Unit-5

V. Banupriya
ECE

1. Explain Multiple-Input Multiple-Output Systems.

* MIMO systems are systems with Multiple Element Antennas (MEAS) at both ends of the link.

* MIMO systems can be used for four different purposes.

1. Beamforming
2. Diversity
3. Interference Suppression
4. Spatial Multiplexing.

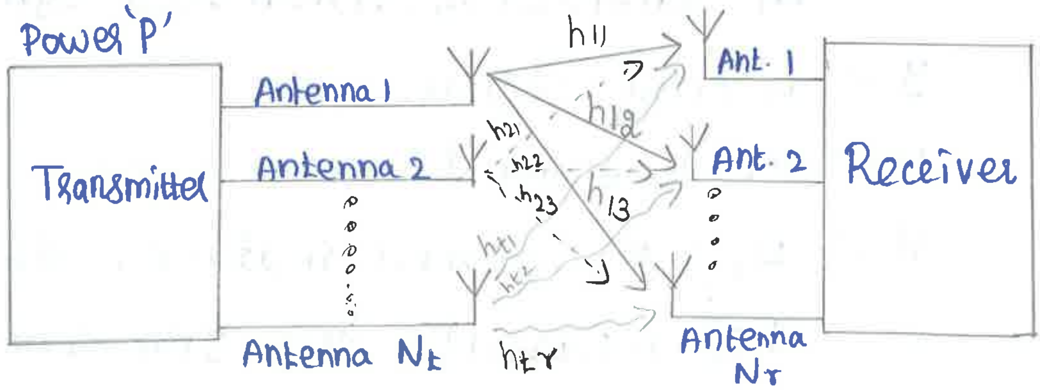
Features of MIMO System:

→ It increases the capacity of the system.

→ It effectively exploits multipath.

→ Spectral efficiency is as high as 20-40 bps.

System Model:



where $\gamma \to$ SNR at each receiver branch.

Fig (a) Block diagram of MIMO system.

* At the transmitter, the data stream enters an encoder, whose outputs are forwarded to N_t transmit antennas.

* From the antennas, the signal is sent through the wireless propagation channel.

* The coherence time of the channel is so long that "large number" of bits can be transmitted within this time.

* The discrete system can be represented by the following discrete-time model:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{N_r} \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1N_t} \\ \vdots & & \vdots \\ h_{N_r1} & \cdots & h_{N_rN_t} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{N_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{N_r} \end{bmatrix}$$

(or) simply,

$$\boxed{y = Hx + n}$$

where, $x \rightarrow N_t$ dimensional transmitted symbol

$y \rightarrow$ received signal.

$n \rightarrow N_r$ dimensional noise vector

$H \rightarrow N_r \times N_t$ channel matrix, where h_{ij} represents the gain from transmit antenna j to receiver antenna i.

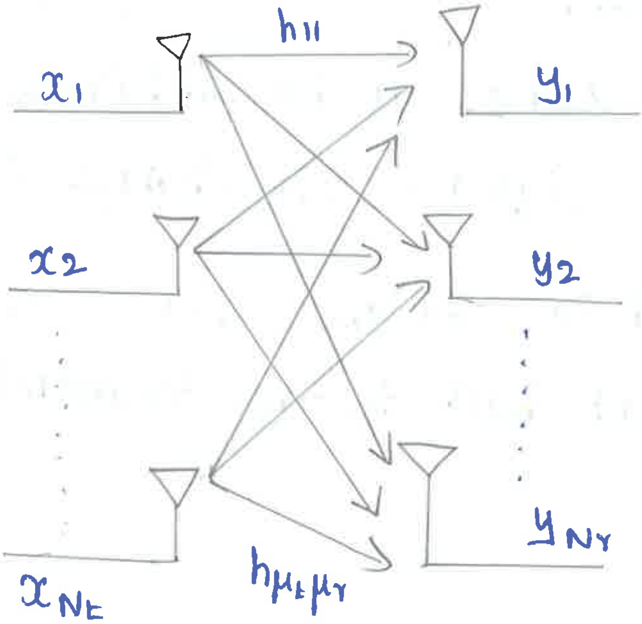


Fig. MIMO system.

* The input symbol in MIMO system should satisfy the following equation under some power constraint.

$$\sum_{i=1}^{N_t} E\left[x_i x_i^*\right] = \rho \quad (\text{or}) \quad T_r(R_x) = \rho$$

where ρ is the average SNR per receive antenna under unity channel gain.

where $T_r(R_x)$ is the trace of the input covariance matrix $R_x = E\left[x x^H\right]$.

