# Assignment Problems:

## Part-B:

1. The specifications of the desired low pass filter is.

$$0.8 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \qquad 0.32\pi \le \omega \le \pi.$$

Design Butterworth filter using impulse invariant transformation.

Solution:

* To find order of filter N:

$$\frac{1}{\sqrt{1+\varepsilon_1^2}} = 0.8.$$

$$\boxed{\varepsilon_1 = 0.75}$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2$$

$$\boxed{\lambda = 4.899}$$

$\omega_p = 0.2\pi$

$\omega_s = 0.32\pi.$

$$\Omega_p = \frac{\omega_p}{T}$$

$$\Omega_s = \frac{\omega_s}{T}$$

$$K = \frac{\Omega_p}{\Omega_s} = \frac{\omega_p/T}{\omega_s/T} = \frac{\omega_p}{\omega_s} = \frac{0.2\pi}{0.32\pi} = \frac{0.2}{0.32}$$

$$N \ge \frac{\log \lambda/\varepsilon_1}{\log 1/K} \simeq \frac{\log \frac{4.899}{0.75}}{\log \frac{0.32}{0.2}} \simeq 3.99$$

$$\boxed{N=4}$$

find the normalized transfer function $H(s)$:

poles = 4.

$$S_k = e^{j\phi_k} \qquad k = 1, 2, \dots, N$$

$$S_k = e^{j\phi_k} \qquad k = 1, 2, 3, 4.$$

$$\phi_k = \pi/2 + \frac{(2k-1)\pi}{2N}$$

$$\phi_k = \pi/2 + \frac{(2k-1)\pi}{8}$$

$\phi_1 = 112.5$, $\phi_2 = 157.5$, $\phi_3 = 202.5$, $\phi_4 = 247.5$

$$s_1 = e^{j\phi_1} = -0.3827 + j0.9239$$

$$s_2 = e^{j\phi_2} = -0.9239 + j0.3827$$

$$s_3 = e^{j\phi_3} = -0.9239 - j0.3827$$

$$s_4 = e^{j\phi_4} = -0.3827 - j0.9239$$

$$H(s) = \frac{1}{(s-(-0.3827+j0.9239))(s-(-0.9239+j0.3827))(s-(-0.9239-j0.3827))(s-(-0.3827-j0.9239))}$$

$$= \frac{1}{(s+0.3827-j0.9239)(s+0.9239-j0.3827)(s+0.9239-j0.3827)(s+0.3827+j0.9239)}$$

$$= \frac{1}{((s+0.3827)^2+0.9239^2)((s+0.9239)^2+0.3827^2)}$$

$$H(s) = \boxed{\frac{1}{(s^2+0.7654s+1)(s^2+1.8478s+1)}}$$

* To find $\Omega_c$:

②

$$\Omega_c = \frac{\Omega_p}{\varepsilon_1^{1/N}} = \frac{0.2\pi}{(0.75)^{1/4}} = 0.6752\ \text{rad/sec}.$$

$$\boxed{\Omega_c = 0.6752\ \text{rad/sec}}$$

* To find actual $H_a(s)$:

$$H_a(s) = H(s)\Big|_{s \to s/\Omega_c}$$

$$= H(s)\Big|_{s \to \frac{s}{0.6752}}$$

$$H_a(s) = \frac{1}{\left[\left(\frac{s}{0.6752}\right)^2 + 0.7654 \times \frac{s}{0.6752} + 1\right]\left[\left(\frac{s}{0.6752}\right)^2 + 1.8478 \times \frac{s}{0.6752} + 1\right]}$$

$$= \frac{0.2078}{[s^2 + 0.5168s + 0.4559][s^2 + 1.2476s + 0.4559]}$$

* To use impulse invariant method:

Express $H_a(s) = \sum_{k=1}^{N} \frac{c_k}{s - p_k}$

$$H_a(s) = \frac{0.2078}{[s-(-0.2584+j0.6238)][s-(-0.2584-j0.6238)][s-(-0.6238+j0.2584)][s-(-0.6238-j0.2584)]}$$

* The formula for digital filter using impulse invariance method.

