DIGITAL SIGNAL PROCESSING

UNIT-I SIGNALS AND SYSTEM

Basic elements of DSP – Concepts of frequency in Analog and Digital Signals – Sampling theorem – Discrete – time signals Systems – Analysis of discrete time LTI systems – Z transform – Convolution (linear and circular) – Correlation.

Signal:

* Anything that carries some information is called a signal. A signal is defined as any physical quantity that varies with time, space (or) any other independent variable.

eg: Electrocardiogram (ECG), Electroencephalogram (EEG), Seismic signal, speech signal, AC power supply signal.

* A signal can be a function of one (or) more independent variable. Depending on this the signal can be classified as

One dimensional Signal:

If a signal depends on only one variable, then it is known as one dimensional signal.

eg: AC power supply signal, Speech signal.

Two dimensional signal:

If a signal depends on two independent variables, then the signal is known as two dimensional signal.

eg: X-ray images, sonograms, picture

Multi dimensional signal:

If a signal depends on more than two independent variables, then the signal is known as multidimensional signal.

eg: speed of wind, air pressure.

Classification of Signals:

Continuous time Signal:

A continuous time signal is one that is defined for every instant of time.

A continuous time signal is called as analog signal.

eg: $A \sin(\Omega t)$, e^{-at}

It is denoted by $x(t)$.

Discrete time signal:

These signals are defined over discrete instant of time. The discrete time signals are continuous in amplitude and discrete time. It is denoted by $x(n)$.

eg: Runs scored by a team in one day cricket match.

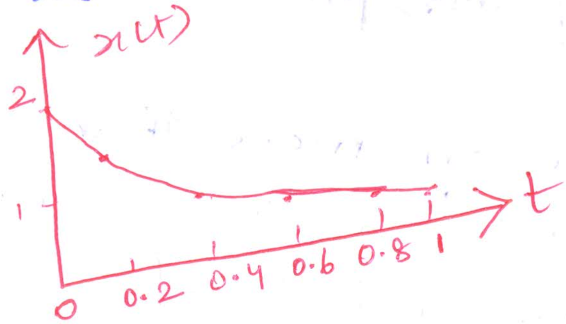
Example:

* Sketch the continuous time signal $x(t) = 2e^{-2t}$ for an interval $0 \le t \le 2$.

Soln:

$x(t) = 2e^{-2t}$

$x(0) = 2$, $x(0.2) = 1.34$, $x(0.4) = 0.8987$, $x(0.6) = 0.6024$, $x(0.8) = 0.4038$, $x(1) = 0.2707$

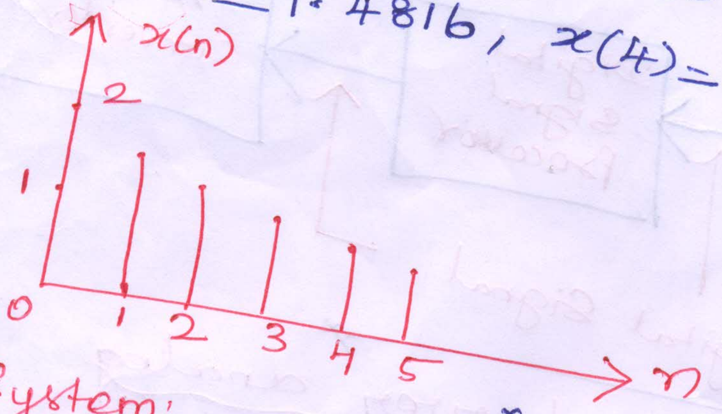


-time signal,

$x(n) = 2e^{-0.1n}$

$x(0) = 2$, $x(1) = 1.8097$, $x(2) = 1.6375$

$x(3) = 1.4816$, $x(4) = 1.3406$, $x(5) = 1.2131$



System:

A system is defined as a physical device that performs an operation on a signal.

eg: filter.

Deterministic signal:

Any signal whose past, present and future values are precisely known without any uncertianity.

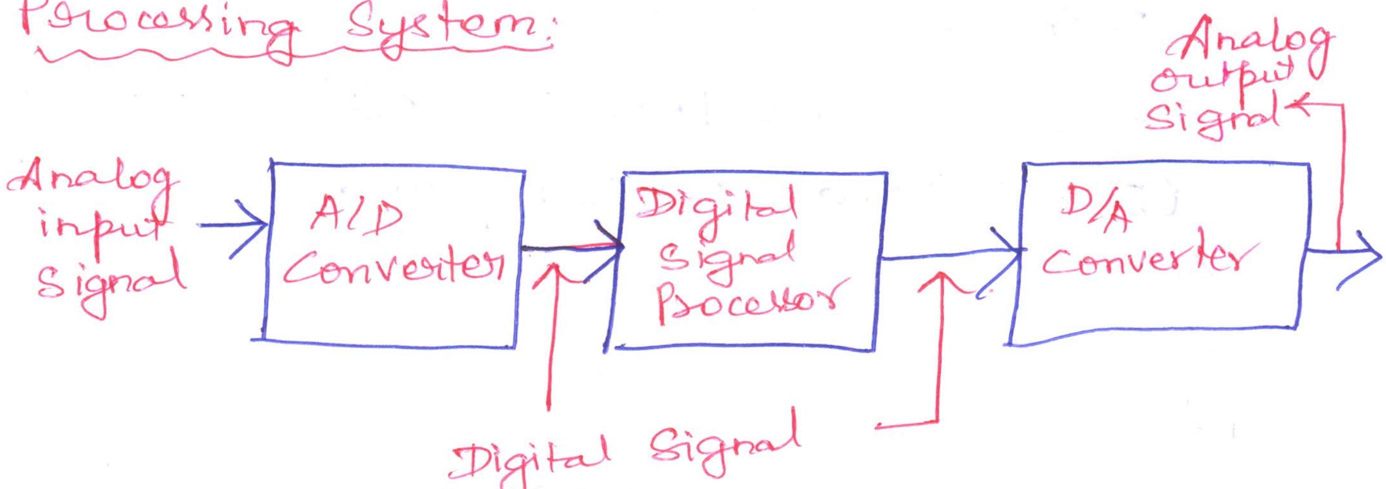
eg: sine signal.

