UNIT-I

Discrete Fourier Transform

The DFT computes the values of the z-transform for evenly spaced points around the unit circle for a given sequence.

If the sequence to be represented is of finite duration ie) has only a finite number of non zero values, the transform used is Discrete Fourier Transform.

DFT Applications :-

* linear filtering
* Correlation analysis
* Spectrum Analysis.

Definition of DFT :-

Let $x(n)$ be a finite duration sequence. The N-point DFT of the sequence $x(n)$ is expressed by

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}, \quad k = 0, 1, \ldots N-1$$

and the corresponding IDFT is

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi nk/N}, \quad n = 0, 1, \ldots N-1$$

Discrete Time Fourier Transform :- (DTFT)

- The Fourier transform of a discrete time sequence $x(n)$ is represented by the complex exponential sequence $e^{-j\omega n}$

$\omega \rightarrow$ real frequency variable.

Definition of DTFT:-

The DTFT $X(e^{j\omega})$ of a sequence $x(n)$ is given by

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

IDTFT

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Relationship between DFT and DTFT:-

Let $x(n)$ be an aperiodic finite energy sequence. The Fourier Transform is given by.

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

If $X(e^{j\omega})$ is sampled at N equally spaced frequencies.

$$\omega_k = \frac{2\pi k}{N} = 0, 1, 2 \ldots N-1$$

then

$$X(k) = X(e^{j\omega})\Big|_{\omega = 2\pi k/N}$$

$$= \sum_{n=-\infty}^{\infty} x(n)\, e^{-j2\pi nk/N} \qquad k = 0, 1, \ldots N-1.$$

Note:- The DTFT and the z-transform are applicable to any arbitrary sequences, where the DFT can be applied only to finite sequences.

Problems:

1) Compute the DFT of the sequence $x(n) = \{1, j, -1, -j\}$ for $N = 4$;

Soln: The DFT of the sequence $x(n)$ is given by

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \quad ; \quad k = 0, 1, 2 \ldots N-1$$

for $k=0$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{0}$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + j - 1 - j$$

$$X(0) = 0$$

for $k=1$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2}$$

$$= x(0)e^{0} + x(1)e^{-j\pi/2} + x(2)e^{-j\pi} + x(3)e^{-j3\pi/2}$$

$$= 1 + j(-j) + (-1)(-1) + (-j)(j)$$

$$= 1 + 1 + 1 + 1$$

$$= 4$$

for $k = 2$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0)e^{0} + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1 + (j)(-1) + (-1)(1) + (-j)(-1)$$

$$= 1 - j - 1 + j$$

$$X(2) = 0$$

for $k=3$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j3\pi n/2}$$

$$= x(0) + x(1) e^{-j3\pi/2} + x(2) e^{-j3\pi} + x(3) e^{-j9\pi/2}$$

$$= 1 + (j)(j) + (-1)(-1) + (-j)(-j)$$

$$= 1 + (-1) + 1 - 1$$

$$X(3) = 0$$

$$\therefore \quad X(k) = \{0, 4, 0, 0\}$$

Exercise:-

1) Find 8-point DFT of $x(n) = \{1, -1, 1, -1, 1, -1, 1, -1\}$

Zero padding:-

N - length of the DFT

L - length of the sequence $x[n]$

If $N < L$ time domain aliasing occurs due to undersampling and in the process we could miss out some important details and get misleading information.

To avoid this the no of samples of $x(n)$ is increased by adding some dummy sample of 0 value. This addition of dummy samples is known as zero padding.

Problems:

1) Compute the 4-pt DFT of the sequence

$$x(n) = \{1 \qquad 0 \le n < 2$$

Soln:

for the given sequence $L = 3$

$N = 4$

$\therefore$ By adding dummy samples of 0 values.

$$x(n) = \{1, 1, 1, 0\}.$$

$$W_4 = \begin{array}{c} \\ k=0 \\ k=1 \\ k=2 \\ k=3 \end{array} \begin{array}{c} \begin{array}{cccc} n=0 & n=1 & n=2 & n=3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \end{array}$$

$$\therefore X_4 = W_4 (x_4)$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$X_4 = \begin{bmatrix} 1+1+1+0 \\ 1-j-1+0 \\ 1-1+1+0 \\ 1+j-1+0 \end{bmatrix} = \begin{bmatrix} 3 \\ -j \\ 1 \\ j \end{bmatrix}$$

$$X(k) = \{3, -j, 1, j\}.$$

Exercise:-

2. Compute the 4-point DFT of the following sequence:-

i) $x(n) = \{1, 1, 1, 1\}$

ii) $x(n) = \{1, 1, 0, 0\}$

iii) $x(n) = \cos \pi n$

iv) $x(n) = \sin\left(\frac{n\pi}{2}\right)$

Example:-

1) find the N-point DFT of the following sequences for $0 \le n \le N-1$

$$x(n) = \delta(n)$$

Soln:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$$

$$x(n) = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$$

$$X(0) = \sum_{n=0} x(0)\, e^{0}$$

$$X(0) = 1$$

Problems based on IDFT

1) Find the IDFT of the following functions with $N=4$.

i) $X(k) = \{1, 0, 1, 0\}$.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j2\pi nk/N}$$

$n=0$

$$x(0) = \frac{1}{4}\sum_{k=0}^{3} X(k) e^{0}$$

$$= \frac{1}{4}\left[X(0) + X(1) + X(2) + X(3)\right]$$

$$= \frac{1}{4}\left[1+1\right]$$

$$x(0) = 1/2$$

$n=1$

$$x(1) = \frac{1}{4}\sum_{k=0}^{3} X(k)\, e^{j2\pi k/4}$$

$$= \frac{1}{4}\left[\sum_{k=0}^{3} X(k)\, e^{j\pi k/2}\right]$$

$$= \frac{1}{4}\left[X(0) + X(1) e^{j\pi/2} + X(2) e^{j\pi} + X(3) e^{j3\pi/2}\right]$$

$$= \frac{1}{4}\left[1 + 0 + (1)(-1) + 0\right]$$

$$= \frac{1}{4}(0)$$

$$x(1) = 0$$

$n=2$

$$x(2) = \frac{1}{4}\sum_{k=0}^{3} X(k)\, e^{j\pi k}$$

$$= \frac{1}{4}\left[X(0) + X(1) e^{j\pi} + X(2) e^{j2\pi} + X(3) e^{j3\pi}\right]$$

$$x(2) = \frac{1}{4}\left[1 + 0 + 1 + 0\right] = 1/2$$

$n = 3.$

$$x(3) = \frac{1}{4} \sum_{k=0}^{4} X(k)\, e^{j3\pi k/2}$$

$$= \frac{1}{4} \sum_{k=0}^{4} X(k)\, e^{\frac{j3\pi k}{2}}$$

$$= \frac{1}{4} \left[X(0) + X(1)\, e^{j3\pi/2} + X(2)\, e^{j3\pi} + X(3)\, e^{j9\pi/2} \right]$$

$$= \frac{1}{4} \left[1 + 0 + 1(-1) + 0 \right]$$

$$x(3) = 0$$

$$\therefore \quad x(n) = \left\{ \tfrac{1}{2}, 0, \tfrac{1}{2}, 0 \right\}.$$

Exercise :-

1) Find the IDFT of the following sequence with $N = 4$.

$$X(k) = \{ 6, (-2+j2), -2, (-2-j2) \}.$$

Plot of Magnitude and phase spectrum :-

1) Compute the 4-point DFT of causal three sample sequence given by

$$x(n) = \begin{cases} \tfrac{1}{3}, & 0 \le n \le 2 \\ 0 & \text{else} \end{cases}$$

plot the magnitude and phase spectrum :-

Soln :- N-point DFT of $x(n)$ is

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}, \quad k = 0, 1 \ldots N-1$$

Here $N = 4$

$$\therefore X(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi kn/4} = \sum_{n=0}^{3} x(n) e^{j\pi kn/2}, \quad k = 0,1,2,3.$$

$$X(k) = x(0) + x(1)\, e^{-j\pi k/2} + x(2)\, e^{-j\pi k} + x(3)\, e^{-j3\pi k/2}.$$

$$= \frac{1}{3} + \frac{1}{3} e^{-j\pi k/2} + \frac{1}{3} e^{-j\pi k} + 0$$

$$X(k) = \frac{1}{3}\left[1 + e^{-j\pi k/2} + e^{-j\pi k}\right]$$

$$X(k) = \frac{1}{3}\left[1 + \cos \pi k/2 - j \sin \pi k/2 + \cos \pi k - j \sin \pi k\right]$$

