

UNIT – I**SIGNALS AND SYSTEMS****PART – A****1. What is DSP? (May 2014)**

DSP is defined as changing or analyzing information which discrete sequences of numbers.

2. What are the advantages and limitations of digital signal processing? (Nov 2014)

The digital signal processing systems has many advantages. Even though there are certain disadvantages as follows:

1. Bandwidth limitations: In case of DSP, if input signal is having wide bandwidth then it demands for high speed ADC. This is because to avoid aliasing effect, the sampling rate should be atleast twice the bandwidth. Thus such signals require fast digital signal processors. But always there is a practical limitation in the speed of processors and ADC.
2. System complexity: The digital signal processing system makes use of converters like ADC and DAC. This increases the system complexity compared to analog systems. Similarly in many applications the time required for this conversion is more.
3. Power Consumption: A typical digital signal processing chip contains more than 4 lakh transistors. Thus power dissipation is more in caps systems compared to analog systems.
4. DSP systems are expensive as compared to analog system.

Advantages of digital Signal processing: 1. Digital programmable systems are reconfigurable.

2. DSP provides better accuracy. 3. Digital signals can be easily stored. 4. DSP systems are cheaper.

3. What are the applications of DSP?

1. Image processing like pattern recognition, animation, robotic vision, image enhancement.
2. Instrumentation and control like spectral analysis, noise reduction, data compression.
3. Speech/Audio like speech recognition, speech synthesis, equalization.
4. Biomedical like scanners ECG analysis, patient monitoring
5. Telecommunication like in echo cancellation, spread spectrum and data communication.
6. Military like Sonar processing, radar processing, secure communication.
7. Consumer applications like digital audio and video, power like monitor.
8. Automotive applications like vibration analysis, voice commands and cellular telephones.
9. Industrial applications like robotics and CNC, power line monitors.

4. Write the major classification of signals?

There are various types of signals. Every signal is having its own characteristic The processing of signal mainly depends on the characteristics of that particular signal So classification of signal is necessary Broadly the signal are classified as follows: 1. Continuous and discrete time signals.

2. Continuous valued and discrete valued signals. 3. Periodic and non periodic signals. 4. Even and odd signals. 5. Energy and power signals. 6. Deterministic and random signals. 7. Multichannel and multidimensional signals.

5. What are energy and power signals?

Ans: The energy E of a signal $x(n)$ is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The energy of a signal can be finite or infinite. If E is finite i.e $0 < E < \infty$ then $x(n)$

is called an energy signal. Many signals that possess infinite energy, have a finite average power. The average Power of a discrete time signal $x(n)$ is defined as

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

If E is finite, $P = 0$. On the other hand, If E is infinite; the average power may be either finite or infinite. If P is finite (and non zero), the signal is called a power signal.

6. Differentiate: Linear and Nonlinear systems.

A system is called linear, if superposition principle applies to that system. This means that linear system may be defined as one whose response to the sum of the weighted inputs is same as the sum of the weighted responses.

Linearity property for discrete time systems may be written as:

$$\mathcal{L}[a_1 x_1(n) + a_2 x_2(n)] \rightarrow a_1 y_1(n) + a_2 y_2(n)$$

For any non-linear system, the principle of super-position does not hold true and the above are not satisfied. Few examples of linear system are filters, communication channels etc.

7. State the necessary and sufficient condition for stability of LTI systems

LTI system is stable if its impulse response is absolutely summable i.e.

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Here $h(k) = h(n)$ is the impulse response of LTI system. Thus, the above equation gives the condition of stability in terms of impulse response of the system.

8. What is the causality condition for an LTI system?

The necessary and sufficient condition for causality of an LTI system is, its unit sample response $h(n) = 0$ for negative values of n i.e., $h(n) = 0$ for $n < 0$

9. State sampling theorem. (Nov/Dec 2013)

A continuous time signal $x(t)$ can be completely represented in its sampled form and recovered back from the sample form if the sampling frequency

$$f_s \geq 2\omega$$

where ' ω ' is the maximum frequency of the continuous time signal $x(t)$

10. Convolve {1,3,1} and {1,2,2}

$$x(n) = \{1, 3, 1\} \quad h(n) = \{1, 2, 2\}$$

	$x(0)$	$x(1)$	$x(2)$
$h(0)$	1	3	1
$h(1)$	2	6	2
$h(2)$	2	6	2

Range of n is
and

$$y_l = x_l + h_l = 0 + 0 = 0$$

$$y_h = x_h + h_h = 2 + 2 = 4$$

$$y(0) = h(0) x(0) = 1$$

$$y(1) = h(1) x(0) + h(0) x(1) \\ = 2 + 3 = 5$$

$$y(2) = h(2) x(0) + h(1) x(1) + h(0) x(2) \\ = 2 + 6 + 1 = 9$$

$$y(3) = h(2) x(1) + h(1) x(2) \\ = 6 + 2 = 8$$

$$y(4) = h(2) x(2) = 2$$

$$y(n) = \{1, 5, 9, 8, 2\}$$

$y(n)$ is output of the convolution.

11. Differentiate time variant from time invariant system. (May 2014)

A system is called time invariant if its input output characteristics do not change with time. A LTI discrete time system satisfies both the linearity and the time invariance properties. To test if any given system is time invariant, first apply an arbitrary sequence $x(n)$ and find $y(n)$.

$$y(n) = T[x(n)]$$

Now delay the input sequence by k samples and find output sequence denote it as $y(n, k) = T[x(n-k)]$

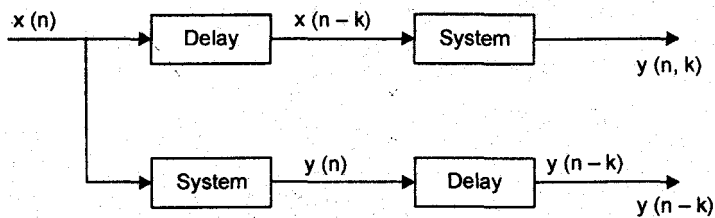
Delay the output sequence by k samples denote it as

$$y(n, k) = y(n-k)$$

For all possible values of k , the system is time invariant. on the other hand

$$y(n, k) \neq y(n-k)$$

Even for one value of k , the system is time variant.

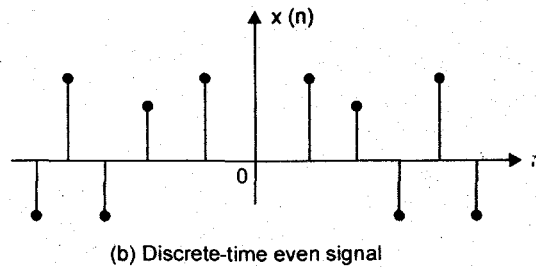


12. What are symmetric and asymmetric signals?

An even signal is that type of signal which exhibits symmetry in the time domain. This type of signal is identical about the origin. Mathematically, an even signal must satisfy the following condition.

For a discrete-time signal, $x[n] = x[-n]$

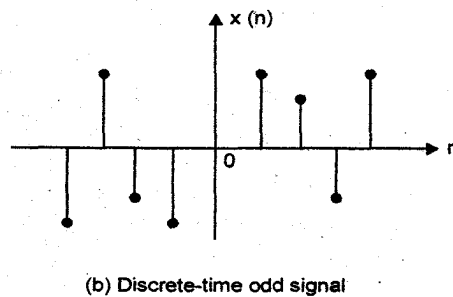
Figure shows continuous-time and discrete-time even signals.



Similarly, an odd signal is that type of signal which exhibits anti-symmetry. This type of signal is not identical about the origin actually, the signal is identical to its negative mathematically, and an odd signal must satisfy the following condition

For a discrete-time signal, $x[n] = -x[-n]$

Figure shows continuous-time and discrete-time odd signals.



13. Determine the power and energy of the unit step sequence.

The average power of the unit step signal is

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N |u(n)|^2$$

$$= \lim_{N \rightarrow \infty} \frac{N+1}{2N+1} = \lim_{N \rightarrow \infty} \frac{1+1/N}{2+1/N} = \frac{1}{2}$$

Consequently, the unit step sequence is a power signal. Its energy is infinite.

14. Determine if the system $y[n] = x[-n]$ is time variant or time invariant.

The system is described the input output relation

$$y[n] = T[x[n]] = x[-n]$$

The response of the system to $x[n-k]$ is

$$y[n, k] = T[x[n-k]] = x[-n-k]$$

Now if we delay the output $y[n]$, by k units in time, the result will be

$$y[n-k] = x[-n+k]$$

Since $y[n, k] \neq y[n-k]$ the system is time variant

15. Determine if the system described by the following input-output equation are linear or

nonlinear $y(n) = x(n^2)$

Ans: For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$y_1(n) = x_1(n^2)$$

$$y_2(n) = x_2(n^2)$$

The output of the system to a linear combination of $x_1(n)$ and $x_2(n)$ is

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)] \\ = a_1 x_1(n^2) + a_2 x_2(n^2) \quad (1)$$

Finally a linear combination of the two outputs

$$a_1 x_1(n) + a_2 x_2(n) = a_1 x_1(n^2) + a_2 x_2(n^2) \quad (2)$$

By comparing (1) with (2), we can conclude that the system is linear.

