UNIT - 1.

WIRELESS CHANNELS

LARGE SCALE PATHLOSS:

Introduction:

Tx | free space | Rx ← Wireless Propagation

→ The information s/l from Tx can take multiple paths to reach Rx. (called MPC _ Multipath component)

During Propagation → "Reflection, Diffraction, Scattering, etc" happens to the s/l (i.e) each MPC

Instead of analysing each MPC we choose to study on CHANNEL PARAMETERS ← *Of all

Received Power / Field strength

↓

It decreases with increase in distance b/w Tx & Rx

Also, MPC interfere with one another → leading to

* Fading
  Variations in R'd s/l power

* Delay Dispersion
  if, Tx'd s/l duration = T
  Rx'd s/l duration = T'
  T' > T
  ↓ will lead to
  ISI - Inter Symbol Interference
  (More relevant for digital s/m like GSM, 3G etc.)

Propagation Models → Predict the avg. r'd s/l strength

2 categories,

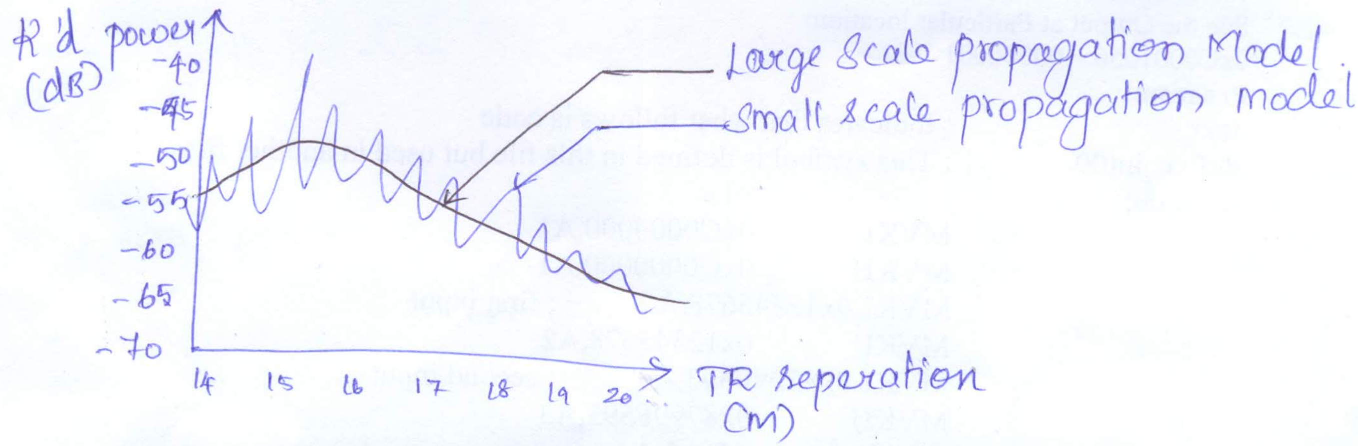
1. Large Scale Propagation Model
   → Predict avg. s/l strength for an arbitrary Tx & Rx pair
   → useful in estimating the coverage area.

2. Small Scale Propagation Model

- Characterize rapid fluctuations of R'd s/l strength (over small T R distances / short time durations)

eg: Mobile phones move over small area
⇓
hence R'd s/l strength fluctuates ⇒ leads to small scale fading.

Graph showing 1 & 2



PATH LOSS MODELS:

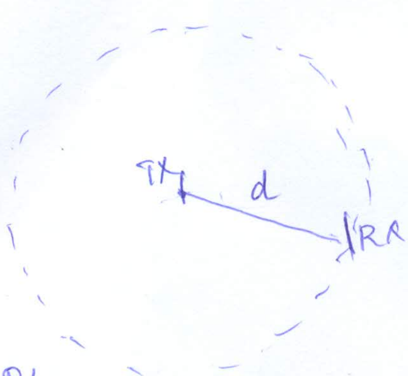
- Used to predict the received s/l strength only when Tx and Rx has LOS (Line of Sight) path b/w them.

eg: Satellite Communication, Microwave LOS links

" The R'd power decays as a function of T-R seperation distance raised to some power "

