DIFFERENTIAL CALCULUS
UNIT-II

1.Radius of Curvature in Cartesian form:

$$\rho = \frac{\left[1+y_1^2\right]^{\frac{3}{2}}}{y_2} \quad \textbf{where } y_1 = \frac{dy}{dx}, y_2 = \frac{d^2y}{dx^2}$$

2. Curvature is $k = \frac{1}{\rho}$**.**

3. If $y_1 = \frac{dy}{dx}$ **=∞ at a point (x,y) then**

$$\rho = \frac{\left[1+x_1^2\right]^{\frac{3}{2}}}{x_2} \quad \textbf{where } x_1 = \frac{dx}{dy}, x_2 = \frac{d^2x}{dy^2}$$

4. Radius of curvature in parametric form
If x=f(t),y=g(t) then

$$\rho = \frac{\left(x'^2+y'^2\right)^{\frac{3}{2}}}{x'y''-y'x''} \quad where \quad x' = \frac{dx}{dt}, y' = \frac{dy}{dt}, x'' = \frac{d^2x}{dt^2}, y'' = \frac{d^2y}{dt^2}$$

(or) Use $\rho = \frac{\left[1+y_1^2\right]^{\frac{3}{2}}}{y_2}$ **where** $y_1 = \left(\frac{dy}{dt}\right)\left(\frac{dt}{dx}\right)$ **and** $y_2 = \left(\frac{dy_1}{dt}\right)\left(\frac{dt}{dx}\right)$

5.Centre of curvature is $\left(\overline{X}, \overline{Y}\right)$ **where** $\overline{X} = x - \frac{y_1\left(1+y_1^2\right)}{y_2}$ **and** $\overline{Y} = y + \frac{\left(1+y_1^2\right)}{y_2}$

6.Equation of circle of curvature is $\left(x-\overline{X}\right)^2 + \left(y-\overline{Y}\right)^2 = \rho^2$

7.**Working Rule for finding Evolute:**

Step:1 Write the parametric equation x=f(t),y=g(t) of the given curve y=f(x) ----(1)

Step:2 Find the co-ordinates of the centre of curvature $\left(\overline{X}, \overline{Y}\right)$--------(2)

Step:3 Eliminate the parameter ‘t’ from (1) and (2) and the equation in terms of $\overline{X}$ *and* $\overline{Y}$

Step:4 Replace $\overline{X}$ *and* $\overline{Y}$ by x and y to get the equation of Evolute .

8. Working Rule for finding Envelope:

(i) Step: 1 Let the family of curves be f(x,y,m) =0 ,where m is the parameter .

Step: 2 Find $\frac{\partial}{\partial m} f(x,y,m) = 0$.

Step:3 Eliminate m from f(x,y,m) =0 and $\frac{\partial}{\partial m} f(x,y,m) = 0$, which gives the equation of envelope.

(ii) If the family of curves is the of the form $A\lambda^2 + B\lambda + C = 0$ where A,B,C are functions of x and y then the envelope of family of curves is B^2-4AC=0

9.Evolute as the envelope of its normals :

Step:1 Find the equation of the normal to the given curve

Step:2 Find the envelope of the normal which is the Evolute