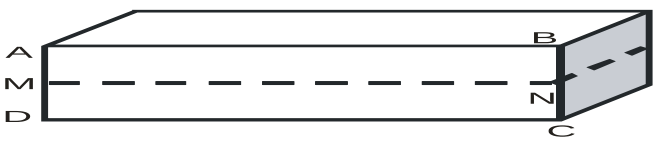
<u>Beam</u>

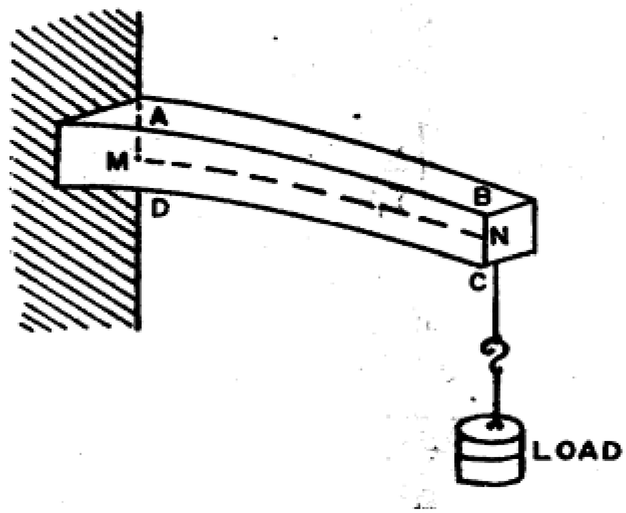
A beam is a rod or bar of uniform cross–section (circular or rectangular) whose length is very large compared to the other dimensions as shown in Figure

A beam is considered to be made up of a large number of thin plane layers called surfaces placed one above the other.

Consider a beam to be bent into an arc of a circle by the application of an external couple. Taking the longitudinal section ABCD of the bent beam, the layers in the upper half are elongated while those in the lower half are compressed. .

In the middle there is a layer (MN) which is not elongated or compressed due to bending of the beam.This layer is called the ***'neutral surface'*** and the line (MN) at which the neutral layer intersects the plane of bending is called the ***'neutral axis'***.

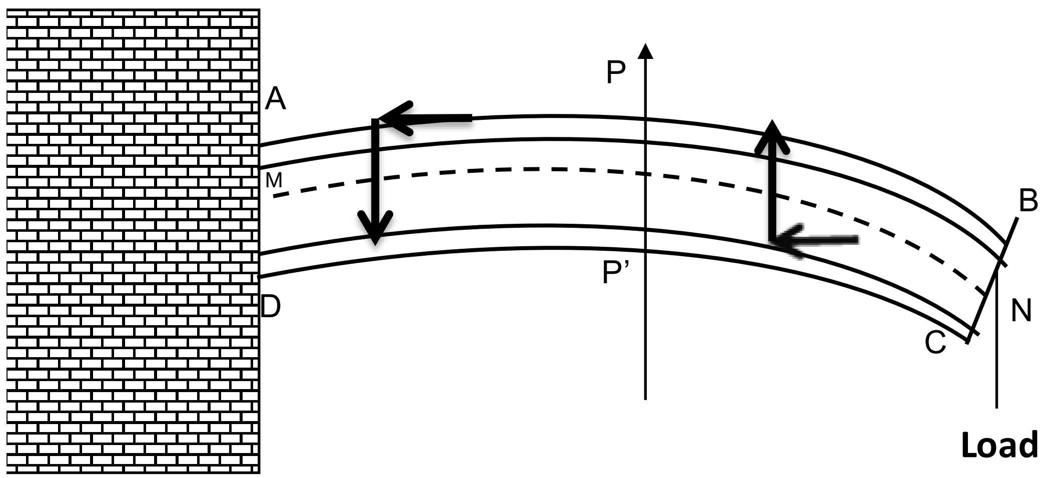




It is found that the length of the filament increases or decreases in proportion to its distance away from the neutral axis MN. The layers below MN are compressed and those above MN are elongated and there will be such pairs of layers one above MN and one below MN experiencing same forces of elongation and compression due to bending and each pair forms a couple.