$k = 0,1,2,3.$

for $k = 0.$

$$X(0) = \frac{1}{3}\left[1 + \cos 0 - j \sin 0 + \cos 0 - j \sin 0\right]$$

$$= \frac{1}{3}\left[1 + 1 + 1\right]$$

$$X(0) = 1 \underline{|0}$$

for $k = 1$

$$X(1) = \frac{1}{3}\left[1 + \cos \pi/2 - j \sin \pi/2 + \cos \pi - j \sin \pi\right]$$

$$= \frac{1}{3}\left[1 + 0 - j + (-1) - 0\right]$$

$$X(1) = \frac{1}{3}\left[-j\right] = -\frac{1}{3} j = \frac{1}{3} \underline{|-\pi/2}$$

for $k=2$

$$X(2) = \frac{1}{3}\left[1+\cos\pi - j\sin\pi + \cos 2\pi - j\sin 2\pi\right]$$

$$= \frac{1}{3}\left[1+(-1) - 0 + (1) - 0\right]$$

$$X(2) = \frac{1}{3}(1) = \frac{1}{3}\underline{\lfloor 0}$$

for $k=3$

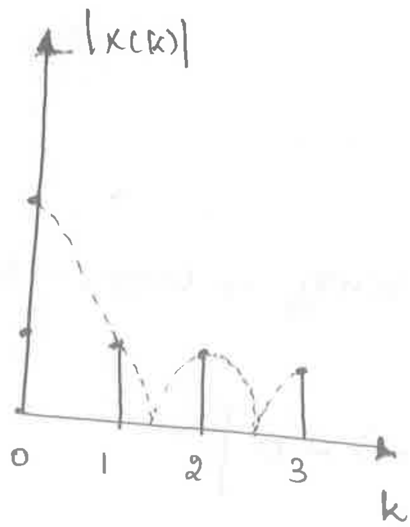
$$X(3) = \frac{1}{3}\left[1+\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2} + \cos 3\pi - j\sin 3\pi\right]$$

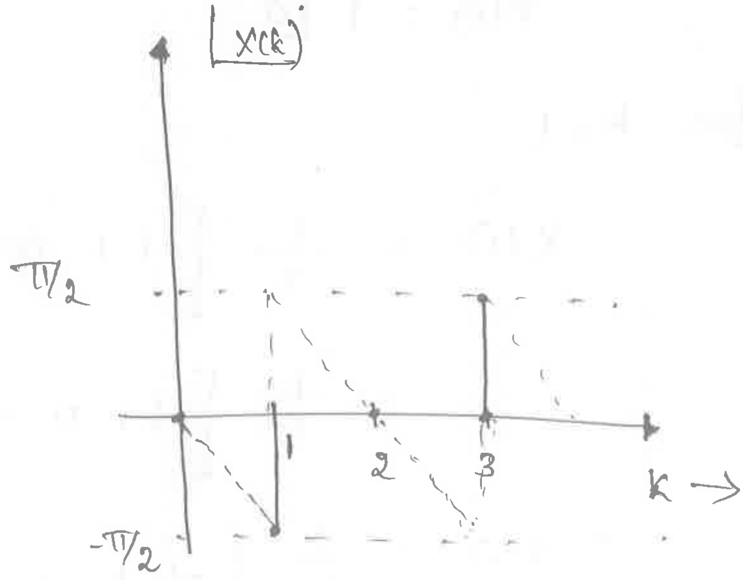
$$= \frac{1}{3}\left[1+0+j+(-1)\right]$$

$$= \frac{1}{3}\left[j\right]$$

$$X(3) = \frac{1}{3}\underline{\lfloor \pi/2}$$

$$\therefore \quad X(k) = \left\{1\underline{\lfloor 0},\ \frac{1}{3}\underline{\lfloor -\pi/2},\ \frac{1}{3}\underline{\lfloor 0},\ \frac{1}{3}\underline{\lfloor \pi/2}\right\}$$





Problems:

1 Find the 4-point DFT of the sequence $x(n) = \cos\frac{n\pi}{4}$

Ans:

$N = 4$

$$x(n) = \{\cos(0),\ \cos\pi/4,\ \cos\pi/2,\ \cos 3\pi/4\}$$

$$x(n) = \{1,\ 0.707,\ 0,\ -0.707\}$$

The N-point DFT of the sequence $x(n)$ is given as

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N} \quad ; \quad k = 0, 1, \ldots N-1$$

∴ The DFT is

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j2\pi nk/4} \ ;$$

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\pi nk/2} \ ;$$

for $k=0$

$$X(0) = \sum_{n=0}^{3} x(n) e^{0} \ ;$$

$$= x(0) + x(1) + x(2) + x(3)$$

$$= 1 + 0.707 + 0 - 0.707$$

$$X(0) = 1$$

for $k=1$

$$X(1) = \sum_{n=0}^{3} x(n) e^{-j\pi k/2}.$$

$$X(1) = x(0)e^{0} + x(1)e^{j\pi/2} + x(2)e^{-j\pi} + x(3)e^{-j3\pi/2}$$

$$= 1 + 0.707(-j) + 0 + (-0.707)(j)$$

$$X(1) = 1 - j1.414$$

for $k=2$.

$$X(2) = \sum_{n=0}^{3} x(n) e^{-j\pi n}$$

$$= x(0)e^{0} + x(1)e^{-j\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1 + 0.707 e^{-j\pi} + 0 - 0.707 e^{-j3\pi}$$

$$X(2) = 1$$

for $k=3$

$$X(3) = \sum_{n=0}^{3} x(n) e^{-j3\pi n/2}.$$

$$= x(0)e^{0} + x(1)e^{-j3\pi/2} + x(2)e^{-j3\pi} + x(3)e^{-j9\pi/2}.$$

$$= 1 + 0.707(j) + 0 + (-0.707)(-j)$$

$$X(3) = 1 + j1.414$$

Ans :-

$$X(k) = \{1,\ 1-j1.414,\ 0,\ 1+j1.414\}$$

Exercise :-

2. Compute the DFT of the sequence whose values for one period is given by $x(n) = \{1, 1, -2, -2\}$.

Exercise! -

1) Compute the 4 point DFT of the sequence.

$$x(n) = \{0, 1, 2, 3\}.$$

sketch the magnitude and phase spectrum:-

Ans: $|X(k)| = \{6, 2.8, 2, 2.8\}$

$\angle X(k) = \{0, 0.75\pi, \pi, -0.75\pi$

2) Find the DFT of the sequence

$$x(n) = \begin{cases} 1, & \text{for } 0 \le n \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

for (i) $N = 4$ and

ii) $N = 8$

Plot $|X(k)|$ and $\angle X(k)$. Comment on the result.

Ans: for $N = 4$ $|X(k)| = \{3, 1, 1, 1\}$.

$\angle X(k) = \{0, -\pi/2, 0, \pi/2\}$

for $N = 8$

$|X(k)| = \{3, 2.414, 1, 0.414, 1, 0.414, 1, 2.41$

$\angle X(k) = \{0, -\pi/4, -\pi/2, \pi/4, 0, -\pi/4, \pi/2, \pi/4\}$

Properties of DFT:

1) **Periodicity:**

Statement: If a sequence $x(n)$ is periodic with periodicity of N samples, then N-point DFT of the sequence is also periodic with periodicity of N samples.

If $x(n)$ and $X(k)$ are an N point DFT pair then

$$x(n+N) = x(n) \text{ for all } n$$

$$X(k+N) = X(k) \text{ for all } k.$$

Proof:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$$

$$X(k+N) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi(k+N)n}{N}}$$

$$= \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N} \cdot e^{-j2\pi n}$$

$$e^{-j2\pi n} = 1 \quad \text{for all } n$$

$$\therefore X(k+N) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$$\boxed{X(k+N) = X(k)}$$

2) **Linearity:**

Statement: If $x_1(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X_1(k)$ and $x_2(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X_2(k)$ then for any real valued or complex valued constants a_1 and a_2

$$a_1 x_1(n) + a_2 x_2(n) \xleftrightarrow[N]{DFT} a_1 X_1(k) + a_2 X_2(k)$$

Proof :-

$$DFT\{a_1 x_1(n) + a_2 x_2(n)\} = \sum_{n=0}^{N-1} \{a_1 x_1(n) + a_2 x_2(n)\} e^{-j2\pi nk/N}$$

$$= a_1 \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi nk/N} + a_2 \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi nk/N}$$

$$\boxed{DFT\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 X_1(k) + a_2 X_2(k)}$$

3) Multiplication of Two DFT's and circular convolution :-

Let $x_1(n)$ and $x_2(n)$ are finite duration sequence of length N. then

$$x_1(n) \, \text{N} \, x_2(n) \xleftrightarrow{DFT} X_1(k) X_2(k)$$

Statement :-

Multiplication of the DFT's of two sequence is equivalent to the DFT of the circular convolution of the two sequences.

