St. JOSEPH'S COLLEGE OF ENGINEERING, CHENNAI-119.
St. JOSEPH'S INSTITUTE OF TECHNOLOGY, CHENNAI-119.
B.E./B.TECH (COMMON TO ALL BRANCHES) - FIRST SEMESTER

MA6151 / MATHEMATICS – I

ASSIGNMENT I - UNIT I - MATRICES

PART - A

1. State Cayley Hamilton theorem.
2. The product of two eigen values of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigen value.
3. If λ_1 , λ_2 , λ_3 , … , λ_n are the eigen values of an n x n matrix A, then show that λ_1^3, λ_2^3 , λ_3^3,… , λ_n^3 are the eigen values of A^3.
4. If the eigen values of a matrix of order 3 × 3 are 2, 3 and 1, then find the eigen values of A^T and Adjoint of A.
5. If 2, 3 are the eigen values of $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ b & 0 & 2 \end{pmatrix}$, then find the value of b.
6. Determine the nature of the following quadratic form: $f(x_1,x_2,x_3) = x_1^2 + 2x_2^2$.
7. Find the Rank, index and signature of the Quadratic form whose Canonical form is $\mathbf{x_1^2 + 2x_2^2 - 3x_3^2}$
8. Write down the quadratic form for the given matrix $\mathbf{A} = \begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{-2} \\ \mathbf{1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{-2} & \mathbf{-1} & \mathbf{-4} \end{bmatrix}$

PART B

1. (i) Find the eigen values and eigen vectors of $\begin{pmatrix} \mathbf{7} & \mathbf{-2} & \mathbf{0} \\ \mathbf{-2} & \mathbf{6} & \mathbf{-2} \\ \mathbf{0} & \mathbf{-2} & \mathbf{5} \end{pmatrix}$

   (ii) Find the Eigen values and Eigen vectors of the matrix A = $\begin{pmatrix} \mathbf{2} & \mathbf{2} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} & \mathbf{1} \\ \mathbf{1} & \mathbf{2} & \mathbf{2} \end{pmatrix}$
2. Verify Cayley-Hamilton theorem and hence find A^{-1} and A^4 , if A = $\begin{pmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-2} \\ \mathbf{-1} & \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} & \mathbf{1} \end{pmatrix}$
3. (i) Using Cayley-Hamilton theorem, Evaluate the matrix equation

   $\boldsymbol{A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 - 8A^2 + 2A - I}$ for A = $\begin{pmatrix} \mathbf{2} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} \end{pmatrix}$

   (ii) Diagonalise the matrix $\begin{pmatrix} \mathbf{3} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{3} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{3} \end{pmatrix}$ by means of an orthogonal transformation.
4. Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ into a canonical form by an orthogonal reduction. Hence find the rank, index, signature and nature of the quadratic form.