UNIT-IV NOTES

UNIT IV ASSOCIATION RULE MINING AND CLASSIFICATION 11
Mining Frequent Patterns, Associations and Correlations – Mining Methods – Mining Various Kinds of Association Rules – Correlation Analysis – Constraint Based Association Mining – Classification and Prediction – Basic Concepts – Decision Tree Induction – Bayesian Classification – Rule Based Classification – Classification by Backpropagation – Support Vector Machines – Associative Classification – Lazy Learners – Other Classification Methods – Prediction

Basic Concepts

Frequent pattern mining searches for recurring relationships in a given data set. It introduces the basic concepts of frequent pattern mining for the discovery of interesting associations and correlations between itemsets in transactional and relational databases.

Market Basket Analysis: A Motivating Example

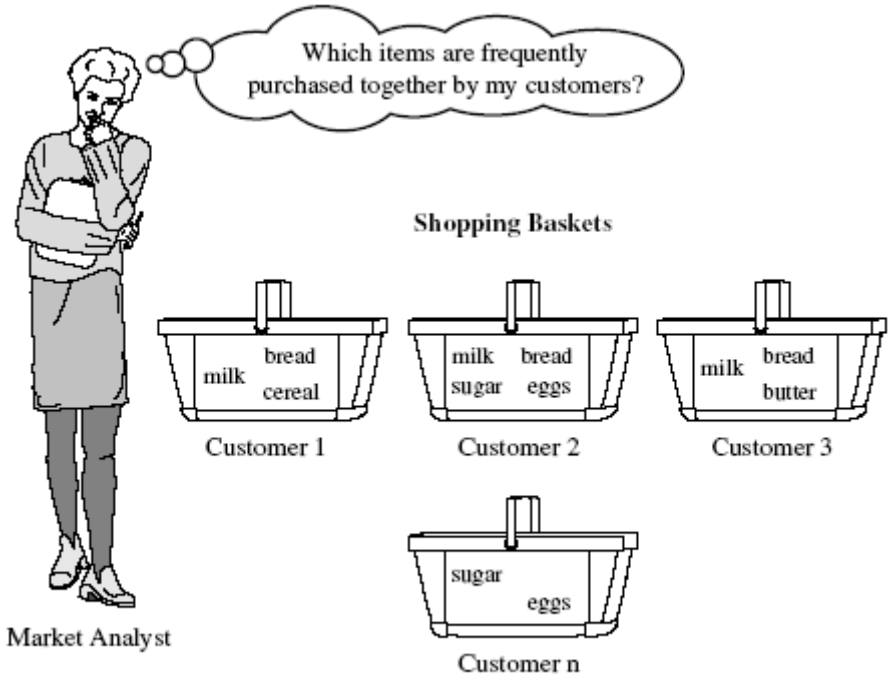


Figure 5.1 Market basket analysis.

A typical example of frequent itemset mining is market basket analysis. This process analyzes customer buying habits by finding associations between the different items that customers place in their "shopping baskets" (Figure 5.1). The discovery of such associations can help retailers develop marketing

strategies by gaining insight into which items are frequently purchased together by customers. For instance, if customers are buying milk, how likely are they to also buy bread (and what kind of bread) on the same trip to the supermarket? Such information can lead to increased sales by helping retailers do selective marketing and plan their shelf space.

If we think of the universe as the set of items available at the store, then each item has a Boolean variable representing the presence or absence of that item. Each basket can then be represented by a Boolean vector of values assigned to these variables. The Boolean vectors can be analyzed for buying patterns that reflect items that are frequently *associated* or purchased together. These patterns can be represented in the form of association rules. For example, the information that customers who purchase computers also tend to buy antivirus software at the same time is represented in Association Rule (5.1) below:

$$Computer \Rightarrow antivirus\ software\ [support = 2\%;\ confidence = 60\%] \qquad (5.1)$$

Rule support and confidence are two measures of rule interestingness. They respectively reflect the usefulness and certainty of discovered rules. A support of 2% for Association Rule (5.1) means that 2% of all the transactions under analysis show that computer and antivirus software are purchased together. A confidence of 60% means that 60% of the customers who purchased a computer also bought the software. Typically, association rules are considered interesting if they satisfy both a minimum support threshold and a minimum confidence threshold. Such thresholds can be set by users or domain experts. Additional analysis can be performed to uncover interesting statistical correlations between associated items.

Frequent Itemsets, Closed Itemsets, and Association Rules

- A set of items is referred to as an **itemset.**
- An itemset that contains *k* items is a ***k*-itemset**.
- The set {*computer, antivirus software*} is a **2-itemset**.
- The occurrence frequency of an itemset is the number of transactions that contain the itemset. This is also known, simply, as the **frequency, support count,** or **count** of the itemset.

$$\begin{aligned} support(A \Rightarrow B) &= P(A \cup B) \\ confidence(A \Rightarrow B) &= P(B|A). \end{aligned}$$

$$confidence(A \Rightarrow B) = P(B|A) = \frac{support(A \cup B)}{support(A)} = \frac{support\_count(A \cup B)}{support\_count(A)}.$$

- Rules that satisfy both a minimum support threshold (*min sup*) and a minimum confidence threshold (*min conf*) are called **Strong Association Rules.**

In general, association rule mining can be viewed as a **two-step process**:

1. Find all frequent itemsets: By definition, each of these itemsets will occur at least as frequently as a predetermined minimum support count, *min_sup*.

2. Generate strong association rules from the frequent itemsets: By definition, these rules must satisfy minimum support and minimum confidence.

Association Rules

- Implication: X ➔ Y where X,Y ⊆ I and X ∩Y = ∅;
- Support of AR (s) X ➔ Y:
  - Percentage of transactions that contain X ∪ Y
  - Probability that a transaction contains X∪Y.
- Confidence of AR (a) X ➔ Y:
  - Ratio of number of transactions that contain X ∪ Y to the number that contain X
  - Conditional probability that a transaction having X also contains *Y*.

- Example:

Transaction	Items
t_1	Bread,Jelly,PeanutButter
t_2	Bread,PeanutButter
t_3	Bread,Milk,PeanutButter
t_4	Beer,Bread
t_5	Beer,Milk

X➔ Y	Support	Confidence
Bread ➔ Peanutbutter	= 3/5 %= 60%	= (3/5)/(4/5)%=75%
Peanutbutter ➔ Bread	60%	= (3/5)/(3/5)%=100%
Jelly ➔ Milk	0%	0%
Jelly ➔ Peanutbutter	=1/5 % = 20%	= (1/5)/(1/5) % = 100%

Apriori algorithm:

- It is used to mine Boolean, single-level, and single-dimension ARs.
- Apriori Principle
- Uses prior knowledge of frequent itemset properties.
  - It is an iterative algorithm known as level-wise search.

- The search proceeds level-by-level as follows:
  - First determine the set of frequent 1-itemset; L1
  - Second determine the set of frequent 2-itemset using L1: L2
  - Etc.

- The complexity of computing Li is O(n) where n is the number of transactions in the transaction database.

- Reduction of search space:
  - In the worst case what is the number of itemsets in a level Li?
  - Apriori uses "**Apriori Property**":

- **Apriori Property**:
  - It is an **anti-monotone** property: if a set cannot pass a test, all of its supersets will fail the same test as well.
  - It is called **anti-monotone** because the property is monotonic in the context of failing a test.
  - All nonempty subsets of a frequent itemset must also be frequent.
  - An itemset I is not frequent if it does not satisfy the minimum support threshold:

$$P(I) < min\_sup$$

  - If an item A is added to the itemset I, then the resulting itemset $I \cup A$ cannot occur more frequently than I:

$$I \cup A \text{ is not frequent}$$

$$\text{Therefore, } P(I \cup A) < min\_sup$$

- How Apriori algorithm uses "Apriori property"?
  - In the computation of the itemsets in L_k using L_{k-1}
  - It is done in two steps:
    - Join
    - Prune

1.1. Join Step:

- The set of candidate k-itemsets (element of L_k), C_k, is generated by joining L_{k-1} with itself:

$$L_{k-1} \infty L_{k-1}$$

- Given l_1 and l_2 of L_{k-1}

$$L_i=l_{i1},l_{i2},l_{i3},\ldots,l_{i(k-2)},l_{i(k-1)}$$
$$L_j=l_{j1},l_{j2},l_{j3},\ldots,l_{j(k-2)},l_{j(k-1)}$$

Where L_i and L_j are sorted.

- L_i and L_j are joined if there are different (no duplicate generation). Assume the following:

$$l_{i1}=l_{j1},\ l_{i2}=l_{j1},\ \ldots,\ l_{i(k-2)}=l_{j(k-2)} \text{ and } l_{i(k-1)} < l_{j(k-1)}$$

- The resulting itemset is:

$$l_{i1},l_{i2},l_{i3},\ldots,l_{i(k-1)},l_{j(k-1)}$$

- Example of Candidate-generation:

L3={abc, abd, acd, ace, bcd}

Self-joining: L3 ∞ L3

abcd from ab**c** and ab**d**
acde from ac**d** and ac**e**

1.2. Prune step

- C_k is a superset of L_k ➔ some itemset in C_k may or may not be frequent.
- L_k: Test each generated itemset against the database:
  - Scan the database to determine the count of each generated itemset and include those that have a count no less than the minimum support count.
  - This may require intensive computation.

- Use Apriori property to reduce the search space:
  - Any (k-1)-itemset that is not frequent cannot be a subset of a frequent k-itemset.
  - Remove from C_k any k-itemset that has a (k-1)-subset not in L_{k-1} (itemsets that are not frequent)
  - Efficiently implemented: maintain a hash table of all frequent itemset.

- Example of Candidate-generation and Pruning:

L3={abc, abd, acd, ace, bcd}

Self-joining: L3 ∞ L3

abcd from ab**c** and ab**d**
acde from ac**d** and ac**e**

Pruning:

acde is removed because ade is not in L_3

C_4={abcd}

1.3. Pseudo-code

C_k: Candidate itemset of size k
L_k : frequent itemset of size k
L_1 = {frequent items};
for (k = 1; L_k !=Æ; k++) **do**
- C_{k+1} = candidates generated from L_k;
- **for each** transaction t in database **do**
  increment the count of all candidates in C_{k+1} that are contained in t;
  endfor;
- L_{k+1} = candidates in C_{k+1} with min_support

endfor;
return $\cup_k L_k$;

1.4. Challenges

- Multiple scans of transaction database
- Huge number of candidates
- Tedious workload of support counting for candidates
- Improving Apriori:
  - general ideas
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates
  - Easily parallelized

Apriori is a seminal algorithm proposed by R. Agrawal and R. Srikant in 1994 for mining frequent itemsets for Boolean association rules. The name of the algorithm is based on the fact that the algorithm uses *prior knowledge* of frequent itemset properties, as we shall see following. Apriori employs an iterative approach known as a *level-wise* search, where *k*-itemsets are used to explore (*k*+1)-itemsets. First, the set of frequent 1-itemsets is found by scanning the database to accumulate the count for each item, and collecting those items that satisfy minimum support. The resulting set is denoted *L*1.Next, *L*1 is used to find *L*2, the set of frequent 2-itemsets, which is used to find *L*3, and so on, until no more frequent *k*-itemsets can be found. The finding of each *Lk* requires one full scan of the database.

To improve the efficiency of the level-wise generation of frequent itemsets, an important property called the Apriori property, presented below, is used to reduce the search space. We will first describe this property, and then show an example illustrating its use.

Apriori property: *All nonempty subsets of a frequent itemset must also be frequent.*

A two-step process is followed, consisting of join and prune actions

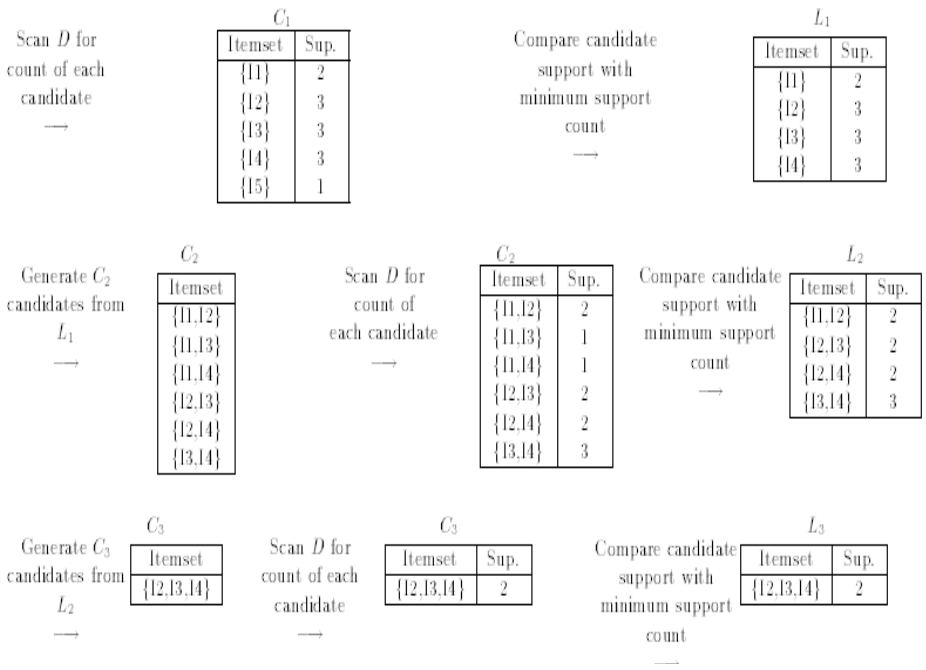
1. **The join step**: To find L_k, a set of **candidate** k-itemsets is generated by joining L_{k-1} with itself. This set of candidates is denoted C_k. Let l_1 and l_2 be itemsets in L_{k-1}. The notation $l_i[j]$ refers to the jth item in l_i (e.g., $l_1[k-2]$ refers to the second to the last item in l_1). By convention, Apriori assumes that items within a transaction or itemset are sorted in lexicographic order. For the $(k-1)$-itemset, l_i, this means that the items are sorted such that $l_i[1] < l_i[2] < \ldots < l_i[k-1]$. The join, $L_{k-1} \bowtie L_{k-1}$, is performed, where members of L_{k-1} are joinable if their first $(k-2)$ items are in common. That is, members l_1 and l_2 of L_{k-1} are joined if $(l_1[1] = l_2[1]) \wedge (l_1[2] = l_2[2]) \wedge \ldots \wedge (l_1[k-2] = l_2[k-2]) \wedge (l_1[k-1] < l_2[k-1])$. The condition $l_1[k-1] < l_2[k-1]$ simply ensures that no duplicates are generated. The resulting itemset formed by joining l_1 and l_2 is $l_1[1], l_1[2], \ldots, l_1[k-2], l_1[k-1], l_2[k-1]$.
2. **The prune step**: C_k is a superset of L_k, that is, its members may or may not be frequent, but all of the frequent k-itemsets are included in C_k. A scan of the database to determine the count of each candidate in C_k would result in the determination of L_k (i.e., all candidates having a count no less than the minimum support count are frequent by definition, and therefore belong to L_k). C_k, however, can be huge, and so this could involve heavy computation. To reduce the size of C_k, the Apriori property is used as follows. Any $(k-1)$-itemset that is not frequent cannot be a subset of a frequent k-itemset. Hence, if any $(k-1)$-subset of a candidate k-itemset is not in L_{k-1}, then the candidate cannot be frequent either and so can be removed from C_k. This **subset testing** can be done quickly by maintaining a hash tree of all frequent itemsets.

Table 5.1 Transactional data for an *AllElectronics* branch.

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Example 5.3 **Apriori.** Let's look at a concrete example, based on the *AllElectronics* transaction database, D, of Table 5.1. There are nine transactions in this database, that is, $|D| = 9$. We use Figure 5.2 to illustrate the Apriori algorithm for finding frequent itemsets in D.

1. In the first iteration of the algorithm, each item is a member of the set of candidate 1-itemsets, C_1. The algorithm simply scans all of the transactions in order to count the number of occurrences of each item.
2. Suppose that the minimum support count required is 2, that is, $min\_sup = 2$. (Here, we are referring to *absolute* support because we are using a support count. The corresponding relative support is 2/9 = 22%). The set of frequent 1-itemsets, L_1, can then be determined. It consists of the candidate 1-itemsets satisfying minimum support. In our example, all of the candidates in C_1 satisfy minimum support.
3. To discover the set of frequent 2-itemsets, L_2, the algorithm uses the join $L_1 \bowtie L_1$ to generate a candidate set of 2-itemsets, C_2.[8] C_2 consists of $\binom{|L_1|}{2}$ 2-itemsets. Note that no candidates are removed from C_2 during the prune step because each subset of the candidates is also frequent.
4. Next, the transactions in D are scanned and the support count of each candidate itemset in C_2 is accumulated, as shown in the middle table of the second row in Figure 5.2.
5. The set of frequent 2-itemsets, L_2, is then determined, consisting of those candidate 2-itemsets in C_2 having minimum support.
6. The generation of the set of candidate 3-itemsets, C_3, is detailed in Figure 5.3. From the join step, we first get $C_3 = L_2 \bowtie L_2 = \{\{I1, I2, I3\}, \{I1, I2, I5\}, \{I1, I3, I5\}, \{I2, I3, I4\}, \{I2, I3, I5\}, \{I2, I4, I5\}\}$. Based on the Apriori property that all subsets of a frequent itemset must also be frequent, we can determine that the four latter candidates cannot possibly be frequent. We therefore remove them from C_3, thereby saving the effort of unnecessarily obtaining their counts during the subsequent scan of D to determine L_3. Note that when given a candidate k-itemset, we only need to check if its $(k-1)$-subsets are frequent since the Apriori algorithm uses a level-wise search strategy. The resulting pruned version of C_3 is shown in the first table of the bottom row of Figure 5.2.
7. The transactions in D are scanned in order to determine L_3, consisting of those candidate 3-itemsets in C_3 having minimum support (Figure 5.2).
8. The algorithm uses $L_3 \bowtie L_3$ to generate a candidate set of 4-itemsets, C_4. Although the join results in $\{\{I1, I2, I3, I5\}\}$, this itemset is pruned because its subset $\{\{I2, I3, I5\}\}$ is not frequent. Thus, $C_4 = \phi$, and the algorithm terminates, having found all of the frequent itemsets. ■



Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
  - To find frequent itemset $i_1 i_2 \ldots i_{100}$
    - # of scans: 100
    - # of Candidates: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100}-1 = 1.27*10^{30}$!
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?

