UNIT– II

PART-B

1.Define circular convolution. How can linear convolution be realized using circular convolution?

Ans. Consider we have two finite duration sequences of lengths N, x1 (n) & x2 (n).

Their respective N-point DFT are

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) e^{\frac{-j2\pi kn}{N}} \quad k = 0, 1, \ldots, N-1 \qquad \ldots(1)$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n) e^{\frac{-j2\pi kn}{N}} \quad k = 0, 1, \ldots, N-1 \qquad \ldots(2)$$

. If we multiply the two DFT's together, the results in a DFT say X_3 (k) of a sequence $x_3(n)$ of length N. Let us determine the relationship between $x_3(n)$ and the sequences $x_1(n)$ & $x_2(n)$.

$$X_3(k) = X_1(k)\, X_2(k) \qquad k = 0, 1, \ldots, N-1 \qquad \ldots(3)$$

The IDFT of $\{X_3(k)\}$ is

$$X_3(m) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k)\, e^{\frac{j2\pi km}{N}}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X_1(k)\, X_2(k)\, e^{\frac{j2\pi km}{N}} \qquad \ldots(4)$$

Suppose that we substitute for $X_1(k)$ & $X_2(k)$ is (4) using the DFT's given in (1) & (2). Thus we obtain.

$$x_3(m) = \frac{1}{N} \sum_{k=0}^{N-1} \left[\sum_{n=0}^{N-1} x_1(n) e^{\frac{-j2\pi kn}{N}} \right] \left[\sum_{l=0}^{N-1} x_2(l) e^{\frac{-j2\pi kl}{N}} \right] \times e^{\frac{j2\pi km}{N}} \qquad \ldots(5)$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \left[\sum_{k=0}^{N-1} e^{\frac{j2\pi k(m-n-l)}{N}} \right]$$

The inner sum in brackets in (5) has form.

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N & a=1 \\ \frac{1-a^N}{1-a} & a \neq 1 \end{cases} \quad \ldots(6)$$

where a is defined as

$$a = e^{\frac{j2\pi(m-n-l)}{N}}$$

We observe that a = I when m - n - 1 is multiple of N. On the other hand ar" = I for any value of $a \neq 0$. Consequently (6) reduces to

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N & l = m-n+PN \quad = ((m-n))_N \\ 0 & \text{otherwise} \quad \text{P is an integer} \end{cases} \quad \ldots(7)$$

If we substitute the result (7) in (6) we obtam the desired expression for x3 (m) in 'form

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2((m-n))_N \;;\; m = 0,1,\ldots, N-1 \quad \ldots(8)$$

The expression (8) has form of a convolution sum. However it is not the ordinary linear convolution which relates the output sequence y (n) of a linear system to input sequence x (n) and the impulse response h (n). Instead the convolution sumin (8) involves the index ((m—n))N and is called circular convolution.

The basic difference between these two types of convolution is that in circular convolution the folding & shifting operations are performed in circular fashion by computing the index of one of the sequences module N. In linear convolution, there is no module N operation. Basically circular convolution y (n) contains same number of samples as that of x (n) & h (n). In case of linear convolution no. of samples are given by

N.

N=L+M-1

L = No. of samples in x (n)

M = No. of samples in h (n)

N = No. of samples in the result of linear convolution.

That's why the result of linear and circular convolution are not same.

To obtain the same result both convolutions, the following, steps are used

1 Calculate the value of N that means no of samples contained in linear convolution

2. By doing zero padding make the length of every sequence equal to N. That means in this case we need to add zeros in x (n) as well as Ii (n).

3. Perform the circular convolution. The result of circular convolution & linear convolution will be same.

2. Discuss various properties of DFT.

Ans. Properties of the DFT.

1. Linearity : The DFT obeys the law of linearity. If

$$x_1(n) \xleftrightarrow[N]{DFT} X_1(k)$$

$$x_2(n) \xleftrightarrow[N]{DFT} X_2(k)$$

then for any two constants a and b,

$$ax_1(n)+bx_2(n) \xleftrightarrow[N]{DFT} aX_1(k)+bX_2(k) \qquad ...(1)$$

1. Periodicity

If $\quad x(n) \xleftrightarrow[N]{DFT} X(k).$

then $\quad x(n+N) = x(n)$

then $\quad X(k+N) = X(k) \qquad ...(2)$

Proof. $\quad X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi k}{N}n}$

$$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi(k+N)}{N}n}$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}n} \cdot e^{-j2\pi n} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}}$$

$X(k + N) = X(k)$.

3. Circular shift of a sequence : This property is analogous to the time shifting

property of the DFT, but with some difference.

Let us consider a sequence x (n) of length N which is defined for 0 n N - 1. The sample value of such sequence is zero for n <0 and n N. For any arbitrary integer k, the shift sequence x1 (n) = x (n — k) is no longer defined for the range 0 n N - 1. Therefore we need to define another type of shift that will always keep the shifted sequence in the range 0 n N - 1.

This shift is known as circular shift that can be represented as the index modulo N. Thus we can write,

$$x_c(n) = x_p(n-k) \qquad \text{...(3)}$$

$$x_c(n) = x[(n-k), \text{modulo N}] \qquad 0 \le n \le N-1$$

$$x_c(n) = x[((n-k))_N] \qquad \text{...(4)}$$

where x (n) is represented the circular shift of x (n). Or more generally we can

define

$$x_c(n) = \begin{cases} x_p(n-k) = x[((n-k))_N] & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases} \qquad \text{...(5)}$$

Equations (6.4) and (6.5) tell us how to construct x (n).

1. Circular convolution and multiplication of two DFTs : Consider two finite duration sequences $x_1(n)$ and $x_2(n)$ both of length, with their N - point DFTs $X_1(k)$ and $X_2(k)$ i.e.,

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) e^{-j\frac{2\pi k}{N}n} \; ; k = 0, 1, 2, \ldots N-1 \qquad \text{...(6)}$$

$$X_2(k) = \sum_{n=0}^{N-1} x_2(n) e^{-j\frac{2\pi k}{N}n} \; ; k = 0, 1, 2, \ldots N-1 \qquad \text{...(7)}$$

Here we wish to determine the sequence x3 (n) for which the DFT is,

$$X_3(k) = X_1(k)\,X_2(k),\ k = 0, 1, 2 \ldots\ldots N-1 \qquad \ldots(8)$$

The DFT of $X_3(k)$ is,

$$x_3(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_3(k)e^{\frac{j2\pi k}{N}n}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X_1(k)X_2(k)e^{\frac{j2\pi k}{N}n}$$

Using equations (6.6) and (6.7) in (6.8), we have,

$$x_3(n) = \frac{1}{N}\sum_{k=0}^{N-1}\left[\sum_{m=0}^{N-1} x_1(m)e^{\frac{-j2\pi k}{N}m}\right]\left[\sum_{l=0}^{N-1} x_2(l)e^{\frac{-j2\pi k}{N}l}\right]e^{j\frac{2\pi k}{N}n}$$

$$x_3(n) = \frac{1}{N}\sum_{m=0}^{N-1} x_1(m)\sum_{l=0}^{N-1} x_2(l)\left[\sum_{k=0}^{N-1} e^{j2\pi k[n-m-l]/N}\right] \qquad \ldots(9)$$

The brackets term in eqn. (6.9) has the form,

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N & \text{for } a=1 \\ \dfrac{1-a^N}{1-a} & \text{for } a \neq 1 \end{cases} \qquad \ldots(10)$$

where, we have defined 'a' as, $a = e^{j2\pi\,(n-m-1)}/N$.

See that for $a = 1$, the $(n - m - l)$ must be integral multiple of N.

$$a^N = e^{j\frac{2\pi(n-m-l)}{N}\times N} = 1$$

i.e., $a^N = 1$ for any value of $a \neq 0$.

Therefore, eqn. (6.10) reduces to,

$$\sum_{k=0}^{N-1} a^k = \begin{cases} N & n-m-l = qN; \quad [l = n-m-qN] = ((n-m))_N \\ 0 & \text{otherwise} \qquad q \text{ an integer} \end{cases} \qquad \ldots(11)$$

If we substitute the eqn. (6.11) into eqn. (6.12), we have,

$$x_3(n) = \frac{1}{N}\sum_{m=0}^{N-1} x_1(m)\sum_{l=0}^{N-1} x_2(l)[N] \quad l = ((m-n))_N$$

$$x_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2[((n-m))_N] \quad ; m = 0, 1, \ldots\ldots N-1 \qquad \ldots(12)$$

The equation (6.12) has the form of convolution sum. However it differs from a linear convolution of x1 (n) and x2 (n) as defined in unit-TI. In linear convolution the computation of the sequence x3 (n) involves multiplying one sequence by a folder and linear shifted version of the other and then summing the values of the product x1 (m) x2 (n - m) for all values of n.

Instead the convolution sum in eqn. (6.12), the second sequence is circularly time reversed and circularly shifted w.r.t. of first. The equation (6.12) is called circular convolution of two finite duration sequences.

Thus we conclude that multiplication of the DFTs of two sequences is equivalent to the circular convolution of the two sequences in time domain. The operation of forming

a sequences $x_3(n)$ for $0 \le n \le N-1$, using eqn. (6.12) is denoted as,

$$x_3(n) = x_1(n) \, N \, x_2(n) \quad \text{...(13)}$$

Since the DFT of $x_3(n)$ is $X_3(k) = X_1(k) X_2(k)$ and $X_1(k) X_2(k) = X_2(k) X_1(k)$.

We have, $x_3(n) = x_1(n) \, N \, x_2(n) = x_2(n) \, N \, x_1(n)$.

or
$$x_3(n) = \sum_{m=0}^{N-1} x_2(m) \, [((n-m))_N] \quad \text{...(14)}$$

$$\therefore \quad x_1(n) N x_2(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X_1(k) X_2(k) \quad \text{...(15)}$$

where, $x_1(n) \, N \, x_2(n)$ denotes the circular convolution of two N length sequences, x1 (n) and x2 (n).