T-R seperation distance - d

For tx antenna power density $= P_{TX} \cdot \frac{1}{4\pi d^2}$



The Rx has effective area A_{RX}

then, power impinging on that area is R'd

$$P_{RX}(d) = P_{TX} \cdot \frac{1}{4\pi d^2} \cdot A_{RX} \quad \text{--- (1)}$$

Consider if Tx antenna → not isotropic

$$\text{Power Density} = \frac{P_{TX}}{4\pi d^2} * G_{TX} \quad \text{— (2)}$$

$$P_{RX}(d) = \frac{P_{TX}}{4\pi d^2} \cdot G_{TX} \cdot A_{RX} \quad \text{— (3)}$$

Relationship b/w effective area & Gain

$$G_{RX} = \frac{4\pi}{\lambda^2} A_{RX}$$

$$A_{RX} = \frac{\lambda^2}{4\pi} G_{RX} \quad \text{— (4)}$$

substitute (4) in (3)

$$P_{RX} = P_{TX} \cdot G_{TX} \cdot G_{RX} \left(\frac{\lambda}{4\pi d}\right)^2 \quad \text{— (5)}$$

$\left(\frac{\lambda}{4\pi d}\right)^2$ → called as Free Space Loss Factor

(5) ⇒ called Friis Law

According to Friis Law,

- Attenuation in free space increase with frequency.
- energy is never lost but redistributed over a sphere of surface area $4\pi d^2$.
- This mechanism is independent of wavelength

Validity of Friis law,

- Restricted to the farfield of the antenna i.e) Tx & Rx are atleast one Rayleigh distance apart. (also known as Fraunhofer Distance.

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Rayleigh Distance,

$$d_R = \frac{2L_a^2}{\lambda} \quad —(6)$$

where, $L_a \rightarrow$ Largest Dimension of Antenna

further, $d_R \gg L_a$ & $d_R \gg \lambda$

For setting up link Budgets, the logarithmic form of Friis law is,

$\boxed{10 \log_{10} (P \text{ (in watts)}) = P \text{ (in dB)}}$

$$P_{Rx}\Big|_{dBm} = P_{Tx}\Big|_{dBm} + G_{Tx}\Big|_{dB} + G_{Rx}\Big|_{dB} + 20\log\left(\frac{\lambda}{4\pi d}\right) \quad —(7)$$

If $d = 1m$, then R'd power at 1m distance,

$$P_{Rx}(1m)\Big|_{dBm} = P_{Tx}\Big|_{dBm} + G_{Tx}\Big|_{dB} + G_{Rx}\Big|_{dB} + 20\log\left(\frac{\lambda}{4\pi}\right) \quad —(8)$$

When R'd power for Tx-Rx seperation of 1m is known, Rx'd power for distance 'd' is given as,

$$P_{Rx}(d)\Big|_{dBm} = P_{Rx}(1m)_{dBm} - 20\log d \quad —(9)$$

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2 RAY MODEL / GROUND REFLECTION MODEL

Tx(Mobile Radio Channel)Rx

- There is no single path b/w tx & Rx.
- 2 Ray model is based on geometric optics.
- Considers 2 paths of propagation.
  * Ground Reflected path
  * Direct / LOS path

- Reasonably accurate in propagating large scale s/l strength over distances of several kilometers.
- Mostly T-R seperation distance is only a few tens of kms and earth is assumed to be flat.

Total Received E field,

$$E_{TOT} = E_{LOS} + E_g$$

where, E_{LOS} - Direct Line of Sight component.
E_g - ground Reflected Component

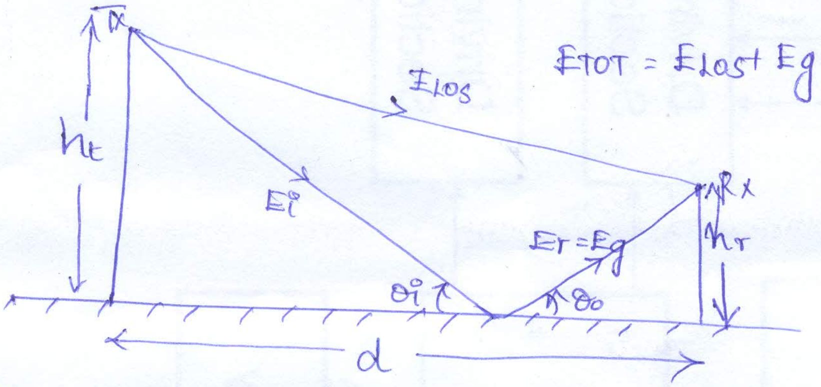


Fig - 2-Ray ground reflection Model.

