

**ST.JOSEPH'S COLLEGE OF ENGINEERING
&
ST.JOSEPH'S INSTITUTE OF TECHNOLOGY
DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
&
DEPARTMENT OF INFORMATION TECHNOLOGY
Chennai-119**

CS 6201: Digital Principles and Systems Design

ASSIGNMENT-I QUESTIONS

PART A:

1. What is the abbreviation of ASCII and EBCDIC code?
2. Give the different types of binary codes.
3. Define De Morgan's Theorem?
4. What is meant by tabulation method?
5. Define Karnaugh Map? State the limitations of Karnaugh map.
6. Simplify the function using K-Map $F(A, B) = \sum m(0, 2, 3)$
7. Simplify the function using K-Map $F(A, B) = \pi M(0, 1, 3)$
8. How many bits are required to represent the decimal numbers in the range 0 to 999 using straight binary code? Using BCD codes?
9. Show that the excess-3 code is self-complementing.
10. What is meant by weighted and non-weighted code?
11. Write the two properties of Gray code & mention the application of Gray code
12. What is meant by Fast Adder?
13. Define distributive law.
14. Give the canonical product form of $F = x_1'x_2'x_3 + x_1'x_2x_3' + x_1x_2'x_3' + x_1'x_2x_3$
15. What is meant by prime implicant?
16. What are Universal Gates? Why are they called so?
17. Define positive logic and negative logic system.
18. Define bit, byte and nibble.
19. Define Combinational circuit?
20. Define SSI and MSI.

PART B:

1. a) (i) solve the following using k-map.

$$1) f(A, B, C, D) = \sum m (0, 2, 3, 8, 11, 12) + d (1, 9, 14)$$

AB \ CD	CD			
	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 0	X 1	1 3	1 2
$\bar{A}B$	0 4	0 5	0 7	0 6
AB	1 12	0 13	0 15	X 14
$A\bar{B}$	1 8	X 9	1 11	0 10

$$f = \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{B}D + ABD$$

$$2) f(A, B, C, D) = \sum m (0, 1, 2, 3, 5, 7, 8, 9, 11, 14)$$

CD \ AB	AB			
	00	01	11	10
00	1 0	0 4	0 12	1 8
01	1 1	1 5	0 13	1 9
11	1 3	1 7	0 15	1 11
10	1 2	0 6	1 14	0 10

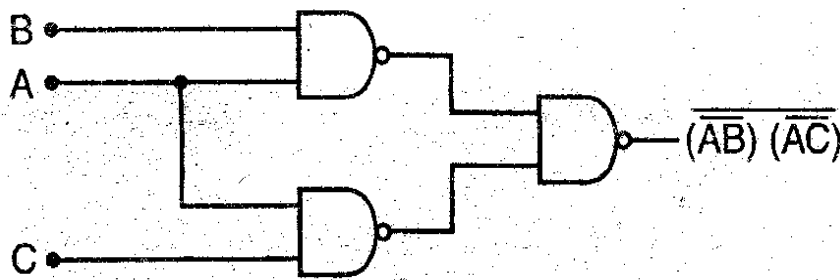
$$Y = \bar{A}\bar{B} + \bar{A}D + \bar{B}D + \bar{B}\bar{C} + ABCD$$

ii) Simplify the expression $Z = AB + AC + ABC(AB + C)$. Implement using Minimum number of NAND gates.

$$\begin{aligned}
 Z &= AB + AC + ABC(AB + C) \\
 &= AB + AC + ABC \cdot AB + ABCC \\
 &= AB + AC + ABC + ABC \\
 &= AB + AC + ABC \\
 &= AB(1 + C) + AC \\
 &= AB + AC
 \end{aligned}$$

Implementation using minimum number & NAND gates:

$$\overline{\overline{Z}} = \overline{AB+AC} = (\overline{AB})(\overline{AC})$$



2. Simplify the function $f(A, B, C, D) = (0,1,2,3,5,7,8,10,12,13,15)$, using tabulation method (Quine–McCluskey method).

First List	Second List	Third List																																																																																																																																																																																																																			
<table><tr><th>A</th><th>B</th><th>C</th><th>D</th></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td><td>✓</td></tr><tr><td>1</td><td>0</td><td>0</td><td>1</td><td>✓</td></tr><tr><td>2</td><td>0</td><td>0</td><td>1</td><td>0</td><td>✓</td></tr><tr><td>8</td><td>1</td><td>0</td><td>0</td><td>0</td><td>✓</td></tr><tr><td>3</td><td>0</td><td>0</td><td>1</td><td>1</td><td>✓</td></tr><tr><td>5</td><td>0</td><td>1</td><td>0</td><td>1</td><td>✓</td></tr><tr><td>10</td><td>1</td><td>0</td><td>1</td><td>0</td><td>✓</td></tr><tr><td>12</td><td>1</td><td>1</td><td>0</td><td>0</td><td>✓</td></tr><tr><td>7</td><td>0</td><td>1</td><td>1</td><td>1</td><td>✓</td></tr><tr><td>13</td><td>1</td><td>1</td><td>1</td><td>1</td><td>✓</td></tr><tr><td>15</td><td>1</td><td>1</td><td>1</td><td>1</td><td>✓</td></tr></table>	A	B	C	D	0	0	0	0	✓	1	0	0	1	✓	2	0	0	1	0	✓	8	1	0	0	0	✓	3	0	0	1	1	✓	5	0	1	0	1	✓	10	1	0	1	0	✓	12	1	1	0	0	✓	7	0	1	1	1	✓	13	1	1	1	1	✓	15	1	1	1	1	✓	<table><tr><th>A</th><th>B</th><th>C</th><th>D</th></tr><tr><td>0,1</td><td>0</td><td>0</td><td>0</td><td>✓</td></tr><tr><td>0,2</td><td>0</td><td>0</td><td>-</td><td>0</td><td>✓</td></tr><tr><td>0,8</td><td>-</td><td>0</td><td>0</td><td>0</td><td>✓</td></tr><tr><td>1,3</td><td>0</td><td>0</td><td>-</td><td>1</td><td>✓</td></tr><tr><td>1,5</td><td>0</td><td>-</td><td>0</td><td>1</td><td>✓</td></tr><tr><td>2,3</td><td>0</td><td>0</td><td>1</td><td>-</td><td>✓</td></tr><tr><td>2,10</td><td>-</td><td>0</td><td>1</td><td>0</td><td>✓</td></tr><tr><td>8,10</td><td>1</td><td>0</td><td>-</td><td>0</td><td>✓</td></tr><tr><td>8,12</td><td>1</td><td>-</td><td>0</td><td>0</td><td>$\overline{A} \overline{C} \overline{D}$</td></tr><tr><td>3,7</td><td>0</td><td>-</td><td>1</td><td>1</td><td>✓</td></tr><tr><td>5,7</td><td>0</td><td>1</td><td>-</td><td>1</td><td>✓</td></tr><tr><td>5,13</td><td>-</td><td>1</td><td>0</td><td>1</td><td>✓</td></tr><tr><td>12,13</td><td>1</td><td>1</td><td>0</td><td>-</td><td>$A B \overline{C}$</td></tr><tr><td>7,15</td><td>-</td><td>1</td><td>1</td><td>1</td><td>✓</td></tr><tr><td>13,15</td><td>1</td><td>1</td><td>-</td><td>1</td><td>✓</td></tr></table>	A	B	C	D	0,1	0	0	0	✓	0,2	0	0	-	0	✓	0,8	-	0	0	0	✓	1,3	0	0	-	1	✓	1,5	0	-	0	1	✓	2,3	0	0	1	-	✓	2,10	-	0	1	0	✓	8,10	1	0	-	0	✓	8,12	1	-	0	0	$\overline{A} \overline{C} \overline{D}$	3,7	0	-	1	1	✓	5,7	0	1	-	1	✓	5,13	-	1	0	1	✓	12,13	1	1	0	-	$A B \overline{C}$	7,15	-	1	1	1	✓	13,15	1	1	-	1	✓	<table><tr><th>A</th><th>B</th><th>C</th><th>D</th></tr><tr><td>0,1,2,3</td><td>0</td><td>0</td><td>-</td><td>$\overline{A} \overline{B}$</td></tr><tr><td>0,2,1,3</td><td>0</td><td>0</td><td>-</td><td>-</td></tr><tr><td>0,2,8,10</td><td>-</td><td>0</td><td>-</td><td>0</td><td>$\overline{B} \overline{D}$</td></tr><tr><td>0,8,2,10</td><td>-</td><td>0</td><td>-</td><td>0</td><td>-</td></tr><tr><td>1,3,5,7</td><td>0</td><td>-</td><td>-</td><td>1</td><td>$\overline{A} D$</td></tr><tr><td>1,5,3,7</td><td>0</td><td>-</td><td>-</td><td>1</td><td>-</td></tr><tr><td>5,7,13,15</td><td>-</td><td>1</td><td>-</td><td>1</td><td>$B D$</td></tr><tr><td>5,13,7,15</td><td>-</td><td>1</td><td>-</td><td>1</td><td>-</td></tr></table>	A	B	C	D	0,1,2,3	0	0	-	$\overline{A} \overline{B}$	0,2,1,3	0	0	-	-	0,2,8,10	-	0	-	0	$\overline{B} \overline{D}$	0,8,2,10	-	0	-	0	-	1,3,5,7	0	-	-	1	$\overline{A} D$	1,5,3,7	0	-	-	1	-	5,7,13,15	-	1	-	1	$B D$	5,13,7,15	-	1	-	1	-
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The prime implicants are: $\overline{A}\overline{B} + \overline{B}\overline{D} + \overline{A}D + BD + A\overline{C}\overline{D} + AB\overline{C}$

