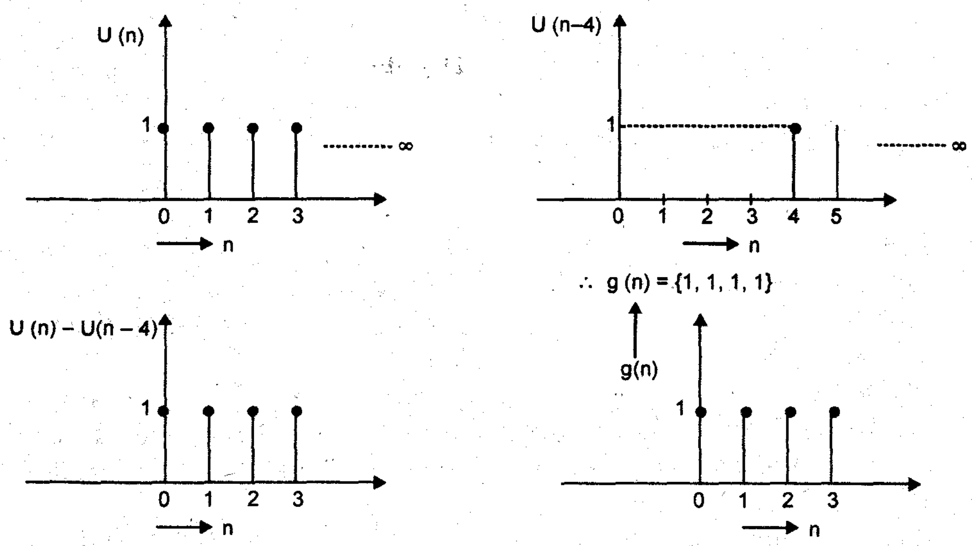
UNIT – I SIGNALS AND SYSTEMS

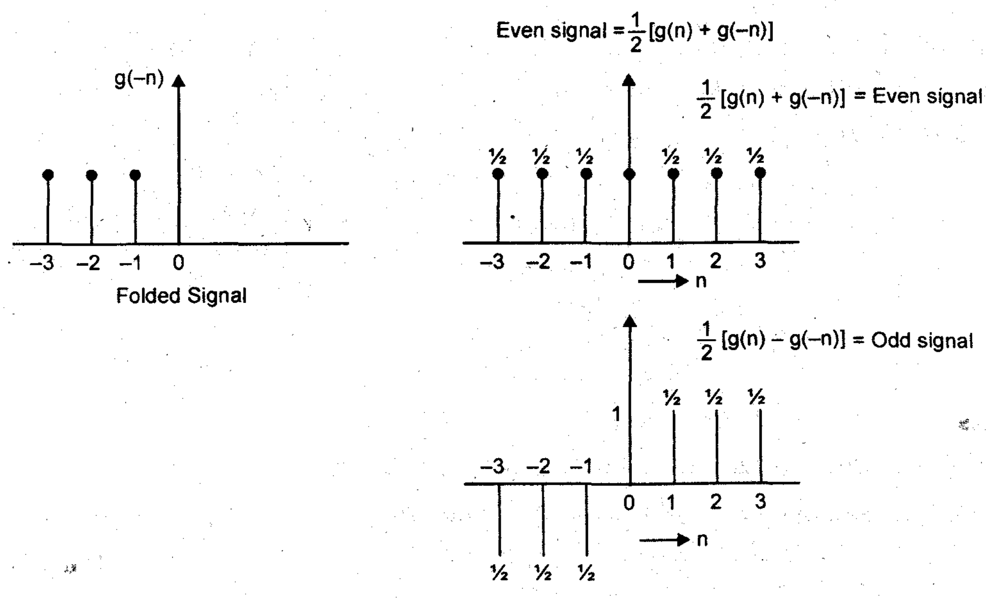
PART-B

1. Find the even and odd parts of the function g[n] = u [n] - u[n -4].

   Ans:



The calculations of even and odd parts are shown in fig below



2. Compute convolution of y (n) of the signals.

$$X(n) = \begin{cases} \alpha^n, -3 \leq n \leq 5 \\ 0, \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, 0 \leq n \leq 4 \\ 0, \text{elsewhere} \end{cases}$$

Ans:

$$X(n) = \begin{cases} \alpha^n, -3 \le n \le 5 \\ 0, \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, 0 \le n \le 4 \\ 0, \text{elsewhere} \end{cases}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$x(n) = \left[\frac{1}{\alpha^3}, \frac{1}{\alpha^2}, \frac{1}{\alpha}, \underset{\uparrow}{1}, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5\right]$$

$$h(n) = \underset{\uparrow}{1}\ 1\ 1\ 1\ 1$$

$$y(n) = \sum_{k=-3}^{5} x(k)\, h(n-k)$$

$$y(0) = x(-3)\,h(3) + x(-2)\,h(2) + x(-1)\,h(1) + x(0)\,h(0) + x(1)\,h(-1) + x(2)\,h(-2) + x(3)\,h(-3) + x(4)\,h(-4) + x(5)\,h(-5)$$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1$$

$$y(1) = \sum_{k=-3}^{5} x(k) h(1-k)$$

$$= x(-3)\ h(4) + x(-2)\ h(3) + x(-1)\ h(2) + x(0)\ h(1) + x(1)\ h(0) + x(2)\ h(-1) + x(3) h(-2) + \ldots..$$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha$$

$$y(2) = \sum_{k=-3}^{5} x(k)\ h(2-k)$$

$$= x(-3)\,h(5) + x(-2)\,h(4) + x(-1)\,h(3) + x(0)\,h(2) + x(1)\,h(1) + x(2)\,h(0) + x(3)h(-1) + \ldots..$$

$$= \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha + \alpha^2$$

$$y(3) = \sum_{k=-3}^{5} x(k) h(3-k)$$

$$= x(-3)\,h(6) + x(-2)\,h(5) + x(-1)\,h(4) + x(0)\,h(3) + x(1)\,h(2) + x(2)\,h(1)\,x(3) + h(0) + x(4)\,h(-1) + x(5)\,h(-2)$$

$$= 0 + 0 + \frac{1}{\alpha} + 1 + \alpha + \alpha^2 + \alpha^3$$

$$= 1 + \alpha + \alpha^2 + \alpha^3 + \frac{1}{\alpha}$$

$$y(4) = \sum_{k=-3}^{5} x(k)h(4-k)$$

$= x(-3)\ h(7) + x(-2)\ h(6) + x(-1)\ h(5) + x(0)\ h(4) + x(1)$
$h(3) + x(2)\ h(2)\ x(3) + h(1) + x(4)\ h(0) + x(5)\ h(-1)$
$= 0 + 0 + 0 + 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4$
$= 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4$

$$y(5) = \sum_{k=-3}^{5} x(k)n(5-k)$$

$= x(-3)\ h(5) + x(-2)\ h(7) + x(-1)\ h(6) + x(0)\ h(5) + x(1)\ h(4)$
$+ x(2)\ h(3) + x(3)\ h(2) + x(4)\ h(1) + x(5)\ h(0)$
$= 0 + 0 + 0 + 0 + 1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$
$= 1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$$y(6) = \sum_{k=-3}^{5} x(k)h(6-k)$$

