

1. Branch and Bound - ² Assignment Problem

	Job 1	Job 2	Job 3	Job 4
Person 1	9	2	7	8
Person 2	6	4	3	7
Person 3	5	8	1	8
Person 4	7	6	9	4

1) Construct Root with least lower bound value.

Find minimum value in each row

$$lb = 2 + 3 + 1 + 4 = 10$$

Find minimum value in each column.

$$lb = 5 + 2 + 1 + 4 = 12$$

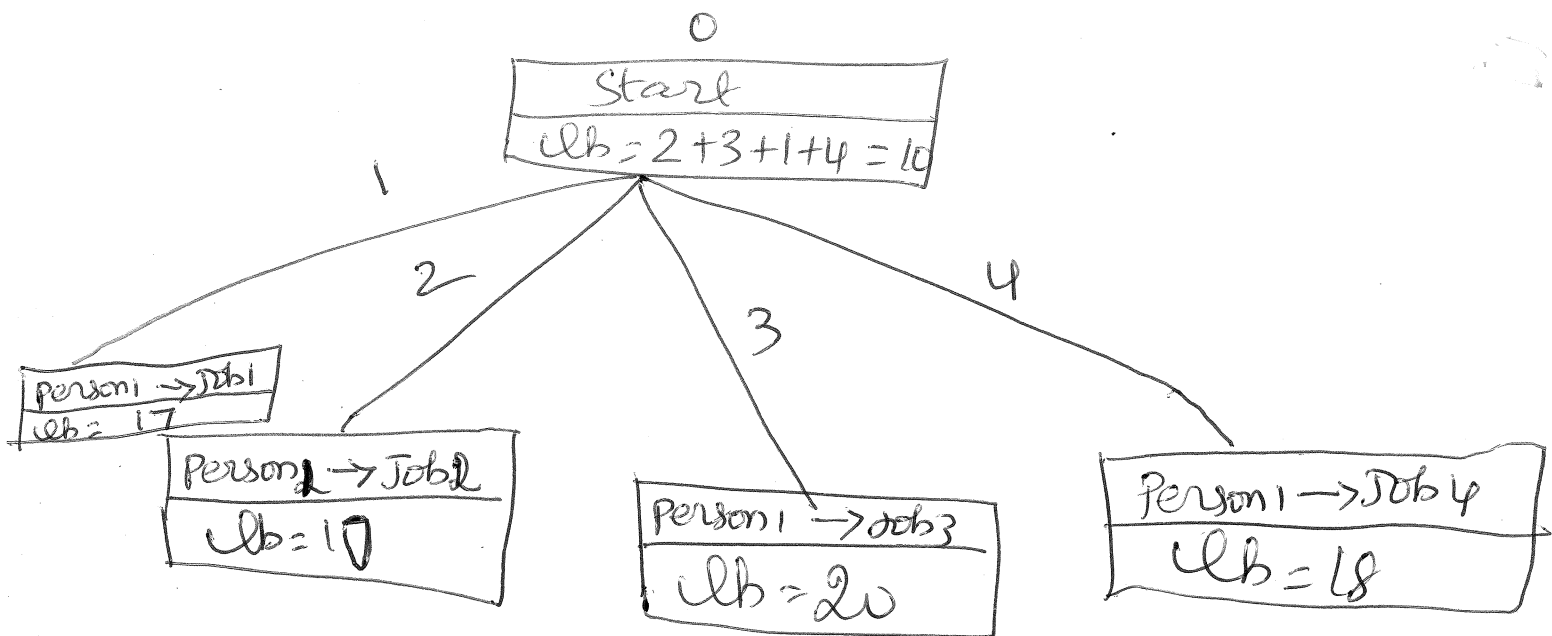
Out of these two values minimum is 10.

So root node is constructed with $lb = 10$

$$\begin{array}{|c|} \hline \text{Start} \\ \hline lb = 2 + 3 + 1 + 4 \\ \hline \end{array} = 10$$

2) using Best first search Branch root node.

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Person1 → Job1

Ulb = 9 + (since Person1 is assigned Job1 Value)
3 + 1 + 4 { other persons are assigned minimum value }

$$= 9 + 3 + 1 + 4$$

$$\boxed{\text{Ulb} = 17}$$

Person1 → Job2

Ulb = 2 + (since Person1 is assigned Job2 Value)
3 + 1 + 4 { other persons are assigned minimum value }

$$\boxed{\text{Ulb} = 10}$$

Person1 → Job3

Ulb = 7 + (since Person1 is assigned Job3 Value)
③ + ① + 4 (Person2 is not assigned Job3. Since it is assigned Person1. So choose next minimum value)

$$= 7 + 4 + 5 + 4$$

$$\boxed{\text{Ulb} = 20}$$

(Person3 is not assigned Job3. Since it is assigned Person1. So choose next minimum value)

