Anna University Exams April/May 2015 – Regulation 2013
Rejinpaul.com Unique Important Questions – 4th Semester BE/BTECH
MA6453 PROBABILITY AND QUEUING THEORY
Part 1 /2 - Topic Wise Important Questions

UNIT 1	RANDOM VARIABLES	
	PART-B TOPICS	PART-A TOPICS
1	Find MGF ,mean variances of (i) Binomial (ii)poisson (iii) geometric (iv)exponential (v)gamma distribution	Find mean ,variences, mgf for pdf or pmf
2	Problems based on (i) Binomial (ii)poisson (iii) geometric	Definition of all distribution
3	Problems based on (iv)exponential (v)gamma distribution (vi) normal distribution	
4	Find probabilities from pdf or pmf	
UNIT 2	TWO DIMENSIONAL RANDOM VARIABLES	
	PART-B TOPICS	PART-A TOPICS
1	Find marginal, conditional probability from pdf or pmf	Check independent or not
2	Find correlation and regression line from pdf or pmf	Marginal distribution
3	Transformation of two dimensional R.V	Regression line and its coefficient
4	Find probabilities from pdf or pmf	
UNIT 3	RANDOM PROCESSES	
	PART-B TOPICS	PART-A TOPICS
1	Stationary process problem WSS,SSS or not	Definition of WSS,SSS
2	Poisson process derivation, mean, variances,auto correlation, sum of two poisson process is poisson, differences is not.	Markov process problem
3	Poisson process problem	Invariant probabilities
4	Markovian process	
UNIT 4	QUEUEING MODELS	
	PART-B TOPICS	PART-A TOPICS
1	Find L_s, L_q, W_s, W_q MODEL I (M/M/1): (∞/ FIFO)	Littles formula
2	Find L_s, L_q, W_s, W_q MODEL II: (M/M/c) : (∞ / FIFO)	Steady state probabilites
3	Find L_s, L_q, W_s, W_q MODEL III : (M/M/1) (K/FIFO)	
4	Problems in MODEL I (M/M/1): (∞/ FIFO), (M/M/c) : (∞ / FIFO), MODEL III : (M/M/1) (K/FIFO)	
UNIT 5	ADVANCED QUEUING MODELS	
	PART-B TOPICS	PART-A TOPICS
1	Pollaczek Khinchin formula	Definition of all models
2	Series queues	Simples formulas
3	M/D/1 and M/EK/1 as special cases	
4	Open Jackson networks	

Part 2/2 – Question Wise Important Questions

UNIT 1 RANDOM VARIABLES

1.Find the *mgf*. mean and variance of a binomial distribution, poisson , geometric, exponential, gamma distribution.

2.The probability that an individual suffers from a bad reaction from a certain injection is 0.001,determine the probability that out of 2000 individuals (a)exactly 3 (b)more than 2, individuals will suffer from a bad reaction.

3.In a distribution exactly normal 7% of the items are under 35 and 89% under 63. What are the mean and standard deviation of the distribution.

4.Let X be a discrete r.v whose cumulative distribution function is

$$F(x)=\begin{cases}0 & for\ x<-3\\ 1/6 & for -3\le x<6\\ 1/2 & for\ 6\le x<10\\ 1 & for\ x\ge 10\end{cases}$$

a)Find $P(X\le 4), P(-5<X\le 4), P(X=-3), P(X=4).$

(b)Find the probability mass function.

5. A Random Variable X has the following probability distribution function

X	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2k	3K	k^2	$2k^2$	$7k^2+k$

(i) Find the value of k (ii) Evaluate P(X<6), P(X≥ 6) (iii) If $P(X\le c)>\frac{1}{2}$ find the minimum value of c.

6. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the 4th trial (2) in fewer than 4 trials?

UNIT 2 TWO DIMENSIONAL RANDOM VARIABLES

1.The joint probability mass function of (X, Y) is given by $P(x,y)=k(2x+3y);\ x=0,1,2.\ y=1,2,3.$ Find all the marginal and conditional probability distributions. Also find the probability distribution of (X+Y) and $P[X+Y>3]$

2.The joint p.d.f of random variables X and Y is given by

$$f(x,y)=3(x+y);\ 0<x\le 1;\ 0\le y\le 1, x+y\le 1$$
$$=0,\ otherwise$$

Find (i) the marginal P.d.f of X (ii) P[X+Y<1/2] (iii) cov.(X,Y) (iv) regression coefficient

3. If the joint probability density function of *X* and *Y* is

$$f(x,y)=\begin{cases}e^{-(x+y)} & \text{for } x>0, y>0\\ 0, & \text{elsewhere}\end{cases}$$

