.012, but when the n=8 or 12 the error is eliminated other than computational error. The error is eliminated because when n is 6 or higher the error is 0, so we know the line is an exact fit.

For the second problem we will never get an exact fit because no matter the order of derivative sin(x) will never equal 0. However, the larger the n the larger the error's denominator is. As the denominator increases the error gets closer to 0. So when n increases we'll get a better line.

The third problem is discontinuous so we can't derivative it. The error is unreliable. We won't be able to find a good line for this function.

In the last problem the error increases with n. When the order of derivative increases this function increases. It increases at a faster rate then (n+1)! So the error increases when n rises and the line gets worse.

```
function [  ] = main( )

    figure
    hold on
    y1=linspace(0,1);
    f1=@(x) x.^6;
    fz14=interp(linspace(0,1,5),f1,y1)
    fz18=interp(linspace(0,1,9),f1,y1)
    fz112=interp(linspace(0,1,13),f1,y1)
    axis([0,1,f1(0),f1(1)]);
    hold off

    figure
    hold on
    y2=linspace(0,2*pi);
    f2=@(x) sin(x);
    fz24=interp(linspace(0,2*pi,5),f2,y2)
    fz28=interp(linspace(0,2*pi,9),f2,y2)
    fz212=interp(linspace(0,2*pi,13),f2,y2)
    axis([0,2*pi,-1.5,1.5]);
    hold off

    figure
    hold on
    y3=linspace(0,2);
    f3=@(x) .45*(x<1) + .5*(x==1) + .55*(x>1);
    fz34=interp(linspace(0,2,5),f3,y3)
    fz38=interp(linspace(0,2,9),f3,y3)
    fz312=interp(linspace(0,2,13),f3,y3)
    axis([0,2,.4,.6]);
    hold off

    figure
    hold on
    y4=linspace(-1,1);
    f4=@(x) 1./(1 + 10*x.^2);
    fz44=interp(linspace(-1,1,5),f4,y4)