$= x(-3)\ h(9) + x(-2)\ h(8) + x(-1)\ h(7) + x(0)\ h(6) + x(1)\ h(5) +$
$x(2)\ h(4) + x(3)\ h(3) + x(4)\ h(2) + x(5)\ h(1)$
$= 0 + 0 + 0 + 0 + 0 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$
$= \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

$$y(7) = \sum_{k=-3}^{5} x(k)h(7-k)$$

$= x(-3)\ h(10) + x(-2)\ h(9) + x(-1)\ h(8) + x(0)\ h(7) + x(1)\ h(6) +$
$x(2)\ h(5) + x(3)\ h(4) + x(4)\ h(3) + x(5)\ h(2)$
$= 0 + 0 + 0 + 0 + 0 + 0 + 0 + \alpha^3 + \alpha^4 + \alpha^5$
$= \alpha^3 + \alpha^4 + \alpha^5.$

$$y(8) = \sum_{k=-3}^{5} x(k)h(8-k)$$

$= x(-3)\ h(11) + x(-2)\ h(10) + x(-1)\ h(9) + x(0)\ h(8) + x(1)\ h(7) +$
$x(2)\ h(6)\ x(3)\ h(5) + x(4)\ h(4) + x(5)\ h(3)$
$= 0 + 0 + 0 + 0 + 0 + 0 + \alpha^4 + \alpha^5 = \alpha^4 + \alpha^5$

$$y(9) = \sum_{k=-3}^{5} x(k)h(9-k)$$

$= x(-3)\ h(12) + x(-2)\ h(11) + x(-1)\ h(10) + x(0)\ h(9) + x(1)\ h(8)$
$+ x(2)\ h(7) + x(3) + h(6) + x(4)\ h(5) + x(5)\ h(4)$
$= 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + \alpha^5 = \alpha^5.$

$$y(-1) = \sum_{k=-3}^{5} x(k)\,h(-1-k)$$

$= x(-3)\ h(2) + x(-2)\ h(1) + x(-1)\ h(0) + x(0)\ h(-1) + x(1)\ h(-2)$
$+ x(1)\ h(-3) + \ldots\ldots$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 0 = \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha}$$

$$y(-2) = \sum_{k=-3}^{5} x(k)\,h(-2-k)$$

$= x(-3)\ h(1) + x(-2)\ h(0) + x(-1)\ h(-1) + x(0)\ h(-2) + x(1)\ h(-3) + \ldots$

$$= \frac{1}{\alpha^3} + \frac{1}{\alpha^2}$$

$$y(-3) = \sum_{k=-3}^{5} x(k)h(-3-k)$$

$$= x(-3)\,h(0) + x(-2)\,h(-1) + x(-1)\,h(-2) + x(0)\,h(-3) + x(1)\,h(-4) + x(2)\,h(-5) + \ldots$$

$$= \frac{1}{\alpha^3}$$

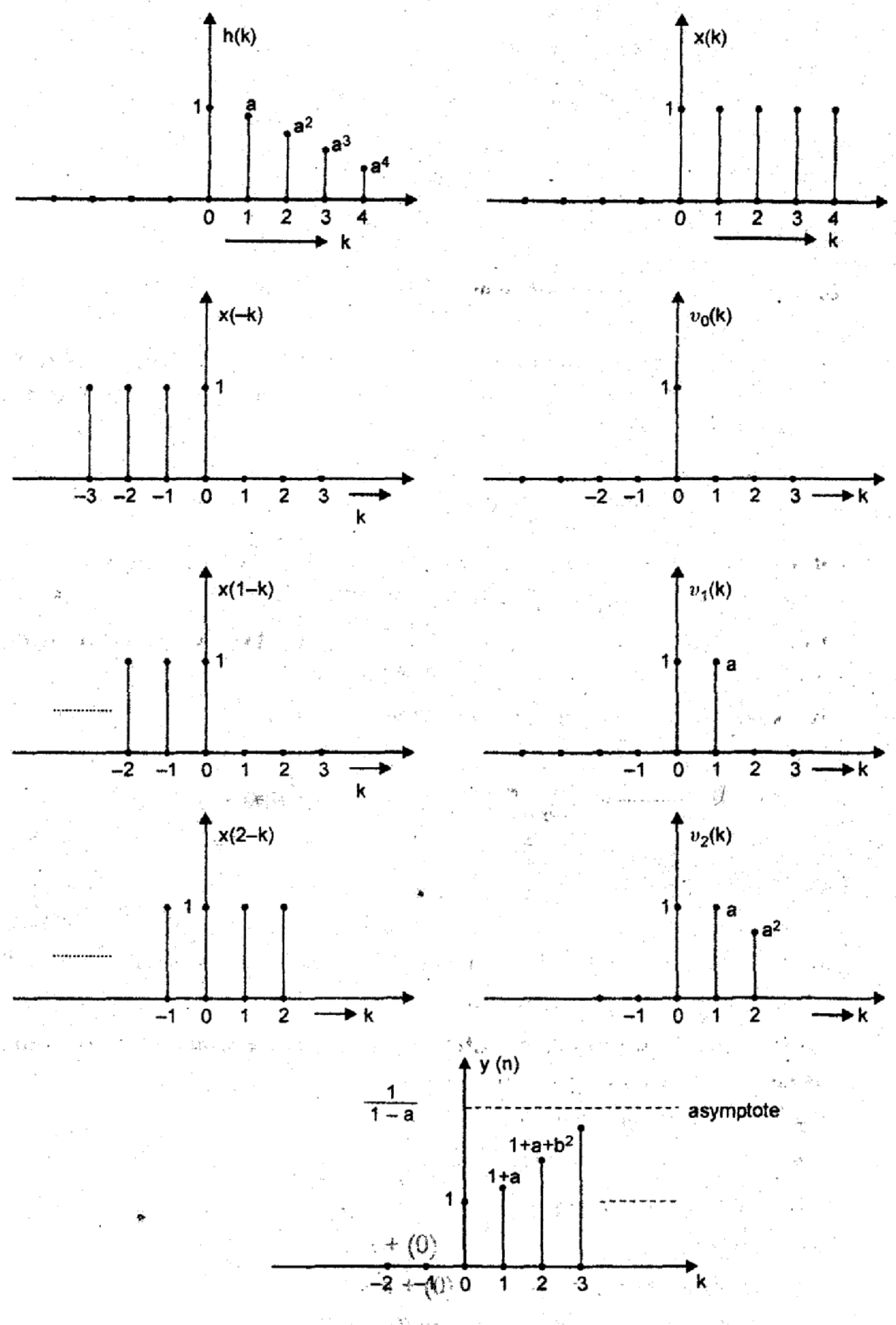
$$y(n) = \left(\frac{1}{\alpha^3}, \frac{1}{\alpha^3} + \frac{1}{\alpha^2}, \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha}, \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1, \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha,, \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha + \alpha^2, \frac{1}{\alpha} + 1 + \alpha + \alpha^2 + \alpha^3, 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4, 1 + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5, \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5, \alpha^3 + \alpha^4 + \alpha^5, \alpha^4 + \alpha^5, \alpha^5\right)$$

3. Determine the output y(n) of linear time invariant system units impulse response, $h(n) = a^n\, U(n),\ |a| < 1$ when the input is a unit step sequence x(n) = U(n).

