St. JOSEPH'S COLLEGE OF ENGINEERING, CHENNAI-119.
St. JOSEPH'S INSTITUTE OF TECHNOLOGY, CHENNAI-119.
COURSE: B.E./B.TECH (COMMON TO ALL BRANCHES) - FIRST SEMESTER

MA6151 / MATHEMATICS I - ASSIGNMENT QUESTIONS

UNIT – II – SEQUENCE AND SERIES

PART - A

1. Define Monotonically increasing and Monotonically decreasing sequence with examples
2. State Comparison tests for convergence
3. Test the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ for convergence
4. State D' Alembert's Ratio Test for convergence
5. Test the convergence of the series 5 – 4 – 1+ 5 – 4 –1 +5 – 4 – 1+.....
6. Test the convergence of the series $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{7.8} - \ldots\ldots$
7. Test the convergence of the series $\log 2 + \log\left(\frac{3}{2}\right) + \log\left(\frac{4}{3}\right) + \log\left(\frac{5}{4}\right) + \ldots\ldots$
8. Prove that $\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \ldots\ldots$ converges absolutely.
9. State Leibnitz's Test for convergence
10. Test the convergence of $\sum \frac{n!\,2^n}{n^n}$

PART B

1a) Show that the series $1 + x + x^2 + x^3 + x^4 + \ldots\ldots$to ∞

(i) Converges if | x | < 1 , (ii) Divergent if x ≥ 1
(ii) Oscillates finitely if x = –1 (iv) Oscillates infinitely if x < – 1

1b) Show that the series $\sum \frac{1}{n^p}$ converges if p >1 and diverges if p ≤ 1

2a) Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n\,(\log n)^p}$, $(p > 0)$

2b) Discuss the Convergence of the series $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$

3a) Test the convergence of the series $1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{16}{17}x^3 + \ldots + \frac{2^n - 2}{2^n + 1}x^{n-1} + \ldots, x > 0$

3b) Discuss the Convergence of the series $\sum \frac{1}{n^2+1}$

4a) Examine the convergence of the series $\sum_{1}^{\infty} \frac{n}{(n+1)(n+2)} x^n$

4b) Test the convergence of the series $\frac{1}{1.2.3}+\frac{1}{2.3.4}+\frac{1}{3.4.5}+...$

5a) Test the convergence of series $\sum_{n=1}^{\infty}\left[\frac{1}{\sqrt{n}+\sqrt{n+1}}\right]$

5b) Test the convergence of series $1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+...$

6a) Examine the series $1-\frac{x^2}{2^2}+\frac{x^4}{2^2 4^2}-\frac{x^6}{2^2 4^2 6^2}+...$ for absolute convergence

6b) Test for convergence , absolute convergence and conditional convergence of the series $1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+...$

7a) Test the Convergence of the series $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+...$

7b) Show tha the Exponential series $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...$ converges absolutely for all values of x

UNIT – IV – DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES

PART - A

1. Find $\frac{du}{dt}$ when $u=\sin\left(\frac{x}{y}\right)$ *and* $x=e^t,\ \ y=t^2$
2. If $x^y+y^x=c, find\ \frac{dy}{dx}$
3. If $u=\frac{y^2}{2x}, v=\frac{x^2+y^2}{2x}, find\ \frac{\partial(u,v)}{\partial(x,y)}$
4. If $x=r\cos\theta, y=r\sin\theta$, verify that $\frac{\partial(x,y)}{\partial(r,\theta)}\times\frac{\partial(r,\theta)}{\partial(x,y)}=1$
5. Prove that $JJ'=1$
6. Show that $f(z)=\frac{x^2y}{x^4+y^2},\ z\neq 0\ \ and\ \ f(0)=0$ is discontinuous at $z=0$.
7. Find the stationary points of $f(x,y)=x^2+y^2+xy+ax+by$
8. State Taylor's series expansion for a function of two variables
9. Write the Taylor's series expansion of x^y near the point (1,1) up to first degree terms .
10. State the sufficient conditions for f(x,y) to have a maximum value at (a,b).

PART B

11. (i) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$

(ii) If $g(x,y) = \psi(u,v)$ where $u = x^2 - y^2$ & $v = 2xy$, prove that

$$\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} = 4(x^2+y^2)\left[\frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2}\right]$$

12. (i) Expand $e^x \cos y$ about $\left(1, \frac{\pi}{4}\right)$ upto third degree terms using Taylor's series.

