Mini Pretty Proofs

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This is my notes template.

Contents

§1 Sets

Sets.

§1.1 Introduction

Definition 1.1

A **set** is an unordered collection of objects.

Example 1.2

Below are some finite sets:

- $\{1, 2, 4, 7, -3\}$ is a set of five numbers.
- $\{a, b, c\}$ is a set of three letters.
- $\{\text{red}, \text{blue}, \text{green}\}$ is a set of of three colors.

Here are some infinite sets!

- $\mathbb{N} = \{1, 2, 3, \dots\}$ is the set of natural numbers.
- $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ is the set of integers.
- $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z},\ b \neq 0\}$ is the set of rational numbers.
- $\mathbb{R}$ is the set of real numbers (think number line).

Question 1.3. Prove that for any set H, $|H| < |2^H|$.

That is, the cardinality of the powerset of H is strictly larger than the cardinality of H itself.

We'll refer to 1.3 as a theorem once we prove it. Notice that once we prove it, we can construct a hierarchy of infinities, each provably larger than before.

Remark 1.4. The prove of 1.3 uses what a technique analogous to "diagonalization" which appears in many places. In mathematics, you can see it used in Cantor's diagonalization argument, Russell's paradox, and Godel's incompleteness theorem. In computer science to prove the undecidability of the halting problem. In other places too :)