Unit I - Vector Calculus

Formula List

1. The vector differential operator , $\nabla \equiv \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}$
2. Gradient of a scalar point function , $\nabla\phi = \vec{i}\frac{\partial \phi}{\partial x} + \vec{j}\frac{\partial \phi}{\partial y} + \vec{k}\frac{\partial \phi}{\partial z}$
3. Directional Derivative of ϕ in the direction of $\vec{a}$ is $\dfrac{\nabla\phi.\vec{a}}{|\ \vec{a}\ |}$
4. Maximum Directional Derivative of ϕ is $|\ \nabla\phi\ |$
5. Unit normal vector $\hat{n}$ = $\dfrac{\nabla\phi}{|\nabla\phi|}$
6. Angle between two surfaces is $\theta = \cos^{-1}\left(\dfrac{\nabla\phi_1 \ . \ \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}\right)$
7. Two surfaces are cut orthogonal then $\nabla\phi_1 . \nabla\phi_2$ =0
8. Divergence of $\vec{F}$ =$\nabla.\ \vec{F}$ = $\dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y} + \dfrac{\partial F_3}{\partial z}$ where $\vec{\mathrm{F}} = F_1\ \vec{i} + F_2\ \vec{j} + F_3\ \vec{k}$
9. Curl of $\vec{F}$ = ∇ x $\vec{F}$ = $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$ where $\vec{\mathrm{F}} = F_1\ \vec{i} + F_2\ \vec{j} + F_3\ \vec{k}$
10. If $\nabla.\ \vec{F}$ =0 then $\vec{F}$ is said to be Solenoidal Vector
11. If ∇ x $\vec{F}$ = $\vec{0}$ then $\vec{F}$ is said to be Irrotational Vector
12. If ∇ x $\vec{F}$ = $\vec{0}$ then there exists a scalar point function (Scalar Potential) s.t $\vec{F}$ = $\nabla\phi$
13. Laplacian operator $\nabla^2 \equiv \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$

14. Green's Theorem

If P, Q, $\frac{\partial P}{\partial y}, \frac{\partial Q}{\partial x}$ are continuous and single valued functions of x, y in a closed region R enclosed by a curve C, then $\int_C (Pdx + Qdy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy$

15. Stoke's Theorem

The surface integral of the normal component of the curl of a vector function $\vec{F}$ over an open surface S is equal to the line integral of the tangential component of $\vec{F}$ around the closed curve C bounding S. (i.e.) $\int_C \vec{F} . dr = \iint_S curl\ \vec{F} . \hat{n}\, dS$

16. Gauss Divergence Theorem

The surface integral of the normal component of a vector function $\vec{F}$ over a closed surface S enclosing a volume V is equal to the volume integral of the divergence of $\vec{F}$ taken throughout the volume V. (i.e.) $\iint_S \vec{F} . \hat{n}\, ds = \iiint_V div\ \vec{F}\ dV$, where $\hat{n}$ is the unit outward normal to the surface S.