Random Signal:

Any signal whose values are in unpredictable manner.

eg: Speech signal.

Basic Elements of Digital Signal Processing System:



Advantages of Digital over analog Signal processing:

1. * Reconfiguration of digital system is simple by changing the program.
   * Reconfiguration of analog system involves redesign of hardware.
2. * A digital system provides good accuracy.
   * Analog circuit components don't have much accuracy due to tolerances.
3. * Digital signals can be easily stored, transported and processed.
   * Processing analog signals is difficult.
4. Digital signal processing systems are cheaper.

Limitation of Digital Signal Processing:

* The speed of operation of A/D converters and digital signal processor limits processing high bandwidth signal.

* ————→ *.

Continuous Time Signal

⊛ Representation:

$$\boxed{x_a(t) = A\cos(\Omega t + \theta)}$$

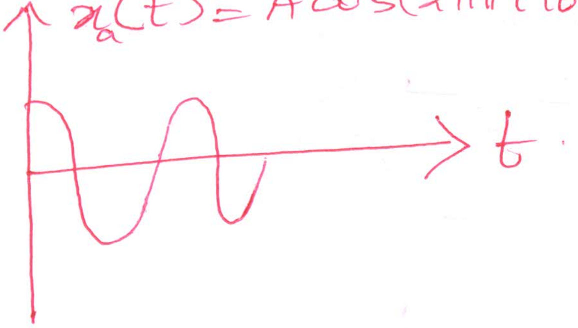
$-\infty < t < \infty$.

* A – amplitude of signal
  Ω – Frequency in radians/sec
  θ – phase in radians

⊛ $\boxed{\Omega = 2\pi F}$

⇒ F – frequency in cycles per sec.

$x_a(t) = A\cos(2\pi Ft + \theta)$



* $x_a(t)$ is periodic if

$$\boxed{x_a(t + T_p) = x_a(t)}$$

Discrete time Signal

⊛ Representation:

$$\boxed{x(n) = A\cos(\omega n + \theta)}$$

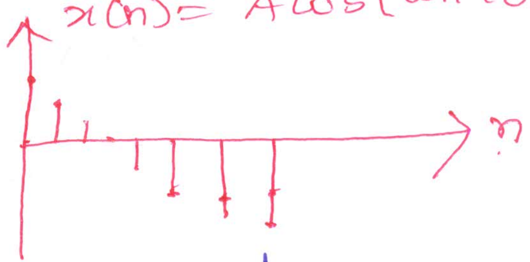
$-\infty < n < \infty$.

* A – amplitude of Signal
  ω – frequency in radians/sample
  θ – phase in radians.

⊛ $\boxed{\omega = 2\pi f}$

⇒ f – frequency in cycles per sample.

$x(n) = A\cos(\omega n + \theta)$.



* A sine signal x(n) is periodic if its frequency f is a rational number.

$$\boxed{f = k/N}$$

where $F \rightarrow$ fundamental frequency

$T_p = 1/F \rightarrow$ fundamental period.

(*) Signals with distinct frequencies are distinct

$\therefore -\infty < \Omega < \infty$

$-\infty < F < \infty$

$\boxed{x(n+N) = x(n)}$ $\forall n$.

$\Rightarrow$ N is the fundamental period.

(*) Discrete-time sine signals whose frequencies are separated by multiple of 2π are identical.

$\therefore -\pi \leq \omega \leq \pi$

f

$-\frac{1}{2} \leq f \leq \frac{1}{2}$

* All frequencies $|\omega| > \pi$, $|f| > 1/2$ are aliases.

* fundamental range:

$\Rightarrow 0 \leq \omega \leq 2\pi$ (or)

$-\pi \leq \omega \leq \pi$

$\Rightarrow 0 \leq f \leq 1$ (or)

$-1/2 \leq f \leq 1/2$.

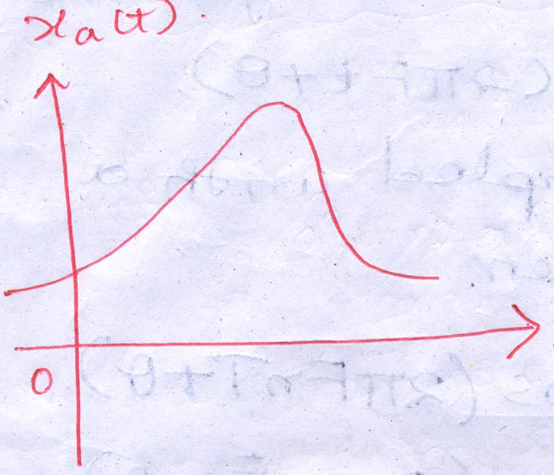
Convert continuous-time signal to discrete-time signal