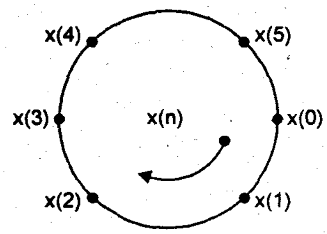
1. Time reversal of a sequence:

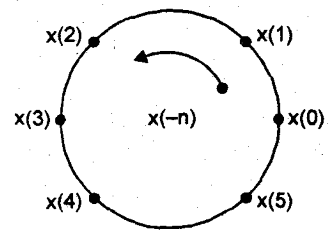
If
$$x(n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X(k)$$

then
$$x((-n))_N = x(N-n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X((-k))_N = X(N-k).$$

or
$$x(N-n) \overset{DFT}{\underset{N}{\longleftrightarrow}} X(N-k).$$

Hence, when the N-point sequence is reverse in time, it is equivalent to reversing the DFT values. The time reversal is illustrated in Fig.





Proof. Since we know that,

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N} \quad ; k = 0, 1, 2 \dots N-1.$$

$$\therefore \text{DFT}\{x(N-n)\} = \sum_{n=0}^{N-1} x(N-n)e^{-j2\pi nk/N}$$

If we change the index from n to m by defining m = N - n, then,

$$\text{DFT }\{x\,(\mathrm{N}-n)\} = \sum_{m=\mathrm{N}}^{1} x(m)e^{\frac{-j2\pi(\mathrm{N}-m)k}{\mathrm{N}}}, \text{ put N = 0}$$

$$= \sum_{m=0}^{\mathrm{N}-1} x(m)e^{\frac{j2\pi km}{\mathrm{N}}}$$

$$\text{DFT }\{x\,(\mathrm{N}-n)\} = \sum_{m=0}^{\mathrm{N}-1} x(m)e^{\frac{j2\pi km}{\mathrm{N}}}.e^{\frac{-j2p\mathrm{N}m}{\mathrm{N}}}$$

Note. $e^{-j2\pi \mathrm{N}m/\mathrm{N}} = e^{-j2\pi m}$

i.e., $e^{-j2\pi} = \cos 2\pi - j \sin 2\pi = 1.$

$e^{-j2\pi m} = \cos m2\pi - j\sin m\,2\pi = 1.$ $\quad m = 0, 1, \ldots.. \mathrm{N}-1$

all even 2π of cosine values are '1' and even 2π of sine values are '0'.

$$\text{DFT }\{x\,(\mathrm{N}-n)\} = \sum_{m=0}^{\mathrm{N}-1} x(m)e^{\frac{-j2\pi(\mathrm{N}-k)}{\mathrm{N}}}$$

$$\text{DFT }\{x\,(\mathrm{N}-n)\} = \mathrm{X}\,(\mathrm{N}-k)$$

$$x(\mathrm{N}-n) \xleftrightarrow[\mathrm{N}]{\mathrm{DFT}} \mathrm{X}(\mathrm{N}-k) \qquad \ldots(16)$$

6. Circular frequency shift :

If $\quad x(n) \xleftrightarrow[\mathrm{N}]{\mathrm{DFT}} \mathrm{X}(k)$

then $\quad x(n)e^{\frac{j2\pi ln}{\mathrm{N}}} \xleftrightarrow[\mathrm{N}]{\mathrm{DFT}} \mathrm{X}((k-l))_{\mathrm{N}}$

Proof. $\text{DFT }[x\,(n)e^{j2\pi ln/\mathrm{N}}] = \sum_{n=0}^{\mathrm{N}-1} [x(n)e^{j2\pi ln/\mathrm{N}}.e^{-j2\pi kn/\mathrm{N}}]$

$$= \sum_{n=0}^{\mathrm{N}-1} x(n)e^{-j2\pi n(k-l)/\mathrm{N}} = \sum_{n=0}^{\mathrm{N}-1} x(n)e^{-j2\pi n(\mathrm{N}+k-l)/\mathrm{N}}$$

$$= \mathrm{X}\,(\mathrm{N}+k-l)$$

$$\text{DFT }[x\,(n)\,e^{j2\pi ln/\mathrm{N}}] = \mathrm{X}\,((k-l))_{\mathrm{N}} \qquad \ldots(17)$$

7. Circular time shift:

If $\quad x(n) \xleftrightarrow[\mathrm{N}]{\mathrm{DFT}} \mathrm{X}(k)$

then $\quad x((n-l))_{\mathrm{N}} \xleftrightarrow[\mathrm{N}]{\mathrm{DFT}} \mathrm{X}(k)e^{-j2\pi kl/\mathrm{N}}$

Proof.

$$. \text{DFT}\ [x\ ((n-1))_N] = \sum_{n=0}^{N-1} x(n-l)_N\, e^{\frac{-j2\pi kn}{N}}.$$

If we change the index from n to m = N + n - 1, then

$$= \sum_{n=0}^{N-1} x(m)\, e^{-j2\pi km/N},$$

$$= \sum_{n=0}^{N-1} x(N+n-l)\, e^{\frac{-j2\pi kn}{N}} \qquad ...(18)$$

Equation (6.18) can be written as

$$= \sum_{n=0}^{l-1} x(N+n-l)\, e^{\frac{-j2\pi kn}{N}} + \sum_{n=0}^{N-1} x(N+n-l)\, e^{\frac{-j2\pi k(n)}{N}}$$

$$= \sum_{m=n-1}^{N-1} x(m)\, e^{\frac{-j2\pi k(m+l-N)}{N}} + \sum_{m=N}^{2N-l-1} x(m)\, e^{-j2\pi k\frac{(m+l-N)}{N}}$$

$$= \sum_{m=n-1}^{N-1} x(m)\, e^{\frac{-j2\pi k(m+l)}{N}} + \sum_{m=N}^{2N-l-1} x(m)\, e^{\frac{-j2\pi k(m+l)}{N}}$$

$$= \sum_{m=N-l}^{N-1} x(m)\, e^{\frac{-j2\pi k(m+l)}{N}} + \sum_{m=0}^{N-l-1} x(m)\, e^{\frac{-j2\pi k(m+l)}{N}}$$

$$= \sum_{m=0}^{N-1} x(m)\, e^{-j2\pi kl/N} e^{-j2\pi km/N} = e^{-j2\pi kl/N}\, X(k).$$

$$\text{DFT}\ \{x\ ((n-l))_N\} = e^{-j2\pi kl/N}\, X(k) \qquad ...(19)$$

8. Multiplication of two sequences:

If $\quad x_1(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X_1(k)$

and $\quad x_2(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X_2(k)$

then $\quad x_1(n)x_2(n) \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} \frac{1}{N} X_1(k)X_2(k) \qquad ...(20)$

The proof of this property is similar to circular convolution.

9. Circular correlator : For complex valued sequence x (n) and y (n).

If $x(n) \xleftrightarrow[N]{DFT} X(k)$

and $y(n) \xleftrightarrow[N]{DFT} Y(k)$

then $r_{xy}(l) \xleftrightarrow[N]{DFT} R_{xy}(k) = X(k)\, Y^*(k)$

where $r_{xy}(l)$ is the circular cross-correlation sequence, given as,

$$r_{xy}(l) = \sum_{n=0}^{N-1} x(n) y^*((n-l))_N \qquad \text{...(21)}$$

Proof.

Since $x(l)\, \text{N} y^*(l) = \sum_{m=0}^{N-1} x(m) y^*((l-m))_N$

then, $r_{xy}(l) = x(l)\, \text{N} y^*(-l)$

$$= \sum_{m=0}^{N-1} x(m) y^*[(-(l-m))_N]$$

$$r_{xy}(l) = \sum_{m=0}^{N-1} x(m) y^*(n-l)_N$$

$$r_{xy}(l) = x(l)\, \text{N}\, y^*(-l).$$

We know circular convolution of the two sequences is just equal to the multiplication of the their DFTs and from complex conjugate property

$$y^*(-l) \xleftrightarrow[N]{DFT} y^*(k)$$

Then N-point DFT of $r_{xy}(l)$ is,

$$R_{xy}(l) = X(k) Y^*(k). \qquad \text{...(22)}$$

10. Parseval's theorem:

For the complex-valued sequence x (n) and y (n), if

$$x(n) \xleftrightarrow[N]{DFT} X(k)$$

and $$y(n) \xleftrightarrow[N]{DFT} Y(k).$$

then $$\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(k)y^*(k)$$

Proof. From the correlation property,

$$r_{xy}(l) = \sum_{n=0}^{N-1} x(n)y^*(n-l)_N$$

then $$r_{xy}(0) = \sum_{n=0}^{N-1} x(n)y^*(n)$$

and $$r_{xy}(l) = \frac{1}{N}\sum_{k=0}^{N-1} R_{xy}(k)e^{\frac{j2\pi kl}{N}}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)e^{\frac{j2\pi kl}{N}} = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\left(Y(k)e^{\frac{-j2\pi kl}{N}}\right)$$

$$r_{xy}(0) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k)$$

$$r_{xy}(0) = \sum_{n=0}^{N-1} x(n)y^*(n)$$

Thus, $$\sum_{n=0}^{N-1} x(n)y^*(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)Y^*(k) \qquad ...(23)$$

3. Develop a Radix-2, 8-point DIF FFT algorithm with neat flow chart.

Ans. Decimation in frequency stands for splitting the sequences in terms of frequency. That means we have split output sequences into smaller subsequences. This decimation is done as follows.

First stage of decimation:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad ...(1)$$

We can divide the stmmation into two parts as follows.

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{kn} + \sum_{n=\frac{N}{2}}^{N-1} x(n)\, W_N^{kn} \qquad ...(2)$$

Now consider the second summation that means

$$\sum_{n=\frac{N}{2}}^{N-1} x(n)\, W_N^{kn}$$

Put $n = n + \frac{N}{2}$, the limits will change as follows.