Algorithm: Apriori. Find frequent itemsets using an iterative level-wise approach based on candidate generation.

Input:

- D, a database of transactions;
- min_sup, the minimum support count threshold.

Output: L, frequent itemsets in D.

Method:

```
(1)   L1 = find_frequent_1-itemsets(D);
(2)   for (k = 2; Lk−1 ≠ ϕ; k++) {
(3)       Ck = apriori_gen(Lk−1);
(4)       for each transaction t ∈ D { // scan D for counts
(5)           Ct = subset(Ck, t); // get the subsets of t that are candidates
(6)           for each candidate c ∈ Ct
(7)               c.count++;
(8)       }
(9)       Lk = {c ∈ Ck | c.count ≥ min_sup}
(10)  }
(11)  return L = ∪k Lk;

procedure apriori_gen(Lk−1:frequent (k − 1)-itemsets)
(1)   for each itemset l1 ∈ Lk−1
(2)       for each itemset l2 ∈ Lk−1
(3)           if (l1[1] = l2[1]) ∧ (l1[2] = l2[2]) ∧ ... ∧ (l1[k − 2] = l2[k − 2]) ∧ (l1[k − 1] < l2[k − 1]) then {
(4)               c = l1 ⋈ l2; // join step: generate candidates
(5)               if has_infrequent_subset(c, Lk−1) then
(6)                   delete c; // prune step: remove unfruitful candidate
(7)               else add c to Ck;
(8)           }
(9)   return Ck;

procedure has_infrequent_subset(c: candidate k-itemset;
          Lk−1: frequent (k − 1)-itemsets); // use prior knowledge
(1)   for each (k − 1)-subset s of c
(2)       if s ∉ Lk−1 then
(3)           return TRUE;
(4)   return FALSE;
```

Generating Association Rules from Frequent Itemsets

Once the frequent itemsets from transactions in a database D have been found, it is straightforward to generate strong association rules from them (where *strong* association rules satisfy both minimum support and minimum confidence). This can be done using Equation (5.4) for confidence, which we show again here for completeness:

$$confidence(A \Rightarrow B) = P(B|A) = \frac{support\_count(A \cup B)}{support\_count(A)}.$$

The conditional probability is expressed in terms of itemset support count, where $support\_count(A \cup B)$ is the number of transactions containing the itemsets $A \cup B$, and $support\_count(A)$ is the number of transactions containing the itemset A. Based on this equation, association rules can be generated as follows:

- For each frequent itemset l, generate all nonempty subsets of l.
- For every nonempty subset s of l, output the rule "$s \Rightarrow (l - s)$" if $\frac{support\_count(l)}{support\_count(s)} \geq min\_conf$, where min_conf is the minimum confidence threshold.

Because the rules are generated from frequent itemsets, each one automatically satisfies minimum support. Frequent itemsets can be stored ahead of time in hash tables along with their counts so that they can be accessed quickly.

Example 5.4 **Generating association rules.** Let's try an example based on the transactional data for *AllElectronics* shown in Table 5.1. Suppose the data contain the frequent itemset $l = \{I1, I2, I5\}$. What are the association rules that can be generated from l? The nonempty subsets of l are $\{I1, I2\}$, $\{I1, I5\}$, $\{I2, I5\}$, $\{I1\}$, $\{I2\}$, and $\{I5\}$. The resulting association rules are as shown below, each listed with its confidence:

$$
\begin{aligned}
&I1 \wedge I2 \Rightarrow I5, && confidence = 2/4 = 50\% \\
&I1 \wedge I5 \Rightarrow I2, && confidence = 2/2 = 100\% \\
&I2 \wedge I5 \Rightarrow I1, && confidence = 2/2 = 100\% \\
&I1 \Rightarrow I2 \wedge I5, && confidence = 2/6 = 33\% \\
&I2 \Rightarrow I1 \wedge I5, && confidence = 2/7 = 29\% \\
&I5 \Rightarrow I1 \wedge I2, && confidence = 2/2 = 100\%
\end{aligned}
$$

If the minimum confidence threshold is, say, 70%, then only the second, third, and last rules above are output, because these are the only ones generated that are strong. Note that, unlike conventional classification rules, association rules can contain more than one conjunct in the right-hand side of the rule. ■

1.5. Improving the Efficiency of Apriori

- Several attempts have been introduced to improve the efficiency of Apriori:
  - Hash-based technique
    - Hashing itemset counts
    - Example:

  - Transaction DB:

TID	List of Transactions
T100	I1,I2,I5

T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

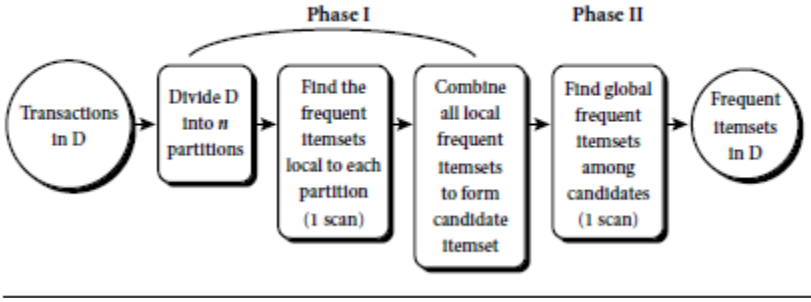
- Create a hash table for candidate 2-itemsets:
  - Generate all 2-itemsets for each transaction in the transaction DB
  - H(x,y) = ((order of x) * 10 + (order of y)) mod 7
  - A 2-itemset whose corresponding bucket count is below the support threshold cannot be frequent.

Bucket @	0	1	2	3	4	5	6
Bucket count	2	2	4	2	2	4	4
Content	{I1,I4}	{I1,I5}	{I2,I3}	{I2,I4}	{I2,I5}	{I1,I2}	{I1,I3}
	{I3,I5}	{I1,I5}	{I2,I3}	{I2,I4}	{I2,I5}	{I1,I2}	{I1,I3}
			{I2,I3}			{I1,I2}	{I1,I3}
			{I2,I3}			{I1,I2}	{I1,I3}

  - Remember: support(x➔y) = percentage number of transactions that contain x and y. Therefore, if the minimum support is 3, then the itemsets in buckets 0, 1, 3, and 4 cannot be frequent and so they should not be included in C_2.

- Transaction reduction
  - Reduce the number of transactions scanned in future iterations.
  - A transaction that does not contain any frequent k-itemsets cannot contain any frequent (k+1)-itemsets: Do not include such transaction in subsequent scans.

- Other techniques include:
  - Partitioning (partition the data to find candidate itemsets)



- Sampling (Mining on a subset of the given data)
- Dynamic itemset counting (Adding candidate itemsets at different points during a scan)

FP-Growth Method: Mining Frequent Itemsets without Candidate Generation

The Apriori Algorithm: Finding Frequent Itemsets Using Candidate Generation

- Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
  - highly condensed, but complete for frequent pattern mining
  - avoid costly database scans
- Develop an efficient, FP-tree-based frequent pattern mining method
  - A divide-and-conquer methodology: decompose mining tasks into smaller ones
  - Avoid candidate generation: sub-database test only!

Principles of Frequent Pattern Growth

- Pattern growth property
  - Let α be a frequent itemset in DB, B be α's conditional pattern base, and β be an itemset in B. Then $\alpha \cup \beta$ is a frequent itemset in DB iff β is frequent in B.
- "*abcdef*" is a frequent pattern, if and only if
  - "*abcde*" is a frequent pattern, and
  - "*f*" is frequent in the set of transactions containing "*abcde*"

Benefits of the FP-tree Structure

- Completeness:
  - never breaks a long pattern of any transaction
  - preserves complete information for frequent pattern mining
- Compactness
  - reduce irrelevant information—infrequent items are gone
  - frequency descending ordering: more frequent items are more likely to be shared
  - never be larger than the original database (if not count node-links and counts)
  - Example: For Connect-4 DB, compression ratio could be over 100

- Objectives:
  - The bottleneck of *Apriori*: candidate generation
  - Huge candidate sets:

        - For 10^4 frequent 1-itemset, Apriori will generate 10^7 candidate 2-itemsets.
        - To discover a frequent pattern of size 100, e.g., {a1, a2, …, a100}, one needs to generate
          $2^{100} \approx 10^{30}$ candidates.
    - Multiple scans of database:
        - Needs (n +1) scans, n is the length of the longest pattern.

- Principal
    - Compress a large database into a compact, Frequent-Pattern tree (FP-tree) structure
        - Highly condensed, but complete for frequent pattern mining
        - Avoid costly database scans
    - Develop an efficient, FP-tree-based frequent pattern mining method
        - A divide-and-conquer methodology: decompose mining tasks into smaller ones
        - Avoid candidate generation: sub-database test only!
- Algorithm:
    1. Scan DB once, find frequent 1-itemset (single item pattern)
    2. Order frequent items in frequency descending order, called ***L order***: (in the example below: F(4), c(4), a(3), etc.)
    3. Scan DB again and construct FP-tree
        a. Create the root of the tree and label it null or {}
        b. The items in each transaction are processed in the L order (sorted according to descending support count).
        c. Create a branch for each transaction
        d. Branches share common prefixes

Mining Frequent patterns using FP-Tree

- General idea (divide-and-conquer)
    - Recursively grow frequent pattern path using the FP-tree
- Method
    - For each item, construct its conditional pattern-base, and then its conditional FP-tree
    - Recursion: Repeat the process on each newly created conditional FP-tree
    - Until the resulting FP-tree is empty, or it contains only one path (single path will generate all the combinations of its sub-paths, each of which is a frequent pattern)

Major steps to mine FP-trees

- Main Steps:
    1. Construct conditional pattern base for each node in the FP-tree
    2. Construct conditional FP-tree from each conditional pattern-base
    3. Recursively mine conditional FP-trees and grow frequent patterns obtained so far If the conditional FP-tree contains a single path, simply enumerate all the patterns

- Step 1: From FP-tree to Conditional Pattern Base
    - Starting at the frequent header table in the FP-tree
    - Traverse the FP-tree by following the link of each frequent item, starting by the item with the highest frequency.
    - Accumulate all of transformed ***prefix paths*** of that item to form a conditional pattern base

Properties of FP-tree for Conditional Pattern Base Construction:

- Node-link property
    - For any frequent item a_i, all the possible frequent patterns that contain a_i can be obtained by following a_i's node-links, starting from a_i's head in the FP-tree header.
- Prefix path property
    - To calculate the frequent patterns for a node a_i in a path P, only the prefix sub-path of a_i in P need to be accumulated and its ***frequency count should carry the same count as node*** a_i.

- Step 2: Construct Conditional FP-tree
    - For each pattern-base
        - Accumulate the count for each item in the base
        - Construct the FP-tree for the frequent items of the pattern base
- Step 3: Recursively mine the conditional FP-tree

- **Why is FP-Tree mining fast?**
    - The performance study shows FP-growth is an order of magnitude faster than Apriori
    - Reasoning:
        - No candidate generation, no candidate test
        - Use compact data structure
        - Eliminate repeated database scan
        - Basic operation is counting and FP-tree building

As we have seen, in many cases the Apriori candidate generate-and-test method significantly reduces the size of candidate sets, leading to good performance gain.

An interesting method in this attempt is called frequent-pattern growth, or simply FP-growth, which adopts a *divide-and-conquer* strategy as follows. First, it compresses the database representing frequent items into a frequent-pattern tree, or FP-tree, which retains the itemset association information. It then divides the compressed database into a set of *conditional databases* (a special kind of projected database), each associated with one frequent item or "pattern fragment," and mines each such database separately. You'll see how it works with the following example.

Example 5.5 FP-growth (finding frequent itemsets without candidate generation). We re-examine the mining of transaction database, *D*, of Table 5.1 in Example 5.3 using the frequent pattern growth approach.

The first scan of the database is the same as Apriori, which derives the set of frequent items (1-itemsets) and their support counts (frequencies). Let the minimum support count be 2. The set of frequent items is sorted in the order of descending support count. This resulting set or *list* is denoted *L*. Thus, we have $L = \{\{I2: 7\}, \{I1: 6\}, \{I3: 6\}, \{I4: 2\}, \{I5: 2\}\}$.

An FP-tree is then constructed as follows. First, create the root of the tree, labeled with "null." Scan database *D* a second time. The items in each transaction are processed in *L* order (i.e., sorted according to descending support count), and a branch is created for each transaction. For example, the scan of the first transaction, "T100: I1, I2, I5," which contains three items (I2, I1, I5 in *L* order), leads to the construction of the first branch of the tree with three nodes, ⟨I2: 1⟩, ⟨I1:1⟩, and ⟨I5: 1⟩, where I2 is linked as a child of the root, I1 is linked to I2, and I5 is linked to I1. The second transaction, T200, contains the items I2 and I4 in *L* order, which would result in a branch where I2 is linked to the root and I4 is linked to I2. However, this branch would share a common **prefix**, I2, with the existing path for T100. Therefore, we instead increment the count of the I2 node by 1, and create a new node, ⟨I4: 1⟩, which is linked as a child of ⟨I2: 2⟩. In general, when considering the branch to be added for a transaction, the count of each node along a common prefix is incremented by 1, and nodes for the items following the prefix are created and linked accordingly.

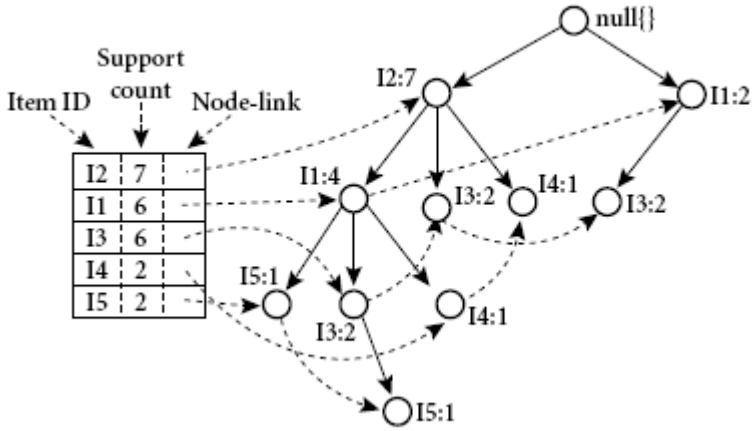


Figure 5.7 An FP-tree registers compressed, frequent pattern information.

Mining the FP-tree by creating conditional (sub-)pattern bases.

Item	*Conditional Pattern Base*	*Conditional FP-tree*	*Frequent Patterns Generated*
I5	{{I2, I1: 1}, {I2, I1, I3: 1}}	⟨I2: 2, I1: 2⟩	{I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}
I4	{{I2, I1: 1}, {I2: 1}}	⟨I2: 2⟩	{I2, I4: 2}
I3	{{I2, I1: 2}, {I2: 2}, {I1: 2}}	⟨I2: 4, I1: 2⟩, ⟨I1: 2⟩	{I2, I3: 4}, {I1, I3: 4}, {I2, I1, I3: 2}
I1	{{I2: 4}}	⟨I2: 4⟩	{I2, I1: 4}

Mining of the FP-tree is summarized in Table 5.2 and detailed as follows. We first consider I5, which is the last item in L, rather than the first. The reason for starting at the end of the list will become apparent as we explain the FP-tree mining process. I5 occurs in two branches of the FP-tree of Figure 5.7. (The occurrences of I5 can easily be found by following its chain of node-links.) The paths formed by these branches are ⟨I2, I1, I5: 1⟩ and ⟨I2, I1, I3, I5: 1⟩. Therefore, considering I5 as a suffix, its corresponding two prefix paths are ⟨I2, I1: 1⟩ and ⟨I2, I1, I3: 1⟩, which form its conditional pattern base. Its conditional FP-tree contains only a single path, ⟨I2: 2, I1: 2⟩; I3 is not included because its support count of 1 is less than the minimum support count. The single path generates all the combinations of frequent patterns: {I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}.

For I4, its two prefix paths form the conditional pattern base, {{I2 I1: 1}, {I2: 1}}, which generates a single-node conditional FP-tree, ⟨I2: 2⟩, and derives one frequent pattern, {I2, I1: 2}. Notice that although I5 follows I4 in the first branch, there is no need to include I5 in the analysis here because any frequent pattern involving I5 is analyzed in the examination of I5.

Similar to the above analysis, I3's conditional pattern base is {{I2, I1: 2}, {I2: 2}, {I1: 2}}. Its conditional FP-tree has two branches, ⟨I2: 4, I1: 2⟩ and ⟨I1: 2⟩, as shown in Figure 5.8, which generates the set of patterns, {{I2, I3: 4}, {I1, I3: 4}, {I2, I1, I3: 2}}. Finally, I1's conditional pattern base is {{I2: 4}}, whose FP-tree contains only one node, ⟨I2: 4⟩, which generates one frequent pattern, {I2, I1: 4}. This mining process is summarized in Figure 5.9. ■

Algorithm: FP_growth. Mine frequent itemsets using an FP-tree by pattern fragment growth.