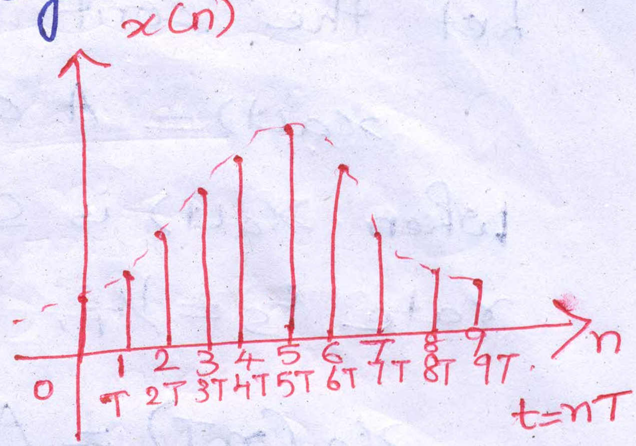
$\omega = \Omega T, \quad f = F/F_s$

$\Omega = \omega/T, \quad F = f \cdot F_s$

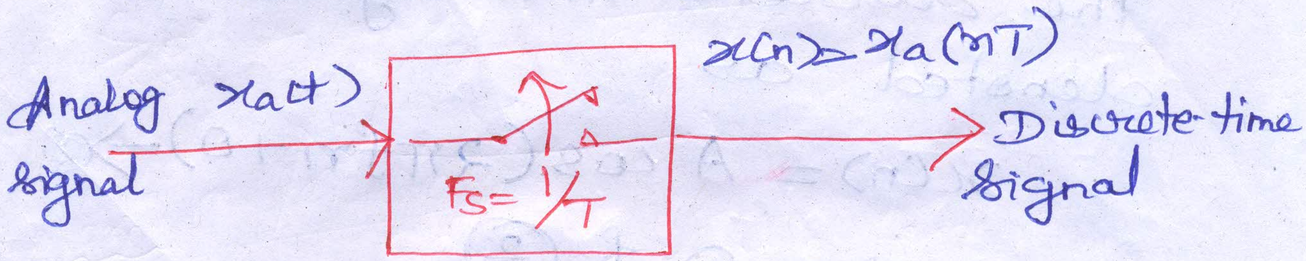
Convert Discrete time signal to Continuous time signal.

Sampling:





Sampled for every T seconds.



Sampler

* $x(n)$ – discrete time signal obtained by taking samples of analog signal $x_a(t)$ every T secs.

$$x(n) = x_a(nT) \quad -\infty < n < \infty.$$

Sampling period (or) sample interval:

Time interval T between successive samples.

Sampling rate (or) sampling frequency:

$$\boxed{F_s = 1/T}$$ (samples/sec (or) Hz).

Relationship between discrete time variable 'n' and continuous time variable 't' is as follows:

$$\boxed{t = nT = n/F_s}$$

Let the continuous time signal be

$$x_a(t) = A\cos(2\pi F t + \theta).$$

When $x_a(t)$ is sampled with a rate $F_s = 1/T$, then

$$x_a(nT) = A\cos(2\pi F nT + \theta).$$

$$= A\cos\left(2\pi n \frac{F}{F_s} + \theta\right) \rightarrow (1)$$

The discrete time signal is denoted as

$$x(n) = A\cos(2\pi f n + \theta) \rightarrow (2)$$

Comparing (1) & (2)

$$\boxed{f = F/F_s.} \rightarrow (3) \quad |||ly \quad 2\pi f = \frac{2\pi F}{F_s}$$

i.e. $\boxed{\omega = \Omega T} \rightarrow (4)$

It is known as relative (or) normalized frequency.

* Range of continuous time frequency variable.

$$-\infty < F < \infty$$

$$-\infty < \Omega < \infty$$

* Constraint for Discrete time frequency variable.

$$-1/2 < f < 1/2$$

$$-\pi < \omega < \pi.$$

* ∴ Frequency that can be properly sampled is

$$\Rightarrow -\frac{1}{2} < f < \frac{1}{2}$$

Substituting (3) we get

$$-\frac{1}{2} \leq \frac{F}{F_S} \leq \frac{1}{2}$$

$$-\frac{1}{2T} = -\frac{F_S}{2} \leq F \leq \frac{F_S}{2} = \frac{1}{2T}$$

$\Rightarrow$ IIIly $-\pi < \omega < \pi$

Substituting (4) we get

$$-\pi < \Omega T < \pi$$

$$-\pi F_S = \frac{-\pi}{T} \leq \Omega \leq \frac{\pi}{T} = \pi F_S$$

* ∴ Highest frequency component that can be uniquely sampled is given as

$$\boxed{\begin{aligned} F_{max} &= \frac{F_S}{2} = \frac{1}{2T} \\ \Omega_{max} &= \pi F_S = \frac{\pi}{T} \end{aligned}}$$

Sampling theorem:

If the highest frequency contained in an analog signal $x_a(t)$ is $F_{max} = B$ and the signal is sampled at a rate $F_S > 2F_{max} = 2B$, then $x_a(t)$ can be exactly recovered from its sample values using the interpolation function,

$$g(t) = \frac{\sin 2\pi Bt}{2\pi Bt}, \quad x_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{n}{F_s}\right)$$

Nyquist rate:

The minimum sampling rate, $F_s = 2B = 2F_{max}$ is called as the Nyquist rate F_N.

Problem:

a) Consider the analog signal

$$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$$

What is the Nyquist rate for this signal.

Soln:

A signal is represented as $\sum A \sin 2\pi ft$.

$x(t) = \sum A \cos 2\pi ft$ (or) $\sum A \sin 2\pi ft$

$$\therefore x_a(t) = 3\cos 2\pi \times 25t + 10\sin 2\pi 150t - \cos 2\pi \times 50t$$

$\therefore F_1 = 25\,Hz, \; F_2 = 150\,Hz, \; F_3 = 50\,Hz$

$\therefore F_{max} = 150\,Hz$.

Nyquist rate $= 2F_{max} = 2 \times 150$

$= \underline{\underline{300\,Hz}}$.

Representation of Discrete time Signal:

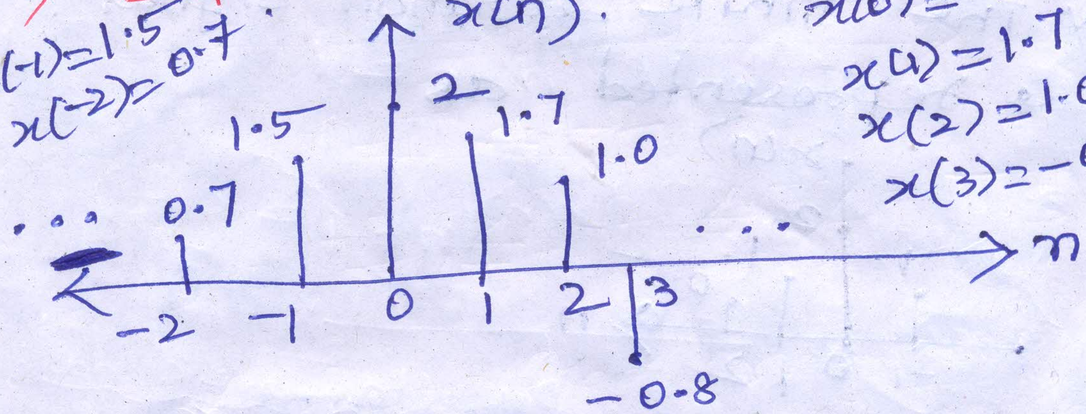
A discrete time signal can be represented in different ways.

