Unit – II Sequence & Series

Formula Sheet

Define Geometric series

The series $a + ar + ar^2 + ar^3 + ar^4 + \ldots\ldots..$ is called geometric series. The series converges if $|r| < 1$, diverges if $r \geq 1$ and oscillatory if $r \leq -1$

Harmonic Series(p- Series)

The series $\sum \frac{1}{n^p}$ is called Harmonic Series and it converges if p >1 and diverges if p ≤ 1

Comparison tests for convergence

(a) If there are two series of positive terms $\sum u_n$ *and* $\sum v_n$ such that

(i) $\sum v_n$ Converges (ii) $u_n \leq v_n$ for all values of n , then $\sum u_n$ also converges.

(b) If there are two series of positive terms $\sum u_n$ *and* $\sum v_n$ such that

(i) $\sum v_n$ Diverges (ii) $v_n \leq u_n$ for all values of n , then $\sum u_n$ also diverges.

Comparison Test (Limit Form)

If $\sum u_n$ *and* $\sum v_n$ are two series of positive terms such that $\underset{n\to\infty}{Lt} \frac{u_n}{v_n} = l$, is a finite non –zero quantity then the two series are either convergent or divergent.

Integral test for convergence

A positive term series $\sum u_n = \sum f(n)$, where f(n) decreases as n increases converges or diverges according as the integral $\int_1^\infty f(x)\, dx$ is finite or infinite.

D' Alembert's Ratio Test for convergence

The series $\sum u_n$ of positive terms is convergent if $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} < 1$ and diverges if $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} > 1$

Alternating series

A series with alternately positive and negative terms is called an alternating series.

Leibnitz's Test for convergence for alternating series

An alternating series $u_1 - u_2 + u_3 - u_4 + \ldots..$converges if

(i) Each term is numerically less than its preceding term (i.e.) $u_1 > u_2 > u_3 \ldots.$

(ii) $\lim_{n\to\infty} u_n = 0$

Absolute convergence

If $\sum u_n$ is convergent and $\sum |u_n|$is convergent, then $\sum u_n$ is said to be Absolute Convergent

Define Conditional convergence

If $\sum u_n$ is convergent and $\sum |u_n|$ is divergent, then $\sum u_n$ is said to be conditionally convergent.

Formulae

$(i)\quad \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$

$(ii)\ \lim_{n\to\infty}\left(1+\frac{x}{n}\right)^{n} = e^{x}$

$(iii)\ \lim_{n\to\infty}(1+n)^{1/n} = e$

$(iv)\ \lim_{n\to\infty}\frac{x^{n}}{n!} = 0$

$(v)\quad \lim_{n\to\infty}\frac{n}{(n!)^{1/n}} = e$

$(vi)\ \lim_{n\to\infty}(n)^{1/n} = 1$