3. Simplify the following expressions by Quine–McCluskey method.

$$\overline{A}.B.C + \overline{A}.\overline{B}.D + A.\overline{C}.D + B.\overline{C}.\overline{D} + \overline{A}.B.\overline{C}.D$$

In the first step, we write the expanded version of the given expression. It can be written as follows:

$$\begin{aligned} &\overline{A}.B.C.D + \overline{A}.B.C.\overline{D} + \overline{A}.\overline{B}.C.D + \overline{A}.\overline{B}.\overline{C}.D + A.B.\overline{C}.D + A.\overline{B}.\overline{C}.D + A.B.\overline{C}.\overline{D} \\ &+ \overline{A}.B.\overline{C}.\overline{D} + \overline{A}.\overline{B}.C.D \end{aligned}$$

The formation of groups, the placement of terms in different groups and the first-round matching are shown as follows:

A	B	C	D	A	B	C	D	A	B	C	D		
0	0	0	1	0	0	0	1	✓	0	0	–	1	✓
0	0	1	1	0	1	0	0	✓	0	–	0	1	✓
0	1	0	0						–	0	0	1	✓
0	1	0	1	0	0	1	1	✓	0	1	0	–	✓
0	1	1	0	0	1	0	1	✓	0	1	–	0	✓
0	1	1	1	0	1	1	0	✓	–	1	0	0	✓
1	0	0	1	1	0	0	1	✓					
1	1	0	0	1	1	0	0	✓	0	–	1	1	✓
1	1	0	1	0	1	1	1	✓	0	1	–	1	✓
				1	1	0	1	✓	–	1	0	1	✓
									0	1	1	–	✓
									1	–	0	1	✓
									1	1	0	–	✓

The second round of matching begins with the table shown on the previous page. Each term in the first group is compared with every term in the second group. For instance, the first term in the first group 00–1 matches with the second term in the second group 01–1 to yield 0—1, which is recorded in the table shown below. The process continues until all terms have been compared for a possible match. Since this new table has only one group, the terms contained therein are all prime implicants.

In the present example, the terms in the first and second tables have all found a match. But that is not always the case.

A	B	C	D
0	—	—	1
—	—	0	1
0	1	—	—
—	1	0	—

The next table is what is known as the prime implicant table. The prime implicant table contains all the original terms in different columns and all the prime implicants recorded in different rows as shown below:

0001	0011	0100	0101	0110	0111	1001	1100	1101	
✓	✓		✓		✓				0— —1
✓			✓			✓		✓	— —01
		✓	✓	✓	✓				01 — —
		✓	✓				✓	✓	—10 —

Therefore, the minimized expression = $\overline{A}.D + \overline{C}.D + \overline{A}.B + B.\overline{C}$.

4. a) (i) Simplify the four variable switching functions. Draw the circuit of the minimal expression using only NAND gates.

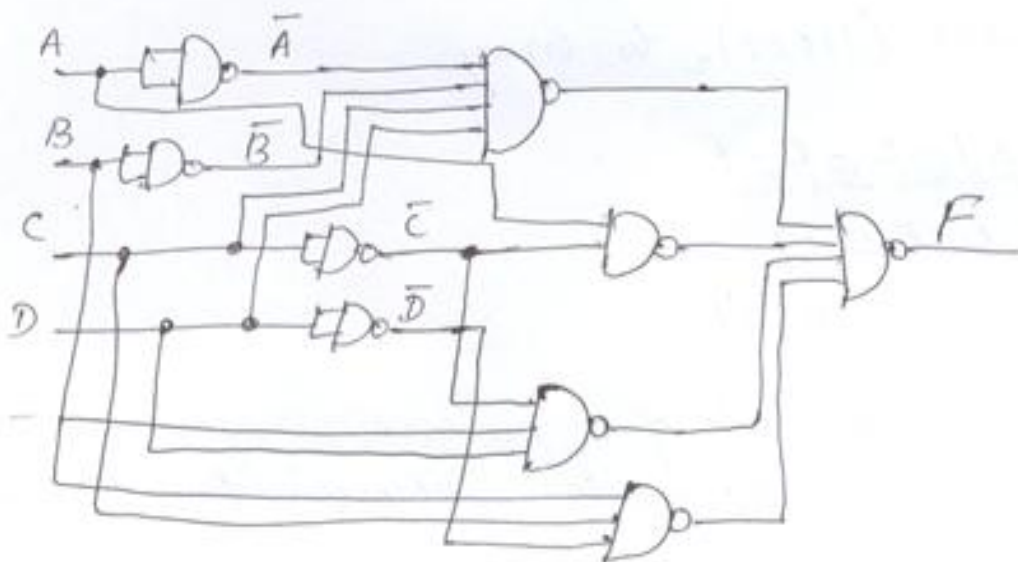
$$F(A,B,C,D) = \sum m(3,5,6,8,9,12,13,14)$$

(a) $F(A,B,C,D) = \sum m(3,5,6,8,9,12,13,14)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	0
$\bar{A}B$	0	1	0	1
AB	1	1	0	1
$A\bar{B}$	1	1	0	0

$$F(A,B,C,D) = \bar{A}\bar{B}CD + B\bar{C}D + BC\bar{D} + AC\bar{C}$$

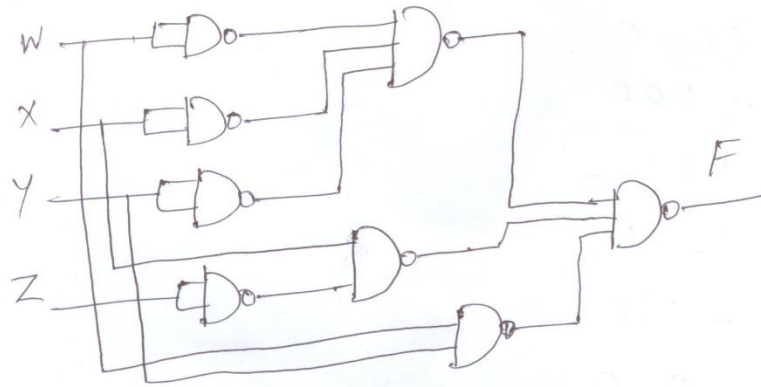
Implementation using NAND gates.



b) (i) Find a minimal sum of products representation for

$$F(W,X,Y,Z) = \sum m(1,4,6,10,12,14) + d(0,11,13,15) \text{ using K-Map. Draw the circuit of the}$$

Minimal expression using only NAND gates



$$F(W,X,Y,Z) = \sum m(1,4,6,10,12,14) + d(0,11,13,15)$$

WX \ YZ	ȲZ	ȲZ	YZ	YZ
W̄X̄	1	0	0	0
W̄X	0	0	0	1
WX̄	1	1	1	1
WX	0	0	1	1

$$F = \bar{W}\bar{X}\bar{Y} + XZ + WY$$

5. a) Simplify using K-Map

$$F(A,B,C,D) = \prod M(3,6,8,10,12,13,15)$$

$$a) F(A,B,C,D) = \prod M(3,6,8,10,12,13,15)$$

AB \ CD	C+D	C+D	C+D	C+D
A+B	1	1	0	1
A+B̄	1	1	1	0
Ā+B̄	0	0	0	1
Ā+B	0	1	1	0

$$F = (A+B+\bar{C}+\bar{D})(A+\bar{B}+\bar{C}+D)(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{D})(\bar{A}+B+D)$$

b) i) Perform subtraction on the given unsigned numbers using the 10's complement.