a) Graphical representation
b) Functional representation
c) Tabular representation
d) Sequence representation.

a) Graphical Representation:

$x(-1) = 1.5$
$x(-2) = 0.7$

$x(0) = 2$
$x(1) = 1.7$
$x(2) = 1.0$
$x(3) = -0.8$



b) Functional representation:

$$x(n) = \begin{cases} 0.7, & n = -2 \\ 1.5, & n = -1 \\ 2, & n = 0 \\ 1.7, & n = 1 \\ 1.0, & n = 2 \\ -0.8, & n = 3 \\ 0, & \text{elsewhere} \end{cases}$$

c) Tabular representation:

n	...	-2	-1	0	1	2	3	...
$x(n)$	...	0.7	1.5	2	1.7	1.0	-0.8	...

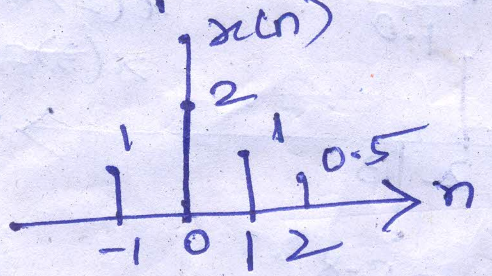
d) Sequence Representation:

$$x(n) = \{\ldots 0.7, 1.5, \underset{\uparrow}{2}, 1.7, 1.0, -0.8, \ldots\}$$

The time origin $(n=0)$ is indicated by $\uparrow$ symbol.

* The above is an infinite duration signal.

* The finite duration signal is represented as



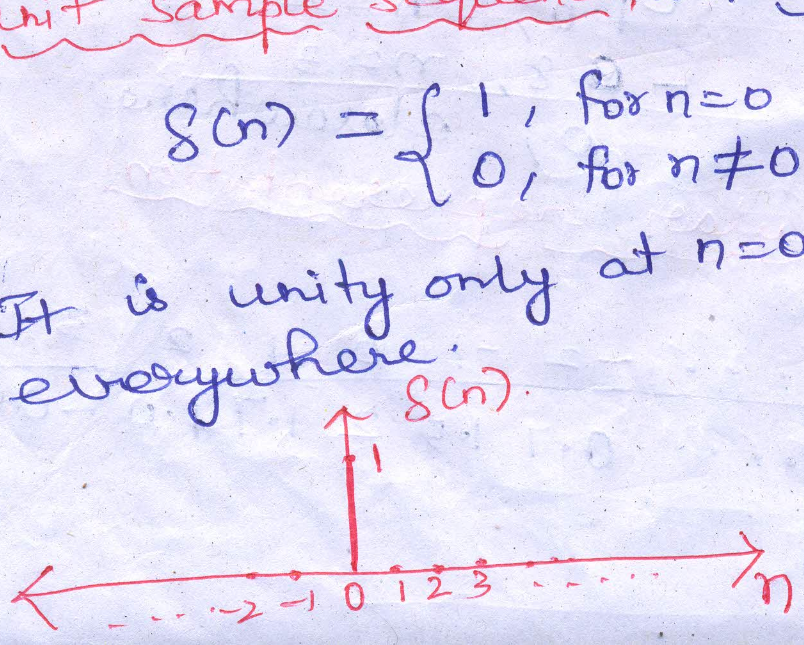
$$x(n) = \{1, \underset{\uparrow}{2}, 1, 0.5\}$$

Elementary Discrete time Signal:

Unit Sample Sequence: (or) Unit impulse:

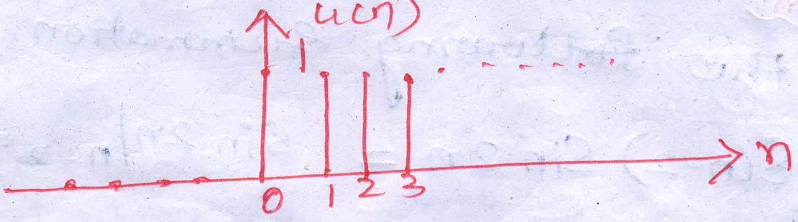
$$\delta(n) = \begin{cases} 1, & \text{for } n=0 \\ 0, & \text{for } n \neq 0 \end{cases}$$

It is unity only at $n=0$ & zero everywhere.



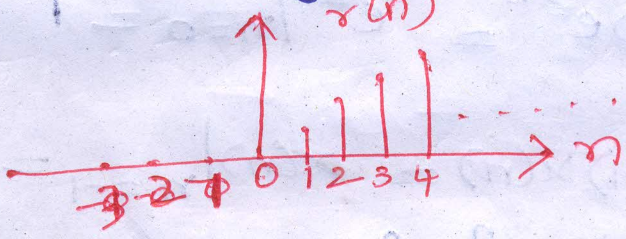
Unit step signal:

$$u(n) = \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0. \end{cases}$$



Unit ramp sequence:

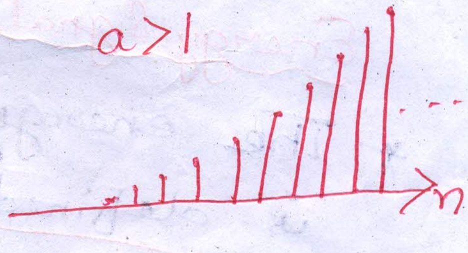
$$r(n) = \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0. \end{cases}$$



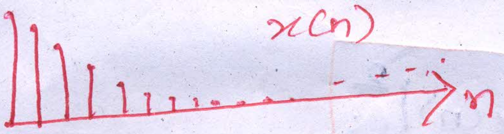
Exponential signal:

$x(n) = a^n$ for all n.

