ST. JOSEPH'S COLLEGE OF ENGINEERING, CHENNAI 600 119.
DEPARTMENT OF MATHEMATICS
MA6453 PROBABILITY AND QUEUEING THEORY
ASSIGNMENT II
UNIT II TWO DIMENSIONAL RANDOM VARIABLES

PART A

1. If the function f(x , y) = c(1 – x) (1– y), 0 < x < 1, 0 < y < 1 is the joint pdf of (X , Y), find the value of c.
2. Prove that Cov(aX , bY) = abCov (X , Y) for any random variables X and Y.
3. If X has mean 4 and variance 9, while Y has mean –2 and variance 5, and the two are independent find E (XY) and E (XY^2)
4. If the probability density function of a random variable X and Y is given by $f(x,y)=\frac{x^3y^3}{16}, 0<x<2, 0<y<2$, find the marginal functions of X and Y.
5. Let X and Y be continuous random variables with joint p.d.f. $f(x,y)=\begin{cases}2xy+\frac{3y^2}{2}, 0<x<1, 0<y<1\\ 0 \quad ,otherwise\end{cases}$.
   Find P (X + Y < 1)
6. If the joint pdf of the random variable (X , Y) is f(x , y) = 8xy , 0 < x < 1 , 0 < y < x, find f (y / x).
7. For λ > 0, let $F(x,y)=\begin{cases}1-\lambda e^{-\lambda(x+y)}, x>0, y>0\\ 0 \quad ,otherwise\end{cases}$. Check whether F(x , y) can be the joint probability distribution function of two random variables X and Y.
8. From the following joint distribution of X and Y, find the marginal distributions.

Y \ X	0	1	2
0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$
1	$\frac{3}{14}$	$\frac{3}{14}$	0
2	$\frac{1}{28}$	0	0

PART B

1.a. If the joint density of X and Y is given by $f(x,y)=\begin{cases}e^{-(x+y)}, x\ge 0, y\ge 0\\ 0 \quad ,elsewhere\end{cases}$. Find (i) the marginal density functions (ii) Test whether X and Y are independent (iii) P(X< 1) (iv) P(X + Y < 1) (v) P (X > Y) (vi) conditional distributions.

b. If X and Y are independent random variables having density functions $f(x) = 2e^{-2x}, x \geq 0$ and $f(y) = 3e^{-3y}, y \geq 0$ respectively. Find the density function of Z = X – Y.

c. The joint pdf of random variables X and Y is given by $f(x, y) = kxye^{-(x^2+y^2)}, x > 0, y > 0$. Find the value of k and also prove that X and Y are independent.

2.a. Obtain the equation of the lines of regression from the data given below:

X:	1	2	3	4	5	6	7
Y:	9	8	10	12	11	13	14

b. Let (X, Y) be a two dimensional continuous random variable having the joint density

$$f(x, y) = \begin{cases} 4xye^{-(x^2+y^2)}, & x \geq 0, y \geq 0 \\ 0 & , otherwise \end{cases}$$

. Find the density function of $U = \sqrt{X^2 + Y^2}$

c. The joint density of X and Y is given by $f(x, y) = \frac{1}{2} ye^{-xy}, 0 < x < \infty, 0 < y < 2$. Calculate the conditional density of X given Y = 1.

3.a. Let X and Y be two random variables having the joint probability function f(x,y) = k(x + 2y), where X and Y can assume only the integer values 0, 1 and 2. Find the marginal and conditional distributions. Find also P(X + Y > 2)

b. Two dimensional random variable (X, Y) have the joint probability density function $f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0 & , elsewhere \end{cases}$. Find (i) $P\left(X < \frac{1}{2} \cap Y < \frac{1}{4} \right)$ (ii) the marginal and conditional distributions. (iii) Are X and Y independent?

c. If the joint pdf of two dimensional random variable (X , Y) given f (x , y) = x + y , 0 < x < 1, 0 < y < 1, find the coefficient of correlation.

4.a. The joint probability density function of a two dimensional random variable (X , Y) is

$$f(x, y) = \frac{1}{8}(6 - x - y), 0 < x < 2, \; 2 < y < 4$$

. Find (i) $P(X < 1 \cap Y < 3)$ (ii) P(X + Y <3) (iii) P(X < 1 / Y < 3) (iv) Check whether X and Y are independent.

b. Two random variables X and Y have joint density function $f(x, y) = \begin{cases} 2 - x - y, & 0 < x < 1, 0 < y < 1 \\ 0 & , otherwise \end{cases}$.

Find Cov(X , Y) and correlation coefficient of X and Y.

c. In a partially destroyed laboratory record only the lines of regressions and variance of X are available. The regression equations are 8x – 10y + 66 = 0 and 40x – 18y = 214 and the variance of X = 9. Find the (i) the correlation coefficient between X and Y.

(ii) Mean values of X and Y.

(iii) Variance of Y.