    fz48=interp(linspace(-1,1,9),f4,y4)
    fz412=interp(linspace(-1,1,13),f4,y4)
    hold off

end

function [ fz ] = interp( x,f,z )
    %divided difference
    c = DivDif(x,f);

    %multiply coefficients by variables and x values
    syms q;
    px = [c(1)];
    xs=1;
    for i=2:length(x)
       xs=xs*(q-x(i-1));
       px=[px,c(i)*xs(end)];
    end

    %simplify px
    simp=0;
    for i=1:length(px)
```

```
        simp=simp+px(i);
    end

    %solve for z's
    p = sym2poly(simp);
    fz=zeros(1,length(z));
    for i=1:length(z)
        %function polyval uses nested multiplication
        fz(i)=polyval(p,z(i));
    end

    s=vpa(simp);
    %plot
    plot(z, f(z));
    ezplot(s,z);

    %calulate error
    maxError=0;
    for i=1:length(z)
        if abs(f(z(i))-subs(s,z(i))) > maxError
            maxError=abs(f(z(i))-subs(s,z(i)));
        end
    end
    maxError=simplify(maxError)
end

function [ c, table ] = DivDif( x, f )
    table = zeros([1 2*length(x)+((((length(x)-1)*(length(x)))/2))]);
    %4 times the size of x fits everything
    for i=1:length(x)                          %put the x's in
        table(i)=x(i);
    end
    for i=1:length(x)                          %put the y's in
        table(i+length(x))=f(x(i));
    end

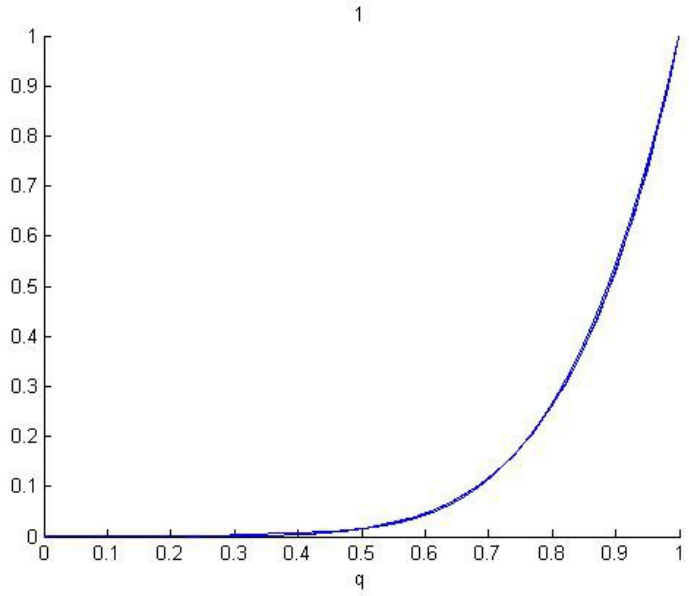
    sub=1;
    index=1;
    num=length(x); %number of elemnts in column
    spacing=0;
    for i=(length(x)*2)+1:length(table)
       if index == num
           spacing=spacing+index;
           num=num-1;
           index=1;

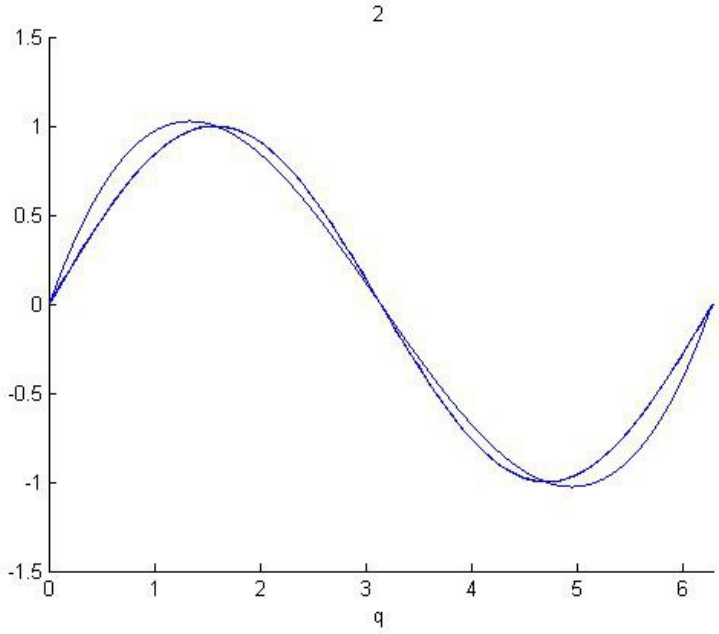
       end

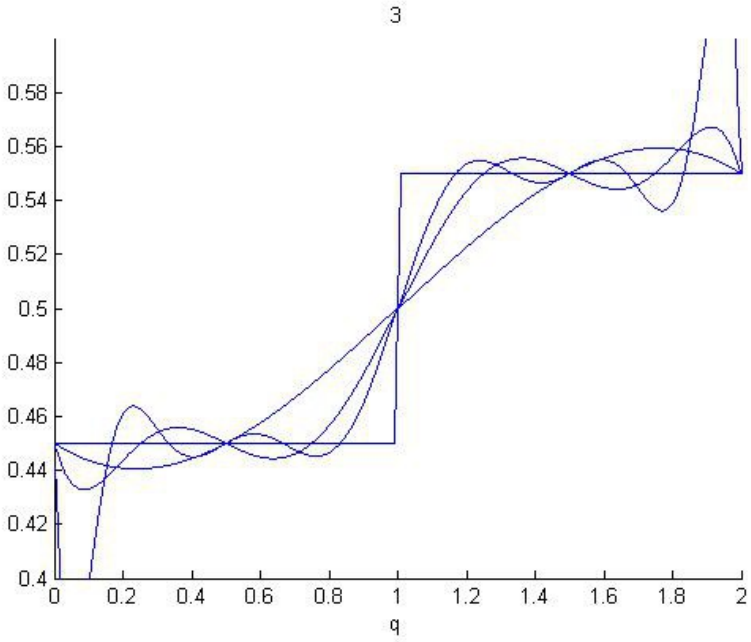
       distance=length(x)-num+1;
       numerator = table(index+length(x)+spacing)...