When $n = \frac{N}{2}$ $\Rightarrow$ $\frac{N}{2} = n + \frac{N}{2}$ $\therefore n = 0$

and when $n = N-1$ $\Rightarrow$ $N-1 = n + \frac{N}{2}$ $\therefore n = N-1-\frac{N}{2} = \frac{N}{2}-1$

$$\therefore \quad \sum_{n=\frac{N}{2}}^{N-1} x(n) W_N^{kn} = \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{k\left(n+\frac{N}{2}\right)}$$

Putting this value in 2equatioñ (2), we get

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right). W_N^{k\left(n+\frac{N}{2}\right)}$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{kn} + \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right). W_N^{kn} . W_N^{k\frac{N}{2}}$$

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n) W_N^{kn} + W_N^{k\frac{N}{2}} \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{kn} \qquad ...(3)$$

Now we have $\quad W_N = e^{\frac{-j2\pi}{N}}$

$$\therefore \quad W_N^{\frac{kN}{2}} = e^{\frac{-j2\pi}{N} \times \frac{kN}{2}} = e^{-j\pi k} = (e^{-j\pi})^k$$

$$\therefore \quad W_N^{\frac{kN}{2}} = (\cos \pi - j \sin \pi)^k = (-1-j0)^k$$

$$\therefore \quad W_N^{\frac{kN}{2}} = (-1)^k$$

Putting this value in equation (3) we get

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1} x(n)\, W_N^{kn} + (-1)^k \sum_{n=0}^{\frac{N}{2}-1} x\left(n+\frac{N}{2}\right) W_N^{kn}$$

Taking the summation common we get

$$X(k) = \sum_{n=0}^{\frac{N}{2}-1}\left[x(n)+(-1)^k x\left(n+\frac{N}{2}\right)\right] W_N^{kn} \qquad \ldots(4)$$

Here we have no split the sequence in terms of frequency. So we will split X(k) in terms of even numbered and add numbered DFT.co-efficients. Let X(2r) represents even numbered DFT and X(2r + 1) represents odd numbered DFT.

Thus puffing k = 2 r in eq (4), we will get even numbered sequence

$$X(2r) = \sum_{n=0}^{\frac{N}{2}-1}\left[x(n)+(-1)^{2r} x\left(n+\frac{N}{2}\right)\right] W_N^{2rn} \qquad \ldots(5)$$

By puffing k = 2r + 1 in eq. (4), we will get odd numbered sequence

$$X(2r+1) = \sum_{n=0}^{\frac{N}{2}-1}\left[x(n)+(-1)^{2r+1} x\left(n+\frac{N}{2}\right)\right] W_N^{(2r+1)n} \qquad \ldots(6)$$

Here 'r' is an integer similar to k and it varies from 0 to N/2 -1

$$\therefore \qquad (-1)^{2r} = 1 \qquad \ldots(7)$$

$$\textit{and} \qquad (-1)^{2r+1} = (-1)^{2r}(-1)^1 = -1 \qquad \ldots(8)$$

Puffing these values in eq (5) & (6) we get

$$X(2r) = \sum_{n=0}^{\frac{N}{2}-1}\left[x(n)+x\left(n+\frac{N}{2}\right)\right] W_N^{2rn} \qquad \ldots(9)$$

$$X(2r+1) = \sum_{n=0}^{\frac{N}{2}-1}\left[x(n)-x\left(n+\frac{N}{2}\right)\right] W_N^{(2r+1)n} \qquad \ldots(10)$$

Now consider the term W_N^{2rn}.

$$W_N^{2rn} = \left(W_N^2\right)^{rn}$$

But we have $W_N^2 = W_{N/2}$

$$\therefore \quad W_N^{2rn} = \left(W_{N/2}\right)^{rn} = W_{N/2}^{rn} \qquad ...(11)$$

Now we can write

$$W_N^{(2r+1)n} = W_N^{2rn} \cdot W_N^n = W_{N/2}^{rn} \cdot W_N^n \qquad ...(12)$$

Puffing these values in eq. (9) & (10) we get

$$X(2r) = \sum_{n=0}^{\frac{N}{2}-1}\left[x(n)+x\left(n+\frac{N}{2}\right)\right] W_{N/2}^{rn} \qquad ...(13)$$

and

$$X(2r+1) = \sum_{n=0}^{\frac{N}{2}-1}\left[x(n)-x\left(n+\frac{N}{2}\right)\right] W_{N/2}^{rn}\, W_N^n \qquad ...(14)$$

Now let

$$g(n) = x(n) + x\left(n+\frac{N}{2}\right) \qquad ...(15)$$

and

$$h(n) = \left[x(n)-x\left(n+\frac{N}{2}\right)\right] W_N^n \qquad ...(16)$$

Putting these values in eq. (13) & (14) we get

$$x(2r) = \sum_{n=0}^{\frac{N}{2}-1} g(n)\, W_{N/2}^{rn} \qquad ...(17)$$

and

$$x(2r+1) = \sum_{n=0}^{\frac{N}{2}-1} h(n)\, W_{N/2}^{rn} \qquad ...(18)$$

Note that at this stage we have decimated the sequence of ‘N’ pomt DFT mto two N/2 point DFT’s given by eq (17) & (18) Let us consider an example of 8 pomt OFT That means N = 8 So considermg (17) & (18) (that means N/2=4) we can obtain N (8-point) DFT. This is first stage of decimation. Note that eq. (17) indicate 4 (N/2) point OFT of IN\ g(n) and eq (18) indicates 4 (N/2) pomt DFT of h(n) For 8 pomt OFT eq (15) becomes

$$g(n) = x(n) + x(n+4) \qquad ...(19)$$

Here we are computing '4' point DFT, So range of 'n' is n=0 to n=3. Putting these values in eq. (19), we get

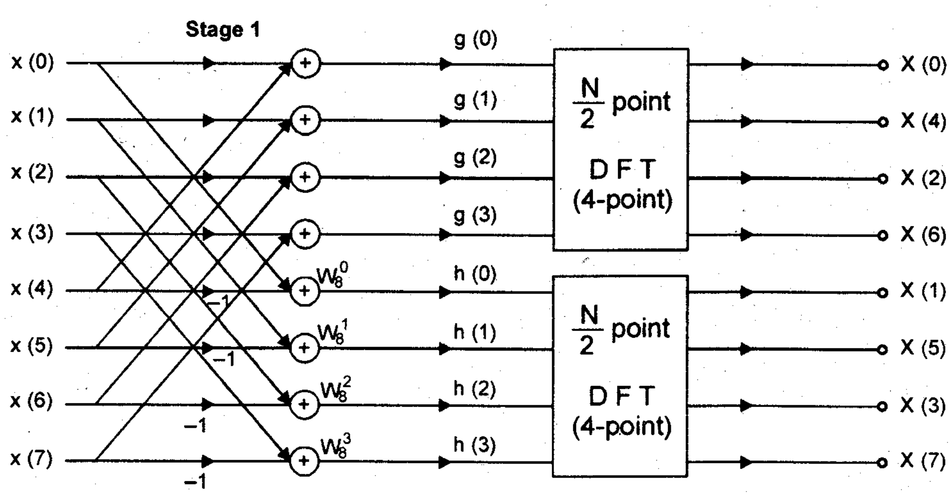
$$\left.\begin{array}{llll} \text{For} & n=0 & \Rightarrow g(0) & = x(0)+x(4) \\ & n=1 & \Rightarrow g(1) & = x(1)+x(5) \\ & n=2 & \Rightarrow g(2) & = x(2)+x(6) \\ & n=3 & \Rightarrow g(3) & = x(3)+x(7) \end{array}\right\} \qquad ...(20)$$

Similarly for 8 point DFT equation (16) becomes

$$h(n) = [x(n) - x(n+4)]\ W_8^n \qquad ...(21)$$

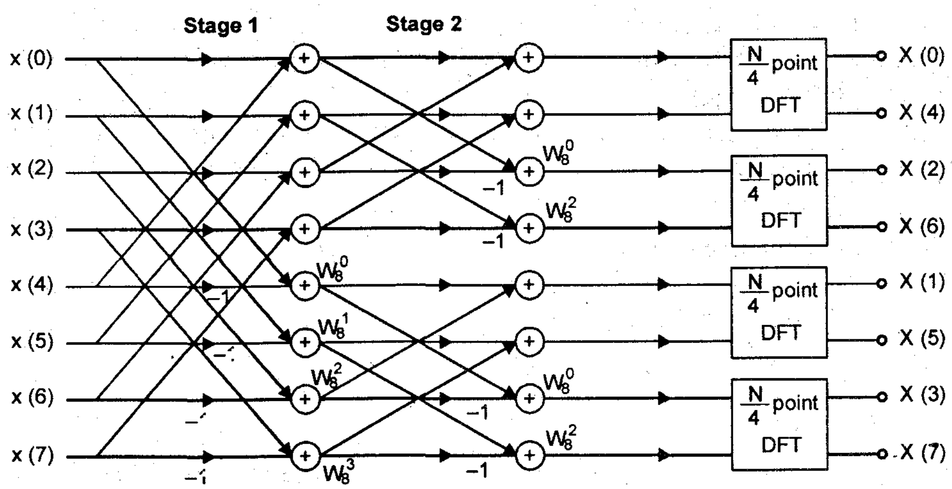
$$\left.\begin{array}{llll} \text{For} & n=0 & \Rightarrow h(0) & = [x(0)-x(4)]W_8^0 \\ & n=1 & \Rightarrow h(1) & = [x(1)-x(5)]W_8^1 \\ & n=2 & \Rightarrow h(2) & = [x(2)-x(6)]W_8^2 \\ & n=3 & \Rightarrow h(3) & = [x(3)-x(7)]W_8^3 \end{array}\right\} \qquad ...(22)$$

Using equations (20) & (22) & eq. (17) & (18) we can draw the flow graph of the first stage of decimation as shown in fig. below.



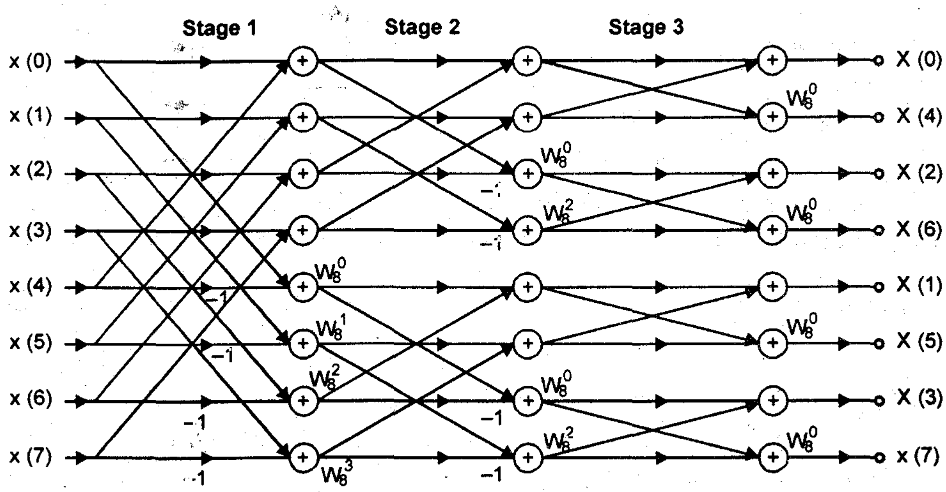
First Stage of decimation

Second stage of decimation: In the first stage of decimation we have used 4-point DFT. We can further decimate the sequence by using 2 point DFT. The second stage of decimation is shown in fig below.