Proof :-

DFT's of $x_1(n)$ & $x_2(n)$ are given by

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi nk/N}, \quad k = 0, 1, \ldots N-1$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi nk/N}, \quad k = 0, 1, \ldots N-1$$

and

$$X_3(k) = X_1(k) \cdot X_2(k) \quad ; \quad k = 0, 1, \ldots N-1$$

IDFT of $X_3(k) = x_3(m)$

By the definition of IDFT,

$$x_3(m) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k)\, e^{j2\pi km/N} \quad ; \quad m = 0, 1, \ldots N-1$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X_1(k) \cdot X_2(k)\, e^{j2\pi km/N}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi nk/N} \right] \left[\sum_{l=0}^{N-1} x_2(l)\, e^{-j2\pi lk/N} \right] e^{j2\pi km/N}$$

$$\therefore x_3(m) = \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) \cdot \sum_{l=0}^{N-1} x_2(l) \cdot \sum_{k=0}^{N-1} e^{\frac{j2\pi k(m-n-l)}{N}}$$

Let $m - n - l = pN$ where p is an integer

$$\therefore e^{\frac{j2\pi k(m-n-l)}{N}} = e^{\frac{j2\pi kpN}{N}} = e^{j2\pi kp} = \left(e^{j2\pi p}\right)^k$$

From finite geometric series sum formula,

$$\sum_{n=0}^{N-1} a^n = \begin{cases} N & \text{for } a = 1 \\ \dfrac{1-a^N}{1-a} & \text{for } a \neq 1 \end{cases}$$

$$\therefore \sum_{k=0}^{N-1} e^{\frac{j2\pi k(m-n-l)}{N}} = \sum_{k=0}^{N-1} \left(e^{j2\pi p}\right)^k = \sum_{k=0}^{N-1} (1)^k = N$$

$\because e^{j0} = 1$

$$x_3(m) = \frac{1}{\cancel{N}} \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \cancel{N}$$

$$x_3(m) : \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l)$$

If $x_2(l)$ is a periodic sequence with periodicity of N Samples then $x_2(l+PN) = x_2(l)$

here $m-n-l = PN$

$\therefore$ $l = m-n-pN$

$$x_2(l) = x_2(m-n-pN) = x_2(m-n)_N$$

$$= x_2((m-n; \bmod N))$$

$$\therefore x_3(n) = \sum_{n=0}^{N-1} x_1(n) \sum_{n=0}^{N-1} x_2(m-n)_N$$

$$= \sum_{n=0}^{N-1} x_1(n)\, x_2(m-n)_N$$

$$x_3(n) = x_1(n) \; N \; x_2(n)$$

$$\therefore \text{IDFT}\left[X_1(K) \cdot X_2(K)\right] = x_1(n) \; N \; x_2(n)$$

$$X_1(k) \cdot X_2(k) = \text{DFT}\left[x_1(n) \; N \; x_2(n)\right]$$

$$\therefore x_1(n) \; N \; x_2(n) \xleftrightarrow{\text{DFT}} X_1(k) \cdot X_2(k)$$

4. Time reversal of a sequence:-

Statement:- If $x(n) \xleftrightarrow[N]{\text{DFT}} X(k)$ then

$$x((-n))_N = x(N-n) \xleftrightarrow[N]{\text{DFT}} X((-k))_N = X(N-k)$$

Proof :-

$$DFT\{x(N-n)\} = \sum_{n=0}^{N-1} x(N-n)\, e^{-j2\pi nk/N}$$

$N-n = m$.

$\therefore n = N-m$

$$DFT\{x(m)\} = \sum_{m=0}^{N-1} x(m)\, e^{-j2\pi(N-m)k/N}$$

$$= \sum_{m=0}^{N-1} x(m) \cdot e^{-j2\pi Nk/N} \cdot e^{+j2\pi mk/N}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{-j2\pi k} \cdot e^{j2\pi mk/N}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{j2\pi mk/N} \qquad \left[\because e^{-j2\pi k} = 1\right]$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{j2\pi mk/N} \cdot e^{j2\pi m} \cdot e^{-j2\pi m}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{j2\pi mk/N} \cdot e^{-j2\pi mN/N}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{-j2\pi(N-k)m/N}$$

$$= X(N-k)$$

$$= X((-k))_N$$

5. Circular time shift of a sequence:-

If $$x(n) \xleftrightarrow[N]{DFT} X(k) \text{ then}$$

$$x(n-l)_N \xleftrightarrow[N]{DFT} X(k)e^{-j2\pi kl/N}$$

6. Circular frequency shift:-

If $$x(n) \xleftrightarrow[N]{DFT} X(k) \text{ then}$$

$$x(n)\,e^{j2\pi ln/N} \xleftrightarrow[N]{DFT} X((k-l))_N$$

Fast Fourier Transform:- (FFT)

- For large values of N, DFT becomes tedious because of the huge no of mathematical operations required to perform.
- In general for an N point DFT, N^2 multiplications and $N(N-1)$ additions are required.
- Several algorithms have been developed to reduce the computation burden and ease the implementation of DFT.
- The algorithm developed by Cooley and Tukey in 1965 is the most effecient one and is called Fast Fourier Transform. (FFT)

Radix - 2 FFT Algorithm:-

- For efficient computation of DFT several algorithms have been developed based on divide and conquer methods.
- However the method is applicable for N not being a prime number.
- Consider the case when $N = r_1 r_2 r_3 \dots r_v$

If $r_1 = r_2 = r_3 = \dots = r$, then $N = r^v$. In such a case the DFT's are of size r.
- The number 'r' is called the radix of the

FFT algorithm.

- For performing radix-2 FFT the value of N should be such that $N = 2^m$.

- Here the decimation can be performed 'm' times, where $m = \log_2 N$.

Phase factor or twiddle factor:-

W_N be the complex valued phase factor, which is an N^{th} root of unity expressed by

$$W_N = e^{-j2\pi/N}.$$

$$\therefore \quad X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}, \quad 0 \le k \le N-1$$

&

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-nk}, \quad 0 \le n \le N-1.$$

Phase factor properties:-

Symmetry property :- $W_N^{k+N/2} = -W_N^{k}$

Periodicity property :- $W_N^{k+N} = W_N^{k}$.

FFT algorithms exhibit the above two properties.

Decimation in time algorithm:-

This algorithm is also known as Radix-2 DIT FFT algorithm which means the no of output points N can be expressed as a power of 2, (i) $N = 2^M$, M is an integer.

Let $x(n)$ is an N-point sequence, where N is assumed to be a power of 2.

Break this sequence into two $N/2$ sequences, where one sequence consisting of the even indexed values and the other of odd indexed values. $x_e(n) = x(2n)$
$x_o(n) = x(2n+1)$

The N-point DFT of $x(n)$ can be written as,

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk/N}$$

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} \quad ; \quad k = 0, 1, \ldots N-1. \qquad (1)$$

Breaking $x(n)$ into odd and even sequences.

$$X(k) = \sum_{\substack{n=0 \\ \text{even}}}^{N-1} x(n) W_N^{nk} + \sum_{\substack{n=0 \\ \text{odd}}}^{N-1} x(n) W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{(2n+1)k}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2nk} + W_N^{k} \sum_{n=0}^{N/2-1} x(2n+1) W_N^{2nk}$$

$$X(k) = \sum_{n=0}^{N/2-1} x_e(n) W_N^{2nk} + W_N^{k} \sum_{n=0}^{N/2-1} x_o(n) W_N^{2nk} \qquad (2)$$

$$W_N^2 = \left(e^{-j2\pi/N}\right)^2 = e^{-j2\pi/N/2} = W_{N/2}.$$

$$\therefore X(k) = \underbrace{\sum_{n=0}^{N/2-1} x_e(n) W_{N/2}^{nk}}_{N/2 \text{ pt DFT of even sequence}} + W_N^k \underbrace{\sum_{n=0}^{N/2-1} x_o(n) W_{N/2}^{nk}}_{N/2 \text{ pt DFT of odd sequence.}}$$

$$\therefore X(k) = X_e(k) + W_N^k X_o(k) \quad \text{—— (3)}$$

Now let us take $N=8$. Then $X_e(k)$ and $X_o(k)$ are 4-point DFT's.

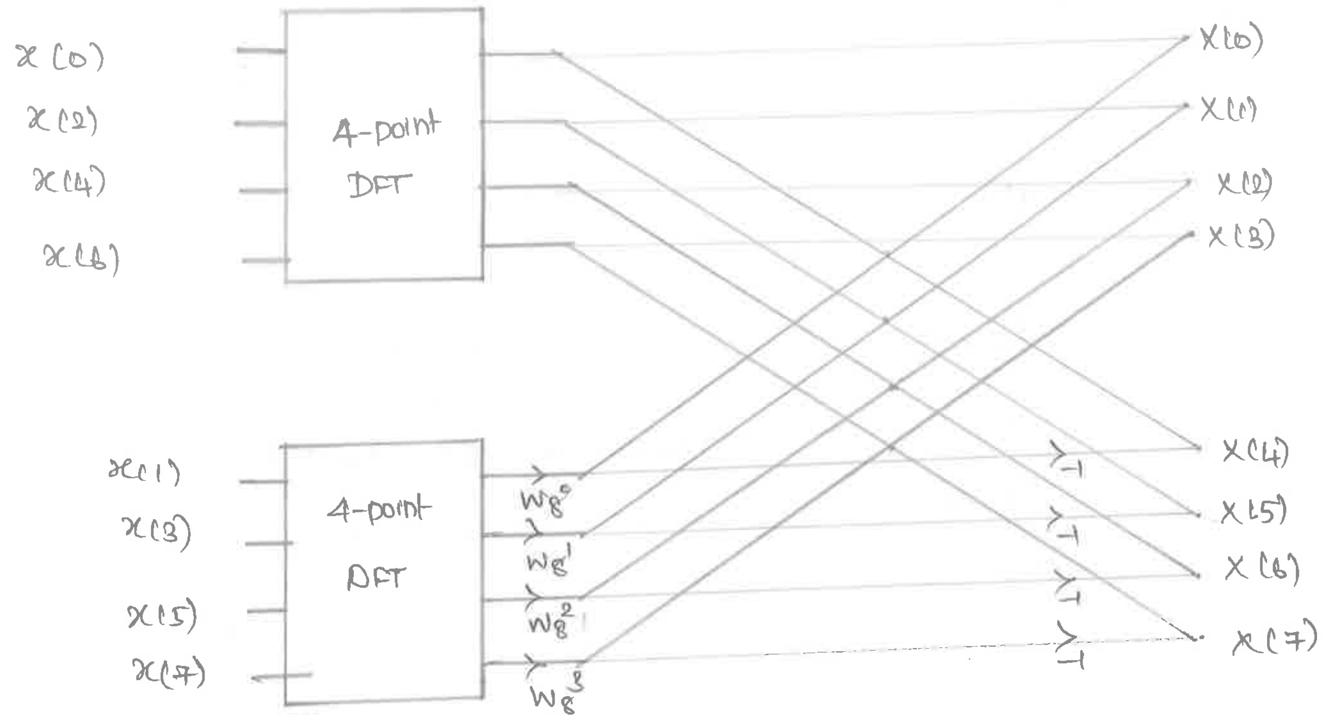
$$\begin{aligned} x_e(0) &= x(0) & x_o(0) &= x(1) \\ x_e(1) &= x(2) & x_o(1) &= x(3) \\ x_e(2) &= x(4) & x_o(2) &= x(5) \\ x_e(3) &= x(6) & x_o(3) &= x(7) \end{aligned}$$

