ST. JOSEPH'S COLLEGE OF ENGINEERING

ST. JOSEPH'S INSTITUTE OF TECHNOLOGY

ASSIGNMENT FOR MATHEMATICS – I (MA 6151)

I-YEAR B.E/ B.TECH. (COMMON TO ALL BRANCHES)

PART A & B QUESTIONS

UNIT – I MATRICES

PART – A

1 **Find the sum and product of the eigen values of the matrix** $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & 3 \end{bmatrix}$

2 **If 3 and 15 are the two eigen values of** $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$**, find** $|A|$**, without expanding the determinant.**

3 **The product of two eigen values of the matrix** $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **is 16. Find the third eigen value.**

4 **Find the eigen values of** A^2**, if** $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

5 **If** $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ **is an eigen vector of** $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$**, find the corresponding eigen value.**

6 **Find the constants a and c such that the matrix** $\begin{pmatrix} a & 4 \\ 1 & c \end{pmatrix}$ **has 3 & –2 as eigen values.**

7 **Determine the nature of the following quadratic form :** $f(x_1,x_2,x_3) = x_1^2 + 2x_2^2$.

8 **If 1 & 2 are the eigen values of a 2x2 matrix A, what are the eigen values of** A^2**, adj A and A+7I.**

9 **Show that** $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ **is orthogonal.**

10 **Write the matrix of the quadratic form** $3x_1^2 + 5x_2^2 - 5x_3^2 - 2x_1x_2 + 2x_2x_3 + 6x_3x_1$

PART – B

1. a)Find the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$

b)Find the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{pmatrix}$

2. a)Find the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

b)Find the Eigen values and Eigen vectors of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}$

3. Diagonalise the matrix $A=\begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$ by means of an orthogonal transformation.

4. Diagonalise the matrix $A=\begin{pmatrix} 2 & 2 & -7 \\ 2 & 1 & 2 \\ 0 & 1 & -3 \end{pmatrix}$ by similarity transformation and hence find A^4.

5. a)Verify Cayley- Hamilton theorem for the matrix $A=\begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and also find A^4

b)Verify Cayley- Hamilton theorem for the matrix $A=\begin{pmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ and hence find A^{-1}

6. Reduce quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_3x_1$ to canonical form through an orthogonal transformation. Also find the index, nature and rank of the quadratic form.

7. Reduce the quadratic form $x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to canonical form through an orthogonal transformation and hence show that it is positive semi definite. Also gives a non-zero

set of values (x_1, x_2, x_3) which makes the quadratic form zero.

8. Verify Cayley- Hamilton theorem for the matrix A=$\begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ and hence evaluate

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 - 8A^2 + 2A - I$ **.**

UNIT – III APPLICATIONS OF DIFFERENTIAL CALCULUS

PART-A

1. **Find the radius of curvature of $y=e^x$ at the point where the curve cuts the Y- axis.**
2. **Find the radius of curvature at any point on y = c log sec (x/c).**
3. **Find the curvature of the curve $2x^2+2y^2+5x-2y+1=0$.**
4. **Define evolute.**
5. **Find the co-ordinates of the centre of curvature of the curve $y = x^2$ at the point (1,-1).**
6. **Define envelope.**
7. **Find the envelope of the family of straight lines xcosα + ysinα = a secα, α being the parameter.**
8. **Find the envelope of the family of straight line y = mx + a/m , m being the parameter**
9. **Find the envelope of the family of circles $(x-\alpha)^2 + y^2 = r^2$, α being the parameter.**
10. **For the catenary y = c cosh (x/c), find the curvature.**

PART-B

1.a) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $(a/4, a/4)$.

b) Find the evolute of the rectangular hyperbola $xy = c^2$

2.a) Find the equation of the circle of curvature of the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$

b) Find the radius of curvature at any point θ on the cycloid $x = a(\theta + \sin\theta), y = a(1-\cos\theta)$.

3.a) Find the envelope of the system of lines $\frac{x}{l} + \frac{y}{m} = 1$ where 'l' & 'm' are connected by the

relation $\frac{l}{a}+\frac{m}{b}=1$, l and m are the parameters.

b) Find the radius of curvature of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ at any point $\left(a\cos^3\theta, a\sin^3\theta\right)$.

4. a) Find the evolute of the parabola $y^2=4ax$

b) Find the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

5. a) Find the envelope of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ where the parameters a and b are connected by the relation $ab=c^2$.

b) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, treating it as the envelope of its normals.

6. a) Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$ where ‘a’ & ‘b’ are parameters that are connected by the relation $a^n+b^n=c^n$, c being a constant.

b) Find the evolute of the parabola $x^2=4ay$ as the envelope of its normals

7. a) Find the equation of circle of curvature of the rectangular hyperbola xy=12 at the point (3,4).

b) Find ρ for the curve x=a (cost + tsint); y=a(sint - tcost).

8. a) Find the radius of curvature of the curve $xy^2=a^3-x^3$ at (a,0).

b) Find the equation of the evolute of the curve $x^{2/3}+y^{2/3}=a^{2/3}$