Ans: In this case both h(n) and x (n) are infinite duration sequences. We use the form of the convolution formula

$$y(n) = \sum_{k=-\infty}^{\infty} x(n-k)h(k)$$

in which $x(k)$ is folded. The sequences $h(k)$, $x(k)$ and $x(-k)$ are shown in fig. The product of sequence $v_0(k)$, $v_1(k)$ and $v_2(k)$ corresponding to $x(-k)\ h(k)$, $x(1-k)\ h(k)$ and x(2-k) h(k) are also illustrated in Fig.


h(k)
1
a
a2
a3
a4
0 1 2 3 4
k
x(k)
1
0 1 2 3 4
k
x(–k)
1
–3 –2 –1 0 1 2 3
k
v0(k)
1
–2 –1 0 1 2 3
k
x(1–k)
1
–2 –1 0 1 2 3
k
v1(k)
1
a
–1 0 1 2 3
k
x(2–k)
1
–1 0 1 2
k
v2(k)
1
a
a2
–1 0 1 2
k
y (n)
1/(1 – a)
asymptote
1+a+b2
1+a
1
–2 –1 0 1 2 3
k

Thus the outputs are

$$y(0) = 1$$
$$y(1) = 1 + a$$
$$y(2) = 1 + a + a^2$$

for $n > 0$, the output is

$$y(n) = 1 + a + a^2 + \dots\dots\dots\dots\dots + a^n$$
$$= \frac{1 - a^{n+1}}{1 - a}$$

for $n < 0$, the product sequences consist of all zeroes. Hence

$$y(n) = 0\,;\ n < 0$$

A graph of output $y(n)$ is also illustrated in Fig for case $0 < a < 1$. Note the exponential in the output as a function of n. Since $|a| < 1$, the final value of the output as n approaches infinity is

$$y(\infty) = \lim_{n \to \infty} y(n) = \frac{1}{1 - a}$$

4. An LTI system is described by $y(n) = y(n-1) - 0.24\, y(n-2) + x(n)$. Find the response of this system for an input of x(n) = 10 cos (0.05 πn).

Ans:

$$y(n) = y(n-1) - 0.24y(n-2) + x(n)$$

For homogeneous solution

Put $x(n) = 0$, then $y(n) = y(n-1) - 0.24y(n-2)$

Let $y_n(n) = \lambda^n$

$$y(n) - y(n-1) + 0.24\, y(n-2) = 0$$
$$\lambda^n - \lambda^{n-1} + 0.24\, \lambda^{n-2} = 0$$
$$\lambda^{n-2}(\lambda^2 - \lambda - 0.24) = 0$$
$$\lambda^2 - \lambda - 0.24 = 0$$

$$\lambda = \frac{1 \pm \sqrt{1 + 4 \times 1 \times 0.24}}{2}$$

$$= \frac{1 \pm \sqrt{1 + 0.96}}{2} = \frac{1 \pm \sqrt{1.96}}{2} = \frac{1 \pm 1.4}{2}$$

$$= \frac{\cancel{2.4}^{12}}{\cancel{2}} = \frac{-0.4}{2}$$

$$\lambda = 1.2, -0.2$$

$$y\,h(n) = C_1 \lambda_1^n + C_2 \lambda_2^n$$
$$= C_1 (1.2)^n + C_2 (-0.2)^n$$

$$y(0) = C_1 + C_2 \quad \dots(1)$$
$$y(1) = 1.2\, C_1 - 0.2\, C_2 \quad \dots(2)$$
$$y(n) = y(n-1) - 0.24\, y(n-2)$$
$$y(0) = y(-1) - 0.24\, y(-2) \quad \dots(3)$$
$$y(1) = y(0) - 0.24y(-1)$$
$$= [y(-1) - 0.24\, y(-2)] - 0.24y(-1)$$

$$= y(-1) - 0.24\, y(-2) - 0.24\, y(-1)$$
$$= y(-1)(1-0.24) - 0.24 y(-2)$$
$$= 0.76\, y(-1) - 0.24\, y(-2) \quad \ldots(4)$$

By equating [(1) and (3)] and [(2) and (4)]

$$C_1 + C_2 = y(-1) - 0.24 y(-2) \quad \ldots(5)$$
$$1.2C_1 - 0.2C_2 = 0.76 y(-1) - 0.24 y(-2) \quad \ldots(6)$$

Multiply eq. (5) by 1.2. and subtract (6) from (5)

$$1.2\, C_1 + 1.2\, C_2 = 1.2 y(-1) - 0.2\,88\, y(-2)$$
$$1.2\, C_1 - 0.2\, C_2 = 0.76 y(-1) - 0.24\, y(-2)$$
$$- \quad + \qquad - \quad +$$

$$1.4 C_2 = 0.44 y(-1) - 0.048\, y(-2)$$

$$C_2 = \frac{0.44}{1.4} y(-1) - \frac{0.048}{1.4} y(-2)$$
$$= 0.31 y(-1) - 0.034 y(-2)$$

Multiply (5) by 0.2 and add with eq. (6)

$$0.2\, C_1 + 0.2\, C_2 = 0.2\, y(-1) - 0.48\, y(-2)$$
$$1.2\, C_1 - 0.2\, C_2 = 0.76\, y(-1) - 0.24\, y(-2)$$
$$1.4\, C_1 = 0.96\, y(-1) - 0.72\, y(-2)$$

$$C_1 = \frac{0.96}{1.4}\, y(-1) - \frac{0.72}{1.4}\, y(-2)$$

$$C_1 = 0.68 y(-1) - 0.51 y(-2)$$
$$y_{zi}(n) = [0.68 y(-1) - 0.51 y(-2)]\,(1.2)^n + [0.31\, y(-1) - 0.034 y(-2)]\,(-0.2)^n$$