Second Stage of decimation

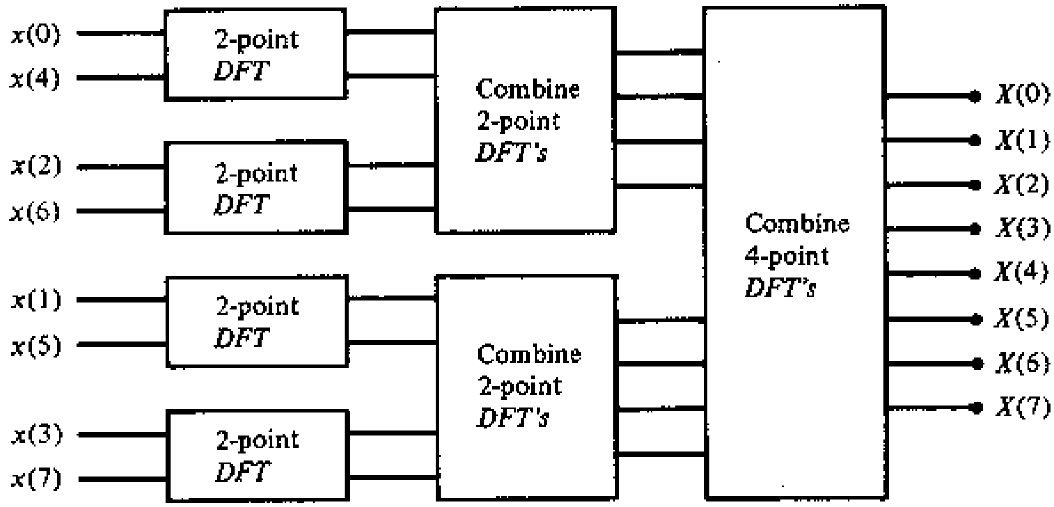
Third stage of decimation : In the second stage of decimation we have used 2- point DFT. So further decimation is not possible. Now we will use a butterfly structure to obtain 2-point DFT. Thus the total flow graph of 8 point DIF-FFT is shown below.



Total Flow graph of 8 point DIF–FFT

4. Develop a Radix-2, 8-point DIT FFT algorithm with neat flow chart.

Ans: Basic butterfly diagram,



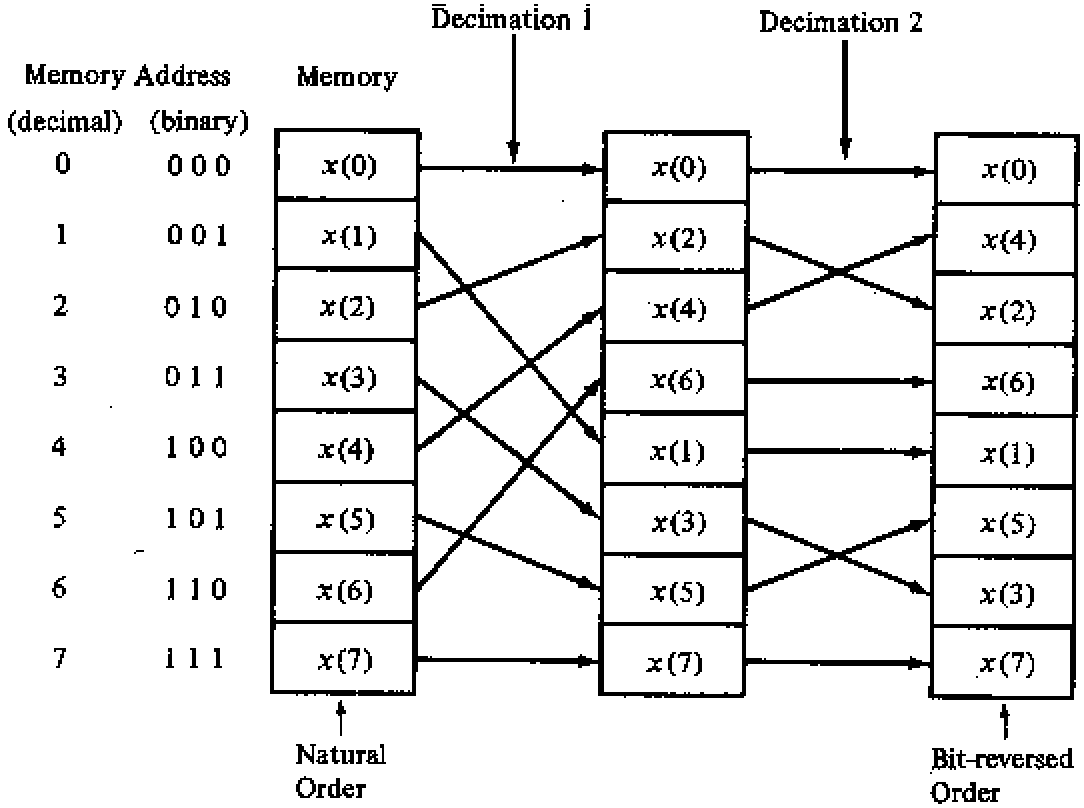
$$F_1(k) = \mathrm{F}\{f_1(2n)\} + W_{N/2}^{k}\,\mathrm{F}\{f_1(2n+1)\}, \qquad k = 0,1,\ldots,\frac{N}{4}-1; \qquad n = 0,1,\ldots,\frac{N}{4}-1$$

$$F_1\left(k+\frac{N}{4}\right) = \mathrm{F}\{f_1(2n)\} - W_{N/2}^{k}\,\mathrm{F}\{f_1(2n+1)\}, \qquad k = 0,1,\ldots,\frac{N}{4}-1; \qquad n = 0,1,\ldots,\frac{N}{4}-1$$

$$F_2(k) = \mathrm{F}\{f_2(2n)\} + W_{N/2}^{k}\,\mathrm{F}\{f_2(2n+1)\}, \qquad k = 0,1,\ldots,\frac{N}{4}-1; \qquad n = 0,1,\ldots,\frac{N}{4}-1$$

$$F_2\left(k+\frac{N}{4}\right) = \mathrm{F}\{f_2(2n)\} - W_{N/2}^{k}\,\mathrm{F}\{f_2(2n+1)\}, \qquad k = 0,1,\ldots,\frac{N}{4}-1; \qquad n = 0,1,\ldots,\frac{N}{4}-1$$

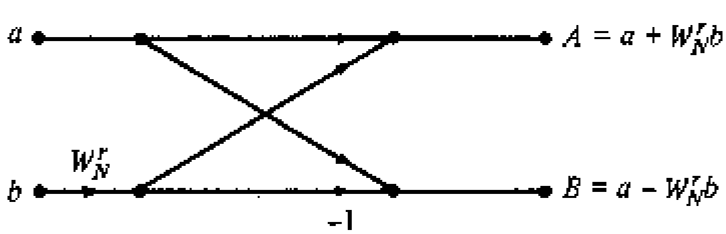
$\mathrm{F}\{*\}$ represents Fourier tr ansform

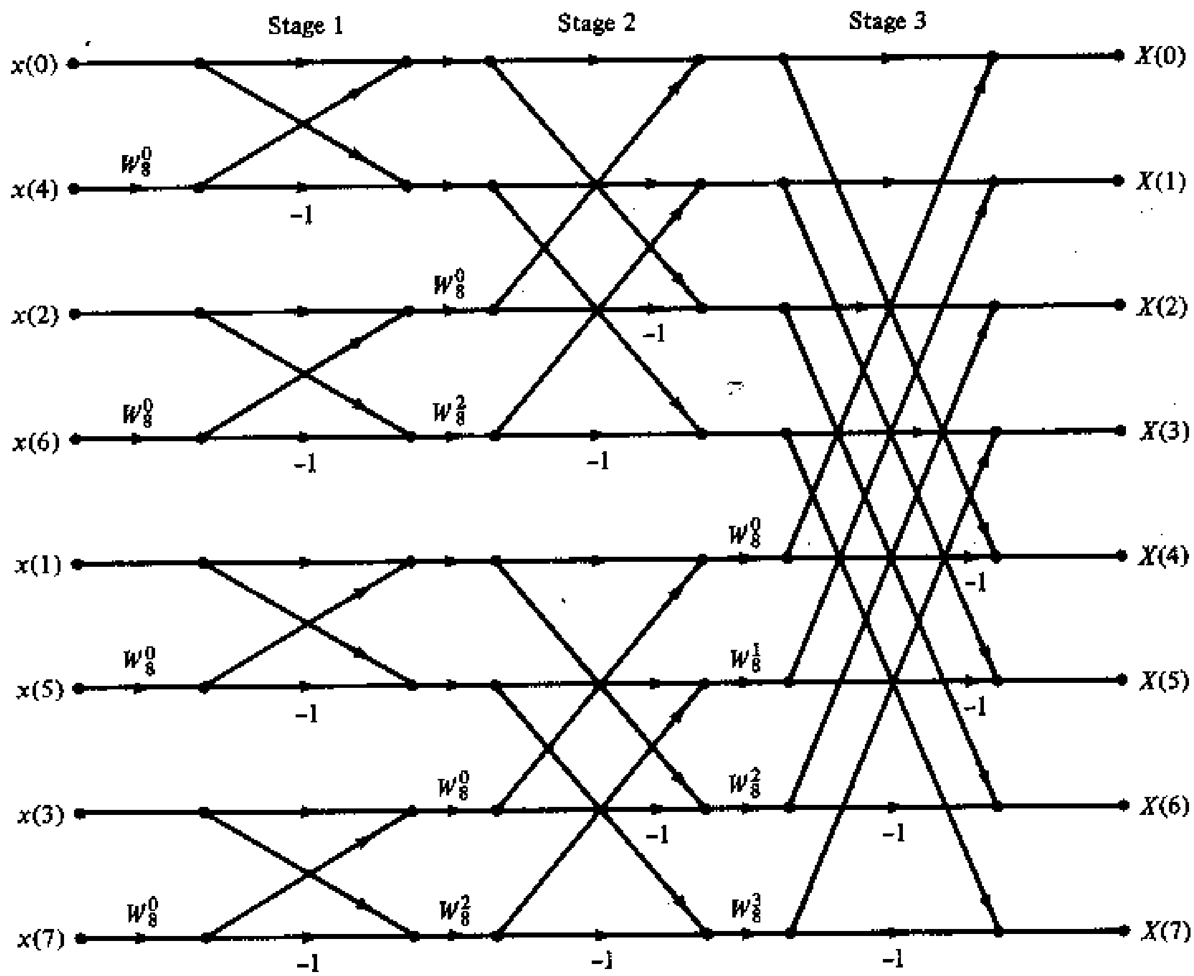


(a)

