Unit-IV
ANALYTIC FUNCTIONS

1. C-R Equations In Cartesian Coordinates :

$$\boxed{u_x = v_y \text{ and } u_y = -v_x}$$

$$f\,'(z) = u_x + iv_x$$

C-R Equations In Polar Coordinates :

$$\boxed{u_r = \frac{1}{r}v_\theta \text{ and } v_r = -\frac{1}{r}u_\theta}$$

$$f\,'(z) = e^{-i\theta}\,[u_r + iv_r]$$

2. Milne Thomson Method

If u is given $f(z) = \int [u_x(z,0) - iu_y(z,0)]\,d\,z + ic$

If v is given $f(z) = \int [v_y(z,0) + i\,v_x(z,0)]\,d\,z + c$

3. $\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} = 4\dfrac{\partial^2}{\partial z\partial \bar{z}}$

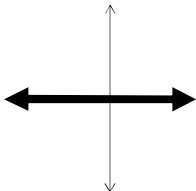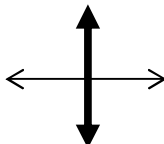**4. Re[f(z)] = u =** $\dfrac{f(z) + f(\bar{z})}{2}$ **5.** $\left| f(z) \right|^2 = f(z)f(\bar{z})$

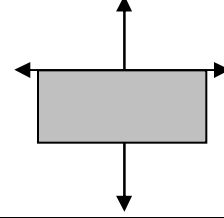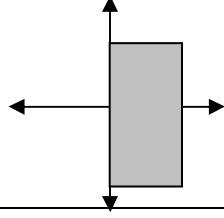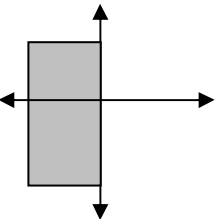**6. Fixed point (Invariant points) are obtained by replacing w = z**
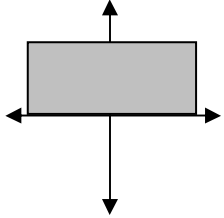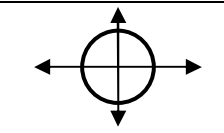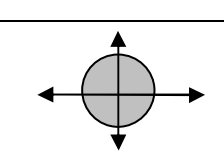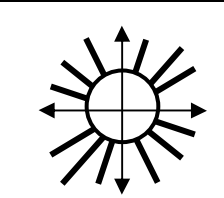
7. Critical points are given by $\dfrac{dw}{dz} = 0 \ \ and \ \ \dfrac{dz}{dw} = 0$

8. Bilinear transformation which maps the points $z_1, z_2 \ and \ z_3$ **of Z – plane onto the points** $w_1, w_2 \ and \ w_3$ **of W- plane is given by**

$$\frac{(w - w_1)(w_2 - w_3)}{(w_1 - w_2)(w_3 - w)} = \frac{(z - z_1)(z_2 - z_3)}{(z_1 - z_2)(z_3 - z)}$$

9.

S.No.	Description	Z – plane	W - plane	Fig
1	Real axis	y = 0	v = 0	
2	Imaginary axis	x = 0	u = 0	

3	Upper half plane	$y > 0$	$v > 0$	
4	Lower half plane	$y < 0$	$v < 0$	
5	Right half plane	$x > 0$	$u > 0$	
6	Left half plane	$x < 0$	$u < 0$	
7	Unit circle with centre at the origin	$\left| z \right| = 1$	$\left| w \right| = 1$	
8	Interior of Unit circle with centre at the origin	$\left| z \right| < 1$	$\left| w \right| < 1$	
9	Exterior of Unit circle with centre at the origin	$\left| z \right| > 1$	$\left| w \right| > 1$	
10	Circle with centre at a and radius r	$\left| z - a \right| = r$	$\left| w - a \right| = r$	