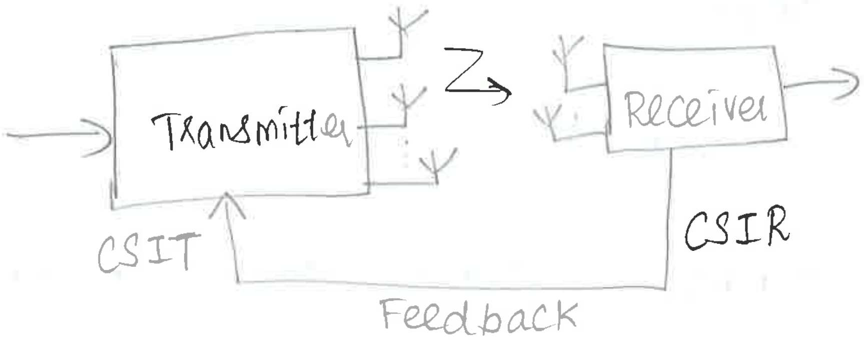
* The knowledge of channel gain matrix H at the transmitter and receiver is referred to as

→ CSIT - Channel state Information at transmitter.

→ CSIR - Channel state Information at receiver.

* For a static channel, CSIR is typically assumed, since the channel gains can be obtained easily by sending a pilot sequence for channel estimation.

* If a feedback path is available then CSIR from the receiver can be sent back to the transmitter to provide CSIT.



* when the channel is not known to either the transmitter (or) receiver then some distribution on the channel gain matrix must be assumed.

* The most common model for this distribution is a zero-mean spatially white Model [ZMSW].

* In this method in the channel matrix H, the entries are assumed to be independent and identically distributed (i.i.d) zero mean, unit variance, complex circularly symmetric Gaussian random variable.

In general,

→ Different assumptions about CSI and about the distribution of H entries lead to different channel capacities.

→ optimal decoding of received signal requires maximum likelihood demodulation.

→ Decoding complexity is reduced if the channel is known to the transmitter.

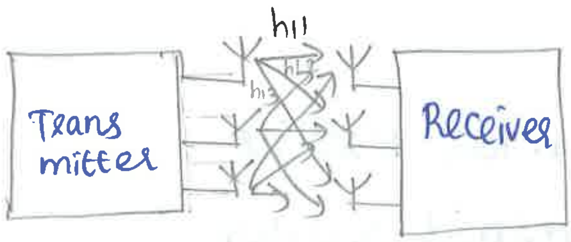
---

2) Explain in detail about precoding techniques.

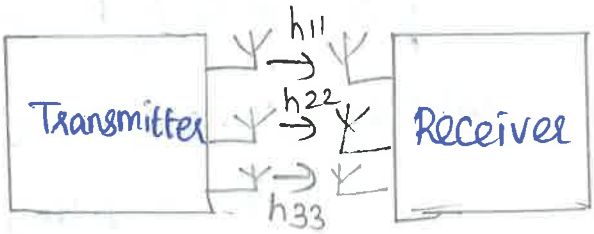
(or)

Explain the parallel decomposition of MIMO channel.

* Precoding improves the SNR value of the MIMO system which states that the effect of the interference can be cancelled by proper coding.



a) Lots of ~~independent~~ Interfering Propagation Paths.



b) Independent Propagation Paths

↓

Here,

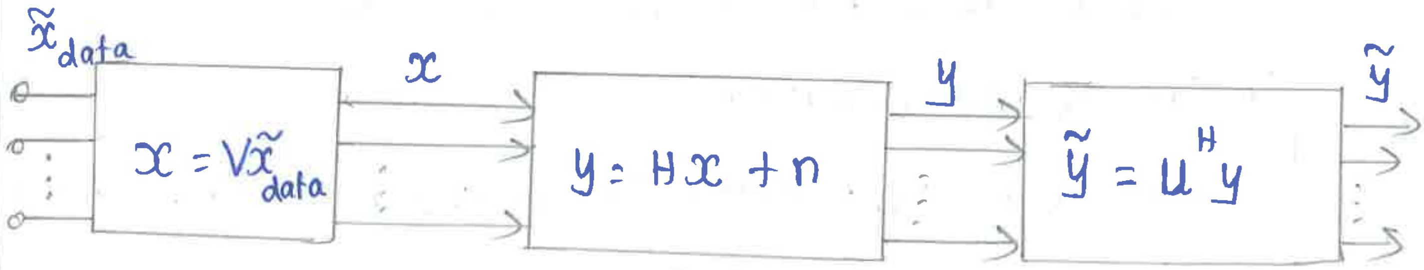
$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