Input:

- D, a transaction database;
- *min_sup*, the minimum support count threshold.

Output: The complete set of frequent patterns.

Method:

1. The FP-tree is constructed in the following steps:
   (a) Scan the transaction database D once. Collect F, the set of frequent items, and their support counts. Sort F in support count descending order as L, the *list* of frequent items.
   (b) Create the root of an FP-tree, and label it as "null." For each transaction *Trans* in D do the following. Select and sort the frequent items in *Trans* according to the order of L. Let the sorted frequent item list in *Trans* be $[p|P]$, where p is the first element and P is the remaining list. Call insert_tree($[p|P], T$), which is performed as follows. If T has a child N such that $N.item\text{-}name = p.item\text{-}name$, then increment N's count by 1; else create a new node N, and let its count be 1, its parent link be linked to T, and its node-link to the nodes with the same *item-name* via the node-link structure. If P is nonempty, call insert_tree(P, N) recursively.
2. The FP-tree is mined by calling FP_growth(*FP_tree*, *null*), which is implemented as follows.

```
procedure FP_growth(Tree, α)
(1)  if Tree contains a single path P then
(2)     for each combination (denoted as β) of the nodes in the path P
(3)        generate pattern β∪α with support_count = minimum support count of nodes in β;
(4)  else for each a_i in the header of Tree {
(5)     generate pattern β = a_i ∪α with support_count = a_i.support_count;
(6)     construct β's conditional pattern base and then β's conditional FP_tree Tree_β;
(7)     if Tree_β ≠ ∅ then
(8)        call FP_growth(Tree_β, β); }
```

Figure 5.9 The FP-growth algorithm for discovering frequent itemsets without candidate generation.

Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not count node-links and the *count* field)
  - For Connect-4 DB, compression ratio could be over 100

Mining Various Kinds of Association Rules

1) Mining Multilevel Association Rules

For many applications, it is difficult to find strong associations among data items at low or primitive levels of abstraction due to the sparsity of data at those levels. Strong associations discovered at high levels of abstraction may represent commonsense knowledge. Moreover, what may represent common sense to one user may seem novel to another. Therefore, data mining systems should provide capabilities for mining association rules at multiple levels of abstraction, with sufficient flexibility for easy traversal among different abstraction spaces.

Let's examine the following example.

Mining multilevel association rules. Suppose we are given the task-relevant set of transactional data in Table for sales in an *AllElectronics* store, showing the items purchased for each transaction. The concept hierarchy for the items is shown in Figure 5.10. A concept hierarchy defines a sequence of mappings from a set of low-level concepts to higher level, more general concepts. Data can be generalized by replacing low-level concepts
within the data by their higher-level concepts, or *ancestors*, from a concept hierarchy.

TID	*Items Purchased*
T100	IBM-ThinkPad-T40/2373, HP-Photosmart-7660
T200	Microsoft-Office-Professional-2003, Microsoft-Plus!-Digital-Media
T300	Logitech-MX700-Cordless-Mouse, Fellowes-Wrist-Rest
T400	Dell-Dimension-XPS, Canon-PowerShot-S400
T500	IBM-ThinkPad-R40/P4M, Symantec-Norton-Antivirus-2003
...	...

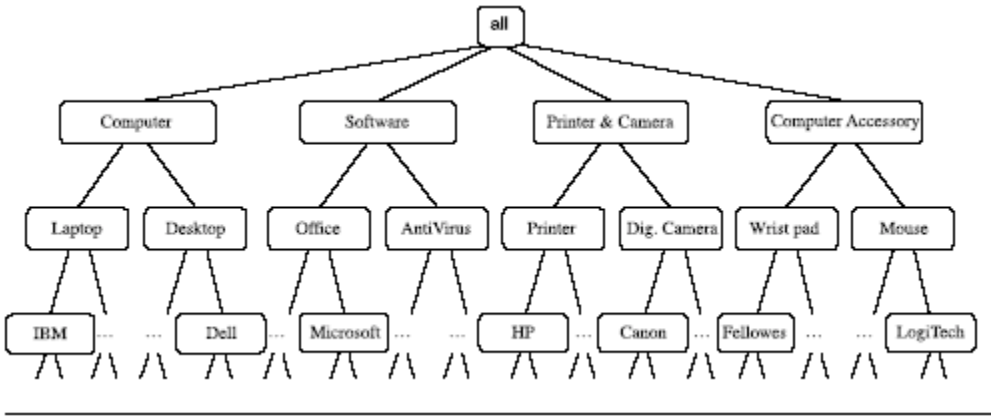
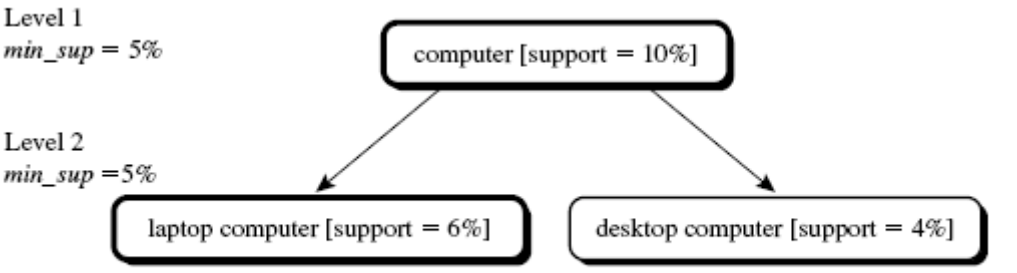


Figure 5.10 A concept hierarchy for *AllElectronics* computer items.

Association rules generated from mining data at multiple levels of abstraction are called multiple-level or multilevel association rules. Multilevel association rules can be mined efficiently using concept hierarchies under a support-confidence framework. In general, a top-down strategy is employed, where counts are accumulated for the calculation of frequent itemsets at each concept level, starting at the concept level 1 and working downward in the hierarchy toward the more specific concept levels, until no more frequent itemsets can be found. For each level, any algorithm for discovering frequent itemsets may be used, such as Apriori or its variations.

- **Using uniform minimum support for all levels (referred to as uniform support):** The same minimum support threshold is used when mining at each level of abstraction. For example, in Figure 5.11, a minimum support threshold of 5% is used throughout (e.g., for mining from *"computer"* down to *"laptop computer"*). Both *"computer"* and *"laptop computer"* are found to be frequent, while *"desktop computer"* is not.

  When a uniform minimum support threshold is used, the search procedure is simplified. The method is also simple in that users are required to specify only one minimum support threshold. An Apriori-like optimization technique can be adopted, based on the knowledge that an ancestor is a superset of its descendants: The search avoids examining itemsets containing any item whose ancestors do not have minimum support.



Multilevel mining with uniform support.

- **Using reduced minimum support at lower levels (referred to as reduced support):** Each level of abstraction has its own minimum support threshold. The deeper the level of abstraction, the smaller the corresponding threshold is. For example, in Figure, the minimum support thresholds for levels 1 and 2 are 5% and 3%, respectively. In this way, *"computer," "laptop computer,"* and *"desktop computer"* are all considered frequent.

- **Using item or group-based minimum support (referred to as group-based support):** Because users or experts often have insight as to which groups are more important than others, it is sometimes more desirable to set up user-specific, item, or group based minimal support thresholds when mining multilevel rules. For example, a user could set up the minimum support thresholds based on product price, or on items of interest, such as by setting particularly low support thresholds for *laptop computers* and *flash drives* in order to pay particular attention to the association patterns containing items in these categories.

2) Mining Multidimensional Association Rules from Relational Databases and DataWarehouses

We have studied association rules that imply a single predicate, that is, the predicate *buys*. For instance, in mining our *AllElectronics* database, we may discover the Boolean association rule

$$buys(X, \text{“}digital\ camera\text{”}) \Rightarrow buys(X, \text{“}HP\ printer\text{”}).$$

Following the terminology used in multidimensional databases, we refer to each distinct predicate in a rule as a dimension. Hence, we can refer to Rule above as a single dimensional or intra dimensional association rule because it contains a single distinct predicate (e.g., *buys*)with multiple occurrences (i.e., the predicate occurs more than once within the rule). As we have seen in the previous sections of this chapter, such rules are commonly mined from transactional data.

Considering each database attribute or warehouse dimension as a predicate, we can therefore mine association rules containing *multiple* predicates, such as

$$age(X, \text{“}20...29\text{”}) \wedge occupation(X, \text{“}student\text{”}) \Rightarrow buys(X, \text{“}laptop\text{”}).$$

Association rules that involve two or more dimensions or predicates can be referred to as multidimensional association rules. Rule above contains three predicates (*age, occupation*, and *buys*), each of which occurs *only once* in the rule. Hence, we say that it has no repeated predicates. Multidimensional association rules with no repeated predicates are called inter dimensional association rules. We can also mine multidimensional association rules with repeated predicates, which contain multiple occurrences of some predicates. These rules are called hybrid-dimensional association rules. An example of such a rule is the following, where the predicate *buys* is repeated:

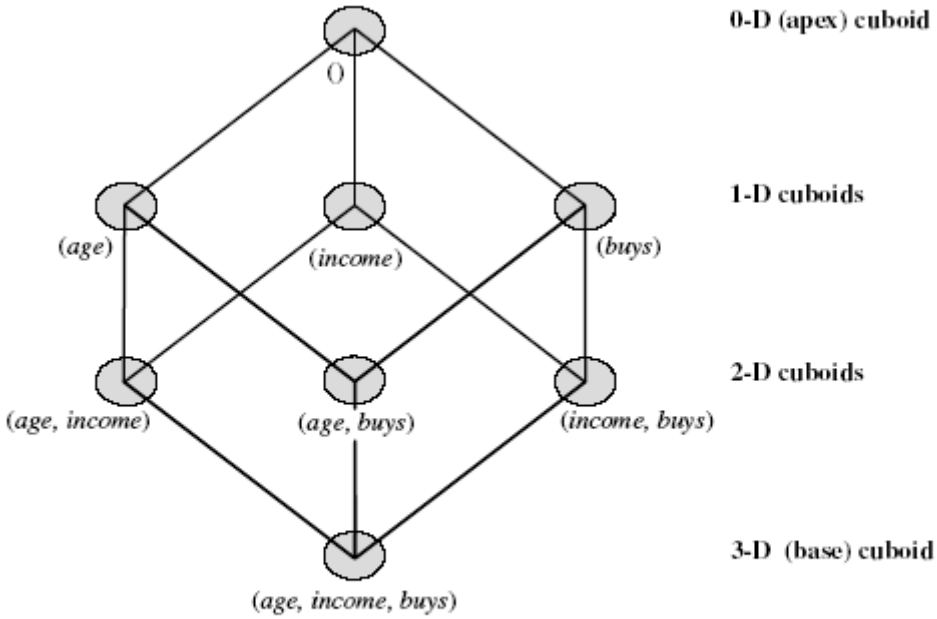
$$age(X, \text{“}20...29\text{”}) \wedge buys(X, \text{“}laptop\text{”}) \Rightarrow buys(X, \text{“}HP\ printer\text{”})$$

Note that database attributes can be categorical or quantitative. Categorical attributes have a finite number of possible values, with no ordering among the values (e.g., *occupation, brand*, *color*). Categorical attributes are also called nominal attributes, because their values are "names of things." Quantitative attributes are numeric and have an implicit ordering among values (e.g., *age, income*, *price*).

Techniques for mining multidimensional association rules can be categorized into two basic approaches regarding the treatment of quantitative attributes.

Mining Multidimensional Association Rules Using Static Discretization of Quantitative Attributes

Quantitative attributes, in this case, are discretized before mining using predefined concept hierarchies or data discretization techniques, where numeric values are replaced by interval labels. Categorical attributes may also be generalized to higher conceptual levels if desired. If the resulting task-relevant data are stored in a relational table, then any of the frequent itemset mining algorithms we have discussed can be modified easily so as to find all frequent predicate sets rather than frequent itemsets. In particular, instead of searching on only one attribute like *buys*, we need to search through all of the relevant attributes, treating each attribute-value pair as an itemset.



Lattice of cuboids, making up a 3-D data cube. Each cuboid represents a different group-by. The base cuboid contains the three predicates *age*, *income*, and *buys*.

Mining Quantitative Association Rules

Quantitative association rules are multidimensional association rules in which the numeric attributes are *dynamically* discretized during the mining process so as to satisfy some mining criteria, such as maximizing the confidence or compactness of the rules mined. In this section, we focus specifically on how to mine quantitative association rules having two quantitative attributes on the left-hand side of the rule and one categorical attribute on the right-hand side of the rule. That is,

$$A_{quan1} \wedge A_{quan2} \Rightarrow A_{cat}$$

where *Aquan*1 and *Aquan*2 are tests on quantitative attribute intervals (where the intervals are dynamically determined), and *Acat* tests a categorical attribute from the task-relevant data. Such rules have been referred to as two-dimensional quantitative association rules, because they contain two quantitative dimensions. For instance, suppose you are curious about the association relationship between pairs of quantitative attributes, like customer age and income, and the type of television (such as *high-definition TV,* i.e., *HDTV*) that customers like to buy. An example of such a 2-D quantitative association rule is

$$age(X, \text{“30...39”}) \wedge income(X, \text{“42}K\text{...48}K\text{”}) \Rightarrow buys(X, \text{“}HDTV\text{”})$$

Binning: Quantitative attributes can have a very wide range of values defining their domain. Just think about how big a 2-D grid would be if we plotted *age* and *income* as axes, where each possible value of *age* was assigned a unique position on one axis, and similarly, each possible value of *income* was assigned a unique position on the other axis! To keep grids down to a manageable size, we instead partition the ranges of quantitative attributes into intervals. These intervals are dynamic in that they may later be further combined during the mining process. The partitioning process is referred to as binning, that is, where the intervals are considered “bins.” Three common binning strategies area as follows:

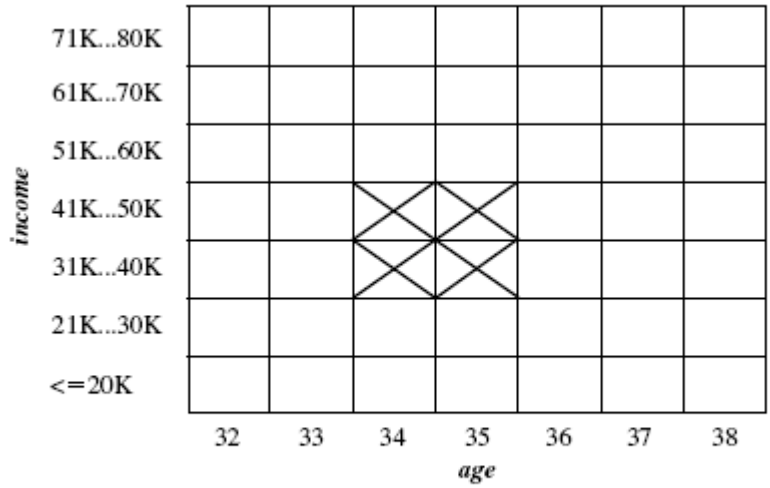
- **Equal-width binning**, where the interval size of each bin is the same
- **Equal-frequency binning**, where each bin has approximately the same number of tuples assigned to it,
- **Clustering-based binning**, where clustering is performed on the quantitative attribute to group *neighboring points* (judged based on various distance measures) into the same bin

Finding frequent predicate sets: Once the 2-D array containing the count distribution for each category is set up, it can be scanned to find the frequent predicate sets (those satisfying minimum support) that also satisfy minimum confidence. Strong association rules can then be generated from these predicate sets, using a rule generation algorithm.

Clustering the association rules: The strong association rules obtained in the previous step are then mapped to a 2-D grid. Figure 5.14 shows a 2-D grid for 2-D quantitative association rules predicting the condition *buys*(*X*, “*HDTV*”) on the rule right-hand side, given the quantitative attributes *age* and *income*. The four Xs correspond to the rules

$$age(X, 34) \wedge income(X, \text{“31}K\text{...40}K\text{”}) \Rightarrow buys(X, \text{“}HDTV\text{”}) \quad (5.16)$$
$$age(X, 35) \wedge income(X, \text{“31}K\text{...40}K\text{”}) \Rightarrow buys(X, \text{“}HDTV\text{”}) \quad (5.17)$$
$$age(X, 34) \wedge income(X, \text{“41}K\text{...50}K\text{”}) \Rightarrow buys(X, \text{“}HDTV\text{”}) \quad (5.18)$$
$$age(X, 35) \wedge income(X, \text{“41}K\text{...50}K\text{”}) \Rightarrow buys(X, \text{“}HDTV\text{”}). \quad (5.19)$$

“Can we find a simpler rule to replace the above four rules?” Notice that these rules are quite “close” to one another, forming a rule cluster on the grid. Indeed, the four rules can be combined or “clustered” together to form the following simpler rule, which subsumes and replaces the above four rules:



A 2-D grid for tuples representing customers who purchase high-definition TVs.

From Association Mining to Correlation Analysis

Most association rule mining algorithms employ a support-confidence framework. Often, many interesting rules can be found using low support thresholds. Although minimum support and confidence thresholds *help* weed out or exclude the exploration of a good number of uninteresting rules, many rules so generated are still not interesting to the users. Unfortunately, this is especially true *when mining at low support thresholds or mining for long patterns*. This has been one of the major bottlenecks for successful application of association rule mining.

