ST. JOSEPH'S COLLEGE OF ENGINEERING,CHENNAI-119

ST. JOSEPH'S INSTITUTE OF TECHNOLOGY, CHENNAI-119

SUB NAME & CODE: TRANSFORMS & PARTIAL DIFFERENTIAL EQUATIONS - MA 6351

ASSIGNMENT – III

UNIT – IV FOURIER TRANSFORM

PART - A

1 **State Fourier integral theorem**

2 **Find the Fourier transform of** $\mathrm{f(x)} = \begin{cases} 1; \text{ for } |\mathrm{x}| < 2 \\ 0; \text{ for } |\mathrm{x}| > 2 \end{cases}$

3 **Define Self-reciprocal under Fourier transform. Give an example.**

4 **Write the Fourier sine transform pair and Fourier Cosine transform pair.**

5 **If** $F_C(f(x)) = F_C(s)$ *and* $F_S(f(x)) = F_S(s)$, **prove that** $F_c(f(x)\sin ax) = \frac{1}{2}\left[F_s(s+a) + F_s(a-s)\right]$

6 **State and prove the change of scale property of Fourier Transform.**

7 **If F(s) is the Fourier transform of f(x), then show that** $F\{f(x-a)\} = e^{ias}\, F(s)$

8 **State Parseval's identity theorem in Fourier Transform.**

PART - B

1 **Show that the Fourier transforms of** $f(x) = \begin{cases} a^2 - x^2; & |x| < a \\ 0; & |x| > a > 0 \end{cases}$ **is** $2\sqrt{\frac{2}{\pi}}\left[\frac{\sin as - as\cos as}{s^3}\right]$. **Hence deduce that** $\int_0^\infty \frac{\sin t - t\cos t}{t^3} dt = \frac{\pi}{4}$. **Using Parseval's Identity, show that** $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt = \frac{\pi}{15}$.

2 a. **Show that** $e^{-\left(x^2/2\right)}$ **is a self-reciprocal under Fourier cosine Transform.**

b. **Express** $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ **as Fourier integral, hence evaluate** $\int_0^\infty \frac{\sin s\ \cos sx}{s} ds$ **. Also find the value of** $\int_0^\infty \frac{\sin s}{s} ds$

3 a. **Evaluate** $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ **using Fourier Transforms ; a>0, b>0.**

b. **Find the Fourier sine transform of** $\frac{e^{-ax}}{x}$ **where** $a > 0$.

4 a. **Find the Fourier transform of** $f(x) = \begin{cases} a - |x|, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$ **and hence evaluate** $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$

b. **Find the Fourier transform of** $e^{-a|x|}$**, a >0. Deduce that** $\int_0^\infty \frac{\cos xt}{a^2+t^2} = \frac{\pi}{2a} e^{-a|x|}$