16. Determine the system $y(n) = x(2n)$ is causal or non causal system.

$$y(n) = x(2n)$$

Put $n=0,1,2$

$$y(0) = x(0)$$

$$y(1) = x(2)$$

$$y(2) = x(4)$$

For output, need of advance value of input, so $y(n)$ is non-causal system.

17. Consider a system with impulse response $h(n) = 3^{-n}u(n)$. Determine whether the system is stable or unstable.

$$S = \sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} 3^{-n} u(n) \\ \sum_{n=-\infty}^{\infty} 3^{-n} u(n) = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2} < \infty$$

Since $S < \infty$, the system is stable.

18. What is Region of convergence?

The z-transform is an infinite power series, it exists only for those values of z for which the series converges. The region of convergence (ROC) of $X(z)$ is set of all values of z for which $X(z)$ attain a finite value. The ROC of a finite duration signal is the entire z -plane, except possibly the point $z=0$ and $z=\infty$.

19. What are the conditions for the region of convergence of a non causal LTI system?

The condition for non-causal discrete time LTI system is that the impulse response of a causal discrete time LTI system is given as $h(n) \neq 0$, for $n < 0$. This means that $h(n)$ is two sided. The ROC of $H(z)$ of non-causal discrete time LTI system is the entire z -plane except $|z| = \infty$.

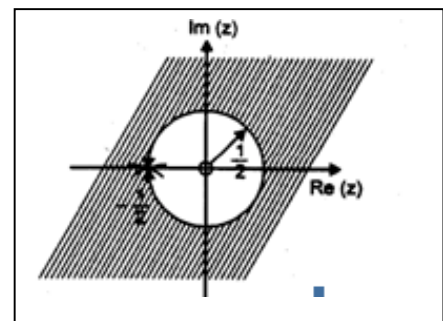
20. Determine to z-transform of the following signal and sketch the pole zero pattern:

$$x(n) = (-1)^n (2)^{-n} u(n)$$

$$\text{Ans: } X(z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (-1)^n (2)^{-n} u(n) Z^{-n}$$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(-\frac{1}{2}\right)^n u(n) Z^{-n} = \frac{Z}{Z + \frac{1}{2}} \text{ with ROC } |z| > \frac{1}{2}$$



21. Find the z-transforms of $x(n) = \delta(n)$.

$$\begin{aligned}
 X(z) &= \sum_{n=-\infty}^{\infty} x(n)z^{-n} = \sum_{n=-\infty}^{\infty} \delta(n)z^{-n} \\
 &= \dots + \delta(-1)z^1 + \delta(0)z^0 + \delta(1)z^{-1} + \dots \\
 &= 1
 \end{aligned}$$

The ROC is entire z-plane.

22. List the properties of discrete time sinusoidal signals. (May/June 2013)

- A discrete time sinusoidal is periodic only if its frequency is a rational Number.
- Discrete time sinusoidal whose frequencies are separated by an integer multiple of 2π are identical
- The highest rate of oscillation in a discrete time sinusoidal is attained when $\omega = \pi$

23. What is correlation? What are its types? (May/June 2013)

It is a measure of similarity between two signals.

- Cross correlation - similarity between different signals
- Autocorrelation - similarity between time shifted version of same signal

24. State and prove convolution property of z transform? (Nov/Dec 2013)

The convolution property for Z-transforms is very important for systems analysis and design. The transform of the convolution is the product of the transforms.

i.e. $z[f_1(k) * f_2(k)] = F_1(z)F_2(z)$

$$\begin{aligned}
 Z[f_1[k] * f_2[k]] &= Z\left[\sum_{m=-\infty}^{\infty} f_1[m]f_2[k-m]\right] \\
 &= \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} f_1[m]f_2[k-m]z^{-k} \\
 &= \sum_{m=-\infty}^{\infty} f_1[m] \sum_{k=-\infty}^{\infty} f_2[k-m]z^{-k} \\
 &= \sum_{m=-\infty}^{\infty} f_1[m] \sum_{r=-\infty}^{\infty} f_2[r]z^{-(r+m)} \\
 &= \sum_{m=-\infty}^{\infty} f_1[m]z^{-m} \sum_{r=-\infty}^{\infty} f_2[r]z^{-r} = F_1[z] F_2[z].
 \end{aligned}$$

25. Determine whether the following sinusoids are periodic. If periodic, then compute their fundamental period. (Nov/Dec 2014)

(a) $\cos(0.01\pi n)$ (b) $\sin(62\pi n/10)$

(a) $\cos(0.01\pi n)$

$$2\pi f = \pi 0.01$$

$$f = 0.01/2 = 1/200 \quad (\text{since } f = K/N)$$

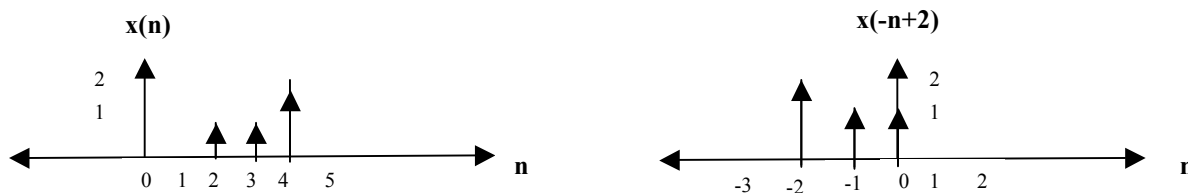
It is periodic. Fundamental Period, $N = 200$ hz

(b) $\sin(62\pi n/10)$

$$2\pi f = 62\pi/10$$

$f = 31/10$, which is not rational. Therefore it is not periodic.

26. A discrete time signal $x(n) = \{0, 0, 1, 1, 2, 0, 0, \dots\}$. Sketch the $x(n)$ and $x(-n+2)$ signals. (Nov/Dec 2014)



27. What do you understand by the term signal processing? (May/June 2014)

Signal processing is performing some operations like amplification, filtration to signal and getting a signal output.

PART-B

1. Find the even and odd parts of the function $g[n] = u[n] - u[n-4]$.
2. Compute convolution of $y(n)$ of the signals.

$$X(n) = \begin{cases} \alpha^n, & -3 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

3. Determine the output $y(n)$ of linear time invariant system units impulse response, $h(n) = a^n U(n)$, $|a| < 1$ when the input is a unit step sequence $x(n) = U(n)$.
4. An LTI system is described by $y(n) = y(n-1) - 0.24 y(n-2) + x(n)$. Find the response of this system for an input of $x(n) = 10 \cos(0.053n)$.
5. Define convolution theorem as applied to discrete time signals. Find the inverse z-transform of $X(z) = z / (z-1)^2$ using convolution theorem.
6. What are the various realization techniques of linear time invariant systems? Mention.
7. Determine the impulse response for the cascade of two linear time invariant systems having impulse responses.

$$h_1(n) = \left(\frac{1}{2}\right)^n U(n)$$

$$h_2(n) = \left(\frac{1}{4}\right)^n U(n)$$

8. List various properties of z-transform with proof
9. Determine the z-transform of the signal

$$x(n) = (\cos \omega_0 n) U(n).$$

10. Find the z-transform and region of convergence for the following sequence. (May 2014)

$$x(n) = 7 \left(\frac{1}{3}\right)^n u(n) - 6 \left(\frac{1}{2}\right)^n u(n)$$

Apply initial value theorem and check the z-transform whether it is correct or not.