↓

Here,

$$H = \begin{bmatrix} h_{11} & 0 & 0 \\ 0 & h_{22} & 0 \\ 0 & 0 & h_{33} \end{bmatrix}$$

* The transmit precoding and receiver shaping transform the MIMO channel into parallel single-input - single output (SISO) channels with input $\tilde{x}$ and $\tilde{y}$ using singular value decomposition (SVD).



a) Transmit Precoding and receiver shaping

SVD for MIMO:

The singular value decomposition for any matrix 'H' is given by,

$$\boxed{H = U \Sigma V^H} \rightarrow (1)$$

The MIMO received signal is given by,

$$\boxed{\bar{y} = H\bar{x} + \bar{n}} \rightarrow (2)$$

where, $H \rightarrow$ MIMO channel Matrix

$\bar{x} \rightarrow$ MIMO transmit vector.

Sub. (1) in (2)

$$\boxed{\bar{y} = U \Sigma V^H \bar{x} + \bar{n}} \rightarrow (3)$$

At receiver, multiply $\bar{y}$ by U^H.

$$\therefore U^H \bar{y} = \tilde{y} = U^H \left[U \Sigma V^H \bar{x} + \bar{n} \right]$$

$$\tilde{y} = u^H u \Sigma V^H \bar{x} + u^H \bar{n}$$

$$\tilde{y} = \Sigma V^H \bar{x} + u^H \bar{n} \qquad \left[\because u^H u = 1,\ u^H \bar{n} \simeq \tilde{n}\right]$$

$$\boxed{\tilde{y} = \Sigma V^H \bar{x} + \tilde{n}} \longrightarrow (4)$$

At the transmitter, precoding is done.

$$\bar{x} = V\tilde{x} \longrightarrow (5)$$

Sub. (5) in (4)

$$\tilde{y} = \Sigma V^H V \tilde{x} + \tilde{n} \qquad \left[\because V^H V = 1\right]$$

$$\tilde{y} = \Sigma \tilde{x} + \tilde{n}$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} \sigma_1 & & & \\ & \sigma_2 & & \\ & & \ddots & \\ & & & \sigma_t \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_t \end{bmatrix}$$

Solving this, we get,

$$\left.\begin{aligned} \tilde{y}_1 &= \sigma_1 \tilde{x}_1 + \tilde{n}_1 \\ \tilde{y}_2 &= \sigma_2 \tilde{x}_2 + \tilde{n}_2 \\ &\vdots \\ \tilde{y}_t &= \sigma_t \tilde{x}_t + \tilde{n}_t \end{aligned}\right\}$$ seperate collection of 't' parallel channels.

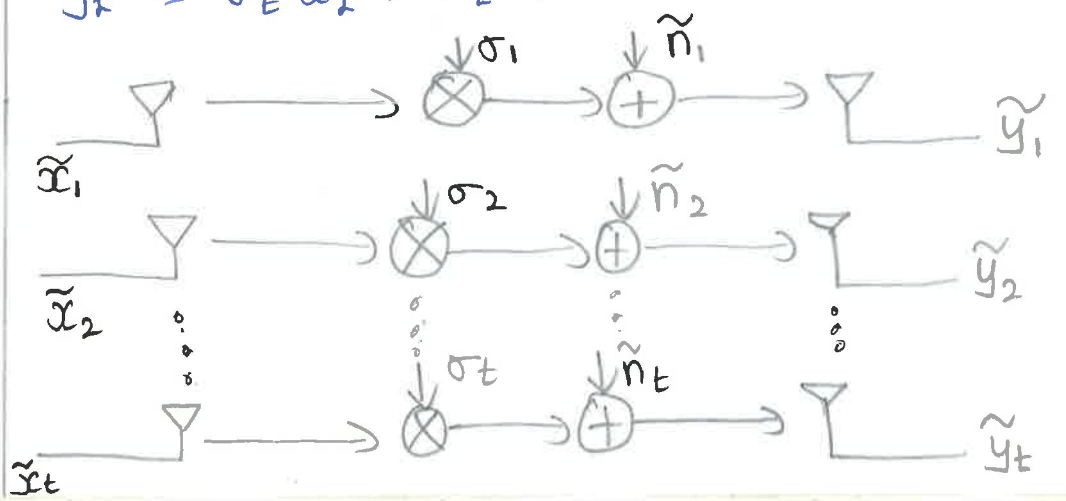


Fig. Parallel decomposition of MIMO channels.