$$H(z) = \sum_{k=1}^{N} \frac{c_k}{1 - e^{p_k T} z^{-1}}.$$

$$* \; H_a(s) = \frac{c_1}{(s-(-0.2584+j0.6238))} + \frac{c_2}{(s-(-0.2584-j0.6238))} + \frac{c_3}{(s-(-0.6238+j0.2584))} + \frac{c_4}{(s-(-0.6238-j0.2584))}$$

$$c_1 = (s-(-0.2584+j0.6238)) \cdot H_a(s) \Big|_{s=-0.2584+j0.6238}$$

$$= -0.3118 + j0.1292$$

$$c_2 = (s-(-0.2584-j0.6238)) \cdot H_a(s) \Big|_{s=-0.2584-j0.6238}$$

$$= -0.3118 - j0.1292$$

$$c_3 = (s-(-0.6238+j0.2584)) \cdot H_a(s) \Big|_{s=-0.6238+j0.2584}$$

$$= 0.3118 - j0.7528$$

$$c_4 = (s-(-0.6238-j0.2584)) \, H_a(s) \Big|_{s=-0.6238-j0.2584}$$

$$= 0.3118 + j0.7528$$

$$\therefore \boxed{H_a(s) = \frac{-0.3118+j0.1292}{(s-(-0.2584+j0.6238))} + \frac{(-0.3118-j0.1292)}{(s-(-0.2584-j0.6238))} + \frac{0.3118-j0.7528}{(s-(-0.6238+j0.2584))} + \frac{0.3118+j0.7528}{(s-(-0.6238-j0.2584))}}$$

$$\therefore H(z) = \frac{-0.3118 + j0.1292}{1 - e^{(-0.2584 + j0.6238)T} z^{-1}} + \frac{(-0.3118 - j0.1292)}{1 - e^{(-0.2584 - j0.6238)T} z^{-1}} + \frac{(0.3118 - j0.7528)}{(1 - e^{(-0.6238 + j0.2584)T} z^{-1})} + \frac{(0.3118 + j0.7528)}{(1 - e^{(-0.6238 - j0.2584)T} z^{-1})}$$

$\Rightarrow$ sub $T = 1$ sec.

$$H(z) = \frac{-0.3118 + j0.1292}{1 - e^{-0.2584} e^{+j0.6238} z^{-1}} + \frac{(-0.3118 - j0.1292)}{(1 - e^{-0.2584} e^{-j0.6238} z^{-1})} + \frac{0.3118 - j0.7528}{(1 - e^{-0.6238} e^{j0.2584} z^{-1})} + \frac{(0.3118 + j0.7528)}{(1 - e^{(-0.6238)} e^{-j0.2584} z^{-1})}$$

$$= \frac{-0.3118 + j0.1292}{1 - 0.6291 z^{-1} - j0.4527 z^{-1}} + \frac{(-0.3118 - j0.1292)}{(1 - 0.6291 z^{-1} + j0.4527 z^{-1})} + \frac{0.3118 - j0.7528}{(1 - 0.5181 z^{-1} + j0.1369 z^{-1})} + \frac{0.3118 + j0.7528}{(1 - 0.5181 z^{-1} - j0.1369 z^{-1})}$$

$$= \frac{-0.6236 + 0.0351 z^{-1}}{(1 - 0.6291 z^{-1})^2 + (0.4527 z^{-1})^2} + \frac{0.6236 - 0.5292 z^{-1}}{(1 - 0.5181 z^{-1})^2 + (0.1369 z^{-1})^2}.$$

$$= \frac{-0.6236 + 0.0351 z^{-1}}{1 - 1.2582 z^{-1} + 0.6062 z^{-2}} + \frac{0.6236 - 0.5292 z^{-1}}{1 - 1.0362 z^{-1} + 0.2872 z^{-2}}$$

2) i) obtain the direct form I, direct form II, cascade, parallel form realization of the system