11. Find the inverse Z-transform of the function,

$$X(z) = \frac{(z-4)}{(z-1)(z-3)^2} \text{ for } |z| > 2$$

12. Determine the inverse z-transform of

$$X(z) = \frac{1 + 2z^{-1}}{1 - 2z^{-1} + z^{-2}}$$

- (a) ROC $|z| > 1$
- (b) ROC $|z| < 1$

13. Determine the z-transform of the signal

$$x(n) = -\alpha^n U(-n-1) = \begin{cases} 0 & n \geq 0 \\ -\alpha^n & n \leq -1 \end{cases}$$

14. i) Consider the analog signal $x(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 1200\pi t$
What is the Nyquist rate for this signal?
ii) Derive the equation for convolution sum and summarize the steps involved in computing convolution (May/June 2013)
15. i) Determine the Z Transform and ROC of the signal $x(n) = -a^n u(-n-1)$
ii) Check whether the discrete time system $y(n) = \cos[x(n)]$ is
a) Static or Dynamic b) Linear or Nonlinear c) Time invariant or Time varying d) Causal or non causal e) Stable or unstable (May/June 2013)
16. i) Compute the convolution of the signals $x(n) = \{1, 2, 3, 4, 5, 3, -1, -2\}$ and $h(n) = \{3, 2, 1, 4\}$ using tabulation

- ii) Check whether the following systems are Static or Dynamic, Linear or Nonlinear, Time invariant or Time varying, Causal or non causal, Stable or unstable
 1) $y(n) = \cos[x(n)]$ 2) $y(n) = x(-n+2)$ 3) $y(n) = x(2n)$ 4) $y(n) = x(n)\cos\omega_0(n)$ (Nov/Dec 2013/ May 2014)
17. i) Describe the different types of digital signal representation.
 ii) What is Nyquist rate? Explain its significance while sampling the analog signals. (Nov/Dec 2013)
18. Check whether the system described by the following equations are Linear, Shift invariant, Causal and Stable.
 (i) $y(n) = x(n)\cos\omega n$ (ii) $y(n) = x(n)$ (iii) $y(n) = \text{sgn}(x(n))$ (Nov 2014)
19. Compare the linear convolution of the following sequences $x(n) = \{0, 2, 2, 3\}$ and $h(n) = \sin(3\pi n/8)$ with $n = (0-4)$ using mathematical equation, multiplication and tabulation method. (Nov 2014)

UNIT II FREQUENCY TRANSFORMATIONS

PART-A

1. Define DFT.

It is a finite duration discrete frequency sequence which is obtained by sampling one period of fourier transform. Sampling is done 'N' equally spaced points over the period extending from $\omega = 0$ to 2π . The DFT of discrete sequence $x(n)$ is denoted by $X(k)$ and it is given by.

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N}$$

where $k = 0, 1, 2 \dots N-1$.

2. Define the Discrete Time Fourier Transform.

The Discrete Time Fourier Transform (DTFT) $X(e^{j\omega})$ of a discrete line signal $x(n)$ is expressed as

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

Symbolically, this may be expressed as

$$x(n) \xleftarrow{\text{DTFT}} X(e^{j\omega})$$

DTFT is periodic units period 2π . So any interval of length 2π is sufficient for the complete specification of the spectrum. Generally, we draw the spectrum in the fundamental interval $(-\pi, \pi)$.

3. Explain the symmetry properties of DFTs which provide basis for fast algorithms. (May 2014)

Most approaches for improving the efficiency of computation of DFT, exploits the symmetry and periodicity property of W_N^{kn} i.e.

$$W_N^{(k + \frac{N}{2})} = -W_N^k \quad [\text{Symmetry property}]$$

$$W_N^{k+N} = W_N^k \quad [\text{Periodicity property}]$$

4. What is zero padding in DFT?

The process of lengthening a sequence by adding zero valued samples is called appending with zeros or zero padding. This is done to equate linear convolution with circular convolutions in case of DFT.

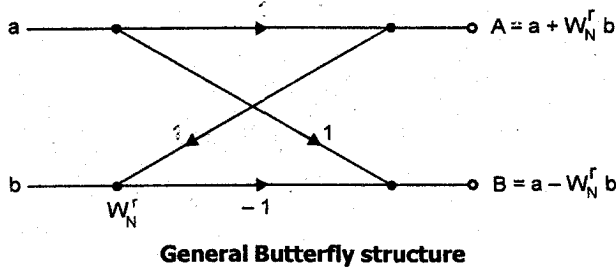
5. What is the importance of FFT's?

Fast Fourier Transform (FFT) is to decompose successively the N-point DFT computation into computations of smaller size DFT's and to take advantage of the periodicity and symmetry properties of the complex number W_N^{kn} . Such decompositions, if properly carried out, can result in a significant surveying in the computational complexity given by the total number of multiplications and the total number of additions needed to compute all N

DFT samples. The total no. of complex multiplications is reduced to $(N/2) \log_2 N$ w.r.t. DFT and the total no. of complex additions is $N \log_2 N$.

6. What is the advantage of in-place computation? (Nov 2014)

The main advantage of in-place computation is reduction in the memory size in-place computation reduces the memory size.



'a' & 'b' are inputs and 'A' and 'B' are outputs of butterfly. For anyone input 'a' and 'b' two memory locations are required for each. One memory location to store real part and other memory location to store imaginary part. So for both inputs 'a' & 'b' = 2 + 2 = 4 memory location are required.

Thus outputs 'A' & 'B' are calculated by using the values 'a' & 'b' stored in memory.

'A' & 'B' complex numbers, so 2 + 2 = 4 memory location are required.

Once the computation of 'A' & 'B' done then values of 'a' & 'b' are not required. Instead of storing 'A' & 'B' at other memory locations, there values are stored at the same place where 'a' & 'b' were stored. That means 'A' & 'B' are stored in the place of 'a' & 'b'. This is called as in-place computation.

7. Indicate the number of stages, the number of complex multiplications at each stage, and the total number of multiplications required to compute 64-point FFT using radix-2 algorithm.

$$\text{Number of stages} = \log_2 N = \log_2 64 = 6$$

$$\text{Number of complex multiplication} = N_2 \log_2 N = \frac{64}{2} \times 6 = 192$$

$$\text{Total number of multiplications} = N \log_2 N = 64 \times 6 = 384.$$

8. Write application of FFT algorithm.

Linear filtering, correlation analysis and spectrum analysis are same important applications of FFT algorithm.

9. What is a decimation in time algorithm?

DIT algorithm is used to calculate the DFT of a N point sequence. Initially the N point sequence is divided into two N/2 point sequences $X_{\text{even}}(n)$ and $X_{\text{odd}}(n)$. The N/2 point DFTs of these two sequences are evaluated and combined to give the N point DFT. Similarly the N/2 point DFTs can be expressed as a combination of N/4 point DFTs. This process is continued until left with 2 point DFT. This algorithm is called decimation in time because the sequence $X(n)$ is often splitted into smaller sequences.

10. Compute the DFT of $x(n) = \delta(n)$.

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn} = \sum_{n=0}^{N-1} \delta(n) W_N^{kn} = 1.$$

11. What is meant by radix-2 FFT? (May2014)

The FFT algorithm is most efficient in calculating N point DFT. If the number of point N can be expressed as a power of 2 ie $N = 2^M$ where M is an integer, then this algorithm is known as radix-2 FFT algorithm.

12. What is decimation in frequency algorithm?

It is one of the FFT algorithms. In this the output sequence $X(k)$ is divided into smaller subsequences, that is why the name decimation in frequency. Initially the input sequence is divided into two consisting of the first N/2 samples of $X(n)$ and the last N/2 samples of $X(n)$. The above procedure can now be iterated to express each N/2 point DFT as a combination of two N/4 point DFTs. This process is continued until we are left with 2 point and 1 DFT.

13. What are the applications of FFT algorithms?

The applications of FFT algorithms includes Linear filtering, Correlation, Spectrum analysis.

14. Calculate the number of multiplications needed in the calculation of DFT and FFT with 64 point sequence. (Nov2014)

Number of complex multiplications required using direct computation is

$$N^2 = 64^2 = 4096$$

Number of complex multiplications required using FFT is

$$(N/2) \log N = ((64/2) \log 64 = 192$$

Speed improvement factor $(4096/192) = 21.33$.

15. What are the properties of DIT FFT?

1. Computation are done in place. Once a butterfly structure operation is performed on a pair of complex numbers (a,b) to produce (A,B) there is no need to save the input pair (a,b). Hence we can store the results (A,B) in the same location as(a,b).
2. Data x (n) after decimation is stored in reverse order.

16. What are the advantages of FFT algorithm?

Fast fourier transform reduces the computation time. In DFT computation, number of multiplication is N^2 and the number of addition is $N(N-1)$. In FFT algorithm, number of multiplication is only $N/2(\log_2 N)$. Hence FFT reduces the number of elements (adder, multiplier Z & delay elements). This is achieved by effectively utilizing the symmetric and periodicity properties of Fourier transform.

17. What is meant by radix 4 FFT?(May/June2013)

FFT algorithm used to compute DFT when the number of data points N in the DFT is a power of 4.

18. Give transforms pair equation of DCT. (May/June2013)

Forward DCT

$$c(k) = a(k) \sum_{n=0}^{N-1} x(n) \cos \left[\frac{\pi((2n+1)k)}{2N} \right] \quad 0 \leq k \leq N-1$$

Inverse DCT

$$x(n) = \sum_{k=0}^{N-1} a(k) c(k) \cos \left[\frac{\pi((2n+1)k)}{2N} \right] \quad 0 \leq n \leq N-1$$

19. Compute the IDFT of $X(N) = \{1,0,1,0\}$.(Nov/Dec 2013)

$$x_N = \frac{1}{N} [W_N^*] X_N$$

$$X_N = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$W_N^* = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

For $N = 4$

$$x_N = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$x[n] = \{0.5, 0, 0.5, 0\}$$

20. What are the differences and similarities between DIF and DIT algorithms?

Differences:

For DIT the input is bit reversed while the output is in natural order, whereas for DIF the input is in natural order while the output is bit reversed.