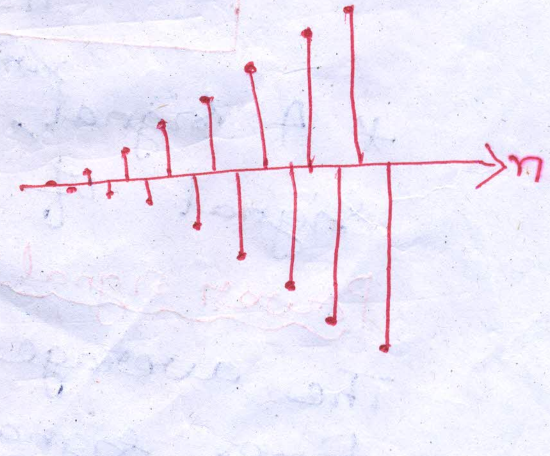
$a > 1$



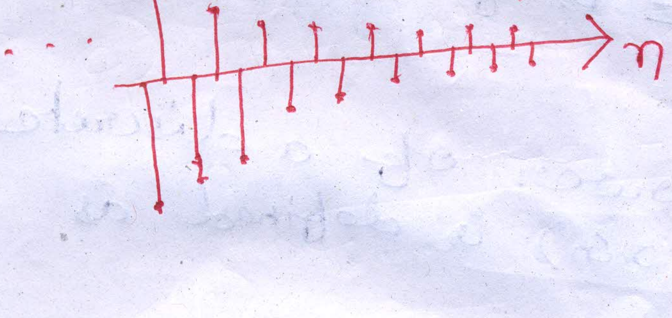
$0 < a < 1$



$a < -1$



$-1 < a < 0$



Sinusoidal Signal:

$$x(n) = A\cos(\omega_0 n + \phi)$$

Problem:

Find the following summation

(i) $\sum_{n=-\infty}^{\infty} \delta(n-2)\sin 2n = \sin 2n\big|_{n=2} = \sin 4$

$\delta(n-2) = 1$ for $n=2$

$= 0$ elsewhere

(ii) $\sum_{n=0}^{\infty} \delta(n) e^{2n} = e^{2n}\big|_{n=0} = 1$

(iii) $\sum_{n=-\infty}^{\infty} \delta(n+1) x(n) = x(n)\big|_{n=-1} = x(-1)$

(iv) $\sum_{n=0}^{\infty} \delta(n+1) e^{-2n} = 0$

Classification of Discrete Time signal:

Energy Signal:

* The energy E of a signal $x(n)$ is defined as

$$\boxed{E = \sum_{n=-\infty}^{\infty} |x(n)|^2}$$

* A signal $x(n)$ is called as energy signal if E is finite. (i.e. $0 < E < \infty$)

Power signal:

The average power of a discrete time signal $x(n)$ is defined as

$$\boxed{P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N} |x(n)|^2}$$

If P is finite, the signal is called a power signal.

Problem:

Determine the value of power and energy of the following signals.

(i) $x(n) = (1/3)^n u(n)$.

$$\text{Energy, } E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$E = \sum_{n=-\infty}^{\infty} \left[(1/3)^n u(n)\right]^2$$

$$= \sum_{n=0}^{\infty} \left[(1/3)^n\right]^2 \quad \left[\because u(n) = 1 \text{ for } 0 \le n < \infty\right]$$

$$= \sum_{n=0}^{\infty} (1/3)^{2n} = \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n$$

$$= \frac{1}{1 - 1/9} = \boxed{9/8} \quad \left[1 + a + a^2 + \dots \infty = \frac{1}{1-a}\right].$$

$$\text{Power, } P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left[(1/3)^n u(n)\right]^2$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1/3)^{2n}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^{n}$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \left(\frac{1-(1/9)^{N+1}}{1-(1/9)} \right)$$

$$= \boxed{0}$$

∴ The signal is a energy signal as it has finite energy

(ii) $x(n) = e^{j\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)}$.

Energy,

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$\therefore E = \sum_{n=-\infty}^{\infty} \left| e^{j\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)} \right|^2$$

$$E = \sum_{n=-\infty}^{\infty} 1 \qquad [\because |e^{j\theta}|^2 = 1]$$

$$\therefore \boxed{E = \infty}$$

Power,

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left| e^{j\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)} \right|^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} 1$$

$$= \lim_{N \to \infty} \frac{2N+1}{2N+1} = \lim_{N \to \infty} 1 = \boxed{1}$$

Symmetric (even) & Antisymmetric (odd) signal:

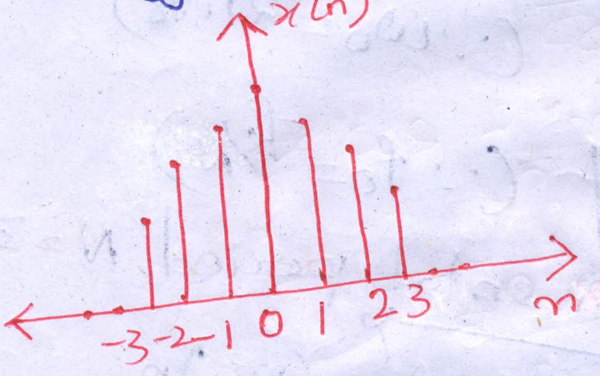
* A real-valued signal $x(n)$ is called symmetric (even) if

$$\boxed{x(-n) = x(n).}$$ eg: cos ωn

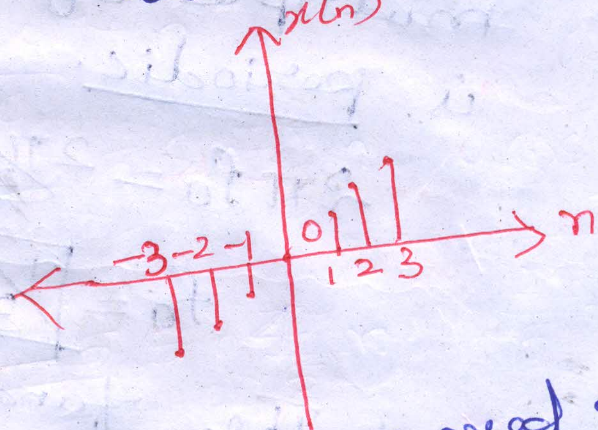
* A signal $x(n)$ is called antisymmetric (odd) if

$$\boxed{x(-n) = -x(n).}$$ eg: sin ωn

Even (or) Symmetric signal



Odd (or) Antisymmetric signal:



* A signal $x(n)$ can be expressed as sum of odd and even components.

$$\boxed{x(n) = x_e(n) + x_o(n)}$$

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

Causal and Non causal signal:

* A signal $x(n)$ is causal if its value is zero for $n<0$.

eg: $a^n u(n) \Rightarrow \{\underset{\uparrow}{1}, 2, -3, -1\}$

* If a signal $x(n)$ has its value non-zero for $n<0$, then its non-causal signal. $a^n u(-n+1)$, $\{1, -2, \underset{\uparrow}{1}, 4, 3\}$

* A signal that is zero for all $n \geq 0$ is called anticausal signal.

Problem:

Consider the analog signal

$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$.

(a) What is the Nyquist rate for this signal?

General format $x_a(t) = \cos 2\pi F t$ (or) $\sin 2\pi F t$

$*\ 2\pi F_1 = 2000\pi$

$F_1 = 1000$

$*\ 2\pi F_2 = 6000\pi$

$F_2 = 3000$

$*\ 2\pi F_3 = 12000\pi$

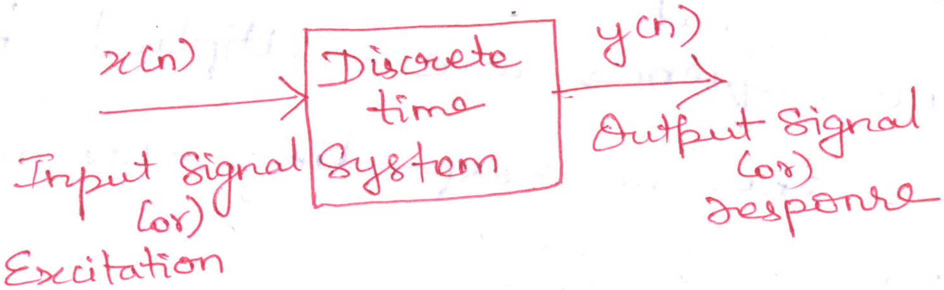
$F_3 = 6000$

$F_{max} = 6000$

$\therefore F_N = 2 * F_{max} = 2 \times 6000 = 12000 = \boxed{12\ kHz}$

Discrete time System:

Discrete time system is a device (or) algorithm that operates on a discrete time signal, called the input (or) excitation, according to some well defined rule, to produce another discrete time signal called the output (or) response of the system.



$$y(n) = T[x(n)]$$

↑ Transformation (operator).

eg: Determine the response of the following system to input signal

$$x(n) = \begin{cases} |n|, & -3 \le n \le 3 \\ 0, & \text{otherwise.} \end{cases}$$

(a) $y(n) = x(n)$ (identity system).

$$x(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}.$$

$$y(n) = \{\ldots, 0, 3, 2, 1, \underset{\uparrow}{0}, 1, 2, 3, 0, \ldots\}.$$

Classification of Discrete-time system:

Discrete time system based upon its characteristics can be classified as follows:

① Static and Dynamic systems.
② Causal and Non-causal system.
③ Linear and Non-linear system.
④ Time variant and Time Invariant system.

Static and Dynamic System:

Static System: A discrete-time system is said to be static (or) memoryless if its output at instant 'n' depends only on input sample at the same time, but not on past (or) future samples of input.

* The systems which don't obey this are called dynamic system (or) with memory.

* If the o/p of system depends on input samples from interval "n-N" to n, then if $0 < N < \infty$, the system has finite memory, if $N = \infty$, the system has infinite memory.

Problem:

Find whether the following system are static (or) dynamic.

(i) $y(n) = ax(n)$,
$y(0) = ax(0)$; $y(1) = ax(1)$, o/p depends on only present input.
i.e. Static system

Causal and Non causal system:

* A system is said to be causal if the output of the system depends only on present and past input $\{x(n), x(n-1), x(n-2)..\}$ but not on future inputs $\{x(n+1), x(n+2),...\}$.
* Mathematically it can be represented as

$$y(n) = F\left[x(n), x(n-1), x(n-2), \ldots\right]$$

* If a system does not satisfy above condition, it is called non-causal system.

Problem:

Determine whether the following systems are causal (or) non-causal.

(i) $y(n) = x(n) - x(n-1)$.

$$y(-1) = x(-1) - x(-2)$$
$$y(0) = x(0) - x(-1)$$
$$y(1) = x(1) - x(0)$$

$\therefore$ o/p depends on present & past i/ps

$\therefore$ Causal system.

(ii) $y(n) = x(2n)$

$$y(-1) = x(-2)$$
$$y(0) = x(0)$$
$$y(1) = x(2)$$

$\therefore$ o/p depends on future i/ps.

$\therefore$ Non causal system.

(iii) $y(n) = \sum_{k=-\infty}^{n} x(k)$ — causal system.

$y(-1) = \sum_{k=-\infty}^{-1} x(k)$, $y(0) = \sum_{k=-\infty}^{0} x(k)$ — Causal system.

(ii) $y(n) = x(n^2)$

$y(2) = x(4)$ ∴ o/p depends on future i/p

∴ Noncausal System.

(ii) $y(n) = x(-n)$

$y(-1) = x(1)$

$y(0) = x(0)$ ∴ o/p depends on present and past i/p.

$y(1) = x(-1)$

∴ Non causal system.

