UNIT 3

QPSK

A Quadrature-Phase Shift Keying (QPSK)-modulated signal is a PAM where the signal carries 1 bit per symbol interval on both the in-phase and quadrature-phase component. The original data stream is split into two streams, $b1_i$ and $b2_i$: $b1_i = b2_i$ $b2_i = b2_{i+1}$ each of which has a data rate that is half that of the original data stream: $R_S = 1/T_S = R_B/2 = 1/(2T_B)$ Let us first consider the situation where basis pulses are rectangular pulses, $g(t) = g_R(t, T_S)$.
Then we can give an interpretation of QPSK as either a phase modulation or as a PAM.

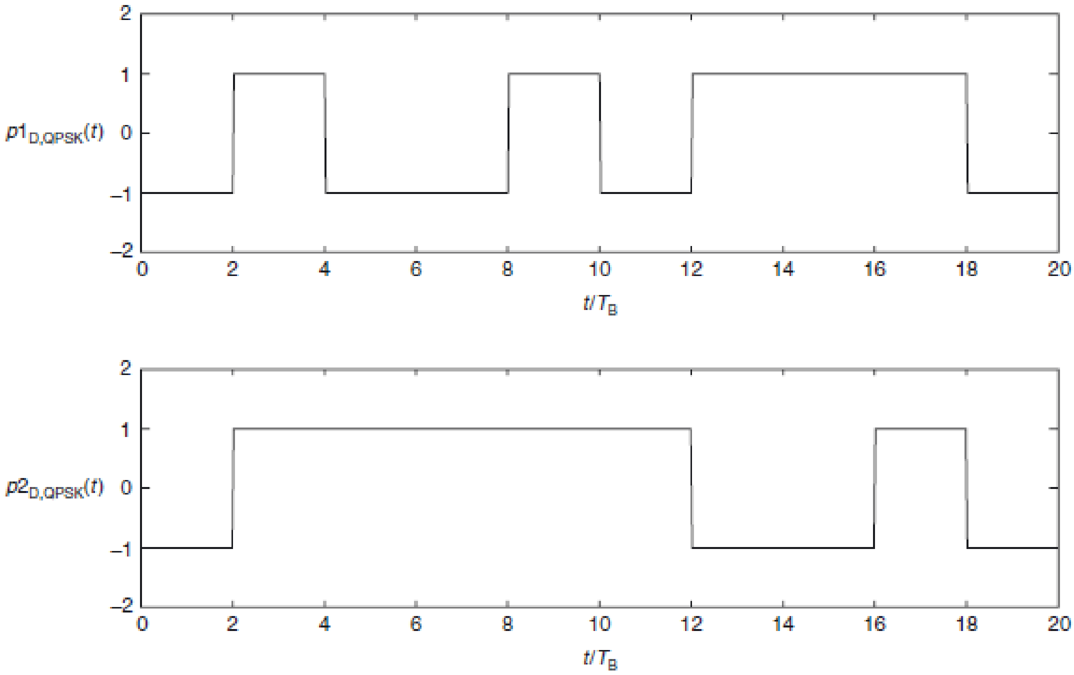
$$\left.\begin{aligned} p1_D(t) &= \sum_{i=-\infty}^{\infty} b1_i g(t - iT_S) = b1_i * g(t) \\ p2_D(t) &= \sum_{i=-\infty}^{\infty} b2_i g(t - iT_S) = b2_i * g(t) \end{aligned}\right\}$$

We first define two sequences of pulses

When interpreting QPSK as a *PAM*, the bandpass signal reads

$$s_{BP}(t) = \sqrt{E_B/T_B}\left[p1_D(t)\cos(2\pi f_c t) - p2_D(t)\sin(2\pi f_c t)\right]$$

Normalization is done in such a way that the energy within one symbol interval is $\int_0^{T_S} s_{BP}(t)^2 dt = 2E_B$, where E_B is the energy expended on transmission of a bit.



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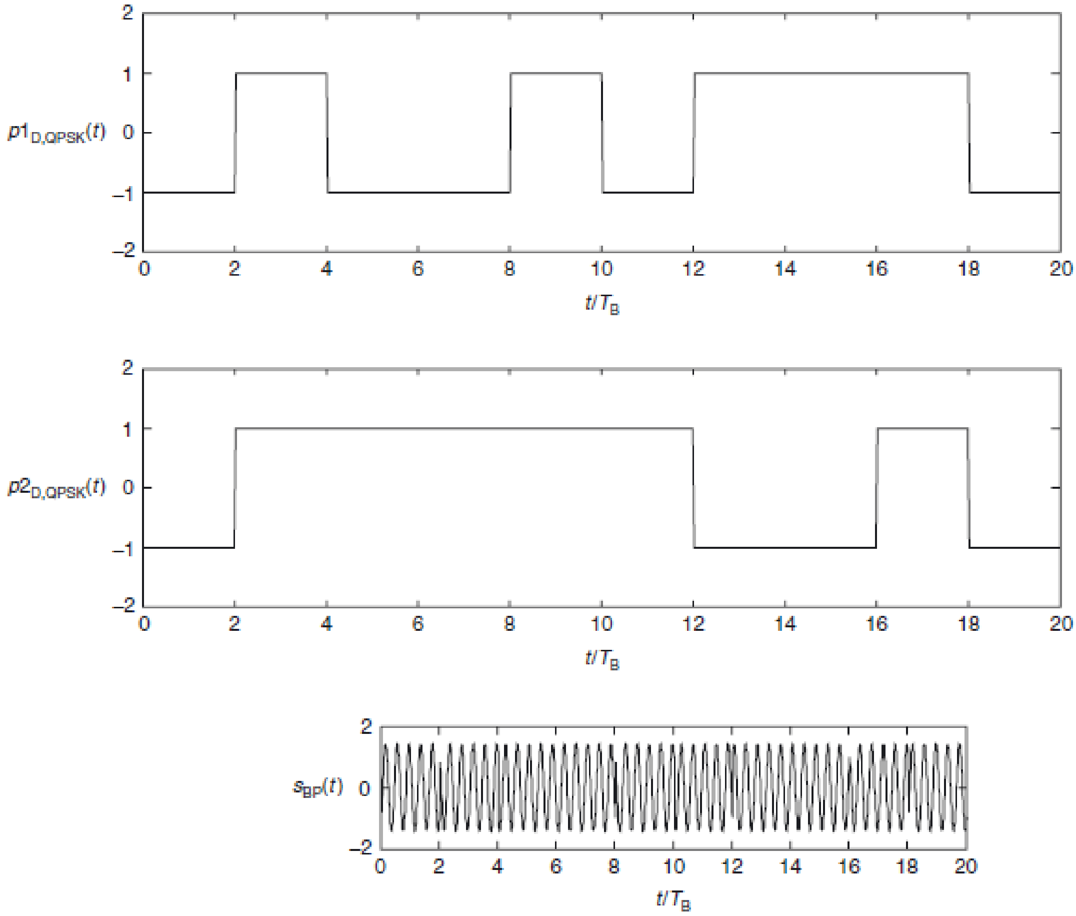