           -table(index+length(x)+spacing+1);
       den = table(index)-table(index+distance);
       table(i)=numerator./den;
       index=index+1;
       sub=sub+1;
   end
   table;

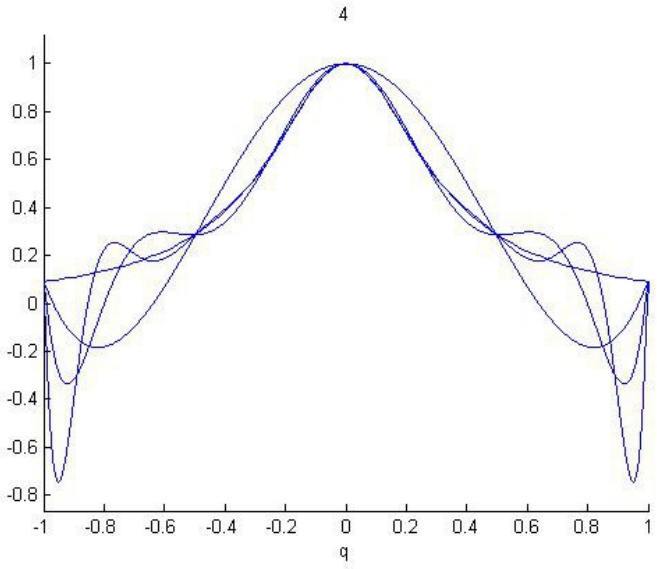
   c=[];
```

```
  i=length(x)+1;
  in=length(x);
  while i<=length(table)
      c=[c,table(i)];
      i=i+in;
      in=in-1;
  end
end
```






3
0.58
0.56
0.54
0.52
0.5
0.48
0.46
0.44
0.42
0.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
q


4
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
q

```
>> main
maxError =
0.012085475041503048470771871461945
fz14 =
  Columns 1 through 9
         0   -0.0022   -0.0040   -0.0055   -0.0067   -0.0077   -0.0084   -0.0088   -0.0091
  Columns 10 through 18
   -0.0092   -0.0091   -0.0089   -0.0086   -0.0081   -0.0076   -0.0070   -0.0063   -0.0056
  Columns 19 through 27
   -0.0049   -0.0041   -0.0033   -0.0026   -0.0018   -0.0010   -0.0003    0.0004    0.0011
  Columns 28 through 36
    0.0018    0.0024    0.0030    0.0035    0.0041    0.0046    0.0050    0.0055    0.0059
  Columns 37 through 45
    0.0063    0.0067    0.0071    0.0075    0.0079    0.0084    0.0089    0.0095    0.0101
  Columns 46 through 54
    0.0108    0.0117    0.0126    0.0137    0.0149    0.0164    0.0180    0.0198    0.0219
  Columns 55 through 63
    0.0243    0.0269    0.0299    0.0332    0.0369    0.0410    0.0455    0.0505    0.0560
  Columns 64 through 72
    0.0621    0.0687    0.0759    0.0837    0.0922    0.1015    0.1115    0.1223    0.1339
  Columns 73 through 81
    0.1464    0.1598    0.1742    0.1896    0.2061    0.2237    0.2424    0.2624    0.2836
  Columns 82 through 90
    0.3061    0.3299    0.3552    0.3820    0.4103    0.4401    0.4716    0.5048    0.5398
  Columns 91 through 99
    0.5766    0.6152    0.6558    0.6984    0.7431    0.7900    0.8390    0.8903    0.9439
  Column 100
    1.0000
maxError =
0.000000000000000029382981965916339404717780081503
fz18 =
  Columns 1 through 9
         0    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
  Columns 10 through 18
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
  Columns 19 through 27
    0.0000    0.0000    0.0001    0.0001    0.0001    0.0002    0.0002    0.0003    0.0003
  Columns 28 through 36
    0.0004    0.0005    0.0006    0.0008    0.0009    0.0011    0.0014    0.0016    0.0020
  Columns 37 through 45
    0.0023    0.0027    0.0032    0.0037    0.0044    0.0050    0.0058    0.0067    0.0077
  Columns 46 through 54
    0.0088    0.0101    0.0114    0.0130    0.0147    0.0166    0.0187    0.0210    0.0235
  Columns 55 through 63
    0.0263    0.0294    0.0328    0.0364    0.0404    0.0448    0.0496    0.0547    0.0603
  Columns 64 through 72
    0.0664    0.0730    0.0801    0.0878    0.0961    0.1050    0.1146    0.1250    0.1361
  Columns 73 through 81
    0.1480    0.1607    0.1744    0.1890    0.2047    0.2214    0.2392    0.2582    0.2784
  Columns 82 through 90
    0.3000    0.3229    0.3473    0.3731    0.4006    0.4297    0.4606    0.4933    0.5279
  Columns 91 through 99
    0.5645    0.6032    0.6440    0.6872    0.7328    0.7808    0.8314    0.8847    0.9409
  Column 100
    1.0000
maxError =
0.000000000000000240273877630952248808413891722 6
fz112 =
```

```
  Columns 1 through 9
         0    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