The resultant of the moments of all these internal couples is called the ***internal bending moment*** and at the equilibrium condition, this is equal to the external bending moment.

BENDING MOMENT OF A BEAM



Consider a beam ***ABCD*** is fixed at one end and loaded at other end. Consider the small section *PBCP*'of the beam; the extended filaments lying above the neutral axis MN are exert an inward pressure. The shortened filaments lying below the neutral axis MN are exert an outward pressure. As a result of inward and outward pressure in the lower and upper halves of the beam, tensile and compressive stresses develop in the respective halves of the beam and setup a couple which opposes to bending of the beam. The moment of this couple is said to be the ***moment of the resistance to bending.*** At equilibrium, the bending moment is equal to the restoring couple or the moment of resistance to the bending.

To find an expression for the moment of the restoring couple, consider a portion of the beam ABMN. The neutral filament MN is at a distance of *r* from the upper filament. Let the radius of curvature be *R* and ϕ be the angle subtended by it at the centre of curvature. In unstrained position of the beam, the length of the filament (beam) C*D* = MN = $R\phi$. The length of the filament when strained is $(R + r)\ \phi$

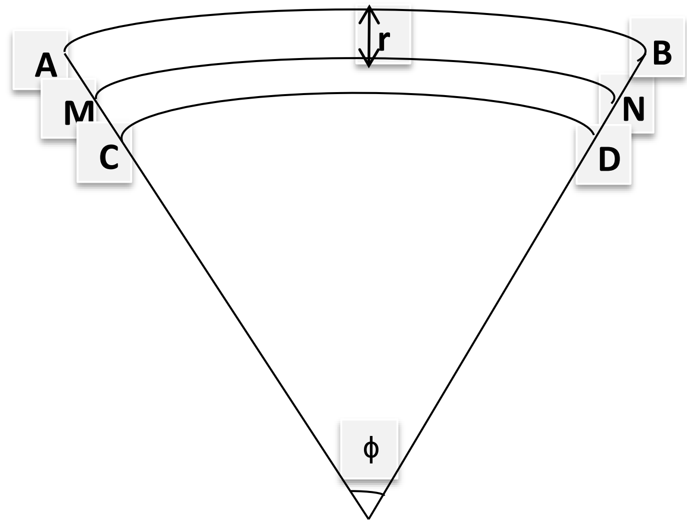
Strain in the filament CD, $= \dfrac{Change\ in\ the\ length}{Original\ length}$

$$Strain = \frac{(R + r)\ \phi - R\phi}{R\phi} = \frac{r}{R}$$

The above equation shows that the strain is proportional to the distance from the neutral axis. The young's modulus is given by,

$$Y = \frac{Stress}{Strain}$$

i.e., Stress = Y x Strain = $Y\,r\,/\,R$



Let A be the area of cross section of the beam,

$$Stress = \frac{Force}{Area}$$

$\therefore$ Force applied on the area A is

$$F = Y(r/R) \times A$$

Therefore the moment of this force about MN

$$= Y(r/R) \times A \times r = Y A r^2 / R$$

As the moment of the forces acting on both the upper and lower halves of the section are in the same direction, the total moment of the force is given by

$$= \Sigma Y \frac{a r^2}{R} = \frac{Y}{R} \Sigma a r^2 = \frac{Y}{R} I_g$$

Where I_g is the geometrical moment of inertia and is equal to $\Sigma a r^2$ or AK^2, A being the total area of the section and K being the radius of gyration.

$$\therefore \text{Moment of the force} = \frac{Y}{R} I_g$$

In equilibrium, bending moment of the beam is equal and opposite to the moment of bending couple due to the load on one end.