Figure QPSK Timing Diagram

space diagram. The baseband signal is

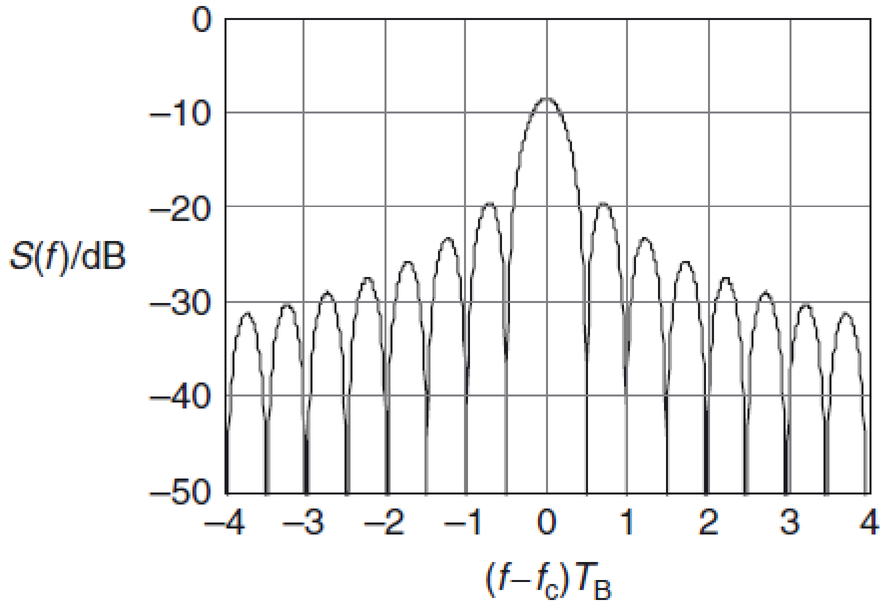
$$s_{LP}(t) = [p1_D(t) + jp2_D(t)]\sqrt{E_B/T_B}$$

When interpreting QPSK as a *phase modulation*, the low-pass signal can be written as $\sqrt{2E_B/T_B}\exp(j\Phi_S(t))$ with:

$$\Phi_S(t) = \pi \cdot \left[\frac{1}{2} \cdot p2_D(t) - \frac{1}{4} \cdot p1_D(t) \cdot p2_D(t)\right]$$

π/4 Differential QPSK

Even though QPSK is nominally a constant envelope format, it has amplitude dips at bit ransitions; this can also be seen by the fact that the trajectories in the I–Q diagram pass through the origin for



some of the bit transitions. The duration of the dips is longer when non-rectangular basis pulses are used. Such variations of the signal envelope are undesirable, because they make the design of suitable amplifiers more difficult. One possibility for reducing these problems lies in the use of $\pi/4$-DQPSK ($\pi/4$ differential quadrature-phase shift keying). This modulation format had great importance for second-generation cellphones – it was used in several American standards (IS-54, IS-136, PWT), as well as the Japanese cellphone (JDC) and cordless (PHS) standards, and the European trunk radio standard (TETRA).
The principle of $\pi/4$-DQPSK can be understood from the signal space diagram of DQPSK (see Figure).

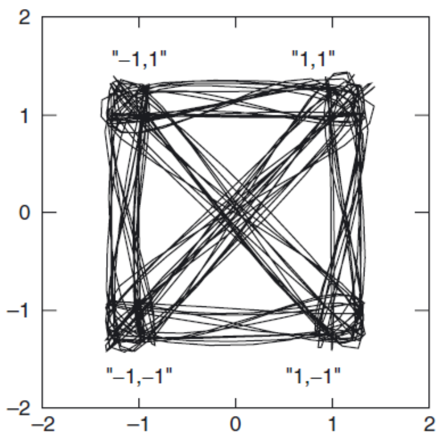


Figure I–Q diagram of quadrature amplitude modulation with raised cosine basis pulses. Also shown are the four normalized points of the normalized signal space diagram, (1, 1), (1, −1), (−1, −1), (−1, 1).

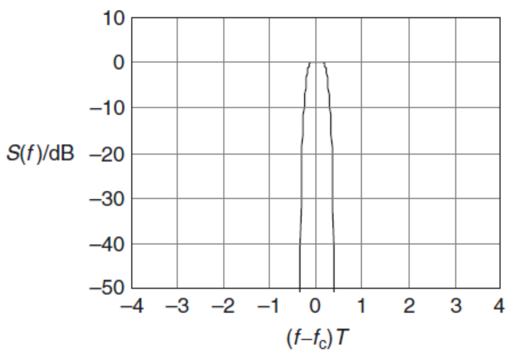


Figure Normalized power-spectral density of quadrature amplitude modulation with raised cosine filters with $\alpha = 0.5$.

There exist *two* sets of signal constellations: (0, 90, 180, 270◦) and (45, 135, 225 , 315◦). All symbols with an even temporal index *i* are chosen from the first set, while all symbols with odd index are chosen from the second set. In other words: whenever *t* is an integer multiple f
the symbol duration, the transmit phase is increased by $\pi/4$, in addition to the change of phase due to the transmit symbol. Therefore, transitions between subsequent signal constellations can never pass through the origin, in physical terms, this means smaller fluctuations of the envelope.
The signal phase is given by

$$\Phi_s(t) = \pi\left[\frac{1}{2}p2_D(t) - \frac{1}{4}p1_D(t)p2_D(t) + \frac{1}{4}\left\lfloor\frac{t}{T_S}\right\rfloor\right]$$

where x denotes the largest integer smaller or equal to *x*. Comparing this, we can clearly see the change in phase at each integer multiple of *T*S.

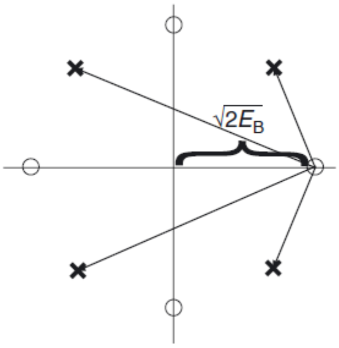


Figure Allowed transitions in the signal space diagram of $\pi/4$ differential quadrature-phase shift keying.