* Thus, the transmit precoding and receiver shaping transform the MIMO channel into 't' parallel independent channels, where ith channel has the input $\tilde{x}_i$, output $\tilde{y}_i$ and noise $\tilde{n}_i$ and channel gain σ_i.

* The capacity of each channel is given by Shanon's capacity theorem.

capacity of ith channel, $C_i = B \log_2(1+SNR)$

(ie) $$\boxed{C_i = B \log_2\left(1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right)}$$

* The total MIMO capacity is given by

$$\boxed{C = \sum_{i=1}^{t} B \log_2\left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right)}$$

3) Explain in detail about the beam forming techniques.

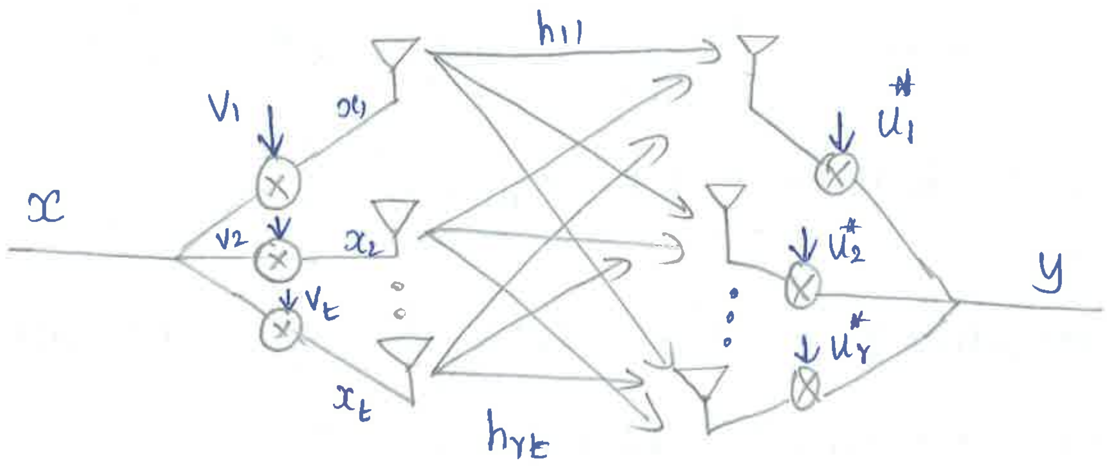
* The multiple antennas at the transmitter and receiver can be used to obtain array and diversity gain instead of capacity gain.

* In this technique, the same symbol weighted by a complex scale factor is sent over each transmit antenna, so that the input covaraiance matrix has unit rank.

* This scheme is referred to as MIMO beamforming.

* A beamforming strategy corresponds to the precoding and shaping matrices being just column vectors:

$$\boxed{V = v} \text{ and } \boxed{U = u}.$$



a) MIMO channel with beamforming

* At the transmitter side, x is sent over i^{th} antenna with weight v_i.

* On the receiver side, the signal received on i^{th} antenna is weighted by u_i^*.

* Both transmit and receive weight vectors are normalized. ie) $\|u\| = \|v\| = 1$.

* The resulting received signal is given by,

$$\boxed{y = u^H H v x + u^H n}$$

WKT, SVD $\Rightarrow$

$y = Hx + n$

$y = u^H y = u^H Hx + u^H n.$

Sub. $x = vx$.

$\therefore y = u^H H v x + u^H n$

* Beamforming provides diversity and array gain via coherent combining of multiple signal paths.

* The capacity of a scalar channel is,

$$C = B \log_2 (1 + SNR)$$

$$\boxed{SNR = \sigma_{max}^2 \rho}$$

where, $\sigma_{max}^2 \rightarrow$ largest singular value of H.