Substituting the values of k.

$$X(k) = X_e(k) + W_8^k X_o(k) \quad \text{for } 0 \le k \le 3.$$

$$X(k) = X_e(k-4) - W_8^k X_o(k-4) \quad \text{for } 4 \le k \le 7.$$

$$\begin{aligned} \therefore X(0) &= X_e(0) + W_8^0 X_o(0) & X(4) &= X_e(0) - W_8^0 X_o(0) \\ X(1) &= X_e(1) + W_8^1 X_o(1) & X(5) &= X_e(1) - W_8^1 X_o(1) \\ X(2) &= X_e(2) + W_8^2 X_o(2) & X(6) &= X_e(2) - W_8^2 X_o(2) \\ X(3) &= X_e(3) + W_8^3 X_o(3) & X(7) &= X_e(3) - W_8^3 X_o(3) \end{aligned}$$



Now we apply the same approach to decompose each of $N/2$ sample DFT.

This can be done by dividing the sequence $x_e(n)$ and $x_o(n)$ into two sequences consisting of even and odd members of the sequences.

The $\frac{N}{2}$ point DFTs can be expressed as $\frac{N}{4}$ point DFT's.

$$\therefore \quad X_e(k) = X_{ee}(k) + W_N^{2k} X_{eo}(k) \quad \text{for } 0 \le k \le N/4 - 1$$

$$X_e(k) = X_{ee}(k - N/4) + W_N^{2(k-N/4)} X_{eo}(k - N/4) \quad \text{for } N/4 \le k \le N/2$$

and

$$X_o(k) = X_{oe}(k) + W_N^{2k} X_{oo}(k) \quad \text{for } 0 \le k \le N/4 - 1$$

$$= X_{oe}(k - N/4) + W_N^{2(k-N/4)} X_{oo}(k - N/4) \quad N/4 \le k \le N/2 - 1$$

For $N=8$.

$x_{ee}(0) = x_e(0)$ $\qquad$ $x_{eo}(0) = x_e(1)$

$x_{ee}(1) = x_e(2)$ $\qquad$ $x_{eo}(1) = x_e(3)$

$\therefore$

$$X_e(0) = X_{ee}(0) + W_8^0 X_{eo}(0)$$

$$X_e(1) = X_{ee}(1) + W_8^2 X_{eo}(1)$$

$$X_e(2) = X_{ee}(0) - W_8^0 X_{eo}(0)$$

$$X_e(3) = X_{ee}(1) - W_8^2 X_{eo}(1)$$

for odd.

$x_{oe}(0) = x_o(0)$ $\qquad$ $x_{oo}(0) = x_o(1)$

$x_{oe}(1) = x_o(2)$ $\qquad$ $x_{oo}(1) = x_o(3)$

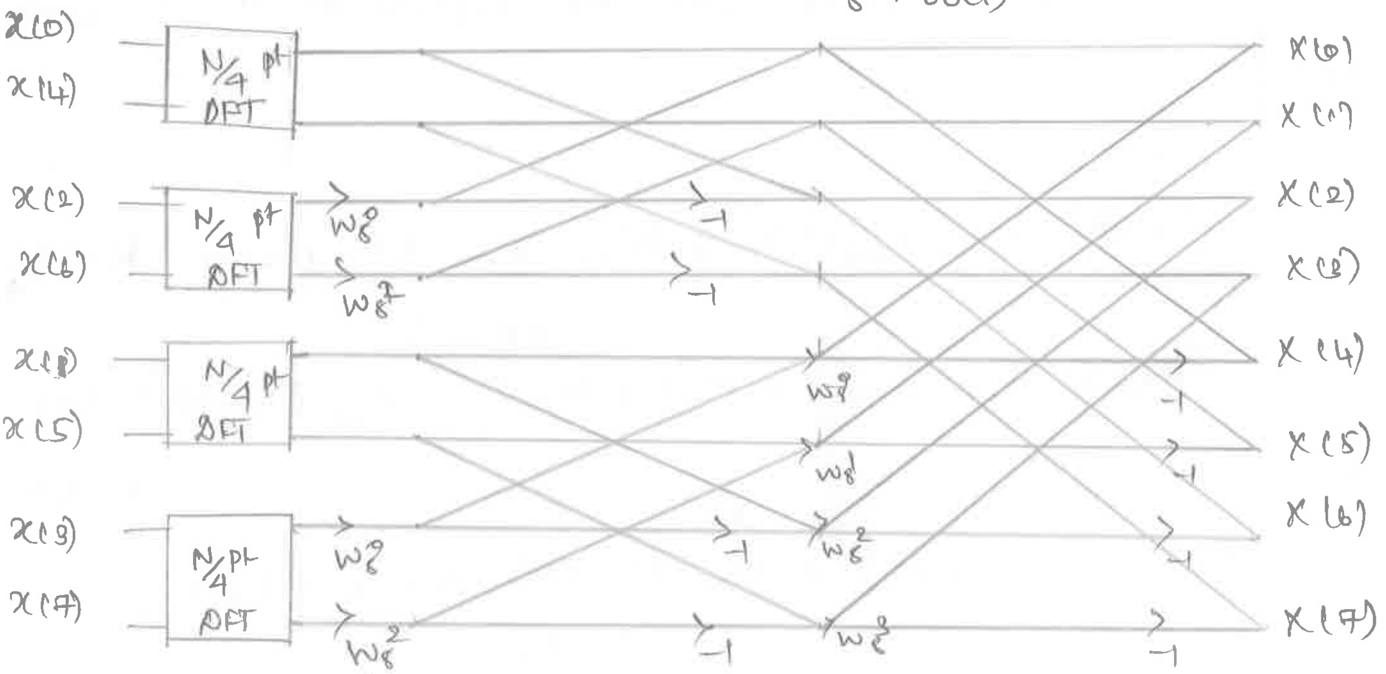
$\therefore$

$$X_o(0) = X_{oe}(0) + W_8^0 X_{oo}(0)$$

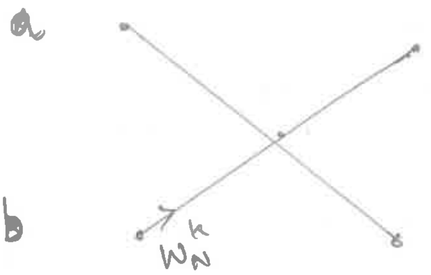
$$X_o(1) = X_{oe}(1) + W_8^2 X_{oo}(1)$$

$$X_o(2) = X_{oe}(0) - W_8^0 X_{oo}(0)$$

$$X_o(3) = X_{oe}(1) - W_8^2 X_{oo}(1)$$



∴ The basic flow graph of DIT algorithm is



Bit reversal:-

In DIT algorithm, the o/p sequence is in a natural order, the input sequence is in a shuffled order.