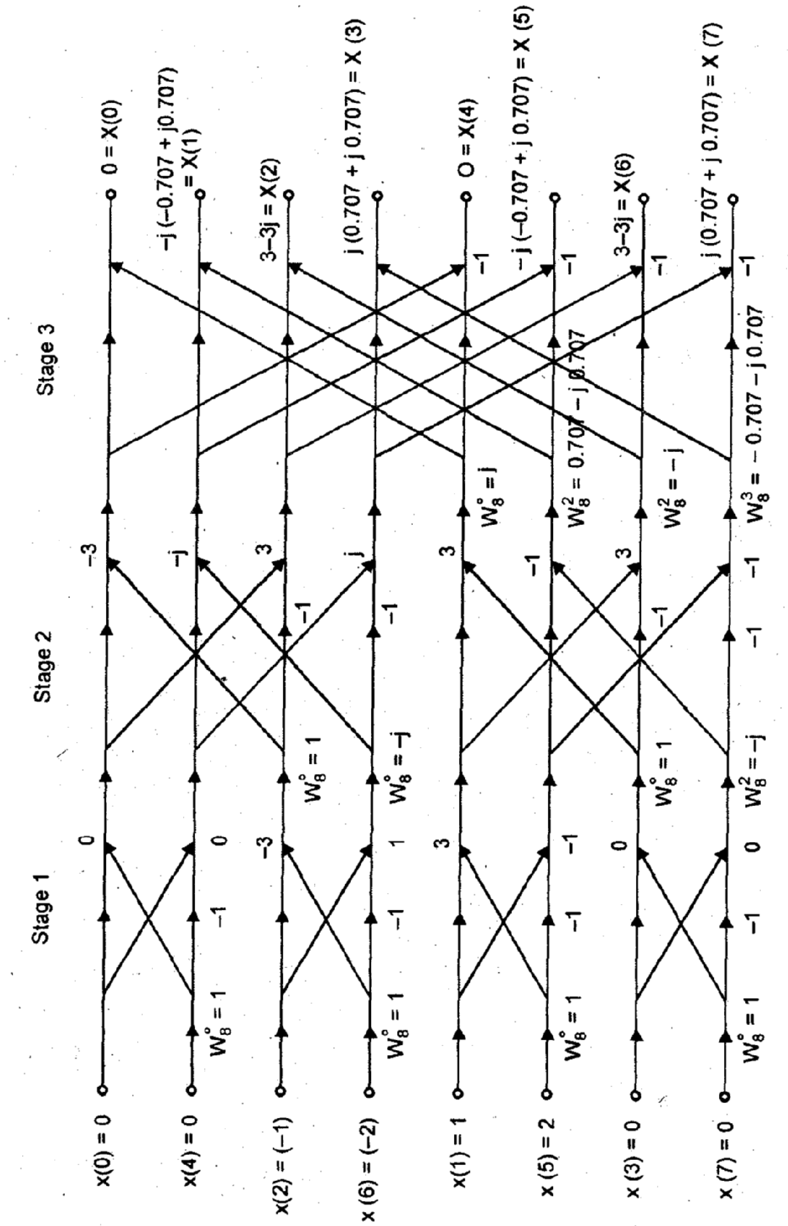
$(n_2n_1n_0)$	$\to$	$(n_0n_2n_1)$	$\to$	$(n_0n_1n_2)$
(0 0 0)	$\to$	(0 0 0)	$\to$	(0 0 0)
(0 0 1)	$\to$	(1 0 0)	$\to$	(1 0 0)
(0 1 0)	$\to$	(0 0 1)	$\to$	(0 1 0)
(0 1 1)	$\to$	(1 0 1)	$\to$	(1 1 0)
(1 0 0)	$\to$	(0 1 0)	$\to$	(0 0 1)
(1 0 1)	$\to$	(1 1 0)	$\to$	(1 0 1)
(1 1 0)	$\to$	(0 1 1)	$\to$	(0 1 1)
(1 1 1)	$\to$	(1 1 1)	$\to$	(1 1 1)

(b)





5. Draw a 8 point radix-2 FFT DIT flow graphs and obtain DFT of the following sequence x(n)=(0,1,-1,0,0,2,-2,0) **(May 2014)**



6. Compute 4-point DFT of causal three sample sequence given by. **(Nov2014)**

$$\mathrm{X}(n) = \frac{1}{3} \quad 0 \le n \le 2$$
$$= 0 \quad \text{else.}$$

Draw its and magnitude and phase spectrum.

Ans. By definition of N-point DFT, the kth is complex co-efficient of X (k) for $0 \le k \le \mathrm{N}-1$ is given by

$$\mathrm{X}(k) = \sum_{n=0}^{\mathrm{N}-1} x(n) e^{\frac{-j2\pi kn}{\mathrm{N}}}$$

Here N = 4, therefore the 4-point DFF is

$$X(k) = \sum_{n=0}^{3} x(n) e^{\frac{-j2\pi kn}{4}} = \sum_{n=0}^{3} x(n) e^{\frac{-j\pi kn}{2}}$$

$$= x(0)\, e^{0} + x(1)\, e^{\frac{-j\pi k}{2}} + x(2)\, e^{-j\pi k} + x(3)\, e^{\frac{-j3\pi k}{2}}$$

$$= \frac{1}{3} + \frac{1}{3} e^{\frac{-j\pi k}{2}} + \frac{1}{3}\, e^{-j\pi k} + 0 = \frac{1}{3}\left[1 + e^{\frac{-j\pi k}{2}} + e^{-j\pi k}\right]$$

$$= \frac{1}{3}\left[1 + \frac{\cos \pi k}{2} - j \sin \frac{\pi k}{2} + \cos \pi k - j \sin \pi k\right]$$

The values of X (k) can be evaluated for k = 0, 1, 2, 3.

When $k = 0$

$$x(0) = \frac{1}{3}[1 + \cos 0 - j \sin 0 + \cos 0 - j \sin 0]$$

$$= \frac{1}{3}[1 + 1 + 1] = 1 < 0$$

When $k = 1$

$$x(1) = \frac{1}{3}\left[1 + \frac{\cos \pi}{2} - j \sin \frac{\pi}{2} + \cos \pi - j \sin \pi\right]$$

$$= \frac{1}{3}\,[1 + 0 - j - 1 - j\,0] = -j\,\frac{1}{3} = \frac{1}{3} < -\frac{\pi}{2}$$

When $k = 2$

$$x(2) = \frac{1}{3}[1 + \cos \pi - j \sin \pi + \cos 2\pi - j \sin 2\pi]$$

$$= \frac{1}{3}[1 - 1 - j\,0 + 1 - j\,0] = \frac{1}{3} < 0$$

When $k = 3$

$$x(3) = \frac{1}{3}\left[1+\frac{\cos 3\pi}{2} - j\sin\frac{3\pi}{2} + \cos 3\pi - j\sin 3\pi\right]$$

$$= \frac{1}{3}\ ]1+0+j-1-j0] = j\frac{1}{3} = \frac{1}{3} < \frac{\pi}{2}$$

$\therefore$ **The 4-point DFT sequence of $x(n)$ is given by**

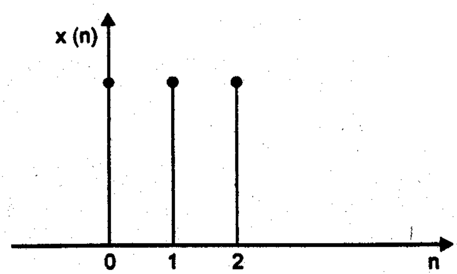
$$X(k) = \left\{1 < 0, \frac{1}{3} < \frac{-\pi}{2} + \frac{1}{3} < 0, \frac{1}{3} < \frac{\pi}{2}\right\}$$

Magnitude function

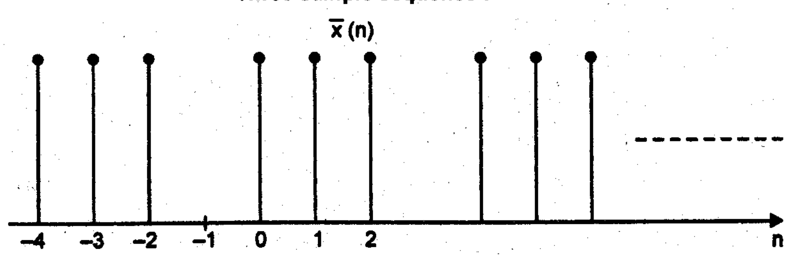
$$|X(k)| = \left\{1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\}$$

Phase function $$< X(k) > = \left\{0, \frac{-\pi}{2}, 0, \frac{\pi}{2}\right\}$$

The three sample sequence and its periodic extension are shown in Fig. below.



Three sample sequence :

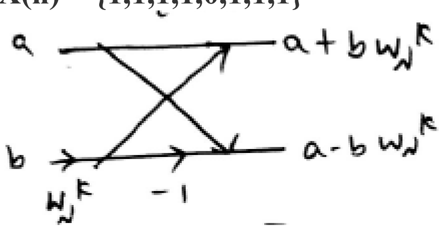


Periodic extension of x (n) for N = 4

7.a)Determine 8- point DFT of sequence.
$x(n) = \{1, -3 \le n \le 3$ using radix-2 DIT-FFT algorithm.

Ans: X(n) = x(n+N)

X(n) = {1,1,1,1,0,1,1,1}



$w_8^0 = 1$

$w_8^1 = 0.707 - j0.707$ **or 1/√2 – j / √2**

$w_8^2 = -j$

$w_8^1 = -0.707 - j0.707$ **or -1/√2 – j / √2**

Draw the butterfly structure

X(k) = {7,1,-1,1,-1,1,-1,1}

b) A finite duration sequence of length L is given as

$$X(n) = \begin{cases} 1, 0 \le n \le L-1 \\ 0, \text{otherwise} \end{cases}$$

Determine N-point DFT of this sequence for N ≥ L.