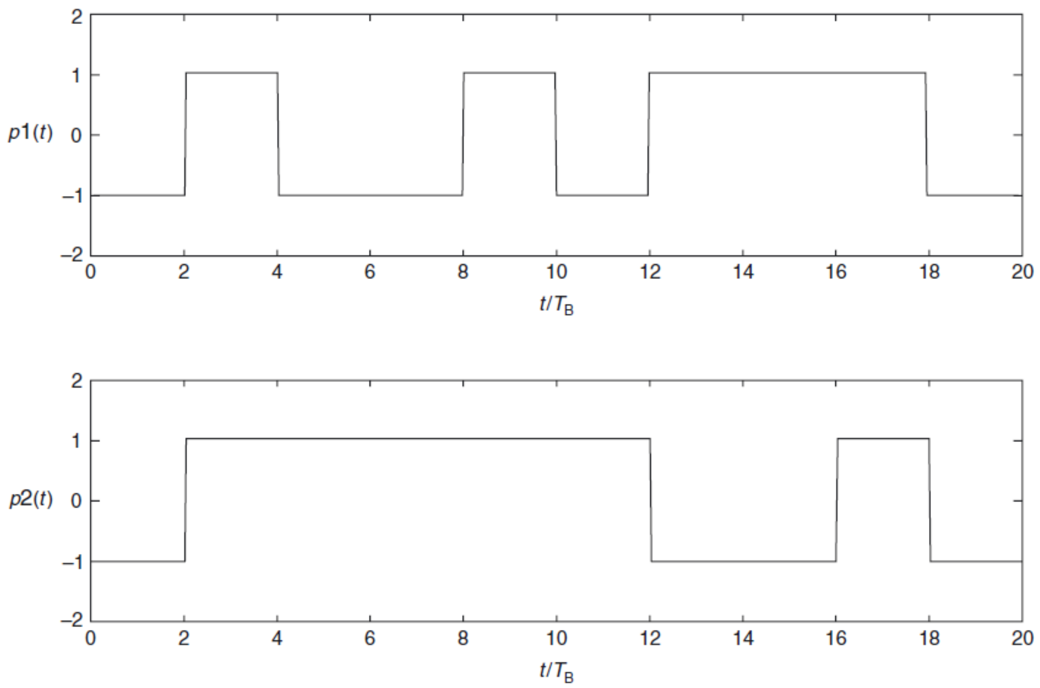


Figure Sequence of basis pulses for $\pi/4$ differential quadrature-phase shift keying.

Offset QPSK

Another way of improving the peak-to-average ratio in QPSK is to make sure that bit transitions for the in-phase and the quadrature-phase components occur at different time instants. This method is called OQPSK (offset QPSK). The bitstreams modulating the in-phase and quadrature-phase components are offset half a symbol duration with respect to each other so

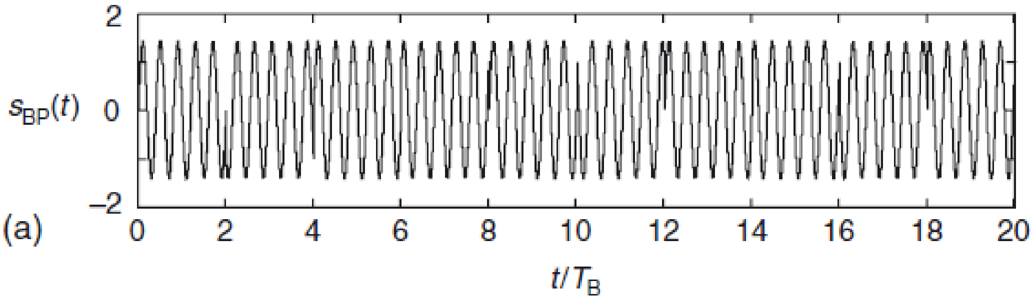


Figure $\pi/4$ differential quadrature-phase shift keying signals as function of time for rectangular basis functions (a) and raised cosine basis pulses (b).

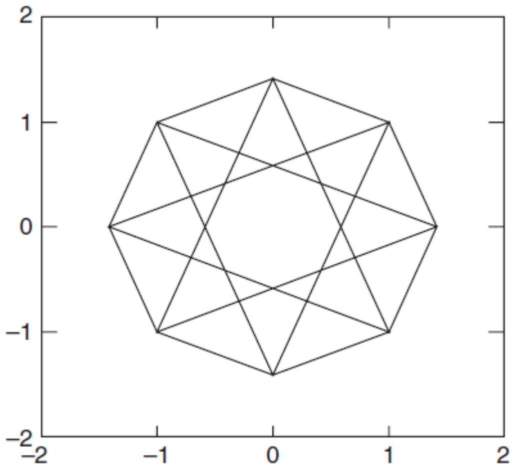


Figure I–Q diagram of a $\pi/4$-differential quadrature-phase shift keying signal with rectangular basis functions.

that transitions for the in-phase component occur at integer multiples of the symbol duration (even integer multiples of the bit duration), while quadrature component transitions occur half a symbol duration (1-bit duration) later. Thus, the transmit pulse streams are

$$\left.\begin{aligned} p1_D(t) &= \sum_{i=-\infty}^{\infty} b1_i g(t - iT_S) = b1_i * g(t) \\ p2_D(t) &= \sum_{i=-\infty}^{\infty} b2_i g\left(t - \left(i + \tfrac{1}{2}\right) T_S\right) = b2_i * g\left(t - \frac{T_S}{2}\right) \end{aligned}\right\}$$

These data streams can again be used for interpretation as PAM or as phase modulation, according to Eq. The resulting bandpass signal is shown in Figure

The representation in the I–Q diagram makes clear that there are no transitions passing through the origin of the coordinate system; thus this modulation format takes care of envelope fluctuations as well.

BFSK- Binary FSK

In *FSK*, each symbol is represented by transmitting (for a time T_S) a sinusoidal signal whose frequency depends on the symbol to be transmitted. FSK cannot be represented as PAM. Rather, it is a form of multipulse modulation: depending on the bit to be transmitted, basis pulses with different center frequencies are transmitted

$$g_m(t) = \cos[(2\pi f_c + b_m 2\pi f_{mod})t/T + \psi]$$

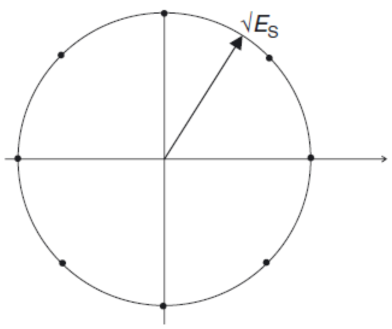


Figure Envelope diagram of 8-PSK. Also shown are the eight points of the signal space diagram.