$$\therefore \boxed{C = B \log_2 (1 + \sigma_{max}^2 \rho)}$$

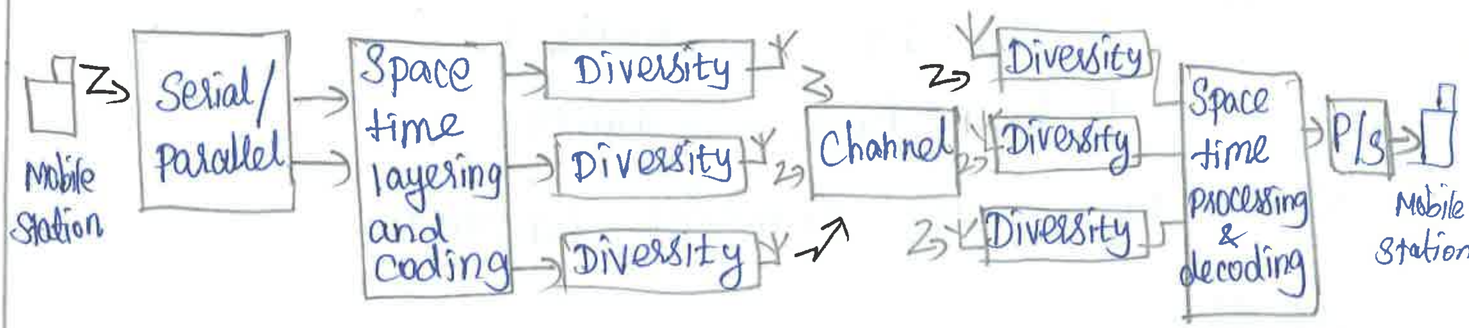
* Beamforming is also achieved by using directional antennas (or) Smart antennas.
* At the receiver side, Maximal ratio combining technique is used to achieve the complete diversity order.

---

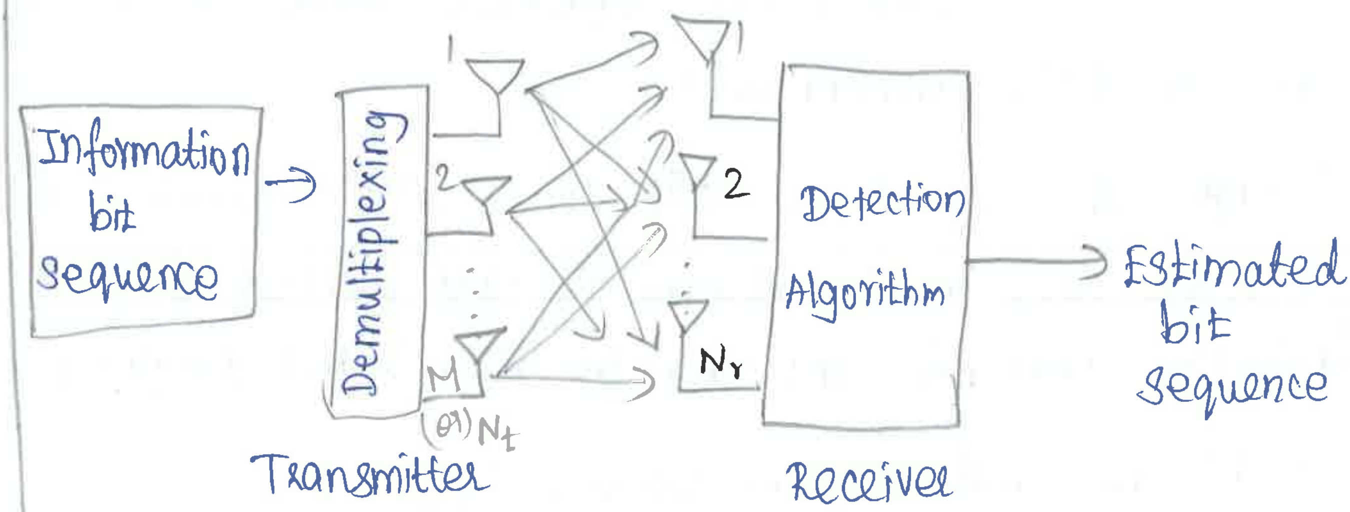
4) Explain the concepts of Spatial Multiplexing :-

* Spatial Multiplexing is a very powerful technique for increasing channel capacity at higher signal to noise ratios (SNR).
* It is used to maximize data rate.
* MIMO Spatial multiplexing achieves this by utilizing the multiple paths and effectively using them as additional 'channels' to carry data.

* But for a single antenna systems it is well known that given a fixed bandwidth, capacity can only be increased logarithmically with the SNR, by increasing the transmit power.



a) Principle behind spatial Multiplexing



b) Representation of Spatial multiplexing

System model :-

* At the transmitter, the information bit sequence is split into M sub-sequences that are modulated and transmitted simultaneously over the transmit antennas using the same frequency band.

* At the receiver, the transmitted sequences are separated by employing an interference cancellation type of algorithm.

* The channel matrix is denoted as,

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ h_{N_r1} & h_{N_r2} & \cdots & h_{N_rN_t} \end{bmatrix}$$

where h_{ij} is the transfer function.

Operation :-

→ Spatial multiplexing techniques simultaneously transmit independent information sequences, often called layers over multiple antennas.