The DIF butterfly is slightly different from the DIT butterfly, the difference being that the complex multiplication takes place after the add-subtract operation in DIF.

Similarities:

Both algorithms require same number of operations to compute the DFT. Both algorithms can be done in place and both need to perform bit reversal at some place during the computation.

21. Find the DTFT of $x(n) = -a^n \cdot u(-n-1)$ (Nov/Dec 2013)

$$\begin{aligned} X[\Omega] &= \sum_{n=-\infty}^{-1} -a^n u[-n-1] e^{-j\Omega n} \\ &= -\sum_{n=1}^{\infty} (a^{-1} e^{j\Omega})^n \end{aligned}$$

$$\begin{aligned}
 &= -\{a^{-1}e^{j\Omega} + (a^{-1}e^{j\Omega})^2 + (a^{-1}e^{j\Omega})^3 + \dots\} \\
 &= -a^{-1}e^{j\Omega} [1 + (a^{-1}e^{j\Omega}) + (a^{-1}e^{j\Omega})^2 + \dots] \\
 &= -a^{-1}e^{j\Omega} / (1 - a^{-1}e^{j\Omega})
 \end{aligned}$$

$$X(\Omega) = 1 / (1 - ae^{-j\Omega}) \quad |a| > 1$$

22. In the direct computation of N-point DFT of a sequence how many multiplication and additions are required? (Nov/Dec 2014)

$$\text{Number of additions required} = N(N-1)$$

$$\text{Number of multiplications required} = N^2$$

23. Using the definition $W = e^{-i(2\pi/N)}$ and the Euler identity $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$, the value of $W^{N/3}$ is ? (Nov/Dec 2014)

$$W^{N/3} = e^{-i(2\pi/N)(N/3)}$$

$$= \cos(2\pi/3) - i \sin(2\pi/3)$$

$$= -0.5 - i(\sqrt{3}/2)$$

PART-B

1. Define circular convolution. How can linear convolution be realized using circular convolution?

2. Discuss various properties of DFT.

3. Develop a Radix-2, 8-point DIF FFT algorithm with neat flow chart.

4. Develop a Radix-2, 8-point DIT FFT algorithm with neat flow chart.

5. Draw a 8 point radix-2 FFT DIT flow graphs and obtain DFT of the following sequence

$$x(n) = (0, 1, -1, 0, 0, 2, -2, 0) \quad (\text{May 2014})$$

6. Compute 4-point DFT of causal three sample sequence given by. (Nov 2014)

$$X(n) = \frac{1}{3} \quad 0 \leq n \leq 2$$

$$= 0 \quad \text{else.}$$

Draw its magnitude and phase spectrum.

7.a) Determine 8-point DFT of sequence.

$$x(n) = \{1, -3 \leq n \leq 3\} \quad \text{using radix-2 DIT-FFT algorithm.}$$

b) A finite duration sequence of length L is given as

$$X(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Determine N-point DFT of this sequence for $N \geq L$.

8. Compute the DFT of sequence defined by : $x(n) = (-1)^n$ for

(a) N = even

(b) N = odd.

9. By means of the DFT & IDFT, determine the sequence $x_3(n)$ corresponding to the circular convolution of the sequence $x_1(n)$ and $x_2(n)$.

$$x_1(n) = \{2, 1, 2, 1\}, \quad x_2(n) = \{1, 2, 3, 4\}$$

10. By means of the DFT & IDFT, determine the response of the FIR filter with impulse response.

$$h(n) = \{1, 2, 3\} \quad \text{to the input sequence} \quad x(n) = \{1, 2, 2\}$$

11. An 8-point sequence is given by $x(n) = \{2, 2, 2, 2, 1, 1, 1, 1\}$. Compute 8-point DFT of $x(n)$ by radix-2 DIT FFT. Also sketch the magnitude and phase spectrum.

12. An 8-point sequence is given by $x(n) = \{2, 2, 2, 2, 1, 1, 1, 1\}$. Compute 8 point DFT of $x(n)$ by radix-2 DIF-FFT.

13. i) Discuss the properties of DFT. (Nov 2014) (May/June 2013)

ii) Discuss the use of FFT algorithm in linear filtering and correlation. (Nov/Dec 2013)

14. Find DFT of $x(n) = \{1, 1, 2, 0, 1, 2, 0, 1\}$ using DIT FFT algorithm and plot the spectrum.

(Nov/Dec 2013)

15. Find 8 point DFT of the following sequence using direct method; $\{1, 1, 1, 1, 1, 1, 0, 0\}$

16. i) Compute the 8 point DFT of the following sequence using radix 2 Decimation in Time FFT

$$\text{Algorithm: } x(n) = \{1, -1, 1, -1, 1, -1, 1, -1\}$$

ii) Discuss the use of FFT in linear filtering. (May/June 2013)

17. a) Find the 4 point DFT of a) $x(n) = 2^n$ b) $x(n) = \{0, 1, 0, -1\}$ (Nov 2014)

18. Find 8- point DFT of sequence $x(n) = n+1$ using radix 2 DIF FFT algorithm (Nov 2014)

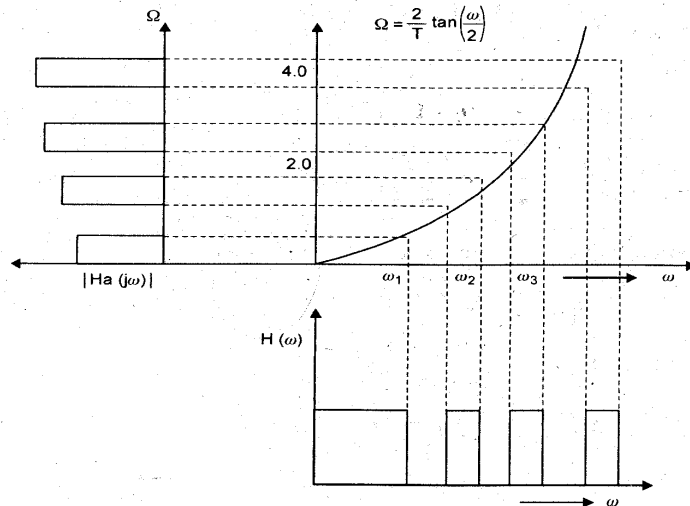
19. Find 8- point DFT of sequence $x(n) = n$ using radix 2 DIT FFT algorithm (May 2014)

UNIT III IIR FILTER DESIGN

PART-A

1. What is frequency warping in bilinear transformation?

The mapping of frequency from Ω to ω is approximately linear for small value of Ω & ω . For the higher frequencies, however the relation between Ω & ω becomes highly non-linear. This introduces the distortion in the frequency scale of digital filter relative to analog filter. This effect is known as warping effect.



2. What are the conditions for distortion less transmission?

The conditions for distortion less transmission are given below.

- Anti-aliasing filter must be used which is a low pass filter to remove high frequency noise contain in input signal. It avoids aliasing effect also.
- Sample and hold circuit is used to keep the voltage level constant.
- Output signal of digital to analog converter is analog i.e. a continuous signal. But it contain high frequency components. Such high frequency components are understood. To remove these components reconstruction filter is used.
- Amplifiers are used sometimes to bring the voltage level of input signal upto required level for distortion less transmission.

3. What are methods used to convert analog to digital filter?

Approximation of derivatives, Impulse invariant method & Bilinear transformation method.

4. Write the pole mapping rule in Impulse invariant method?(Nov2014)

A pole located at $s = s_p$ in the s plane is transferred into a pole in the z plane located at $Z = e^{s_p T_s}$. Each strip of width $2\pi/T$ on left half of s -plane should be mapped to region inside the unit circle in z -plane. The imaginary axis of s -plane is mapped to unit circle in z -plane. Left half of s -plane is mapped to outer region of unit circle.

5. What are the disadvantages of Impulse invariant method?

It provides many to one pole mapping from s -plane to z -plane. So aliasing will occur in IIT.

6. What are the advantages of Bilinear transformation method?

The Bilinear transform method provides non linear one to one mapping of the frequency points on the $j\omega$ axis in the S plane to those on the unit circle in the Z plane. i.e Entire $j\omega$ axis for $-\infty < \omega < \infty$ maps uniquely on to a unit circle $-\pi/T < \omega/T < \pi/T$. This procedure allows us to implement digital high pass filters from their analog counter parts. No aliasing effects.