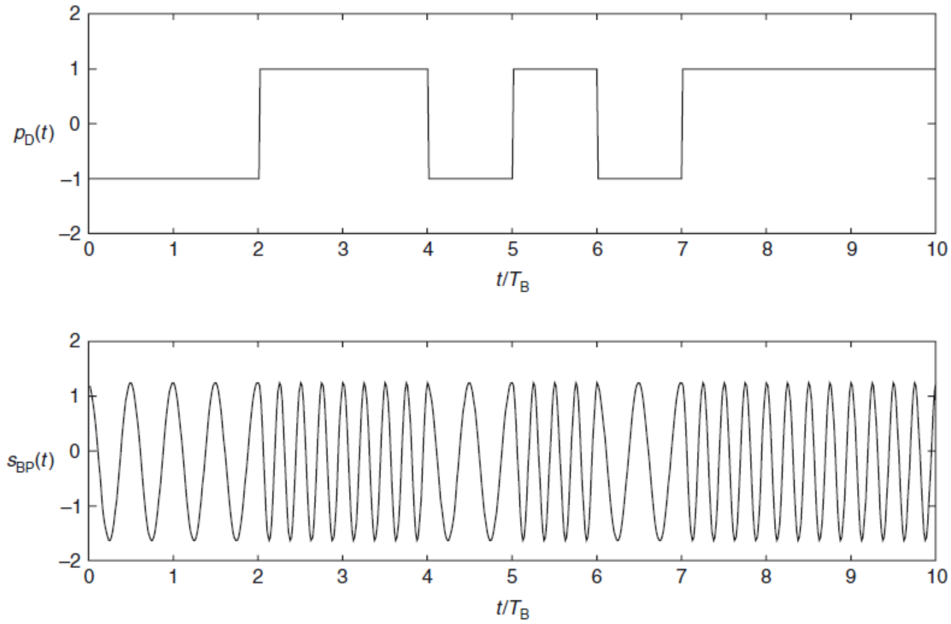


Figure Frequency shift keying signal as a function of time.

Note that the phase of the transmit signal can jump at the bit transitions. This leads to undesirable spectral properties.

The power spectrum of FSK can be shown to consist of a continuous and a discrete (spectral lines) part:

$$S(f) = S_{\text{cont}}(f) + S_{\text{disc}}(f)$$

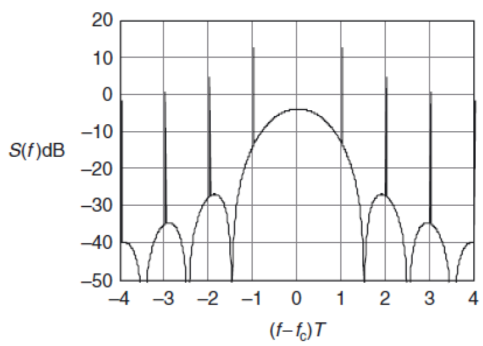


Figure Power-spectral density of (noncontinuous phase) frequency shift keying with $h_{\text{mod}} = 1$.

and

$$S_{\text{disc}}(f) = \frac{1}{(2T)^2} \left| \sum_{m=1}^{2} G_m(f) \right|^2 \sum_n \delta\left(f - \frac{n}{T}\right)$$

where $G_m(f)$ is the Fourier transform of $g_m(t)$. An example is shown in Figure

CPFSK enforces a smooth change of the phase at the bit transitions. The phase of the transmission signal is chosen as

$$\Phi_S(t) = 2\pi h_{mod} \int_{-\infty}^{t} \tilde{p}_{D,FSK}(\tau)\, d\tau$$

The resulting signal has a constant envelope, where h_{mod} is the modulation index. Using the normalization for phase pulses (Eq. 11.13) and assuming rectangular phase basis pulses:

$$\tilde{g}_{FSK}(t) = \frac{1}{2T_B} g_R(t, T_B)$$

the phase pulse sequence is

$$\tilde{p}_{D,FSK}(t) = \sum_{i=-\infty}^{\infty} b_i \tilde{g}_{FSK}(t - iT_B) = b_i * \tilde{g}_{FSK}(t)$$

The instantaneous frequency is given as

$$f(t) = f_c + b_i f_{mod}(t) = f_c + f_D(t) = f_c + \frac{1}{2\pi}\frac{d\Phi_S(t)}{dt}$$

The real and imaginary parts of the equivalent baseband signal are then

$$\mathrm{Re}(s_{LP}(t)) = \sqrt{2E_B/T_B} \cos\left[2\pi h_{mod} \int_{-\infty}^{t} \tilde{p}_{D,FSK}(\tau)\, d\tau\right]$$

$$\mathrm{Im}(s_{LP}(t)) = \sqrt{2E_B/T_B} \sin\left[2\pi h_{mod} \int_{-\infty}^{t} \tilde{p}_{D,FSK}(\tau)\, d\tau\right]$$

MSK Minimum Shift Keying:

Minimum Shift Keying (MSK) is one of the most important modulation formats for wireless communications. However, it can be interpreted in different ways, which leads to considerable confusion:

1. The first interpretation is as CPFSK with a modulation index:

$$h_{mod} = 0.5, \quad f_{mod} = 1/4T$$

   This implies that the phase changes by $\pm\pi/2$ during a 1-bit duration The bandpass signal is shown in Figure

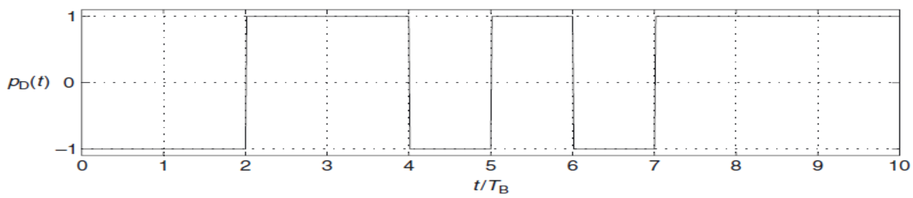
2. Alternatively, we can interpret MSK as *Offset QAM* (OQAM) with basis pulses that are sinusoidal half-waves extending over a duration of $2T_B$

$$g(t) = \sin(2\pi f_{mod}(t + T_B)) g_R(t, 2T_B)$$

Due to the use of smoother basis functions, the spectrum decreases faster than that of "regular" OQPSK:

$$S(f) = \frac{16T_B}{\pi^2}\left(\frac{\cos(2\pi f T_B)}{1 - 16f^2T_B^2}\right)^2$$

On the other hand, MSK is only a binary modulation format, while OQPSK transmits 2 bits per symbol duration. As a consequence, MSK has lower spectral efficiency when considering the 90% energy bandwidth (1.29 bit/s/H_z), but still performs reasonably well when considering the 99% energy bandwidth (0.85 bit/s/H_z).