Particular Solution

$$y_p(n) = \mathrm{K}\, 10 \cos(0.05\pi n)\, \mathrm{U}(n)$$
$$y(n) - y(n-1) + 0.24 y(n-2) = x(n)$$
$$\mathrm{K}\, 10 \cos(0.05\pi n)\, \mathrm{U}(n) - \mathrm{K}\, 10 \cos(0.05\pi(n-1))\, \mathrm{U}(n-1)) + 0.24\, \mathrm{K} 10 \cos(0.05\pi(n-2)).\ \mathrm{U}(n-2) = 10 \cos(0.05\,\pi n)\, \mathrm{U}(n)$$

Put $n \geq 2$

$$10\,\mathrm{K} \cos(0.1\pi) - 10\mathrm{K} \cos(0.05\pi) + 2.4\mathrm{K} \cos(0) = 10 \cos(0.1\pi)$$
$$10\,\mathrm{K} \cos(0.1\pi) - 10\,\mathrm{K} \cos(0.05\pi) + 2.4\mathrm{K} = 10\cos(0.1\pi)$$
$$10\,\mathrm{K}(\cos(0.1\pi) - \cos(0.05\pi) + 0.24) = 10 \cos(0.1\pi)$$
$$\mathrm{K}(\cos(0.1\pi) - \cos(0.05\pi) + 0.24) = \cos(0.1\pi)$$

$$K = \frac{\cos(0.1\pi)}{[\cos(0.1\pi) - \cos(0.05\pi) + 0.24]}$$

$$y_p(n) = K\cdot 10\cos(0.05\pi n)\,U(n)$$

$$= \frac{\cos(0.1\pi)(10\cos(0.05\pi n)U(n)}{[\cos(0.1\pi)-\cos(0.05\pi)+0.24]}$$

$$y(n) = y_{zi}(n) + y_p(n)$$

$$y(n) = [0.68\,y(-1) - 0.51y(-2)]\,(1.2)^n + [0.31\,y(-1) - 0.034y(-2)]\,(-0.2)^n + \frac{10\cos(0.1\pi)\cos(0.05\pi n)\,U(n)}{[\cos(0.1\pi)-\cos(0.05\pi)+0.24]}$$

5. Define convolution theorem as applied to discrete time signals. Find the inverse z-transform of $X(z) = z / (z-1)^2$ using convolution theorem.

Ans: Like convolution integral for continuous time LTI systems, the convolution of two discrete time signals is called discrete convolution. This is used to find the response of a discrete time LTI system with impulse response h(n) for any arbitrary input x(n).

Let us consider that a discrete time input signal x(n) is applied to a discrete time LTI system with unit impulse response h(n).

Then the discrete time output signal y(n) of this system may be expressed as

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \quad \text{...(1)}$$

The above expression for discrete time output signal y(n) is called the convolution

sum as against the convolution integral for continuous time LTI systm.

These equation (1) may be expressed as

$$y(n) = x(n) * h(n)$$

This equation indicates that the output y(n) is the convolution of the discrete time

input signal x(n) and discrete time unit impulse response h(n).

The convolution process can be summarised into the following steps.

a. Choose an initial value of ‘n’ the starting time for evoluting the output sequence y(n). If x(n) starts at $n = n_1$ and h(n) starts at n n_2 then $n = n_1 + n_2$ is good

choice. Then express both sequence in terms of the index k.

b. Folding: Fold the Iz(k) about the origin and obtain h(-k).

c. Time shifting: shift the h(—k) by n unit to right if n is positive and left if n is —ye to obtain h[-. (k — n)] = h(n — k).

d. Multiplication: Multiply x(k) by h(n - k) to obtain $\omega_n(k) = x(k)\, h\,(n - k).$

e. Summation: Sum all the values of the product w(k) to obtain the value of 0/p y(n).

f. Increment the index n, shift the sequence h(n — k) to right by one sample and do step 4.

g. Repeat step 6 until the sum of product is zero all remaining values of n.

Given $$X(z) = \frac{z}{(z-1)^2}$$

$$X(z) = \frac{z}{(z-1)} \cdot \frac{1}{(z-1)} = U(n).n$$

Taking $x_1(n) = U(n)$ and $x_2(n) = U\,(n-1)$

$$x_1(n) * x_2(n) \xleftarrow{z} X_1\,(z)\,.\;X_2(z)$$

using convolution

$$y(n) = x_1\,(n) * x_2(n) = \sum_{k=-\infty}^{\infty} x_2(k).x_1(n-k)$$

$$= \sum_{k_n=-\infty}^{\infty} u(k-1)U(-k+n) = \sum_{k=1}^{n} 1 = \sum_{k=1}^{n} (1)^k$$

where $x_2\,(k) = U(k-1)$

$$x_1\,(k) = U(k)$$

$$x_1\,(n-k) = U\,(-k+n)\}$$

We have standard summation, here $N_2 = n$, $N_1 = 1$

$$\sum_{n=N_1}^{N_2} A^n = N_2 - N_1 + 1 \text{ if } A = 1 \therefore y(n) = n - 1 + 1 = n$$

$\therefore$ $x_1(n) * x_2(n) = n\,U(n)$

6. What are the various realization techniques of linear time invariant systems? Mention.

Ans: The linear time invariant system is described by the difference equation of form.

$$y(n) = -\sum_{k=0}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k) \qquad \text{...(1)}$$

or equivalently.

$$\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k) \qquad \text{...(2)}$$

Here a_k and b_k are constants with $a_0 = 1$.

The methods of realizing digital systems can be divided into the two classes namely

recursive and non-recursive. For the linear time invariant system the non recursive realization has the form

$$y(n) = -\sum_{k=1}^{N} a_k\, y(n-k) + \sum_{k=0}^{M} b_k\, x(n-k) \qquad \text{...(3)}$$

The current output sample y(n) is a function of past output and present and past

inputs samples. For the LTI system the recursive realization has the form of

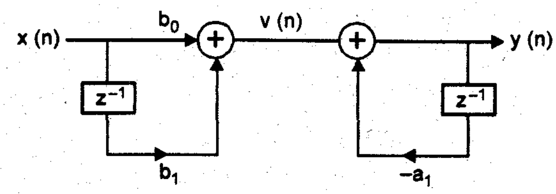
$$y(n) = \sum_{k=0}^{M} b_k\, x(n-k) \qquad \text{...(4)}$$

First order system block diagram representation.

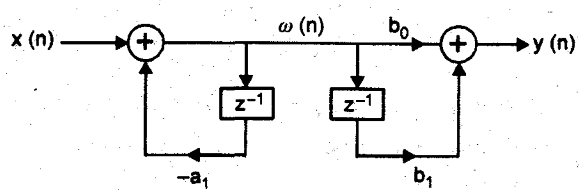
let us consider a first order system given by

$$y(n) = -a_1 y(n-1) + b_0\, x(n) + b_1\, x(n-1) \qquad \text{...(5)}$$

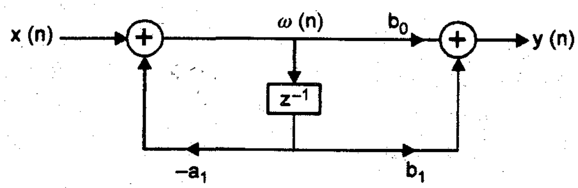
Which is realised as in fig. below. The term y(n —1) is obtained as the output of delay element with y(n) as input, x (n —1) is obtained as the output of delayed element with x(n) as input. This realization uses separate delay for both input and output signal samples. This structure is called as direct form I structure. This systeth can be viewed as two LTI systems in cascade.