7. Define prewarping or prescaling.

For large frequency values the non linear compression that occurs in the mapping of Ω to ω is more apparent. This compression causes the transfer function at high Ω frequency to be highly distorted when it is translated to the ω domain. This compression is being compensated by introducing a prescaling or prewarping to Ω frequency scale. For bilinear transform Ω scale is converted into Ω^* scale (i.e)

$$\Omega^* = \frac{2}{T_s} \tan(\Omega T_s / 2) \text{ (prewarped frequency)}$$

8. Comparison of analog and digital filters. (Nov 2014)

Analog filter	Digital filter
1. In analog filter both input and output continuous time signal	1. In digital filter, both the input and output are discrete time signals.
2. It can be constructed using active and passive components.	2. It can be constructed using adder, multiplier and delay units.
3. These filters operate in infinite freq. Range, theoretically but in practice it is limited by finite max. operating freq. depending upon the devices used.	3. freq. range is restricted to half the sampling range and it is also restricted by max. computational speed available for particular application.
4. It is defined by linear differential eqn.	4. It is defined by linear difference eqn

9. What are the advantages of digital filter?

- 1. Filter coefficient can be changed any time thus it implements the adaptive filter.
- 2. It does not require impedance matching between input and output.
- 3. Multiple filtering is possible. 4. Improved accuracy, stability and dynamic range.

10. What are disadvantages of Digital Filter?

- The bandwidth of the filter is limited by sampling frequency.
- The performance of the digital filter depends on the hardware used to implement the filter.
- The quantization error arises due to finite word length effect in representation of signal and filter coefficient.

11. What are the properties of chebyshev filter?

- For $\omega \geq 1$ $H(j\omega)$ decreases monotonically towards zero.
- For $\omega \leq 1$ $H(j\omega)$ it oscillates between 1 and $1/(1+\epsilon^2)$

12. Compare Butterworth filter and chebyshev filter.**Butterworth filter**

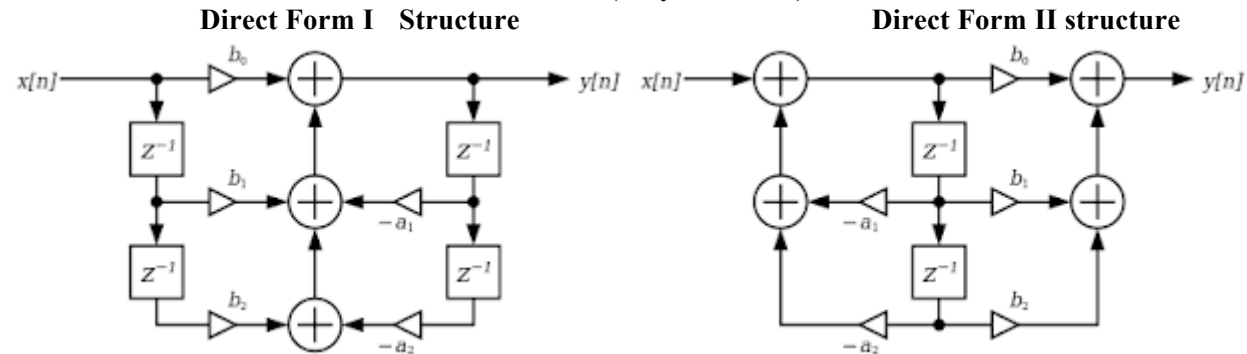
- The Magnitude response of Butterworth filter decreases monotonically as the frequency increases.
- The Transition width is more
- The order of butterworth filter is more, thus it requires more elements to construct and is expensive.
- The Poles of the butterworth filter lies along the circle.
- Magnitude response is flat at $\omega=0$ thus it is known as maximally flat filter.

Chebyshev Filter

- The Magnitude response of Chebyshev filter will not decrease monotonically with frequency because it exhibits ripples in pass band or stop band.
- The Transition width is very small
- For the same specifications the order of the filter is small and is less complex and inexpensive.
- The poles of chebyshev filter lies along the ellipse.
- Magnitude response produces ripples in the pass band or stop band thus it is known as equiripple filter.

13. Compare Bilinear Transformation and Impulse Invariant Transformation (May 2014)

Bilinear Transformation	Impulse Invariant Transformation
1. It is one to one mapping	1. It is many to one mapping
2. The relation between analog and digital frequency is nonlinear, ie $\Omega = 2/T \tan(\omega/2)$	2. The relation between analog and digital frequency is linear, ie $\omega = \Omega T$ or $\Omega = \omega/T$
3. Due to nonlinear relation between ω and Ω distortion occurs in frequency domain of digital filter.	3. The aliasing error occur due to sampling thus this method is suitable for design of only band limited filters such Low pass and Band pass.
4. Due to the warping effect both amplitude and phase response of analog filter are affected but the magnitude response may be preserved by applying pre-warped procedure.	4. The frequency response of analog can be preserved by selecting low sampling time or high sampling frequency.

14. Draw the direct form structure of IIR filter. (May/Jun 2014)**19. Write the transformation equation to convert low pass filter into low pass filter with different cut off frequency and high pass filter.(May/June2013)**

Cut off frequency = Ω_c

Low pass to Low pass transformation: Substitute $s = s/\Omega_c$.

Low pass to High pass transformation: Substitute $s = \Omega_c/s$.

20. What are the characteristics of Chebyshev filter? (May/June2013)

- Magnitude response of Chebyshev filter produces ripples in the pass band or stop band.
- The poles of the filter lie on an ellipse.

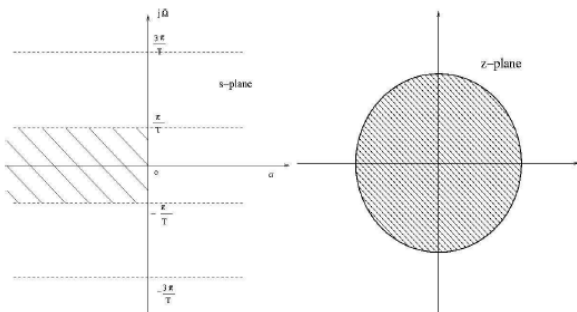
21. Mention the properties of Butterworth filter.(Nov/Dec2013)

- The butterworth filter have all poles design.
- At the cut off frequency Ω_c , the magnitude of normalized butterworth filter is $1/\sqrt{2}$.
- The filter order N , completely specifies the filter and as the value of N increases the magnitude response approaches the ideal response.

22. Define bilinear transformation with expressions.(Nov/Dec2013)

The bilinear transformation is a conformal mapping that transforms the s -plane to z -plane. In this mapping the imaginary axis of s -plane is mapped into the unit circle in z -plane, the left half of s -plane is mapped into interior of unit circle in z -plane. The bilinear mapping is one-to-one mapping and it is accomplished when

$$S = \frac{2}{T} (1 - z^{-1} / 1 + z^{-1})$$

23. Sketch the mapping of s -plane and z -plane in approximation of derivative. (Nov/Dec 2014)**PART -B**

1. Obtain the Direct form I,II and Cascade realization of the system characterized by transfer function.

$$H(Z) = 2(Z+2) / (Z(Z-0.1) (Z+0.5) (Z+0.4))$$

2. Realize the system with transfer function $H(Z) = (Z^{-1} + 4Z^{-2}) / (5 - 2Z^{-1} + 0.15Z^{-2})$ using cascade and parallel form.

3. Realize the following system: $y(n) = 1.4y(n-1) + 1.4y(n-2) + 0.4y(n-3) = 3x(n) + 5x(n-1)$ using cascade form.

4. What are the limitations of IIR filter design by impulse invariance method? How they are over come?

Convert the analog filter with system function $H(s) = S(S+0.1) / (S+0.1)^2 + 16$ into digital IIR filter by means of bilinear transformation.

5.a) Given an analog transfer function as $H(S) = 1 / (S+1) (S+2)$. Obtain $H(z)$ using impulse invariant method. Take $T=1$ Sec.

b) For given analog filter system function $H(S) = \frac{S+0.1}{(S+0.1)^2 + 16}$ into digital IIR filter by means of bilinear z-transformation. Digital filter is to have resonant frequency $\omega_r = \frac{\pi}{2}$.

6. An IIR low pass filter is to be designed to meet the following specifications.

- (a) Pass-band frequency = 0 to 1.2 kHz
- (b) Stop-band edge = 2KHz
- (c) Pass-band attenuation ≤ 8.5 db
- (d) Stop-band attenuation ≥ 15 db

Using Butterworth approximation and Bilinear transformation obtain the desired IIR digital filter.

7. A chebyshev low pass filter has the following specifications:

- (a) Order of the filter = 3
- (b) Ripple in pass-band = 1 db
- (c) Cut off frequency = 100 Hz
- (d) Sampling frequency = 1 kHz.

Determine $H(z)$ of the corresponding high pass digital filter using bilinear transformation technique.