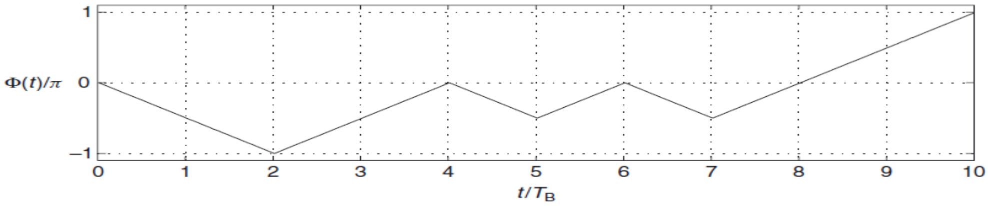


Figure Phase pulse and phase as function of time for minimum shift keying signal.

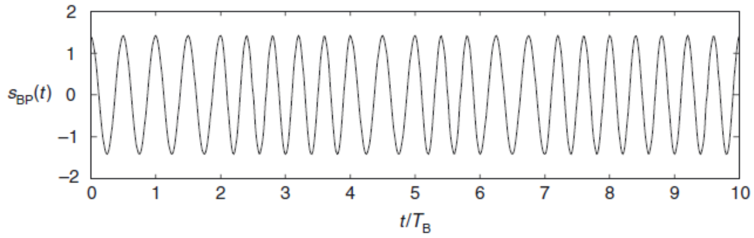
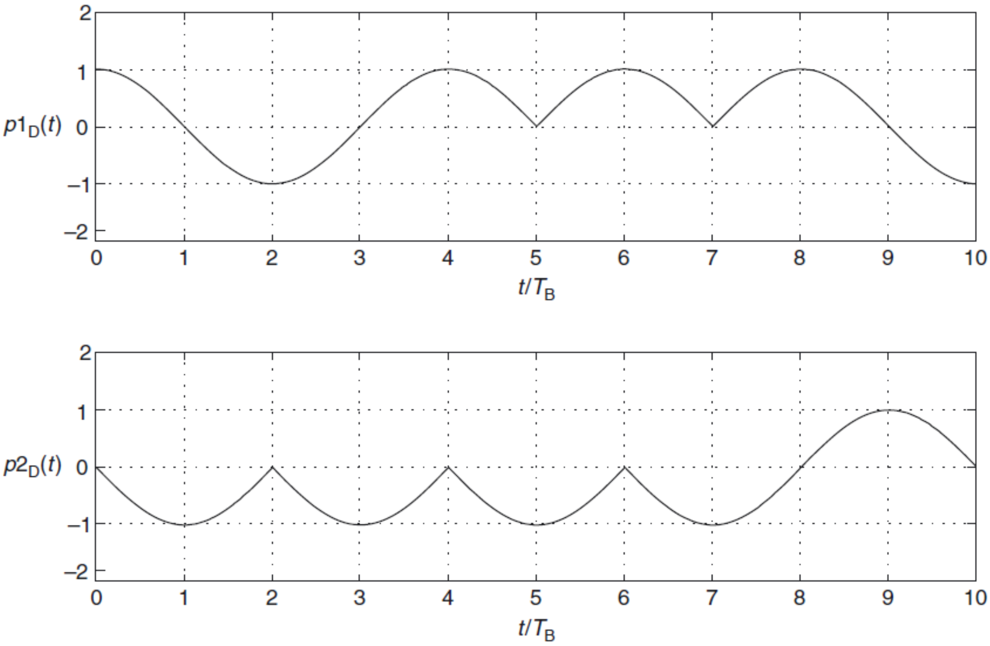


Figure Minimum shift keying modulated signal.



GMSK Gaussian Minimum shift Keying:

GMSK (Gaussian MSK) is CPFSK with modulation index $h_{\mathrm{mod}} = 0.5$ and *Gaussian* phase basis pulses:

$$\tilde{g}(t) = g_{\mathrm{G}}(t, T_{\mathrm{B}}, B_{\mathrm{G}}T)$$

Thus the sequence of transmit phase pulses is

$$p_{\mathrm{D}}(t) = \sum_{i=-\infty}^{\infty} b_i \tilde{g}(t - iT_{\mathrm{B}}) = b_i * \tilde{g}(t)$$

The spectrum is shown in Figure We see that GMSK achieves better spectral efficiency than MSK because it uses the smoother Gaussian phase basis pulses as opposed to the rectangular ones of MSK.

GMSK is the modulation format most widely used in Europe. It is applied in the cellular Global System for Mobile communications (GSM) standard (with $B_G T = 0.3$) and the cordless standard Digital Enhanced Cordless Telecommunications (DECT) (with $B_G T = 0.5$)

It is noteworthy that GMSK *cannot* be interpreted as PAM. However, Laurent [1986] derived equations that allow the interpretation of GMSK as PAM with finite memory.

Error Performance of digital modulation in fading channels.

For AWGN channels, the advantages of the alternative representation of the Q-function are rather limited. They allow a simpler formulation for higher order modulation formats, but do not exhibit significant advantages for the modulation formats that are mostly used in practice. The real advantage emerges when we apply this description method as the basis for computations of the BER in fading channels. We find that we have to average over the pdf of the SNR $pdf_\gamma(\gamma)$, as described in Eq. (12.50). We have now seen that the alternative representation of the Q-function allows us to write the SER (for a given SNR) in the generic form:

$$SER(\gamma) = \int_{\theta_1}^{\theta_2} f_1(\theta)\exp(-\gamma f_2(\theta))\,d\theta$$

Thus, the average SER becomes

$$\begin{aligned}\overline{SER} &= \int_0^\infty pdf_\gamma(\gamma) SER(\gamma)\,d\gamma \\ &= \int_0^\infty pdf_\gamma(\gamma)\int_{\theta_1}^{\theta_2} f_1(\theta)\exp(-\gamma f_2(\theta))\,d\theta\,d\gamma \\ &= \int_{\theta_1}^{\theta_2} f_1(\theta)\int_0^\infty pdf_\gamma(\gamma)\exp(-\gamma f_2(\theta))\,d\gamma\,d\theta\end{aligned}$$

Let us now have a closer look at the inner integral:

$$\int_0^\infty pdf_\gamma(\gamma)\exp(-\gamma f_2(\theta))\,d\gamma$$

We find that it is the moment-generating function of $pdf_\gamma(\gamma)$, evaluated at the point $-f_2(\theta)$. Remember that the moment-generating function is defined as the Laplace transform of the pdf of γ:

$$M_\gamma(s) = \int_0^\infty pdf_\gamma(\gamma)\exp(\gamma s)\,d\gamma$$

and the mean SNR is the first derivative, evaluated at $s = 0$:

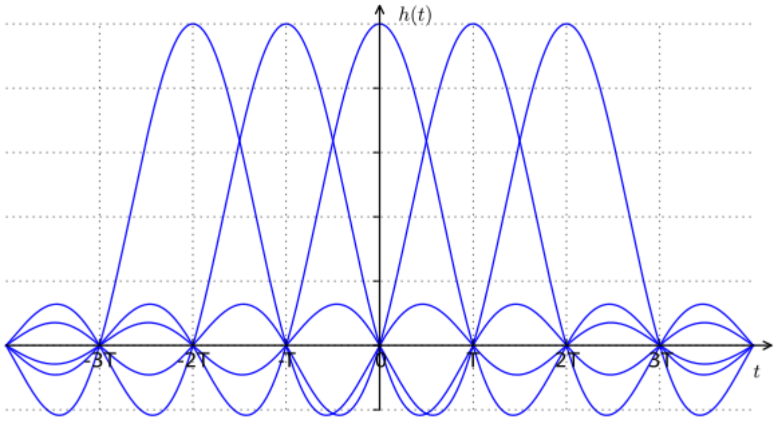
$$\overline{\gamma} = \left.\frac{dM_\gamma(s)}{ds}\right|_{s=0}$$

ISI Cancellation:

In communications, the **Nyquist ISI criterion** describes the conditions which, when satisfied by acommunication channel (including responses of transmit and receive filters), result in no intersymbol interference or ISI. It provides a method for constructing band-limited functions to overcome the effects of intersymbol interference.

When consecutive symbols are transmitted over a channel by a linear modulation (such as ASK, QAM, etc.), the impulse response (or equivalently the frequency response) of the channel causes a transmitted symbol to be spread in the time domain. This causes intersymbol interference because the previously transmitted symbols affect the currently received symbol, thus reducing tolerance for noise. The Nyquist theorem relates this time-domain condition to an equivalent frequency-domain condition.

The Nyquist criterion is closely related to the Nyquist-Shannon sampling theorem, with only a differing point of view.



Raised cosine response meets the Nyquist ISI criterion. Consecutive raised-cosine impulses demonstrate the zero ISI property between transmitted symbols at the sampling instants. At t=0 the middle pulse is at its maximum and the sum of other impulses is zero.

If we denote the channel impulse response as $h(t)$, then the condition for an ISI-free response can be expressed as:

$$h(nT_s) = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases}$$

for all integers n, where T_s is the symbol period. The Nyquist theorem says that this is equivalent to:

$$\frac{1}{T_s} \sum_{k=-\infty}^{+\infty} H\left(f - \frac{k}{T_s}\right) = 1 \quad \forall f,$$

where $H(f)$ is the Fourier transform of $h(t)$. This is the Nyquist ISI criterion.

This criterion can be intuitively understood in the following way: frequency-shifted replicas of *H(f)* must add up to a constant value.

In practice this criterion is applied to baseband filtering by regarding the symbol sequence as weighted impulses (Dirac delta function). When the baseband filters in the communication system satisfy the Nyquist criterion, symbols can be transmitted over a channel with flat response within a limited frequency band, without ISI. Examples of such baseband filters are the raised-cosine filter, or the sinc filter as the ideal case.

To derive the criterion, we first express the received signal in terms of the transmitted symbol and the channel response. Let the function *h(t)* be the channel impulse response, *x[n]* the symbols to be sent, with a symbol period of T_s; the received signal *y(t)* will be in the form (where noise has been ignored for simplicity):

$$y(t) = \sum_{n=-\infty}^{\infty} x[n] \cdot h(t - nT_s).$$

Sampling this signal at intervals of T_s, we can express *y(t)* as a discrete-time equation:

$$y[k] = y(kT_s) = \sum_{n=-\infty}^{\infty} x[n] \cdot h[k-n].$$

If we write the *h[0]* term of the sum separately, we can express this as:

$$y[k] = x[k] \cdot h[0] + \sum_{n \neq k} x[n] \cdot h[k-n],$$

and from this we can conclude that if a response *h[n]* satisfies

$$h[n] = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases},$$

only one transmitted symbol has an effect on the received *y[k]* at sampling instants, thus removing any ISI. This is the time-domain condition for an ISI-free channel. Now we find a frequency-domain equivalent for it. We start by expressing this condition in continuous time:

$$h(nT_s) = \begin{cases} 1; & n = 0 \\ 0; & n \neq 0 \end{cases}$$

for all integer n. We multiply such a *h(t)* by a sum of Dirac delta function (impulses) $\delta(t)$separated by intervals T_s This is equivalent of sampling the response as above but using a continuous time expression. The right side of the condition can then be expressed as one impulse in the origin:

$$h(t) \cdot \sum_{k=-\infty}^{+\infty} \delta(t - kT_s) = \delta(t)$$

Fourier transforming both members of this relationship we obtain:

$$H(f) * \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} \delta\left(f - \frac{k}{T_s}\right) = 1$$

and

$$\frac{1}{T_s} \sum_{k=-\infty}^{+\infty} H\left(f - \frac{k}{T_s}\right) = 1 .$$

This is the Nyquist ISI criterion and, if a channel response satisfies it, then there is no ISI between the different samples.

performance of raised cosine roll off filter:

The **raised-cosine filter** is a filter frequently used for pulse-shaping in digital modulation due to its ability to minimise intersymbol interference (ISI). Its name stems from the fact that the non-zero portion of the frequency spectrum of its simplest form ($\beta = 1$) is a cosine function, 'raised' up to sit above the f(horizontal) axis.

The raised-cosine filter is an implementation of a low-pass Nyquist filter, i.e., one that has the property of vestigial symmetry. This means that its spectrum exhibits odd symmetry about $\frac{1}{2T}$, where $T$is the symbol-period of the communications system.

Its frequency-domain description is a piecewise function, given by:

$$H(f) = \begin{cases} T, & |f| \leq \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}\left[|f| - \frac{1-\beta}{2T}\right]\right)\right], & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$

$$0 \leq \beta \leq 1$$

and characterised by two values; β, the *roll-off factor*, and T, the reciprocal of the symbol-rate.

The impulse response of such a filter is given by:

$$h(t) = \operatorname{sinc}\left(\frac{t}{T}\right) \frac{\cos\left(\frac{\pi \beta t}{T}\right)}{1 - \frac{4\beta^2 t^2}{T^2}}$$

, in terms of the normalised sinc function.

Roll-off factor

The roll-off factor, β, is a measure of the *excess bandwidth* of the filter, i.e. the bandwidth occupied beyond the Nyquist bandwidth of $\frac{1}{2T}$. If we denote the excess bandwidth as Δf, then:

$$\beta = \frac{\Delta f}{\left(\frac{1}{2T}\right)} = \frac{\Delta f}{R_S/2} = 2T\Delta f$$

where $R_S = \frac{1}{T}$ is the symbol-rate.