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2).$$

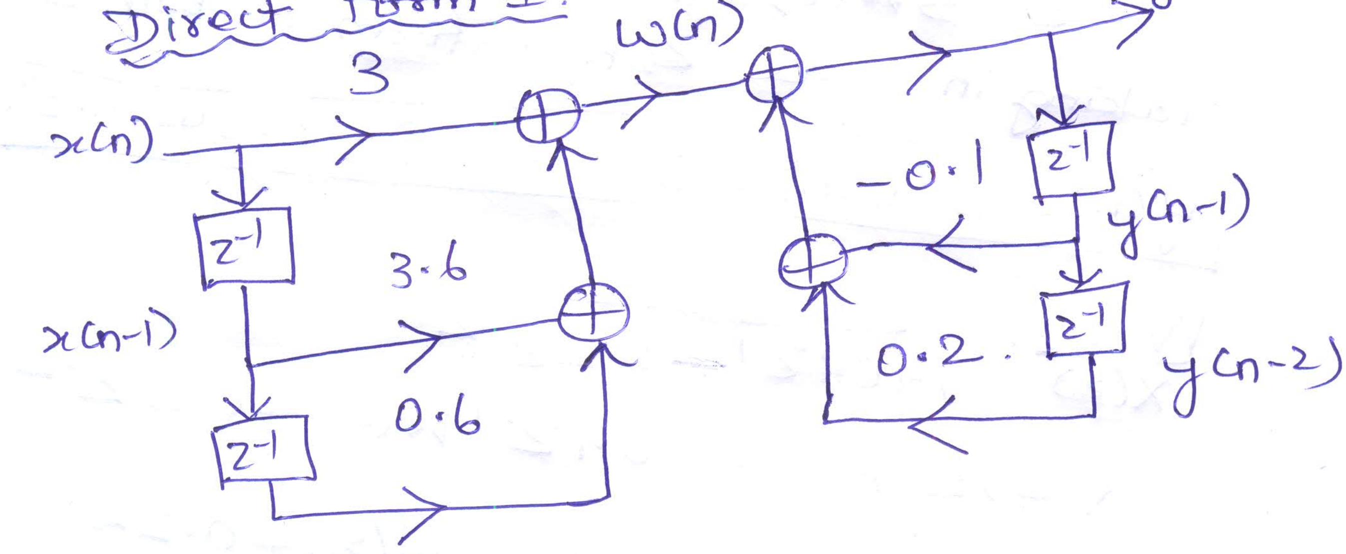
**Solution:**

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2).$$

Let $w(n) = 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + w(n)$$

**Direct form I:**




**Direct form II:**

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

Taking z-transform on both sides

$$Y(z) = -0.1\,z^{-1}Y(z) + 0.2\,z^{-2}Y(z) + 3X(z) + 3.6z^{-1}X(z) + 0.6z^{-2}X(z)$$

$$Y(z)\left(1 + 0.1z^{-1} - 0.2z^{-2}\right) = X(z)\left(3 + 3.6z^{-1} + 0.6z^{-2}\right)$$

$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}.$$

$$\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{Y(z)}{W(z)} = 3+3.6z^{-1}+0.6z^{-2}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+0.1z^{-1}-0.2z^{-2}}$$