(a) Direct form I realization



(b) Rearrangement of two cascaded system



(c) Direct form II realization

The first is non-recursive system described by

$$v(n) = b_0 x(n) + b_1 x(n-1) \qquad \text{...(6)}$$

and second is recursive system described by

$$y(n) = -a_1 y(n-1) + v(n) \qquad \text{...(7)}$$

The block diagram of Fig.(a) can be rearranged or modified in a variety of ways without changing the overall system function. From this figure we obtain two different equations.

$$w(n) = -a_1 w(n-1) + x(n) \qquad \text{...(8)}$$
$$y(n) = b_0 v(n) + b_1 w(n-1) \qquad \text{...(9)}$$

The difference equation (8) and (9) are equivalent to single difference eq. (5)

Fig. (b) reveals that the two elements contain the same input o(n) and hence the same output w(n -1). Therefore, these two elements can be merged into one delay as shown in Fig. (c). In comparison to direct form I realization, the new realization required only one delay for co (n), therefore it is more efficient in term of memory requirements. The realization of Fig. (c) is called as Direct form II structures.

7. Determine the impulse response for the cascade of two linear time invariant systems having impulse responses.

$$h_1(n) = \left(\frac{1}{2}\right)^n U(n)$$

$h_2(n) = \left(\frac{1}{4}\right)^n U(n)$

Ans. To determine the overall impulse response of the two system in cascade, we simply convolve $h_1(n)$ with $h_2(n)$. Hence

$$h(n) = \sum_{k=-\infty}^{\infty} h_1(k) h_2(n-k)$$

where $h_2(n)$ is folded and shifted. We define the product sequence.

$$v_n(k) = h_1(k) \cdot h_2(n-k)$$

$$= \left(\frac{1}{2}\right)^k \left(\frac{1}{4}\right)^{(n-k)}$$

which is non-zero for $k \geq 0$ and $n - k \geq 0$ or $n \geq k \geq 0$. On the other hand, for $n < 0$, we have $v_n(k) = 0$ for all k, and hence

$$h(n) = 0, n < 0$$

For $n \geq k \geq 0$, the sum of the values of the product sequence $v_n(k)$ overall k.

$$h(n) = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{4}\right)^{n-k} = \left(\frac{1}{4}\right)^n \sum_{k=0}^{n} 2^k$$

$$= \left(\frac{1}{4}\right)^n \left(2^{n+1} - 1\right) = \left(\frac{1}{2}\right)^n \left[2 - \left(\frac{1}{2}\right)^n\right] \quad n \geq 0$$

8. List various properties of z-transform with proof

Ans: The various properties of z-transform are given below.

(i) Linearity

$$z\,[a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1(z) + a_2 X_2(z)$$

(ii) Shifting: $m \geq 0$

(a) $$z\,[x(n+m)] = z^m X(z) - \sum_{i=0}^{m-1} x(i) z^{m-i}$$

(b) $$z\,[x(n-m)] = z^{-m} X(z)$$

(iii) Multiplication (by n^m). [or differentiation in z-domain]

$$z\,[n^m x(n)] = \left(-z\frac{d}{dz}\right)^m X(z).$$

(iv) Scaling in z-domain: [Multiplication by a^n]

$$z\,[a^n x(n)] = X(a^{-1}z)$$

(v) Time Reversal:

$$z\,[x\,(-n)] = X\,(z^{-1})$$

(vi) Conjugation:

$$z\,[x^*\,(n)] = X^*\,(z^*)$$

(vii) Convolution:

$$z\left[\sum_{m=0}^{n} h(n-m)\,r(m)\right] = H\,(z)\,R\,(z)$$

(viii) Initial value:

$$z\,[x\,(0)] = \underset{z\to\infty}{\mathrm{Lt}}\, X\,(z)$$

(ix) Final value:

$$z\,[x\,(\infty)] = \underset{z\to 1}{\mathrm{Lt}}\,(1-z^{-1})\,X\,(z)$$

$$= \underset{z\to 1}{\mathrm{Lt}}\,(z-1)\;X\,(z)\ \text{If } (X\,(z) \text{ is analytic for } |z|>1$$

(x) Correlation of two sequence.

$$R_{x_1x_2}(z) = X_1(z)\;X_2\,(z^{-1})$$

11. Parseval's theorem:

$$\sum_{n=-\infty}^{\infty} x_1(n)\,x_2^*\,(n) = \frac{1}{2\pi j}\oint_c X_1(\nu).X_2^*\left(\frac{1}{\nu^*}\right)\nu^{-1}\,d\nu$$

where the contour of integration must be overlap of the regions of convergence of

$x_1\,(\nu)$ and $x_2^*\left(\frac{1}{\nu^*}\right)$.

9.Determine the z-transform of the signal

$x(n) = (\cos \omega_0 n)\;U(n).$

Ans: By using Euler's identity the signal x (n) can be expressed as

$$x(n) = (\cos \omega_0 n)\, U(n) = \frac{1}{2} e^{j\omega_0 n} u(n) + \frac{1}{2} e^{-j\omega_0 n}\, U(n)$$

By simply linearity property.

$$X(z) = \frac{1}{2} z\left\{e^{j\omega_0 n} U(n)\right\} + \frac{1}{2} Z\left\{e^{-j\omega_0 n} u(n)\right\}$$

if we set $\alpha = e^{\pm j\omega_0} \quad \left(|\alpha| = |e^{\pm j\omega_0}| = 1\right)$

we obtain $e^{j\omega_0 n}\, U(n) \xrightarrow{z} \dfrac{1}{1 - e^{j\omega_0} z^{-1}} \quad \text{ROC} : |z| > 1$

and $e^{-j\omega_0 n}\, U(n) \xrightarrow{z} \dfrac{1}{1 - e^{-j\omega_0} z^{-1}} \quad \text{ROC} : |z| > 1$

Thus $X(z) = \dfrac{1}{2} \dfrac{1}{\left(1 - e^{j\omega_0} z^{-1}\right)} + \dfrac{1}{2} \dfrac{1}{\left(1 - e^{-j\omega_0} z^{-1}\right)} \quad \text{ROC} : |z| > 1$

After some simple algebraic manipulations we obtain the desired result namely.

$$(\cos \omega_0 n)\, U(n) \xleftrightarrow{z} \frac{1 - z^{-1} \cos\omega_0}{1 - 2z^{-1} \cos\omega_0 + z^{-2}} \quad \text{ROC}: |z| > 1$$

10.Find the z-transform and region of convergence for the following sequence. **(May 2014)**

$$x(n) = 7\left(\frac{1}{3}\right)^n u(n) - 6\left(\frac{1}{2}\right)^n u(n)$$

Apply initial value theorem and check the z-transform whether it is correct or not.

