Two - Dimensional Random Variables.

TYPE	JOINT DISTRIBUTION	MARGINAL DISTRIBUTIONS		CONDITIONAL DISTRIBUTIONS		EXPECTATION					INDEPENDENCY	
		FOR X	FOR Y	For $X\|Y$	For $Y\|X$	$E(X)$	$E(Y)$	$E(XY)$	$E(X\|Y)$	$E(Y\|X)$		
DISCRETE	$P(X=x_i, Y=y_j) = p_{ij}$ such that (i) $p_{ij} \geq 0 \ \forall i,j$ (ii) $\sum_i \sum_j p_{ij} = 1$	$P(X=x_i) = p_{i.} = \sum_j p_{ij}$	$P(Y=y_j) = p_{.j} = \sum_i p_{ij}$	$P(X=x_i \| Y=y_j) = \frac{p_{ij}}{p_{.j}}$	$P(Y=y_j \| X=x_i) = \frac{p_{ij}}{p_{i.}}$	$E(X) = \sum_i x_i p_{i.} = \overline{X}$ (Mean of X)	$E(Y) = \sum_j y_j p_{.j} = \overline{Y}$ (Mean of Y)	$E(XY) = \sum_i \sum_j x_i y_j p_{ij}$	$E(X\|Y) = \sum_i \frac{x_i p_{ij}}{p_{.j}}$ (Conditional Mean of $X\|Y$)	$E(Y\|X) = \sum_j \frac{y_j p_{ij}}{p_{i.}}$ (Conditional Mean of $Y\|X$)	$p_{ij} = p_{i.} \, p_{.j} \ \forall i,j$	
CONTINU-OUS	$P\{x-\frac{dx}{2} \leq X \leq x+\frac{dx}{2}, y-\frac{dy}{2} \leq Y \leq y+\frac{dy}{2}\} = f_{XY}(x,y)\,dx\,dy$ such that (i) $f_{XY}(x,y) \geq 0$ (ii) $\iint_R f_{XY}(x,y)\,dx\,dy = 1$	$f_X(x) = \int_y f_{XY}(x,y)\,dy$	$f_Y(y) = \int_x f_{XY}(x,y)\,dx$	$f_{X\|Y}(x) = \frac{f_{XY}(x,y)}{f_Y(Y=y)}$	$f_{Y\|X}(y) = \frac{f_{XY}(x,y)}{f_X(X=x)}$	$E(X) = \int_x x f_X(x)\,dx = \overline{X}$ (Mean of X)	$E(Y) = \int_y y f_Y(y)\,dy = \overline{Y}$ (Mean of Y)	$E(XY) = \iint_R xy f_{XY}\,dx\,dy$	$E(X\|Y) = \int_x x f_{X\|Y}\,dx$ (Conditional mean of $X\|Y$)	$E(Y\|X) = \int_y y f_{Y\|X}\,dy$ (Conditional mean of $Y\|X$)	$f_{XY}(x,y) = f_X(x) f_Y(y) \ \forall (x,y) \in R$.	Regression curves of Y on X is $Y = E(Y\|X=x)$ Regression curve of X on Y is $X = E(X\|Y=y)$

$\Rightarrow$ Conditional Variance of $X|Y$ is $E(X^2|Y) - [E(X|Y)]^2$

$\Rightarrow$ Covariance of X, Y is $cov(X,Y) = E\{(X-\overline{X})(Y-\overline{Y})\}$
$= E(XY) - \overline{X}\,\overline{Y}$

$\Rightarrow$ Correlation coefficient $r_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$

$\Rightarrow$ For discrete data,

$$r_{xy} = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - (\sum x)^2}\sqrt{n\sum y^2 - (\sum y)^2}} = \frac{\sum(x-\overline{x})(y-\overline{y})}{\sqrt{\sum(x-\overline{x})^2}\sqrt{\sum(y-\overline{y})^2}}$$

$\Rightarrow$ $-1 \leq r \leq 1$

$\Rightarrow$ If $r=0$, the datas are uncorrelated.

Regression lines.

Equation	X on Y	Y on X.
Equation	$x - \overline{x} = b_{xy}(y - \overline{y})$	$y - \overline{y} = b_{yx}(x - \overline{x})$
Reg. Coefficients	$b_{xy} = r\sigma_x/\sigma_y = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2}$	$b_{yx} = r\sigma_y/\sigma_x = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$
Properties	1. $r = \pm\sqrt{b_{xy} \times b_{yx}}$. 2. If $\|b_{xy}\| > 1$, then $\|b_{yx}\| < 1$ and vice versa. 3. If b_{xy} & b_{yx} are +ve, then r is +ve 4. If b_{xy} & b_{yx} are -ve, then r is -ve 5. Angle b/w the lines $\theta = \tan^{-1}\left(\frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}\right)$	

CENTRAL LIMIT THEOREM

If x_i's are i.i.d RVs,
then (i) $\overline{X} = \frac{\sum x_i}{N} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
(ii) $S_n = \sum x_i \sim N(n\mu, n\sigma^2)$

TRANSFORMATION OF 2D RVs

$g_{uv}(u,v) = f_{xy}(x,y)\,|J|$

where $J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$