  Columns 10 through 18
    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000    0.0000
  Columns 19 through 27
    0.0000    0.0000    0.0001    0.0001    0.0001    0.0002    0.0002    0.0003    0.0003
  Columns 28 through 36
    0.0004    0.0005    0.0006    0.0008    0.0009    0.0011    0.0014    0.0016    0.0020
  Columns 37 through 45
    0.0023    0.0027    0.0032    0.0037    0.0044    0.0050    0.0058    0.0067    0.0077
  Columns 46 through 54
    0.0088    0.0101    0.0114    0.0130    0.0147    0.0166    0.0187    0.0210    0.0235
  Columns 55 through 63
    0.0263    0.0294    0.0328    0.0364    0.0404    0.0448    0.0496    0.0547    0.0603
  Columns 64 through 72
    0.0664    0.0730    0.0801    0.0878    0.0961    0.1050    0.1146    0.1250    0.1361
  Columns 73 through 81
    0.1480    0.1607    0.1744    0.1890    0.2047    0.2214    0.2392    0.2582    0.2784
  Columns 82 through 90
    0.3000    0.3229    0.3473    0.3731    0.4006    0.4297    0.4606    0.4933    0.5279
  Columns 91 through 99
    0.5645    0.6032    0.6440    0.6872    0.7328    0.7808    0.8314    0.8847    0.9409
  Column 100
    1.0000
maxError =
0.18062139140991704531017171669434
fz24 =
  Columns 1 through 9
         0    0.1045    0.2026    0.2944    0.3801    0.4598    0.5337    0.6018    0.6643
  Columns 10 through 18
    0.7213    0.7729    0.8194    0.8608    0.8972    0.9288    0.9557    0.9781    0.9961
  Columns 19 through 27
    1.0098    1.0193    1.0248    1.0264    1.0242    1.0185    1.0092    0.9965    0.9807
  Columns 28 through 36
    0.9617    0.9397    0.9150    0.8875    0.8574    0.8249    0.7901    0.7531    0.7141
  Columns 37 through 45
    0.6732    0.6305    0.5861    0.5402    0.4929    0.4444    0.3948    0.3441    0.2926
  Columns 46 through 54
    0.2404    0.1876    0.1343    0.0807    0.0269   -0.0269   -0.0807   -0.1343   -0.1876
  Columns 55 through 63
   -0.2404   -0.2926   -0.3441   -0.3948   -0.4444   -0.4929   -0.5402   -0.5861   -0.6305
  Columns 64 through 72
   -0.6732   -0.7141   -0.7531   -0.7901   -0.8249   -0.8574   -0.8875   -0.9150   -0.9397
  Columns 73 through 81
   -0.9617   -0.9807   -0.9965   -1.0092   -1.0185   -1.0242   -1.0264   -1.0248   -1.0193
  Columns 82 through 90
   -1.0098   -0.9961   -0.9781   -0.9557   -0.9288   -0.8972   -0.8608   -0.8194   -0.7729
  Columns 91 through 99
   -0.7213   -0.6643   -0.6018   -0.5337   -0.4598   -0.3801   -0.2944   -0.2026   -0.1045
  Column 100
    0.0000
maxError =
0.00120251211160750829262359815649 47
fz28 =
  Columns 1 through 9
         0    0.0641    0.1276    0.1904    0.2524    0.3132    0.3727    0.4306    0.4869
  Columns 10 through 18
    0.5411    0.5932    0.6430    0.6901    0.7345    0.7760    0.8144    0.8495    0.8812
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```
  Columns 19 through 27
    0.9094    0.9340    0.9547    0.9717    0.9847    0.9938    0.9988    0.9999    0.9969
  Columns 28 through 36
    0.9899    0.9789    0.9639    0.9451    0.9224    0.8961    0.8661    0.8326    0.7958
  Columns 37 through 45
    0.7558    0.7127    0.6668    0.6181    0.5670    0.5136    0.4582    0.4009    0.3420
  Columns 46 through 54
    0.2817    0.2203    0.1580    0.0950    0.0317   -0.0317   -0.0950   -0.1580   -0.2203
  Columns 55 through 63
