

UNIT-I CONDUCTING MATERIALS

Microscopic form of Ohm's law

According to classical free electron theory current density,

Resistance $= R = \rho l / A$

According to Ohm's Law, $V = IR$, $V = I(\rho l / A)$

$$I = VA / \rho l, \quad I / A = (1 / \rho)(V / l), \quad J = \sigma E$$

This is microscopic form of Ohm's Law.

Since the resistivity varies with respect to the temperature, the microscopic form of Ohm's law is not true for all the temperatures.

Electrical conductivity

The amount of electrical charges conducted (Q) conducted per unit time (t) across unit area of cross section (A) of the solid per unit applied electrical field (E).

Sources of resistance in metals

The resistance in metals is due to a) Impurities present in the metals b) Temperature of the metal c) Number of free electrons

Free electrons

Free electron is the electron which moves freely in all direction in the absence of external field. These electrons undergo elastic collisions with each other and the lattice with no loss in energy.

Theories based on free electron theory

- a) Classical free electron theory.
- b) Quantum free electron theory.
- c) Band theory of solids.

Postulates of Classical free electron theory

- In an atom electron revolve around the nucleus and a metal is composed of such atoms.
- The valence electrons of atoms are free to move about the whole volume of the metals like the molecules of a perfect gas in a container.
- These free electrons move in random directions and collide with either positive ions fixed to the lattice or other free electrons. All the collisions are elastic in nature.
- The electron velocities in a metal obey the classical Maxwell – Boltzmann distribution of velocities.
- The movements of free electrons obey the laws of the classical kinetic theory of gases.
- The free electrons move in a completely uniform potential field due to ions fixed in the lattice.

Drift velocity

The average velocity acquired by the free electron in a particular direction, due to the application of electric field is called drift velocity.

Relaxation time

Relaxation time can be defined as the time taken by the free electron to reach its equilibrium position from its disturbed position in the presence of applied field.

Mean free path

The average distance traveled by an electron between two successive collisions is known as mean free path.

Mobility of Electrons

The magnitude of the drift velocity per unit electric field is defined as the Mobility of electrons (μ).

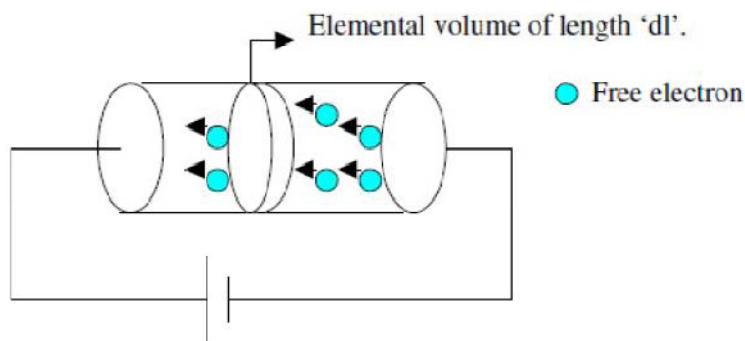
$$\mu = v_d/E$$

Differences between drift velocity and thermal velocity of an electron

S.No	Drift Velocity	Thermal velocity
1	The average velocity acquired by the free electron in the presence of electric field is called drift velocity.	Thermal velocity is the velocity of an electron without any external field
2	The electrons moving with drift velocity moves in the direction opposite to that of the field direction	The direction of the electrons moving with thermal velocity is random

Differences between electrical conductivity and thermal conductivity

S.No	Electrical conductivity	Thermal conductivity
1	The co-efficient of electrical conductivity is defined as the quantity of electricity flowing per unit area per unit time maintained at unit potential gradient	The co-efficient of thermal conductivity is defined as the quantity of heat conducted per unit area per unit time maintained at unit temperature gradient.
2	Electrical conductivity is purely due to number of free electrons	Thermal conductivity is due to both free electrons and phonons.