1) 6428 - 3409

2) 125 - 1800

b) i) 1. $6428 - 3409 = 3019$

Take 9's complement of subtrahend.

$$\begin{array}{r} 9999 \\ - 3409 \\ \hline 6590 \\ + 1 \\ \hline 6591 \rightarrow 10's \text{ complement} \end{array}$$

$$\begin{array}{r} 6428 \\ + 6591 \\ \hline [1]3019 \end{array} \text{ Discard carry.}$$

Result : 3019

(2) $125 - 1800 = -1675$

Take 9's complement of subtrahend.

$$\begin{array}{r} 9999 \\ - 1800 \\ \hline 8199 \\ + 1 \\ \hline 8200 \rightarrow 10's \text{ complement} \end{array}$$

$$\begin{array}{r} 8200 \\ + 0125 \\ \hline 8325 \text{ No carry} \end{array}$$

Take 10's complement of result

ii) Perform subtraction on the given unsigned binary numbers using the 2's complement.

1) 10011 - 10001

2) 100010 - 100011

(ii) 1. $10011 - 10001 = 00010$

Take 2's complement of 10001

$$\begin{array}{r} 01110 \\ + 1 \\ \hline 01111 \\ + 10001 \\ \hline [1]00010 \end{array} \text{ Discard carry}$$

Result = 00010

$$\begin{array}{r} 9999 \\ - 8325 \\ \hline 1674 \\ + 1 \\ \hline 1675 \end{array}$$

Assign negative sign

Result = -1675

2. $100010 - 100011 = -000001$

Take 2's complement of 100011

$$\begin{array}{r} 011100 \\ + 1 \\ \hline 011101 \end{array} \quad \begin{array}{r} 011101 \\ + 100010 \\ \hline 111111 \end{array} \text{ No carry}$$

Take 2's complement of result

$$\begin{array}{r} 000000 \\ + 1 \\ \hline 000001 \end{array} \text{ Assign negative sign}$$

-000001

6. State the postulates and theorems of Boolean algebra.

Principle of duality:

1. Interchanging the OR and AND operations of the expression. 2. Interchanging the 0 and 1 elements of the expression. 3. Not changing the form of the variables.

Commutative Law

$$(a) A + B = B + A$$

$$(b) A B = B A$$

Associative Law

$$(a) (A + B) + C = A + (B + C)$$

$$(b) (A B) C = A (B C)$$

Distributive Law

$$(a) A (B + C) = A B + A C$$

$$(b) A + (B C) = (A + B) (A + C)$$

Identity Law

$$(a) A + A = A$$

$$(b) A A = A$$

Negation Law

$$(a) \overline{(\overline{A})} = A$$

$$(b) \overline{(\overline{\overline{A}})} = A$$

Redundant Law

$$(a) A + A B = A$$

$$(b) A (A + B) = A$$

Postulates:

$$(a) 0 + A = A$$

$$(b) 1 A = A$$

$$(c) 1 + A = 1$$

$$(d) 0 A = 0$$

$$(e) \overline{A} + A = 1$$

$$(f) \overline{A} A = 0$$

$$(g) A + \overline{A} B = A + B$$

$$(h) A (\overline{A} + B) = A B$$

De Morgan's Theorem

$$(a) \overline{(A + B)} = \overline{A} \overline{B}$$

$$(b) \overline{(A B)} = \overline{A} + \overline{B}$$

7. (i) Reduce $AB + (AC)' + AB'C (AB + C)$

$$\begin{aligned}
 AB + (AC)' + AB'C (AB + C) &= AB + (AC)' + AAB'BC + AB'CC \\
 &= AB + (AC)' + AB'CC [A.A' = 0] \\
 &= AB + (AC)' + AB'C [A.A = 1] \\
 &= AB + A' + C' = AB'C [(AB)' = A' + B'] \\
 &= A' + B + C' + AB'C [A + AB' = A + B] \\
 &= A' + B'C + B + C' [A + A'B = A + B] \\
 &= A' + B + C' + B'C \\
 &= A' + B + C' + B' \\
 &= A' + C' + 1 \\
 &= 1 [A + 1 = 1]
 \end{aligned}$$

ii) Show that $(X + Y' + XY)(X + Y')(X'Y) = 0$

$$\begin{aligned}
 (X + Y' + XY)(X + Y')(X'Y) &= (X + Y' + X)(X + Y')(X' + Y) \\
 &\quad [A + A'B = A + B] \\
 &= (X + Y')(X + Y')(X'Y) [A + A = 1] \\
 &= (X + Y')(X'Y) [A.A = 1] \\
 &= X.X' + Y'.X'.Y \\
 &= 0 [A.A' = 0]
 \end{aligned}$$

8. Design a combinational circuit for converting 2421 code to BCD code.

Both the 2421 code and BCD code are 4-bit codes and represent the decimal equivalents 0 to 9. To design the converter circuit for the above, first the truth table is prepared with the input variables W, X, Y, and Z of 2421 code, and the output variables A, B, C, and D. Karnaugh maps to obtain the simplified expressions of the output functions. Unused combinations are considered as don't-care condition.

Decimal Equivalent	Input variables				Output variables			
	2421 code				BCD code			
	W	X	Y	Z	A	B	C	D
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	0
3	0	0	1	1	0	0	1	1
4	0	1	0	0	0	1	0	0
5	1	0	1	1	0	1	0	1
6	1	1	0	0	0	1	1	0
7	1	1	0	1	0	1	1	1
8	1	1	1	0	1	0	0	0
9	1	1	1	1	1	0	0	1

