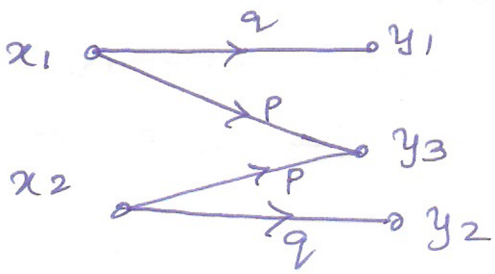
The transition matrix of BEC is

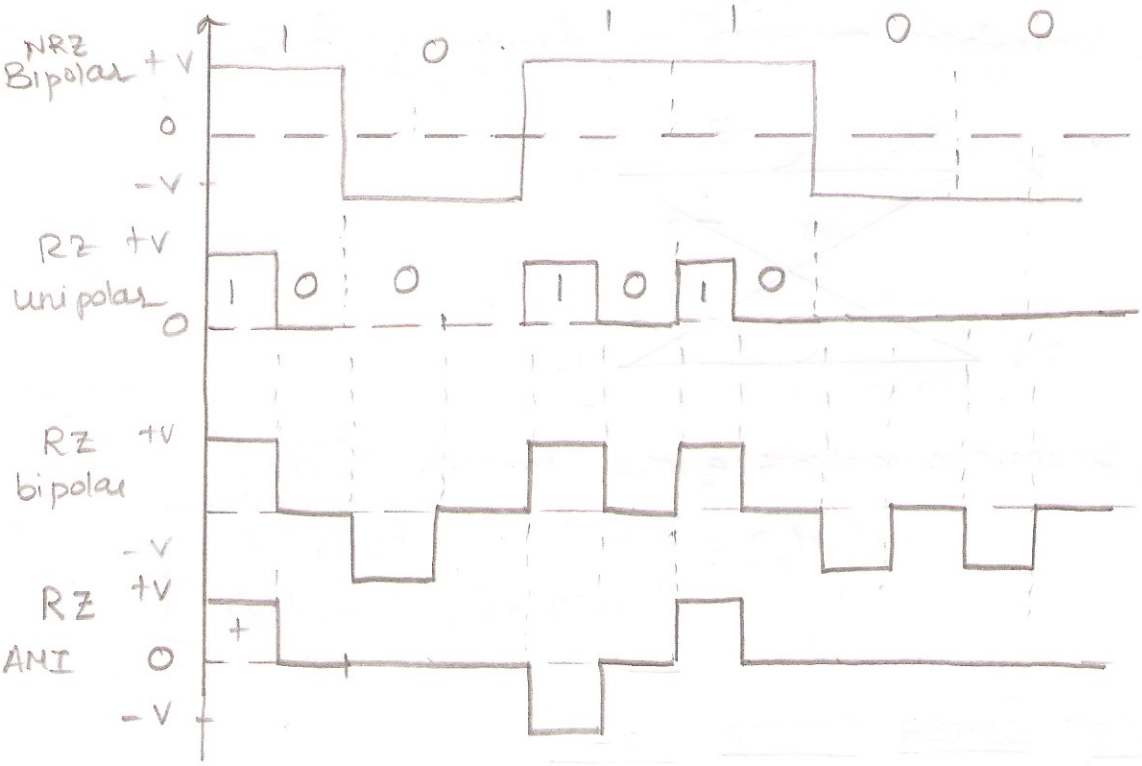


Assume $P(x_1) = P(x_2) = 0.5$.

$$P(Y/X) = \begin{array}{c|ccc} & y_1 & y_2 & y_3 \\ \hline x_1 & q & 0 & p \\ x_2 & 0 & q & p \end{array}$$

$$P(X,Y) = \begin{bmatrix} q/2 & 0 & p/2 \\ 0 & q/2 & p/2 \end{bmatrix}.$$

(ii) Information {1 0 1 1 0 0}



7. Construct a convolutional encoder whose constraint length is 3 and has 3 modulo-2 adders and an o/p multiplexer. The generator sequences of the encoder are $g^{(1)} = (1,0,1)$, $g^{(2)} = (1,1,0)$, $g^{(3)} = (1,1,1)$. Draw the block diagram of the encoder. Find the encoder o/p produced by the message sequence 10111