$$Y(z) = (3+3.6z^{-1}+0.6z^{-2})W(z)$$

$$= 3W(z)+3.6z^{-1}W(z)+W(z)0.6z^{-2}$$

Taking inverse Z-transform on both sides

$$\boxed{y(n) = 3\,w(n)+3.6\,w(n-1)+0.6\,w(n-2)} \rightarrow (1)$$

$$\frac{W(z)}{X(z)} = \frac{1}{1+0.1z^{-1}-0.2z^{-2}}$$

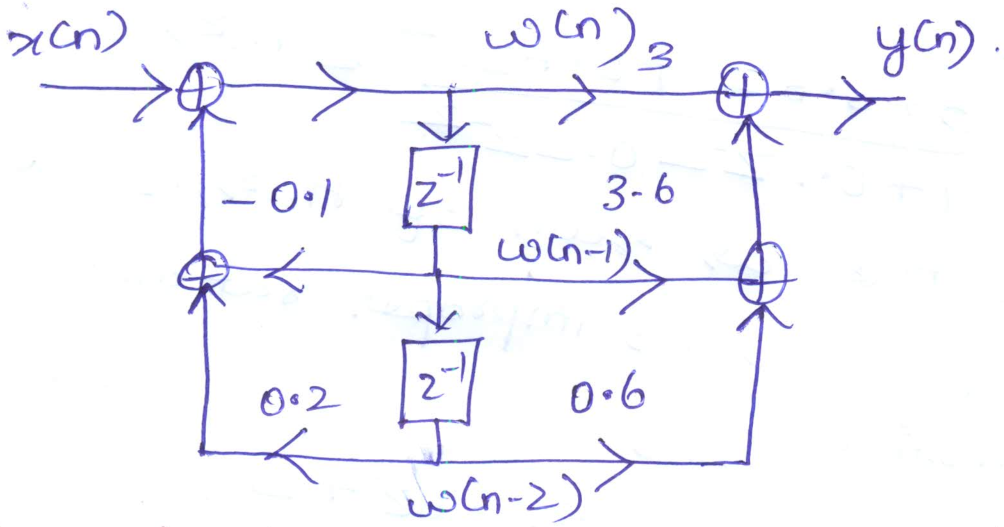
$$X(z) = W(z)+0.1z^{-1}W(z)-0.2z^{-2}.$$

$$W(z) = X(z)-0.1z^{-1}W(z)+0.2z^{-2}$$

Taking inverse Z-transform on both sides

$$\boxed{w(n) = x(n)-0.1\,w(n-1)+0.2\,w(n-2)} \rightarrow (2).$$

## Direct form II:




## Cascade form:

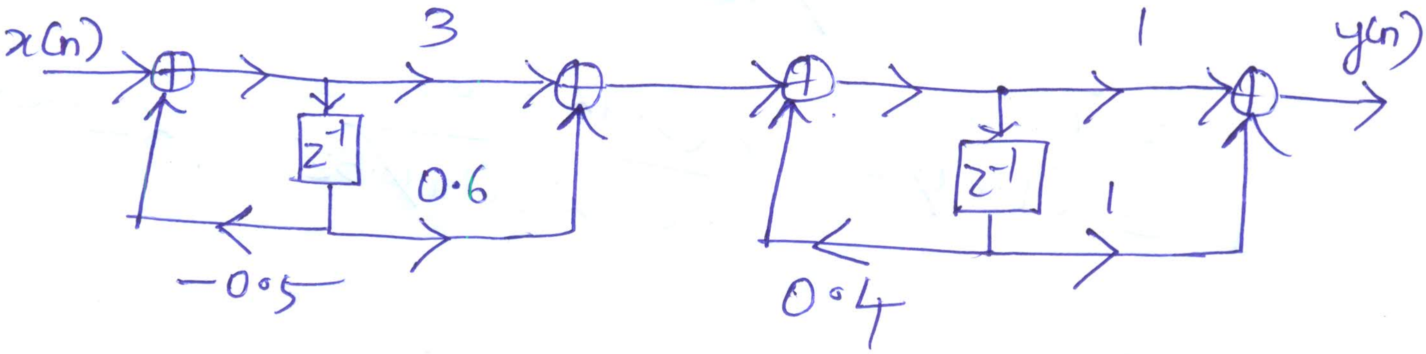
$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}}.$$

$$\frac{Y(z)}{X(z)} = \frac{(3+0.6z^{-1})(1+z^{-1})}{(1+0.5z^{-1})(1-0.4z^{-1})}. = H_1(z)\cdot H_2(z)$$

$$H_1(z) = \frac{3+0.6z^{-1}}{1+0.5z^{-1}}$$

$$H_2(z) = \frac{1+z^{-1}}{1-0.4z^{-1}}$$

## Cascade form Realization:



Parallel form Realization:

$$\frac{Y(z)}{X(z)} = \frac{3+3.6z^{-1}+0.6z^{-2}}{1+0.1z^{-1}-0.2z^{-2}} = H(z)$$

* Divide $H(z)$ as sum of $H_1(z), H_2(z)$ etc

* Making $H(z)$ as improper rational function.

