**St.JOSEPH'S COLLEGE OF ENGINEERING, CHENNAI -119**
**St.JOSEPH'S INSTITUTE OF TECHNOLOGY, CHENNAI-119**

# MA 6453 – PROBABILITY AND QUEUEING THEORY

## ASSIGNMENT – I

## UNIT I RANDOM VARIABLES

### PART-A

1. A coin is tossed 2 times, if *X* denotes the number of heads, find the probability distribution of *X*. **(Nov/Dec 2013)**
2. If a random variable *X* has the distribution function $F(x) = \begin{cases} 1-e^{-\alpha x} & , \ x > 0, \\ 0 & , \ x \le 0 \end{cases}$ where α is the parameter, then find $P(1 \le X \le 2)$. **(Nov/Dec 2010)**
3. A continuous random variable *X* has the probability density function given by $f(x) = \begin{cases} \lambda(1+x^2), & 1 \le x \le 5 \\ 0 & , \text{otherwise} \end{cases}$. Find $\lambda$ and $P(X < 4)$. **(May/June2014)**
4. Every week the average number of wrong-number phone calls received by a certain mail order house is seven. What is the probability that they will receive two wrong calls tomorrow? **(Nov/Dec 2010)**
5. Obtain the mean for a Geometric random variable. **(Ap/May 2010)**
6. If the moment generating function of *X* is given by $M_X(t) = (0.6e^t + 0.4)^8$ find the mean and S.D of *X*.
7. If a random variable *X* has a uniform distribution in (-1,3). Find $P(|X-1| < 1)$.
8. Obtain the moment generating function of Gamma distribution. **(Nov/Dec 2014)**

### PART - B

1. **a)** In a consignment of electric bulbs, 10 percent are defective. A random sample of 20 is taken for inspection. Find the probability that (1) all are good bulbs (2) atmost there are 2 defective bulbs (3) exactly there are 3 defective bulbs. 161 **(May/June 2013)**

   **b)** Find the MGF of the random variable *X* having the pdf $f(x) = \begin{cases} x & , \ 0 < x < 1 \\ 2-x & , 1 < x < 2 \\ 0 & , \text{otherwise} \end{cases}$

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   **c)** Buses arrive at a specified bus stop at 15 minutes intervals starting at 7.a.m. that is 7.a.m., 7.15a.m., 7.30.,etc. If a passenger arrives at the bus stop at a random time which is uniformly distributed between 7 and 7.30a.m. Find the probability that he waits
   (a) less than 5 minutes 226
   (b) at least 12 minutes for a bus.
2. **a)** A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is '*p*'. Find the value of '*p*' so that the probability that an odd number of tosses required is equal to 0.6. Can you find a value of '*p*' so that the probability is 0.5 that an odd number of tosses are required? **(Dec/Nov 2010)**

   **b)** Assume that the reduction of a person's oxygen consumption during a period of Transcendental Meditation (T.M) is a continuous random variable *X* normally distributed with mean 37.6 cc/min and S.D 4.6 cc/min. Determine the probability that during a period of T.M. a person's oxygen consumption will be reduced by
   (1) at least 44.5 cc/min
   (2) atmost 35.0 cc/min
   (3) anywhere from 30.0 to 40.0 cc/mm **(Ap/May 2012)**

**c)** If the density function of a continuous random variable *X* is given by

$$f(x) = \begin{cases} ax & , \ 0 \le x \le 1 \\ a & , \ 1 \le x \le 2 \\ 3a - ax & , \ 2 \le x \le 3 \\ 0 & , \ \text{elsewhere} \end{cases}$$

i) Find the value of $a$

ii) Find the CDF of *X*

iii) Find $P(X \le 1.5)$

**3. a)** The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = \frac{1}{2}$. 237

(1) What is the probability that the repair time exeeds 2h?

(2) What is the conditional probability that a repair takes at least 10h given that its duration exceeds 9h? **(Dec/Nov 2010)**

**b)** The marks obtained by a number of students for a certain subject is assumed to be normally distributed with mean 65 and standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70? **(Ap/May2011)**

**c)** Find the first four moments about the origin for a random variable *X* having probability density function $f(x) = \begin{cases} \dfrac{4x(9-x^2)}{81} & , \ 0 \le x \le 3 \\ 0 & , \ \text{otherwise} \end{cases}$

**4. a)** The distribution function of a continuous random variable *X* is given by

$$F(x) = \begin{cases} 0 & , \ x < 0 \\ x^2 & , \ 0 \le x \le \frac{1}{2} \\ 1 - \frac{3}{25}\left(3 - x^2\right) & , \ \frac{1}{2} \le x \le 3 \\ 1 & , \ x \ge 3 \end{cases}.$$

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(i) Find the PDF of *X*.

(ii) Evaluate $P(|X| \le 1)$ and $P\left(\frac{1}{3} < X < 4\right)$ using both the CDF and PDF. **(Dec/Nov 2010)**

**b)** The probability function of an infinite discrete distribution is given by $P(X = j) = \frac{1}{2^j}$, $j = 1, 2, 3, \ldots, \infty$. Verify that the total probability is 1 and find the mean and variance of the distribution.

Find also (1) P(*X* is even) (2) P(*X* ≥5) and (3) P(*X* is divisible by 3) **(Dec/Nov 2010)**

**c)** If ***X*** is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$.

Find (1) Mean and $E(X^2)$ (2) $P(X \ge 2)$. **(May/June 2012)**