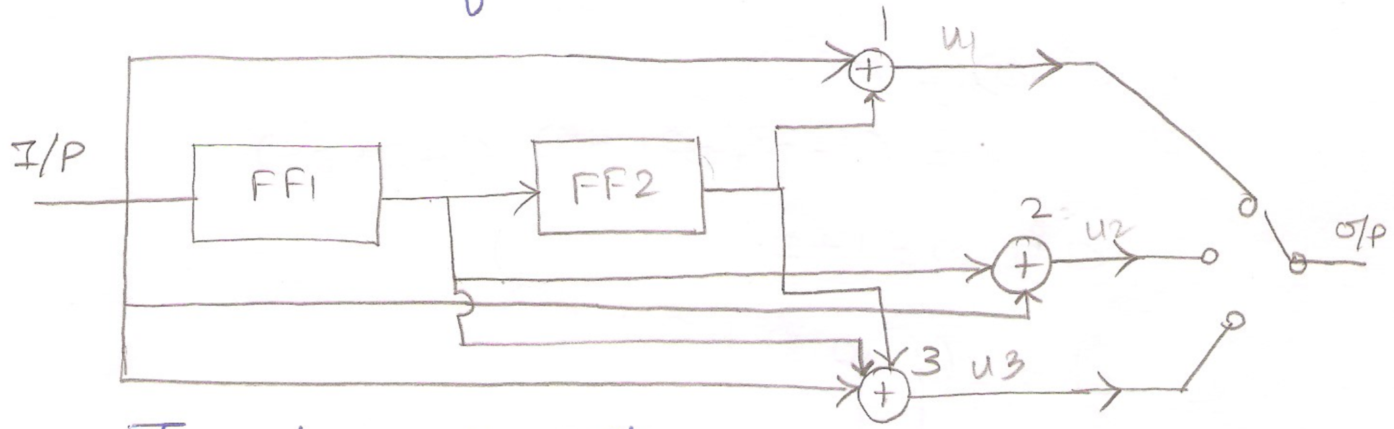
(Refer Notes)

Number of flipflops $m = \text{Constraint length } (k) - 1$.

$= 3-1$.

Encoder diagram

No. of adders $= 3 = n$.



Time domain approach

I/P	FF1	FF2	Adder 1 (I/P+FF2)	Adder 2 (I/P+FF1)	Adder 3 (I/P+FF1+FF2)	o/p
	0	0				
1	1	0	1+0=1	1+0=1	1+0+0=1	111
0	0	1	0+0=0	0+1=1	0+1+0=1	011
1	1	0	1+1=0	1+0=1	1+0+1=0	010
1	1	1	1+0=1	1+1=0	1+1+0=0	100
1	1	1	1+1=0	1+1=0	1+1+1=1	001

(10111) is codes as (111, 011, 010, 100, 001)

"Using algorithm" Transform domain approach.

1. $m(D) = 1 + D^2 + D^3 + D^4$

2. $g^1(D) = 1 + D^2$; $g^2(D) = 1 + D$; $g^3(D) = 1 + D + D^2$.

3. $c^1(D) = m(D) \cdot g^1(D)$

$$= (1+D^2+D^3+D^4)(1+D^2)$$

$$= 1+D^2+D^3+D^4+D^2+D^4+D^5+D^6.$$

$$c^1(D) = 1+D^3+D^5+D^6.$$

$$c^2(D) = m(D) \cdot g^2(D)$$

$$= (1+D^2+D^3+D^4)(1+D)$$

$$= 1+D^2+D^3+D^4+D+D^3+D^4+D^5.$$

$$c^2(D) = 1+D+D^2+D^5.$$

$$c^3(D) = m(D) \cdot g^3(D)$$

$$= (1+D^2+D^3+D^4) \cdot (1+D+D^2)$$

$$c^3(D) = 1+D+D^4+D^6$$

$$c^1 = (1\ 0\ 0\ 1\ 0\ 1\ 1)$$

$$c^2 = (1\ 1\ 1\ 0\ 0\ 1\ 0)$$

$$c^3 = (1\ 1\ 0\ 0\ 1\ 0\ 1)$$

So, code word is

(111, 011, 010, 100, 001, 110, 101)

8. The channel transition matrix is given by

$$\begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

(i) Draw the channel diagram and determine the probabilities associated with outputs assuming equiprobable inputs (8m).

(ii) Explain the properties of mutual information (8 m).