8. Design a chebyshev filter for the following specification using (a) bilinear transformation (b) Impulse invariance method.

$$\begin{aligned} 0.8 \leq |H(e^{j\omega})| &\leq 1 & 0 \leq \omega \leq 0.2\pi \\ |H(e^{j\omega})| &\leq 0.2 & 0.6\pi \leq \omega \leq \pi \end{aligned}$$

9. Why frequency transformation in analog domain is done? Discuss in detail.

10. i) Obtain the direct form I, direct form II, cascade, parallel form realization of the system

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

ii) For the analog transfer function $H(s) = 2/(s+1)(s+2)$, determine $H(z)$ using impulse invariance method. Assume $T=1$ sec. **(May/June 2013)**

11. For the following specifications is required

Pass band -0-500Hz

Stop band -2-4kHz

Pass band ripple -3dB

Stop band ripple - 20Db

Sampling frequency -8kHz. Design a digital butterworth filter. **(May/June 2013)**

12. The specifications of the desired low pass filter is

$$\begin{aligned} 0.8 \leq |H(e^{j\omega})| &\leq 1 & 0 \leq \omega \leq 0.2\pi \\ |H(e^{j\omega})| &\leq 0.2 & 0.32\pi \leq \omega \leq \pi \end{aligned}$$

Design butterworth digital filter using impulse invariant transformation. **(Nov/Dec 2013)**

13. Convert the analog filter with system functions $H_a(S) = 10/(s^2 + 7s + 10)$ into the digital high pass filter by means of the impulse invariance method with $T=0.2$ sec **(Nov 2014)**

14. A digital filter with 3 dB bandwidth of 0.25π is to be designed with the following analog filter

$$H(s) = \Omega_c / (s + \Omega_c) \text{ using BLT and obtain } H(z). \text{ **(Nov 2014)**}$$

15. Design a chebyshev filter for the following specification using BLT **(May 2014)**

$$\begin{aligned} 0.707 \leq |H(e^{j\omega})| &\leq 1 & 0 \leq \omega \leq 0.2\pi \\ |H(e^{j\omega})| &\leq 0.1 & 0.5\pi \leq \omega \leq \pi \end{aligned}$$

16. Obtain direct form-I, direct form-II, cascade and parallel structure for the system described by:

$$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2). \quad \text{(May 2014)}$$

UNIT IV

FIR FILTER DESIGN

PART A

1. What is the basic difference between cascade form and direct form structures for FIR systems?

Cascade form is basically in need of series memory. No of memory space required less in case of direct-2 form of FIR w.r.t. cascade form start use of FIR systems.

2. Compare different form structures of filter realization from the point of view of speed and memory requirement. **(Nov 2013)**

The structural representation provides the relations between some pertinent internal variable with the input and output that in turn provide the keys to implementations. There are various form of structural representations of a digital filter. In

digital implementations, the delay operation can be implemented by providing stronger register for each unit delay that is required.

In case of direct I form structure realization separate delay for both input and output signal samples. So more memory is utilized by this form.

In case of direct-II form structure realization only one delay is required for both input and output signal samples. Therefore it is more efficient in term of memory requirements.

3. Compare the performance of FIR filter and IIR filter.

FIR	IIR
<ol style="list-style-type: none"> 1. It is having linear phase 2. No of necessary multiplications are more. 3. It is a stable filter. 4. Probability of overflow error is very less. 5. Sensitivity to filter coefficient quantization is low. 6. FIR can simulate prototype analog filter. 	<ol style="list-style-type: none"> 1. It is having no linear phase. 2. Less no. of multiplications are required. 3. Stability depend upon the system. 4. More probability of overflow error in case of direct form. 5. High sensitivity to filter co-efficient quantization. 6. It can simulate prototype analog filter.

4. What is the importance of Windowing?

1. The infinite duration impulse response can be converted to a finite duration impulse response by truncating the infinite series at $n=\pm N$. But this results in undesirable oscillations in the pass-band and stop-band of the digital filter. This is due to slow convergence of the Fourier series near the point of discontinuity. These undesirable oscillations can be reduced by using a set of time limited weighing functions $w(n)$ referred as windowing function.

2 The windowing function consists of main lobe which contains most of the energy of window function and side lobes which decay rapidly

3 A major effect of windowing is that the discontinuities in $H(e^{j\omega})$ are converted into transition bands between values on either side of the discontinuity

4 Window function have side lobes that decrease in energy rapidly as ω tends to π .

5. In what cases FIR filters will be preferred over IIR filters? (Nov 2014)

Table IIR and FIR characteristics comparison

Characteristic	IIR	FIR
Number of necessary multiplications	Least	Most
Sensitivity to filter coefficient quantization	Can be high for Direct Form	Very low
Probability of overflow errors	Can be high for Direct Form	Very low
Stability	Depends upon system design	Guaranteed
Linear phase	No	Guaranteed
Can simulate prototype analog filters	Yes	No
Required hardware memory	Least	Most
Hardware filter control complexity	Moderate	Simple
Availability of design software	Good	Very good
Ease of design or complexity of design software	Moderately complicated	Simple
Difficulty of quantization noise analysis	Most complicated	Least complicated
Supports adaptive filtering	Yes	Yes

6. What will happen if length of windows is increased in design of FIR filters?

If length of window is increased in design of FIR filter more coefficients need to be calculated and more memory space used for it.

7. What are the essential features of a good window for FIR filters?(Nov2013)

Features of a good window for FIR filters: 1. Side lobe level should be small. 2. Broaden middle section.

3. Attenuation should be more. 4. Smoother magnitude response. 5. The tradeoff between main lobe widths and side lobe level can be adjusted. 6. Smoother ends. 7. If cosine term is used then side lobes are reduced further.

8. Why FIR digital filters cannot have linear phase?

For FIR filter unit impulse response for symmetric system are given by:

$$h(n) = h(m-1-n) \quad (1), \quad n = 0, 1, 2, \dots, m-1$$

$$n = 0, \quad h(0) = h(8-1-0) = h(7)$$

$$n = 1, \quad h(1) = h(8-1-1) = h(6)$$

If $h(n)$ is symmetric then filter is symmetric. For antisymmetric sequence.

$$h(n) = -h(m-1-n); \quad n = 0, 1, 2, \dots, m-1$$

i.e. condition for linear phase. For FIR filters m is finite i.e. may be odd, symmetric and antisymmetric conditions so in FIR filters m is infinite. So it does not satisfies linear phase condition of eq. (1) and (2). So FIR filters cannot have linear phase.

9. Define Ripple ratio

The Ripple ratio is defined as , the ratio of maximum sidelobes amplitude to the mainlobe amplitude. i.e. $\%RR = (\text{maximum side lobe amplitude/main lobe amplitude}) \times 100$

10. What is Gibb's Oscillation? (or) State the effect of having abrupt discontinuity in frequency response of FIR filters.(May 2014)

The truncation of Fourier series is known to introduce the unwanted ripples in the frequency response characteristics $H(\omega)$ due to non uniform convergence of Fourier series at a discontinuity These ripples or oscillatory behaviour near the band edge of the filter is known as "Gibb's phenomenon or Gibb's oscillation".

11. What are the methods used to reduce Gibb's phenomenon?(May 2014)

There are two methods to reduce Gibb's phenomenon

1. The discontinuity between pass band and stop band in the frequency response is avoided by introducing the transition between the pass band and stop band.

2. Another technique used for the reduction of Gibb's phenomenon is by using window function that contains a taper which decays towards zero gradually instead abruptly.

12. What are FIR filters?

The specifications of the desired filter will be given in terms of ideal frequency response $H_d(\omega)$. The impulse response $h_d(n)$ of desired filter can be obtained by inverse fourier transform of $H_d(\omega)$ which consists of infinite samples. The filters designed by selecting finite no of samples of impulse response are called FIR filters.

13. What are the disadvantages of FIR filter? (May 2013)

The duration of impulse response should be large to realize sharp cut off filters. The non-integral delay can lead to problems in some signal processing applications. A large amount of processing is required to realize the filter if slow convolution is used.

14. What is the necessary and sufficient conditions for linear phase characteristics of a FIR filter?

The necessary and sufficient conditions for linear phase characteristics of a FIR filter is that the phase function should be a linear function of ω , which in turn requires constant phase delay or constant phase and group delay.

15. What are the possible types of impulse response for linear phase FIR filter?

4 types: i. Symmetric impulse response when N is odd ii. Symmetric impulse response when N is even
iii. Antisymmetric impulse response when N is odd iv. Antisymmetric impulse response when N is even.

17. List the factors that are to be specified in the filter design problem.

i. The maximum tolerable passband ripple. ii. The max tolerable stopband ripple.
iii. The passband edge freq ω_p iv. The stopband edge freq ω_s .

18. What are the conditions that are to be satisfied for const phase delay in linear phase FIR filter?

The conditions for const phase delay are,

Phase delay, $\alpha = (N-1)/2$ (i.e phase delay is const)

Impulse response $h(n) = h(N-1-n)$ (i.e. impulse response is symmetric).