1) Strong Rules Are Not Necessarily Interesting: An Example

Whether or not a rule is interesting can be assessed either subjectively or objectively. Ultimately, only the user can judge if a given rule is interesting, and this judgment, being subjective, may differ from one user to another. However, objective interestingness measures, based on the statistics "behind" the data, can be used as one step toward the goal of weeding out uninteresting rules from presentation to the user.

The support and confidence measures are insufficient at filtering out uninteresting association rules. To tackle this weakness, a correlation measure can be used to augment the support-confidence framework for association rules. This leads to *correlation rules* of the form

$$A \Rightarrow B\,[support, confidence. correlation].$$

That is, a correlation rule is measured not only by its support and confidence but also by the correlation between itemsets A and B. There are many different correlation measures from which to

choose. In this section, we study various correlation measures to determine which would be good for mining large data sets.

Constraint-Based Association Mining

A data mining process may uncover thousands of rules from a given set of data, most of which end up being unrelated or uninteresting to the users. Often, users have a good sense of which "direction" of mining may lead to interesting patterns and the "form" of the patterns or rules they would like to find. Thus, a good heuristic is to have the users specify such intuition or expectations as *constraints* to confine the search space. This strategy is known as constraint-based mining. The constraints can include the following:

- **Knowledge type constraints:** These specify the type of knowledge to be mined, such as association or correlation.
- **Data constraints:** These specify the set of task-relevant data.
- **Dimension/level constraints:** These specify the desired dimensions (or attributes) of the data, or levels of the concept hierarchies, to be used in mining.
- **Interestingness constraints:** These specify thresholds on statistical measures of rule interestingness, such as support, confidence, and correlation.
- **Rule constraints:** These specify the form of rules to be mined. Such constraints may be expressed as metarules (rule templates), as the maximum or minimum number of predicates that can occur in the rule antecedent or consequent, or as relationships among attributes, attribute values, and/or aggregates.

1) Metarule-Guided Mining of Association Rules

"How are metarules useful?" Metarules allow users to specify the syntactic form of rules that they are interested in mining. The rule forms can be used as constraints to help improve the efficiency of the mining process. Metarules may be based on the analyst's experience, expectations, or intuition regarding the data or may be automatically generated based on the database schema.

Metarule-guided mining:- Suppose that as a market analyst for *AllElectronics*, you have access to the data describing customers (such as customer age, address, and credit rating) as well as the list of customer transactions. You are interested in finding associations between customer traits and the items that customers buy. However, rather than finding *all* of the association rules reflecting these relationships, you are particularly interested only in determining which pairs of customer traits promote the sale of office software.A metarule can be used to specify this information describing the form of rules you are interested in finding. An example of such a metarule is

$$P_1(X, Y) \wedge P_2(X, W) \Rightarrow buys(X, \text{"}office\ software\text{"}),$$

where $P1$ and $P2$ are predicate variables that are instantiated to attributes from the given database during the mining process, X is a variable representing a customer, and Y and W take on values of the

attributes assigned to $P1$ and $P2$, respectively. Typically, a user will specify a list of attributes to be considered for instantiation with $P1$ and $P2$. Otherwise, a default set may be used.

2) Constraint Pushing: Mining Guided by Rule Constraints

Rule constraints specify expected set/subset relationships of the variables in the mined rules, constant initiation of variables, and aggregate functions. Users typically employ their knowledge of the application or data to specify rule constraints for the mining task. These rule constraints may be used together with, or as an alternative to, metarule-guided mining. In this section, we examine rule constraints as to how they can be used to make the mining process more efficient. Let's study an example where rule constraints are used to mine hybrid-dimensional association rules.

Our association mining query is to *"Find the sales of which cheap items (where the sum of the prices is less than $100) may promote the sales of which expensive items (where the minimum price is $500) of the same group for Chicago customers in 2004."* This can be expressed in the DMQL data mining query language as follows,

(1) mine associations as
(2) $lives\_in(C, \_, \text{"}Chicago\text{"}) \wedge sales^{+}(C, ?\{I\}, \{S\}) \Rightarrow sales^{+}(C, ?\{J\}, \{T\})$
(3) from sales
(4) where S.year = 2004 and T.year = 2004 and I.group = J.group
(5) group by C, I.group
(6) having sum(I.price) $<$ 100 and min(J.price) $\geq$ 500
(7) with support threshold = 1%
(8) with confidence threshold = 50%

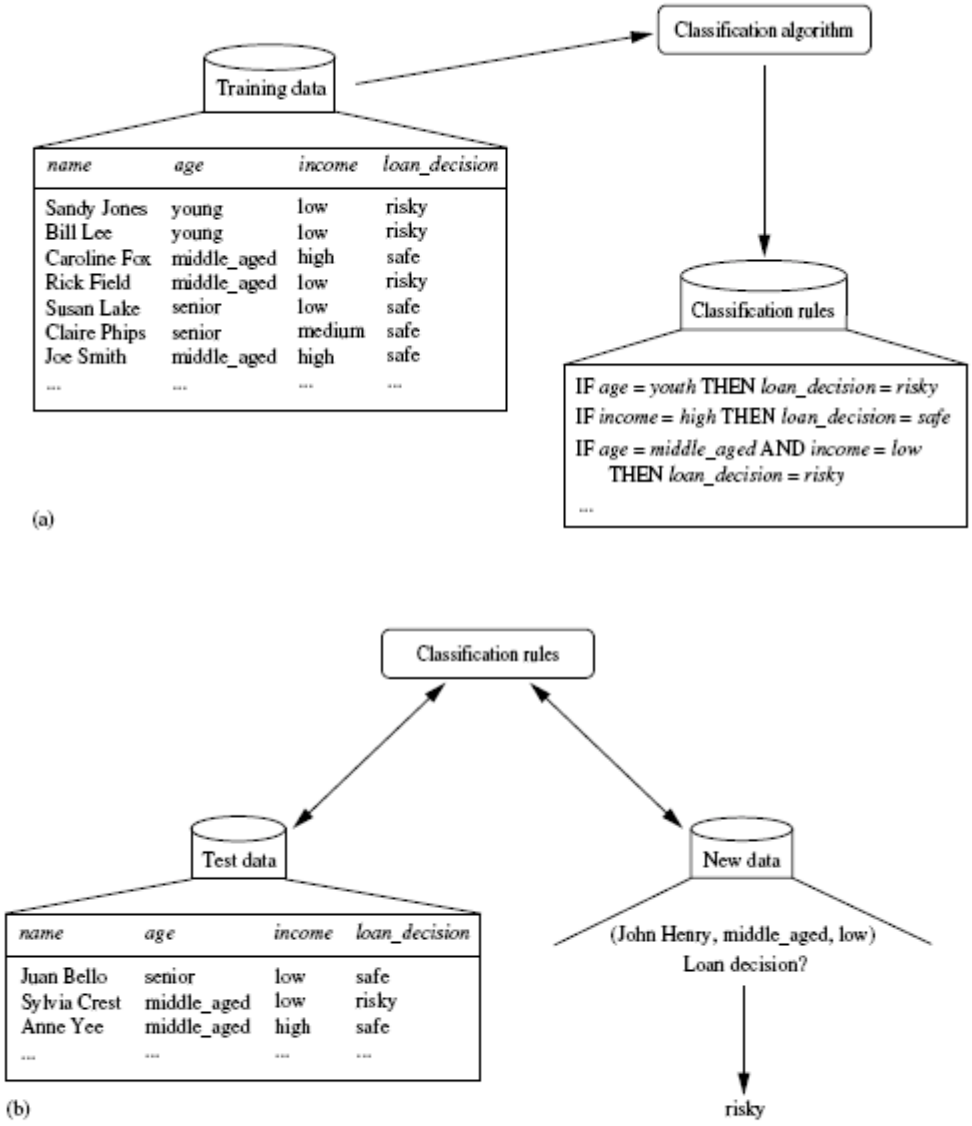
Classification and Prediction

What Is Classification? What Is Prediction?

A bank loans officer needs analysis of her data in order to learn which loan applicants are "safe" and which are "risky" for the bank. A marketing manager at *AllElectronics* needs data analysis to help guess whether a customer with a given profile will buy a new computer. A medical researcher wants to analyze breast cancer data in order to predict which one of three specific treatments a patient should receive. In each of these examples, the data analysis task is classification, where a model or classifier is constructed to predict *categorical labels*, such as "safe" or "risky" for the loan application data; "yes" or "no" for the marketing data; or "treatment A," "treatment B," or "treatment C" for the medical data. These categories can be represented by discrete values, where the ordering among values has no meaning. For example, the values 1, 2, and 3 may be used to represent treatments A, B, and C, where there is no ordering implied among this group of treatment regimes.

Suppose that the marketing manager would like to predict how much a given customer will spend during a sale at *AllElectronics*. This data analysis task is an example of numeric prediction, where the model constructed predicts a *continuous-valued function*, or *ordered value*, as opposed to a categorical label. This model is a predictor

"How does classification work? Data classification is a two-step process, as shown for the loan application data of Figure 6.1. (The data are simplified for illustrative purposes. In reality, we may expect many more attributes to be considered.) In the first step, a classifier is built describing a predetermined set of data classes or concepts. This is the learning step (or training phase), where a classification algorithm builds the classifier by analyzing or "learning from" a training set made up of database tuples and their associated class labels.



The data classification process: (a) *Learning*: Training data are analyzed by a classification algorithm. Here, the class label attribute is *loan_decision*, and the learned model or classifier is represented in the form of classification rules. (b) *Classification*: Test data are used to estimate the accuracy of the classification rules. If the accuracy is considered acceptable, the rules can be applied to the classification of new data tuples.

Why Classification? A motivating application

- Credit approval

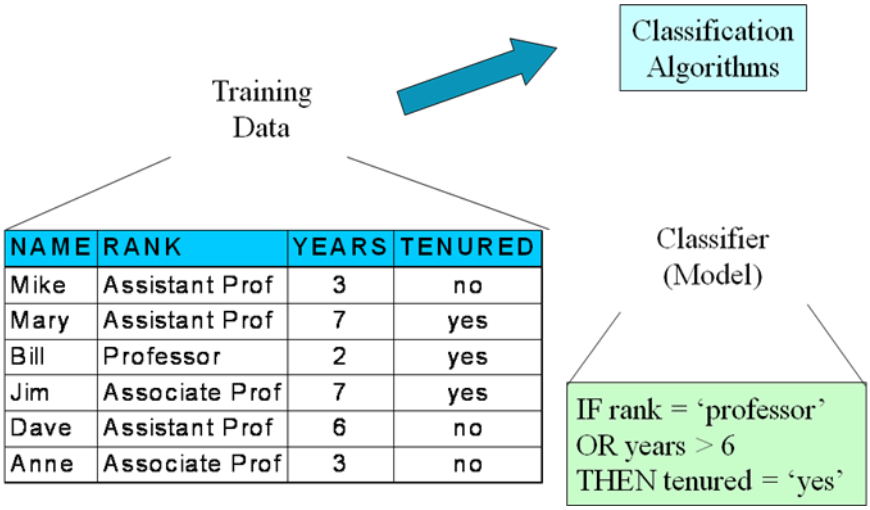
- A bank wants to classify its customers based on whether they are expected to pay back their approved loans
- The history of past customers is used to train the classifier
- The classifier provides rules, which identify potentially reliable future customers
- Classification rule:
  - If age = "31...40" and income = high then credit_rating = excellent
- Future customers
  - Paul: age = 35, income = high ⇒ excellent credit rating
  - John: age = 20, income = medium ⇒ fair credit rating

- **Classification:**
  - predicts categorical class labels
  - classifies data (constructs a model) based on the training set and the values (class labels) in a classifying attribute and uses it in classifying new data
- **Prediction**
  - models continuous-valued functions, i.e., predicts unknown or missing values
- **Typical applications**
  - Credit approval
  - Target marketing
  - Medical diagnosis
  - Fraud detection
  -

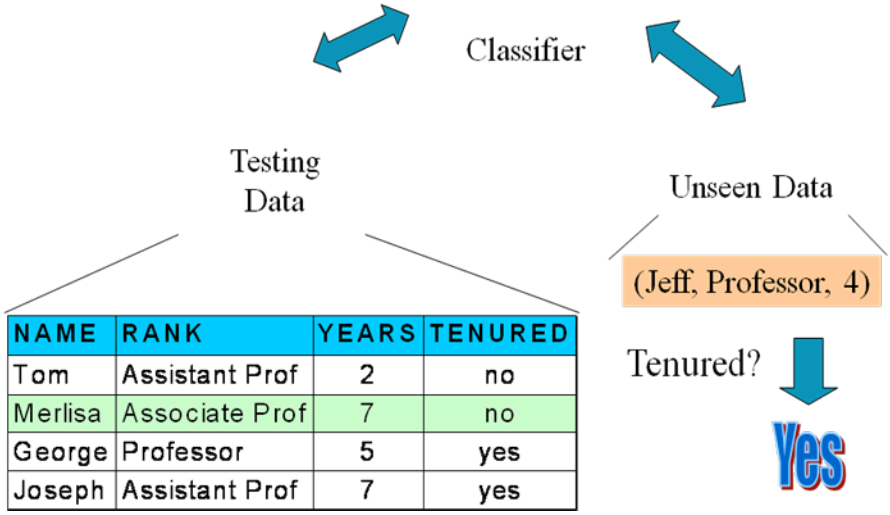
Classification—A Two-Step Process

- Model construction: describing a set of predetermined classes
  - Each tuple/sample is assumed to belong to a predefined class, as determined by the class label attribute
  - The set of tuples used for model construction: training set
  - The model is represented as classification rules, decision trees, or mathematical formulae
- Model usage: for classifying future or unknown objects
  - Estimate accuracy of the model
    - The known label of test sample is compared with the classified result from the model
    - Accuracy rate is the percentage of test set samples that are correctly classified by the model
    - Test set is independent of training set, otherwise over-fitting will occur

Process (1): Model Construction



Process (2): Using the Model in Prediction



Supervised vs. Unsupervised Learning

- Supervised learning (classification)
  - Supervision: The training data (observations, measurements, etc.) are accompanied by labels indicating the class of the observations
  - New data is classified based on the training set
- Unsupervised learning (clustering)
  - The class labels of training data is unknown
  - Given a set of measurements, observations, etc. with the aim of establishing the existence of classes or clusters in the data

Issues Regarding Classification and Prediction

- **Data cleaning**: This refers to the preprocessing of data in order to remove or reduce *noise* (by applying smoothing techniques, for example) and the treatment of *missing values* (e.g., by replacing a missing value with the most commonly occurring value for that attribute, or with the most probable value based on statistics). Although most classification algorithms have some mechanisms for handling noisy or missing data, this step can help reduce confusion during learning.

- **Relevance analysis**: Many of the attributes in the data may be *redundant*. Correlation analysis can be used to identify whether any two given attributes are statistically related. For example, a strong correlation between attributes *A*1 and *A*2 would suggest that one of the two could be removed from further analysis. A database may also contain *irrelevant* attributes. Attribute subset selection4 can be used in these cases to find a reduced set of attributes such that the resulting probability distribution of the data classes is as close as possible to the original distribution obtained using all attributes. Hence, relevance analysis, in the form of correlation analysis and attribute subset selection, can be used to detect attributes that do not contribute to the classification or prediction task. Including such attributes may otherwise slow down, and possibly mislead, the learning step. Ideally, the time spent on relevance analysis, when added to the time spent on learning from the resulting "reduced" attribute (or feature) subset, should be less than the time that would have been spent on learning from the original set of attributes. Hence, such analysis can help improve classification efficiency and scalability.

- **Data transformation and reduction:** The data may be transformed by normalization, particularly when neural networks or methods involving distance measurements are used in the learning step. Normalization involves scaling all values for a given attribute so that they fall within a small specified range, such as -1.0 to 1.0, or 0.0 to 1.0. In methods that use distance measurements, for example, this would prevent attributes with initially large ranges (like, say, *income*) from out weighing attributes with initially smaller ranges (such as binary attributes).

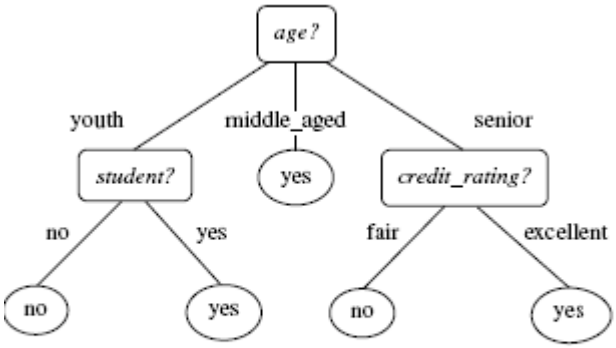
Comparing Classification and Prediction Methods

Classification and prediction methods can be compared and evaluated according to the following criteria:

- **Predictive accuracy**
- **Speed**
  - **time to construct the model**
  - **time to use the model**
- **Robustness**
  - **handling noise and missing values**
- **Scalability**
  - **efficiency in disk-resident databases**
- **Interpretability:**
  - **understanding and insight provided by the model**
- **Goodness of rules (quality)**
  - **decision tree size**
  - **compactness of classification rules**

Classification by Decision Tree Induction (16 Mark Question)

Decision tree induction is the learning of decision trees from class-labeled training tuples. A decision tree is a flowchart-like tree structure, where each internal node (nonleaf node) denotes a test on an attribute, each branch represents an outcome of the test, and each leaf node (or *terminal node*) holds a class label. The topmost node in a tree is the root node.