(i) Given $P(Y/X) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$

$$P(Y) = P(Y/X) \cdot P(X)$$

Since it is given as equiprobable, $P(x_1) = P(x_2) = 0.5$.

$$P(X) = [P(x_1) \cdot P(x_2)]$$

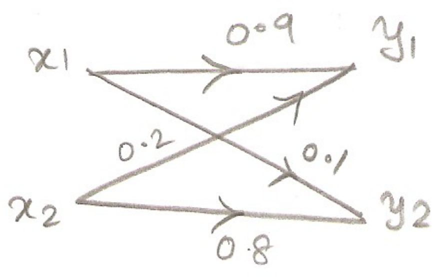
$$= [0.5 \quad 0.5]$$

Above given $P(Y/X)$ is in the form of

$$\begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & y_1/x_1 & y_2/x_1 \\ x_2 & y_1/x_2 & y_2/x_2 \end{array}$$

$$P(Y) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix} [0.5 \quad 0.5]$$

$$= \cancel{[(0.9)(0.5)} = [0.55 \quad 0.45].$$



(ii) Properties of mutual information.

1) The mutual information of a channel is symmetric, that is

$$I(X;Y) = I(Y;X).$$

Where $I(X;Y)$ is a measure of the uncertainty about the channel input that is resolved by observing the channel o/p and ~~for~~ $I(Y;X)$ is a measure of uncertainty about the channel o/p that is resolved by sending the channel input.

$$I(X;Y) = \sum_{j=0}^{j-1} \sum_{k=0}^{K-1} P(x_i, y_k) \log_2\left[\frac{P(x_j/y_k)}{P(x_j)}\right] \quad -①$$

From Baye's rule for conditional probabilities,

$$\frac{P(x_j/y_k)}{P(x_j)} = \frac{P(y_k/x_j)}{P(y_k)} \quad -②$$

Substituting eq ② in ①,

$$I(X;Y) = \sum_{k=0}^{k-1} \sum_{j=0}^{J-1} P(x_j, y_k) \log_2\left[\frac{P(y_k/x_j)}{P(y_k)}\right]$$

$$I(X;Y) = I(Y;X)$$

Hence proved.

2) The mutual information is always non-negative.

$$I(X;Y) \geq 0$$

The joint probability

$$P(x_i, y_k) = P\left(\frac{x_i}{y_k}\right) P(y_k).$$

$$P\left(\frac{x_j}{y_k}\right) = \frac{P(x_j y_k)}{P(y_k)} \quad -③$$

Substituting ③ in ①,

$$I(X;Y) = \sum_{j=0}^{J-1} \sum_{k=0}^{K-1} P(x_i, y_k) \log_2\left[\frac{P(x_j, y_k)}{P(x_j) \cdot P(y_k)}\right]$$

with fundamental inequality,

$$I(X;Y) \geq 0;$$

$P(X_j, Y_k) = P(X_j) \cdot P(Y_k)$ for all j and k (with equality if and only if)

9. Explain coding and decoding process of block codes (16m)

<u>Block codes</u>: The channel encoder accepts info. from the Source encoder in successive k bit blocks. For each block it adds (n-k) redundant bits that are algebraically related to the k-message bits thereby producing an overall encoded block of n where $n > k$.

The channel encoder produces bits at the rate

$$R_o = \left(\frac{n}{k}\right) R_s.$$

R_o = Bit rate of the channel.

R_s = Bit rate of the Source.

Types of Block Codes:

1) Linear Block codes.
2) Cyclic codes.

Encoder of a linear Block Code (n, k):-

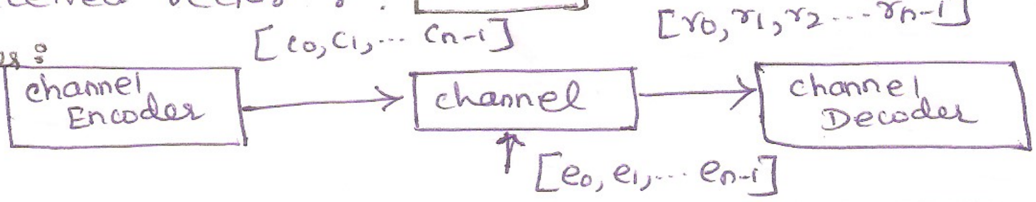
<u>Coding algorithm</u>:

1) Write the message vectors $m_{(1 \cdot k)}$
2) Compute the parity matrix $P_{k \cdot (n \cdot k)}$.
3) Compute the generator matrix $G_{k \cdot n} = [P_{k \cdot (n-k)} \vdots I_{k \times k}]$
4) Compute the Codeword $C_{1 \times n} = M_{1 \times k} G_{k \times n}$.