For, a T-R reference distance d_0, free space E-field is E_0 V/m.

for $d > d_0$

1. Free space propagating E-field,

$$E(d,t) = \frac{E_0 d_0}{d} \cos(\omega_c(t - t_d)) \quad \text{---(1)}$$

where $t_d = \frac{d}{c}$

2. Direct/LOS wave travels a distance d' then, Efield due to LOS component,

$$E_{LOS}(d', t) = \frac{E_0 d_0}{d'} (\cos(\omega_c(t - t_d'))) \quad \text{---(2)}$$

where $t_d' = \frac{d'}{c}$

3. ground reflected wave travels a distance d' then efield due to it,

$$E_g(d'', t) = \Gamma \frac{E_0 d_0}{d''} \cos(\omega_c(t - t_{d''})) \quad \text{—(3)}$$

where, $t_d'' = \frac{d''}{c}$

Γ - Fresnel reflection coefficient for ground.

From the laws of reflection,

$$\left.\begin{aligned} \theta_i &= \theta_0 \\ E_g &= \Gamma E_i \\ E_t &= (1+\Gamma) E_i \end{aligned}\right\} \text{—(4)}$$

Assuming, perfect horizontal E-field & E-field ⊥r to the plane of incidence

then,

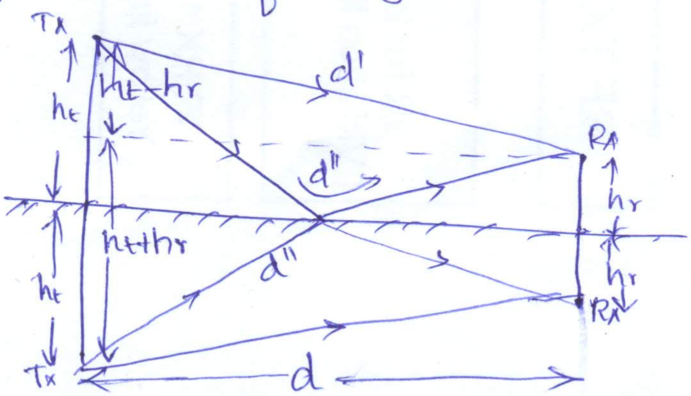
$\Gamma_\perp = -1$, & $E_t = 0$

The resultant E-field is,

$$|E_{TOT}| = |E_{LOS} + E_g|$$

$$E_{TOT}(d,t) = \frac{E_0 d_0}{d'} \cos \omega_c(t - t_d') + (-1)\frac{E_0 d_0}{d''} \cos \omega_c(t + t_d'') \quad \text{—(5)}$$

To find the difference b/w LOS & ground reflected path using method of images -



Path Difference,

$$\Delta = d'' - d'$$

$$\Delta = \sqrt{(h_t+h_r)^2 + d^2} - \sqrt{(h_t-h_r)^2 + d^2} \quad \text{—— (6)}$$

When T-R distance $d \gg h_t + h_r$ —(*)

upon simplification using Taylor's series & considering first 2 terms

Taylor series of

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \dots$$

Taking 1st 2 terms

$$\Delta = d\left(\sqrt{\frac{(h_t+h_r)^2}{d^2} + 1} - \sqrt{\frac{(h_t-h_r)^2}{d^2} + 1}\right)$$

$$\Delta = \frac{d}{2}\left(\frac{(h_t+h_r)^2}{d^2} - \frac{(h_t-h_r)^2}{d^2}\right) \quad \text{—— (7)}$$

applying condition (*)

$$\Delta \simeq \frac{2h_t h_r}{d}$$

phase Difference

$$\theta_\Delta = \frac{2\pi\Delta}{\lambda} \quad \text{—— (8)}$$

$$\lambda = \frac{c}{f_c} \qquad = \frac{2\pi\Delta f_c}{c}$$

$$\Delta = \frac{\omega_c \Delta}{c}$$

$$\Delta = \frac{\theta_\Delta c}{\omega_c} \quad \text{—— (9)}$$

propagation time

$$\tau_d = \frac{\Delta}{c} = \frac{\theta_\Delta}{2\pi f_c} \quad \text{—— (10)}$$

When d becomes large, $d'' - d'$ is very small.

ie) Δ decreases.