Ans. Length L is given as

$$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$$

of this sequence for N ≥ L.

Form of this sequence is

$$\sum_{n=0}^{L-1} x(n)e^{-j\omega n} = \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated in Fig for L = 10. The N point of x(n) is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies ω

κ =0, 1, ……, N -1. Hence

$$X(\kappa) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}}, \; k = 0, 1, \ldots\ldots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)N}$$

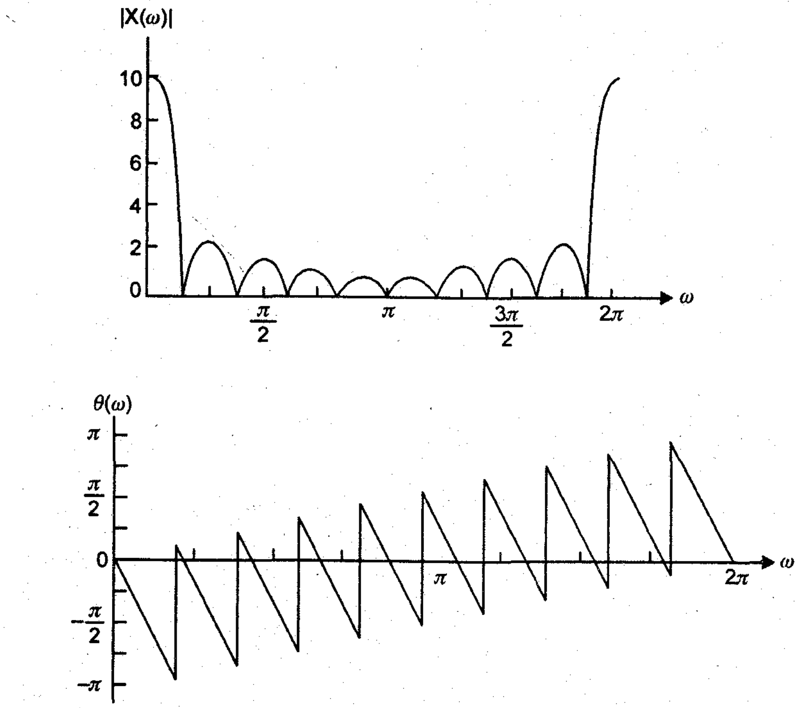


Fig. Magnitude and phase characteristics of the Fourier transform for signal

8. Compute the DFT of sequence defined by : $x(n) = (-1)^n$ for
   (a) N = even
   (b)N= odd.

Ans.

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}} = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$$= \sum_{n=0}^{N-1} (-1)^n W_N^{kn} \qquad \left[\because W_N^N = 1\right]$$

$$= \sum_{n=0}^{N-1} \left[(-1)\times W_N^k\right]^n = \sum_{n=0}^{N-1} \left[-W_N^k\right]^n \qquad \left[\because W_N^k = 1\right]$$

$$= \frac{1-\left(-W_N^k\right)^N}{1+W_N^k} = \frac{1-(-1)^N}{1+W_N^k}$$

1. For N even

$$W_N^k = -1 \quad \text{if} \quad k = \frac{N}{2}$$

$$X(k) = \frac{1-(-1)^N}{1+W_N^k} = \frac{0}{1+W_N^k} \quad \text{for} \quad k \neq \frac{N}{2}$$

But for $k = \frac{N}{2}$

$$X\left(\frac{N}{2}\right) = \sum_{n=0}^{N-1}\left(-W_N^{N/2}\right)^n = \sum_{n=0}^{N-1}[-(-1)^n] = \sum_{n=0}^{N-1} 1 = N$$

(B)For N odd

$$W_N^k = -1 \text{ if } \frac{N}{2} = 2 \text{ or } N = 2k$$

Since N is odd, therefore, no k exist so we can write.

$$X(k) = \frac{2}{1+W_N^k} = \frac{2}{1+\frac{\cos 2\pi k}{N} - \frac{j\sin 2\pi k}{N}}$$

9.By means of the DFT & IDFT, determine the sequence x_3 (n) corresponding to the circular convolution of the sequence $x_1(n)$ and $x_2(n)$.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\},\ x_2(n) = \{\underset{\uparrow}{1}\ 2, 3, 4,\}$$

Ans. First we compute the DFT's of x_1 (n) and x_2(n).

The four point DFT of x_1 (n) is

$$X_1(k) = \sum_{n=0}^{3} x_1(n) e^{\frac{-j2\pi nk}{4}} \quad k = 0, 1, 2, 3$$

$$= 2 + e^{\frac{-j\pi k}{2}} + 2\, e^{-j\pi k} + e^{\frac{-3\pi k}{2}}$$

Thus

$$X_1(0) = 6,\ X_1(1) = 0,\ X_1(2) = 2,\ X_1(3) = 0$$

The DFT of $x_2(n)$ **is**

$$X_2(k) = \sum_{n=0}^{3} x_2(n) e^{\frac{-j2\pi kn}{4}} \quad k = 0, 1, 2, 3,..$$

$$= 1 + 2\, e^{\frac{-j\pi k}{2}} + 3\, e^{-j\pi k} + 4\, e^{\frac{-j3\pi k}{2}}$$

Thus

$$X_2(0) = 10,\ X_2(1) = -2 + j\,2,\ X_2(2) = -2,\ X_2(3) = -2-j2$$

When we multiply the two DFT's we obtain the product.

$$X_3(k) = X_1(k)\, X_2(k)$$

$$X_3(0) = 60,\ X_3\,(1) = 0,\ X_3\,(2) = -4,\ X_3\,(3) = 0$$

Now, the IDFT of $x_3\,(k)$ is

$$x_3(n) = \sum_{n=0}^{3} X_3(k) e^{\frac{-j2\pi kn}{4}} \qquad [n = 0, 1, 2, 3]$$

$$= \frac{1}{4}\,(60-4\, e^{j\pi n})$$

Thus $\quad x_3\,(0) = 14,\ x_3\,(1) = 16,\ x_3\,(2) = 14,\ x_3\,(3) = 16$

10. By means of the DFT & IDFT, determine the response of the FIR filter with impulse response.

$h\,(n) = \{\underset{\uparrow}{1}, 2, 3\}$ to the input sequence $\quad$ **$x\,(n) = \{\underset{\uparrow}{1}, 2, 2\}$**

Ans. The input sequence has length L = 4 and the impulse response has length M = 3. Linear convolution of these two sequences produces a sequence of length N = 6. Consequently, the size of the DFT must be at least six.

For simplicity we compute eight point DFT's. We should also mention that the efficient computation of the DFT via the Fast Fourier Transform (FFT) algorithm is usually performed for a length N, that is power of 2. Hence eight point DFT of x(n) is

$$X(k) = \sum_{n=0}^{7} x(n) e^{\frac{-j2\pi kn}{8}}$$

$$= 1 + 2\, e^{\frac{-j\pi k}{4}} + 2\, e^{\frac{-j\pi k}{2}} + e^{\frac{-j3\pi k}{4}} \quad k = 0,1..7.$$

The computation yields.

$$X(0) = 6 \qquad X(1) = \frac{2+\sqrt{2}}{2} - j\left(\frac{4+3\sqrt{2}}{2}\right)$$

$X(2) = -1 - j$ $\qquad X(3) = \frac{2-\sqrt{2}}{2} + j\left(\frac{4-3\sqrt{2}}{2}\right)$

$X(4) = 0$ $\qquad X(5) = \frac{2-\sqrt{2}}{2} - j\left(\frac{4-3\sqrt{2}}{2}\right)$

$X(6) = -1 + j$ $\qquad X(7) = \frac{2+\sqrt{2}}{2} + j\left(\frac{4+3\sqrt{2}}{2}\right)$

The eight point DFT of $h(n)$ is

$$H(k) = \sum_{n=0}^{7} h(n)\, e^{\frac{-j2\pi kn}{8}}$$

$$= 1 + 2\, e^{\frac{-j\pi k}{4}} + 3\, e^{\frac{-j\pi k}{2}}$$

Hence

$$H(0) = 6,\ H(1) = 1 + \sqrt{2} - j(3 + \sqrt{2})$$

$$H(2) = -2 - j2,\ H(3) = 1 - \sqrt{2} + j(3 - \sqrt{2})$$

$$H(4) = 2,\ H(5) = 1 - \sqrt{2} - j(3 - \sqrt{2})$$

$$H(6) = -2 + j2,\ H(7) = 1 + \sqrt{2} + j(3 + \sqrt{2})$$

The product of these two DFT's yield $Y(k)$ *i.e.*

$$Y(0) = 36,\ Y(1) = -14.07 - j\,17.48,\ Y(2) = j4$$

$$Y(3) = 0.07 + j0.515,\ Y(4) = 0,\ Y(5) = 0.07 - j0.515.$$

$$Y(6) = -j4,\ Y(7) = -14.07 + j17.48$$

Finally the eight point IDFT is

$$y(n) = \sum_{k=0}^{7} Y(k)\, e^{\frac{j2\pi kn}{8}} \quad n = 0, 1, \ldots 7.$$

This computation yields a result.

$$y(n) = \{\underset{\uparrow}{1}, 4, 9, 11, 8, 3, 0, 0\}$$

11.An 8-point sequence is given by x (n) = {2, 2, 2, 2, 1, 1, 1, 1}. Compute 8-point DFT of x(n) by radix-2 DIT FFT. Also sketch the magnitude and phase spectrum.

Ans. The given sequence is first arranged in the bit reversed order.