$$\begin{array}{r|l} & \quad -3 \\ \hline -0.2z^{-2}+0.1z^{-1}+1 & 0.6z^{-2}+3.6z^{-1}+3 \\ & +0.6z^{-2}-0.3z^{-1}-3 \\ & (-) \quad (+) \quad (+) \\ \hline & \quad 3.9z^{-1}+6 \end{array}$$

$$H(z) = -3 + \frac{3.9z^{-1}+6}{1+0.1z^{-1}-0.2z^{-2}}$$

$$= -3 + \frac{3.9z^{-1}+6}{(1-0.4z^{-1})(1+0.5z^{-1})}$$

$$= -3 + \frac{A}{1-0.4z^{-1}} + \frac{B}{1+0.5z^{-1}}.$$

$$A = \left.\frac{\cancel{(1-0.4z^{-1})}(3.9z^{-1}+6)}{\cancel{(1-0.4z^{-1})}(1+0.5z^{-1})}\right|_{z=0.4}$$

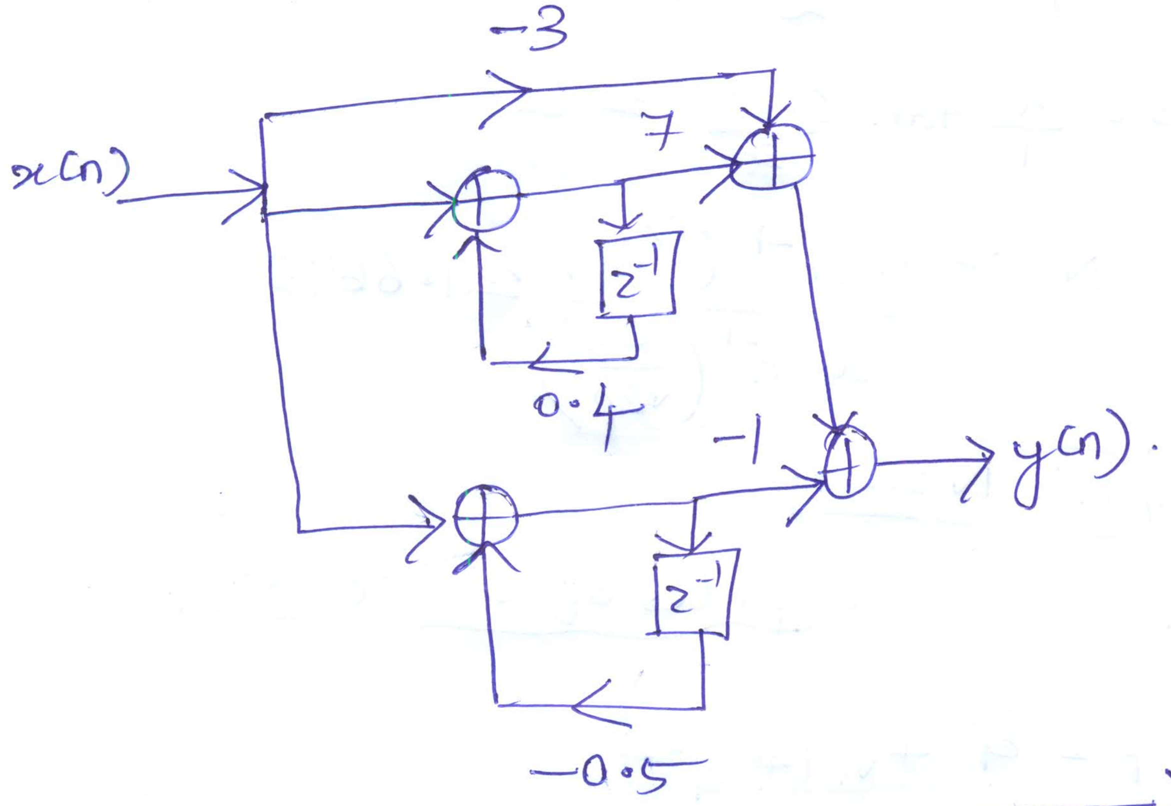
$= 7$

$$B = \left.\frac{\cancel{(1+0.5z^{-1})}(3.9z^{-1}+6)}{(1-0.4z^{-1})\cancel{(1+0.5z^{-1})}}\right|_{z=-0.5}$$

