Unit - IV

MARKOVIAN QUEUEING MODELS.

S. No	Operating Characteristics	MODEL -I (M/M/1):(∞/FCFS)	MODEL-II (M/M/S):(∞/FCFS)
1	Probability of **n** customers in the system	$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$	$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0, & n = 1,2,\dots s \\ \frac{1}{s!\, s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0, & n = s+1,\dots \end{cases}$
2	Probability of **no** customers in the system	$P_0 = 1 - \left(\frac{\lambda}{\mu}\right)$	$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!\,(1-\frac{\lambda}{s\mu})}\right]^{-1}$
3	Probability of busy system = traffic intensity = Probability that an arrival has to wait	$\frac{\lambda}{\mu}$	$P(n \geq 2) = \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\,(1-\frac{\lambda}{s\mu})}$
4	Probability of empty(idle) system = Probability that an arrival enters the service without waiting	1-Prob(busy system)	1-Prob(busy system)
5	Expected(Average) number of customers in the system= queue size = line length	$L_s = \frac{\lambda}{\mu - \lambda}$	$L_s = L_q + \frac{\lambda}{\mu}$
6	Expected number of customers in the queue = queue length	$L_q = L_s - \frac{\lambda}{\mu}$	$L_q = \frac{\left(\frac{\lambda}{\mu}\right)^{s+1} P_0}{s\,(s!)\left(1-\frac{\lambda}{s\mu}\right)^2}$
7	Expected waiting time of a customer in the system	$W_s = \frac{L_s}{\lambda}$	$W_s = \frac{L_s}{\lambda}$
8	Expected waiting time of a customer in the queue	$W_q = \frac{L_q}{\lambda}$	$W_q = \frac{L_q}{\lambda}$
9	Average length of non empty queue = length of queue that form from time to time	$L_n = \frac{\mu}{\mu - \lambda}$	$L_n = \frac{\lambda}{s\mu - \lambda}$
10	Average waiting time of a customer in a non empty queue = waiting time of an arrival who has to wait	$W_n = \frac{\mu}{\mu - \lambda}$	$W_n = \frac{1}{s\mu - \lambda}$

S. No	MODEL -I (M/M/1):(∞/FCFS)	MODEL-II (M/M/S):(∞/FCFS)
11	(i)Probability that the number of customers in the system ≥ r $= P(n \geq r) = \left(\frac{\lambda}{\mu}\right)^r$ (ii)Probability that the number of customers in the system > r $= P(n > r) = \left(\frac{\lambda}{\mu}\right)^{r+1}$	Probability that some one will be waiting $= \frac{\left(\frac{\lambda}{\mu}\right)^{s+1} P_0}{s\,(s!)\left(1-\frac{\lambda}{s\mu}\right)}$
12	(i)Probability that the waiting time of customers in the system > t $= P(T_s > t) = e^{-(\mu-\lambda)t}$ (ii)Probability that the waiting time of customers in the queue > t $= P(T_q > t) = \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}$	(i) Efficiency of M/M/S model $= \frac{s - \sum_{n=0}^{s-1}(s-n)P_n}{s}$ (ii) Probability of any server being idle $= \frac{\sum_{n=0}^{s-1}(s-n)P_n}{s}$ (iii) Average number of idle servers $= \sum_{n=0}^{s-1}(s-n)P_n$

S. No.	Operating Characteristics	MODEL -III (M/M/1):(N/FCFS)	MODEL-IV (M/M/S):(N/FCFS)
1	Probability of **n** customers in the system	$P_n = \left(\frac{\lambda}{\mu}\right)^n P_0$	$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0, & n = 1,2,\dots s \\ \frac{1}{s!\, s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0, & n = s+1,\dots N \end{cases}$
2	Probability of **no** customers in the system	$P_0 = \begin{cases} \frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{N+1}}, & \lambda \neq \mu \\ \frac{1}{N+1}, & \lambda = \mu \end{cases}$	$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!} \sum_{n=s}^{N} \left(\frac{\lambda}{s\mu}\right)^{n-s}\right]^{-1}$
3	Expected number of customers in the system	$L_s = \frac{\frac{\lambda}{\mu}}{1-\frac{\lambda}{\mu}} - \frac{(N+1)\left(\frac{\lambda}{\mu}\right)^{N+1}}{1-\left(\frac{\lambda}{\mu}\right)^{N+1}}$ = N/2 for $\lambda = \mu$	$L_s = L_q + \frac{\lambda'}{\mu}$ where $\lambda' = \mu(s - \sum_{n=0}^{s-1}(s-n)P_n)$
4	Expected number of customers in the queue	$L_q = L_s - \frac{\lambda'}{\mu}$ where $\lambda' = \mu(1 - P_0)$	$L_q = \frac{1}{s!}\left(\frac{\lambda}{\mu}\right)^s \left[\frac{\rho(1-\rho^{N-S})}{(1-\rho)^2} - \frac{(N-S)\rho^{N-S+1}}{1-\rho}\right] P_0$ where $\rho = \lambda/s\mu$
5	Expected waiting time of a customer in the system	$W_s = \frac{L_s}{\lambda'}$	$W_s = \frac{L_s}{\lambda'}$
6	Expected waiting time of a customer in the queue	$W_q = \frac{L_q}{\lambda'}$	$W_q = \frac{L_q}{\lambda'}$
7	Probability of a customer turned away = probability of overflow	$\left(\frac{\lambda}{\mu}\right)^k P_0$	$\frac{1}{s!\, s^{k-s}}\left(\frac{\lambda}{\mu}\right)^k P_0$

Utilization factor $= \frac{\lambda}{\mu}$ for (M/M/1)

$= \frac{\lambda}{s\mu}$ for (M/M/S)