The graph shows the amplitude response as β is varied between 0 and 1, and the corresponding effect on the impulse response. As can be seen, the time-domain ripple level increases as β decreases. This shows that the excess bandwidth of the filter can be reduced, but only at the expense of an elongated impulse response.

$$\beta = 0$$

As β approaches 0, the roll-off zone becomes infinitesimally narrow, hence:

$$\lim_{\beta \to 0} H(f) = \mathrm{rect}(fT)$$

where $\mathrm{rect}(.)$ is the rectangular function, so the impulse response approaches $\mathrm{sinc}\left(\frac{t}{T}\right)$. Hence, it converges to an ideal or brick-wall filter in this case.

$$\beta = 1$$

When $\beta = 1$, the non-zero portion of the spectrum is a pure raised cosine, leading to the simplification:

$$H(f)|_{\beta=1} = \begin{cases} \frac{T}{2}\left[1 + \cos(\pi f T)\right], & |f| \le \frac{1}{T} \\ 0, & \text{otherwise} \end{cases}$$

Bandwidth

The bandwidth of a raised cosine filter is most commonly defined as the width of the non-zero portion of its spectrum, i.e.:

$$BW = \frac{1}{2} R_S(\beta + 1)_{(0<T<1)}$$

Auto-correlation function

The auto-correlation function of raised cosine function is as follows:

$$R(\tau) = T\left[\mathrm{sinc}\left(\frac{\tau}{T}\right)\frac{\cos\left(\beta\frac{\pi\tau}{T}\right)}{1 - \left(\frac{2\beta\tau}{T}\right)^2} - \frac{\beta}{4}\mathrm{sinc}\left(\beta\frac{\tau}{T}\right)\frac{\cos\left(\frac{\pi\tau}{T}\right)}{1 - \left(\frac{\beta\tau}{T}\right)^2}\right]$$

The auto-correlation result can be used to analyze various sampling offset results when analyzed with auto-correlation.

OFDM: Orthogonal Frequency Division Multiplexing:

If a high data rate is transmitted over a frequency-selective radio channel with a large maximum multi-path propagation delay τ_{max} compared to the symbol duration, an alternative to the classical SC approach is given by the OFDM transmission technique. The general idea of the OFDM transmission technique is to split the total available bandwidth B into many narrowband sub-channels at equidistant frequencies. The sub-channel spectra overlap each other but the subcarrier signals are still orthogonal. The single high-rate data stream is subdivided into many low-rate data streams for the sub-channels. Each sub-channel is modulated individually and will be transmitted simultaneously in a superimposed and parallel form. An OFDM transmit signal therefore consists of N adjacent and orthogonal subcarriers spaced by the frequency distance Δf on the frequency axis. All subcarrier signals are mutually orthogonal within the symbol duration of length T_S, if the subcarrier distance and the symbol duration are chosen such that $T_S = \frac{1}{\Delta f}$. For OFDM-based systems, the symbol duration T_S is much larger compared to the maximum multipath delay τ_{max}. The k-th unmodulated subcarrier signal is described analytically by a complex valued exponential function with carrier frequency $k\Delta f, \tilde{g}_k(t), k = 0, \ldots, N-1$.

$$\tilde{g}_k(t) = \begin{cases} e^{j2\pi k\Delta f t} & \forall t \in [0, T_S] \\ 0 & \forall t \notin [0, T_S] \end{cases}$$

Since the system bandwidth B is subdivided into N narrowband sub-channels, the OFDM symbol duration T_S is N times larger than in the case of an alternative SC transmission system covering the same bandwidth B. Typically, for a given system bandwidth, the number of subcarriers is chosen in such a way that the symbol duration T_S is sufficiently large compared to the maximum multi-path delay τ_{max} of the radio channel. Additionally, in a time-variant radio channel, the Doppler spread imposes restrictions on the subcarrier spacing Δf. In order to keep the resulting Inter-Carrier Interference (ICI) at a tolerable level, the system parameter of the subcarrier spacing Δf must be large enough compared to the maximum Doppler frequency f_{Dmax}. In the appropriate range for choosing the symbol duration T_S as a rule of thumb in practical systems is given as

$$4\tau_{max} \leq T_S \leq 0.03\frac{1}{f_{Dmax}}$$

The duration T_S of the subcarrier signal $\tilde{g}_k(t)$ is additionally extended by a cyclic prefix (so-called guard interval) of length T_G, which is larger than the maximum multi-path delay τ_{max} in order to avoid ISI completely which could occur in multi-path channels in the transition interval between two adjacent OFDM symbols

$$g_k(t) = \begin{cases} e^{j2\pi k\Delta f t} & \forall t \in [-T_G, T_S] \\ 0 & \forall t \notin [-T_G, T_S] \end{cases}$$

The guard interval is directly removed in the receiver after the time synchronization procedure. From this point of view, the guard interval is a pure system overhead. The total OFDM symbol duration is therefore $T = T_S + T_G$. It is an important advantage of the OFDM transmission technique that ISI can be avoided completely or can be reduced at least considerably by a proper choice of OFDM system parameters.

-quentially transmitted on the same axis.

The orthogonality of all subcarrier signals is completely preserved in the receiver even in frequency-selective radio channels, which is an important advantage of OFDM. The radio channel behaves linearly and in a short time interval of a few OFDM symbols even time-invariant. Hence, the radio channel behavior can be described completely by a Linear and Time Invariant (LTI) system model, characterized by the impulse response $h(t)$. The LTI system theory gives the reason for this important system behavior that all subcarrier signals are orthogonal in the receiver even when transmitting the signal in frequency-selective radio channels. All complex-valued exponential signals (e.g., all subcarrier signals) are eigenfunctions of each LTI system and therefore eigenfunctions of the considered radio channel, which means that only the signal amplitude and phase will be changed if a subcarrier signal is transmitted over the linear and time-invariant radio channel.

The subcarrier frequency is not affected at all by the radio channel transmission, which means that all subcarrier signals are even orthogonal in the receiver and at the output of a frequency-selective radio channel. The radio channel disturbs only amplitudes and phases individually, but not the subcarrier frequency of all received sub-channel signals. Therefore all subcarrier signals are still mutually orthogonal in the receiver. Due to this important property, the received signal which is superimposed by all subcarrier signals can be split directly into the different sub-channel components by a Fourier transform and each subcarrier signal can be restored by a single-tap equalizer and demodulated individually in the receiver.