	Y'Z'	Y'Z	YZ	YZ'
W'X'				
W'X		X	X	X
WX			1	1
WX'	X	X		X

	Y'Z'	Y'Z	YZ	YZ'
W'X'				
W'X	1	X	X	X
WX	1	1		
WX'	X	X	1	X

	Y'Z'	Y'Z	YZ	YZ'
W'X'			1	1
W'X		X	X	X
WX	1	1		
WX'	X	X		X

	Y'Z'	Y'Z	YZ	YZ'
W'X'		1	1	
W'X		X	X	X
WX		1	1	
WX'	X	X	1	X

The Boolean expressions for the output functions are

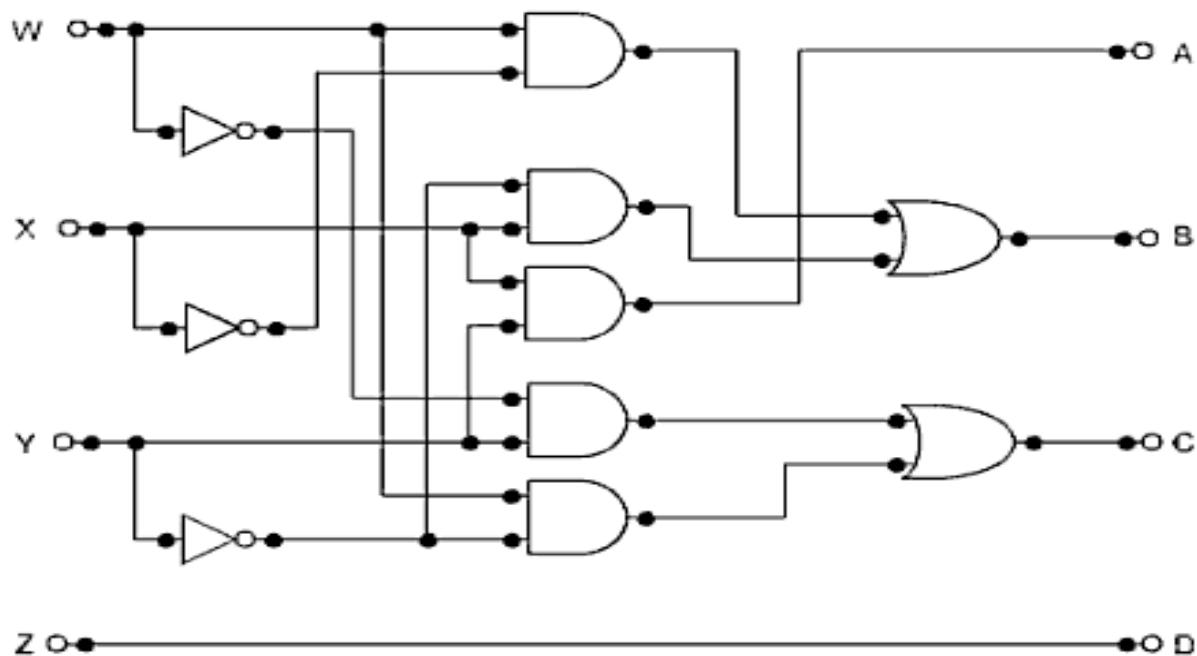
$$A = XY$$

$$B = XY' + WX'$$

$$C = W'Y + WY'$$

$$D = Z$$

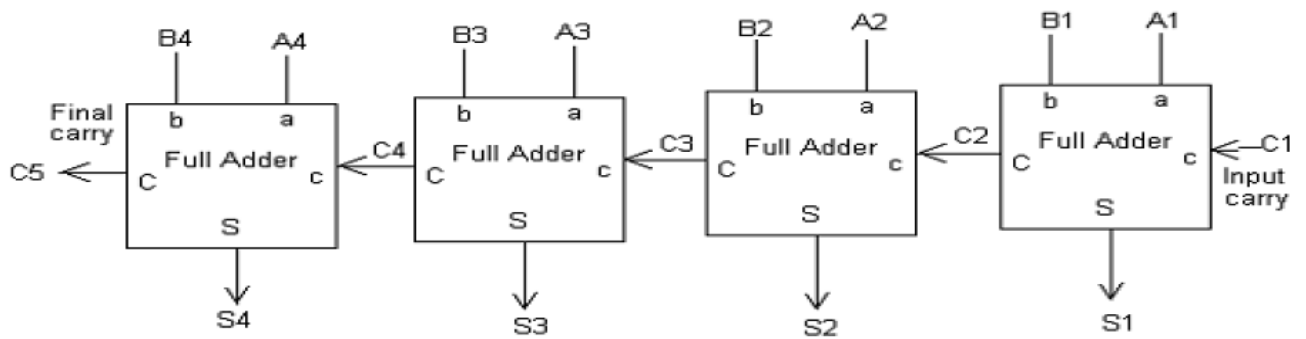
LOGIC DIAGRAM:



9.Design a Binary Adder and Subtractor.

Binary Adder:

In this adder we need n full adders for n bit adder. In this adder we use the n full adders in cascaded from to implement the ripple carry adder. This type of adder is also called carry propagation adder. The circuit for 4-bit parallel adder is as follow:



For example:

To add $A = 1011$ and $B = 0011$

Subscript i: 3 2 1 0

Input carry: 0 1 1 0 C_i

Augend: 1 0 1 1 A_i

Addend: 0 0 1 1 B_i

Sum: 1 1 1 0 S_i

Output carry: 0 0 1 1 C_{i+1}

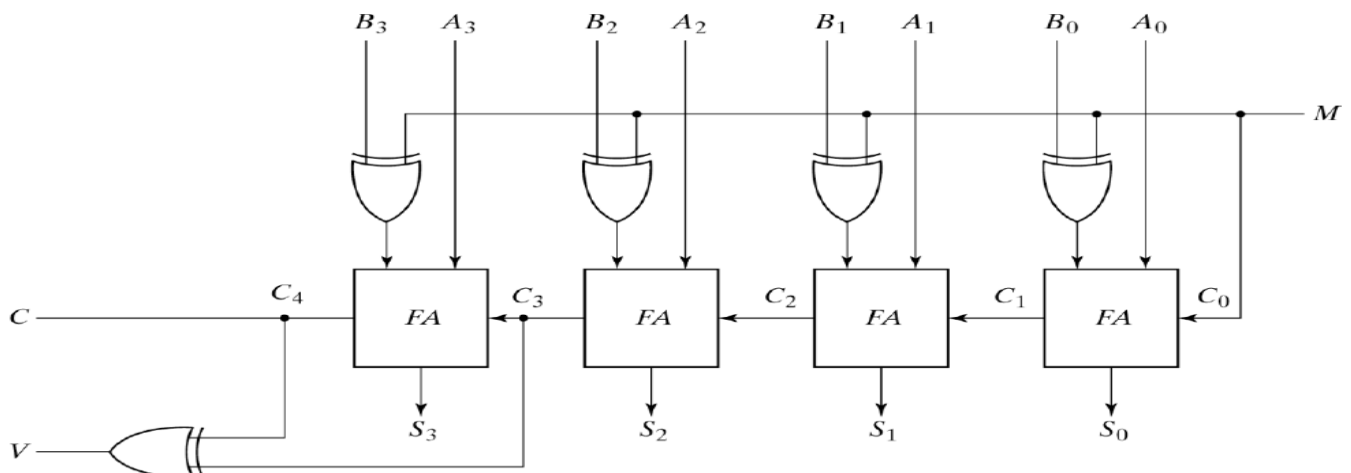
Binary Subtractor:

The subtraction $A - B$ can be done by taking the 2's complement of B and adding it to A because $A - B = A + (-B)$. It means if we use the inverters to make 1's complement of B (connecting each B_i to an inverter) and then add 1 to the least significant bit (by setting carry C_0 to 1) of binary adder, then we can make a binary subtractor.

Binary Adder Subtractor:

- The addition and subtraction can be combined into one circuit with one common binary adder (see next slide).

- The mode M controls the operation. When $M=0$ the circuit is an adder when $M=1$ the circuit is subtractor. It can be done by using exclusive-OR for each B_i and M . Note that $1 \oplus x = x'$ and $0 \oplus x = x$.

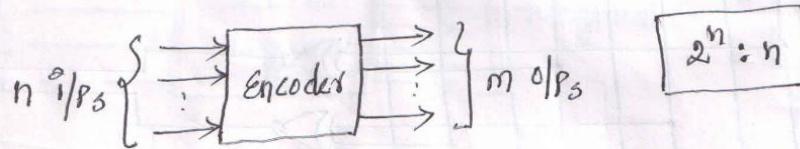


10. Design a combinational circuit for ENCODER and DECODER.

Encoder:-

↳ An encoder is a digital circuit that performs the inverse operation of a decoder.

↳ An encoder is a combinational logic circuit that converts an active input signal into a coded output signal.



Octal to binary Encoder (8:3)

It accepts eight inputs and produces 3-bit output code corresponding to the activated input.

Truth Table:-

Inputs								Outputs		
D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	1	0	0	0	1	0	0
0	0	0	0	0	1	0	0	1	0	1
0	0	0	0	0	0	1	0	1	1	0
0	0	0	0	0	0	0	1	1	1	1

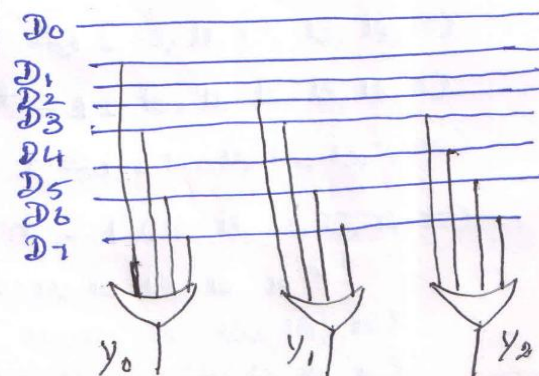
Boolean expression

$$Y_0 = D_1 + D_3 + D_5 + D_7$$

$$Y_1 = D_2 + D_3 + D_6 + D_7$$

$$Y_2 = D_4 + D_5 + D_6 + D_7$$

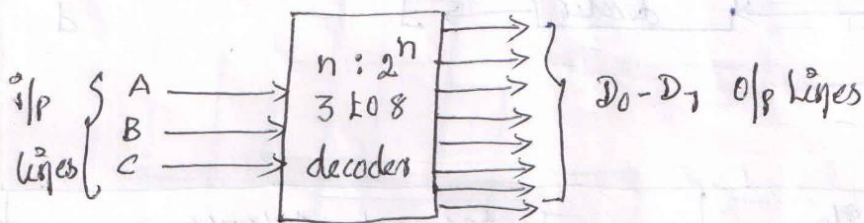
Logic diagram:-



Decoder:-

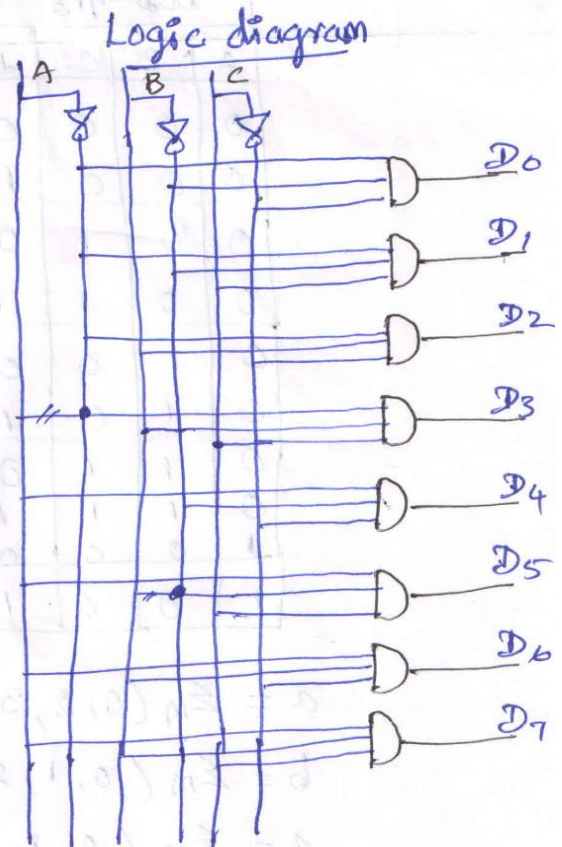
→ A decoder is a logic circuit that converts an n -bit binary input code (data) into 2^n output lines, such that each output line will be activated for only one of the possible combinations of inputs.

