UNIT-I CONDUCTING MATERIALS
ASSISGNMENT-I QUESTIONS
PART – A

1) Write the differences between drift velocity and thermal velocity of an electron.

S.No	Drift Velocity	Thermal velocity
1	The average velocity acquired by the free electron in the presence of electric field is called drift velocity.	Thermal velocity is the velocity of an electron without any external field
2	The electrons moving with drift velocity moves in the direction opposite to that of the field direction	The direction of the electrons moving with thermal velocity is random

2) Define drift velocity & relaxation time.
The average velocity acquired by the free electron in a particular direction, due to the application of electric field is called drift velocity.
Relaxation time can be defined as the time taken by the free electron to reach its equilibrium position from its disturbed position in the presence of applied field.

3) Define mean free path & mobility of electrons.
The average distance traveled by an electron between two successive collisions is known as mean free path.
The magnitude of the drift velocity per unit electric field is defined as Mobility of electrons (μ). [$\mu = v_d/E$]

4) Distinguish between electrical conductivity and thermal conductivity.

S.No	Electrical conductivity	Thermal conductivity
1	The co-efficient of electrical conductivity is defined as the quantity of electricity flowing per unit area per unit time maintained at unit potential gradient	The co-efficient of thermal conductivity is defined as the quantity of heat conducted per unit area per unit time maintained at unit temperature gradient.
2	Electrical conductivity is purely due to number of free electrons	Thermal conductivity is due to both free electrons and phonons.

5) What are the drawbacks of classical free electron theory?
a) According to this theory, the value of electronic specific heat is$(3/2)R$. But experimentally it is about $0.01\ R$ only.
b) The ratio between thermal conductivity and electrical conductivity is not constant at low temperatures.
c) The theoretical value of paramagnetic susceptibility is greater than experimental value.

d) The electrical conductivity of semiconductors, ferromagnetism, photoelectric effect and blackbody radiation cannot be explained.

6) What are the merits of classical free electron theory?
a) It verifies Ohms law.
b) It explains the electrical conductivity and thermal conductivity of metals.
c) It is used to derive Wiedmann – Franz law.

7) Calculate the drift velocity of the free electrons [with a mobility of 3.5 x 10^{-3} $m^2V^{-1}s^{-1}$] in copper for electric field strength of 0.5 V/m.

$$\mu = v_d/E\,,\ v_d = E/\mu = 0.00175\, m/s$$

8) Use Fermi distribution function to obtain the value of F(E) for E-E_F = 0.01 eV at 200 K.

$$F(E) = 1 + e^{(E-E_F)/kT} = 0.359.$$

9) Calculate the electrical conductivity in copper if the mean free path of electrons is 4x10^{-8} m, electron density is 8.4x10^{28} m^{-3} and average thermal velocity of electron is 1.6x10^{6} m/s.

$$\sigma = ne^2\lambda/mv = 5.9\times 10^7 mho/m$$

10) Find the drift velocity of the free electrons in a copper wire whose cross sectional area is 1.0 mm^2 when the wire carries a current of 1 A. Assume that each copper atom contributes one electron to the electron gas. Given n = 8.5x10^{28} m^3

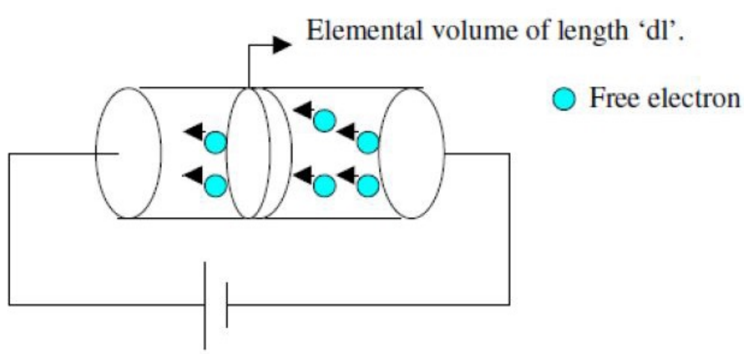
$$v_d = I/neA = 7.4\times 10^{-5} m/s$$

Part – B

UNIT - I

1. Deduce mathematical expression for electrical conductivity and thermal conductivity of a conductor and hence obtain Wiedemann – Franz law. 16

Derivation for electrical conductivity



Consider an electric field (E) applied to a metal then the electric Lorentz force given as follows.