A decision tree for the concept *buys_computer*, indicating whether a customer at *AllElectronics* is likely to purchase a computer. Each internal (nonleaf) node represents a test on an attribute. Each leaf node represents a class (either *buys_computer* = *yes* or *buys_computer* = *no*).

A typical decision tree is shown in Figure. It represents the concept *buys computer*, that is, it predicts whether a customer at *AllElectronics* is likely to purchase a computer. Internal nodes are denoted by rectangles, and leaf nodes are denoted by ovals. Some decision tree algorithms produce only *binary* trees (where each internal node branches to exactly two other nodes), whereas others can produce non binary trees.

"How are decision trees used for classification?" Given a tuple, ***X***, for which the associated class label is unknown, the attribute values of the tuple are tested against the decision tree. A path is traced from the root to a leaf node, which holds the class prediction for that tuple. Decision trees can easily be converted to classification rules.

Decision tree

- A flow-chart-like tree structure
- Internal node denotes a test on an attribute
- Branch represents an outcome of the test
- Leaf nodes represent class labels or class distribution

- Decision tree generation consists of two phases
  - Tree construction
    - At start, all the training examples are at the root
    - Partition examples recursively based on selected attributes
  - Tree pruning
    - Identify and remove branches that reflect noise or outliers
- Use of decision tree: Classifying an unknown sample
  - Test the attribute values of the sample against the decision tree

Training Dataset
This follows an example from Quinlan's ID3

age	income	student	credit_rating
<=30	high	no	fair
<=30	high	no	excellent
31…40	high	no	fair
>40	medium	no	fair
>40	low	yes	fair
>40	low	yes	excellent
31…40	low	yes	excellent
<=30	medium	no	fair
<=30	low	yes	fair
>40	medium	yes	fair
<=30	medium	yes	excellent
31…40	medium	no	excellent
31…40	high	yes	fair
>40	medium	no	excellent

Algorithm for Decision Tree Induction

- Basic algorithm (a greedy algorithm)
  - Tree is constructed in a top-down recursive divide-and-conquer manner
  - At start, all the training examples are at the root
  - Attributes are categorical (if continuous-valued, they are discretized in advance)
  - Examples are partitioned recursively based on selected attributes
  - Test attributes are selected on the basis of a heuristic or statistical measure (e.g., information gain)
- Conditions for stopping partitioning
  - All samples for a given node belong to the same class
  - There are no remaining attributes for further partitioning – majority voting is employed for classifying the leaf
  - There are no samples left

Extracting Classification Rules from Trees

- Represent the knowledge in the form of IF-THEN rules
- One rule is created for each path from the root to a leaf
- Each attribute-value pair along a path forms a conjunction
- The leaf node holds the class prediction
- Rules are easier for humans to understand
- Example

IF *age* = "<=30" AND *student* = "*no*" THEN *buys_computer* = "*no*"
IF *age* = "<=30" AND *student* = "*yes*" THEN *buys_computer* = "*yes*"
IF *age* = "31…40" THEN *buys_computer* = "*yes*"
IF *age* = ">40" AND *credit_rating* = "*excellent*" THEN *buys_computer* = "*yes*"
IF *age* = ">40" AND *credit_rating* = "*fair*" THEN *buys_computer* = "*no*"

Decision Tree Induction

The algorithm is called with three parameters: *D*, *attribute list*, and *Attribute selection method*. We refer to *D* as a data partition. Initially, it is the complete set of training tuples and their associated class labels. The parameter *attribute list* is a list of attributes describing the tuples. *Attribute selection method* specifies a heuristic procedure for selecting the attribute that "best" discriminates the given tuples

accordingto class. This procedure employs an attribute selection measure, such as information gain or the gini index. Whether the tree is strictly binary is generally driven by the attribute selection measure. Some attribute selection measures, such as the gini index, enforce the resulting tree to be binary. Others, like information gain, do not, therein allowing multiway splits (i.e., two or more branches to be grown from a node).

Algorithm: Generate_decision_tree. Generate a decision tree from the training tuples of data partition D.

Input:

- Data partition, D, which is a set of training tuples and their associated class labels;
- *attribute_list*, the set of candidate attributes;
- *Attribute_selection_method*, a procedure to determine the splitting criterion that "best" partitions the data tuples into individual classes. This criterion consists of a *splitting_attribute* and, possibly, either a *split point* or *splitting subset*.

Output: A decision tree.

Method:

```
(1)  create a node N;
(2)  if tuples in D are all of the same class, C then
(3)      return N as a leaf node labeled with the class C;
(4)  if attribute_list is empty then
(5)      return N as a leaf node labeled with the majority class in D; // majority voting
(6)  apply Attribute_selection_method(D, attribute_list) to find the "best" splitting_criterion;
(7)  label node N with splitting_criterion;
(8)  if splitting_attribute is discrete-valued and
         multiway splits allowed then // not restricted to binary trees
(9)      attribute_list ← attribute_list − splitting_attribute; // remove splitting_attribute
(10) for each outcome j of splitting_criterion
     // partition the tuples and grow subtrees for each partition
(11)     let Dj be the set of data tuples in D satisfying outcome j; // a partition
(12)     if Dj is empty then
(13)         attach a leaf labeled with the majority class in D to node N;
(14)     else attach the node returned by Generate_decision_tree(Dj, attribute_list) to node N;
     endfor
(15) return N;
```

Basic algorithm for inducing a decision tree from training tuples.

Algorithm for Decision Tree Induction (pseudocode)
Algorithm GenDecTree(Sample S, Attlist A)

1. create a node N
2. If all samples are of the same class C then label N with C; terminate;
3. If A is empty then label N with the most common class C in S (majority voting); terminate;
4. Select a∈A, with the highest information gain; Label N with a;
5. For each value v of a:
    a. Grow a branch from N with condition a=v;
    b. Let S_v be the subset of samples in S with a=v;
    c. If S_v is empty then attach a leaf labeled with the most common class in S;

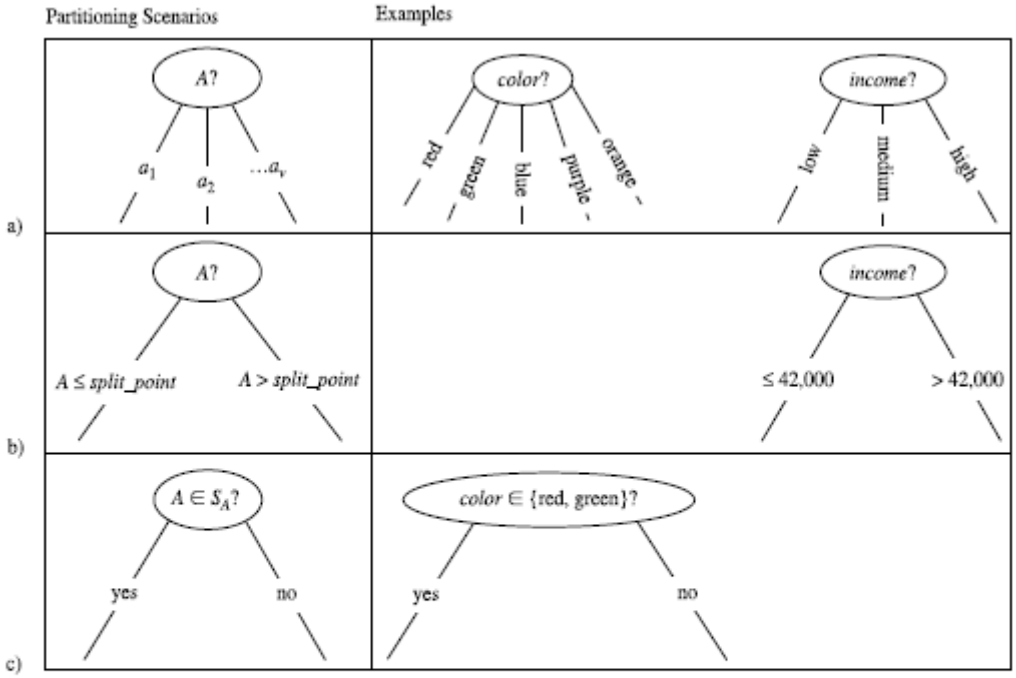
d. Else attach the node generated by GenDecTree(S_v, A-a)

- The tree starts as a single node, *N*, representing the training tuples in *D* (step 1)

- If the tuples in *D* are all of the same class, then node *N* becomes a leaf and is labeled with that class (steps 2 and 3). Note that steps 4 and 5 are terminating conditions. All of the terminating conditions are explained at the end of the algorithm.

- Otherwise, the algorithm calls *Attribute selection method* to determine the splitting criterion. The splitting criterion tells us which attribute to test at node *N* by determining the "best" way to separate or partition the tuples in *D* into individual classes(step 6). The splitting criterion also tells us which branches to grow from node *N* with respect to the outcomes of the chosen test. More specifically, the splitting criterion indicates the splitting attribute and may also indicate either a split-point or a splitting subset. The splitting criterion is determined so that, ideally, the resulting partitions at each branch are as "pure" as possible. A partition is pure if all of the tuples in it belong to the same class. In other words, if we were to split up the tuples in *D* according to the mutually exclusive outcomes of the splitting criterion, we hope for the resulting partitions to be as pure as possible.

- The node *N* is labeled with the splitting criterion, which serves as a test at the node (step 7). A branch is grown from node *N* for each of the outcomes of the splitting criterion. The tuples in *D* are partitioned accordingly (steps 10 to 11). There are three possible scenarios, as illustrated in Figure. Let *A* be the splitting attribute. *A* has *v* distinct values, {*a*1, *a*2, : : : , *av}*, based on the training data.



Three possibilities for partitioning tuples based on the splitting criterion, shown with examples. Let A be the splitting attribute. (a) If A is discrete-valued, then one branch is grown for each known value of A. (b) If A is continuous-valued, then two branches are grown, corresponding to $A \leq split\_point$ and $A > split\_point$. (c) If A is discrete-valued and a binary tree must be produced, then the test is of the form $A \in S_A$, where S_A is the splitting subset for A.

Avoid Overfitting in Classification

- The generated tree may overfit the training data
  - Too many branches, some may reflect anomalies due to noise or outliers
  - Result is in poor accuracy for unseen samples
- Two approaches to avoid overfitting
  - Prepruning: Halt tree construction early—do not split a node if this would result in the goodness measure falling below a threshold
    - Difficult to choose an appropriate threshold
  - Postpruning: Remove branches from a "fully grown" tree—get a sequence of progressively pruned trees
    - Use a set of data different from the training data to decide which is the "best pruned tree"

Attribute Selection Measure: Information Gain (ID3/C4.5)

- Select the attribute with the highest information gain
- Let p_i be the probability that an arbitrary tuple in D belongs to class C_i, estimated by $|C_{i,D}|/|D|$

Expected information (entropy) needed to classify a tuple in D:

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i)$$

- **Information needed (after using A to split D into v partitions) to classify D:**

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times I(D_j)$$

- **Information gained by branching on attribute A**

$$Gain(A) = Info(D) - Info_A(D)$$

- Class P: buys_computer = "yes"
- Class N: buys_computer = "no"

$$Info(D) = I(9,5) = -\frac{9}{14}\log_2(\frac{9}{14}) - \frac{5}{14}\log_2(\frac{5}{14}) = 0.940$$

$$Info_{age}(D) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) = 0.694$$

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

$$Gain(income) = 0.029$$
$$Gain(student) = 0.151$$
$$Gain(credit\_rating) = 0.048$$

Attribute Selection Measures

An attribute selection measure is a heuristic for selecting the splitting criterion that "best" separates a given data partition, *D*, of class-labeled training tuples into individual classes. If we were to split *D* into smaller partitions according to the outcomes of the splitting criterion, ideally each partition would be pure (i.e., all of the tuples that fall into a given partition would belong to the same class). Conceptually, the "best" splitting criterion is the one that most closely results in such a scenario. Attribute selection measures are also known as splitting rules because they determine how the tuples at a given node are to be split. The attribute selection measure provides a ranking for each attribute describing the given training tuples. The attribute having the best score for the measure6 is chosen as the *splitting attribute* for the given tuples. If the splitting attribute is continuous-valued or if we are restricted to binary trees then, respectively, either a *split point* or a *splitting subset* must also be determined as part of the splitting criterion. The tree node created for partition *D* is labeled with the splitting criterion, branches are grown for each outcome of the criterion, and the tuples are partitioned accordingly. This section describes three popular attribute selection measures—*information gain, gain ratio*, and *gini index*

Information gain

ID3 uses information gain as its attribute selection measure.

$$Info(D) = -\sum_{i=1}^{m} p_i \log_2(p_i),$$

$$Info_A(D) = \sum_{j=1}^{v} \frac{|D_j|}{|D|} \times Info(D_j).$$

Information gain is defined as the difference between the original information requirement (i.e., based on just the proportion of classes) and the new requirement (i.e., obtained after partitioning on *A*). That is,

$$Gain(A) = Info(D) - Info_A(D).$$

In other words, *Gain(A)* tells us how much would be gained by branching on *A*. It is the expected reduction in the information requirement caused by knowing the value of *A*. The attribute *A* with the highest information gain, (*Gain*(*A*)), is chosen as the splitting attribute at node *N*.

Example 6.1 Induction of a decision tree using information gain.

Table 6.1 presents a training set, *D*, of class-labeled tuples randomly selected from the *AllElectronics* customer database. (The data are adapted from [Qui86]. In this example, each attribute is discrete-valued. Continuous-valued attributes have been generalized.) The class label attribute, *buys computer*, has two distinct values (namely, {*yes, no*}); therefore, there are two distinct classes (that is, *m* = 2). Let class *C*1 correspond to *yes* and class *C*2 correspond to *no.* There are nine tuples of class *yes* and five tuples of class *no*. A (root) node *N* is created for the tuples in *D*. To find the splitting criterion for

these tuples, we must compute the information gain of each attribute. We first use Equation (6.1) to compute the expected information needed to classify a tuple in D:

$$Info(D) = -\frac{9}{14}\log_2\left(\frac{9}{14}\right) - \frac{5}{14}\log_2\left(\frac{5}{14}\right) = 0.940 \text{ bits}.$$

Table 6.1 Class-labeled training tuples from the *AllElectronics* customer database.

RID	age	income	student	credit_rating	Class: buys_computer
1	youth	high	no	fair	no
2	youth	high	no	excellent	no
3	middle_aged	high	no	fair	yes
4	senior	medium	no	fair	yes
5	senior	low	yes	fair	yes
6	senior	low	yes	excellent	no
7	middle_aged	low	yes	excellent	yes
8	youth	medium	no	fair	no
9	youth	low	yes	fair	yes
10	senior	medium	yes	fair	yes
11	youth	medium	yes	excellent	yes
12	middle_aged	medium	no	excellent	yes
13	middle_aged	high	yes	fair	yes
14	senior	medium	no	excellent	no

The expected information needed to classify a tuple in D if the tuples are partitioned according to *age* is

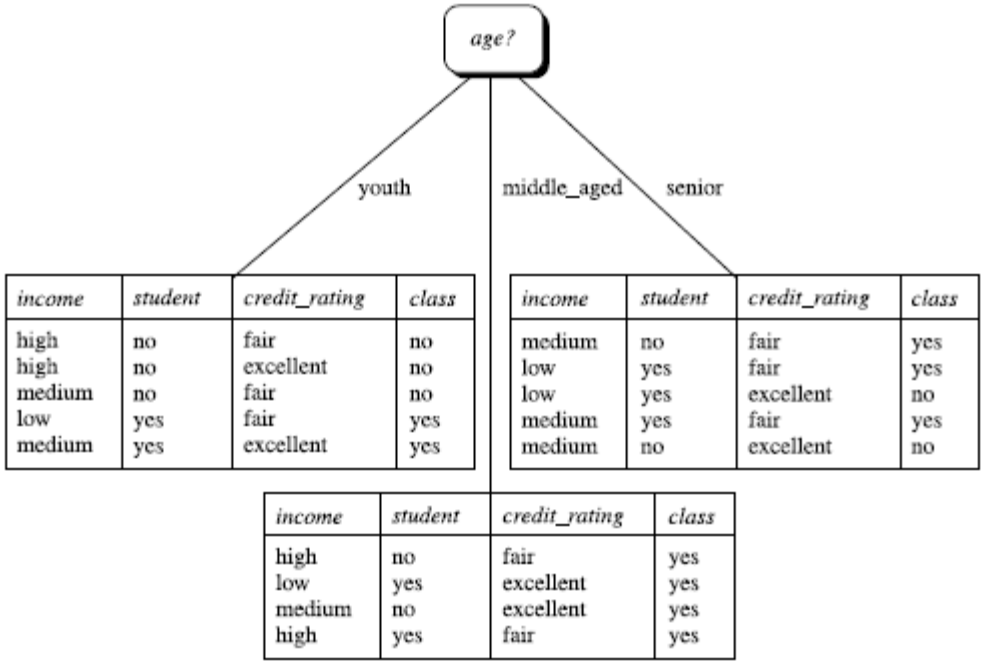
$$\begin{aligned} Info_{age}(D) &= \frac{5}{14} \times \left(-\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5}\right) \\ &+ \frac{4}{14} \times \left(-\frac{4}{4}\log_2\frac{4}{4} - \frac{0}{4}\log_2\frac{0}{4}\right) \\ &+ \frac{5}{14} \times \left(-\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5}\right) \\ &= 0.694 \text{ bits}. \end{aligned}$$

Hence, the gain in information from such a partitioning would be

$$Gain(age) = Info(D) - Info_{age}(D) = 0.940 - 0.694 = 0.246 \text{ bits}.$$

Similarly, we can compute *Gain*(*income*) = 0.029 bits, *Gain*(*student*) = 0.151 bits, and *Gain*(*credit rating*) = 0.048 bits. Because *age* has the highest information gain among the attributes, it is selected as the splitting attribute. Node N is labeled with *age*, and branches are grown for each of the attribute's values. The tuples are then partitioned accordingly, as shown in Figure 6.5. Notice that the tuples falling

into the partition for *age* = *middle aged* all belong to the same class. Because they all belong to class *"yes,"* a leaf should therefore be created at the end of this branch and labeled with *"yes."* The final decision tree returned by the algorithm is shown in Figure 6.5.



income	*student*	*credit_rating*	*class*
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

income	*student*	*credit_rating*	*class*
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

income	*student*	*credit_rating*	*class*
high	no	fair	yes
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

Figure 6.5 The attribute *age* has the highest information gain and therefore becomes the splitting attribute at the root node of the decision tree. Branches are grown for each outcome of *age*. The tuples are shown partitioned accordingly.