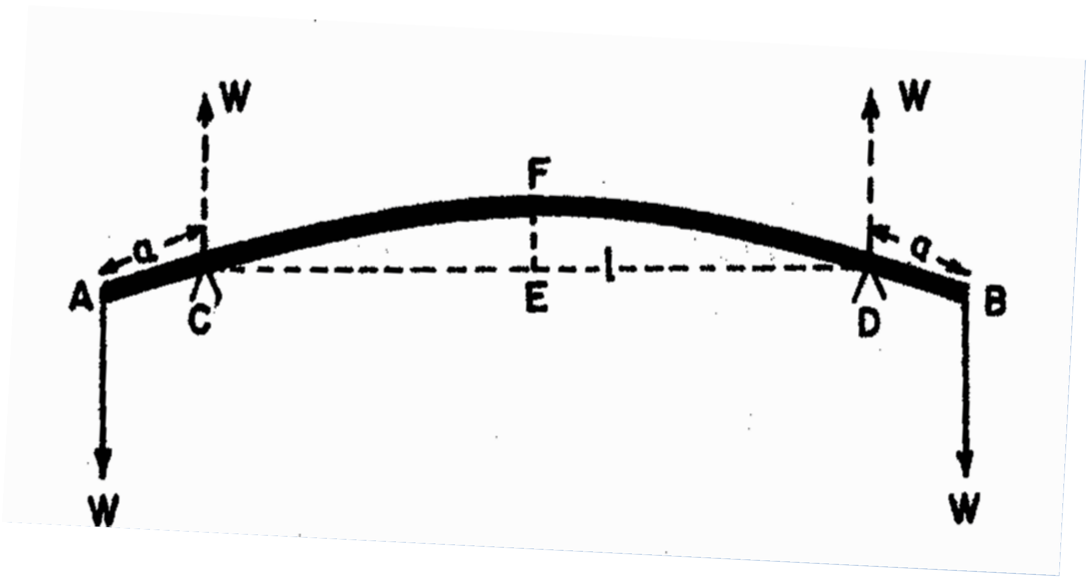
$$\therefore \text{Bending moment of the beam} = \frac{Y}{R} I_g$$

The quantity YI_g ($=YAK^2$) is called the flexural rigidity of the beam. ***Flexural rigidity*** is defined as the bending moment required produces a unit radius of curvature.

UNIFORM BENDING (THEORY AND EXPERIMENT)

The beam is loaded uniformly on its both ends; the bent beam forms an arc of a circle. An elevation in the beam is produced. This type of bending is called as ***uniform bending***.

Consider a beam (or bar) AB arranged horizontally on two knife – edges C and D symmetrically so that AC = BD = a,



The beam is loaded with equal weights W and W at the ends A and B.

The reactions on the knife edges at C and D are equal to W and they are acting vertically upwards.

The external bending moment on the part AF of the beam is

= W x AF – W x CF = W (AF – CF)

= W x AC = W x a

Internal bending moment = $\frac{YI_g}{R}$

- Y - Young's' modulus of the material of the bar
- I_g - Geometrical moment of inertia of the cross-section of beam
- R - Radius of curvature of the bar at F

In the equilibrium position,

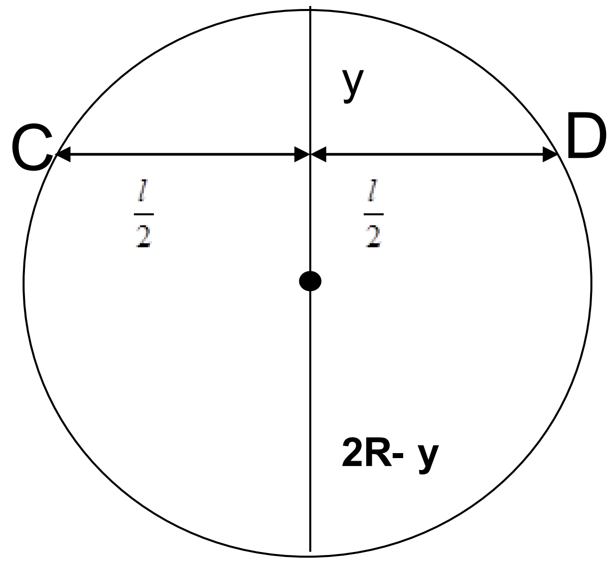
External bending moment = Internal bending moment

$$Wa = \frac{YI_g}{R}$$

Since for a given value of W, the values of a, Y and I_g are constants, R is constant so that the beam is bent uniformly into an arc of a circle of radius R.