$F_L = eE$

According to Newton's third law,

$F = ma$

$eE = ma$

$a = {}^{eE}/_{m}$

Drift velocity, $v_d = a\tau = \frac{eE\tau}{m}$

Relation between current density and drift velocity is

$J = nevd$

$J = ne(eE\tau/m) = ne^2\tau E \,/\, m$

$J = \sigma E$ so

$\boldsymbol{\sigma = ne^2\tau \,/\, m}$

Derivation for thermal conductivity

Consider a metal of length equal to mean free path (λ). Let A and B be the two ends of the rod. The heat energy is flown from A to B end. Let T be the temperature at A and T – dT be the temperature at B.

The average kinetic energy at A = $\frac{3}{2}kT$ ---- 1

The average kinetic energy at B = $\frac{3}{2}\mathrm{k(T - dT)}$ ---- 2

The kinetic energy carried from A to B = $\frac{3}{2}\mathrm{kdT}$ ---- 3 (Eqn. 1 – 2)

Number of electrons crossing per unit time per unit area from A to B = $\frac{1}{6}\mathrm{nv}$ --------- 4

The energy carried from A to B per unit area in unit time = $\frac{1}{4}\mathrm{nvkdT}$---5

Similarly the energy carried from B to A per unit area in unit time = $-\frac{1}{4}\mathrm{nvkdT}$ ------- 6

Hence the net amount of energy carried from A to B per unit area per unit time Q = $\frac{1}{2}\mathrm{nvkdT}$ -- 7

Since K = $\frac{\mathrm{Q}}{\mathrm{At}\frac{\mathrm{dT}}{\mathrm{dx}}}$ if t = 1 A = 1

Q = K dT/ λ ------- 8

Comparing equation 8 and 7,

K = $\frac{1}{2}\mathrm{nv^2k\tau}$

Wiedemann – Franz law

It states that in metals, the ratio of thermal conductivity to electrical is directly proportional to the absolute temperature.

$$\frac{K}{\sigma} \propto T$$

$$\frac{K}{\sigma} = L\,T$$

$$K = \frac{nv^2k\tau}{2}$$

$\sigma = \frac{ne^2\tau}{m}$

$\frac{K}{\sigma} = \frac{1}{2}mv^2\left(\frac{k}{e^2}\right)$

$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k}{e}\right)^2\ T$

where, $\boldsymbol{L} = \frac{3}{2}\left(\frac{k}{e}\right)^2$

2. i) Write an expression for the Fermi distribution function and discuss its behavior with change in temperature .Plot the curve fir T = 0K and T > 0K. 8

It is the probability function F(E) of an electron occupying a given energy level at absolute temperature. It is given by

$$\boldsymbol{F(E) = \frac{1}{1 + e^{E-EF/kT}}}$$

<u>Effect of temperature on Fermi function</u>

The variation of Fermi distribution function on temperature as follows

Case 1: Probability occupation for $E < E_F$ at T = 0 K

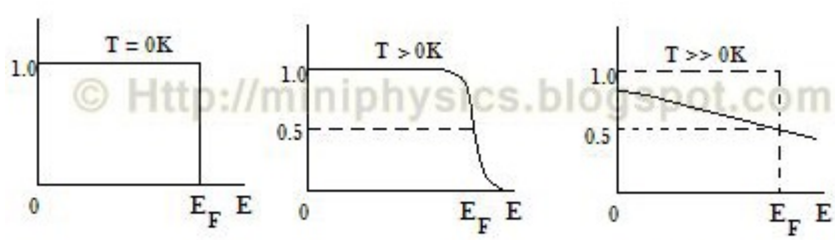
F(E) = 1, 100 % possibility is there for an electron to occupy energy level less than E_F

Case 2: Probability occupation for $E > E_F$ at T = 0 K

F(E) = 0, 0 % possibility is there for an electron to occupy energy level greater than E_F

Case 3: Probability occupation for $E = E_F$ at T > 0 K

F(E) = 0.5, 50 % possibility is there for an electron to occupy Fermi energy level E_F



ii) Using the Fermi function, evaluate the temperature at which there is 1 % probability that an electron in a solid will have an energy 0.5 eV above E_F of 5 eV. 8

$F(E) = 1\ \% = \frac{1}{100}$

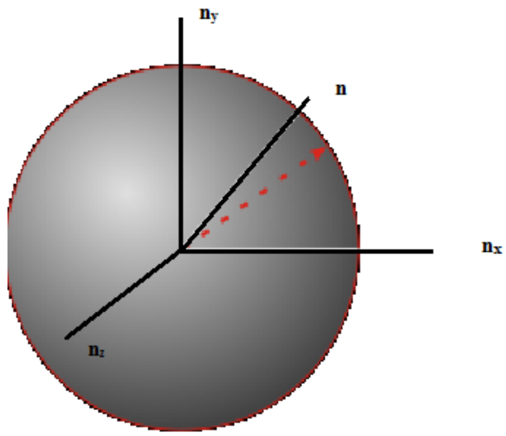
$\frac{1}{100} = \frac{1}{1+e^{E-E_F/kT}}$