At the transmitter side, each subcarrier signal is modulated independently and individually by the complex valued modulation symbol $S_{n,k}$, where the subscript n refers to the time interval and k to the subcarrier signal number in the considered OFDM symbol. Thus, within the symbol duration time interval T the time-continuous signal of the n-th OFDM symbol is formed by a superposition of all N simultaneously modulated subcarrier signals.

$$s_n(t) = \sum_{k=0}^{N-1} S_{n,k} g_k(t - nT)$$

The total time-continuous transmit signal consisting of all OFDM symbols se-

$$s(t) = \sum_{n=0}^{\infty} \sum_{k=0}^{N-1} S_{n,k} e^{j2\pi k \Delta f (n-nT)} \text{rect}\left(\frac{2(t-nT)+T_G-T_S}{2T}\right)$$

The analytical signal description shows that a rectangular pulse shaping is applied for each subcarrier signal and each OFDM symbol. Due to the rectangular pulse shaping, the spectra of all the considered subcarrier signals are sinc-functions which are equidistantly located on the frequency axis, e.g., for the k-th subcarrier signal, the spectrum is described

$$G_k(f) = T \cdot \text{sinc}\left[\pi T(f - k\Delta f)\right]$$

The typical OFDM spectrum shown in Fig. 1.6 consists of N adjacent sinc-functions, which are shifted by Δf in the frequency direction.

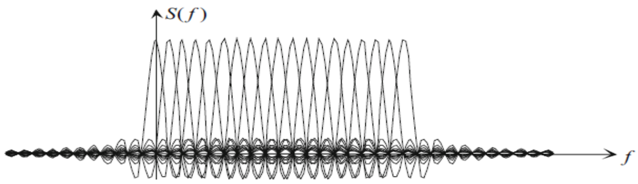


Figure OFDM spectrum which consists of N equidistant sinc functions

The spectra of the considered subcarrier signals overlap on the frequency axis, but the subcarrier signals are still mutually orthogonal, which means the transmitted modulation symbols $S_{n,k}$ can be recovered by a simple correlation technique in each receiver if the radio channel is assumed to be ideal in a first analytical step:

$$\frac{1}{T_S}\int_0^{T_S} g_k(t)g_l^*(t)dt = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases}$$

$$S_{n,k} = \frac{1}{T_S}\int_0^{T_S} s_n(t)g_k^*(t)dt = \frac{1}{T_S}\int_0^{T_S} s_n(t)e^{-j2\pi k\Delta f t}dt$$

where $g_k^*(t)$ is the conjugate complex version of the subcarrier signal $g_k(t)$. shows the correlation process in detail:

$$Corr = \frac{1}{T_S}\int_0^{T_S} s_n(t)g_k^*(t)dt = \frac{1}{T_S}\int_0^{T_S}\sum_{m=0}^{N-1} S_{n,m}g_m(t)g_k^*(t)dt$$

$$= \sum_{m=0}^{N-1} S_{n,m}\frac{1}{T_S}\int_0^{T_S} g_m(t)g_k^*(t)dt = \sum_{m=0}^{N-1} S_{n,m}\delta_{m,k} = S_{n,k}$$

In practical applications, the OFDM transmit signal $s_n(t)$ is generated as a time-discrete signal in the digital baseband. Using the sampling theorem while considering the OFDM transmit signal inside the bandwidth $B = N\Delta f$, the transmit signal must be sampled with the sampling interval $\Delta t = 1/B = 1/N\Delta f$. The individual samples of the transmit signal are denoted by $s_{n,i}, i = 0, 1, \ldots, N-1$ and can be calculated as follows :

$$s(t) = \sum_{k=0}^{N-1} S_{n,k}e^{j2\pi k\Delta f t}$$

$$s(i\Delta t) = \sum_{k=0}^{N-1} S_{n,k}e^{j2\pi k\Delta f i\Delta t}$$

$$s_{n,i} = \sum_{k=0}^{N-1} S_{n,k}e^{j2\pi ik/N}$$

Equation above exactly describes the inverse discrete Fourier transform (IDFT) applied to the complex valued modulation symbols $S_{n,k}$ of all subcarrier signals inside a single OFDM symbol.

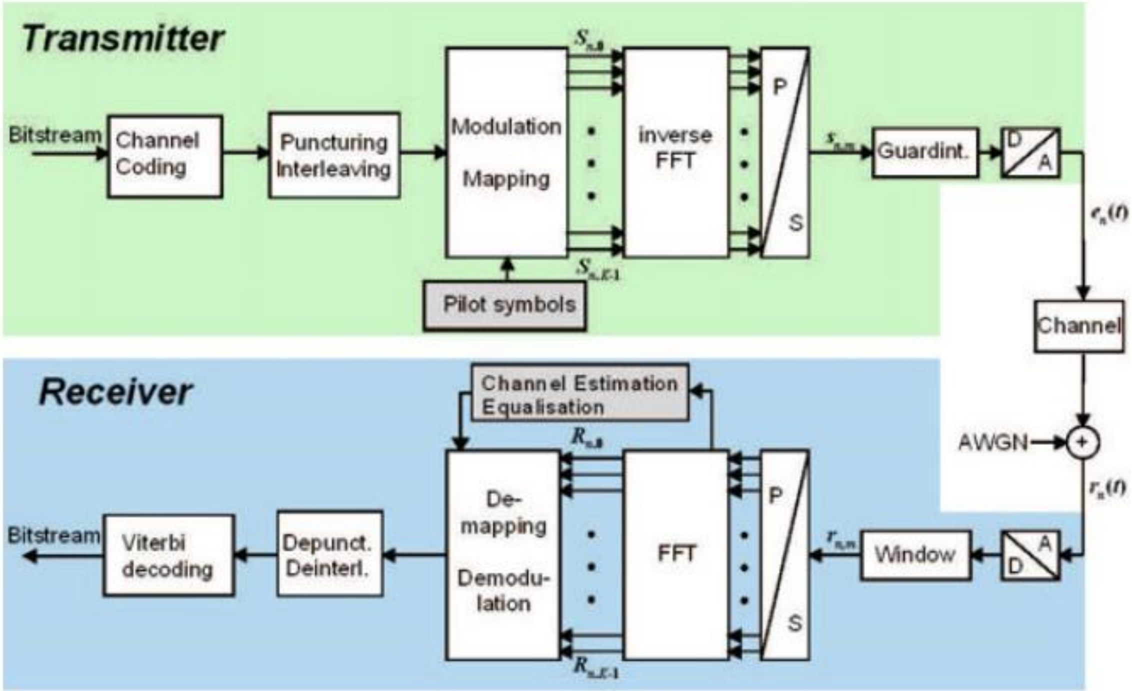


Figure : OFDM system structure in a block diagram