(ii) $y(n) = a\,x(n)$

* Causal System.

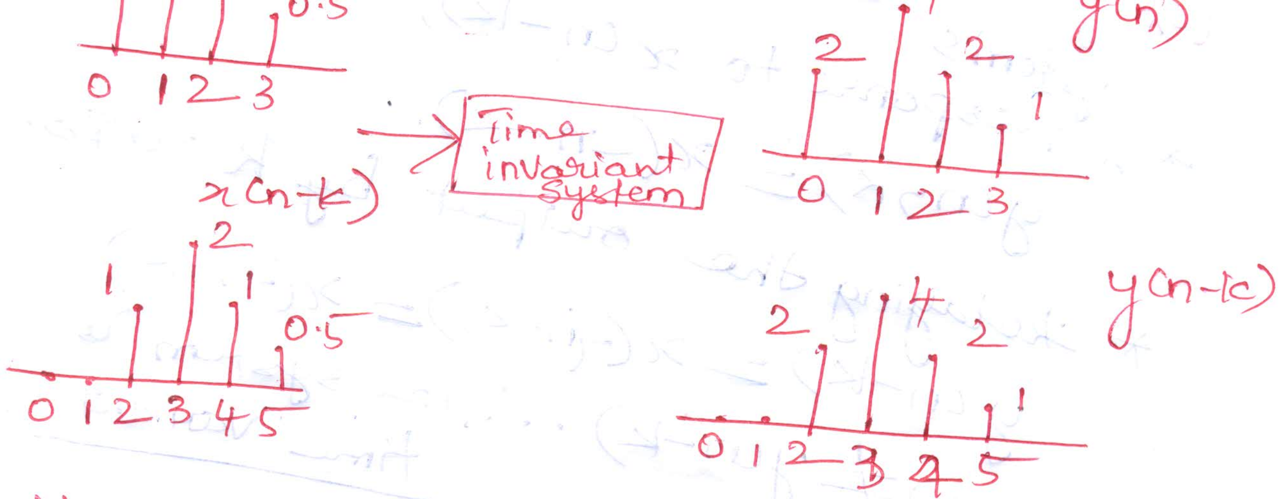
Time invariant and Time variant system

A system is time invariant (or) shift invariant if & only if

$$x(n) \xrightarrow{T} y(n)$$

$$x(n-k) \xrightarrow{T} y(n-k).$$

* Let $y(n,k)$ be response to the input $x(n-k)$. i.e. $y(n,k) = T(y(n-k))$.
* Let $y'(n-k)$ be $y(n)$ delayed by k samples.
* If $y(n-k) = y(n,k)$, then system is time invariant.
* otherwise the system is time variant.



Problem:

Determine if the systems are time invariant (or) time variant.

(a) $y(n) = \mathcal{T}[x(n)] = x(n) - x(n-1)$.

The response to the input $x(n-k)$,

$$\boxed{y(n,k) = x(n-k) - x(n-k-1)}$$

The output $y(n)$ delayed by k units,

$$\boxed{y(n-k) = x(n-k) - x(n-k-1)}$$

$$y(n,k) = y(n-k).$$

$\therefore$ The system is time invariant

(b) $y(n) = \mathcal{T}[x(n)] = n\,x(n)$.

The response to the input $x(n-k)$,

$$\boxed{y(n,k) = n\,x(n-k).}$$

The output $y(n)$ delayed by k units.

$$\boxed{y(n-k) = (n-k)\,x(n-k)}$$

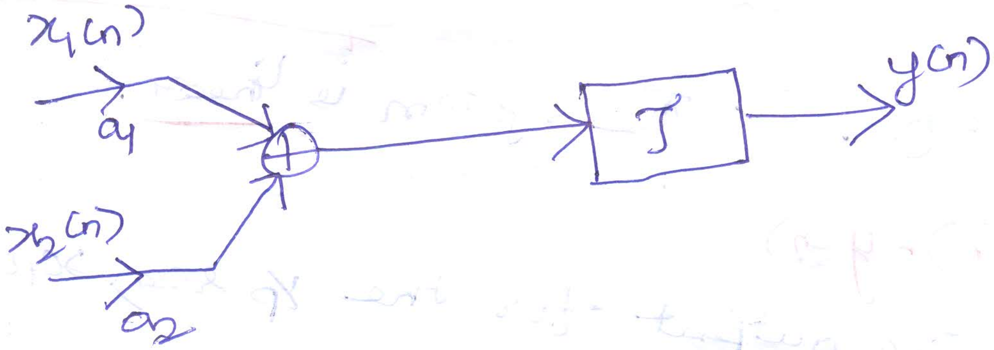
$y(n-k) \neq y(n,k)$ $\therefore$ time variant System.

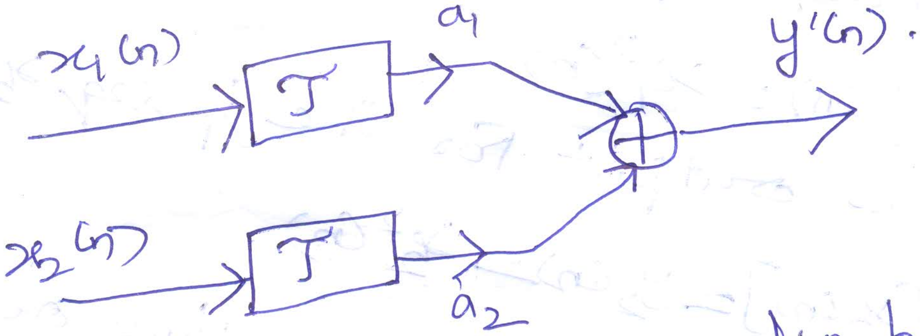
Linear and non linear system:

A System is linear if and only if

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

for any arbitrary input sequences $x_1(n)$ & $x_2(n)$, and any arbitrary constants a_1 and a_2.





* The above property can also be called as Superposition Principle.

Problem:

Determine if the following systems are linear (or) non-linear.

① $y(n) = n\,x(n)$.

