St. JOSEPH'S COLLEGE OF ENGINEERING, CHENNAI 119

St. JOSEPH'S INSTITUTE OF TECHNOLOGY, CHENNAI 119

MATHEMATICS I (MA6151) COMMON TO ALL

ASSIGNMENT III

UNIT – IV DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES

PART – A

1. If $u = \frac{y}{z} + \frac{z}{x}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$
2. If $x^y + y^x = c$, find $\frac{dy}{dx}$
3. State any two properties of Jacobian
4. Find the Jacobian of the transformation $x = r\cos\theta,\ y = r\sin\theta$
5. If $u = \frac{y^2}{x},\ v = \frac{x^2}{y}$, find $\frac{\partial(u,v)}{\partial(x,y)}$
6. Find Taylor's series expansion of $e^x \sin y$ near the point $\left(-1, \frac{\pi}{4}\right)$ up to first degree terms.
7. Write the sufficient conditions for $f(x,y)$ to have a maximum value at (a,b)
8. Find the stationary points of $f(x,y) = x^2 - xy + y^2 - 2x + y$

PART – B

1(a) If $z = f(x,y)$ where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$$

(b) If $z = f(x,y)$, where $x = u^2 - v^2,\ y = 2uv$, prove that $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2 + v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$

2(a) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1},\ y_2 = \frac{x_1 x_3}{x_2},\ y_3 = \frac{x_1 x_2}{x_3}$

(b) If $u = xy + yz + zx,\ v = x^2 + y^2 + z^2$ and $w = x + y + z$, prove that they are functionally dependent and also determine the functional relationship between u, v, w

3(a) Expand $e^x \log(1+y)$ in powers of x and y up to third degree using Taylor's series.

(b) Find the maximum and minimum values of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$

4(a) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction

(b) Find the volume of the largest rectangular solid which can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$