E_{LOS} & E_g are virtually identical & differ only in phase

i) $\left|\frac{E_0 d_0}{d}\right| \approx \left|\frac{E_0 d'}{d}\right| \approx \left|\frac{E_0 d}{d}\right|$

The received E field at some time say at $t = \frac{d''}{c}$

from (5)

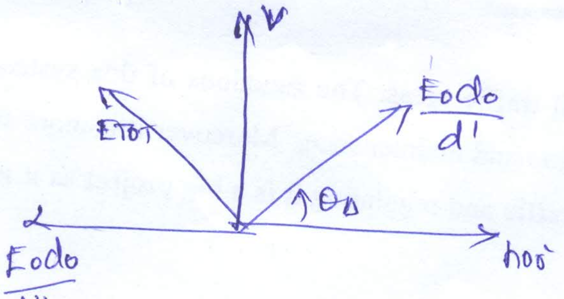
$$E_{TOT}\left(d, t = \frac{d''}{c}\right) = \frac{E_0 d_0}{d'} \cos\left(\omega_c \overbrace{\left(\frac{d'' - d'}{c}\right)}^{\Delta}\right) - \frac{E_0 d_0}{d''} \underbrace{\cos 0}_{1}$$

$$= \frac{E_0 d_0}{d'} \angle \theta_\Delta - \frac{E_0 d_0}{d''}$$

for greater d, $d' \approx d'' \approx d$

$$E_{TOT}\left(d, t = \frac{d''}{c}\right) = \frac{E_0 d_0}{d}\left[\angle \theta_\Delta - 1\right] \quad \text{—(11)}$$

Phasor Diagram,



from the above diagram,

$$\left|E_{TOT}(d)\right| = \sqrt{\left(\frac{E_0 d_0}{d}\right)^2 \left(\cos(\theta_\Delta - 1)^2\right) + \left(\frac{E_0 d_0}{d}\right)^2 \underbrace{\left(\sin\theta_\Delta - 1\right)^2}_{\to \sin^2\theta \text{ for small } \theta_\Delta}}$$

$$= \sqrt{\left(\frac{E_0 d_0}{d}\right)^2 \left[(\cos\theta_\Delta - 1)^2 + \sin^2\theta_\Delta\right]}$$

$$= \frac{E_0 d_0}{d}\sqrt{\cos^2\theta_\Delta - 2\cos\theta_\Delta + 1 + \sin^2\theta_\Delta}$$

$$E_{TOT}(d) = \frac{E_0 d_0}{d}\sqrt{2 - 2\cos\theta_\Delta} \quad \text{—— (12)}$$

If plane of incidence is ||el to E-field ie vertical polarization $\Gamma_{\parallel} = 1$. then, change − as + in previous eqn.)

then,

$$|E_{TOT}(d)| = \frac{E_0 d_0}{d}\sqrt{2+2\cos\theta_\Delta}$$

$$= \frac{E_0 d_0}{d}\sqrt{2(1+\cos\theta_\Delta)}$$

$$= \frac{E_0 d_0}{d}\sqrt{2\left(2\sin^2\theta_\Delta/2\right)}$$

$$|E_{TOT}(d)| = \frac{2E_0 d_0}{d}\sin\frac{\theta_\Delta}{2} \quad \text{— (13)}$$

when $d \uparrow \rightarrow |E_{TOT}(d)| \downarrow$ in oscillatory fashion.

when $\sin\theta/2 \simeq \theta/2$ (occurs when $\theta/2 < 0.3$ rad).

from (7) & (8)

$$\frac{\theta_\Delta}{2} \simeq \frac{2\pi h_t h_r}{\lambda d}$$

$$0.3 > \frac{2\pi h_t h_r}{\lambda d}$$

$$d > \frac{2\pi h_t h_r}{0.3\lambda} \quad \text{— (14)} \quad \simeq \frac{20 h_t h_r}{\lambda}$$

as long as d satisfies (14),

$$|E_{TOT}(d)| = \frac{2E_0 d_0}{d}\,\frac{2\pi h_t h_r}{\lambda d} \simeq \frac{k}{d^2} \quad \text{— (15)}$$

k − const. related to E_0, antenna heights & wavelength.