19. Characteristic features of rectangular window.

i. The mainlobe width is equal to $4\pi/N$. ii. The max sidelobe magnitude is -13 dB.
iii. The sidelobe magnitude does not decrease significantly with increasing ω .

20. List features of hanning window spectrum.

i. The mainlobe width is equal to $8\pi/N$. ii. The max sidelobe magnitude is -31 dB.
iii. The sidelobe magnitude decreases with increasing ω .

21. List features of hamming window spectrum.

i. The mainlobe width is equal to $8\pi/N$. ii. The max sidelobe magnitude is -41 dB.
iii. The sidelobe magnitude remains constant for increasing ω .

22. What are the advantages of Kaiser window?

1. It provides flexibility for the designer to select side lobe level and N
2. It has the attractive property that the side level can be varied continuously from the value in the Blackman window to the high value in the rectangular window.

23. What are the properties of FIR filter ? (Nov 2013)

i. FIR filter is stable. ii. Linear phase which is accompanied by constant time or group delay.

24. Define Group delay.

Defined as derivative of phase with respect to frequency.

25. Define phase delay.

Defined as phase divided by frequency

26. What are the methods used to design FIR filter?

1. Window Method: It involves straight forward analytical procedure however in some cases iteration is required to obtain the desired result
2. Frequency Sampling: A desired frequency response is uniformly sampled and filter coefficients are then determined from these samples using the discrete fourier transform.
3. Optimal or minimal design: Minimizing the maximum error between the desired and the actual frequency response by spreading the error in PB and SB.

27. Why direct Fourier series method is not used in FIR filter design?

The impulse response $h(n)$ is infinite in duration. The filter is unrealizable since the impulse response begins at $-\infty$ i.e no finite amount of delay can make the impulse response realizable. Therefore the filter which results from a Fourier series representation of $h(e^{j\omega})$ is an unrealizable FIR Filter.

28. Give the equations of Hamming and Blackman window (Nov 2014)

Hamming window:

$$w(n) = \alpha - \beta \cos\left(\frac{2\pi n}{N-1}\right), \alpha = 0.54, \beta = 1 - \alpha = 0.46,$$

Blackman window: (May/June 2013)

$$W_B(n) = \begin{cases} 0.42 - 0.5 \cos \frac{2\pi n}{N-1} + 0.08 \cos \frac{4\pi n}{N-1} ; & 0 \leq n \leq N-1 \\ 0 ; & \text{otherwise} \end{cases}$$

29. What are the characteristic feature of FIR filter? (Nov/Dec 2014)

1. They have no feedback.
2. They are inherently stable system
3. The rounding off noise is reduced.
4. They can be realized with linear phase

30. What is the reason that FIR is always stable? (May 2014)

As the system function of the FIR filter is non-recursive, it is stable.

$$H(z) = \sum_{k=0}^{N-1} b_k z^{-k}$$

31. What do you understand that linear phase response of the filters? (May 2014)

For a linear phase FIR filter the phase response is given by $\theta(\omega) = -\omega\alpha$

PART B

1. a) Obtain a cascade realization using minimum number of multiplications for the system.

$$H(z) = \left(1 + \frac{1}{4}z^{-1} + z^{-2}\right) \left(1 + \frac{1}{8}z^{-1} + z^{-2}\right).$$

- b) Realize the system function.

$$H(z) = 1 + \frac{2}{4}z^{-1} + \frac{3}{8}z^{-2} + \frac{3}{4}z^{-3} + \frac{7}{2}z^{-4}$$

by using direct form structure.

2. Design an ideal low pass filter with a frequency response

$$H_d(e^{j\omega}) = 1 \text{ for } \frac{-\pi}{2} \leq \omega \leq \frac{\pi}{2}.$$

$$= 0 \text{ for } \frac{\pi}{2} \leq |\omega| \leq \pi$$

Find the values of $h(n)$ for $N=11$ using triangular window.

3. Design an ideal band pass filter with a frequency response.

$$H_d(e^{j\omega}) = 1 \text{ for } \pi/4 \leq |\omega| \leq 3\pi/4$$

$$= 0 \text{ otherwise}$$

Find the values of $h(n)$ for $N=7$ using rectangular window.

4. Design an ideal band reject filter with a desired frequency response (May 2014)

$$H_d(e^{j\omega}) = 1 \quad \text{for } |\omega| \leq \frac{\pi}{3} \text{ and } |\omega| \geq \frac{2\pi}{3}$$

$$= 0 \text{ otherwise}$$

Find the value of $h(n)$ for $N = 7$ and also find $H(z)$ using blackman window.

5. Design an ideal highpass filter with a frequency response (May 2014)

$$H_d(e^{j\omega}) = 1 \text{ for } \pi/4 \leq |\omega| \leq \pi$$

$$= 0 \text{ for } |\omega| < \pi/4.$$

Find the value of $h(n)$ for $N = 11$ using (a) Hamming window (b) Hanning window.

6. Discuss various steps for the design of linear phase FIR filters using window method and explain the characteristics of window function.

7. Determine the coefficients of a linear phase FIR filter of length $M = 15$ which has a symmetric unit sample response and a frequency response that satisfies the conditions

$$H(2\pi K/15) = \begin{cases} 1 & K = 0, 1, 2, 3 \\ 0.4 & K = 4 \\ 0 & K = 5, 6, 7 \end{cases}$$

8. Determine the filter coefficients $h(n)$ obtained by sampling

$$Hd(e^{j\omega}) = \begin{cases} e^{-j(N-1)\omega/2} & 0 \leq |\omega| \leq \pi/2 \\ 0 & \pi/2 \leq |\omega| \leq \pi \end{cases} \text{ for } N=7. \text{ (Nov 2014)}$$

10. Using Hanning window technique, design a LPF which approximates an ideal filter with cutoff frequency of 1000 Hz and sampling frequency of 8KHZ. Order of filter is 7.

11. Explain the digital FIR filter design using frequency sampling method. (May 2014)

12. i) state and explain the properties of FIR filters. State their importance.

ii) Explain linear phase FIR structures. What are the advantages of such structures? (May 2014)

13. Design band pass filter with cut off frequencies 0.2 rad/sec and 0.3 rad/sec with $M=7$. Use the Hanning window function.

14. Design a low pass FIR filter using rectangular window to meet the following specifications.

$$H(\omega) = \begin{cases} e^{-j(N-1)\omega/2} & \text{for } 0 \leq |\omega| \leq \frac{\pi}{6} \\ 0 & \text{for } \frac{\pi}{6} \leq |\omega| \leq \pi \end{cases}$$

Use 5 tap filters and obtain the impulse response of the desired filter. (Nov 2014)

15. Realize the following FIR using direct and linear phase structure

$$h(n) = \delta(n) + \frac{1}{3} \delta(n-1) + \frac{1}{4} \delta(n-2) + \frac{1}{3} \delta(n-3) + \delta(n-4) \quad \text{(Nov 2014)}$$

16. Design the symmetric FIR low pass whose desired frequency response is given as

$$H_d(\omega) = \begin{cases} e^{-j\omega n} & \text{for } |\omega| \leq \omega_c \\ 0 & \text{otherwise} \end{cases}$$

The length of the filter should be 5 and $\omega_c = 1$ radian/sample using rectangular window. (Nov/Dec 2014)

UNIT V FINITE WORD LENGTH EFFECTS IN DIGITAL FILTERS

PART A

1. What do you understand by a fixed point number?

In fixed point arithmetic the position of the binary point is fixed. The bits to the right represent the fractional part and those to the left represent the integer part. For eg. The binary number 01.1100 has the value 1.75 in decimal.

2. Brief on coefficient inaccuracy.

The filter coefficients are computed to infinite precision in the design. But in digital computation the filter coefficients are represented in binary and are stored in registers. The filter coefficients must be rounded or truncated to 'b' bits which produces an error. Due to quantization of coefficients the frequency response of a filter may differ appreciably from the desired response and sometimes the filter may fail to meet the desired specification. If the poles of the filter are close to the unit circle then those of the filter quantized coefficients may be just outside the unit circle leading to instability.

3. What is meant by (zero input) limit cycle oscillation? (May 2013)

For an IIR filter implemented with infinite precision arithmetic the output should approach zero in the steady state if the input is zero and it should approach a constant value if the input is a constant. However, with an implementation using a finite length register an output can occur even with zero input. The output may be a fixed value or it may oscillate between finite positive and negative values. This effect is referred to as (zero input) limit cycle oscillation.

4. What are the assumptions made concerning the statistical independence of various noise sources that occur in realizing the filter?

Assumptions

1. for any n , the error sequence $e(n)$ is uniformly distributed over the range
2. $(-q/2)$ and $(q/2)$. This implies that the mean value of $e(n)$ is zero and its variance is
3. The error sequence $e(n)$ is a stationary white noise source.
4. The error sequence $e(n)$ is uncorrelated with the signal sequence $x(n)$.