3 to 8 Decoder:-



Truth Table:-

Inputs			Outputs							
A	B	C	D_7	D_6	D_5	D_4	D_3	D_2	D_1	D_0
0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	1	0
0	1	0	0	0	0	0	0	1	0	0
0	1	1	0	0	0	0	1	0	0	0
1	0	0	0	0	0	1	0	0	0	0
1	0	1	0	0	1	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0



Boolean Expression:-

$$D_0 = \bar{A}\bar{B}\bar{C}$$

$$D_1 = \bar{A}\bar{B}C$$

$$D_2 = \bar{A}B\bar{C}$$

$$D_3 = \bar{A}BC$$

$$D_4 = AB\bar{C}$$

$$D_5 = ABC$$

$$D_6 = AB\bar{C}$$

$$D_7 = ABC$$

11. Design a combinational circuit that converts a decimal digit from BCD to Excess 3 code.

BCD to Excess-3

Excess-3 code is a modified form of a BCD number. The Excess-3 code can be derived from the natural BCD code by adding 3 to each coded number. For example, decimal 12 can be represented in BCD as 0001 0010. Now adding 3 to each digit we get Excess-3 code as 0100 0101 (12 in decimal). With this information the truth table for BCD to Excess-3 code converter

Decimal	B ₃	B ₂	B ₁	B ₀	E ₃	E ₂	E ₁	E ₀
0	0	0	0	0	0	0	1	1
1	0	0	0	1	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	1	0	1	1	0
4	0	1	0	0	0	1	1	1
5	0	1	0	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

K-map simplification

For E₃

B ₃ B ₂ \ B ₁ B ₀	00	01	11	10
00	0	0	0	0
01	0	1	1	1
11	X	X	X	X
10	1	1	X	X

$$\therefore E_3 = B_3 + B_2(B_0 + B_1)$$

For E₂

B ₃ B ₂ \ B ₁ B ₀	00	01	11	10
00	0	1	1	1
01	1	0	0	0
11	X	X	X	X
10	0	1	X	X

$$\therefore E_2 = B_2 \bar{B}_1 \bar{B}_0 + \bar{B}_2(B_0 + B_1)$$

For E₁

B ₃ B ₂ \ B ₁ B ₀	00	01	11	10
00	1	0	1	0
01	1	0	1	0
11	X	X	X	X
10	1	0	X	X

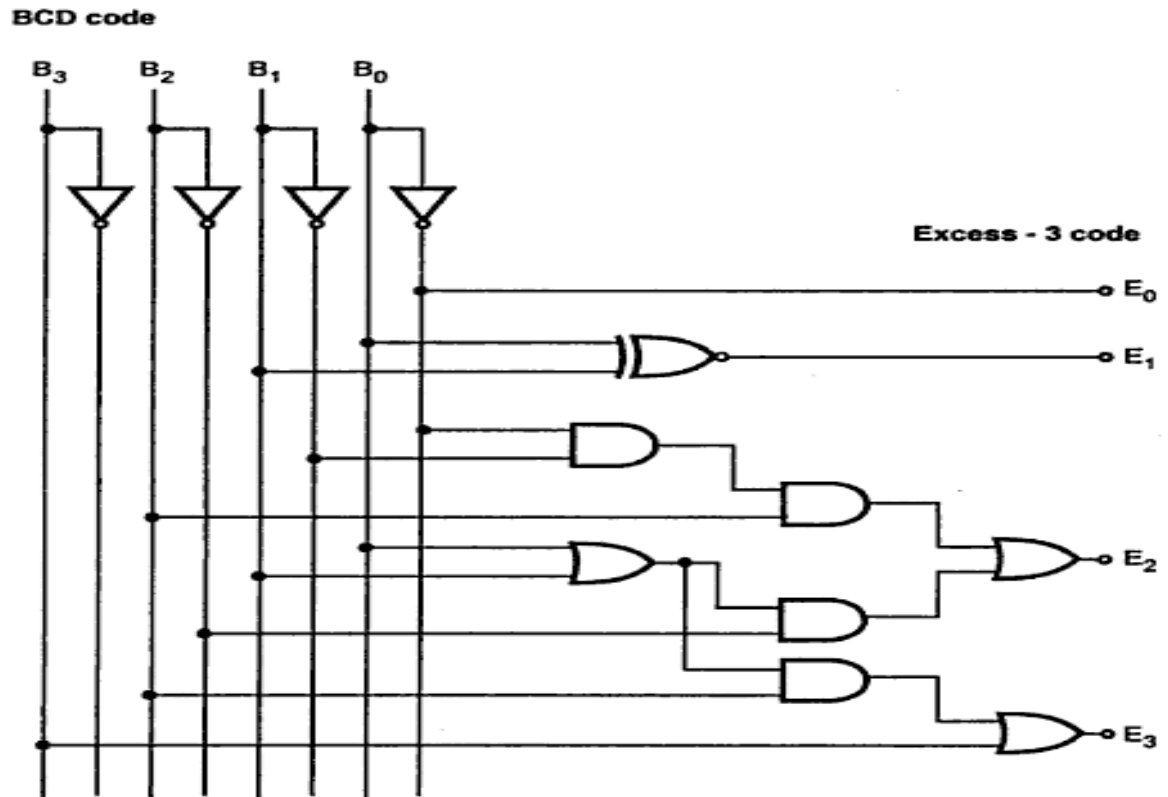
$$E_1 = \bar{B}_1 \bar{B}_0 + B_1 B_0 \\ = B_1 \odot B_0$$

For E₀

B ₃ B ₂ \ B ₁ B ₀	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	X	X	X	X
10	1	0	X	X

$$E_0 = \bar{B}_0$$

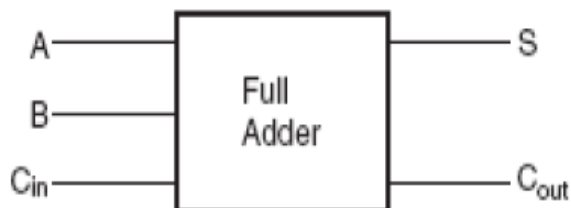
Logic diagram



12.Design a combinational circuit of Full adder& Subtractor.

A full adder circuit is an arithmetic circuit block that can be used to add three bits to produce a SUM and a CARRY output. Such a building block becomes a necessity when it comes to adding binary numbers with a large number of bits. The full adder circuit overcomes the limitation of the half-adder, which can be used to add two bits only. Let us recall the procedure for adding larger binary numbers. We begin with the addition of LSBs of the two numbers. We record the sum under the LSB column and take the carry, if any, forward to the next higher column bits. As a result, when we add the next adjacent higher column bits, we would be required to add three bits if there were a carry from the previous addition. We have a similar situation for the other higher column bits. Also until we reach the MSB. A full adder is therefore essential for the hardware implementation of an adder circuit capable of adding larger binary numbers. A half-adder can be used for addition of LSBs only.

SYMBOL:



TRUTH TABLE:

A	B	C _{in}	SUM (S)	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Figure shows the truth table of a full adder circuit showing all possible input combinations and corresponding outputs. In order to arrive at the logic circuit for hardware implementation of a full adder, we will firstly write the Boolean expressions for the two output variables, that is, the SUM and CARRY outputs, in terms of input variables. These expressions are then simplified by using any of the simplification techniques described in the previous chapter. The Boolean expressions for the two output variables are given in Equation below for the SUM output (S) and in above Equation for the CARRY output (Cout):

$$S = \bar{A}.\bar{B}.C_{in} + \bar{A}.B.\bar{C}_{in} + A.\bar{B}.\bar{C}_{in} + A.B.C_{in}$$