$e^{x/kT}=99$

T= 1260K

3. i) Explain the meaning of density of states. Derive an expression for the number of allowed states per unit volume of a solid. 10

Density of energy states



It is defined as the number of available electron states per unit volume in an energy interval E and E + dE.

Z (E) dE = $\frac{N(E)dE}{V}$

Let us consider a cubical metal piece of side length a. An imaginary sphere is constructed with n as radius where $n^2 = n_x^2 + n_y^2 + n_z^2$ and E is energy corresponds to that quantum no. Let another sphere is drawn with radius n + dn and E + dE is the energy corresponds to that sphere.

The no of energy states within the sphere of radius of radius n = $\frac{4}{3}\pi n^3$

Since the quantum no n_x^2, n_y^2, n_z^2 can have only positive integer values, 1/8 of the total volume of the sphere alone considered for calculation of electron states.

So the no of energy states within the sphere of radius of radius n = $\frac{1}{8}\left[\frac{4}{3}\pi n^3\right]$ ----- 1

The no of energy states within the sphere of radius of radius n + dn = $\frac{1}{8}\left[\frac{4}{3}\pi(n+dn)^3\right]$ ----- 2

Therefore the no energy levels between the spheres of radius n & n + dn,

N (E) = $\frac{\pi}{2}n(ndn)$ ---------- 3 (subtract eqn 1 from 2 & use $(a + b)^3$ formula)

$E = \frac{n^2h^2}{8ma^2}$ n = $\left(\frac{8ma^2E}{h^2}\right)^{1/2}$ ndn = $\frac{8ma^2}{2h^2}\, dE$

Substitute n and ndn in eqn. 3

N (E) = $\frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2} E^{1/2}dE$ ---------- 4

To satisfy Pauli's exclusion principle multiply by 2 (each electron states consist of two electron one spin up and spin down),

$$N(E) = 2 \times \frac{\pi}{4}\left(\frac{8ma^2}{h^2}\right)^{3/2} E^{1/2} dE \quad \text{---------- 5}$$

$$N(E) = \frac{4\pi}{h^3}(2m)^{3/2} a^3 E^{1/2} dE \quad \text{---------- 6}$$

$$Z(E) = \frac{N(E)dE}{V}$$

$$Z(E) = \frac{\frac{4\pi}{h^3}(2m)^{3/2} a^3 E^{1/2} dE}{a^3}$$

$$Z(E) = \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} dE$$

This is the expression for the density of charge carriers in the energy interval E and E + dE.

ii) Calculate the number of states lying in an energy interval of 0.01 eV above the Fermi level for a crystal of unit volume with Fermi energy E_F = 3 eV. **6**

$$\Delta E = E - E_F$$

$$E = E_F + \Delta E = \boldsymbol{4.816 \times 10^{-19} J}$$

$$n_c = \int_{E_F}^{E} \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} dE$$

$$= \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2}\left[\frac{2}{3}\left(E^{3/2}\right)\right]_{E_F}^{E}$$

$$\boldsymbol{n_c = 4.14 \times 10^{25}\, m^{-3}}$$

4. i) What if Fermi energy? **2**

It is that state at which the probability of electron occupation is 50% at any temperature above 0 K and it is the level of maximum energy of the filled states at 0 K.

ii) Starting with density of energy states, obtain an expression for carrier concentration in metals and the expression for Fermi energy of the electron at absolute zero. **10**

<u>Carrier concentration</u>

$$n_c = \int Z(E)\, F(E) dE$$

$$n_c = \int \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2} \frac{1}{1 + e^{\frac{E-E_F}{kT}}} dE$$

Expression for carrier concentration in terms of Fermi energy

For energy level E = 0 to E = E_F, F (E) = 1

$$n_c = \int_0^{E_F} \frac{4\pi}{h^3}(2m)^{3/2} E^{1/2}\, dE$$

Solving we get $n_c = \frac{8\pi}{3h^3}(2mE_F)^{3/2}$

$$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3n_c}{8\pi}\right)^{2/3}$$