5. What is the difference between fixed point arithmetic and floating point arithmetic?

Fixed point arithmetic	Floating point arithmetic
1. fast operation	slow operation

- | | |
|--|--|
| 2. small dynamic range | increased dynamic range |
| 3. relatively economical | more expensive due to costlier hardware |
| 4. round-off errors occur only in addition | round-off errors can occur with both multiplication and addition |
| 5. overflow occurs in addition | overflow does not arise |
| 6. used in small computers | used in larger general purpose computers. |

6. What are the 3 quantization errors due to finite word length register in digital filters?.

1. Input quantization error 2. Coefficient quantization error 3. Product quantization error

7. Explain briefly the need for scaling in the digital filter implementation.

To prevent overflow, the signal level at certain points in the digital filter must be scaled so that no overflow occurs in the adder.

8. What is limit cycles due to overflow? Or What is overflow oscillations?

The addition of two fixed point arithmetic numbers cause overflow when the sum exceeds the word size available to store the sum. This overflow caused by adder make the filter output to oscillate between maximum amplitude limits. Such limit cycles have been referred to as overflow oscillations.

9. Define 'dead band' of the filter. (Nov 2014)

The limit cycles occur as a result of quantization effect in multiplication. The amplitudes of the output during a limit cycle are confined to a range of values called the dead band of the filter.

10. Express the fraction (7/8) and (-7/8) in sign magnitude, 2's complement and 1's complement.

fraction (7/8) = (0.111) in sign magnitude, 1's complement and 2's complement

Fraction (-7/8) = (1.111) in sign magnitude
 = (1.000) in 1's complement
 = (1.001) in 2's complement

11. The filter coefficient H = -0.673 is represented by sign magnitude fixed point arithmetic. If the word length is 6 bits, compute the quantization error due to truncation.

(0.673) = (0.1010110...)

(-0.673) = (1.1010110...)

after truncating to 6 bits we get

(1.101011) = -0.671875

Quantization error = $x_q - x$

= (-0.671875) - (-0.673)

= 0.001125

12. Give the expression for the signal to quantization noise ratio and calculate the improvement with an increase of 2 bits to the existing bit.

$SNR = 6b - 1.24\text{dB}$, where b = number of bits for representation.

With an increase of 2 bits, increase in SNR is approximately 12dB.

13. Why rounding is preferred over truncation in realizing digital filters?

1. The quantization error due to rounding is independent of the type of arithmetic.
2. The mean of rounding error is Zero.
3. The variance of rounding error signal is low.

14. What is product quantization error? (May 2014) or What is round-off noise error?

Product quantization error arise at the output of a multiplier. Multiplication of a 'b' bit data with a 'b' bit coefficient results in a product having 2b bits. Since a 'b' bit register is used, the multiplier output must be rounded or truncated to 'b' bits which produces an error. This error is known as product quantization error.

15. Why the limit cycle problem does not exist when FIR filter is realized in direct form or cascade form?

In FIR filters there are no limit cycle oscillations if the filter is realized in direct form or cascade form since these structures have no feedback.

17. What do you understand by input quantization error? (Nov 2013)

In DSP the continuous time input signals are converted into digital using a 'b' bit ADC. The representation of continuous signal amplitude to digital introduces an error known as input quantization error.

18. What is rounding effect?

Rounding is the process of reducing size of a binary number to finite size of 'b' bits such that the rounded b-bit number is closest to the original unquantized number. The rounding process consists of truncation and addition. In rounding of a number to b-bits, first the unquantized number is truncated to b-bits by retaining the most significant b-bits. Then zero or one is added to the least significant bit of the truncated number depending on the bit that is next to the least significant bit that is retained.

Rounding error, $e_r = N_r - N$

where N_r – quantized i.e. rounded number

N – unquantized number.

In fixed point representation the range of error made by rounding a number to 'b' bits is

$$\frac{-2^{-b}}{2} \leq e_r \leq \frac{2^{-b}}{2}$$

19. What is fixed point representation?

In fixed point representation the bits allowed for integer part and fractional part and so the position of binary point is fixed. The main drawback of this representation is that, due to the fixed integer and fraction part, too large and too small values cannot be represented. The bits to the right represent the fractional part of the number and those to the left represent the integer part. The negative numbers are represented in three different forms for fixed point arithmetic: 1. Sign-magnitude form. 2. One's-complement form. 3. Two's-complement form. 1. Sign Magnitude form: In this form, the MSB is used to represent the given no. as positive or negative. Let 'N' be the length of binary bits, then (N-1) bit will represent magnitude and MS represents sign. 2. One's complement form: In this form the positive number is represented as in the sign magnitude notation. But the negative number is obtained by complementing all the bits of the positive number. 3. Two's complement form: In this form positive numbers are represented as in sign magnitude and one's complement. The negative number is obtained by complementing all the bits of the +ve number and adding one to the least significant bit.

20. What is floating point representation?

In floating point representation, a positive number is represented as

$$N_f = M \times 2^E$$

Where M is called mantissa and it will be in binary fraction format. The value of M will be in the range of $0.5 \leq M \leq 1$ and E is called exponent and it is either a positive or negative integer. In this form, both mantissa and exponent use one bit for representing sign.

21. What are the assumptions made concerning the statistical independence of various noise sources that occur in realizing the filter?

Assumptions: For any n, the error sequence e(n) is uniformly distributed over the range $(-q/2)$ and $(q/2)$. This implies that the mean value of e(n) is zero and its variance is $q^2/12$. The error sequence e(n) is a stationary white noise source. The error sequence e(n) is uncorrelated with the signal sequence x(n).

22. State the method to prevent overflow. (Nov 2013)

1. Saturation Arithmetic 2. Scaling

23. What are the two types of Quantization? (May 2013)

1. Truncation and 2. Rounding

24. State the need for scaling in filter implementation (May 2014)

With fixed-point arithmetic it is possible for filter calculations to overflow. This happens when two numbers of the same sign add to give a value having magnitude greater than one. Since numbers with magnitude greater than one are not representable, the result overflows. It is used to eliminate overflow limit cycle in FIR filters.

25. What is scaling? (Nov 2014)

A process of readjusting certain internal gain parameters in order to constrain internal signals to a range appropriate to the hardware with the constraint that the transfer function from input to output should not be changed. Overflow oscillations require recursion to exist and do not occur in nonrecursive FIR filters. There are several ways to prevent overflow oscillations in fixed-point filter realizations. The most obvious is to scale the filter calculations so as to render overflow impossible.

PART B

1. For a system described by the equation $y(n) = 0.8 y(n-1) + x(n)$ with the range of input $(-1, +1)$ and represented by 5 bits. Compute the output noise power due to input quantization.

2. A second order system is described by $y(n) = 0.35 y(n-2) + 0.92 y(n-1) + x(n)$

Study the effect of shift in pole locations with 4 bit coefficient representation in direct and cascade form realization.

(May 2014)

3. The transfer function of an IIR system is given by, $H(z) = 1 / (1 - 0.48z^{-1})(1 - 0.79z^{-1})$ Find the output roundoff noise power in direct form realization. (Assume that the products are rounded to 3 bits)

4. i) Draw the quantization noise model for second order system in direct and cascade form

- ii) Study the limit cycle oscillation of the system which is defined as $y(n) = 0.9y(n-1) + x(n)$ with zero input and $y(-1) = 12$. Determine the deadband of the system. **(May 2014)**
5. Study the limit cycle behavior of the system described by the equation $y(n) = 0.95y(n-1) + x(n)$. Determine the dead band of the filter
6. For a system described by the equation $y(n) = 0.93y(n-1) + x(n)$ the range of input has a peak value of 10V, represented by 6 bits. Compute the variance of output due to A/D conversion process.
7. i) Write short notes on overflow and zero input limit cycle oscillation
ii) Derive an expression for quantization error of input **(Nov 2014)**
8. Study the effects of shift in pole location of second order IIR filter with $b=3$ bits

$$H(z) = \frac{1}{(1 - 0.5z^{-1})(1 - 0.45z^{-2})}$$

9. a) Explain Briefly about various number representation in digital computer
b) Explain the finite word length effects in digital filters **(NOV 2013)**
10. Consider the transfer function $H(Z) = H_1(Z)H_2(Z)$ where $H_1(Z) = 1/1 - a_1Z^{-1}$; $H_2(z) = 1/1 - a_2Z^{-1}$. Find the o/p round of noise power. Assume $a_1=0.5$ and $a_2=0.6$. **(Nov 2014)**
12. Discuss the various common methods of quantization **(NOV 2013/May 2014/Nov 2014)**
13. Explain how signal scaling is used to prevent overflow LCO **(May 2013)**
14. Study the limit cycle oscillation and find the dead band of the filter $y(n) = 0.2y(n-1) + 0.5y(n-2) + x(n)$ **(MAY 2013)**
15. Derive the signal to quantization noise ratio of A/D converter. **(May 2014)**