St.JOSEPH'S COLLEGE OF ENGINEERING, CHENNAI-119
St.JOSEPH'S INSTITUTE OF TECHNOLOGY, CHENNAI-119
MA6453 – PROBABILITY AND QUEUEING THEORY
UNIT I RANDOM VARIABLES
FORMULAE SHEET

S.NO	DISCRETE RANDOM VARIABLE:	CONTINUOUS RANDOM VARIABLE:
1.	**Discrete Random Variable:** Let X be a discrete random variables with values $x_1, x_2, x_3, ...$ The function $P(X = x_i)$ is said to to be probability mass function if i) $P(X = x_i) \geq 0$, $\forall i$ ii) $\sum_{i=1}^{\infty} P(X = x_i) = 1$	**Continuous Random Variable:** Let X be a continuous random variable. A function f(x) is said to be probability density function of X if i) $f(x) \geq 0$, $\forall x$ ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$
2.	**Distribution Function (or)** **Cumulative Distribution Function (C.D.F):** $F(x) = P(X \leq x) = \sum_{x_i} P(X \leq x_i)$	**Distribution Function (or)** **Cumulative Distribution Function (C.D.F):** $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx$ **Properties of C.D.F:** **i)** $F(-\infty) = \lim_{x \to -\infty} F(x) = 0$ and $F(\infty) = \lim_{x \to \infty} F(x) = 1$ **ii)** $P(a < X < b) = P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = F(b) - F(a)$ **iii)** $P(X \leq a) = F(a)$ **vi)** $P(X > a) = 1 - P(X \leq a) = 1 - F(a)$ **v)** $P(X = a) = 0$ **vi)** $f(x) = \frac{d}{dx}\left[F(x)\right]$
3.	**Mean (or) Expectation of** $X = E(X) = \overline{X} = \mu_1'$ $E(X) = \sum_{x_i} x_i\, P(X = x_i)$	**Mean (or) Expectation of** $X = E(X) = \overline{X} = \mu_1'$ $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$
4.	**Variance of *X:*** $Var(X) = E\left(X^2\right) - \left[E(X)\right]^2 \quad (or)\ Var(X) = \mu_2' - \left(\mu_1'\right)^2$	**Variance of *X:*** $Var(X) = E\left(X^2\right) - \left[E(X)\right]^2 \quad (or)\ Var(X) = \mu_2' - \left(\mu_1'\right)^2$

	Where $E(X^2)=\mu_2'=\sum_{x_i} x_i^2\, P(X=x_i)$	Where $E(X^2)=\mu_2'=\int_{-\infty}^{\infty} x^2 f(x)\,dx$
5.	r^{th} **Order raw Moment :** $\mu_r'=E(X^r)=\sum_{x_i} x_i^r P(X=x_i)$	r^{th} **Order raw Moment :** $\mu_r'=E(X^r)=\int_{-\infty}^{\infty} x^r f(x)\,dx$
6.	r^{th} **Central Moment :** $\mu_r=E(X-\bar{X})^r=\sum_{x_i}(x_i-\bar{X})^r P(X=x_i)$	r^{th} **Central Moment :** $\mu_r=E\left[X-\bar{X}\right]^r=\int_{-\infty}^{\infty}(x-\bar{X})^r f(x)dx$
7.	**Moment generating function:** $M_X(t)=E(e^{tX})=\sum_{x_i} e^{tx} P(X=x_i)$	**Moment generating function:** $M_X(t)=E(e^{tX})=\int_{-\infty}^{\infty} e^{tx} f(x)\,dx$
8.	**First four central moments [Relationship between raw moment and central moment]** $\mu_1=0$ (always) $\mu_2=\mu_2'-(\mu_1')^2$ $\mu_3=\mu_3'-3\mu_2'\mu_1'+2(\mu_1')^3$ $\mu_4=\mu_4'-4\mu_3'\mu_1'+6\mu_2'(\mu_1')^2-3(\mu_1')^4$ In general $\mu_n=\mu_n'-n_{C_1}\mu_{n-1}'\mu_1'+n_{C_2}\mu_{n-2}'\left(\mu_1'\right)^2-n_{C_3}\mu_{n-3}'\left(\mu_1'\right)^3+\ldots+(-1)^{n+1}(n-1)\left(\mu_1'\right)^n$	