(ii) Expand the function sin(xy) in powers of (x – 1) and $\left(y - \frac{\pi}{2}\right)$ upto 2nd degree terms

13. (i) Expand $e^x \log(1 + y)$ in powers of x and y upto terms of third degree

(ii) Find the shortest and longest distances from (1,2,-1) to the sphere $x^2 + y^2 + z^2 = 24$, using Lagrange's method of maxima and minima

14. (i) Show that the functions $u = x + y - z, v = x - y + z, w = x^2 + y^2 + z^2 - 2yz$ are functionally dependent. Find the relation between them.

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if

$$y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}, y_3 = \frac{x_1 x_2}{x_3}$$

15.(i) If $x + y + z = u, y + z = uv, z = uvw$, prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$

(ii) If $u = 2xy, v = x^2 - y^2, x = r\cos\theta, y = r\sin\theta$, compute $\frac{\partial(u,v)}{\partial(r,\theta)}$

16. (i) Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

(ii) Examine $f(x,y) = x^3 + y^3 - 3xy$ for maximum and minimum values.

17. (i) A rectangular box open at the top is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction.

(ii) Find the maximum volume of the largest rectangular parallelopiped that can be inscribed in an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

18. (i) Find the extreme values of $x^2y^2(1-x-y)$

(ii) Find the maximum value of $x^m y^n z^p$ when $x+y+z=a$

UNIT – V – MULTIPLE INTEGRALS

PART - A

1). Evaluate $\int_1^2 \int_0^x \frac{1}{x^2+y^2}\,dx\,dy$

2). Evaluate $\int_2^a \int_2^b \frac{dx\,dy}{xy}$

3). Shade the region of integration $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} dxdy$

4). Change the order of integration in $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)\,dx\,dy$

5). Evaluate $\int_0^a \int_0^{\sin\theta} r\,drd\theta$

6). Transform the integration $\int_0^\infty \int_0^y dx\,dy$ into polar coordinates.

7). Compute the entire area bounded by $r^2 = a^2 \cos 2\theta$.

8). Express the region bounded by $x \geq 0,\ y \geq 0,\ z \geq 0,\ x^2+y^2+z^2 \leq 1$ as a triple integral.

9). Evaluate $\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\,dx\,dy\,dz$

10). Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \ \ dx\,dy\,dz$

PART - B

11). (i) Evaluate $\iint_R \frac{e^{-y}}{y}\,dx\,dy,$ where R is the region bounded by the lines $x=0,\ x=y,$ and $y=\infty$

(ii) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dx\, dy$ and hence evaluate it.

12). (i) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate it.

(ii) By transforming into polar coordinates, Evaluate $\iint_R \frac{x^2y^2}{\sqrt{x^2+y^2}}\, dx\, dy$ over the annular region R between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, $(b>a)$.

13). (i) Transform the integral into polar co-ordinates and hence evaluate $\int_0^\infty \int_0^\infty e^{-\left(x^2+y^2\right)}\, dx\, dy$

(ii) Transform the integral into polar co-ordinates and hence evaluate $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\, dx\, dy$

14). (i) Evaluate $\iint_R (x+y)^2\, dx\, dy$, where R is the parallelogram in the xy-plane with vertices $(1,0),(3,1),(2,2),(0,1)$ using the transformation $u=x+y$ and $v=x-2y$

(ii) By using the transformation $x+y=u,\ y=uv$, show that $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}}\, dy\, dx = \frac{e-1}{2}$.

15). (i) Find the smaller of the area bounded by $y=2-x$ and $x^2+y^2=4$

(ii) Find the area lying inside the circle $r=a\sin\theta$ and outside the cardioid $r=a(1-\cos\theta)$

16). (i) Find the surface area of the sphere $x^2+y^2+z^2=a^2$

(ii) Find the area of the portion of the cylinder $x^2+z^2=4$ lying inside the cylinder $x^2+y^2=4$

17). (i) Find the common area between the curves $y^2=4x$ *and* $x^2=4y$

(ii) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

18). (i) Find the volume of the tetrahedron bounded by the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and the coordinate planes.

(ii) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz\, dy\, dx}{\sqrt{a^2-x^2-y^2-z^2}}$