Person1 → job4

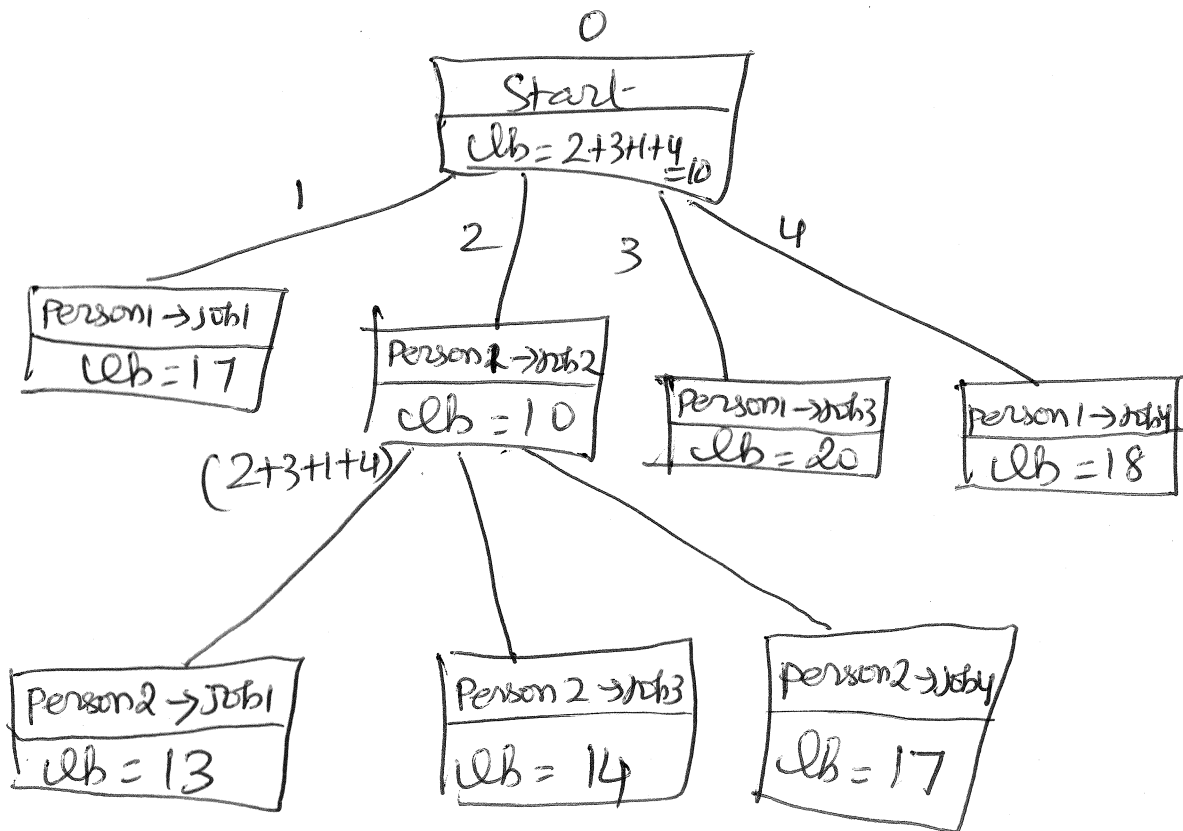
$lb = 8 +$ (Since Person1 is assigned with job4)

$3 + 1 + ④$ (Person4 is not assigned with job4.
Since it is assigned with person1. So
Choose next minimum value)

$$= 8 + 3 + 1 + 6$$

$$\boxed{lb = 18}$$

Out of these four lb values, Choose ^{minimum} value. So $lb = 10$ value is chosen. So Branch node 2.



Person2 → job1

$lb = 2$ (Since Person1 is assigned job2. because lb is
minimum)
 $+ 6$ (person2 is assigned to job1)
 $+ 1 + 4$ (other persons are assigned minimum
value)

$$= 2 + 6 + 1 + 4$$
$$\boxed{lb = 13}$$

Person 2 → Job 3

lb = 2 (since Person 1 is assigned Job 2 because lb is minimum
+ 3 (Person 2 is assigned Job 3)
+ 1 + 4 (other persons are assigned minimum value. Person 3 is not assigned Job 3. Since it is already assigned Person 2. Choose next minimum value)

$$= 2 + 3 + 5 + 4$$

$$\boxed{\text{lb} = 14}$$

Person 2 → Job 4

lb = 2 (since it is assigned Person 1 Job 2)
+ ~~7~~ (Person 2 is assigned Job 4)
+ 1 + 4 (since Job 4 is assigned Person 2. So choose next minimum value. Next minimum is 6. It is already assigned for Person 1. So choose next minimum value)

$$= 2 + 7 + 1 + 7$$

$$\boxed{\text{lb} = 17}$$

Out of these three lb values minimum value will be taken for further Branching.

