

ASSIGNMENT -II

UNIT- III - APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

PART-A

1. Write down the partial differential equation governing one dimensional wave equation?
2. Write all possible solutions of the transverse vibration of the string in one dimension.
3. State Fourier law of heat conduction.
4. In steady state condition derive the solution of one dimension heat flow equation.
5. An insulated rod of length 60 cm has its ends A and B maintained at 20°C and 80°C respectively. Find the steady state temperature in the rod.
6. The boundary value problem governing the steady state temperature distribution in a flat thin plate is given by $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $u(x, 0) = 0$, $u(0, y) = 0$, $u(a, y) = 0$ and $u(x, a) = 4 \sin^3 \left[\frac{\pi x}{a} \right]$.

Find C_n when the most general solution is $u(x, y) = \sum_{n=1}^{\infty} c_n \sin \frac{n\pi x}{a} \sinh \frac{n\pi y}{a}$.

7. Given the boundary conditions on a square or rectangular plate, how will you identify the proper solutions?
8. An infinitely long plate is bounded by two parallel edges and an end at right angles to them. The breadth of the edge $y = 0$ is π and it is maintained at constant temperature u_0 at all points and the other edges are kept at zero temperatures. Formulate the boundary value problem to determine the steady state temperature.

PART-B

1. A uniform string is stretched and fastened to two points ' l ' apart. Motion is started by displacing the string into the form of the curve $y = kx(l - x)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at time t .
2. If a string of length l is initially at rest in its equilibrium position and each of its points is given a

velocity v such that $v = \begin{cases} cx; & 0 < x < \frac{l}{2} \\ c(l - x); & \frac{l}{2} < x < l \end{cases}$, find the displacement at any time t .

3. The ends A and B of a rod l cm long have their temperatures kept at 30°C and 80°C , until steady state conditions prevail. The temperature of the end B is suddenly reduced to 60°C and that of A is increased to 40°C . Find the steady state temperature distribution in the rod after time t .
4. A square plate is bounded by the lines $x = 0$, $y = 0$ and $x = y = 20$, its faces are insulated. The temperature along upper horizontal line is given by $u(x, 20) = x(20 - x)$ where $0 < x < 20$. While other 3 edges are kept at 0°C . Find the steady state temperature in the plate.