   -0.2817   -0.3420   -0.4009   -0.4582   -0.5136   -0.5670   -0.6181   -0.6668   -0.7127
  Columns 64 through 72
   -0.7558   -0.7958   -0.8326   -0.8661   -0.8961   -0.9224   -0.9451   -0.9639   -0.9789
  Columns 73 through 81
   -0.9899   -0.9969   -0.9999   -0.9988   -0.9938   -0.9847   -0.9717   -0.9547   -0.9340
  Columns 82 through 90
   -0.9094   -0.8812   -0.8495   -0.8144   -0.7760   -0.7345   -0.6901   -0.6430   -0.5932
  Columns 91 through 99
   -0.5411   -0.4869   -0.4306   -0.3727   -0.3132   -0.2524   -0.1904   -0.1276   -0.0641
  Column 100
   -0.0000
maxError =
0.000001574793257342270246812405771049 7
fz212 =
  Columns 1 through 9
         0    0.0634    0.1266    0.1893    0.2511    0.3120    0.3717    0.4298    0.4862
  Columns 10 through 18
    0.5406    0.5929    0.6428    0.6901    0.7346    0.7761    0.8146    0.8497    0.8815
  Columns 19 through 27
    0.9096    0.9341    0.9549    0.9718    0.9848    0.9938    0.9989    0.9999    0.9969
  Columns 28 through 36
    0.9898    0.9788    0.9638    0.9450    0.9224    0.8960    0.8660    0.8326    0.7958
  Columns 37 through 45
    0.7557    0.7127    0.6668    0.6182    0.5671    0.5137    0.4582    0.4009    0.3420
  Columns 46 through 54
    0.2817    0.2203    0.1580    0.0951    0.0317   -0.0317   -0.0951   -0.1580   -0.2203
  Columns 55 through 63
   -0.2817   -0.3420   -0.4009   -0.4582   -0.5137   -0.5671   -0.6182   -0.6668   -0.7127
  Columns 64 through 72
   -0.7557   -0.7958   -0.8326   -0.8660   -0.8960   -0.9224   -0.9450   -0.9638   -0.9788
  Columns 73 through 81
   -0.9898   -0.9969   -0.9999   -0.9989   -0.9938   -0.9848   -0.9718   -0.9549   -0.9341
  Columns 82 through 90
   -0.9096   -0.8815   -0.8497   -0.8146   -0.7761   -0.7346   -0.6901   -0.6428   -0.5929
  Columns 91 through 99
   -0.5406   -0.4862   -0.4298   -0.3717   -0.3120   -0.2511   -0.1893   -0.1266   -0.0634
  Column 100
   -0.0000
maxError =
0.048821617528892296495711087931554
fz34 =
  Columns 1 through 9
    0.4500    0.4484    0.4470    0.4457    0.4445    0.4436    0.4427    0.4420    0.4415
  Columns 10 through 18
    0.4411    0.4408    0.4406    0.4406    0.4407    0.4409    0.4413    0.4417    0.4423
  Columns 19 through 27
    0.4429    0.4437    0.4446    0.4456    0.4466    0.4478    0.4490    0.4503    0.4517
  Columns 28 through 36
    0.4532    0.4548    0.4564    0.4581    0.4599    0.4617    0.4636    0.4655    0.4675
```

```
  Columns 37 through 45
    0.4695    0.4716    0.4737    0.4759    0.4781    0.4803    0.4826    0.4848    0.4871
  Columns 46 through 54
    0.4894    0.4918    0.4941    0.4965    0.4988    0.5012    0.5035    0.5059    0.5082
  Columns 55 through 63
    0.5106    0.5129    0.5152    0.5174    0.5197    0.5219    0.5241    0.5263    0.5284
  Columns 64 through 72
    0.5305    0.5325    0.5345    0.5364    0.5383    0.5401    0.5419    0.5436    0.5452
  Columns 73 through 81
    0.5468    0.5483    0.5497    0.5510    0.5522    0.5534    0.5544    0.5554    0.5563
  Columns 82 through 90
    0.5571    0.5577    0.5583    0.5587    0.5591    0.5593    0.5594    0.5594    0.5592
  Columns 91 through 99
    0.5589    0.5585    0.5580    0.5573    0.5564    0.5555    0.5543    0.5530    0.5516
  Column 100
    0.5500
maxError =
0.047437252077025713138849802891 52
fz38 =