$\boxed{\therefore \text{lb} = 13}$ is chosen for Branching

Person 3 → job 3

$$\begin{aligned} \text{clb} &= 2 + (\text{Person 1 is assigned job 2}) \\ &\quad 6 + (\text{Person 2 is assigned job 1}) \\ &\quad 1 + (\text{Person 3 is assigned job 3}) \\ &\quad 4 + (\text{other persons choose minimum value}) \\ &= 2 + 6 + 1 + 4 \end{aligned}$$

$$\boxed{\text{clb} = 13}$$

Person 3 → job 4

$$\begin{aligned} \text{clb} &= 2 + (\text{Person 1 is assigned job 2}) \\ &\quad 6 + (\text{Person 2 is assigned job 1}) \\ &\quad 8 + (\text{Person 3 is assigned job 4}) \\ &\quad 9 + (\text{others choose minimum value}) \end{aligned}$$

$$= 2 + 6 + 8 + 9$$

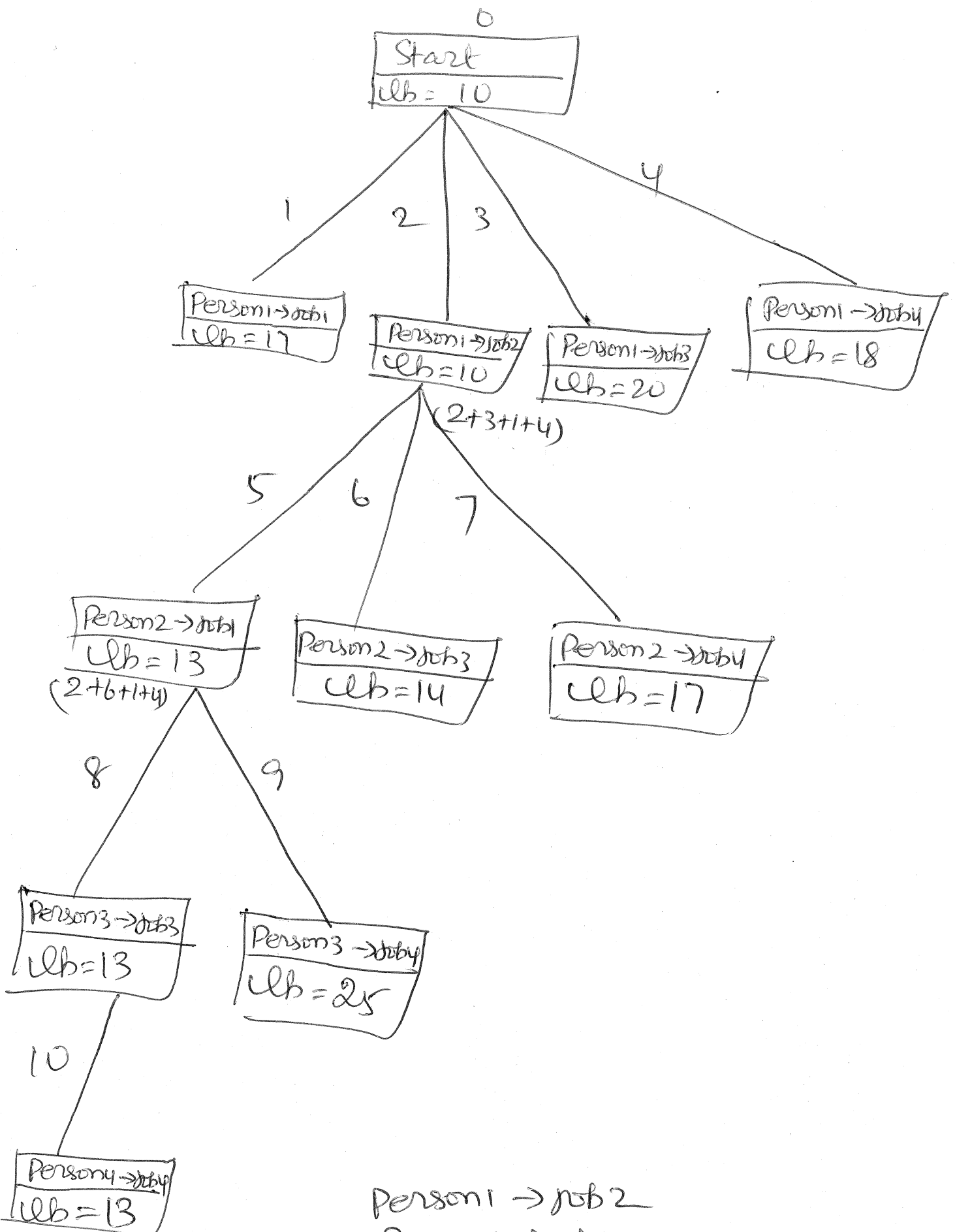
$$\boxed{\text{clb} = 25}$$

Since this minimum value is assigned to other jobs i.e. job 4 is assigned Person 3. So choose next minimum value (5) that value is also assigned to Person 1. Choose next minimum value (7). This value is also assigned to Person 2. So choose next minimum value.

out of these ~~min~~ clb values
Choose minimum clb value.

∴ $\text{clb} = 13$ is for branching.

∴ Person 1 is assigned job 2
Person 2 is assigned job 1
Person 3 is assigned job 3
Person 4 is assigned job 4



Conclusion

Person1 → job2
 Person2 → job1
 Person3 → job3
 ∴ Person4 → job4
 $lbb = 2 + 6 + 1 + 4$
(lbb = 13)