PROPERTIES OF MEAN & VARIANCE

1) $E(c)=c$ **where c is any constant.**
2) $E(aX\pm b)=aE(X)\pm b$
3) $\text{var}(c)=0$ **where c is any constant.**
4) $\text{var}(aX\pm b)=a^2\,\text{var}(X)$

PROPERTIES OF MOMENT GENERATING FUNCTION:

1) $E(X^r)=\mu_r'=$ coefficient of $\frac{t^r}{r!}$ in the expansion of $M_X(t)$
2) $E(X^r)=\mu_r'=\left[\frac{d^r}{dt^r}\left(M_X(t)\right)\right]_{t=0}$
3) $M_{CX}(t)=M_X(Ct)$, **where C being a constant.**
4) $M_{X_1+X_2}(t)=M_{X_1}(t)M_{X_2}(t)$ **if** X_1 & X_2 **are independent.**
5) $M_{aX+b}(t)=e^{bt}M_X(at)$

STANDARD DISTRIBUTIONS:

Discret Distributions	Probability Mass Function	MGF $\{M_X(t)\}$	MEAN $\{E(X)\}$	VARIANCE $\{Var(X)\}$	Condition for applying (or) Remarks
Binomial Distribution $X \sim B(n,p)$	$P(X=x)={}^nC_x p^x q^{n-x}$, $x=0,1,2,3,\ldots,\mathrm{n}$ Where n – number of trials p – probability of Success q – probability of failures X – Number of Success (out of 'n' trials) and $p+q=1$	$(q+pe^t)^n$	np	npq	i) The n trials are independent ii) n is small ($n<30$)
Poisson Distribution $X \sim P(\lambda)$	$P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$, $x=0,1,2,3,\ldots$ Where n – number of trials p – probability of Success X – Number of Success (out of 'n' trials) and $p+q=1$	$e^{\lambda(e^t-1)}$	λ	λ	i) n is infinitely large (i.e) $n\to\infty$ ii) p is very small (i.e) $p\to 0$ iii) $\lambda=np$
Geometric Distribution $X \sim G(\mathrm{p})$	**Form I** $P(X=x)=q^{x-1}p$, $x=1,2,3,\ldots,n$ X – number of trials required to get a first success	$\frac{pe^t}{1-qe^t}$	$\frac{1}{p}$	$\frac{q}{p^2}$	**Memoryless property:** If X is a random variable such that for all $s,t>0$ then X is said to have memory less property $P(X>s+t/X>s)=P(X>t),\ \forall\ s\,\&\,t>0$
	Form II $P(X=x)=q^x p$, $x=0,1,2,3,\ldots$ X – number of failures before the first success	$\frac{p}{1-qe^t}$	$\frac{q}{p}$	$\frac{q}{p^2}$	

Continuous Distributions	**Probability density Function**	**MGF** $\{M_X(t)\}$	**MEAN** $\{E(X)\}$	**VARIANCE** $\{Var(X)\}$	
Uniform Distribution $X \sim U(a,b)$	$f(x)=\begin{cases}\frac{1}{b-a}, & a<x<b\\ 0, & otherwise\end{cases}$	$\frac{e^{bt}-e^{at}}{(b-a)t}, t\neq 0$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	-
Exponential Distribution $X \sim e(\lambda)$	$f(x)=\begin{cases}\lambda e^{-\lambda x}, & x\geq 0\\ 0, & otherwise\end{cases}$	$\frac{\lambda}{\lambda-t}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	**Memoryless property:** If X is a random variable with exponential distribution, then X lacks memory, in the sense that $P(X>s+t/X>s)=P(X>t), \forall s,t>0$
Gamma Distribution $X \sim G(\lambda,\alpha)$	$f(x)=\begin{cases}\lambda e^{-\lambda x}\frac{(\lambda x)^{\alpha-1}}{\Gamma\alpha}, & x\geq 0\\ 0, & otherwise\end{cases}$ The parameters λ and α are positive	$\left[\frac{\lambda}{\lambda-t}\right]^{\alpha}, \lambda>t$	$\frac{\alpha}{\lambda}$	$\frac{\alpha^2}{\lambda^2}$	If α=1 Gamma distribution becomes exponential distribution
Normal Distribution $X \sim N\left(\mu,\sigma^2\right)$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}, -\infty<x<\infty$	$e^{\mu t+\frac{t^2\sigma^2}{2}}$	μ	σ^2	-