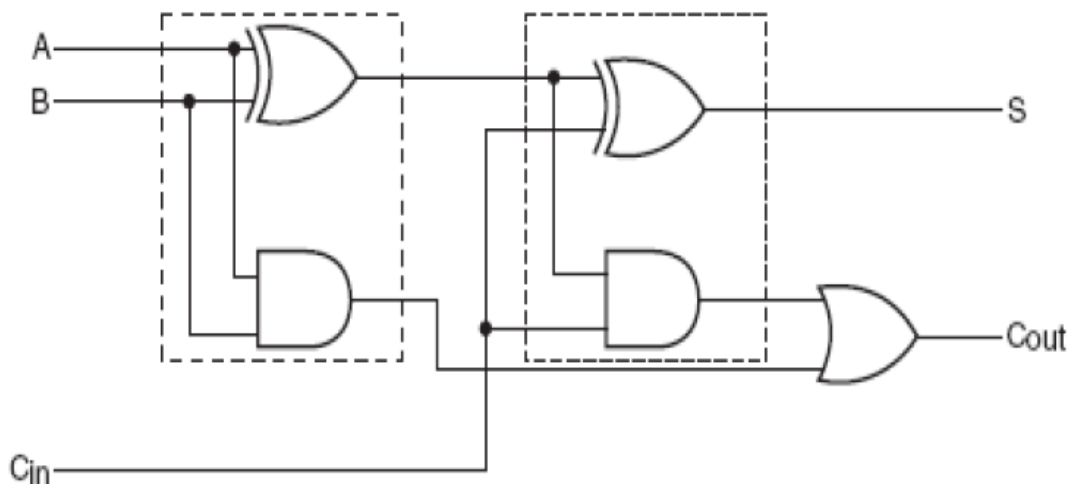
$$C_{out} = \bar{A}.B.C_{in} + A.\bar{B}.C_{in} + A.B.\bar{C}_{in} + A.B.C_{in}$$

The next step is to simplify the two expressions.

$$S = \bar{C}_{in}.(\bar{A}.B + A.\bar{B}) + C_{in}.(\bar{\bar{A}.B + A.\bar{B}})$$

$$C_{out} = A.B + C_{in}.(\bar{A}.B + A.\bar{B})$$

LOGIC DIAGRAM:



FULL SUBTRACTOR:

A combinational circuit of full-subtractor performs the operation of subtraction of three bits—the minuend, subtrahend, and borrow generated from the subtraction operation of previous significant digits and produces the outputs difference and borrow. Let us designate the input variables minuend as X, subtrahend as Y, and previous borrow as Z, and outputs difference as D and borrow as B. Eight different input combinations are possible for three input variables. The truth table is shown in Figure 5.10(a) according to its functions.

Input variables			Outputs	
X	Y	Z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

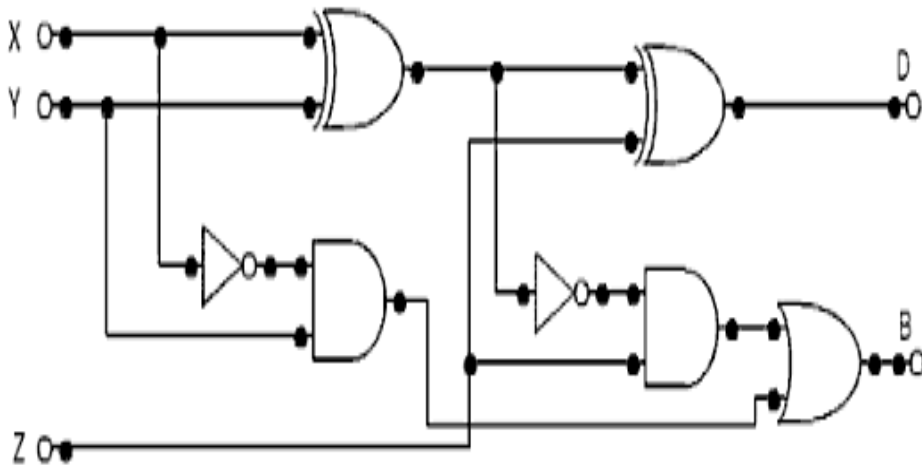
	Y'Z'	Y'Z	YZ	YZ'
X'		1		1
X	1		1	

	Y'Z'	Y'Z	YZ	YZ'
X'		1	1	1
X			1	

$$\begin{aligned}
 D &= X'Y'Z + X'YZ' + XY'Z' + XYZ \\
 &= X' (Y'Z + YZ') + X (Y'Z' + YZ) \\
 &= X' (Y \oplus Z) + X (Y \oplus Z)' \\
 &= X \oplus Y \oplus Z
 \end{aligned}$$

$$\begin{aligned}
 B &= X'Z + X'Y + YZ = X'Y + Z (X' + Y) \\
 &= X'Y + Z(X'Y + X'Y' + XY + X'Y) \\
 &= X'Y + Z(X'Y + X'Y' + XY) \\
 &= X'Y + X'YZ + Z(X'Y' + XY) \\
 &= X'Y + Z(X \oplus Y)'
 \end{aligned}$$

LOGIC DIAGRAM:



13. Design a combinational circuit to perform BCD addition.

A BCD adder adds two BCD digits and produces a BCD digit. If the sum of BCD digits is less than or equal to 9, then it is a valid BCD form. If the sum is greater than 9, then 6 (six) i.e. (0110)₂ is added to the sum to make it valid.

Let us 4-bit binary adder A₀ A₁ A₂ A₃ and B₀ B₁ B₂ B₃ produces sum S₀ S₁ S₂ S₃. A combinational circuit is there to check if the sum is 9 or more than 9 so that six can be added to it. For the combinational circuit, the truth-table is as:

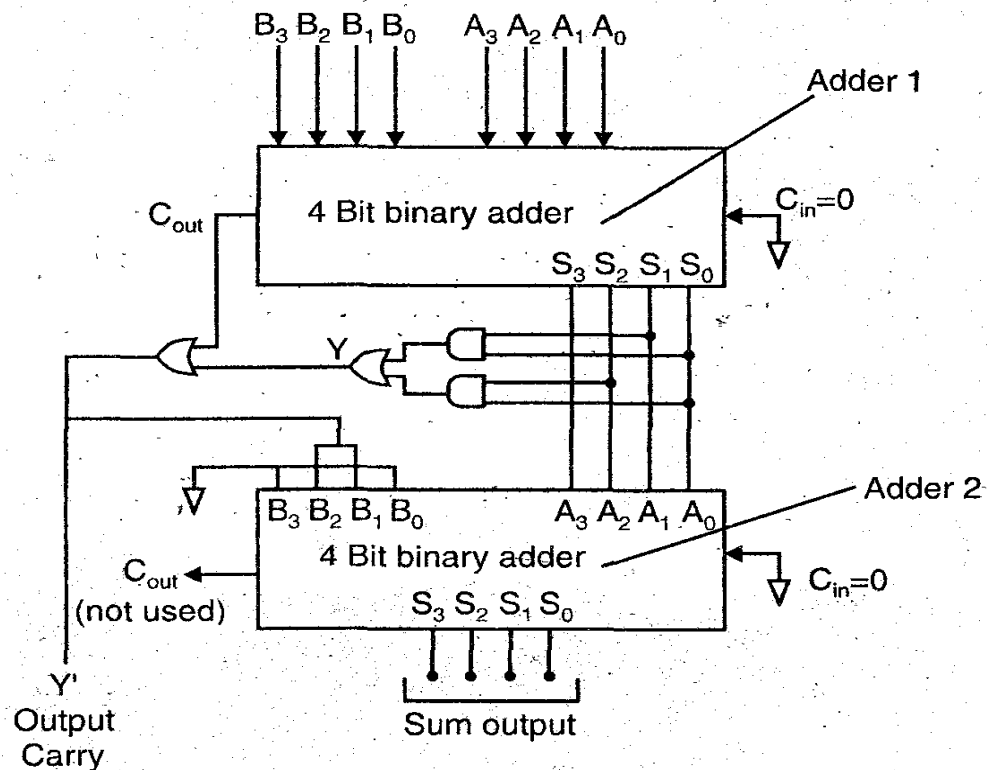
S ₃	S ₂	S ₁	S ₀	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

K-map

S ₀ S ₁		S ₂ S ₃			
		00	01	11	10
S ₀ S ₁	00	0 ₀	0 ₄	1 ₁₂	0 ₈
	01	0 ₁	0 ₅	1 ₁₃	0 ₉
S ₀ S ₁	11	0 ₃	0 ₇	1 ₁₅	1 ₁₁
	10	0 ₂	0 ₆	1 ₁₄	1 ₁₀

$$Y = S_0S_2 + S_0S_1$$

BCD circuit diagram:



We will get correct BCD sum from output of Adder-2. Which will be in BCD form.