Computing Information-Gain for Continuous-Value Attributes

- Let attribute A be a continuous-valued attribute
- Must determine the *best split point* for A
  - Sort the value A in increasing order
  - Typically, the midpoint between each pair of adjacent values is considered as a possible *split point*
    - $(a_i+a_{i+1})/2$ is the midpoint between the values of a_i and a_{i+1}
  - The point with the *minimum expected information requirement* for A is selected as the split-point for A
- Split:D1 is the set of tuples in D satisfying A ≤ split-point, and D2 is the set of tuples in D satisfying A > split-point

Gain Ratio for Attribute Selection (C4.5)

- Information gain measure is biased towards attributes with a large number of values
- C4.5 (a successor of ID3) uses gain ratio to overcome the problem (normalization to information gain)

$$SplitInfo_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \times \log_2\left(\frac{|D_j|}{|D|}\right)$$

$$SplitInfo_A(D) = -\frac{4}{14} \times \log_2\left(\frac{4}{14}\right) - \frac{6}{14} \times \log_2\left(\frac{6}{14}\right) - \frac{4}{14} \times \log_2\left(\frac{4}{14}\right) = 0.926$$

- GainRatio(A) = Gain(A)/SplitInfo(A)

- Ex. gain_ratio(income) = 0.029/0.926 = 0.031
- The attribute with the maximum gain ratio is selected as the splitting attribute

Gini index

- If a data set *D* contains examples from *n* classes, gini index, *gini*(*D*) is defined as

$$gini(D) = 1 - \sum_{j=1}^{n} p_j^2$$

where p_j is the relative frequency of class *j* in *D*

- If a data set *D* is split on A into two subsets D_1 and D_2, the *gini* index *gini*(*D*) is defined as

$$gini_A(D) = \frac{|D_1|}{|D|} gini(D_1) + \frac{|D_2|}{|D|} gini(D_2)$$

- Reduction in Impurity:

$$\Delta gini(A) = gini(D) - gini_A(D)$$

- The attribute provides the smallest $gini_{split}(D)$ (or the largest reduction in impurity) is chosen to split the node (*need to enumerate all the possible splitting points for each attribute*)

Bayesian Classification (16 mark Question)

"What are Bayesian classifiers?" Bayesian classifiers are statistical classifiers. They can predict class membership probabilities, such as the probability that a given tuple belongs to a particular class.

Bayesian classification is based on Bayes' theorem, described below. Studies comparing classification algorithms have found a simple Bayesian classifier known as the *naïve Bayesian classifier* to be comparable in performance with decision tree and selected neural network classifiers. Bayesian classifiers have also exhibited high accuracy and speed when applied to large databases.

1) Bayes' Theorem

Bayes' theorem is named after Thomas Bayes, a nonconformist English clergyman who did early work in probability and decision theory during the 18th century. Let ***X*** be a data tuple. In Bayesian terms, ***X*** is considered "evidence." As usual, it is described by measurements made on a set of *n* attributes. Let *H* be some hypothesis, such as that the data tuple ***X*** belongs to a specified class *C*. For classification problems, we want to determine *P*(*H*j***X***), the probability that the hypothesis *H* holds given the "evidence"

or observed data tuple ***X***. In other words, we are looking for the probability that tuple ***X*** belongs to class *C*, given that we know the attribute description of ***X***.

"How are these probabilities estimated?" $P(H)$, $P(\mathbf{X}|H)$, and $P(\mathbf{X})$ may be estimated from the given data, as we shall see below. Bayes' theorem is useful in that it provides a way of calculating the posterior probability, $P(H|\mathbf{X})$, from $P(H)$, $P(\mathbf{X}|H)$, and $P(\mathbf{X})$. Bayes' theorem is

$$P(H|X) = \frac{P(X|H)P(H)}{P(X)}.$$

2) Naïve Bayesian Classification

The **naïve Bayesian** classifier, or **simple Bayesian** classifier, works as follows:

1. Let D be a training set of tuples and their associated class labels. As usual, each tuple is represented by an n-dimensional attribute vector, $X = (x_1, x_2, \ldots, x_n)$, depicting n measurements made on the tuple from n attributes, respectively, $A_1, A_2, \ldots, A_n$.
2. Suppose that there are m classes, $C_1, C_2, \ldots, C_m$. Given a tuple, X, the classifier will predict that X belongs to the class having the highest posterior probability, conditioned on X. That is, the naïve Bayesian classifier predicts that tuple X belongs to the class C_i if and only if

   $$P(C_i|X) > P(C_j|X) \quad \text{for } 1 \le j \le m, j \ne i.$$

   Thus we maximize $P(C_i|X)$. The class C_i for which $P(C_i|X)$ is maximized is called the *maximum posteriori hypothesis*. By Bayes' theorem (Equation (6.10)),

   $$P(C_i|X) = \frac{P(X|C_i)P(C_i)}{P(X)}. \tag{6.11}$$

3. As $P(X)$ is constant for all classes, only $P(X|C_i)P(C_i)$ need be maximized. If the class prior probabilities are not known, then it is commonly assumed that the classes are equally likely, that is, $P(C_1) = P(C_2) = \cdots = P(C_m)$, and we would therefore maximize $P(X|C_i)$. Otherwise, we maximize $P(X|C_i)P(C_i)$. Note that the class prior probabilities may be estimated by $P(C_i) = |C_{i,D}|/|D|$, where $|C_{i,D}|$ is the number of training tuples of class C_i in D.
4. Given data sets with many attributes, it would be extremely computationally expensive to compute $P(X|C_i)$. In order to reduce computation in evaluating $P(X|C_i)$, the naive assumption of **class conditional independence** is made. This presumes that the values of the attributes are conditionally independent of one another, given the class label of the tuple (i.e., that there are no dependence relationships among the attributes). Thus,

$$P(X|C_i) = \prod_{k=1}^{n} P(x_k|C_i)$$
$$= P(x_1|C_i) \times P(x_2|C_i) \times \cdots \times P(x_n|C_i).$$

5. In order to predict the class label of X, $P(X|C_i)P(C_i)$ is evaluated for each class C_i. The classifier predicts that the class label of tuple X is the class C_i if and only if

$$P(X|C_i)P(C_i) > P(X|C_j)P(C_j) \quad \text{for } 1 \leq j \leq m, j \neq i. \tag{6.15}$$

In other words, the predicted class label is the class C_i for which $P(X|C_i)P(C_i)$ is the maximum.

Example 6.4 **Predicting a class label using naïve Bayesian classification.** We wish to predict the class label of a tuple using naïve Bayesian classification, given the same training data as in Example 6.3 for decision tree induction. The training data are in Table 6.1. The data tuples are described by the attributes *age*, *income*, *student*, and *credit_rating*. The class label attribute, *buys_computer*, has two distinct values (namely, {*yes, no*}). Let C_1 correspond to the class *buys_computer* = *yes* and C_2 correspond to *buys_computer* = *no*. The tuple we wish to classify is

$$X = (age = youth, income = medium, student = yes, credit\_rating = fair)$$

We need to maximize $P(X|C_i)P(C_i)$, for $i = 1, 2$. $P(C_i)$, the prior probability of each class, can be computed based on the training tuples:

$$P(buys\_computer = yes) = 9/14 = 0.643$$
$$P(buys\_computer = no) = 5/14 = 0.357$$

To compute $PX|C_i)$, for $i = 1, 2$, we compute the following conditional probabilities:

$$P(age = youth \mid buys\_computer = yes) = 2/9 = 0.222$$
$$P(age = youth \mid buys\_computer = no) = 3/5 = 0.600$$
$$P(income = medium \mid buys\_computer = yes) = 4/9 = 0.444$$
$$P(income = medium \mid buys\_computer = no) = 2/5 = 0.400$$
$$P(student = yes \mid buys\_computer = yes) = 6/9 = 0.667$$
$$P(student = yes \mid buys\_computer = no) = 1/5 = 0.200$$
$$P(credit\_rating = fair \mid buys\_computer = yes) = 6/9 = 0.667$$
$$P(credit\_rating = fair \mid buys\_computer = no) = 2/5 = 0.400$$

Using the above probabilities, we obtain

$$P(X|buys\_computer = yes) = P(age = youth \mid buys\_computer = yes) \times$$
$$P(income = medium \mid buys\_computer = yes) \times$$
$$P(student = yes \mid buys\_computer = yes) \times$$
$$P(credit\_rating = fair \mid buys\_computer = yes)$$
$$= 0.222 \times 0.444 \times 0.667 \times 0.667 = 0.044.$$

Similarly,

$$P(X|buys\_computer = no) = 0.600 \times 0.400 \times 0.200 \times 0.400 = 0.019.$$

To find the class, C_i, that maximizes $P(X|C_i)P(C_i)$, we compute

$$P(X|buys\_computer = yes)P(buys\_computer = yes) = 0.044 \times 0.643 = 0.028$$

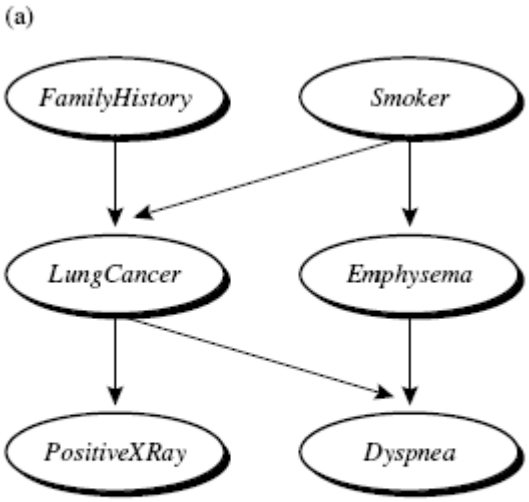
$$P(X|buys\_computer = no)P(buys\_computer = no) = 0.019 \times 0.357 = 0.007$$

Therefore, the naïve Bayesian classifier predicts *buys_computer = yes* for tuple X. ■

3) Bayesian Belief Networks

The naïve Bayesian classifier makes the assumption of class conditional independence, that is, given the class label of a tuple, the values of the attributes are assumed to be conditionally independent of one another. This simplifies computation. When the assumption holds true, then the naïve Bayesian classifier is the most accurate in comparison with all other classifiers. In practice, however, dependencies can exist between variables. Bayesian belief networks specify joint conditional probability distributions. They allow class conditional independencies to be defined between subsets of variables. They provide a graphical model of causal relationships, on which learning can be performed. Trained Bayesian belief networks can be used for classification. Bayesian belief networks are also known as belief networks, Bayesian networks, and probabilistic networks. For brevity, we will refer to them as belief networks.

A belief network is defined by two components—a *directed acyclic graph* and a set of *conditional probability tables* (Figure 6.11). Each node in the directed acyclic graph represents a random variable. The variables may be discrete or continuous-valued. They may correspond to actual attributes given in the data or to "hidden variables" believed to form a relationship (e.g., in the case of medical data, a hidden variable may indicate a syndrome, representing a number of symptoms that, together, characterize a specific disease). Each arc represents a probabilistic dependence. If an arc is drawn from a node Y to a node Z, then Y is a parent or immediate predecessor of Z, and Z is a descendant of Y. *Each variable is conditionally independent of its non descendants in the graph, given its parents.*



(b)

	FH, S	FH, ~S	~FH, S	~FH, ~S
LC	0.8	0.5	0.7	0.1
~LC	0.2	0.5	0.3	0.9

A simple Bayesian belief network: (a) A proposed causal model, represented by a directed acyclic graph. (b) The conditional probability table for the values of the variable *LungCancer (LC)* showing each possible combination of the values of its parent nodes, *FamilyHistory (FH)* and *Smoker (S)*. Figure is adapted from [RBKK95].

A belief network has one conditional probability table (CPT) for each variable. The CPT for a variable Y specifies the conditional distribution $P(Y|Parents(Y))$, where $Parents(Y)$ are the parents of Y. Figure(b) shows a CPT for the variable *LungCancer*. The conditional probability for each known value of *LungCancer* is given for each possible combination of values of its parents. For instance, from the upper leftmost and bottom rightmost entries, respectively, we see that

$$P(LungCancer = yes \mid FamilyHistory = yes, Smoker = yes) = 0.8$$
$$P(LungCancer = no \mid FamilyHistory = no, Smoker = no) = 0.9$$

Let $\boldsymbol{X} = (x1, : : : , xn)$ be a data tuple described by the variables or attributes $Y1, : : : , Yn$, respectively. Recall that each variable is conditionally independent of its non descendants in the network graph, given its parents. This allows the network to provide a complete representation of the existing joint probability distribution with the
following equation:

$$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i | Parents(Y_i)),$$

Rule-Based Classification

We look at rule-based classifiers, where the learned model is represented as a set of IF-THEN rules. We first examine how such rules are used for classification. We then study ways in which they can be generated, either from a decision tree or directly from the training data using a *sequential covering algorithm.*

1) Using IF-THEN Rules for Classification

Rules are a good way of representing information or bits of knowledge. A rule-based classifier uses a set of IF-THEN rules for classification. An IF-THEN rule is an expression of the form

IF *condition* THEN *conclusion*.

An example is rule *R*1,

R1: IF *age* = *youth* AND *student* = *yes* THEN *buys computer* = *yes*.

The "IF"-part (or left-hand side)of a rule is known as the rule antecedent or precondition. The "THEN"-part (or right-hand side) is the rule consequent. In the rule antecedent, the condition consists of one or more *attribute tests* (such as *age* = *youth*, and *student* = *yes*) that are logically ANDed. The rule's consequent contains a class prediction (in this case, we are predicting whether a customer will buy a computer). *R*1 can also be written as

$$\text{R1: } (age = youth) \wedge (student = yes) \Rightarrow (buys\_computer = yes).$$

If the condition (that is, all of the attribute tests) in a rule antecedent holds true for a given tuple, we say that the rule antecedent is satisfied (or simply, that the rule is satisfied) and that the rule covers the tuple.

A rule *R* can be assessed by its coverage and accuracy. Given a tuple, ***X***, from a class labeled data set *D*, let *ncovers* be the number of tuples covered by *R*; *ncorrect* be the number of tuples correctly classified by *R*; and |*D*| be the number of tuples in *D*. We can define the coverage and accuracy of *R* as

$$coverage(R) = \frac{n_{covers}}{|D|}$$

$$accuracy(R) = \frac{n_{correct}}{n_{covers}}.$$

That is, a rule's coverage is the percentage of tuples that are covered by the rule (i.e. whose attribute values hold true for the rule's antecedent). For a rule's accuracy, we look at the tuples that it covers and see what percentage of them the rule can correctly classify.

2) Rule Extraction from a Decision Tree

We learned how to build a decision tree classifier from a set of training data. Decision tree classifiers are a popular method of classification—it is easy to understand how decision trees work and they are known for their accuracy. Decision trees can become large and difficult to interpret. In this subsection, we look at how to build a rule based classifier by extracting IF-THEN rules from a decision tree. In comparison with a decision tree, the IF-THEN rules may be easier for humans to understand, particularly if the decision tree is very large.

To extract rules from a decision tree, one rule is created for each path from the root to a leaf node. Each splitting criterion along a given path is logically ANDed to form the rule antecedent ("IF" part). The leaf node holds the class prediction, forming the rule consequent ("THEN" part).

Extracting classification rules from a decision tree. The decision tree of Figure 6.2 can be converted to classification IF-THEN rules by tracing the path from the root node to each leaf node in the tree. The rules extracted from Figure 6.2 are

*R*1: IF *age* = *youth* AND *student* = *no* THEN *buys_computer* = *no*
*R*2: IF *age* = *youth* AND *student* = *yes* THEN *buys_computer* = *yes*
*R*3: IF *age* = *middle_aged* THEN *buys_computer* = *yes*
*R*4: IF *age* = *senior* AND *credit_rating* = *excellent* THEN *buys_computer* = *yes*
*R*5: IF *age* = *senior* AND *credit_rating* = *fair* THEN *buys_computer* = *no*

■

Classification by Backpropagation

"What is backpropagation?" Backpropagation is a neural network learning algorithm. The field of neural networks was originally kindled by psychologists and neurobiologists who sought to develop and test computational analogues of neurons. Roughly speaking, a neural network is a set of connected input/output units in which each connection has a weight associated with it. During the learning phase, the network learns by adjusting the weights so as to be able to predict the correct class label of the input tuples. Neural network learning is also referred to as connectionist learning due to the connections between units.

Neural networks involve long training times and are therefore more suitable for applications where this is feasible. They require a number of parameters that are typically best determined empirically, such as the network topology or "structure." Neural networks have been criticized for their poor interpretability. For example, it is difficult for humans to interpret the symbolic meaning behind the learned weights and of "hidden units" in the network. These features initially made neural networks less desirable for data mining.