Ans:

$$x(n) = 7\left(\frac{1}{3}\right)^n u(n) - 6\left(\frac{1}{2}\right)^n u(n)$$

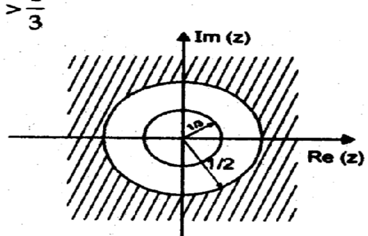
Let $x_1(n) = 7\left(\frac{1}{3}\right)^n u(n)$

$$x_1(n) = 7\left\{\frac{1}{3}, \left(\frac{1}{3}\right)^2, \left(\frac{1}{3}\right)^3, \ldots \left(\frac{1}{3}\right)^n\right\}$$

then $X_1(z) = 7\left[\frac{1}{3} + \frac{1}{3} z^{-1} + \frac{1}{3} z^{-2} + \ldots + \left(\frac{1}{3}\right) z^{-n}\right]$

$$X_1(z) = 7 \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} = 7\left[\sum_{n=0}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n\right]$$

$$X_1(z) = 7\left(\frac{1}{1 - \frac{1}{3} z^{-1}}\right) = 7\left(\frac{z}{z - \frac{1}{3}}\right) \qquad |z| > \frac{1}{3}$$



Let $x_2(n) = 6\left(\frac{1}{2}\right)^n U(n)$

$$x_2(n) = 6\left[\frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \ldots \left(\frac{1}{2}\right)^n\right]$$

then $X_2(z) = 6\left[\frac{1}{2} + \frac{1}{2} z^{-1} + \frac{1}{2} z^{-2} + \ldots + \left(\frac{1}{2}\right) z^{-n}\right]$

$$= 6\left[\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n} z^{-n}\right] = 6\left[\sum_{n=0}^{\infty}\left(\frac{1}{2}z^{-1}\right)^{n}\right]$$

$$= 6\left(\frac{1}{1-\frac{1}{2}z^{-1}}\right) = 6\left(\frac{z}{z-\frac{1}{2}}\right) \quad |z| > \frac{1}{2}$$

$$X(z) = 7\left(\frac{z}{z-\frac{1}{3}}\right) - 6\left(\frac{z}{z-\frac{1}{2}}\right)$$

$$\text{ROC}: z > \frac{1}{2} \;\&\; z > \frac{1}{3}$$

Initial value theorem, $x(0) = \lim_{z\to\infty} X(z)$

$$x(n) = 7\left(\frac{1}{3}\right)^{n} U(n) - 6\left(\frac{1}{2}\right)^{n} U(n)$$

$$x(0) = 7\left(\frac{1}{3}\right)^{0} U(0) - 6\left(\frac{1}{2}\right)^{0} U(0)$$

$$x(0) = 1$$

$$x(0) = \lim_{z\to\infty} \frac{7z}{z-\frac{1}{3}} - \frac{6z}{z-\frac{1}{2}}$$

Dividing numerator & denominator by z we get

$$x(0) = \lim_{z\to\infty} \frac{7}{1-\frac{1}{3z}} - \frac{6}{1-\frac{1}{2z}}$$

$$x(0) = 7-6 = 1$$

Thus calculated z transfer is corrects.

11. Find the inverse Z-transform of the function,

$$X(z) = \frac{(z-4)}{(z-1)(z-3)^2} \text{ for } |z| > 2$$

Ans: Given:

$$X(z) = \frac{1+2z^{-1}+z^{-2}}{1-z^{-1}+0.356z^{-2}}$$

By using long division method.

$$
\begin{array}{r|l}
 & 1+3z^{-1}+3.644z^{-2}+2.576z^{-3}+\ldots \\
1-z^{-1}+0.356z^{-2} & 1+2z^{-1}+z^{-2} \\
 & \underline{\_1\mp z^{-1}\pm 0.356z^{-2}} \\
 & 3z^{-1}+0.644z^{-2} \\
 & \underline{\_3z^{-1}\mp 3z^{-2}\pm 1.068z^{-3}} \\
 & 3.644z^{-2}-1.068z^{-3} \\
 & \underline{\_3.644z^{-2}\mp 3.644z^{-3}\pm 1.297z^{-4}} \\
 & 2.576z^{-3}-1.297z^{-4}
\end{array}
$$

$\therefore$ $\quad X(z) = 1 + 3z^{-1} + 3.644z^{-2} + 2.57z^{-3} + \ldots\ldots\ldots\ldots$ ...(1)

where $\quad X(z) = x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + \ldots\ldots$ ...(2)

Comparing (1) & (2)

$$x(0) = 1,\ x(1) = 3,\ x(2) = 3.644,\ x(3) = 2.57$$

$\therefore$ $\quad x(n) = \{1, 3, 3.644, 2.57, \ldots\}$

12. Determine the inverse z-transform of

$$X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+z^{-2}}$$

(a) ROC $|z| > 1$
(b) ROC $|z| < 1$

Ans: (a) Since the ROC is exterior of a circle, we except x (n) to be a causal sequence. This we divide so as to obtain a series in negative power of z. Carrying out the long division, we obtain

$$
\begin{array}{r|l}
 & 1+4z^{-1}+7z^{-2}+10z^{-3}+\ldots \\
1-2z^{-1}+z^{-2} & 1+2z^{-1} \\
 & \underline{\_1\mp 2z^{-1}\pm z^{-2}} \\
 & 4z^{-1}-z^{-2} \\
 & \underline{\_4z^{-1}\mp 8z^{-2}\pm 4z^{-3}} \\
 & 7z^{-2}-4z^{-3} \\
 & \underline{\_7z^{-2}\mp 14z^{-3}\pm 7z^{-4}} \\
 & 10z^{-3}-7z^{-4} \\
 & \underline{\_10z^{-3}\mp 20z^{-4}\pm 10z^{-5}} \\
 & 13z^{-4}-10z^{-5}
\end{array}
$$

Thus $\quad X(z) = 1 + 4z^{-1} + 7z^{-2} + 10z^{-3} + \ldots$

$\therefore$ $\quad x(n) = \{\underset{\uparrow}{1}, 4, 7, 10, \ldots\}$

(b)When the ROC is interior of the circle, the signal x (n) is anticausal signal. Thus we divide so as to obtain a series in power of z as follows.