$= -1$

$$\therefore H(z) = -3 + \frac{7}{1-0.4z^{-1}} - \frac{1}{1+0.5z^{-1}}$$

## Parallel Realization:




3. Design a chebyshev filter for the following specification using bilinear transformation.

$$0.707 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$0 \le |H(e^{j\omega})| \le 0.1 \qquad 0.5\pi \le \omega \le \pi.$$

Solution:

$$\frac{1}{\sqrt{1+\varepsilon_1^2}} = 0.707 \qquad \frac{1}{\sqrt{1+\lambda^2}} = 0.1$$

$$\boxed{\varepsilon_1 = 1} \qquad \boxed{\lambda = 9.9499}$$

$\omega_p = 0.2\pi$

$\omega_s = 0.5\pi.$

* To perform bilinear transformation the frequencies must be prewarped.

$$\boxed{\Omega = \frac{2}{T}\tan\frac{\omega}{2}.}$$

Let

$\Omega p = \frac{2}{1} \tan \frac{0.2\pi}{2} = 0.6498$

$\Omega s = \frac{2}{1} \tan \frac{0.5\pi}{2} = 2$

$$N \geq \frac{\cosh^{-1}(\lambda/\epsilon)}{\cosh^{-1}\left(\frac{\Omega s}{\Omega p}\right)} \simeq 1.6695$$

order of filter. $\boxed{\therefore N = 2}$

* To find the poles of the filter:

$$\mu = \epsilon^{-1} + \sqrt{1 + \epsilon^{-2}}$$
$$= 2.4142$$

$$a = \Omega p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.6498\left[\frac{2.4142^{1/2} - 2.4142^{-1/2}}{2}\right]$$

$\boxed{a = 0.2957}$

$$b = \Omega p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.6498\left[\frac{2.4142^{1/2} + 2.4142^{-1/2}}{2}\right]$$

$\boxed{b = 0.7139}$

$$\phi_K = \pi/2 + \frac{(2K-1)\pi}{2N} \quad K = 1, 2, \ldots, N$$

$$\phi_K = \pi/2 + \frac{(2K-1)\pi}{4} \quad K = 1, 2.$$

$\phi_1 = \pi/2 + \pi/4 = 135$

$\phi_2 = \pi/2 + 3\pi/4 = 225.$

$S_K = a\cos\phi_K + jb\sin\phi_K. \quad K = 1, 2, \dots, N.$

$S_1 = a\cos\phi_1 + jb\sin\phi_1$

$S_2 = a\cos\phi_2 + jb\sin\phi_2.$

$S_1 = 0.2957 \cos 135^\circ + j\, 0.7139 \sin 135^\circ$

$= -0.2091 + j0.5048$

$S_2 = 0.2957 \cos 225^\circ + j0.7139 \sin 225^\circ.$

$= -0.2091 \quad - j\, 0.5048.$

* The denominator polynomial is

$H(s)_{den} = (S - S_1)(S - S_2)$

$= (S - (-0.2091 + j0.5048))$
$(S - (-0.2091 - j0.5048))$

$= (S + 0.2091 - j0.5048)(S + 0.2091 + j0.5048)$

$= ((S + 0.2091)^2 + 0.5048^2)$

$= S^2 + 0.4182\,S + 0.2985.$

$$H(s)_{Num} = \frac{\text{Substituting } S = 0 \text{ in denominator polynomial}}{\sqrt{1+\epsilon^2}}$$

$$= \frac{0.2985}{\sqrt{1+1}} = 0.2111$$

$$\therefore \boxed{H(s) = \frac{0.2111}{S^2 + 0.4182S + 0.2985}}$$

* Getting digital filter using bilinear transformati

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$T=1$

$$H(z) = H(s)\Big|_{s=2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$H(z) = \frac{0.2111}{\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)0.4182 + 0.2985}$$

$$= \frac{0.2111\,(1+z^{-1})^2}{4-8z^{-1}+4z^{-2}+0.8364-0.8364z^{-2}+0.2985}$$

$$= \frac{0.2111+0.4222z^{-1}+0.2111z^{-2}}{5.1349-8z^{-1}+3.1636z^{-2}}$$

$$\boxed{H(z) = \frac{0.0411+0.0822z^{-1}+0.0411z^{-2}}{1-1.558z^{-1}+0.6161z^{-2}}}$$

3) (ii) For the analog transfer function $H(s) = \left(\frac{s+0.1}{(s+0.1)^2+16}\right)$ into a digital IIR filter by means of the bilinear transformation. The digital filter is to have a resonant frequency, $\omega_r = \pi/2$.