1) A Multilayer Feed-Forward Neural Network

The backpropagation algorithm performs learning on a *multilayer feed-forward* neural network. It iteratively learns a set of weights for prediction of the class label of tuples. A multilayer feed-forward neural network consists of an *input layer*, one or more *hidden layers*, and an *output layer*. An example of a multilayer feed-forward network is shown in Figure 6.15.

Each layer is made up of units. The inputs to the network correspond to the attributes measured for each training tuple. The inputs are fed simultaneously into the units making up the input layer. These inputs pass through the input layer and are then weighted and fed simultaneously to a second layer of "neuron like" units, known as a hidden layer. The outputs of the hidden layer units can be input to another hidden layer, and so on. The number of hidden layers is arbitrary, although in practice, usually only one is used. The weighted outputs of the last hidden layer are input to units making up the output layer, which emits the network's prediction for given tuples.

The units in the input layer are called input units. The units in the hidden layers and output layer are sometimes referred to as neurodes, due to their symbolic biological basis, or as output units. The multilayer neural network shown in Figure 6.15 has two layers

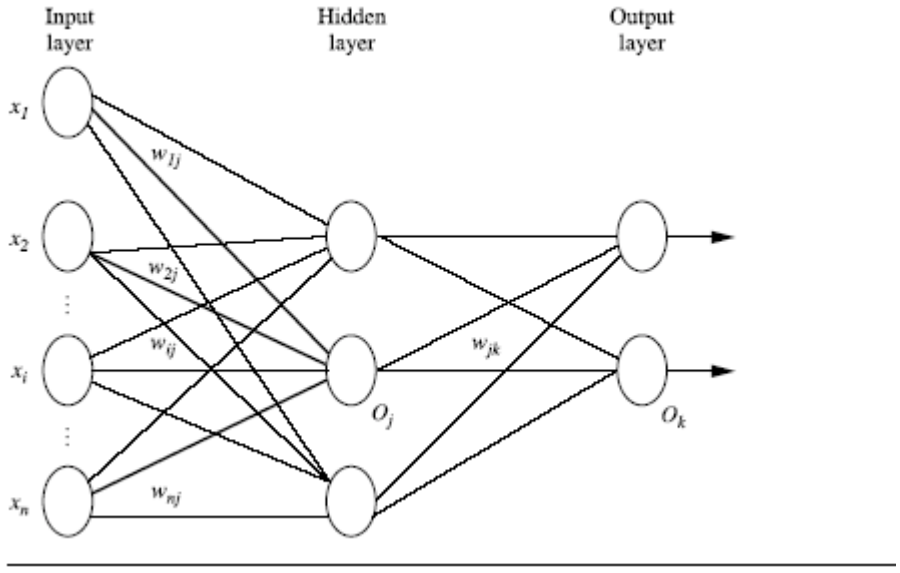


Figure 6.15 A multilayer feed-forward neural network.

2) Defining a Network Topology

"How can I design the topology of the neural network?" Before training can begin, the user must decide on the network topology by specifying the number of units in the input layer, the number of hidden layers (if more than one), the number of units in each hidden layer, and the number of units in the output layer.

Normalizing the input values for each attribute measured in the training tuples will help speed up the learning phase. Typically, input values are normalized so as to fall between 0:0 and 1:0. Discrete-valued attributes may be encoded such that there is one input unit per domain value. For example, if an attribute *A* has three possible or known values, namely f*a*0, *a*1, *a*2g, then we may assign three input units to represent *A*. That is, we may have, say, *I*0, *I*1, *I*2 as input units. Each unit is initialized to 0. If *A=a0*, then I0 is set to 1. If *A* = *a*1, *I*1 is set to 1, and so on. Neural networks can be used for both classification (to predict the class label of a given tuple) or prediction (to predict a continuous-valued output). For classification, one output unit may be used to represent two classes (where the value 1 represents one class, and the value 0 represents the other). If there are more than two classes, then one output unit per class is used.

3) Backpropagation

"How does backpropagation work?" Backpropagation learns by iteratively processing a data set of training tuples, comparing the network's prediction for each tuple with the actual known *target* value. The target value may be the known class label of the training tuple (for classification problems) or a continuous value (for prediction). For each training tuple, the weights are modified so as to minimize the mean squared error between the network's prediction and the actual target value. These modifications are made in the "backwards" direction, that is, from the output layer, through each hidden layer down to the first hidden layer (hence the name *backpropagation*). Although it is not guaranteed, in general the weights will eventually converge, and the learning process stops. The algorithm is summarized in Figure 6.16. The steps involved are expressed in terms of inputs, outputs, and errors, and may seem awkward if this is your

first look at neural network learning. However, once you become familiar with the process, you will see that each step is inherently simple. The steps are described below.

Algorithm: Backpropagation. Neural network learning for classification or prediction, using the backpropagation algorithm.

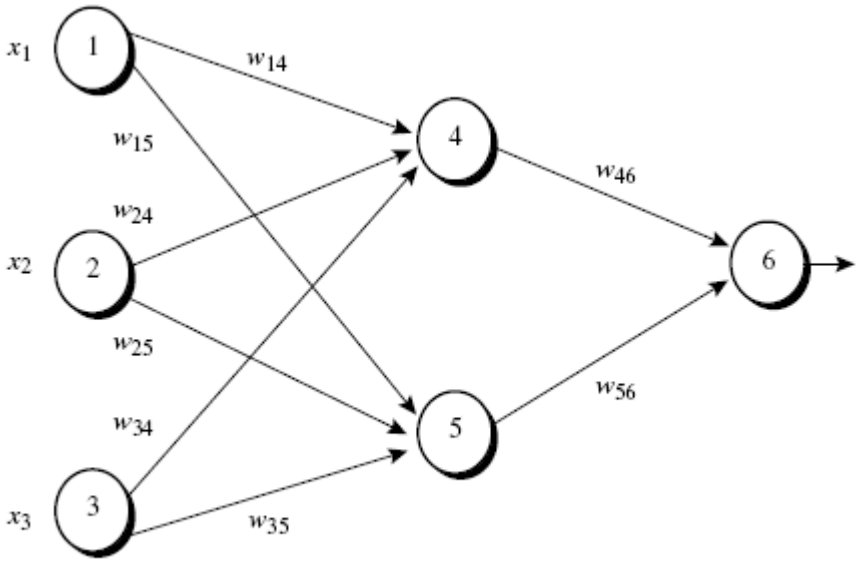
Input:

- D, a data set consisting of the training tuples and their associated target values;
- l, the learning rate;
- *network*, a multilayer feed-forward network.

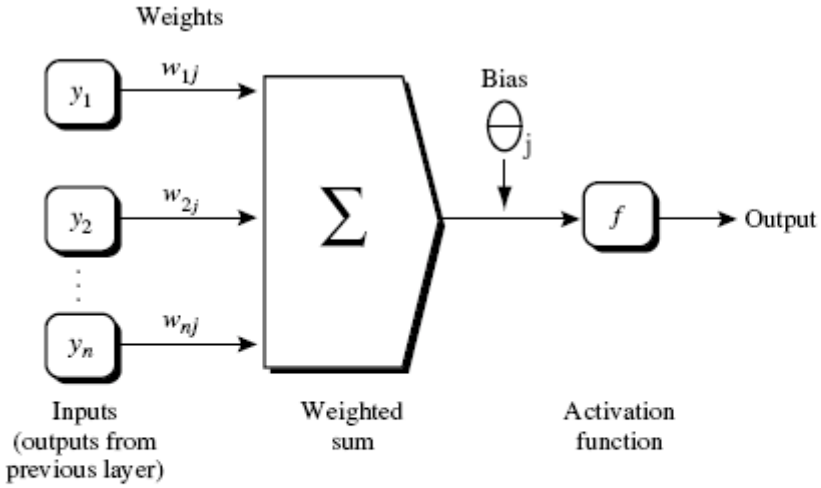
Output: A trained neural network.

Method:

(1) Initialize all weights and biases in *network*;
(2) while terminating condition is not satisfied {
(3) for each training tuple X in D {
(4) // Propagate the inputs forward:
(5) for each input layer unit j {
(6) $O_j = I_j$; // output of an input unit is its actual input value
(7) for each hidden or output layer unit j {
(8) $I_j = \sum_i w_{ij} O_i + \theta_j$; //compute the net input of unit j with respect to the previous layer, i
(9) $O_j = \frac{1}{1+e^{-I_j}}$; } // compute the output of each unit j
(10) // Backpropagate the errors:
(11) for each unit j in the output layer
(12) $Err_j = O_j(1-O_j)(T_j - O_j)$; // compute the error
(13) for each unit j in the hidden layers, from the last to the first hidden layer
(14) $Err_j = O_j(1-O_j)\sum_k Err_k w_{jk}$; // compute the error with respect to the next higher layer, k
(15) for each weight w_{ij} in *network* {
(16) $\Delta w_{ij} = (l) Err_j O_i$; // weight increment
(17) $w_{ij} = w_{ij} + \Delta w_{ij}$; } // weight update
(18) for each bias θ_j in *network* {
(19) $\Delta\theta_j = (l) Err_j$; // bias increment
(20) $\theta_j = \theta_j + \Delta\theta_j$; } // bias update
(21) } }



An example of a multilayer feed-forward neural network.



A hidden or output layer unit j: The inputs to unit j are outputs from the previous layer. These are multiplied by their corresponding weights in order to form a weighted sum, which is added to the bias associated with unit j. A nonlinear activation function is applied to the net input. (For ease of explanation, the inputs to unit j are labeled $y_1, y_2, \ldots, y_n$. If unit j were in the first hidden layer, then these inputs would correspond to the input tuple $(x_1, x_2, \ldots, x_n)$.)

Support Vector Machines

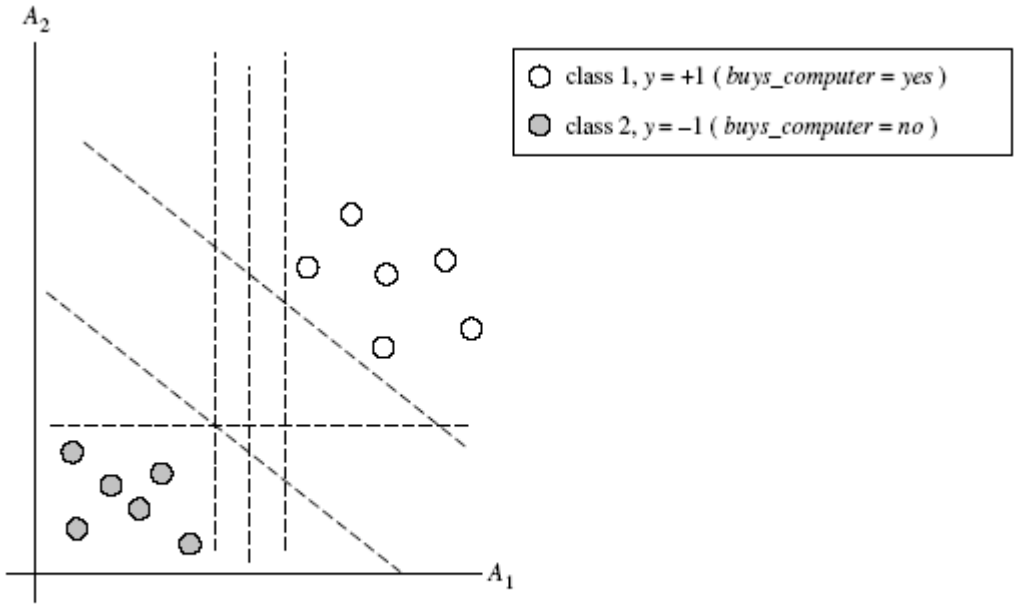
We study Support Vector Machines, a promising new method for the classification of both linear and nonlinear data. In a nutshell, a support vector machine (or SVM) is an algorithm that works as follows. It uses a nonlinear mapping to transform the original training data into a higher dimension. Within this new dimension, it searches for the linear optimal separating hyperplane (that is, a "decision boundary" separating the tuples of one class from another). With an appropriate nonlinear mapping to a

sufficiently high dimension, data from two classes can always be separated by a hyperplane. The SVM finds this hyperplane using *support vectors* ("essential" training tuples) and *margins* (defined by the support vectors).We will delve more into these new concepts further below.

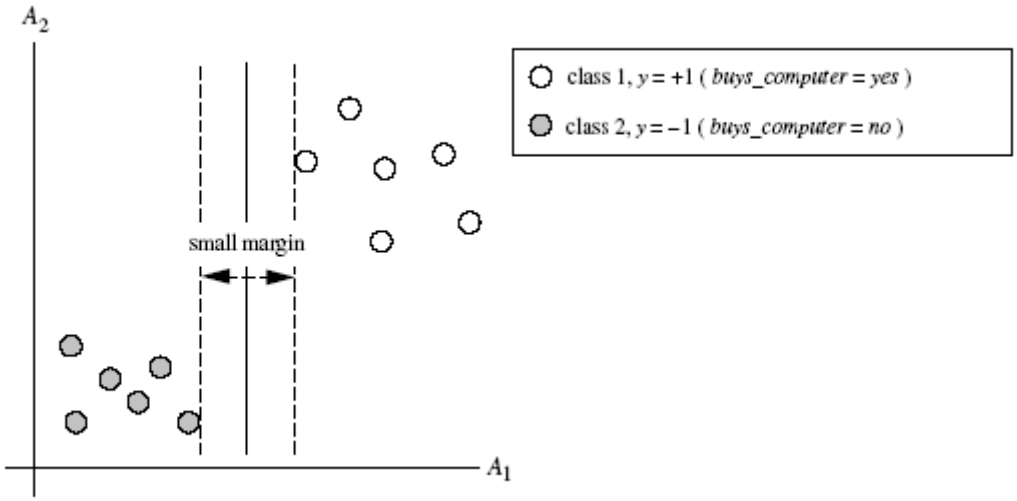
"I've heard that SVMs have attracted a great deal of attention lately. Why?" The first paper on support vector machines was presented in 1992 by Vladimir Vapnik and colleagues Bernhard Boser and Isabelle Guyon, although the groundwork for SVMs has been around since the 1960s (including early work by Vapnik and Alexei Chervonenkis on statistical learning theory). Although the training time of even the fastest SVMs can be extremely slow, they are highly accurate, owing to their ability to model complex nonlinear decision boundaries. They aremuch less prone to overfitting than othermethods. The support vectors found also provide a compact description of the learned model. SVMs can be used for prediction as well as classification. They have been applied to a number of areas, including handwritten digit recognition, object recognition, and speaker identification, as well as benchmark time-series prediction tests.

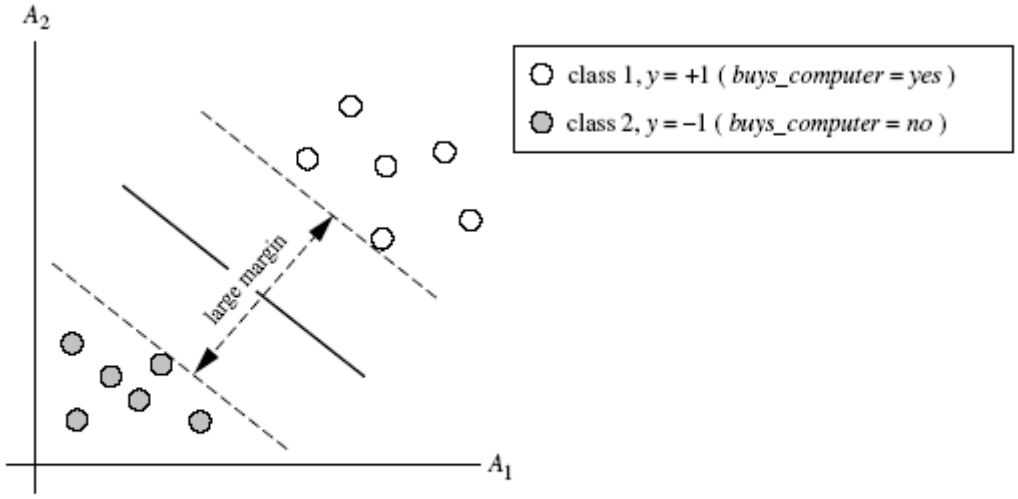
1) The Case When the Data Are Linearly Separable

An SVM approaches this problem by searching for the maximum marginal hyperplane. Consider the below Figure , which shows two possible separating hyperplanes and their associated margins. Before we get into the definition of margins, let's take an intuitive look at this figure. Both hyperplanes can correctly classify all of the given data tuples. Intuitively, however, we expect the hyperplane with the larger margin to be more accurate at classifying future data tuples than the hyperplane with the smaller margin. This is why (during the learning or training phase), the SVM searches for the hyperplane with the largest margin, that is, the *maximum marginal hyperplane* (MMH). The associated margin gives the largest separation between classes. Getting to an informal definition of margin, we can say that the shortest distance from a hyperplane to one side of its margin is equal to the shortest distance from the hyperplane to the other side of its margin, where the "sides" of the margin are parallel to the hyperplane. When dealing with the MMH, this distance is, in fact, the shortest distance from the MMH to the closest training tuple of either class.



The 2-D training data are linearly separable. There are an infinite number of (possible) separating hyperplanes or "decision boundaries." Which one is best?





Here we see just two possible separating hyperplanes and their associated margins. Which one is better? The one with the larger margin should have greater generalization accuracy.