$$
\begin{array}{r|l}
 & 2z+5z^2+8z^3+11z^4 \\
\hline
z^{-2}-2z^{-1}+1 & 2z^{-1}+\ 1 \\
 & \underline{\_2z^{-1} \mp 4 \pm 2z} \\
 & 5-2z \\
 & \underline{\_5 \mp 10z \pm 5z^2} \\
 & 8z-5z^2 \\
 & \underline{\_8z \mp 16z^2 \pm 8z^3} \\
 & 11z^2-8z^3 \\
 & \underline{\_11z^2 \mp 22z^3 \pm 11z^4} \\
 & 14z^3-11z^4
\end{array}
$$

Thus $X(z) = 2z + 5z^2 + 8z^3 + 11z^4 + ...$

In this case $x(n) = 0$ for $n \geq 0$, thus by comparing

The result with eq. $X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n},$

we get $x(n) = \{.... 11, 8, 5, 2, \underset{\uparrow}{0}, \}.$

13. Determine the z-transform of the signal

$$x(n) = -\propto^n U(-n-1) = \begin{cases} 0 & n \geq 0 \\ -\propto^n & n \leq -1 \end{cases}.$$

Ans:

$$X(z) = \sum_{n=-\infty}^{-1} (-\alpha^n)\, z^{-n} = -\sum_{l=1}^{\infty} (\alpha^{-1} z)^l$$

Where $l = -n$. Using the formula

$$A + A^2 + A^3 + ... = A(1 + A + A^2 + ...) = \frac{A}{1-A}$$

When $|A| < 1$ gives

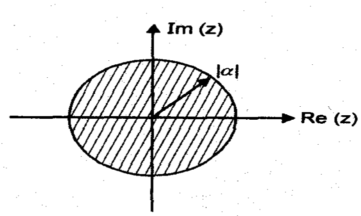
$$X(z) = \frac{-\alpha^{-1} z}{1 - \alpha^{-1} z}$$

$$= \frac{1}{1 - \propto z^{-1}}$$

Provided that $|\propto^{-1} z| < |$ or equivalently $|z| < |\propto|$.

Thus $x(n) = -\alpha^n U(-n-1) \xleftrightarrow{z} X(z) = \frac{-1}{1 - \alpha z^{-1}}$ ROC: $|z| < |\alpha|$

The ROC is now the interior of a circle having radius I . This is shown in fig below.



ROC of $x(n) = -\alpha^n U(-n-1)$

14. i)Consider the analog signal x(t)= 3cos 2000πt+5sin6000πt+10cos1200πt
What is the Nyquist rate for this signal?

Ans:

f1=1000 Hz
f2=3000 Hz
f3=600 Hz

Nyquist rate=2fmax=2*3000=6000Hz

ii) Derive the equation for convolution sum and summarize the steps involved in computing convolution **(May/June2013)**

Consider a DT LTI system, **L**.

$$x(n) \longrightarrow \boxed{\mathbf{L}} \longrightarrow y(n)$$

DT convolution is based on an earlier result where we showed that any signal $x(n)$ can t expressed as a sum of impulses.

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

So let us consider $x(n)$ written in this form to be our input to the LTI system.

$$y(n) = \mathrm{L}\left[x(n)\right] = \mathrm{L}\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right]$$

This looks like our general linear form with a scalar $x(k)$ and a signal in n, $\delta(n-k)$. Reca that for an LTI system:

- Linearity (L): $ax_1(n) + bx_2(n) \longrightarrow \boxed{\mathrm{L}} \longrightarrow ay_1(n) + by_2(n)$
- Time Invariance (TI): $x(n-n_o) \longrightarrow \boxed{\mathrm{L}} \longrightarrow y(n-n_o)$

We can use the property of linearity to distribute the system **L** over our input.

$$y(n) = \mathrm{L}\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right] = \sum_{k=-\infty}^{\infty} x(k)\mathrm{L}\left[\delta(n-k)\right]$$

So now we wonder, what is $\mathrm{L}\left[\delta(n-k)\right]$? Well, we can figure it out. Suppose we know ho **L** acts on one impulse $\delta(n)$, and we call it

$$h(n) = \mathrm{L}\left[\delta(n)\right]$$

then by time invariance we get our answer.

$$h(n-k) = \mathrm{L}\left[\delta(n-k)\right]$$

$$\delta(n-k) \longrightarrow \boxed{\mathbf{L}} \longrightarrow h(n-k)$$

This means that if we know *one* input-output pair for this system, namely

$$\delta(n) \longrightarrow \boxed{\mathbf{L}} \longrightarrow h(n)$$

then we can infer

$$x(n) \longrightarrow \boxed{\mathbf{L}} \longrightarrow y(n)$$

which gives us the following.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

This is the *convolution sum* for DT LTI systems.

The convolution sum for $x(n)$ and $h(n)$ is usually written as shown here.

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

15.i)Determine the Z Transform and ROC of the signal $x(n)=-a^n u(-n-1)$

The *bilateral* or *two-sided* Z-transform of a discrete-time signal *x[n]* is the formal power series *X(z)* defined as

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

If $x(n) = a^n u(n)$

$$X(z) = \frac{1}{1+az^{-1}}$$

$$z\,[x\,(n-m)] = z^{-m}\,X(z)$$

$$z\,[x\,(-n)] = X\,(z^{-1})$$

$X(z) = \frac{-1}{(1-az^{-1})}$ ROC: $|z| < |a|$

16.Check whether the following systems are Static or Dynamic, Linear or Nonlinear, Time invariant or Time varying, Causal or non causal, Stable or unstable

1)$y(n)=\cos[x(n)]$ 2)$y(n)=x(-n+2)$ 3)$y(n)=x(2n)$ 4)$y(n)=x(n)\cos\omega_0(n)$**(Nov/Dec 2013/ May2014)**

(i) $y(n) = \cos[x(n)]$.

* $y(n) = \cos[x(n)]$

$y(0) = \cos[x(0)]$

$y(-1) = \cos[x(-1)]$

$y(-2) = \cos[x(-2)]$

∴ The system is static

* o/p depends only on present input.

* ∴ The system is Causal.

* $y_1(n) = \cos[x_1(n)]$, $T[x_2(n)] = y_2(n) = \cos[x_2(n)]$

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 \cos[x_1(n)] + a_2 \cos[x_2(n)] \rightarrow (1)$

$T[a_1 x_1(n) + a_2 x_2(n)] = \cos[a_1 x_1(n) + a_2 x_2(n)] \rightarrow (2)$

$(1) \neq (2)$

∴ The system is non-linear

* $y(n,k) = T[x(n-k)] = \cos[x(n-k)] \rightarrow (1)$

$y(n-k) = \cos[x(n-k)] \rightarrow (2)$.

$(1) = (2)$. ∴ The system is time invariant.

x ———— y.

(2) $y(n) = x(-n+2)$.