Find the probability density function of $Z=\frac{X}{X+Y}$

4. Calculate the correlation co-efficient and obtain the lines of regression from the data given below:

X	62	64	65	69	70	71	72	74
Y	126	125	139	145	165	152	180	208

5. If X and Y are random variables having the joint density function

$$f(x,y)=\frac{1}{8}(6-x-y);\ 0<x<2;\ 2<y<4$$
$$=0,\ otherwise$$

Find (i) $P(X<1\cap Y<3)$ (ii) $P(X+Y<3)$ (iii) $P(X<1/Y<3)$

UNIT 3 RANDOM PROCESSES

1.The process {X(t)} whose probability distribution under certain conditions is given by

$$P\{X(t)=n\}=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2\\ \dfrac{at}{(1+at)}, & n=0\end{cases}$$. Show that it is not stationary

2. Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide sense stationary if A and ω are constant and is a uniformly distributed random variable in $(0, 2\pi)$.
3. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train, but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work iff a 6 appeared. Find (i) the probability that he takes a train on the third day and (ii) the probability that he drives to work in the long run.
4. Let $\{X_n : n = 1,2,3 \ldots\ldots\}$ be a Markov chain on the space S = {1,2,3 } with one step t.p.m

$$P=\begin{bmatrix}0 & 1 & 0\\ ½ & 0 & ½\\ 1 & 0 & 0\end{bmatrix}$$

i)Sketch the transition diagram
ii) Is the chain irreducible? Explain.
iii)Is the chain ergodic? Explain.

5. The transition probability matrix of a Markov chain $\{X_n\}$, n = 1,2,3, ... having 3 states 1,2 and 3 is

$$P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$$ and the initial distribution is P(0) = (0.7, 0.2, 0.1)

Find i) $P(X_2=3)$ ii) $P(X_3=2, X_2=3, X_1=3, X_0=2)$

6. Define Poisson process and obtain the probability distribution for that. Also find the auto correlation function for the process, Find mean and variances, prove that sum of two poisson process is poisson ,but differences is not.
7. Suppose that customers arrive at a bank according to a Poisson process with a mean rate of 3 per minute. Find the probability that during a time interval of 2 mins (i) exactly 4 customers arrive (ii) more than 4 customers arrive.

UNIT 4 QUEUEING MODELS

1. Customers arrive at a one – man barber shop according to a Poisson process with a mean interarrival time of 20 *minutes*. Customers spend an average of 15 *minutes* in the barber's chair. If an hour is used as a unit of time, then (i)What is the probability that a customer need not wait for a haircut? What is the expected number of customers in the barber shop and in the queue?How much time can a customer expect to spend in the barber shop? Find the average time that the customer spends in the queue.?The owner of the shop will provide another chair and hire another barber when a customer's average time in the shop exceeds 1.25 *hr*. By how much should the average rate of arrivals increase in order to justify a second barber?Estimate the fraction of the day that the customer will be idle.?What is the probability that there will be more than 6 customers waiting for service? Estimate the percentage of customers who have to wait prior to getting into the barber's chair. What is the probability that the waiting time (a) in the system (b) in the queue, is greater than 12 *minutes?*
2. Suppose there are 3 typists in a typing pool. Each typist can type an average of 6 *letters/hr.* If the letters arrive to be typed at the rate of 15 *letter / hr,* what fraction of the time are all three typists busy?

what is the average number of letters waiting to be typed?what is the probability that there is one letter in the system?what is the average time a letter spends in the system (waiting and being typed)? what is the probability a letter will take longer than 20 *minutes* waiting to be typed and being typed? Suppose that each individual typist receives letters at the average rate of 5 / *hr* Assume each typist can type at the average rate of 6 letters / *hr*. What is the average time a letter spends in the system waiting and being typed?

3. The railway marshalling yard is sufficient only for trains (there being 11 lines, one of which is earmarked for the shunting engine to reverse itself from the crest of the hump to the rear of the train). Trains arrive at the rate of 25 trains per day, inter – arrival time and service time follow exponential with an average of 30 *minutes*. Determine the probability that the yard is empty. Also find average queue length.
4. Find L_s, L_q, W_s, W_q (M/M/1): (∞/ FIFO), (M/M/c): (∞/ FIFO), (M/M/1): (k/ FIFO)

UNIT 5 ADVANCED QUEUING MODELS

1. Derive Pollaczek-Khinchin formula
2. In a network of 3 service station 1,2, 3 customer arrive at 1,2,3 from outside in accordance with Poisson process having rate 5, 10, 15 res. The service time at the stations are exponential with respect rate 10, 50, 100, A customer completing service at station -1 is equally likely to (i) go to station 2 (ii) go to station 3 or (iii) leave the system. A customer departing from service at station 2 always goes to station 3. A departure from service at station 3 is equally likely to go station 2 or leave the system. (a) What is the average number customer in the system consisting of all the three stations? (b) What is the average time a customer spend in the system?
3. A repair facility by a large number of machines has two sequential stations with respective rates one per hour and two per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behavior may be approximated by the two-stage tandem queue determine the average repair time.
4. A car wash facility operates with only one bay. Cars arrive according to a Poisson fashion with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. Find the average time a car spends in the facility, if the time for washing and cleaning a car is constant of 10 minutes