CD = l and y is the elevation of the midpoint E of the beam so that y = EF

Then the property of circle



EF X EG = CE X ED

EF (2R – EF) = (CE)2

$$y\,(2R - y) = \left(\frac{l}{2}\right)\left(\frac{l}{2}\right)$$

$$y\,2R = \frac{l^2}{4} \quad (y^2 \text{ is negligible})$$

$$y = \frac{l^2}{8R}$$

$$\frac{1}{R} = \frac{8y}{l^2}$$

$$\text{Therefore } Wa = \frac{8y}{\ell^2} YI_g$$

$$Y = \frac{W\,l^2 a}{8 I_g y}$$

If the beam is of rectangular cross-section, $I_g = \frac{bd^3}{12}$, where b is the breath and d is the thickness of beam.

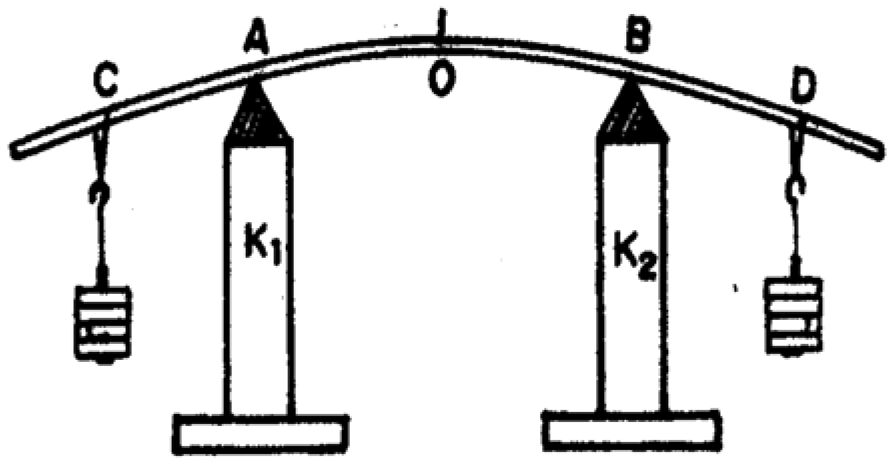
If M is the mass, the corresponding weight W = Mg

$$\text{Hence} \quad Y = \frac{3}{2}\frac{Mgl^2 a}{bd^3 y}$$

From which Y the Young’s modulus of the material of the bar is determined.

Experiment

A rectangular beam AB of uniform – section is supported horizontally on two knife – edges A and B as shown in Figure



Two weight hangers of equal masses are suspended from the ends of the beam. A pin is arranged vertically at the mid-point of the beam. A microscope is focused on the tip of the pin.

Initial reading of the microscope in the vertical scale is noted.

Equal weights are added to both hangers simultaneously and the reading of the microscope in the vertical scale in noted.

The experiment is repeated for decreasing order of magnitude of the equal masses.

The observations are then tabulated and the mean elevation (y) at the mid point of the bar is determined.

s.no	*Load*	*Microscope reading*						*mean*	*Elevation*
		loading			*Unloading*				
		MSR mm	***VSC div***	***TR div***	***MSR mm***	***VSC div***	***TR div***		
1	***W***								
2	***W+50***								
3	***W+100***								
4	***W+150***								
5	***W+200***								

The length of the bar between the knife edges '*l*' is measured. The distance of one of the weight hangers from the nearest knife edge p is measured. The breadth (*b*) and thickness (*d*) of the bar are measured by using vernier calipers and screw gauge.

The young's modulus of the material of the beam is determined by the relation

- $Y = \frac{3}{2}\frac{Mgl^2a}{bd^3y}$

DEPRESSION OF A CANTILEVER

Cantilever:

It is a beam fixed horizontally at one end and loaded at the other end.