  Columns 1 through 9
    0.4500    0.4420    0.4368    0.4339    0.4328    0.4331    0.4345    0.4365    0.4389
  Columns 10 through 18
    0.4416    0.4443    0.4468    0.4492    0.4512    0.4529    0.4543    0.4552    0.4557
  Columns 19 through 27
    0.4558    0.4556    0.4551    0.4543    0.4533    0.4522    0.4510    0.4497    0.4484
  Columns 28 through 36
    0.4473    0.4462    0.4454    0.4448    0.4444    0.4444    0.4447    0.4454    0.4464
  Columns 37 through 45
    0.4479    0.4497    0.4520    0.4546    0.4576    0.4610    0.4647    0.4688    0.4731
  Columns 46 through 54
    0.4776    0.4824    0.4873    0.4923    0.4974    0.5026    0.5077    0.5127    0.5176
  Columns 55 through 63
    0.5224    0.5269    0.5312    0.5353    0.5390    0.5424    0.5454    0.5480    0.5503
  Columns 64 through 72
    0.5521    0.5536    0.5546    0.5553    0.5556    0.5556    0.5552    0.5546    0.5538
  Columns 73 through 81
    0.5527    0.5516    0.5503    0.5490    0.5478    0.5467    0.5457    0.5449    0.5444
  Columns 82 through 90
    0.5442    0.5443    0.5448    0.5457    0.5471    0.5488    0.5508    0.5532    0.5557
  Columns 91 through 99
    0.5584    0.5611    0.5635    0.5655    0.5669    0.5672    0.5661    0.5632    0.5580
  Column 100
    0.5500
maxError =
0.080859510961997604288507367015388
fz312 =
  Columns 1 through 9
    0.4500    0.3911    0.3691    0.3696    0.3822    0.3999    0.4183    0.4345    0.4474
  Columns 10 through 18
    0.4564    0.4616    0.4637    0.4633    0.4612    0.4582    0.4548    0.4515    0.4487
  Columns 19 through 27
    0.4466    0.4454    0.4450    0.4453    0.4461    0.4474    0.4489    0.4504    0.4517
  Columns 28 through 36
    0.4527    0.4533    0.4534    0.4531    0.4524    0.4513    0.4500    0.4486    0.4473
  Columns 37 through 45
    0.4462    0.4455    0.4453    0.4458    0.4471    0.4493    0.4524    0.4564    0.4614
  Columns 46 through 54
    0.4671    0.4736    0.4807    0.4882    0.4960    0.5040    0.5118    0.5193    0.5264
```

```
  Columns 55 through 63
    0.5329    0.5386    0.5436    0.5476    0.5507    0.5529    0.5542    0.5547    0.5545
  Columns 64 through 72
    0.5538    0.5527    0.5514    0.5500    0.5487    0.5476    0.5469    0.5466    0.5467
  Columns 73 through 81
    0.5473    0.5483    0.5496    0.5511    0.5526    0.5539    0.5547    0.5550    0.5546
  Columns 82 through 90
    0.5534    0.5513    0.5485    0.5452    0.5418    0.5388    0.5367    0.5363    0.5384
  Columns 91 through 99
    0.5436    0.5526    0.5655    0.5817    0.6001    0.6178    0.6304    0.6309    0.6089
  Column 100
    0.5500
maxError =
0.31537296198563854510557673346261
fz44 =
  Columns 1 through 9
    0.0909    0.0275   -0.0265   -0.0716   -0.1085   -0.1374   -0.1589   -0.1734   -0.1814
  Columns 10 through 18
   -0.1834   -0.1796   -0.1707   -0.1569   -0.1387   -0.1164   -0.0904   -0.0611   -0.0289
  Columns 19 through 27
    0.0060    0.0431    0.0823    0.1230    0.1652    0.2084    0.2524    0.2969    0.3416
  Columns 28 through 36
    0.3864    0.4309    0.4750    0.5184    0.5609    0.6023    0.6425    0.6812    0.7182
  Columns 37 through 45
    0.7536    0.7870    0.8183    0.8475    0.8744    0.8989    0.9209    0.9403    0.9571
  Columns 46 through 54
    0.9712    0.9825    0.9911    0.9968    0.9996    0.9996    0.9968    0.9911    0.9825