That shuffled order is called the bit reversal order and can be explained as follows.

i/p sample index.	Binary representation.	Bit reversed binary.	Bit reversed sample Index
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7

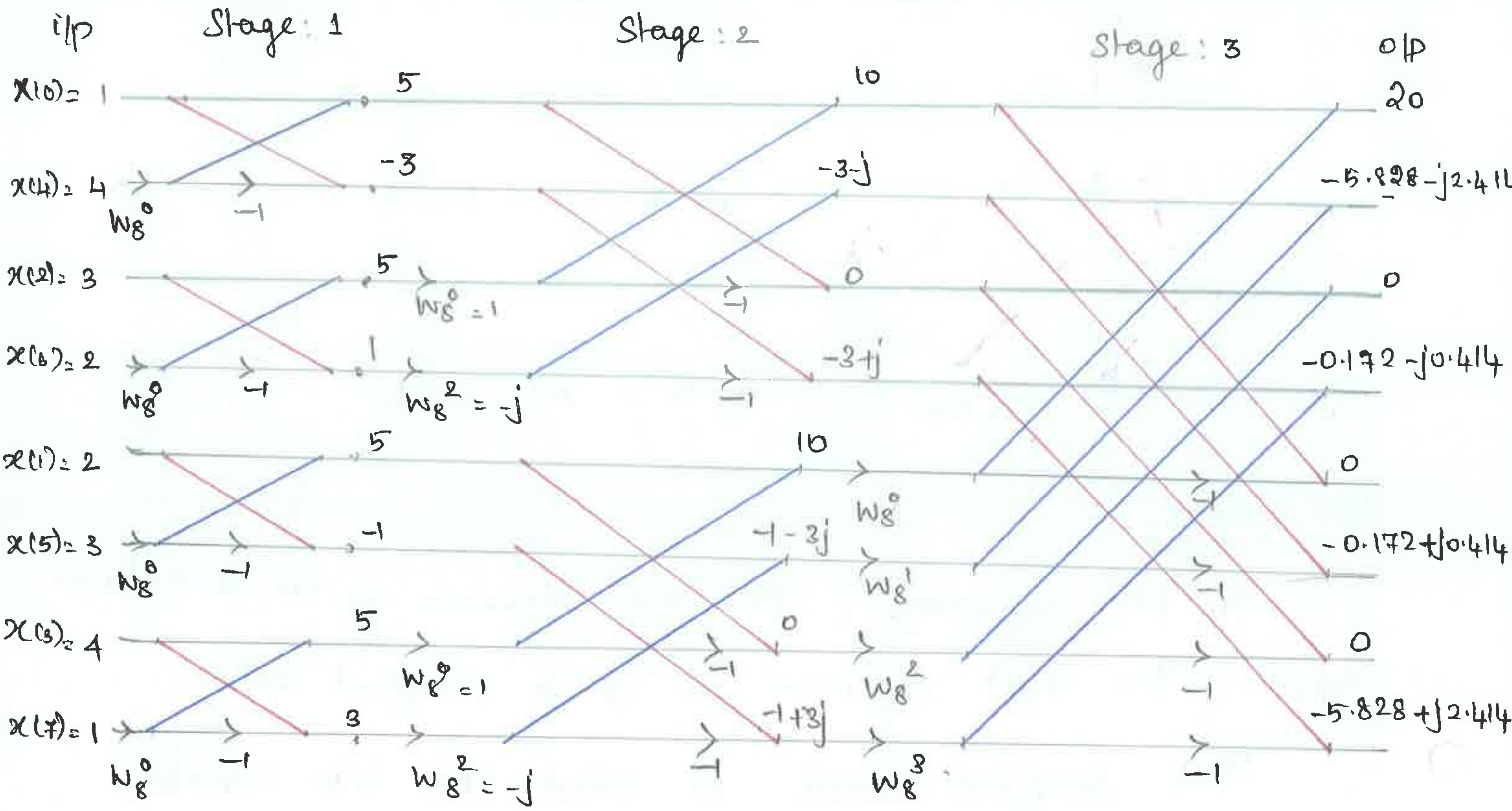
Example:-

1) Find the DFT of a sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIT algorithm:-

The twiddle factors associated with the flow graph are

$W_8^0 = 1$ $\qquad$ $W_8^2 = \left(e^{-j2\pi/8}\right)^2 = e^{-j\pi/2} = -j$

$W_8^1 = 0.707 - j0.707$ $\qquad$ $W_8^3 = \left(e^{-j2\pi/8}\right)^3 = e^{-j3\pi/4} = -0.707 - j0.707$



I/p	O/p of stage 1	O/p of stage 2	O/p of stage 3
1	$1+4=5$	$5+5=10$	$10+10=20$
4	$1-4=-3$	$-3+(-j)(1)=-3-j$	$-3-j+(0.707-j0.707)(-1-3j)$ $=-5.828-j2.414$
3	$3+2=5$	$5-5=0$	0
2	$3-2=1$	$-3-(-j)1=-3+j$	$(-3+j)+(-0.707j0.707)(-1+3j)$ $=-0.172-j0.414$
2	$2+3=5$	$5+5=10$	$10-10=0$
3	$2-3=-1$	$-1+(-j)3=-1-3j$	$(-3-j)-(0.707-j0.707)(-1-3j)$ $=-0.172+j0.414$
4	$4+1=5$	$5-5=0$	0
1	$4-1=3$	$-1-(-j)(3)=-1+3j$	$(-3+j)-(-0.707-j0.707)(-1+3j)$ $=-5.828+j2.414$

Ans:- $X(k) = \{20, -5.828-j2.414, 0, -0.172-j0.414, 0, -0.172+j0.414, 0, -0.528+j2.414\}$.

DIF Algorithm:-

- Decimation in frequency FFT decomposes the DFT by recursively splitting the sequence elements $X(k)$ in the frequency domain into sets of smaller and smaller subsequences.

- To derive the decimation in frequency FFT algorithm for N, a power of 2, the input sequence $x(n)$ is divided into the first half and the last half of the points.

$$X(k) = \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=N/2}^{N-1} x(n) W_N^{nk} \quad \text{——— (1)}$$

$$= \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + \sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right] W_N^{(n+N/2)k}$$

$$= \sum_{n=0}^{N/2-1} x(n) W_N^{nk} + W_N^{N/2\,k} \sum_{n=0}^{N/2-1} x\left[n+\frac{N}{2}\right] W_N^{nk}$$

$$W_N^{(N/2)k} = e^{-j\frac{2\pi}{N}\cdot\frac{N}{2}\cdot k}$$

$$= \cos \pi k - j \sin \pi k = (-1)^k.$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{nk} + (-1)^k \sum_{n=0}^{\frac{N}{2}-1} x\left[n+\frac{N}{2}\right] W_N^{nk}.$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^k x\left[n+\frac{N}{2}\right] \right] W_N^{nk}. \quad \text{——— (2)}$$

Decomposing the sequence in the frequency domain $X(k)$ into an even numbered sequence $X(2r)$ and an odd numbered sequence $X(2r+1)$ yields,

$$X(2r) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^{2r} x\left[n + \frac{N}{2}\right]\right] W_N^{2rn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left[x(n) + (-1)^{2r} x\left[n + \frac{N}{2}\right]\right] W_{N/2}^{rn} \quad \text{——— (3)}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} \left[x(n) + (-1)^{2r+1} x\left[n + N/2\right]\right] W_N^{(2r+1)n}.$$

$$= \sum_{n=0}^{N/2-1} \left[x(n) - x\left[n + N/2\right]\right] W_N^{n} \cdot W_{N/2}^{rn},$$

$$0 \leq r \leq N/2 - 1 \quad \text{——— (4)}$$

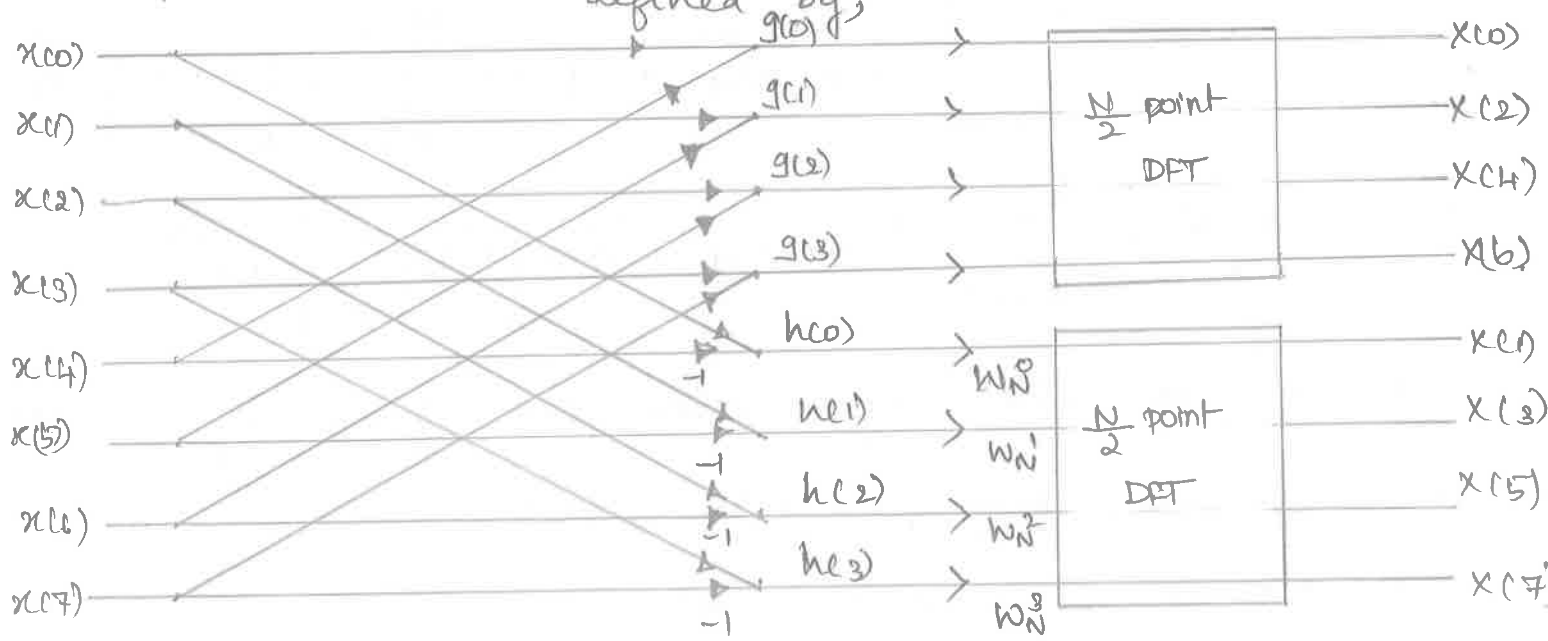
Equations (2) & (4) represent the $N/2$ point DFT's.