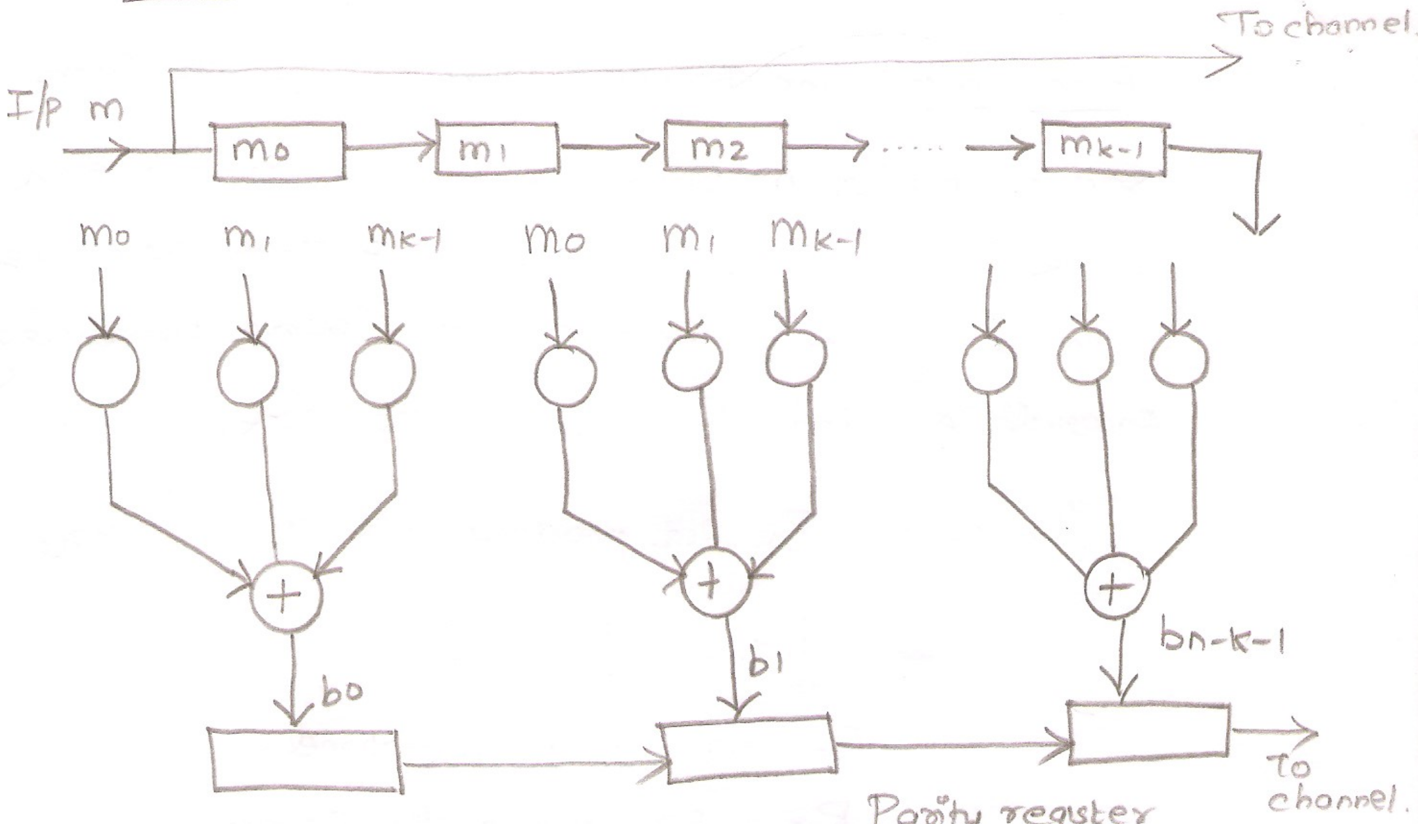
Decoder of a linear block Code:

<u>Syndrome</u>: Syndrome S is $1 \times (n-k)$ vector and it contains information about the error pattern and may therefore used for error detection (ie.,) used to decode the vector 'c' from the received vector 'r'. $\boxed{S = rH^T}$

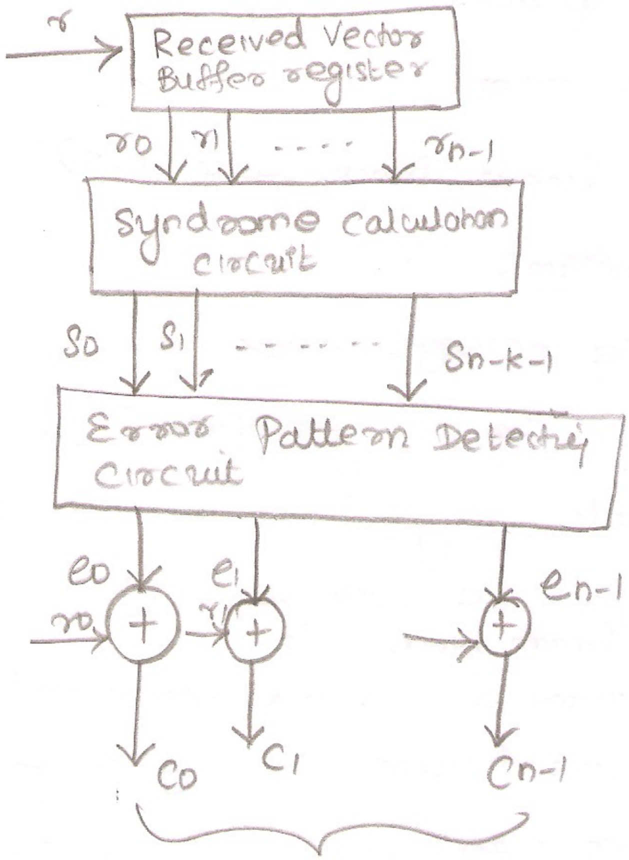
<u>Channel Encoder</u>:



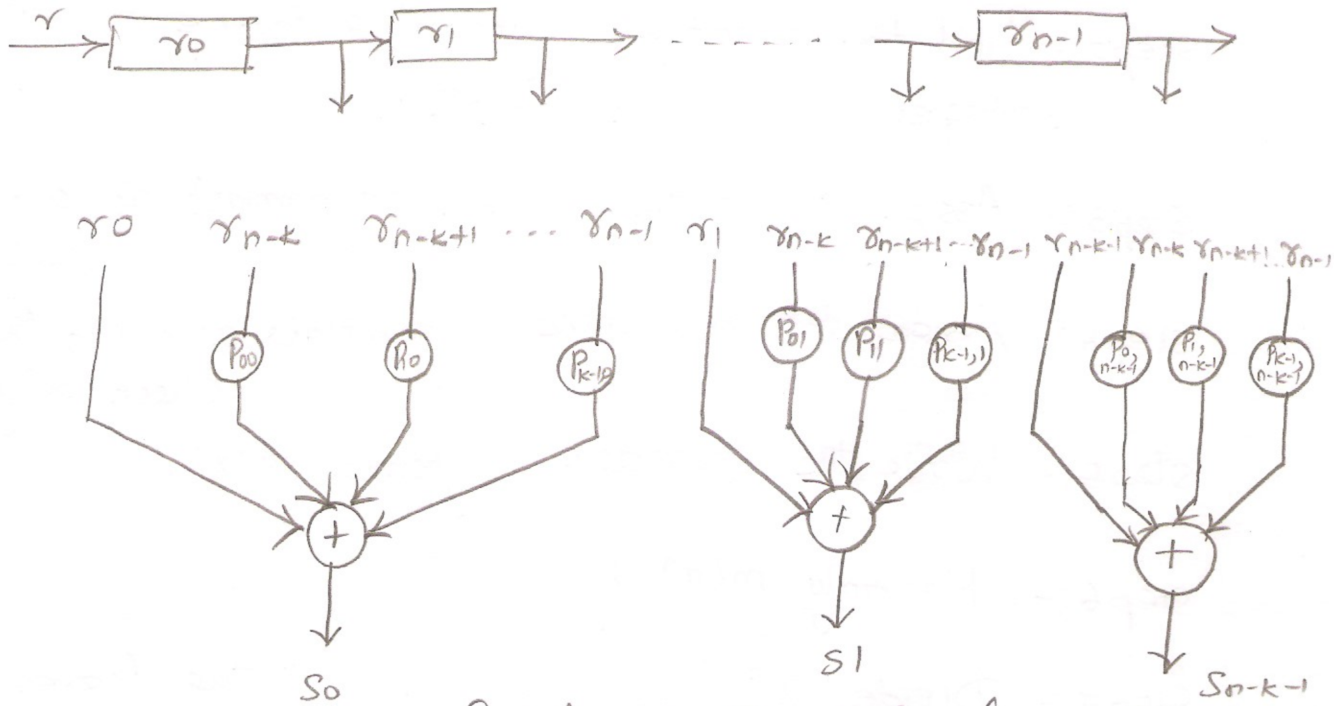
Encoder for linear Block Code.