Solution:

* $$H_a(s) = \frac{s+0.1}{(s+0.1)^2+16} \rightarrow ①$$

General format for $H_a(s) = \frac{s}{s^2+\Omega_c^2}$.

* from eqn ①, $\Omega_c = 4$.

* $\omega_r = \pi/2$.

* To perform bilinear transformation, we do prewarping of frequency.

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

$$4 = \frac{2}{T} \tan \frac{\pi}{2} \cdot \frac{1}{2}$$

$$= \frac{2}{T} \tan \pi/4$$

$$4 = \frac{2}{T}$$

$$\therefore \boxed{T = \frac{2}{4} = \frac{1}{2} \text{ sec}}$$

* Design of digital filter using bilinear transformation:

$$H(z) = H_a(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= H_a(s)\Big|_{s=4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$H(z) = \frac{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1}{\left(4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1\right)^2+16}$$

$$= \frac{4\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.1}{\ldots (1+z^{-1})^2}$$

$$\frac{\ldots}{16(1-z^{-1})^2 + 0.01(1+z^{-1})^2 + 0.8(1-z^{-1})(1+z^{-1}) + 16(1+z^{-1})^2}$$

$$= \frac{4(1-z^{-1})(1+z^{-1}) + 0.1(1+z^{-1})^2}{16 + 16z^{-2} - \cancel{32z^{-1}} + 0.01 + 0.01z^{-2} + 0.02z^{-1} + 0.8 - 0.8z^{-2} + 16 + 16z^{-2} + \cancel{32z^{-1}}}$$

$$= \frac{4 - 4z^{-2} + 0.1 + 0.1z^{-2} + 0.2z^{-1}}{32.81 + 0.02z^{-1} + 31.21z^{-2}}$$

$$= \frac{4.1 + 0.2z^{-1} - 3.9z^{-2}}{32.81 + 0.02z^{-1} + 31.21z^{-2}}$$

$$\boxed{H(z) = \frac{0.125 + 0.006z^{-1} - 0.12z^{-2}}{1 + 0.0006z^{-1} + 0.9512z^{-2}}}$$

dividing Num & Den by 32.81.

Design a chebyshev low pass filter with the pass band attenuation 1 dB ripple in the pass band $0 \le \omega \le 0.2\pi$, stop band attenuation 15 dB ripple in stop band $0.3\pi \le \omega \le \pi$ using bilinear transformation and impulse invariant Technique: (Assume T=1 sec).

Soln:

* Design using Bilinear Transform:

⇒ Pass band attenuation, $\alpha_p = 1\,dB$

⇒ Stop band attenuation, $\alpha_s = 15\,dB$.

$$\omega_p = 0.2\pi$$
$$\omega_s = 0.3\pi$$

* Prewarping frequency values:

$$\Omega = \frac{2}{T}\tan\frac{\omega}{2}$$

* $T = 1\,sec$

$$\therefore \Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2}$$
$$= 2\tan\frac{0.2\pi}{2} = \boxed{0.65}$$
$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2}$$
$$= 2\tan\frac{0.3\pi}{2} = \boxed{1.02}$$