14. (i) what are Magnitude comparators” Explain the design of magnitude comparators with the help of a suitable example

A comparator is a logic circuit used to compare magnitude to two binary numbers or more The result will be, either bits will be equal greater or lesser

SYMBOL:



TRUTH TABLE:

A	B	A > B	A = B	A < B
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

K-MAP:

	B	\bar{B}	B
A			
\bar{A}	0	0	
A	1	0	

$$A > B \Rightarrow AB$$

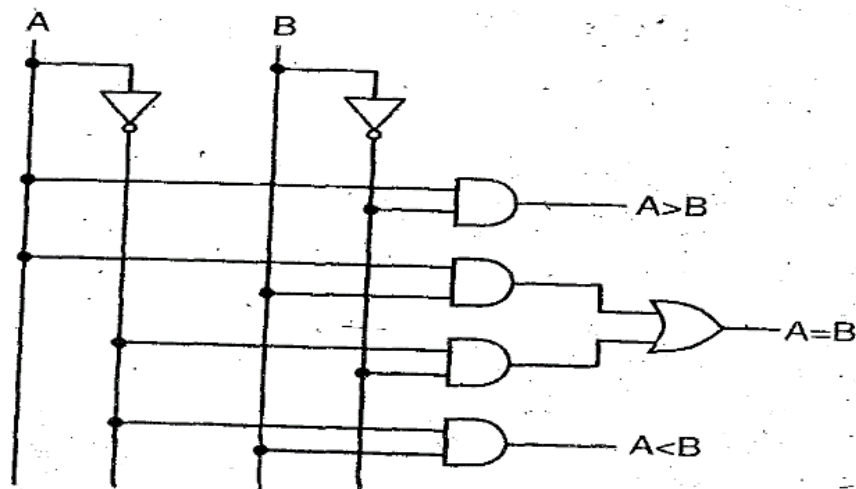
	B	\bar{B}	B
A			
\bar{A}	1	0	
A	0	1	

$$A = B \Rightarrow \bar{A}\bar{B} + AB$$

	B	\bar{B}	B
A			
\bar{A}	0	1	
A	0	0	

$$A < B \Rightarrow \bar{A}B$$

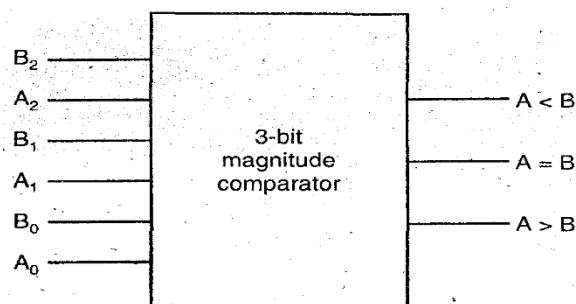
LOGIC DIAGRAM:



Similarly, 7485 is TTL 4 bit magnitude comparator. These inputs can be extended to compare more than 4 bits.

14. (ii) Write note on 3 bit binary magnitude comparator.

SYMBOL:



Where, $A = A_2 A_1 A_0$ having three bits and

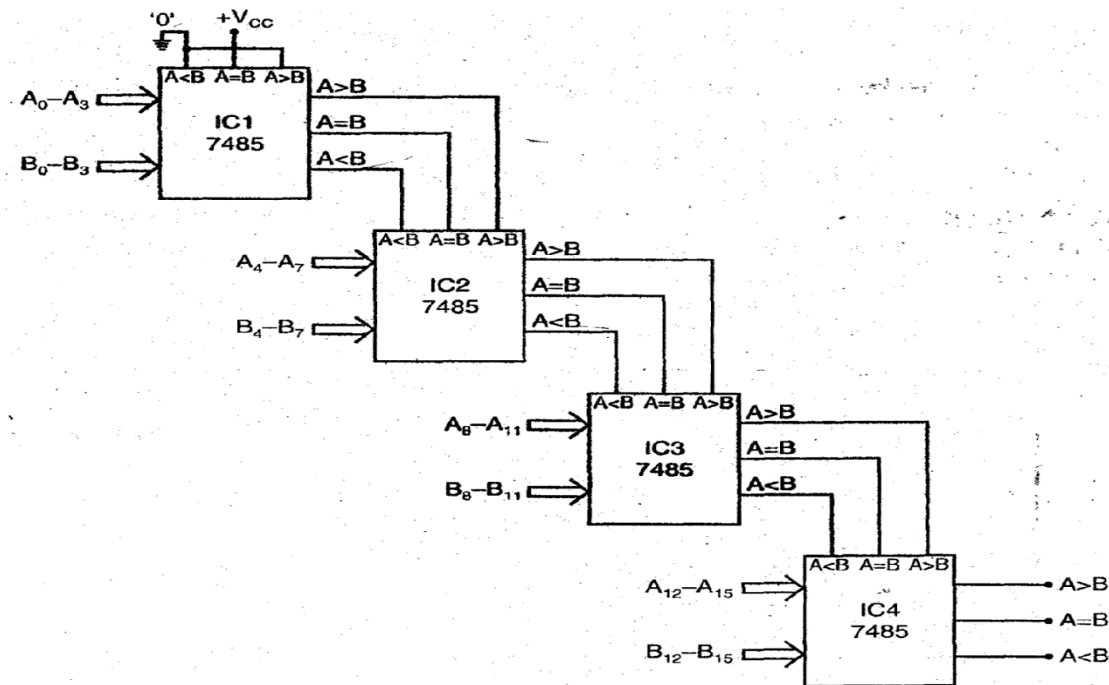
$B = B_2 B_1 B_0$ also having three bits.

$A_2 A_1 A_0$ are compared with another three bits i.e. $B_2 B_1 B_0$

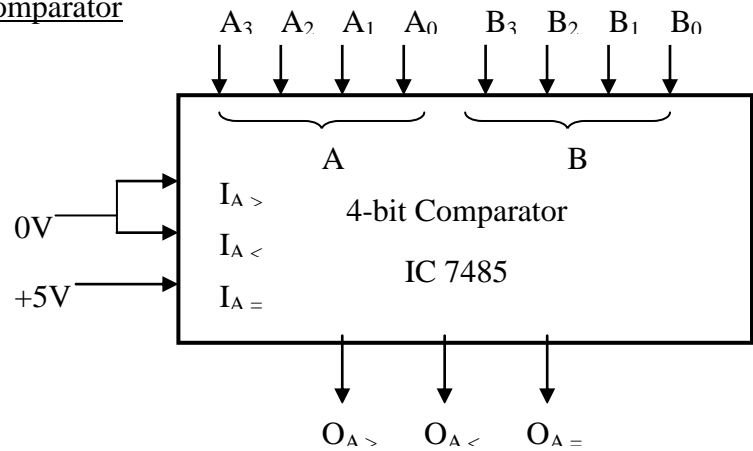
Total combinations are possible. As $2^6 = 64$. Thus, six variable k-maps are required to solve for $A > B$, $A = B$ and $A < B$.

14. (iii) Construct 16-bit comparator using 4-bit comparator as a building block.

4 bit comparator IC in 7485. It is used for 16 bit comparator. Thus, 4 IC'S are used.



Logic diagram of a 4-bit comparator



15. Discuss the need and working principle of Carry Look ahead adder.

Carry Look ahead adder (Fast adder)

The parallel adder is ripple carry type in which the carry output of each full adder stage is connected to the carry i/p to the next higher-order stage. Therefore, the sum and carry outputs of any stage cannot be produced until the input carry occurs, this leads to a time delay in the addition process.

eg.

$$\begin{array}{r} 0101 \\ + 0011 \\ \hline 1000 \end{array}$$

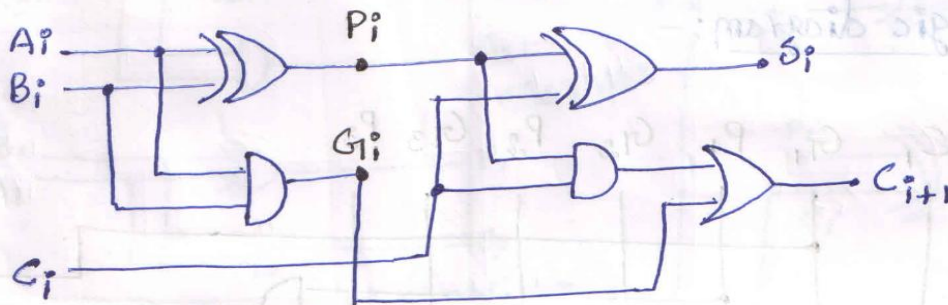
\Rightarrow carry i/p of next MSB

↳ A Method of increasing the speed of this process by eliminating inter stage carry delay is called look ahead carry-addition.