* The output to the input sequence $x_1(n)$,

$T[x_1(n)] = y_1(n) = n\,x_1(n)$.

* The output to the input sequence $x_2(n)$,

$T[x_2(n)] = y_2(n) = n\,x_2(n)$.

A linear combination of two inputs produces the following output.

$T[a_1 x_1(n) + a_2 x_2(n)] = n[a_1 x_1(n) + a_2 x_2(n)]$

$= \boxed{n a_1 x_1(n) + n a_2 x_2(n)} \longrightarrow$ ①.

* $a_1 T[x_1(n)] + a_2 T[x_2(n)] = \boxed{a_1 n x_1(n) + a_2 n x_2(n)} \longrightarrow$ ②

① = ② . ∴ The system is linear.

② $x^2(n) = y(n)$

* The output for the i/p seq $x_1(n)$ is

$T[x_1(n)] = y_1(n) = x_1^2(n)$

* The output for the i/p seq $x_2(n)$ is

$T[x_2(n)] = y_2(n) = x_2^2(n)$

* Linear combination of two outputs.

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1^2(n) + a_2 x_2^2(n)$ → ①

* Output produced for linear combination of input

$T[a_1 x_1(n) + a_2 x_2(n)] = [a_1 x_1(n) + a_2 x_2(n)]^2$

$= a_1^2 x_1^2(n) + 2 a_1 a_2 x_1(n) x_2(n) + a_2^2 x_2^2(n) \longrightarrow$ ②.

① ≠ ② ∴ The system is non-linear

* Check ...
Static (or) dynamic, linear (or) non-linear, time variant (or) invariant, causal (or) non-causal, stable (or) unstable.

① $y(n) = \cos[x(n)]$.

* $y(n) = \cos[x(n)]$
$y(0) = \cos[x(0)]$
$y(-1) = \cos[x(-1)]$
$y(-2) = \cos[x(-2)]$

∴ The system is static
* o/p depends only on present input.
* ∴ The system is causal.

* $y_1(n) = \cos[x_1(n)]$, $T[x_2(n)] = y_2(n) = \cos[x_2(n)]$

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 \cos[x_1(n)] + a_2 \cos[x_2(n)] \rightarrow ①$

$T[a_1 x_1(n) + a_2 x_2(n)] = \cos[a_1 x_1(n) + a_2 x_2(n)]$ ②

① ≠ ②

∴ The system is non-linear

* $y(n,k) = T[x(n-k)] = \cos[x(n-k)] \rightarrow ①$

$y(n-k) = \cos[x(n-k)] \rightarrow ②$.

① = ②. ∴ The system is time invariant.

x ———————— x.

② $y(n) = x(-n+2)$.

* $y(1) = x(-1+2) = x(1)$
$y(0) = x(2)$
$y(-1) = x(3)$.

∴ The o/p depends not on present input alone
∴ The system is dynamic.

The o/p depends on future and past inputs.
∴ The system is non-causal.

* $T[x_1(n)] = y_1(n) = x_1(-n+2)$

$T[x_2(n)] = y_2(n) = x_2(-n+2)$.

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1(-n+2) + a_2 x_2(-n+2) \leftarrow ①$

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 x_1(-n+2) + a_2 x_2(-n+2) \leftarrow ②$

① = ②. Therefore the system is <u>Linear</u>

* $y(n,k) = T[x(n-k)] = x_1(-n+2-k) \rightarrow ①$

$y(n-k) = x_1(-(n-k)+2) = x_1(-n+k+2) \rightarrow ②$

① ≠ ② ∴ Therefore the system is <u>time Variant</u>.

---

③ $y(n) = x(2n)$.

* $y(0) = x(0)$

$y(1) = x(2)$

$y(-1) = x(-2)$

* O/p depends on past and future inputs.
* ∴ The system is <u>dynamic</u> and <u>non-causal</u>

* $T[x_1(n)] = x_1(2n)$, $T[x_2(n)] = x_2(2n)$

$a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1(2n) + a_2 x_2(2n) \leftarrow ①$

$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 x_1(2n) + a_2 x_2(2n) \leftarrow ②$

① = ②. ∴ The system is <u>linear</u>

* $y(n,k) = T[x(n-k)] = x(2n-k) \rightarrow ①$

$y(n-k) = x(2(n-k)) = x(2n-2k) \rightarrow ②$

① ≠ ② ∴ The system is <u>time variant</u>.

(+) $y(n) = x(n) \cos \omega_0 (n)$.

* $y(0) = x(0) \cos \omega_0 (0)$
  $y(1) = x(1) \cos \omega_0 (+1)$
  $y(-1) = x(-1) \cos \omega_0 (-1)$

* The o/p depends on only present input.
  ∴ The system is causal and static

* $T[x_1(n)] = x_1(n) \cos \omega_0 (n)$
  $T[x_2(n)] = x_2(n) \cos \omega_0 (n)$
  $a_1 T[x_1(n)] + a_2 T[x_2(n)] = a_1 x_1(n) \cos \omega_0 (n) + a_2 x_2(n) \cos \omega_0 (n)$ ①

  $T[a_1 x_1(n) + a_2 x_2(n)]$
  $= a_1 x_1(n) \cos \omega_0 n + a_2 x_2(n) \cos \omega_0 (n) \to$ ②
  ① = ②
* The system is ~~time~~ linear.

* $y(n,k) = T[x(n-k)] = x(n-k) \cos \omega_0 n$ ①
  $y(n-k) = x(n-k) \cos (n-k) \to$ ②
  ① ≠ ②
* The system is time variant.

(4) $y[n] = 2x(n-1) + x[-n]$.

* $y(0) = 2x(-1) + x(0)$
  $y(1) = 2x(0) + x(-1)$
  $y(-1) = 2x(-2) + x(1)$.

* o/p depends on future & past input.
  ∴ The system is non-causal and dynamic

* $T[x_1(n)] = 2x_1[n-1] + x_1[-n]$
  $T[x_2(n)] = 2x_2[n-1] + x_2(-n)$