wkt., $G = \frac{4\pi A_e}{\lambda^2}$ — (16)

free space power received

$$P_{RX}(d) = P_{TX}(d) \cdot G_{TX} \cdot G_{RX} \cdot \left(\frac{\lambda}{4\pi d}\right)^2 \quad \text{— (17)}$$

for 2 Ray model

$$P_{RX}(d) = (17) \times E_{TOT}^2 \quad (\text{with } d_0 E_0 = 1)$$

$$P_{RX}(d) = P_{TX}(d) \cdot G_{TX}\, G_{RX} \cdot \frac{h_t^2\, h_r^2}{d^4} \quad \text{—(18)}$$

$$\text{Path Loss} = \frac{1}{P_{RX}(d)}$$

In dB,

$$P_{RX}(d)\Big|_{dB} = P_{TX}(d)\Big|_{dB} + G_{TX}\Big|_{dB} + G_{RX}\Big|_{dB} + 20\log h_t h_r - 40\log d \quad \text{—(19)}$$

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LINK BUDGET DESIGN USING PATHLOSS MODELS :

- Path loss models → derived with assumptions
- Deviations from assumptions like frequencies, environments etc. nullify the validity of expressions in other environments.
- Path loss models calculate received s/l level hence can be used to predict SNR.
- Some pathloss estimation techniques are,
  - Log-Distance Path loss model
  - Log-Normal Shadowing
  - Determination of percentage of coverage area.

Log-Distance Pathloss Model :-

Average R'd s/l power decreases logarithmically with distance. For a large T-R seperation path loss is expressed as a fn. of pathloss exponent 'n'

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n \quad \text{—(1)}$$

in dB,

$$\overline{PL}(d)\Big|_{dB} = \overline{PL}(d_0)\Big|_{dB} + 10n\log\left(\frac{d}{d_0}\right) \quad \text{—(2)}$$

n - path loss exponent - varies with environment.

eg: n=2 for free space

n = 1.6 to 1.8 for in building LOS

d_o - reference distance

d_o = 1 km - for large scale cellular s/m

d_o = 100 km / 1 m for microcellular s/m.

Log Normal Shadowing:-

eqn.(2) doesn't consider the fact that the environment may be vastly different at both T & R points for some seperation d

∴ path loss at a particular location also has the effect of a mean distance-dependent value.

$$P_L(d)[dB] = \overline{P_L}(d) + X_\sigma \quad \text{—(3)}$$

$$P_L(d)\Big|_{dB} = \overline{P_L}(d_o) + 10n\log\left(\frac{d}{d_o}\right) + X_\sigma \quad \text{—(4)}$$

where,

X_σ - zero-mean gaussian distributed random variable

Received pwr = Tx. pwr - Path loss — (5)

$$P_r(d)\Big|_{dBm} = P_t\Big|_{dBm} - P_L(d)\Big|_{dBm} \quad \text{—(6)}$$

The random variable X_σ cause random shadowing effects for locations with same T-R seperation but different levels of clutter in propagation path.

shadowing → follows gaussian distribution.

Standard deviation of shadowing 'σ'

$\Rightarrow d_o, n, \sigma \rightarrow$ determine the path loss.

$P_L(d) \rightarrow$ normal distribution about X_σ

R'd power, $P_r(d) \rightarrow$ normal distribution about $P_L(d)$

<u>Q fn</u> / <u>error fn</u> (erf)

- used to determine the probability that the r'd s/l strength will exceed a particular level.

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{2}\left[1 - erf\left(\frac{z}{\sqrt{2}}\right)\right] \quad \text{—(7)}$$

γ - certain value of r'd power.

$$P[P_r(d) > \gamma] = Q\left[\frac{\gamma - P_r(d)}{\sigma}\right] \quad \text{—(8)}$$

$$P[P_r(d) < \gamma] = Q\left[\frac{P_r(d) - \gamma}{\sigma}\right] \quad \text{—(9)}$$

<u>Determination of Percentage of Coverage area</u>:-

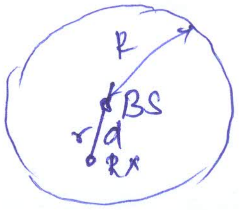
γ - desired s/l strength

There are points within the coverage area where r'd sl strength is below γ.