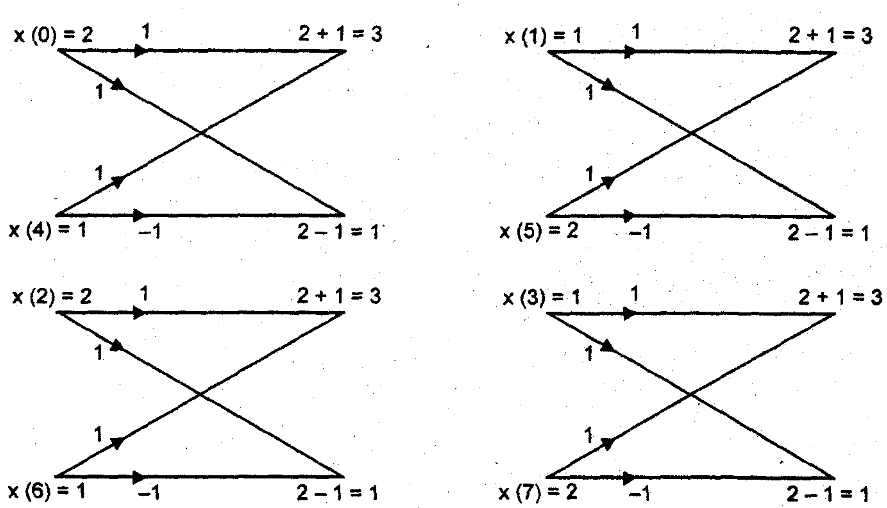
The sequence $x(n)$ in normal order	The sequence $x(n)$ in bit reversed order
$x(0) = 2$	$x(0) = 2$
$x(1) = 2$	$x(4) = 1$
$x(2) = 2$	$x(2) = 2$
$x(3) = 2$	$x(6) = 1$
$x(4) = 1$	$x(1) = 2$
$x(5) = 1$	$x(5) = 1$
$x(6) = 1$	$x(3) = 2$
$x(7) = 1$	$x(7) = 1$

For 8-point DFT by radix 2 FFF we require 3 stages of computation wits 4 butterfly computations in each stage. The sequence rearranged in the bit reversed order forms the input to the first stage. For other stages of computation the 0/P of previous stage will be I/P for current stage.

First stage computation:

The I/P time sequence {2, 1, 2, 1, 2, 1, 2, 1)

The butterfly computations of first stage are shown below.

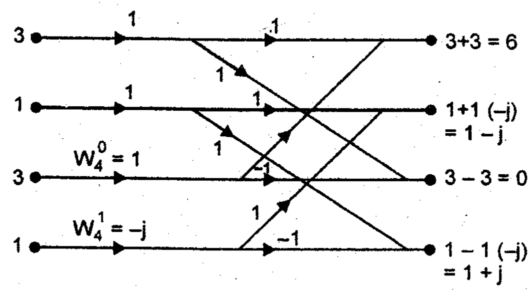


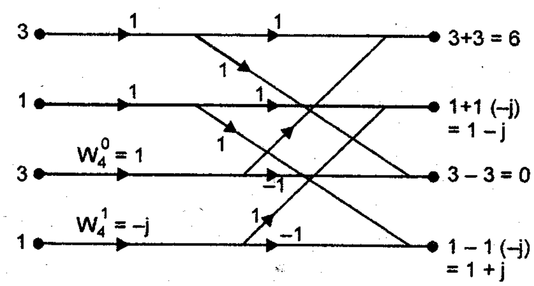
Input sequence = {3, 1, 3, 1, 3, 1, 3, 1)

The phase factors involved in second stage computation are W & W.

The 0/P OFT sequence = {3, 1, 3, 1, 3, 1, 3, 1} Second stage computation are W_4^0 & W_4^1.

The butterfly computation are shown below.





Output DFT sequence = {6, 1–*j*, 0, 1+*j*, 6, 1–*j*, 0 1+*j*}

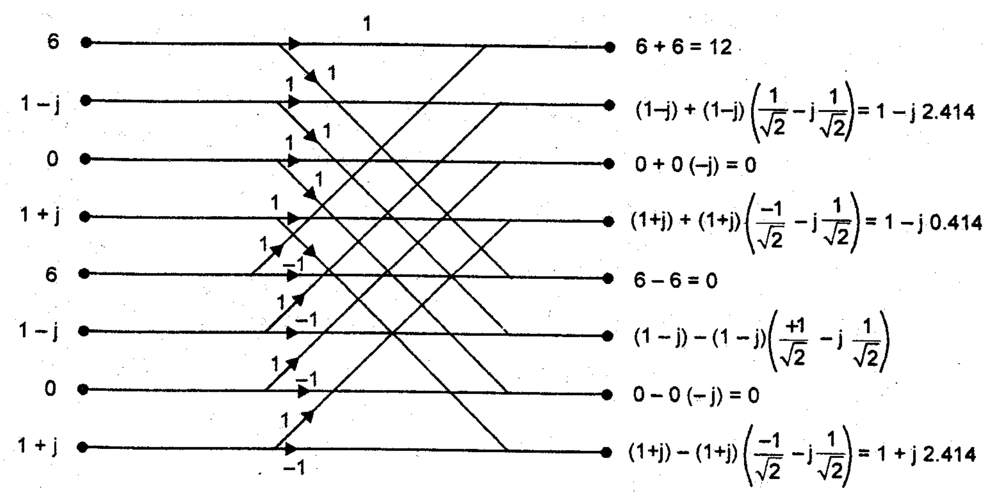
Third stage computation

I/P DFT sequence = {6, 1–*j*, 0, 1+*j*, 6, 1–*j*, 0, 1+*j*}

The phase factors involved in third stage computation are W_8^0, W_8^1, W_8^2 & W_8^3.

$$W_8^0 = 1,\ W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}},\ W_8^2 = -j,\ W_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

The butterfly computations of third stage are shown below



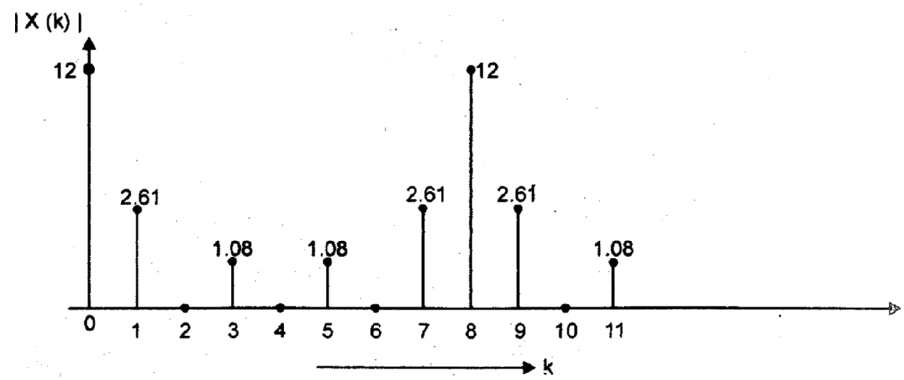
The output DFT sequence = {12, 1–*j* 2.414, 0, 1–*j* 0.414, 0, 1+*j*0.414, 0, 1+*j*2.414}

DFT $x(n)$ = X (k) {12, 1–*j*2.414, 0, 1–*j*0.414, 0, 1+*j*0, 0.414, 0, 1+*j* 2.414}

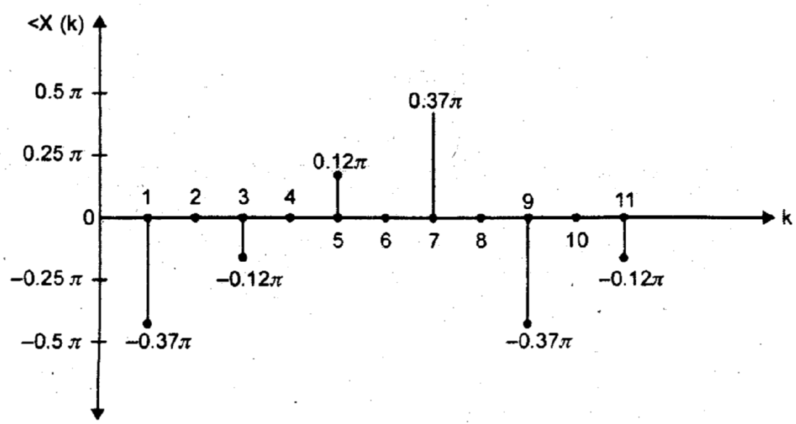
|X (k)| = [12, 2.61,0, 1.08, 0, 1.08, 0, 2.61]

< X (k) = {0, –0.37π, 0, –0.12π, 0, 0.12π, 0, 0.37π}

Magnitude spectrum



Phase spectrum



12. An 8-point sequence is given by x(n) = {2, 2, 2, 2, 1, 1, 1, 1}.Compute 8 point DFT of x(n) by radix -2 DIF-FFT.

Ans. For 8 point DFT by radix 2 FF1 we require 3-stages of computation with 4 butterfly computation in each stage.

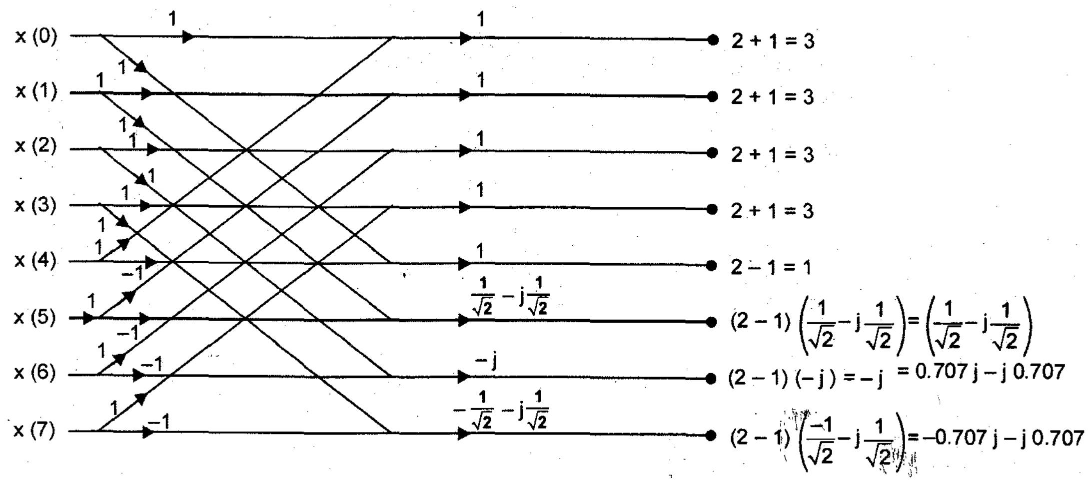
The given sequence is the input to the first stage. For other stages of computation, the output of the previous stage will be the input for the current stage.