The above expression is for Fermi energy level at 0 K.
The expression for Fermi level at any finite temperature above 0 K is

$$\boldsymbol{E_F = E_{F_0}\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{F_0}}\right)^2\right]}$$

iii) Obtain an expression for average energy of an electron at T = 0K. **4**

$$E_{av} = {E_T}/{N}$$

E_T = No. of energy states at 0K x Energy of an electron.

$$E_T = \int_0^{E_{F0}} N(E)dE \times E$$

$$= \frac{4\pi}{h^3}(2m)^{3/2}\int_0^{E_{F0}} E^{1/2}E\, dE$$

$$= \frac{8\pi}{5h^3}(2m)^{3/2}E_{F_0}^{5/2}$$

$$E_{av} = \frac{E_T}{N} = \frac{\frac{8\pi}{5h^3}(2m)^{3/2}E_{F_0}^{5/2}}{\frac{8\pi}{3h^3}(2m)^{3/2}E_{F_0}^{3/2}}$$

$$\boldsymbol{E_{av} = \frac{3}{5}E_{F_0}}$$

5. i) Discuss classical free electron theory. **4**

- In an atom electron revolve around the nucleus and a metal is composed of such atoms.
- The valence electrons of atoms are free to move about the whole volume of the metals like the molecules of a perfect gas in a container.
- These free electrons move in random directions and collide with either positive ions fixed to the lattice or other free electrons. All the collisions are elastic in nature.
- The electron velocities in a metal obey the classical Maxwell – Boltzmann distribution of velocities.
- The movements of free electrons obey the laws of the classical kinetic theory of gases.
- The free electrons move in a completely uniform potential field due to ions fixed in the lattice.

ii) Arrive at the microscopic form of Ohm's law. **4**

According to classical free electron theory current density,
Resistance=$R = \rho l/A$
According to Ohm's Law, $V = IR, V = I(\rho l/A)$
$I = VA/\rho l, \quad I/A = (1/\rho)(V/l), \quad \boldsymbol{J = \sigma E}$

This is microscopic form of Ohm's Law.

iii) Obtain the expression for electrical resistivity. **6**

$R \propto {}^{l}/_{A}$

$R = \rho \, {}^{l}/_{A}$

$\rho = \frac{RA}{l}$ Ohm-m

iv) Discuss its dependence on temperature. **2**

- The resistivity of a conductor remains almost constant at lower temperatures.
- The resistivity proportional to T5 from low temperature to the Debye temperature.
- The resistivity is directly proportional to T above Debye temperature.

6. i) State and prove Wiedemann- Franz law using the expression of electrical and thermal conductivities and find the expression for Lorentz number. The experimental value of Lorentz number does not agree with the value calculated from the classical formula. Why?

<u>Wiedemann-Franz law</u>

It states that in metals, the ratio of thermal conductivity to electrical is directly proportional to the absolute temperature.

$$\frac{K}{\sigma} \propto T$$

$$\frac{K}{\sigma} = L\,T$$

$$K = \frac{nv^2 k\tau}{2}$$

$$\sigma = \frac{ne^2\tau}{m}$$

$$\frac{K}{\sigma} = \frac{1}{2}mv^2\left(\frac{k}{e^2}\right)$$

$$\frac{K}{\sigma} = \frac{3}{2}\left(\frac{k}{e}\right)^2 T$$

where, $\boldsymbol{L} = \frac{3}{2}\left(\frac{k}{e}\right)^2$

The theoretical value and the experimental values of Lorentz number according to classical theory does not agree. Only when the quantum theory is considered and the concept of effective mass is introduced, both the values matches well.

ii) Calculate the electrical and thermal conductivities for a metal with a relaxation time 10^{-14} second at 300 K. Calculate Lorentz number. (Density of free electrons = 6 x 10^{28} / m^3) 6

$\sigma = ne^2\tau/m = \mathbf{1.688\times10^7}$ **mho/m**

$$K = \frac{\pi^2 nk^2 T\tau}{3m} = \mathbf{123.93 W/m/K}$$

$L = K/\sigma T \ = \mathbf{2.44\times10^{-8} Wohm/K^2}$

iii) A conducting rod contains 8.5 x 10^{28} electrons per cubic meter. Calculate the electrical conductivity and mobility of electron if collision time is 2 x 10^{-14} sec. 4

$\sigma = ne^2\tau/m = \mathbf{4.77\times10^7}$ **mho/m**

$\rho = 1/\sigma = 2.09\times10^{-8}\ ohm\ m$

$\mu = \sigma/ne = \mathbf{0.35\times10^{-2}\ m^2/Vs}$