Derivation for electrical conductivity

Consider an electric field (E) applied to a metal then the electric Lorentz force given as follows.

$$F_L = eE$$

According to Newton's third law,

$$F = ma$$

$$eE = ma$$

$$a = eE/m$$

$$\text{Drift velocity, } v_d = a\tau = \frac{eE\tau}{m}$$

Relation between current density and drift velocity is

$$J = nev_d$$

$$J = ne(eE\tau/m) = ne^2\tau E / m$$

$$J = \sigma E \text{ so}$$

$$\sigma = ne^2\tau / m$$

Derivation for thermal conductivity

Consider a metal of length equal to mean free path (λ). Let A and B be the two ends of the rod. The heat energy is flown from A to B end. Let T be the temperature at A and T – dT be the temperature at B.

$$\text{The average kinetic energy at A} = \frac{3}{2}kT \quad \text{---- 1}$$

$$\text{The average kinetic energy at B} = \frac{3}{2}k(T - dT) \quad \text{---- 2}$$

$$\text{The kinetic energy carried from A to B} = \frac{3}{2}kdT \quad \text{---- 3 (Eqn. 1 – 2)}$$

$$\text{Number of electrons crossing per unit time per unit area from A to B} = \frac{1}{6}nv \quad \text{----- 4}$$

$$\text{The energy carried from A to B per unit area in unit time} = \frac{1}{4}nvkdT \quad \text{---5}$$

$$\text{Similarly the energy carried from B to A per unit area in unit time} = -\frac{1}{4}nvkdT \quad \text{----- 6}$$

$$\text{Hence the net amount of energy carried from A to B per unit area per unit time } Q = \frac{1}{2}nvkdT \quad \text{-- 7}$$

$$\text{Since } K = \frac{Q}{At \frac{dT}{dx}} \text{ if } t = 1 \text{ A} = 1$$

$$Q = K dT / \lambda \quad \text{----- 8}$$

Comparing equation 8 and 7,

$$K = \frac{1}{2}nv^2k\tau$$

Wiedemann – Franz law

It states that in metals, the ratio of thermal conductivity to electrical is directly proportional to the absolute temperature.

$$\frac{K}{\sigma} \propto T$$

$$\frac{K}{\sigma} = L T$$

$$K = \frac{nv^2 k \tau}{2}$$

$$\sigma = \frac{ne^2 \tau}{m}$$

$$\frac{K}{\sigma} = \frac{1}{2} m v^2 \left(\frac{k}{e^2} \right)$$

$$\frac{K}{\sigma} = \frac{3}{2} \left(\frac{k}{e} \right)^2 T$$

$$\text{where, } L = \frac{3}{2} \left(\frac{k}{e} \right)^2$$

Merits of classical free electron theory

- a) It verifies Ohms law.
- b) It explains the electrical conductivity and thermal conductivity of metals.
- c) It is used to derive Wiedmann – Franz law.

Drawbacks of classical free electron theory

- a) According to this theory, the value of electronic specific heat is $(3/2)R$. But experimentally it is about $0.01 R$ only.
- b) The ratio between thermal conductivity and electrical conductivity is not constant at low temperatures.
- c) The theoretical value of paramagnetic susceptibility is greater than experimental value.
- d) The electrical conductivity of semiconductors, ferromagnetism, photoelectric effect and blackbody radiation cannot be explained.

Fermi distribution function and its behavior with change in temperature

It is the probability function $F(E)$ of an electron occupying a given energy level at absolute temperature. It is given by

$$F(E) = \frac{1}{1 + e^{E-E_F/kT}}$$

Effect of temperature on Fermi function

The variation of Fermi distribution function on temperature as follows

Case 1: Probability occupation for $E < E_F$ at $T = 0$ K

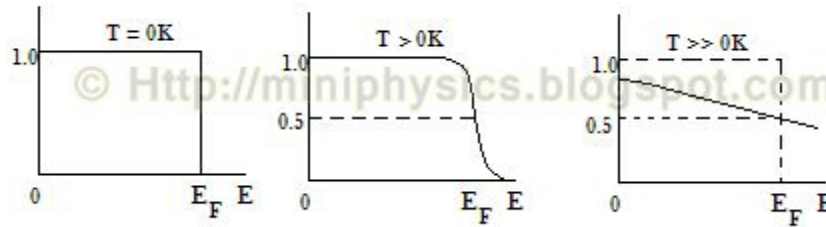
$F(E) = 1$, 100 % possibility is there for an electron to occupy energy level less than E_F