Let

$$g(n) = x(n) + x(n + N/2)$$

$$h(n) = x(n) - x(n + N/2)$$

∴ The flow graph for the first stage 8-point computation scheme defined by,



$$\therefore \quad X(2r) = \sum_{n=0}^{\frac{N}{4}-1} g(n)\, W_N^{2rn} + \sum_{n=\frac{N}{4}}^{(\frac{N}{2})-1} g(n)\, W_N^{2rn}$$

$$= \sum_{n=0}^{\frac{N}{4}+1} g(n)\, W_N^{2rn} + \sum_{n=0}^{\frac{N}{4}-1} g(n+N/4)\, W_N^{2r(n+N/4)}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} g(n)\, W_N^{2rn} + W_N^{2rN/4} \sum_{n=0}^{\frac{N}{4}-1} g(n+N/4)\, W_N^{2rn}$$

$$W_N^{N/2} = -1$$

$$\therefore \quad X(2r) = \sum_{n=0}^{\frac{N}{4}-1} g(n)\, W_N^{2rn} + (-1)^r \sum_{n=0}^{\frac{N}{4}-1} g(n+N/4)\, W_N^{2rn}$$

$$X(2r) = \sum_{n=0}^{\frac{N}{4}-1} \left[g(n) + (-1)^r\, g\left(n+N/4\right) \right] W_N^{2rn}$$

where $r = 2l$

$$X(4l) = \sum_{n=0}^{\frac{N}{4}-1} A(n)\, W_N^{4ln}.$$

where $A(n) = g(n) + g(n+N/4)$

$\therefore \quad A(0) = g(0) + g(2)$

$A(1) = g(1) + g(3)$

where $r = 2l+1$

$$X(4l+2) = \sum_{n=0}^{\frac{N}{4}-1} \left[g(n) - g(n+N/4) \right] W_N^{2n(2l+1)}$$

$$= \sum_{n=0}^{\frac{N}{4}-1} [B(n)]\, W_N^{2n}\, W_N^{4ln},$$

$B(0) = g(0) - g(2)$

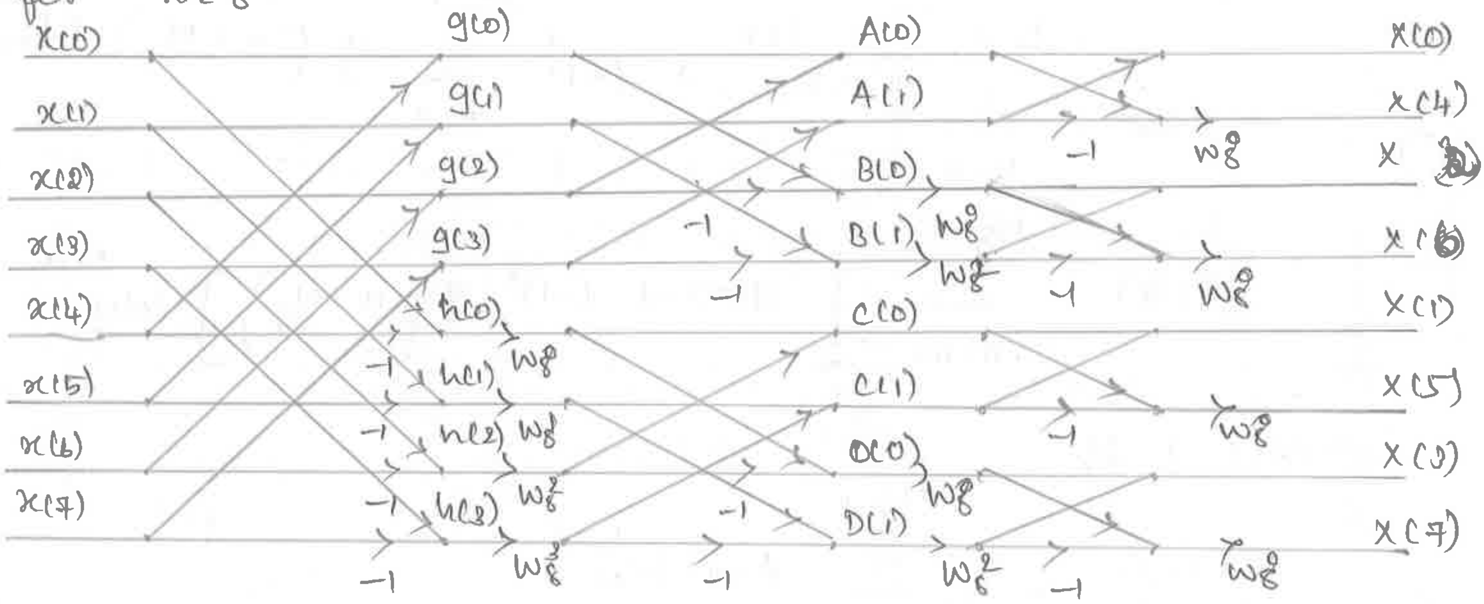
$B(1) = g(1) - g(3)$

Similarly.

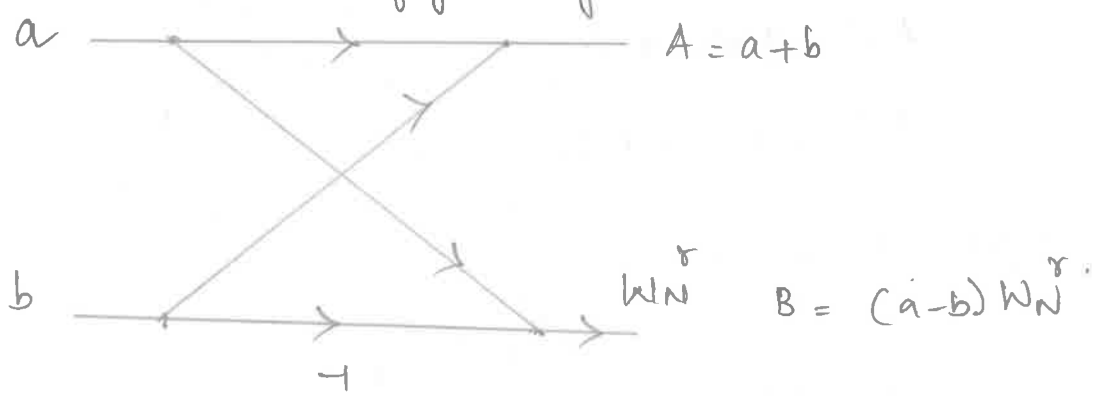
$X(2r+1)$ becomes,

$$\Rightarrow \sum_{n=0}^{N/4-1} \left[h(n) + (-1)^r \, W_{N/2} \, h\left(n+N/2\right) \right] W_N^{(2r+1)n}.$$

∴ The reduced flow graph of final stage DIF FFT for $N=8$.