Syndrome Decoder for a linear Block code :



Syndrome ckt.



Syndrome Circuit for Linear Syndrome code

Decoding Algorithm :-

Step 1 :- Compute the parity check matrix.

$$H_{(n-k)\times n} = I_{(n-k)\times(n-k)} \vdots P^T_{(n-k)\times k}.$$

Step 2 :- Get the received vector r.

Step 3 :- Compute the Syndrome $S = rH^T$.

Step 4 :- Construct the decoding table.

Step 5 :- Locate the coset leader or error pattern.

Step 6 :- Add the error pattern with the received vector to correct the error.

Encoding Algorithm of Cyclic codes

Step1:- Factorize x^n+1 into irreducible polynomials of degree 'm'.

Step2:- Find the primitive polynomial of degree 'm' which satisfies the condition $n = 2^m - 1$.

Step3:- Assign one of the primitive polynomials as generator polynomial $g(x)$.

Step4:- Assign the remaining polynomials as $b(x)$, the parity check polynomial.

Step5:- Write the message polynomial $m(x)$.

Step6:- Multiply $m(x)$ by x^{n-k}.

Step7:- Divide $x^{n-k} m(x)$ by $g(x)$ untill the degree of the remainder polynomial is less than or equal to $(n-k)$.

Step8:- Assign the remainder to the parity polynomial $b(x)$.

Step9:- Find the code polynomial using the expression.

$$c(x) = x^{n-k} m(x) + b(x).$$

Step10:- Derive the codeword from the code polynomial $c(x)$.

Decoding of Cyclic Codes.

Consist of same three steps as that for decoding linear codes.

1) Syndrome Computation.
2) Association of Syndrome to an error pattern.
3) Error Correction.