→ High-rate signal is split into multiple lower-rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel.

→ The received signal vector is,

$$\boxed{y = Hx + n}$$

which contains the signals received by N_r antenna elements, where x is the transmit signal and n is the noise vector.

→ Spatial rate is given by, $\boxed{r_s = N_t}$ and $\boxed{\text{diversity order} = N_r}$; In general $\boxed{N_r \geq N_t}$

(6)

* Spatial multiplexing provides high bandwidth efficiency but at low SNR, error rate increases.

X. Layered Space-time Receiver Structure :-

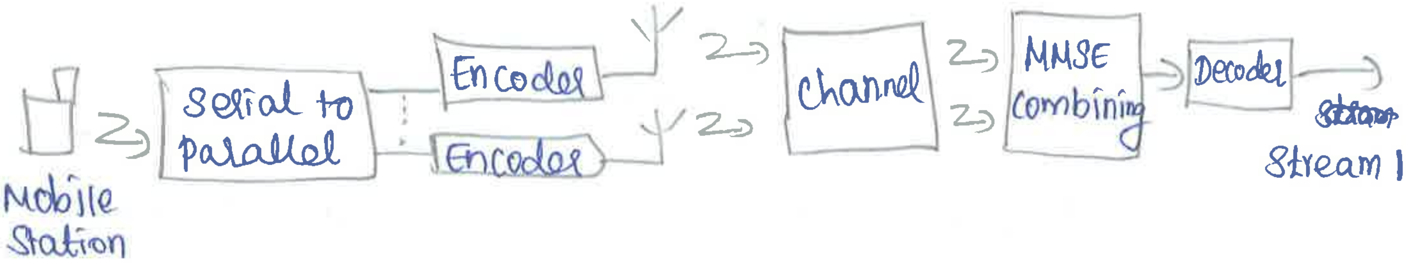
* The layered space time schemes, known as Bell Laboratories layered space time (BLAST) schemes, were developed to acheive transmission rates above one symbol per channel.

* BLAST schemes defines the encoding techniques used in spatial multiplexing.

Horizontal BLAST :- (H-BLAST)

* Horizontal BLAST encoding is the simplest structure.
* The data stream is first converted into N_t parallel streams, each being encoded seperately and then submitted to a different transmit antenna.

$\boxed{N_t = N_r}$ where $l = 1, 2, \ldots N_t$



a) Block diagram of H-Blast transceiver.

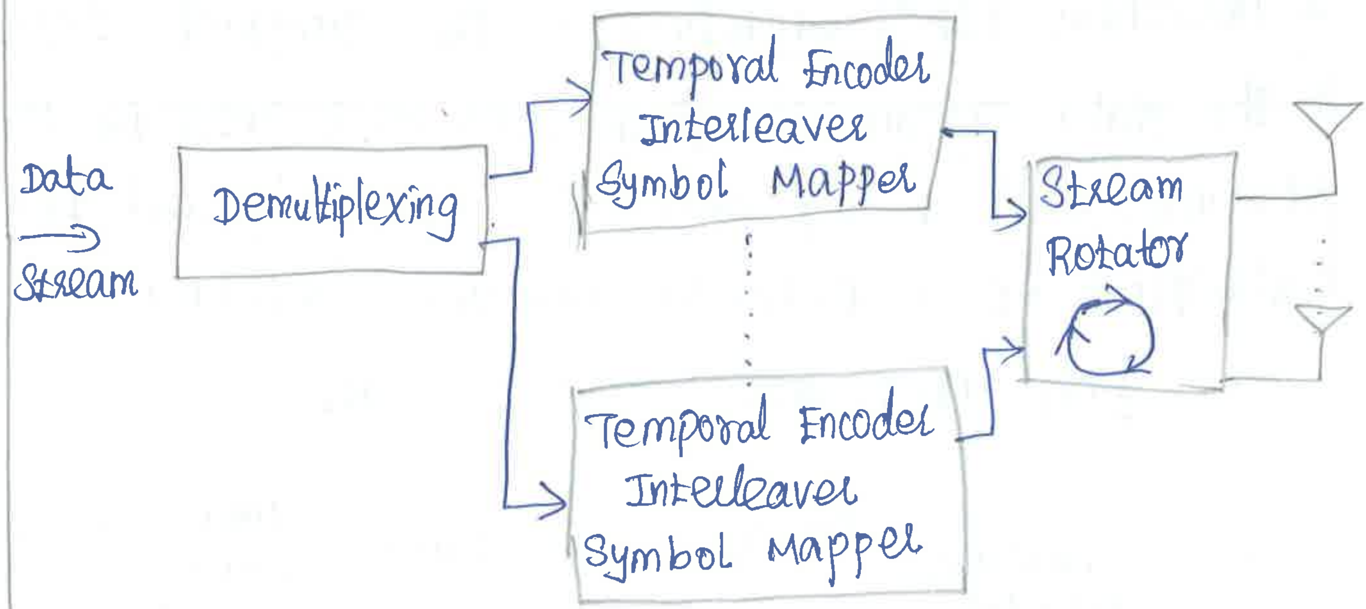
* At the receiver side, these data streams are separated based on optimum combining.