* $y(1) = x(-1+2) = x(1)$

$y(0) = x(2)$

$y(-1) = x(3)$.

∴ The o/p not depends on present input alone

∴ The system is dynamic.

The o/p depends on future and past inputs.

∴ The system is non-causal.

* $T[x_1(n)] = y_1(n) = x_1(-n+2)$
$T[x_2(n)] = y_2(n) = x_2(-n+2)$.

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1(-n+2) + a_2 x_2(-n+2)$ — (1)

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 x_1(-n+2) + a_2 x_2(-n+2)$ — (2)

(1) = (2). Therefore the system is <u>Linear</u>

* $y(n,k) = T[x(n-k)] = x_1(-n+2-k) \rightarrow (1)$
$y(n-k) = x_1(-(n-k)+2) = x_1(-n+k+2) \rightarrow (2)$

(1) $\neq$ (2) ∴ Therefore the system is <u>time Variant</u>.

---

(3) $y(n) = x(2n)$.

* $y(0) = x(0)$
$y(1) = x(2)$
$y(-1) = x(-2)$

* O/p depends on past and future inputs.
* ∴ The system is <u>dynamic</u> and <u>non-causal</u>

* $T[x_1(n)] = x_1(2n)$, $T[x_2(n)] = x_2(2n)$

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1(2n) + a_2 x_2(2n)$ — (1)

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 x_1(2n) + a_2 x_2(2n)$ — (2)

(1) = (2). ∴ The system is <u>linear</u>

* $y(n,k) = T[x(n-k)] = x(2n-k) \rightarrow (1)$
$y(n-k) = x(2(n-k)) = x(2n-2k) \rightarrow (2)$

(1) $\neq$ (2). ∴ The system is <u>time variant</u>

(iv) $y(n) = x(n) \cos \omega_0 (n)$.

* $y(0) = x(0) \cos \omega_0 (0)$
$y(1) = x(1) \cos \omega_0 (+1)$
$y(-1) = x(-1) \cos \omega_0 (-1)$

* The o/p depends on only present input.
∴ The system is causal and static

* $T[x_1(n)] = x_1(n) \cos \omega_0 (n)$
$T[x_2(n)] = x_2(n) \cos \omega_0 (n)$
$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1(n) \cos \omega_0 (n) + a_2 x_2(n) \cos \omega_0 (n)$ → ①

$T[a_1 x_1(n) + a_2 x_2(n)]$
$= a_1 x_1(n) \cos \omega_0 n + a_2 x_2(n) \cos \omega_0 (n)$ → ②
① = ②
* The system is ~~time~~ linear.

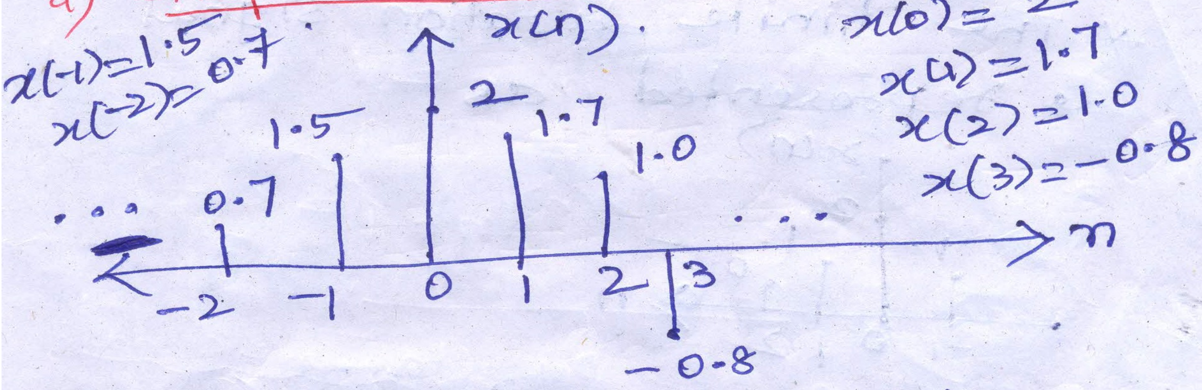
* $y(n,k) = T[x(n-k)] = x(n-k) \cos \omega_0 n$ → ①
$y(n-k) = x(n-k) \cos (n-k)$ → ②
① ≠ ②
* The system is time variant.

17. i)Describe the different types of digital signal representation.

Representation of Discrete time Signal:

A discrete time signal can be represented in different ways.
a) Graphical representation
b) Functional representation
c) Tabular representation
d) Sequence representation.

a) Graphical Representation



b) Functional representation:

$$x(n) = \begin{cases} 0.7, & n=-2 \\ 1.5, & n=-1 \\ 2, & n=0 \\ 1.7, & n=1 \\ 1.0, & n=2 \\ -0.8, & n=3 \\ 0, & \text{elsewhere} \end{cases}$$

c) Tabular representation:

n	...	−2	−1	0	1	2	3	...
$x(n)$	....	0.7	1.5	2	1.7	1.0	−0.8	...

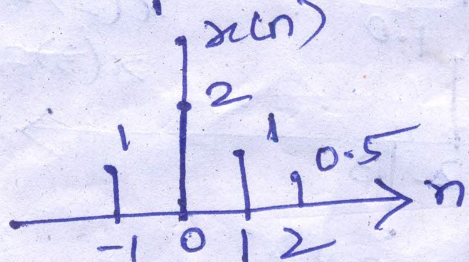
d) Sequence Representation:

$$x(n) = \{\ldots 0.7, 1.5, \underset{\uparrow}{2}, 1.7, 1.0, -0.8, \ldots\}$$

The time origin $(n=0)$ is indicated by $\uparrow$ symbol.

* The above is an infinite duration signal.

* The finite duration signal is represented as



$$x(n) = \{1, \underset{\uparrow}{2}, 1, 0.5\}$$

ii) What is Nyquist rate? Explain its significance while sampling the analog signals. **(Nov/Dec 2013)**

Nyquist rate:

The minimum sampling rate, $F_S = 2B = 2F_{max}$ is called as the Nyquist rate F_N.

Problem:

a) Consider the analog signal

$$x_a(t) = 3\cos 50\pi t + 10 \sin 300\pi t - \cos 100\pi t$$

What is the Nyquist rate for this signal.

Soln:

A signal is represented as

$$x(t) = \sum A \cos 2\pi f t \text{ (or) } \sum A \sin 2\pi f t.$$

$$\therefore x_a(t) = 3\cos 2\pi \times 25t + 10 \sin 2\pi 150 t - \cos 2\pi \times 50 t.$$

$\therefore F_1 = 25\,Hz$, $F_2 = 150\,Hz$, $F_3 = 50\,Hz$

$\therefore F_{max} = 150\,Hz.$

$$\text{Nyquist rate} = 2F_{max} = 2 \times 150 = 300\,Hz.$$