  Columns 55 through 63
    0.9712    0.9571    0.9403    0.9209    0.8989    0.8744    0.8475    0.8183    0.7870
  Columns 64 through 72
    0.7536    0.7182    0.6812    0.6425    0.6023    0.5609    0.5184    0.4750    0.4309
  Columns 73 through 81
    0.3864    0.3416    0.2969    0.2524    0.2084    0.1652    0.1230    0.0823    0.0431
  Columns 82 through 90
    0.0060   -0.0289   -0.0611   -0.0904   -0.1164   -0.1387   -0.1569   -0.1707   -0.1796
  Columns 91 through 99
   -0.1834   -0.1814   -0.1734   -0.1589   -0.1374   -0.1085   -0.0716   -0.0265    0.0275
  Column 100
    0.0909
maxError =
0.43981480807396251339949513903648
fz48 =
  Columns 1 through 9
    0.0909   -0.1270   -0.2576   -0.3209   -0.3340   -0.3110   -0.2636   -0.2011   -0.1310
  Columns 10 through 18
   -0.0589    0.0108    0.0750    0.1318    0.1800    0.2192    0.2495    0.2715    0.2860
  Columns 19 through 27
    0.2943    0.2976    0.2972    0.2945    0.2909    0.2875    0.2856    0.2861    0.2898
  Columns 28 through 36
    0.2974    0.3094    0.3261    0.3475    0.3737    0.4044    0.4393    0.4779    0.5196
  Columns 37 through 45
    0.5638    0.6096    0.6563    0.7030    0.7488    0.7930    0.8346    0.8730    0.9073
  Columns 46 through 54
    0.9369    0.9614    0.9801    0.9928    0.9992    0.9992    0.9928    0.9801    0.9614
  Columns 55 through 63
    0.9369    0.9073    0.8730    0.8346    0.7930    0.7488    0.7030    0.6563    0.6096
  Columns 64 through 72
    0.5638    0.5196    0.4779    0.4393    0.4044    0.3737    0.3475    0.3261    0.3094
```

```
  Columns 73 through 81
    0.2974    0.2898    0.2861    0.2856    0.2875    0.2909    0.2945    0.2972    0.2976
  Columns 82 through 90
    0.2943    0.2860    0.2715    0.2495    0.2192    0.1800    0.1318    0.0750    0.0108
  Columns 91 through 99
   -0.0589   -0.1310   -0.2011   -0.2636   -0.3110   -0.3340   -0.3209   -0.2576   -0.1270
  Column 100
    0.0909
maxError =
0.831722747250207969936884878521 71
fz412 =
  Columns 1 through 9
    0.0909   -0.5260   -0.7338   -0.7053   -0.5578   -0.3673   -0.1797   -0.0200    0.1017
  Columns 10 through 18
    0.1841    0.2317    0.2515    0.2516    0.2395    0.2219    0.2038    0.1891    0.1798
  Columns 19 through 27
    0.1772    0.1812    0.1912    0.2062    0.2249    0.2461    0.2685    0.2914    0.3142
  Columns 28 through 36
    0.3364    0.3581    0.3796    0.4014    0.4239    0.4478    0.4737    0.5021    0.5332
  Columns 37 through 45
    0.5674    0.6044    0.6440    0.6857    0.7286    0.7719    0.8144    0.8551    0.8927
  Columns 46 through 54
    0.9261    0.9543    0.9763    0.9914    0.9990    0.9990    0.9914    0.9763    0.9543
  Columns 55 through 63
    0.9261    0.8927    0.8551    0.8144    0.7719    0.7286    0.6857    0.6440    0.6044
  Columns 64 through 72
    0.5674    0.5332    0.5021    0.4737    0.4478    0.4239    0.4014    0.3796    0.3581
  Columns 73 through 81
    0.3364    0.3142    0.2914    0.2685    0.2461    0.2249    0.2062    0.1912    0.1812
  Columns 82 through 90
    0.1772    0.1798    0.1891    0.2038    0.2219    0.2395    0.2516    0.2515    0.2317
  Columns 91 through 99
    0.1841    0.1017   -0.0200   -0.1797   -0.3673   -0.5578   -0.7053   -0.7338   -0.5260
  Column 100
    0.0909
```