AB is the neutral axis of a cantilever (a beam or rod) of length l is fixed at the end A and loaded at the free end B by a weight W. The end B is depressed to B'.

BB' represents the vertical depression at the free end.

Consider the section of the cantilever P at a distance x from the fixed end A. It is a distance (l-x) from the loaded end B'. Considering the equilibrium of the portion PB', there is a force of reaction W of P.

$\therefore$ external bending moment = $W \; x \; PB' = W\,(l-x)$

Internal bending moment of the cantilever = $\frac{YI}{R}$

Where Y – Young's modulus of the cantilever.

I - Geometrical moment of inertia of its cross-section.

R - Radius of the curvature of the neutral axis at P.

In the equilibrium position,

External bending moment = Internal bending moment

$$W\,(l-x) = \frac{YI}{R} \qquad (1)$$

Q is another point at a distance dx form P i.e., PQ = dx

O is the centre of curvature of the arc PQ

PO = R, $\angle$POQ = $d\theta$

Then, $dx = Rd\theta$ (2)

The tangents are drawn at P and Q meeting the vertical line BB' at C and D.

Vertical depression CD = $dy = (l-x)\, d\theta$ (3)

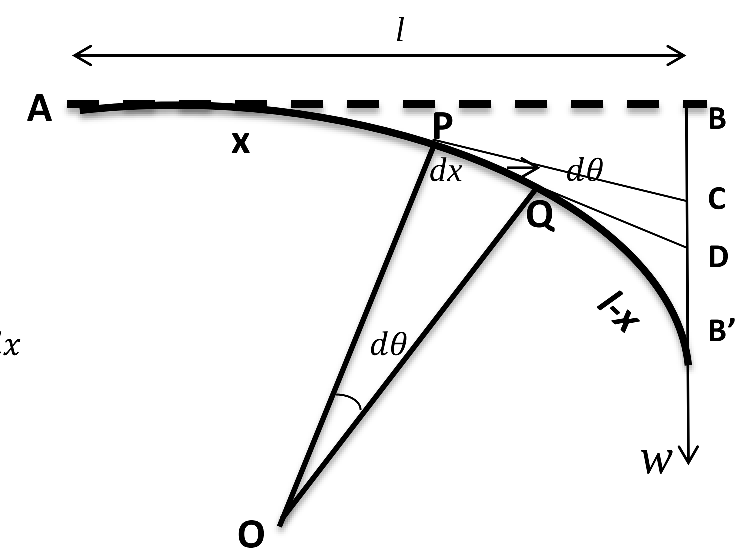
From equations (2) and (3)

$$\frac{dx}{dy} = \frac{R\ \ d\theta}{(l-x)\ d\theta} = \frac{R}{(l-x)}$$

or
$$R = \frac{(l-x)dx}{dy} \qquad (4)$$

Substitute the value of R in eqn. (1), we have

$$w(l-x) = \frac{YI\ dy}{(l-x)\ dx} \qquad (5)$$



or
$$dy = \frac{W}{YI}(l-x)^2 dx$$

The total depression y at the free end is

$$y = \int_0^l \frac{W}{YI}(l-x)^2 dx$$

$$y = \frac{W}{YI}\int_0^l (l^2 + x^2 - 2lx) dx$$

$$y = \frac{W}{YI} \text{ x } \frac{l^3}{3}$$

or
$$y = \frac{W}{YI}\frac{l^3}{3}$$

the young's modulus of the cantilever is determined using the value of depression produced in the cantilever.

The depression at the free end of a single cantilever is

$$y = \frac{W}{YI}\frac{l^3}{3}$$

$$Y = \frac{Wl^3}{3Iy}$$

If the beam is of rectangular cross-section, $I_g = \frac{bd^3}{12}$, where b is the breath and d is the thickness of beam.

If M is the mass, the corresponding weight W = Mg

Hence $$Y = \frac{4}{}\frac{Mgl^3}{bd^3y}$$

From which Y is determined experimentally.