ASSIGNMENT 5

UNIT – III: RANDOM PROCESSES

PART - A

1 Define (a) Continuous-time random process (b) Discrete state random process.

2 Define Wide sense stationary process.

3 Examine whether the Poisson process {X(t)} is stationary or not.

4 Is a Poisson process a continuous time Markov chain? Justify your answer.

5 Define Chapman-Kolmogrov Equation.

6 What are the properties of Poisson process?

7 The number of particles emitted by a radioactive source is Poisson distributed. The source emits particles at the rate of 6 per minute. Each emitted particle has a probability of 0.7 of being counted. Find the probability that 11 particles are counted in 4 minutes.

8 When do you say that a Markov chain is irreducible?

PART B

1 a The process {X(t)} whose probability distribution under certain condition is given by

$$P[X(t)=n]=\begin{cases}\dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n=1,2,3,\cdots\\[2mm] \dfrac{at}{1+at}, & n=0.\end{cases}$$

. Show that {X(t)} is not stationary.

(MAY 2012, DEC 2013)

b Show that the process $X(t)=A\cos\lambda t+B\sin\lambda t$ is wide sense stationary, if E (A) =E(B) = 0, E(A^2) = E(B^2) and E(AB) = 0, where A and B are random variables.

(MAY 2013)

2 a Prove that (i) difference of two independent Poisson processes is not a Poisson process and (ii) Poisson process is a Markov process. (MAY 2013)

b Suppose that customers arrive at a bank according to Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of two minutes
(i) exactly 4 customers arrive (ii) greater than 4 customers arrive
(iii) fewer than 4 customers arrive. (MAY 2012, DEC 2013)

3 a The transition probability matrix of a Markov chain {X(t)}, n = 1,2,3,…. having three states 1, 2 and 3 is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $P^{(0)}=(0.7 \quad 0.2 \quad 0.1)$.

Find $(i)\ P[X_2=3]$ $(ii)\ P[X_3=2, X_2=3, X_1=3, X_0=2]$. (MAY 2012,2014, DEC 2013)

b A gambler has Rs. 2. He bets Re. 1 at a time and wins Re. 1 with probability ½. He stops playing if he loses Rs. 2 or wins Rs. 4. (1) What is the tpm of the related Markov chain?

(2) What is the probability that he has lost his money at the end of 5 plays? (MAY 2013)

4 a Show that the random process $X(t) = A\sin\left(w_0 t + \theta\right)$ is wide-sense stationary, if A and w_0 are constants and θ is uniformly distributed random variable in $(0,2\pi)$

(DEC 2011,MAY 2014)

b Let the Markov Chain consisting of the states 0, 1, 2, 3 have the transition probability matrix

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$ Determine which states are transient and which are recurrent by defining transient and recurrent states. (MAY 2010)