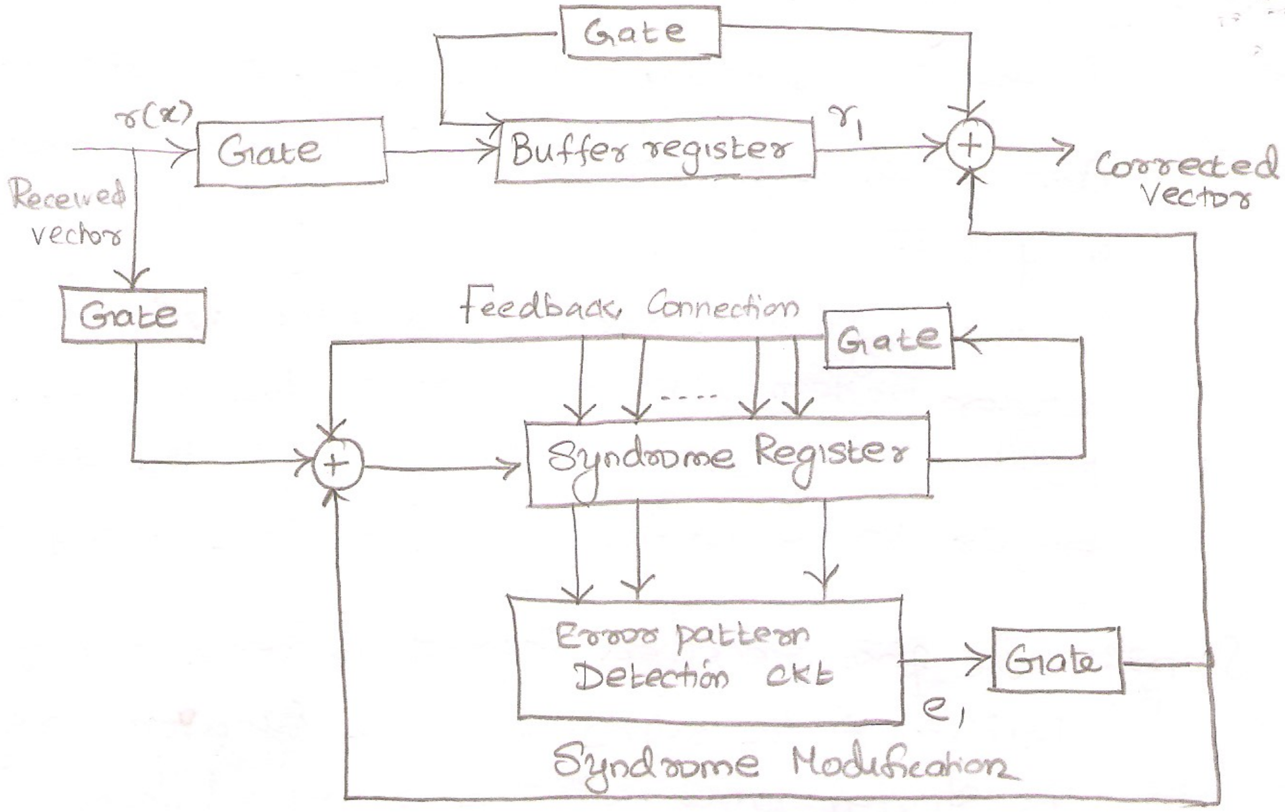
Step1:- The syndrome is formed by shifting the entire received vector into the syndrome register. At the same time received vector is stored into the buffer register.

Step 2 :- The Syndrome is read into the detector and is tested for the Corresponding error pattern. The detector is a Combinational logic ckt which is designed in such a way that its o/p is 1 if and only if the syndrome in the syndrome register Corresponds to a Correctable error pattern with an error at the highest-order position x^{n-1}. Output of the detector is the estimated error value for the symbol to come out of the buffer.

Step 3 :- The first received symbol is read out of the buffer. At the same time the syndrome register is shifted once. If the received symbol is detected to be an erroneous symbol, it is then Corrected by the o/p of the detector. In o/p of detector is fedback to the syndrome register to modify the Syndrome. This results in a new Syndrome.

Step 4 :- The new syndrome formed is used to detect whether or not the second received symbol is an erroneous symbol. The decoder repeats step 2 & 3. The second received symbol is Corrected in exactly the same manner as the first received symbol was Corrected.

Step 5 :- Decoder decodes the received vector, symbol by symbol in the manner outlined above until the entire received vector is read out of the buffer register.



Decoder of Cyclic Codes.

10. (i) With suitable examples explain the various line coding techniques (8m) (ii) Discuss any one of the decoding methods of convolutional coding precisely (8M).

(i) Line Codes :

The channel coded data represented by various dc levels → cannot be directly transmitted.

Symbol digits are mapped to a particular waveform before transmission. This is a line code.

Refer Question No 6(ii) for NRZ bipolar, RZ unipolar, RZ bipolar & RZ AMI example.

(1) AMI : Logic '0' is represented by '0' volts over the whole bit interval. A logic '1' is represented by +V or −V which persists for a fraction of the bit interval. Two successive 1's whether they occur in neighbouring bit intervals or are separated by '0's,

are represented by pulses of opposite polarity.

(2) Unipolar RZ : The excursion is between 0 and +V. But logic 1 is represented by a pulse which returns to zero after a brief period (½ bit period) within bit interval.

(3) Bipolar RZ :- The excursion is between +V and −V. Both logic '1' and '0' are represented by pulses that return to zero within the bit interval.

(4) Bipolar NRZ :- The excursion is between +V and −V. But pulses don't return to zero and stays at that level for the inter bit duration.

(5) HDB : A modified version of AMI is HDB (High Density Bipolar Code) which deliberately introduces 1's when a long string of '0's appear in AMI. The 1's are inserted in such a manner that they generate violations (i.e.,) manner that they generate they do not alternate orginal 1's. This additional 1's is helpful in clk synchronisation.

(ii) Decoding Method of Convolution Coding :-

Sequential Decoding :