$U(\gamma)$ - percentage of useful service area where rx sl strength equals or greater than γ

ie) $P_r(d) > \gamma$

Consider a circular coverage area of radius R.



$$U(\gamma) = \frac{1}{\pi R^2} \int P\,[P_r(d) > \gamma]\, dA$$

$(\because d = r)$

$$= \frac{1}{\pi R^2} \int P\,[P_r(\cancel{d}) > \gamma]\, dA$$

$$= \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^{R} P\,[P_r(r) > \gamma]\, r \cdot dr\, d\theta \quad \text{——— (10)}$$

from (4)

$$P\,[P_r(r) > \gamma] = Q\left[\frac{\gamma - \overline{P_r}(r)}{\sigma}\right]$$

$$= \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{\gamma - P_r(r)}{\sigma\sqrt{2}}\right)\right]$$

$$= \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{\gamma - [P_t - P_L(r)]}{\sqrt{2}\,\sigma}\right)\right]$$

$$= \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left[\frac{\gamma - \left(P_t - \left(\overline{P_L}(d_0) + 10n \log\left(\frac{r}{d_0}\right)\right)\right)}{\sqrt{2}\,\sigma}\right]$$

from (6) & (4)

When path loss is referenced to cell boundary, 'R'

$$\overline{P_L}(r) = \overline{P_L}(d_0) + 10n \log\left(\frac{R}{d_0}\right) + 10n \log\left(\frac{r}{R}\right)$$

then,

$$P\,[P_r(r) > \gamma] = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left[\frac{\gamma - \left(P_t - \left[\overline{P_L}(d_0) + 10n \log\left(\frac{R}{d_0}\right) + 10n \log\left(\frac{r}{R}\right)\right]\right)}{\sqrt{2}\,\sigma}\right]$$

Let,

$$a = \frac{\gamma - P_t + \overline{P_L}(d_0) + 10n \log\left(\frac{R}{d_0}\right)}{\sqrt{2}\,\sigma} \quad \text{——— (12)} \quad \text{——— (11)}$$

$$b = \frac{10n \log e}{\sqrt{2}\,\sigma} \quad \text{——— (13)}$$

then,

$$P[P_r(r) > \gamma] = \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left[a + b \ln \frac{r}{R}\right]$$

from (10)

$$U(\gamma) = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^{R} \frac{1}{2} - \frac{1}{2} \operatorname{erf}\left[a + b \ln \frac{r}{R}\right] r \cdot dr \, d\theta$$

$$= \frac{1}{\pi R^2} \left[\frac{1}{2} \times \frac{R^2}{2} \times 2\pi - \frac{1}{2} 2\pi \int_0^R \operatorname{erf}\left[a + b \ln\left(\frac{r}{R}\right)\right] \cdot r dr \right.$$

$$= \frac{1}{2} - \frac{1}{R^2} \int_0^R \operatorname{erf}\left(a + b \ln\left(\frac{r}{R}\right) r \cdot dr \quad \text{—(14)}\right.$$

perform substitution integration ie) $t = a + b \ln\left(\frac{r}{R}\right)$

& then simplify,

$$U(\gamma) = \frac{1}{2}\left(1 - \operatorname{erf}(a) + \exp\left(\frac{1 - 2ab}{b^2}\right)\left[1 - \operatorname{erf}\left(\frac{1 - ab}{b}\right)\right]\right)$$

When $P_r(R) = \gamma \Rightarrow$ ie) $a = 0$

then $U(\gamma) = \frac{1}{2}\left[1 + \exp\left(\frac{1}{b^2}\right)\left(1 - \operatorname{erf}\left(\frac{1}{b}\right)\right)\right]$ —(15)

$\because b = \frac{10\, n \log e}{\sqrt{2}\, \sigma}$

if n & σ are known $U(\gamma)$ can be calculated

eg: for $n = 4$ & $\sigma = 8$

% of useful coverage area = 75%

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