2) The Case When the Data Are Linearly Inseparable

We learned about linear SVMs for classifying linearly separable data, but what if the data are not linearly separable no straight line cajn be found that would separate the classes. The linear SVMs we studied would not be able to find a feasible solution here. Now what?

The good news is that the approach described for linear SVMs can be extended to create *nonlinear SVMs* for the classification of *linearly inseparable data* (also called *nonlinearly separable data*, or *nonlinear data*, for short). Such SVMs are capable of finding nonlinear decision boundaries (i.e., nonlinear hypersurfaces) in input space.

"So," you may ask, *"how can we extend the linear approach?"* We obtain a nonlinear SVM by extending the approach for linear SVMs as follows. There are two main steps. In the first step, we transform the original input data into a higher dimensional space using a nonlinear mapping. Several common nonlinear mappings can be used in this step, as we will describe further below. Once the data have been transformed into the new higher space, the second step searches for a linear separating hyperplane in the new space.We again end up with a quadratic optimization problem that can be solved using the linear SVM formulation. The maximal marginal hyperplane found in the new space corresponds to a nonlinear separating hypersurface in the original space.

Associative Classification: Classification by Association Rule Analysis

Frequent patterns and their corresponding association or correlation rules characterize interesting relationships between attribute conditions and class labels, and thus have been recently used for effective classification. Association rules show strong associations between attribute-value pairs (or *items*) that occur frequently in a given data set. Association rules are commonly used to analyze the purchasing

patterns of customers in a store. Such analysis is useful in many decision-making processes, such as product placement, catalog design, and cross-marketing.

The discovery of association rules is based on *frequent itemset mining*. Many methods for frequent itemset mining and the generation of association rules were described in Chapter 5. In this section, we look at associative classification, where association rules are generated and analyzed for use in classification. The general idea is that we can search for strong associations between frequent patterns (conjunctions of attribute-value pairs) and class labels. Because association rules explore highly confident associations among multiple attributes, this approach may overcome some constraints introduced by decision-tree induction, which considers only one attribute at a time. In many studies, associative classification has been found to be more accurate than some traditional classification methods, such as C4.5. In particular, we study three main methods: CBA, CMAR, and CPAR.

Lazy Learners (or Learning from Your Neighbors)

The classification methods discussed so far in this chapter—decision tree induction, Bayesian classification, rule-based classification, classification by backpropagation, support vector machines, and classification based on association rule mining—are all examples of *eager learners.* Eager learners, when given a set of training tuples, will construct a generalization (i.e., classification) model before receiving new (e.g., test) tuples to classify. We can think of the learned model as being ready and eager to classify previously unseen tuples.

1) ***k*-Nearest-Neighbor Classifiers**

The *k*-nearest-neighbor method was first described in the early 1950s. The method is labor intensive when given large training sets, and did not gain popularity until the 1960s when increased computing power became available. It has since been widely used in the area of pattern recognition.

Nearest-neighbor classifiers are based on learning by analogy, that is, by comparing a given test tuple with training tuples that are similar to it. The training tuples are described by *n* attributes. Each tuple represents a point in an *n*-dimensional space. In this way, all of the training tuples are stored in an *n*-dimensional pattern space. When given an unknown tuple, a ***k***-nearest-neighbor classifier searches the pattern space for the *k* training tuples that are closest to the unknown tuple. These *k* training tuples are the *k* "nearest neighbors" of the unknown tuple.

"Closeness" is defined in terms of a distance metric, such as Euclidean distance. The Euclidean distance between two points or tuples, say, $\boldsymbol{X}1 = (x11, x12, : : : , x1n)$ and $\boldsymbol{X}2 = (x21, x22, : : : , x2n)$, is

$$dist(X_1, X_2) = \sqrt{\sum_{i=1}^{n} (x_{1i} - x_{2i})^2}.$$

2) **Case-Based Reasoning**

Case-based reasoning (CBR) classifiers use a database of problem solutions to solve new problems. Unlike nearest-neighbor classifiers, which store training tuples as points in Euclidean space,

CBR stores the tuples or "cases" for problem solving as complex symbolic descriptions. Business applications of CBR include problem resolution for customer service help desks, where cases describe product-related diagnostic problems. CBR has also been applied to areas such as engineering and law, where cases are either technical designs or legal rulings, respectively. Medical education is another area for CBR, where patient case histories and treatments are used to help diagnose and treat new patients.

When given a new case to classify, a case-based reasoner will first check if an identical training case exists. If one is found, then the accompanying solution to that case is returned. If no identical case is found, then the case-based reasoner will search for training cases having components that are similar to those of the new case. Conceptually, these training cases may be considered as neighbors of the new case. If cases are represented as graphs, this involves searching for subgraphs that are similar to subgraphs within the new case. The case-based reasoner tries to combine the solutions of the neighboring training cases in order to propose a solution for the new case. If incompatibilities arise with the individual solutions, then backtracking to search for other solutions may be necessary. The case-based reasoner may employ background knowledge and problem-solving strategies in order to propose a feasible combined solution.

Other Classification Methods

1) Genetic Algorithms

Genetic algorithms attempt to incorporate ideas of natural evolution. In general, genetic learning starts as follows. An initial population is created consisting of randomly generated rules. Each rule can be represented by a string of bits. As a simple example, suppose that samples in a given training set are described by two Boolean attributes, $A1$ and $A2$, and that there are two classes,$C1$ and$C2$. The rule "*IF* $A1$ *AND NOT* $A2$ *THEN* $C2$" can be encoded as the bit string "100," where the two leftmost bits represent attributes $A1$ and $A2$, respectively, and the rightmost bit represents the class. Similarly, the rule "*IF NOT* $A1$ *AND NOT* $A2$ *THEN* $C1$" can be encoded as "001." If an attribute has k values, where $k > 2$, then k bits may be used to encode the attribute's values. Classes can be encoded in a similar fashion.

Based on the notion of survival of the fittest, a new population is formed to consist of the *fittest* rules in the current population, as well as *offspring* of these rules. Typically, the fitness of a rule is assessed by its classification accuracy on a set of training samples.

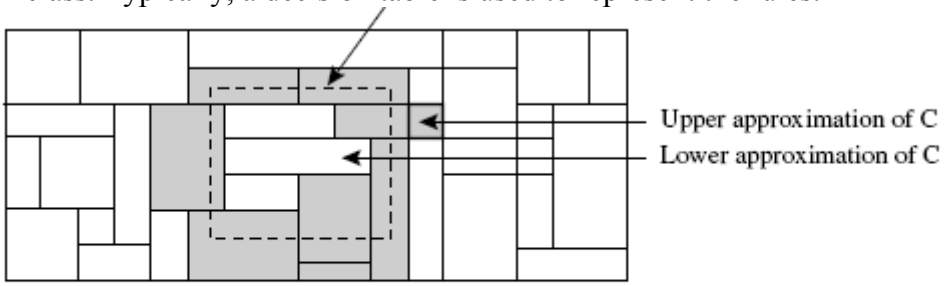
Offspring are created by applying genetic operators such as crossover and mutation. In crossover, substrings from pairs of rules are swapped to form new pairs of rules. In mutation, randomly selected bits in a rule's string are inverted. The process of generating new populations based on prior populations of rules continues until a population, P, evolves where each rule in P satisfies a prespecified fitness threshold.

Genetic algorithms are easily parallelizable and have been used for classification as well as other optimization problems. In data mining, they may be used to evaluate the fitness of other algorithms.

2) Rough Set Approach

Rough set theory can be used for classification to discover structural relationships within imprecise or noisy data. It applies to discrete-valued attributes. Continuous-valued attributes must therefore be discretized before its use.

Rough set theory is based on the establishment of equivalence classes within the given training data. All of the data tuples forming an equivalence class are indiscernible, that is, the samples are identical with respect to the attributes describing the data. Given real world data, it is common that some classes cannot be distinguished in terms of the available attributes. Rough sets can be used to approximately or "roughly" define such classes. A rough set definition for a given class, *C*, is approximated by two sets—a lower approximation of *C* and an upper approximation of *C*. The lower approximation of *C* consists of all of the data tuples that, based on the knowledge of the attributes, are certain to belong to *C* without ambiguity. The upper approximation of *C* consists of all of the tuples that, based on the knowledge of the attributes, cannot be described as not belonging to *C*. The lower and upper approximations for a class *C* are shown in Figure, where each rectangular region represents an equivalence class. Decision rules can be generated for each class. Typically, a decision table is used to represent the rules.



A rough set approximation of the set of tuples of the class *C* using lower and upper approximation sets of *C*. The rectangular regions represent equivalence classes.

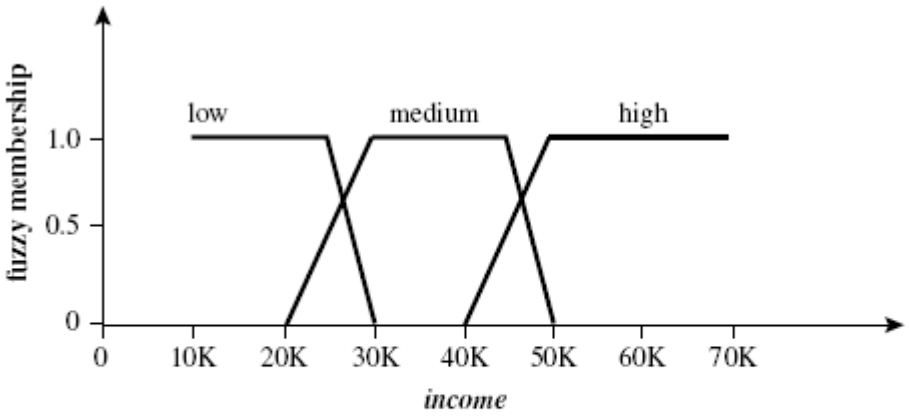
3) Fuzzy Set Approaches

Rule-based systems for classification have the disadvantage that they involve sharp cutoffs for continuous attributes. For example, consider the following rule for customer credit application approval. The rule essentially says that applications for customers who have had a job for two or more years and who have a high income (i.e., of at least $50,000) are approved:

$$IF\ (years\_employed \geq 2)\ AND\ (income \geq 50K)\ THEN\ credit = approved.$$

By above Rule, a customer who has had a job for at least two years will receive credit if her income is, say, $50,000, but not if it is $49,000. Such harsh thresholding may seem unfair. Instead, we can discretize *income* into categories such as {*low income, medium income, high income}*, and then apply fuzzy logic to allow "fuzzy" thresholds or boundaries to be defined for each category (Figure 6.25). Rather than having a precise cutoff between
categories, fuzzy logic uses truth values between 0:0 and 1:0 to represent the degree of membership that a certain value has in a given category. Each category then represents a fuzzy set. Hence, with fuzzy logic, we can capture the notion that an income of $49,000 is, more or less, high, although not as high as an

income of $50,000. Fuzzy logic systems typically provide graphical tools to assist users in converting attribute values to fuzzy truth
values.



Fuzzy truth values for *income*, representing the degree of membership of *income* values with respect to the categories {*low, medium, high*}. Each category represents a fuzzy set. Note that a given income value, *x*, can have membership in more than one fuzzy set. The membership values of *x* in each fuzzy set do not have to total to 1.

Prediction

"What if we would like to predict a continuous value, rather than a categorical label?" Numeric prediction is the task of predicting continuous (or ordered) values for given input. For example, we may wish to predict the salary of college graduates with 10 years of work experience, or the potential sales of a new product given its price. By far, the most widely used approach for numeric prediction (hereafter referred to as prediction) is regression, a statistical methodology that was developed by Sir Frances Galton (1822–1911), a mathematician who was also a cousin of Charles Darwin. In fact, many texts use the terms "regression" and "numeric prediction" synonymously. However, as we have seen, some classification techniques (such as backpropagation, support vector machines, and *k*-nearest-neighbor classifiers) can be adapted for prediction. In this section, we discuss the use of regression techniques for prediction

.

1) **Linear Regression**

Straight-line regression analysis involves a response variable, y, and a single predictor variable, x. It is the simplest form of regression, and models y as a linear function of x. That is,

$$y = b + wx, \tag{6.48}$$

where the variance of y is assumed to be constant, and b and w are **regression coefficients** specifying the Y-intercept and slope of the line, respectively. The regression coefficients, w and b, can also be thought of as weights, so that we can equivalently write,

$$y = w_0 + w_1 x. \tag{6.49}$$

These coefficients can be solved for by the **method of least squares**, which estimates the best-fitting straight line as the one that minimizes the error between the actual data and the estimate of the line. Let D be a training set consisting of values of predictor variable, x, for some population and their associated values for response variable, y. The training set contains $|D|$ data points of the form $(x_1, y_1), (x_2, y_2), \ldots, (x_{|D|}, y_{|D|})$.[12] The regression coefficients can be estimated using this method with the following equations:

$$w_1 = \frac{\sum_{i=1}^{|D|}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{|D|}(x_i - \bar{x})^2} \tag{6.50}$$

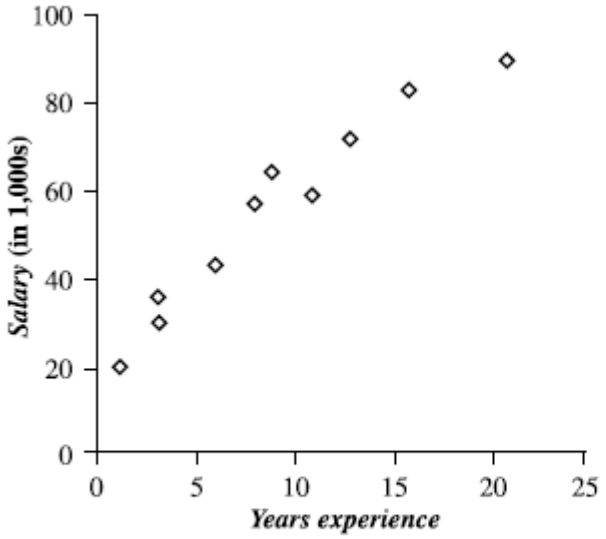
$$w_0 = \bar{y} - w_1 \bar{x} \tag{6.51}$$

where $\bar{x}$ is the mean value of $x_1, x_2, \ldots, x_{|D|}$, and $\bar{y}$ is the mean value of $y_1, y_2, \ldots, y_{|D|}$. The coefficients w_0 and w_1 often provide good approximations to otherwise complicated regression equations.

Straight-line regression using the method of least squares. Table 6.7 shows a set of paired data where x is the number of years of work experience of a college graduate and y is the

Salary data.

x years experience	*y salary (in $1000s)*
3	30
8	57
9	64
13	72
3	36
6	43
11	59
21	90
1	20
16	83



Plot of the data in Table 6.7 for Example 6.11. Although the points do not fall on a straight line, the overall pattern suggests a linear relationship between x (*years experience*) and y (*salary*).

corresponding salary of the graduate. The 2-D data can be graphed on a *scatter plot*, as in Figure 6.26. The plot suggests a linear relationship between the two variables, x and y. We model the relationship that salary may be related to the number of years of work experience with the equation $y = w_0 + w_1 x$.

Given the above data, we compute $\bar{x} = 9.1$ and $\bar{y} = 55.4$. Substituting these values into Equations (6.50) and (6.51), we get

$$w_1 = \frac{(3-9.1)(30-55.4)+(8-9.1)(57-55.4)+\cdots+(16-9.1)(83-55.4)}{(3-9.1)^2+(8-9.1)^2+\cdots+(16-9.1)^2} = 3.5$$

$$w_0 = 55.4-(3.5)(9.1) = 23.6$$

Thus, the equation of the least squares line is estimated by $y = 23.6 + 3.5x$. Using this equation, we can predict that the salary of a college graduate with, say, 10 years of experience is \$58,600. ■

2) Nonlinear Regression

"How can we model data that does not show a linear dependence? For example, what if a given response variable and predictor variable have a relationship that may be modeled by a polynomial function?" Think back to the straight-line linear regression case above where dependent response variable, y, is modeled as a linear function of a single independent predictor variable, x. What if we can get a more accurate model using a nonlinear model, such as a parabola or some other higher-order polynomial? Polynomial regression is often of interest when there is just one predictor variable. It can be modeled by adding polynomial terms to the basic linear model. By applying transformations to the variables, we can convert the nonlinear model into a linear one that can then be solved by the method of least squares.

Example 6.12 Transformation of a polynomial regression model to a linear regression model. Consider a cubic polynomial relationship given by

$$y = w_0 + w_1 x + w_2 x^2 + w_3 x^3. \quad (6.53)$$

To convert this equation to linear form, we define new variables:

$$x_1 = x \qquad x_2 = x^2 \qquad x_3 = x^3 \quad (6.54)$$

Equation (6.53) can then be converted to linear form by applying the above assignments, resulting in the equation $y = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3$, which is easily solved by the method of least squares using software for regression analysis. Note that polynomial regression is a special case of multiple regression. That is, the addition of high-order terms like x^2, x^3, and so on, which are simple functions of the single variable, x, can be considered equivalent to adding new independent variables. ■

Important 16 mark Questions in Unit-IV
➢ **Explain Associations Rule Mining** **Or** **Explain how to generate strong association rule mining** **Or** **Explain the mining methods for association rule generation** **Or** **Explain Apriori and FP-Growth Method with Example.**
➢ **Explain Mining Various Kinds of Association Rules** ➢ **Explain decision tree induction (Attribute Oriented Induction)** ➢ **Explain Bayesian Classification.** ➢ **Explain Rule Based Classification** ➢ **Explain Classification by Backpropagation** ➢ **Write short notes on Support Vector Machines and Lazy Learners** ➢ **Write short notes on Prediction**