↳ It uses 2 functions such as

① carry generate, G_i

② carry propagate, P_i



⇓

$P_i = A_i \oplus B_i \Rightarrow$ carry propagate } 1st stage

$G_i = A_i B_i \Rightarrow$ carry generate }

The o/p sum and carry can be expressed as,

$$S_i = P_i \oplus C_i$$

$$C_{i+1} = G_i + P_i C_i$$

Boolean function:-

$$i=0, \quad C_{0+1} = G_0 + P_0 C_0 \quad \rightarrow \text{stage ①}$$

$$\therefore C_1 = G_0 + P_0 C_0$$

$$i=1, \quad C_{1+1} = G_1 + P_1 C_1$$

$$\therefore C_2 = G_1 + P_1 C_1$$

$$= G_1 + P_1 [G_0 + P_0 C_0]$$

$$= G_1 + P_1 G_0 + P_1 P_0 C_0 \rightarrow \text{②}$$

$$i=2,$$

$$C_{2+1} = G_2 + P_2 C_2$$

$$C_3 = G_2 + P_2 [G_1 + P_1 C_1]$$

$$\therefore C_3 = G_2 + G_1 P_2 + P_1 P_2 C_1 \rightarrow (3)$$

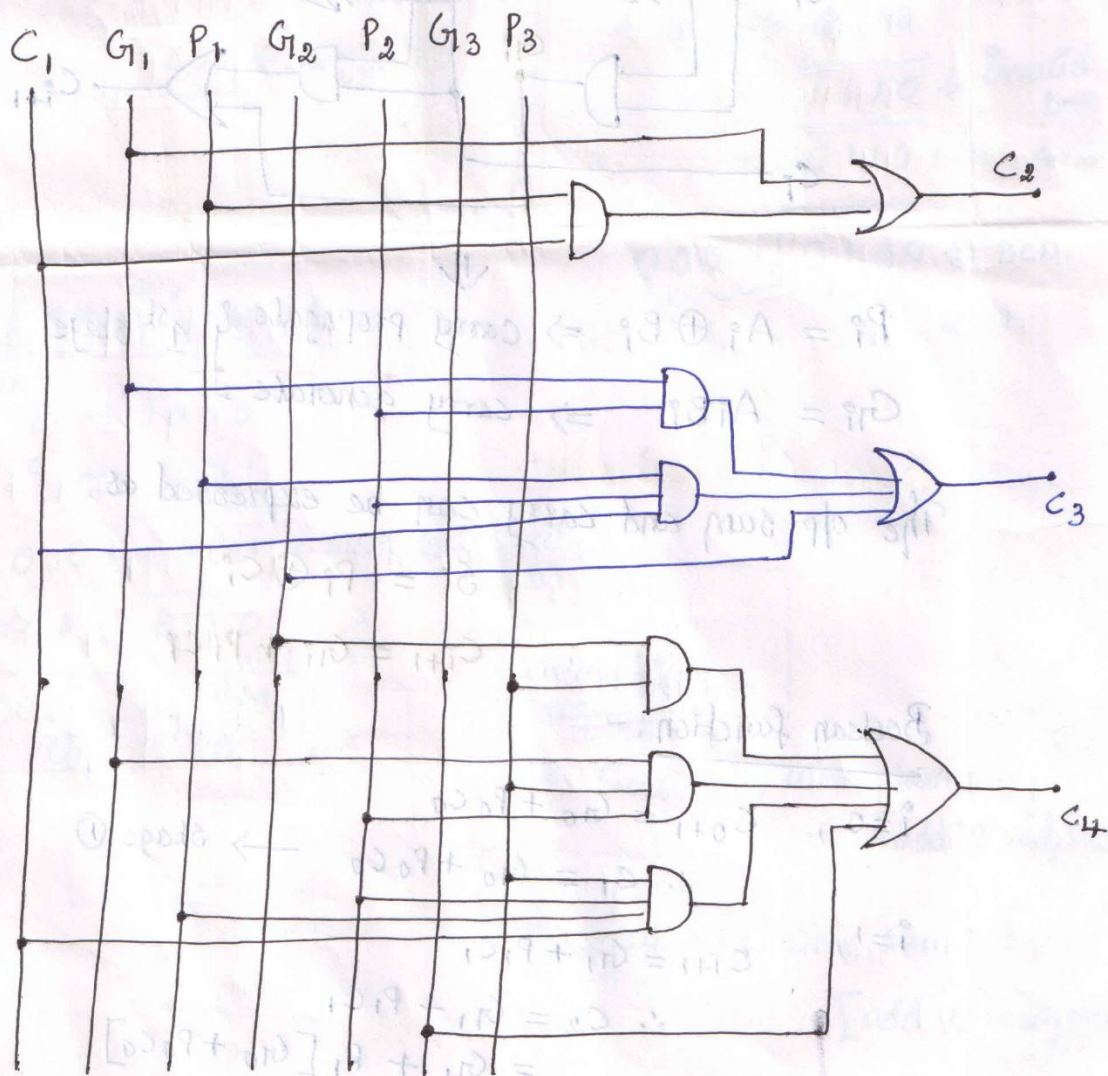
$$i=3,$$

$$C_{3+1} = G_3 + P_3 C_3$$

$$C_4 = G_3 + P_3 [G_2 + G_1 P_2 + P_1 P_2 C_1]$$

$$= G_3 + G_2 P_3 + G_1 P_3 P_2 + P_3 P_2 P_1 C_1 \rightarrow (4)$$

Logic diagram:-



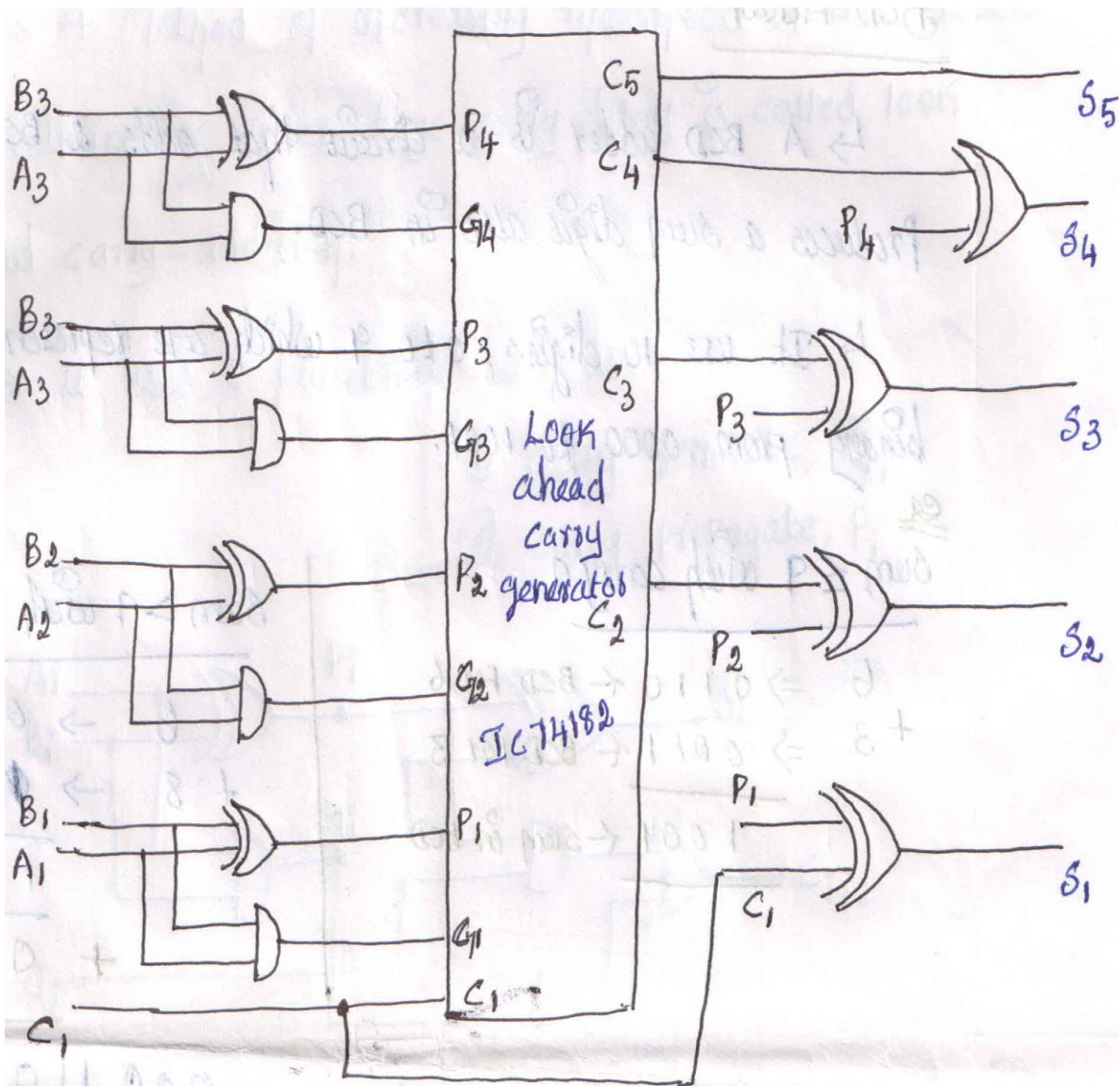


Fig. 4-bit parallel adder with look ahead carry generator