Case 2: Probability occupation for $E > E_F$ at $T = 0$ K

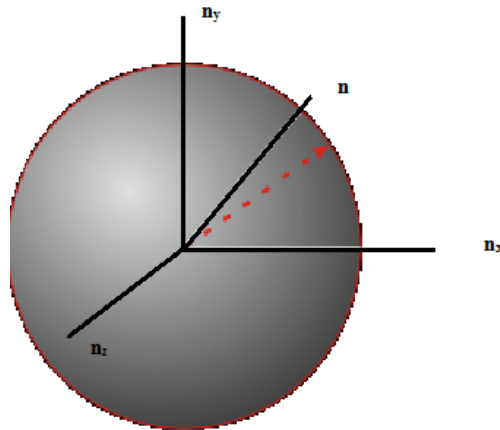
$F(E) = 0$, 0 % possibility is there for an electron to occupy energy level greater than E_F

Case 3: Probability occupation for $E = E_F$ at $T > 0$ K

$F(E) = 0.5$, 50 % possibility is there for an electron to occupy Fermi energy level E_F



Density of states and expression for the number of allowed states per unit volume of a solid



It is defined as the number of available electron states per unit volume in an energy interval E and $E + dE$.

$$Z(E) dE = \frac{N(E)dE}{V}$$

Let us consider a cubical metal piece of side length a . An imaginary sphere is constructed with n as radius where $n^2 = n_x^2 + n_y^2 + n_z^2$ and E is energy corresponds to that quantum no. Let another sphere is drawn with radius $n + dn$ and $E + dE$ is the energy corresponds to that sphere.

The no of energy states within the sphere of radius of radius $n = \frac{4}{3}\pi n^3$

Since the quantum no n_x^2, n_y^2, n_z^2 can have only positive integer values, $1/8$ of the total volume of the sphere alone considered for calculation of electron states.

So the no of energy states within the sphere of radius of radius $n = \frac{1}{8} \left[\frac{4}{3}\pi n^3 \right]$ ----- 1

The no of energy states within the sphere of radius of radius $n + dn = \frac{1}{8} \left[\frac{4}{3}\pi (n + dn)^3 \right]$ ----- 2

Therefore the no energy levels between the spheres of radius n & $n + dn$,

$N(E) = \frac{\pi}{2} n(dn)$ ----- 3 (subtract eqn 1 from 2 & use $(a + b)^3$ formula)

$$E = \frac{n^2 h^2}{8ma^2} \quad n = \left(\frac{8ma^2 E}{h^2} \right)^{1/2} \quad ndn = \frac{8ma^2}{2h^2} dE$$

Substitute n and ndn in eqn. 3

$$N(E) = \frac{\pi}{4} \left(\frac{8ma^2}{h^2} \right)^{3/2} E^{1/2} dE \quad \text{----- 4}$$

To satisfy Pauli's exclusion principle multiply by 2 (each electron states consist of two electron one spin up and spin down),

$$N(E) = 2 \times \frac{\pi}{4} \left(\frac{8ma^2}{h^2} \right)^{3/2} E^{1/2} dE \quad \text{----- 5}$$

$$N(E) = \frac{4\pi}{h^3} (2m)^{3/2} a^3 E^{1/2} dE \quad \text{----- 6}$$

$$Z(E) = \frac{N(E)dE}{V}$$

$$Z(E) = \frac{\frac{4\pi}{h^3} (2m)^{3/2} a^3 E^{1/2} dE}{a^3}$$

$$Z(E) = \frac{4\pi}{h^3} (2m)^{3/2} E^{1/2} dE$$

This is the expression for the density of charge carriers in the energy interval E and E + dE.