A_1	S_1	S_5	S_9	S_{13}
A_2	S_2	S_6	S_{10}	S_{14}
A_3	S_3	S_7	S_{11}	S_{15}
A_4	S_4	S_8	S_{12}	S_{16}

Fig. Assignment of bit streams to 4 different antennas.

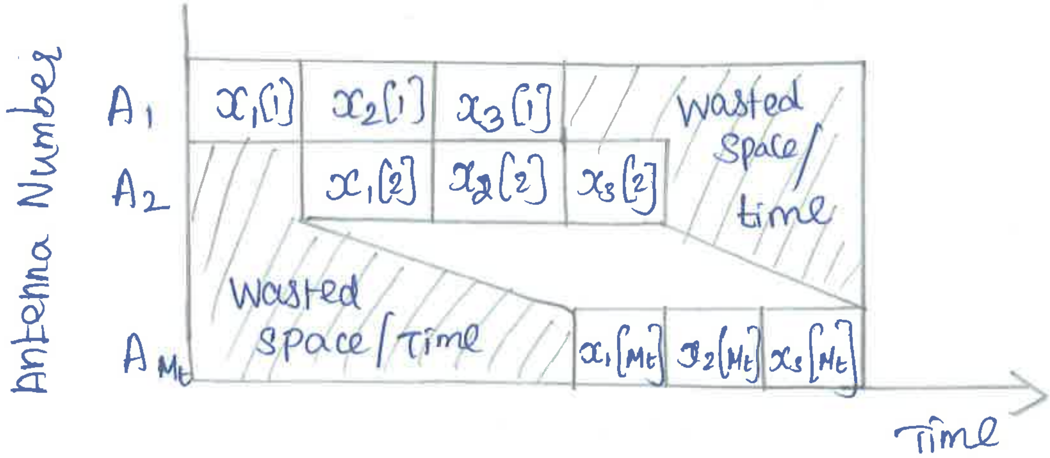
Diagonal BLAST:-

* The data stream is first parallel encoded.
* Each sub-stream is subdivided into many sub-blocks and these sub-blocks are transmitted by different antennas according to a round-robin schedule.

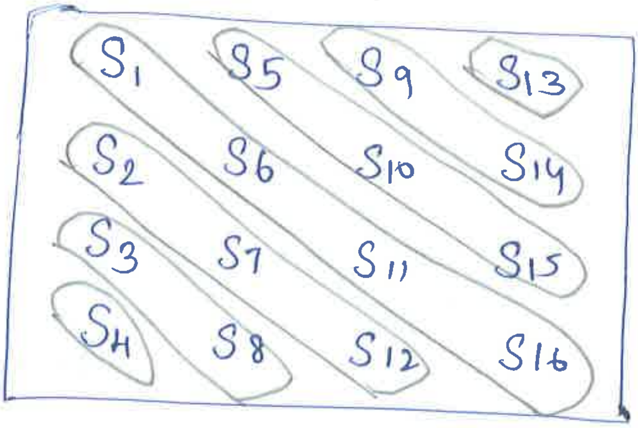


a) Diagonal encoding with stream rotation

* D-BLAST system can acheive maximum capacity with outage. Also D-BLAST acheive full diversity gain $M_t M_r$.