* Order of the filter, N:

order of chebyshev filter, $N \geq \dfrac{\cosh^{-1}\sqrt{\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\cosh^{-1}\dfrac{\Omega_s}{\Omega_p}}$

$$N \geq \frac{\cosh^{-1}\sqrt{\dfrac{10^{1.5}-1}{10^{0.1}-1}}}{\cosh^{-1}\dfrac{1.02}{0.65}} = 3.01$$

$$\therefore \boxed{N = 4}$$

* To determine poles of the filter:

$$\varepsilon_1 = \sqrt{10^{0.1\alpha p} - 1} = \boxed{0.508}$$

$$\mu = \varepsilon_1^{-1} + \sqrt{1+\varepsilon_1^{-2}} = \boxed{4.17}$$

$$a = \Omega p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.65 \left[\frac{4.17^{1/4} - 4.17^{-1/4}}{2}\right]$$

$$= \boxed{0.237}$$

$$b = \Omega p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.65 \left[\frac{4.17^{1/4} + 4.17^{-1/4}}{2}\right]$$

$$= \boxed{0.6918}$$

$$\phi_K = \pi/2 + \frac{(2K-1)\pi}{2N} \quad K = 1, 2, \ldots N$$

$$\phi_K = \pi/2 + \frac{(2K-1)\pi}{8} \quad K = 1, 2, 3, 4.$$

$\phi_1 = 112.5^\circ$, $\phi_2 = 157.5^\circ$, $\phi_3 = 202.5^\circ$, $\phi_4 = 247.5^\circ$.

$$S_K = a\cos\phi_K + jb\sin\phi_K, \quad K = 1, 2, \ldots N$$

$$S_K = a\cos\phi_K + jb\sin\phi_K, \quad K = 1, 2, 3, 4.$$

$$S_1 = 0.237\cos 112.5^\circ + j\,0.6918\sin 112.5^\circ.$$

$$= -0.0907 + j\,0.6391$$

$$S_2 = 0.237\cos 157.5^\circ + j\,0.6918\sin 157.5^\circ$$

$$= -0.2189 + j\,0.2647$$

$$S_3 = 0.237 \cos 202.5^\circ + j\,0.6918 \sin 202.5^\circ$$ (10)

$$= -0.2189 - j\,0.2647$$

$$S_4 = 0.237 \cos 247.5^\circ + j\,0.6918 \sin 247.5^\circ$$

$$= -0.0907 - j\,0.639.$$

* To obtain denominator polynomial:

$$H(s)_{den} = [S - (-0.0907 + j0.639)][S - (-0.2189 + j0.2647)][S + (0.2189 - j0.2647)][S - (-0.0907 - j0.639)]$$

$$= [S + 0.0907 - j0.639][S + 0.2189 - j0.2647][S + 0.2189 + j0.2647][S + 0.0907 + j0.639]$$

$$= [(S+0.0907)^2 + (0.639)^2][(S+0.2189)^2 + (0.2647)^2]$$

$$= [S^2 + 0.1814S + 0.4165)[S^2 + 0.4378S + 0.118].$$

* To obtain Numerator of H(s): (N is even)

$$H(s)_{num} = \frac{\text{Sub } s=0 \text{ in } H(s)_{den}}{\sqrt{1+\varepsilon_1^2}}$$

$$= \frac{(0.4165)(0.118)}{\sqrt{1+0.508^2}} = 0.04381$$

$$\therefore H(s) = \frac{0.04381}{(s^2+0.1814s+0.4165)(s^2+0.4378s+0.118)}$$

* Design of digital filter using Bilinear transformation:

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$ $\boxed{T = 1\,\text{sec}}$

$$H(z) = \frac{0.04381}{(s^2+0.1814s+0.4165)(s^2+0.4378s+0.1180)}\Bigg|_{s=2\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{0.04381}{\left[\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2+0.1814\times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.4165\right)\left[\left[2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2+0.4378\times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.1180\right]}$$

$$= \frac{0.04381\,(1+z^{-1})^4}{\{4(1-z^{-1})^2+0.1814\times 2(1-z^{-2})+0.4165(1+z^{-1})^2\}\{4(1-z^{-1})^2+0.4378\times 2\times(1-z^{-2})+0.1180(1+z^{-1})^2\}}$$

$$= \frac{0.04381\,(1+z^{-1})^4}{\{4.7794-7.1668z^{-1}+4.0538z^{-2}\}\{4.9936-7.764z^{-1}+3.2424z^{-2}\}}$$

$$= \frac{0.001836\,(1+z^{-1})^4}{\{1-1.499z^{-1}+0.8482z^{-2})(1-1.5548z^{-1}+0.6493z^{-2}\}}$$

x ——————— x