Basic butterfly diagram for DIF



DIF Algorithm:-

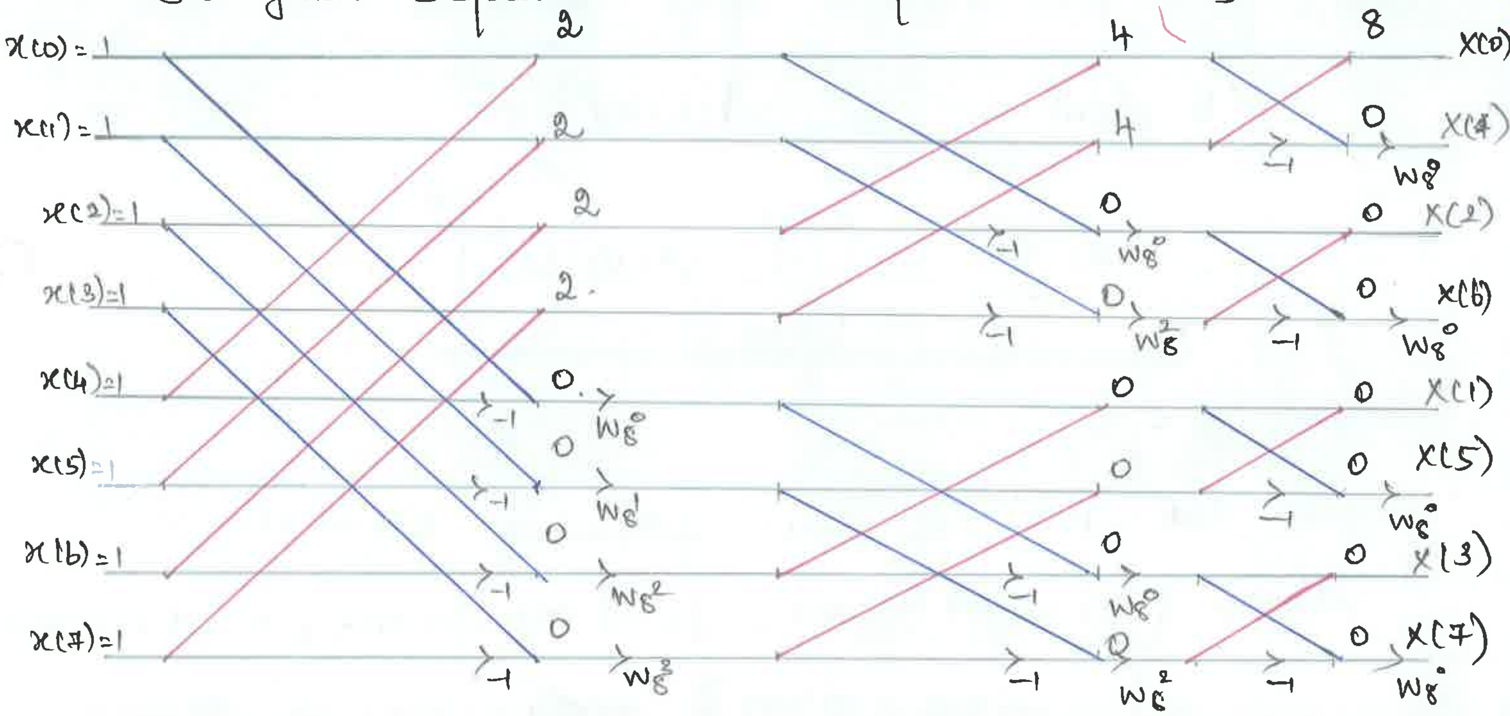
Example:-

1) Compute the eight point DFT of the sequence by using DIF algorithm:-

$$x(n) = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & \text{otherwise} \end{cases}$$

Soln:-

The given sequence $x(n) = \{1, 1, 1, 1, 1, 1, 1, 1\}$



$W_8^0 = 1 \quad W_8^1 = 0.707 - j0.707 \quad W_8^2 = -j \quad W_8^3 = -0.707 - j0.707.$

I/p	o/p of Stage 1	o/p of Stage 2	o/p of Stage 3
1	1+1 = 2	2+2 = 4	4+4 = 8
1	1+1 = 2	2+2 = 4	4-4 = 0
1	1+1 = 2	2-2 = 0	0
1	1+1 = 2	2-2 = 0	0
1	1-1 = 0	0	0
1	1-1 = 0	0	0
1	1-1 = 0	0	0
1	1-1 = 0	0	0

$X(k) = \{8, 0, 0, 0, 0, 0, 0, 0\}$

IDFT using FFT algorithm:-

The inverse DFT of an N- point sequence $X(k)$, is defined as,

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-nk}$$

Take complex conjugate and multiply by N we get,

$$N x^*(n) = \sum_{k=0}^{N-1} X^*(k) W^{nk}$$

$$\therefore x(n) = \frac{1}{N} \left[\sum_{k=0}^{N-1} X^*(k) W_N^{nk} \right]^*$$

Examples:-

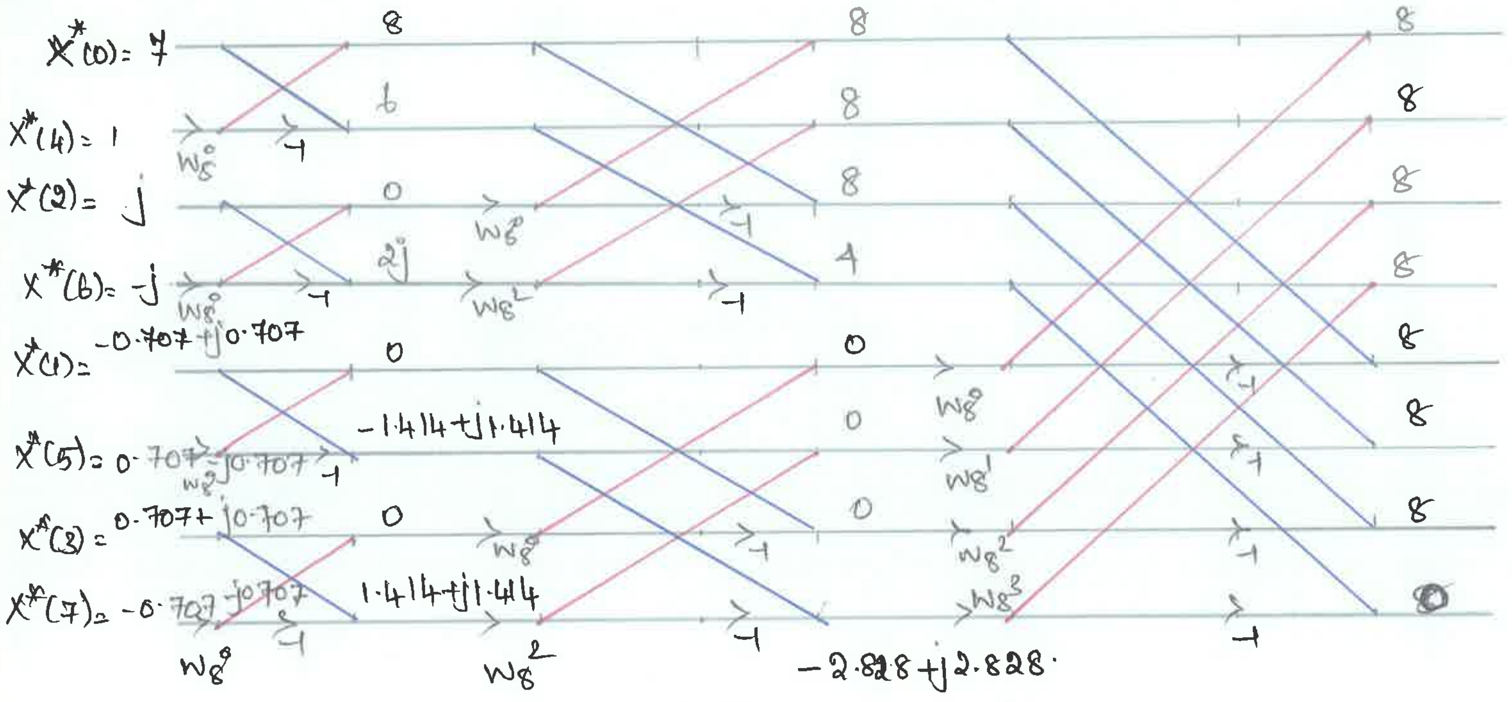
Compute the IDFT of the following sequence.

$X(k) = \{ 7, -0.707 - j0.707, -j, 0.707 - j0.707, 0.707 + j0.707, j, -0.707 + j0.707 \}$ using DIT algorithm:-

Soln:-

The complex conjugate of $X(k)$ is given as

$X^*(k) = \{ 7, -0.707 + j0.707, +j, 0.707 + j0.707, 0.707 - j0.707, -j, -0.707 - j0.707 \}$



o/p :- $N x^*(n) = \{8, 8, 8, 8, 8, 8, 8, 0\}$

$x(n) = \{1, 1, 1, 1, 1, 1, 1, 0\}$

Exercise :-

Compute the IDFT for the above example, using DIF algorithm:.

Differences and similarities between DIT and DIF algorithms:.

DIT	DIF
The i/p is bitreversed, the o/p o/p is in normal order.	The i/p is in normal order The o/p is bitreversed.
The complex multiplication takes place after the add-subtract operation.	The complex multiplication takes place after the add-subtract operation.

Circular convolution:-

The methods to find circular convolution of two sequences are

i) Concentric circle method ii) Matrix multiplication method.

Concentric circle method:-

Given two sequences $x_1(n)$ & $x_2(n)$

i) Graph N samples of $x_1(n)$ as equally spaced points around an outer circle in counter clockwise direction.

ii) Start at the same point as $x_1(n)$ graph N samples of $x_2(n)$ as equally spaced points around an inner circle in clockwise direction.

iii) Multiply corresponding samples on the two circles and sum the products to produce output.

iv) Rotate the inner circle one sample at a time in counterclockwise direction and go to step 3 to obtain the next value of output.

v) Repeat step 4 until the inner circle first sample lines up with the first sample of the exterior circle once again.

