St. Joseph's College of Engineering, Chennai–119
St. Joseph's Institute of Technology, Chennai–119
Department of Mathematics
MA6351- Transforms and Partial Differential Equations
Assignment-III
Unit IV – Fourier Transform

Year II **(Common to all Branches)** **Semester III**

Part-A

1 **State Fourier integral theorem.**
2 **Write the Fourier transform pair**
3 **Find the Fourier transform of** $f(x)=\begin{cases}1 & |x|<2\\ 0 & |x|>2\end{cases}$
4 **If** $F\left[f(x)\right]=F(s)$ **, then show that** $F[f(x-a)]=e^{ias}F(s)$ **.**
5 **Prove that** $F_C\left[f(ax)\right]=\frac{1}{a}F_C\left(\frac{s}{a}\right)$**.**
6 **Find the Fourier sine transform of** e^{-ax}**,** $a>0$**. Hence find** $F_s\left[xe^{-ax}\right]$**.**
7 **State Parseval's identity on Fourier transform.**
8 **Find** $f(x)$ **from the integral equation** $\int_0^\infty f(x)\cos sx\,dx=e^{-s}$.

Part-B

1 **Show that the Fourier transforms of** $f(x)=\begin{cases}a^2-x^2; & |x|<a\\ 0; & |x|>a\end{cases}$ **is** $2\sqrt{\frac{2}{\pi}}\left[\frac{\sin as-as\cos as}{s^3}\right]$**.**

Hence deduce that $\int_0^\infty \frac{\sin t-t\cos t}{t^3}dt=\frac{\pi}{4}$**. Using Parseval's Identity, show that** $\int_0^\infty\left(\frac{\sin t-t\cos t}{t^3}\right)^2dt=\frac{\pi}{15}$

2a) **Find the Fourier transform of** $f(x)=\begin{cases}1-|x|\,; & if\ |x|<1\\ 0\,; & if\ |x|\ge 1\end{cases}$**, Hence Evaluate** $\int_0^\infty\frac{\sin^4 t}{t^4}dt$ **.**

b) **Evaluate** $\int_0^\infty\frac{dx}{(x^2+a^2)(x^2+b^2)}$ **using Fourier Cosine Transforms of** e^{-ax} *and* e^{-bx}, $a, b>0$**.**

3a) **Find the Fourier transform of** $e^{-a^2x^2}$ **and hence show that** $e^{-\frac{x^2}{2}}$ **is self-reciprocal with respect to the Fourier Transform.**

b) **Find the function f(x) if its sine transform is** $\frac{e^{-as}}{s}$**.**

4a) **Find the Fourier transform of** $e^{-a|x|}$, $a>0$ **and hence deduce that i)** $\int_0^\infty\frac{\cos xt}{a^2+t^2}dt=\frac{\pi}{2a}e^{-a|x|}$

ii) $F\left[xe^{-a|x|}\right]=i\sqrt{\frac{2}{\pi}}\frac{2as}{(s^2+a^2)^2}$

b) **Find Fourier sine and cosine transform of** x^{n-1} **and hence prove that** $\frac{1}{\sqrt{x}}$ **is self reciprocal under Fourier sine and cosine transform.**