First stage of computation

The input sequence = {2, 2, 2, 2, 1, 1, 1, 1)

The phase factors involved in first stage of computation are $W_8^0,\ W_8^1,\ W_8^2\ \&\ W_8^3$.

$$W_8^0 = 1,\ W_8^1 = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}},\ W_8^2 = -j,\ W_8^3 = \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$$

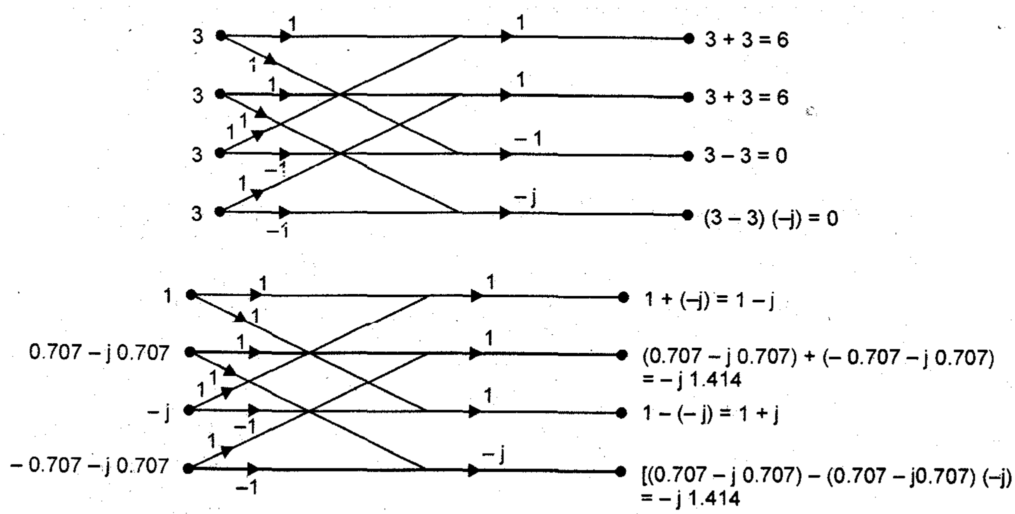


The output sequence of first stage of computation.

= {3, 3, 3, 3, 0.707,–*j* 0.707, –*j*, -0.707–*j* 0.707}

Second stage of computation

The input sequence of 2nd stage = {3, 3, 3, 3, 0.707, –*j* 0.707, –*j*, –0.707–*j*0.707}

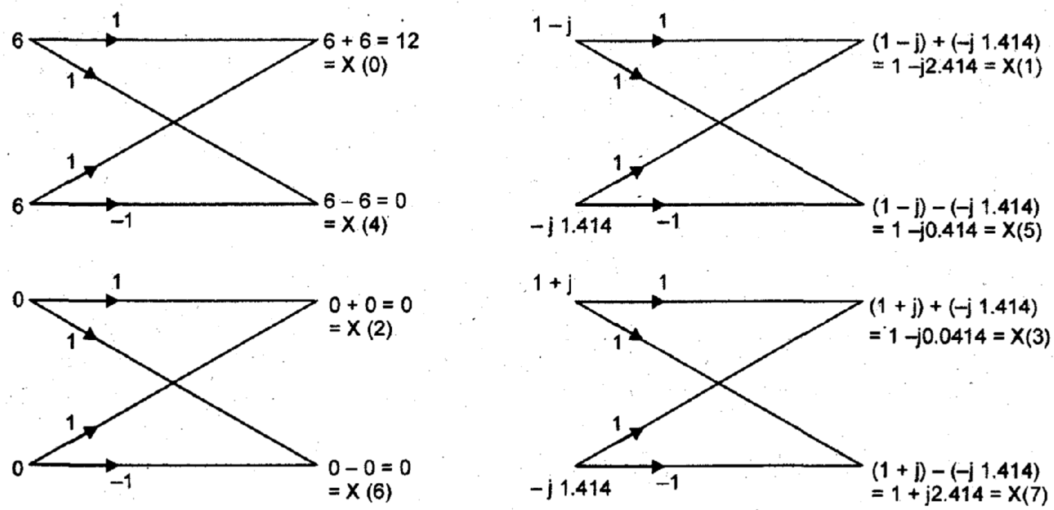


The phase factors involved in 2nd stage of computation are W_4^0 & W_4^1

$W_4^0 = 1$, $W_4^1 = -j$

The butterfly computation of second stage are shown above.

The output sequences = {6, 6, 0, 0, 1–j, –j 1.414, 1+j, –j1.414}



The sequence X(k) is bit reversed order.

X (0) = 12, X (4) = 0, X (2) = 0, X (6) = 0

X (1) = 1–j 2.414, X (5) = 1 + j 0.414

X (3) = 1–j 0.414, X (7) = 1 + j 2.414.

The sequence X(k) is normal order.

X (0) = 12, X (1) = 1–j 2.414, X (2) = 0, X (3) = 1–j0.414

X (4) = 0, X (5) = 1 + j0.414, X (0) = 0, X (7) = 1 + j 2.414

X (k) = {12, 1–j2.414, 1–j0.414, 0, 1+ j0.414, 0, 1+j 2.414}

13. i)Discuss the properties of DFT. (Nov2014) **(May/June2013)**
REF Q.No 2

ii) Discuss the use of FFT algorithm in linear filtering and correlation. **(Nov/Dec2013)**

In the preceding section it was demonstrated that the product of two DFTs is equivalent to the circular convolution of the corresponding time-domain sequences. Unfortunately, circular convolution is of no use to us if our objective is to determine the output of a linear filter to a given input sequence. In this case we seek a frequency-domain methodology equivalent to linear convolution.

Suppose that we have a finite-duration sequence $x(n)$ of length L which excites an FIR filter of length M. Without loss of generality, let

$$\begin{aligned} x(n) &= 0, \qquad n < 0 \text{ and } n \geq L \\ h(n) &= 0, \qquad n < 0 \text{ and } n \geq M \end{aligned}$$

where $h(n)$ is the impulse response of the FIR filter.

The output sequence $y(n)$ of the FIR filter can be expressed in the time domain as the convolution of $x(n)$ and $h(n)$, that is

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k) \tag{7.3.1}$$

Since $h(n)$ and $x(n)$ are finite-duration sequences, their convolution is also finite in duration. In fact, the duration of $y(n)$ is $L + M - 1$.

The frequency-domain equivalent to (7.3.1) is

$$Y(\omega) = X(\omega)H(\omega) \tag{7.3.2}$$

If the sequence $y(n)$ is to be represented uniquely in the frequency domain by samples of its spectrum $Y(\omega)$ at a set of discrete frequencies, the number of distinct samples must equal or exceed $L+M-1$. Therefore, a DFT of size $N \geq L+M-1$ is required to represent $\{y(n)\}$ in the frequency domain.

Now if

$$\begin{aligned} Y(k) &\equiv Y(\omega)|_{\omega=2\pi k/N}, \qquad & k = 0, 1, \ldots, N-1 \\ &= X(\omega)H(\omega)|_{\omega=2\pi k/N}, \qquad & k = 0, 1, \ldots, N-1 \end{aligned}$$

then

$$Y(k) = X(k)H(k), \qquad k = 0, 1, \ldots, N-1$$

where $\{X(k)\}$ and $\{H(k)\}$ are the N-point DFTs of the corresponding sequences $x(n)$ and $h(n)$, respectively. Since the sequences $x(n)$ and $h(n)$ have a duration less than N, we simply pad these sequences with zeros to increase their length to N. This increase in the size of the sequences does not alter their spectra $X(\omega)$ and $H(\omega)$, which are continuous spectra, since the sequences are aperiodic. However, by sampling their spectra at N equally spaced points in frequency (computing the N-point DFTs), we have increased the number of samples that represent these sequences in the frequency domain beyond the minimum number (L or M, respectively).

Since the ($N = L + M - 1$)-point DFT of the output sequence $y(n)$ is sufficient to represent $y(n)$ in the frequency domain, it follows that the multiplication of the N-point DFTs $X(k)$ and $H(k)$, according to (7.3.3), followed by the computation of the N-point IDFT, must yield the sequence $\{y(n)\}$. In turn, this implies that the N-point circular convolution of $x(n)$ with $h(n)$ must be equivalent to the linear convolution of $x(n)$ with $h(n)$. In other words, by increasing the length of the sequences $x(n)$ and $h(n)$ to N points (by appending zeros), and then circularly convolving the resulting sequences, we obtain the same result as would have been obtained with linear convolution. Thus with zero padding, the DFT can be used to perform linear filtering.

14.Find DFT of x(n)={1,1,2,0,1,2,0,1} using DIT FFT algorithm and plot the spectrum. **(Nov/Dec2013)**

Using algorithm method find DFT.

15.Find 8 point DFT of the following sequence using direct method;{1,1,1,1,1,1,0,0}

Equation for computing DFT using direct method

$$X(k) = \sum_{n=0}^{N-1} e^{-j2\pi nk/N} \quad k=0,\ 1, \ldots ,N-1$$

(2 marks)

For the given sequence N=8. By substituting k value and N value in the given equation the following values should be computed.

X(0) = 6

X(1) = -0.707 – j 1.707

X(2) = 1- j

$X(3) = 0.707 + j\,0.293$

$X(4) = 0$

$X(5) = 0.707 - j\,0.293$

$X(6) = 1 + j$

$X(7) = -0.707 + j\,1.707$

16.i)Compute the 8 point DFT of the following sequence using radix 2 Decimation in Time FFT Algorithm: x(n) = {1,-1, 1,-1, 1,-1, 1,-1}

Butterfly diagram should be drawn.

The outputs of each stage are given below:

Input of I stage	Output of I stage	Output of II stage	Output of III stage
1	2	2	0
1	0	2j	-1.414 + j 3.414
-1	0	2	2 – 2j
1	-2	-2j	1.414 – j 0.586
-1	0	-2	4
1	-2	-2	1.414 + j 0.586
-1	-2	2	2 + 2j
-1	0	-2	-1.414 – j 3.414

ii) Discuss the use of FFT in linear filtering. **(May/June2013)**

The overlap-add and overlap-save methods are used for filtering a long data sequence with an

FIR filter based on the use of the DFT.

FFT algorithm can be used for computing DFT and IDFT.

Comparison of overlap-add method and over-lap save method in filtering.

17. a)Find the 4 point DFT of a) $x(n) = 2^n$ b) $x(n) = \{0,1,0,-1\}$ **(Nov2014)**
Use formula method to find DFT.
18. Find 8- point DFT of sequence $x(n)= n+1$ using radix 2 DIF FFT algorithm **(Nov 2014)**
Using algorithm method to find DFT.

19. Find 8- point DFT of sequence $x(n)= n$ using radix 2 DIT FFT algorithm **(May 2014)**
Using algorithm method to find DFT