Matrix Multiplication method:-

$$x_3(n) = x_1(n) \circledN x_2(n)$$

In matrix form,

$$\begin{bmatrix} x_2(0) & x_2(N-1) & x_2(N-2) & \cdots & x_2(1) \\ x_2(1) & x_2(0) & x_2(N-1) & & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & & \\ \vdots & \vdots & \vdots & & \vdots \\ x_2(N-2) & & & & x_2(N-1) \\ x_2(N-1) & x_2(N-2 & x_2(N-3) & & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_2(0) \\ \vdots \\ x_1(N-1) \end{bmatrix} = \begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ \vdots \\ x_3(N-1) \end{bmatrix}$$

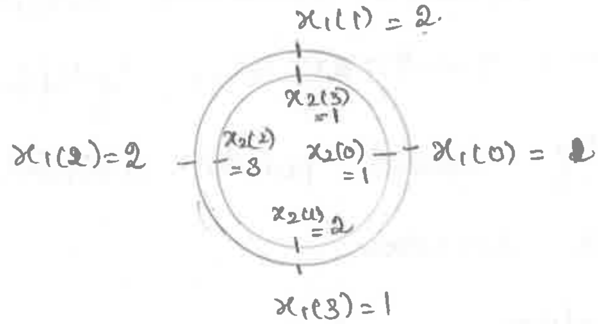
Problems:-

1) Find the circular convolution of the two sequences $x_1(n) = \{1, 2, 2, 1\}$ and $x_2(n) = \{1, 2, 3, 1\}$ using

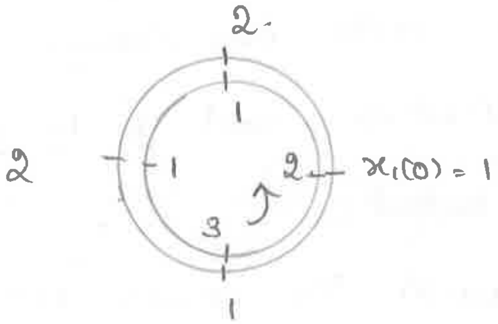
a) concentric circle method. b) matrix multiplication method?

Soln:-

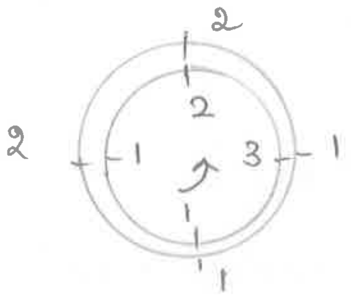
a) concentric circle:-



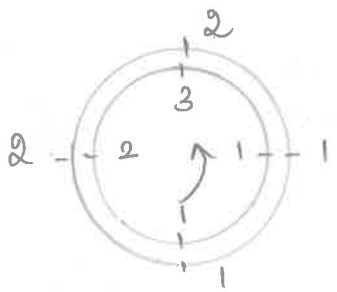
$y(0) = 1(1) + 2(1) + 2(3) + 1(2) = 11$



$y(1) = 1 \times 2 + 1 \times 2 + 1 \times 2 + 1 \times 3 = 9$



$y(2) = 1 \times 3 + 2 \times 2 + 2 \times 1 + 1 \times 1 = 10$



$y(3) = 1 \times 1 + 3 \times 2 + 2 \times 2 + 1 \times 1 = 12$

$\therefore \; y(n) = \{11, 9, 10, 12\}$

b) Matrix method:-

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+6+2 \\ 2+2+2+3 \\ 3+4+2+1 \\ 1+6+4+1 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 10 \\ 12 \end{bmatrix}$$

$y(n) = \{11, 9, 10, 12\}$

Exercise:-

1) Perform circular convolution for the following sequences:-

i) $x_1(n) = \{1, 2, 3, 1\}$ $x_2(n) = \{4, 3, 2, 2\}$.

ii) $x(n) = \{1, 1, 1, 2\}$; $y(n) = \{1, 2, 3, 2\}$

Linear convolution from circular convolution:-

The duration of sequence $x(n)$ is L samples and that of $h(n)$ is M samples.

The resultant sequence $y(n)$ is of $L+M-1$ no of samples.

To get the linear convolution result from circular convolution append $M-1$ no of zeros to $x(n)$ and $L-1$ no of zeros to $h(n)$.

Problems:-

1) Determine the o/p response $y(n)$ if $h(n) = \{1, 1, 1\}$ $x(n) = \{1, 2, 3, 1\}$ using circular convolution with zero padding:-

Soln:-

$L = 4$

$M = 3$.

M-1 no of zeros to $x(n)$

$\therefore x(n) = \{1, 2, 3, 1, 0, 0\}$

L-1 no of zeros to $h(n)$

$\therefore h(n) = \{1, 1, 1, 0, 0, 0\}$.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 3 & 2 \\ 2 & 1 & 0 & 0 & 1 & 3 \\ 3 & 2 & 1 & 0 & 0 & 1 \\ 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 2 & 1 & 0 \\ 0 & 0 & 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 6 \\ 4 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore y(n) = \{1, 3, 6, 6, 4, 1\}$.

Using linear convolution.

	1	1	1
1	1	1	1
2	2	2	2
3	3	3	3
1	1	1	1

$y(n) = \{1, 3, 6, 6, 4, 1\}$

Filtering long duration sequences:-

* Suppose an input sequence of long duration is to be processed with a system having impulse response of finite duration by convolving the two sequences.

* Because of the length of the input sequence, it would not be practical to store it all before performing linear convolution.

* Therefore the i/p sequence must be divided into

blocks, one at a time processing takes place.

* 2 methods that are commonly used for filtering the sectioned data and combining the results:

i) overlap-save

ii) overlap-add.

Overlap save method:-

- Let the length of the i/p sequence be L_s and the length of the impulse response is M.
- The i/p sequence is divided into $N = L + M - 1$
- Each block consists of last M-1 data points of previous block followed by L new data points to form a data sequence.
- For first block of data the first M-1 points are set to zero.
- In the resultant convoluted sequence first M-1 points will not agree with the linear convolution of $x_i(n)$ and $h(n)$ because of aliasing, Hence we discard the first M-1 points of the filtered section.

Examples:-

1. Find the output $y(n)$ of a filter whose impulse response is $h(n) = \{1, 1, 1\}$ and input signal $x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$ using overlap save method.

The input sequence can be divided into blocks as follows.

1) Let us assume the input sequence length L after dividing into subblocks is 3.

$$\therefore N = L + M - 1$$
$$= 3 + 3 - 1$$
$$N = 5$$

2) Last M-1 datapoints from the previous data block must be added.

$\therefore x_1(n) = \{\underbrace{0, 0}_{3-1=2 \text{ zeros}}, \underbrace{3, -1, 0}_{L=3 \text{ datapoints}}\}$.

$x_2(n) = \{\underbrace{-1, 0}_{2 \text{ last data points}}, 1, 3, 2\}$

$x_3(n) = \{3, 2, 0, 1, 2\}$

$x_4(n) = \{1, 2, 1, 0, 0\}$.

& $h(n) = \{1, 1, 1\}$

padding zeros $h(n) = \{1, 1, 1, 0, 0\}$.

3) Circular convolution

$y_1(n) = x_1(n) \; N \; h(n) = \{-1, 0, 3, 2, 2\}$.

$y_2(n) = x_2(n) \; N \; h(n) = \{4, 1, 0, 4, 6\}$

$y_3(n) = x_3(n) \; N \; h(n) = \{6, 7, 5, 3, 3\}$.

$y_4(n) = x_4(n) \; N \; h(n) = \{1, 3, 4, 3, 1\}$

4) discarding first M-1 points i.e. 2 points from the previous section we get

$\underbrace{-1, 0}_{\text{discard}}, 3, 2, 2.$

$\underbrace{4, 1}_{\text{discard}}, 0, 4, 6$

$\underbrace{6, 7}_{\text{discard}}, 5, 3, 3$

$\underbrace{1, 3}_{\text{discard}}, 4, 3, 1$

$\therefore y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}.$

Overlap add method:-

1) Let the length of the sequence be L_s and length of the impulse response is M.

2) The sequence is divided into blocks of datasize having length L and M-1 zeros are appended to it to make the datasize of L+M-1.

3) L-1 zeros are added to the impulse response h(n).

4) The last M-1 points from each o/p block must be overlapped and added to the first M-1 points of the succeeding block.

Example:-

Repeat the last example using overlap add method.

Soln:

Let the length of the datablock L = 3.

M = 3.

N = L+M-1 = 5

Add 2 zeros at the end of each block.

$x_1(n) = \{3, -1, 0, 0, 0\}$

$x_2(n) = \{1, 3, 2, 0, 0\}$

$x_3(n) = \{0, 1, 2, 0, 0\}$

$x_4(n) = \{1, 0, 0, 0, 0\}$

$y_1(n) = x_1(n) \,Ⓝ\, h(n) = \{3, 2, 2, -1, 0\}$

$y_2(n) = x_2(n) \,Ⓝ\, h(n) = \{1, 4, 6, 5, 2\}$

$y_3(n) = x_3(n) \,Ⓝ\, h(n) = \{0, 1, 3, 3, 2\}$

$y_4(n) = x_4(n) \,Ⓝ\, h(n) = \{1, 1, 1, 0, 0\}$

```
3  2  2  -1  0   add
          1  4   6  5  2
                    0  1   3  3  2
                              1  1   1  0  0
```

$y(n) = \{3, 2, 2, 0, 4, 6, 5, 3, 3, 4, 3, 1\}$

<u>Exercise:-</u>

1. Using linear convolution find $y(n)$ for the sequences $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$ and $h(n) = (1, 2)$. Compare the result by solving the problem using overlap add and overlap save method.

Ans: $\{1, 4, 3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2\}$