a) stream rotation

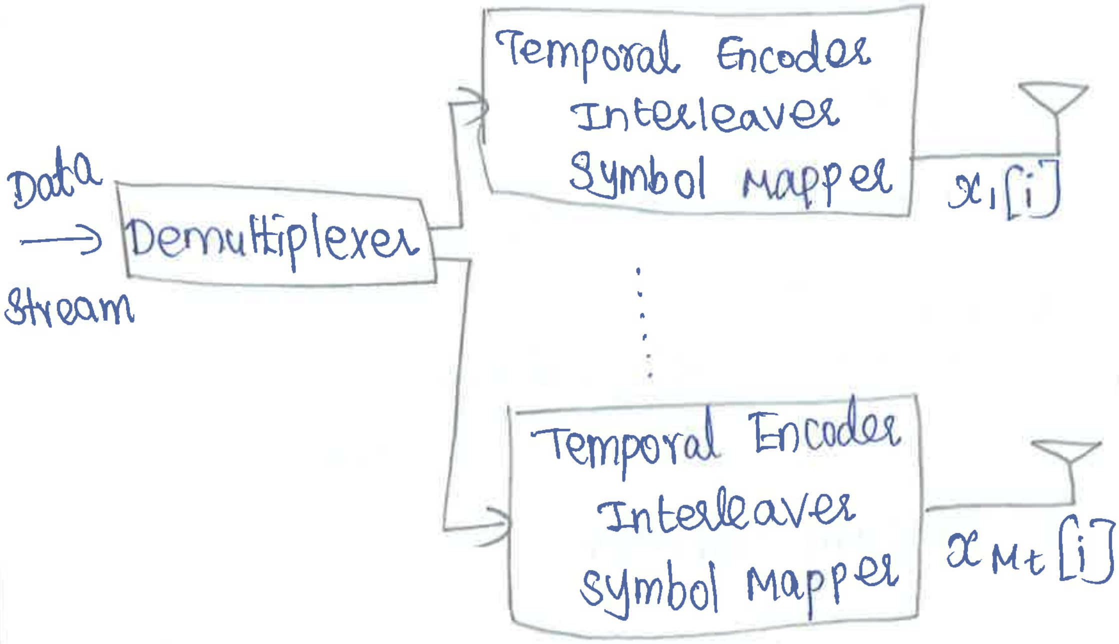


b) Assignment of bit streams to 4 different antennas

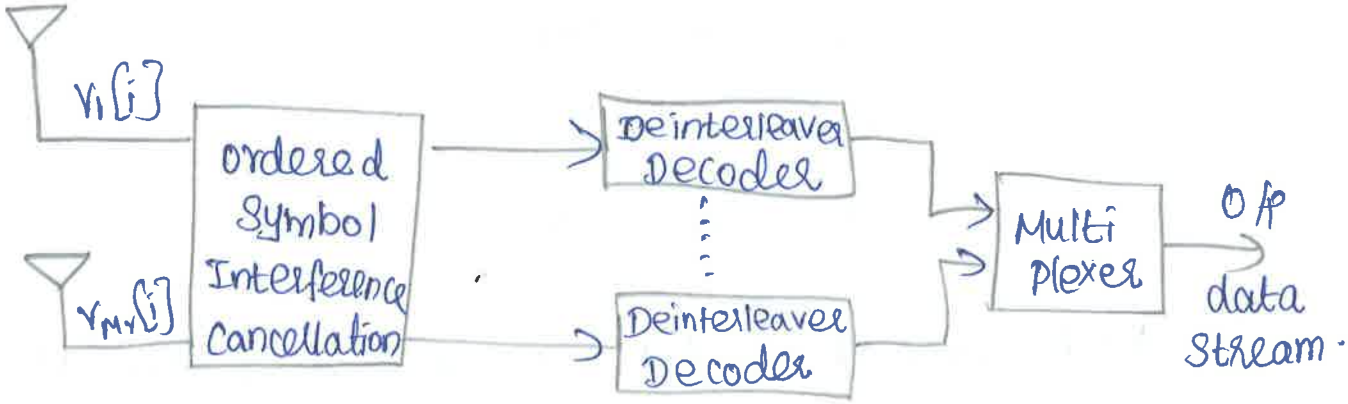
* The receiver decodes each diagonal code independently.

Vertical BLAST: (V-BLAST)

* In V-blast, the bit stream is first temporally encoded, interleaved and symbol mapped.
* The resulting N_s symbols are then demultiplexed into N_t substreams and transmitted over the antennas.
* At the receiver, symbol interference cancellation technique is used.



a) V-BLAST at transmitter.



b) V-BLAST at Receiver.

* Decoding using V-Blast architecture is complex and not suitable for cellular environments.

Explain transmitter Diversity in detail.

* In transmitter diversity N_t transmit antennas and one receiver antenna are used. The transmit power is divided among these antennas.

* ~~Transmit~~ Transmit diversity is desirable in systems where more space, power and processing capability is available on the transmit side. (ex) cellular systems.

Types of transmit diversity :-

* open loop → If the channel state Information (CSI) is unknown at transmitter.

* closed loop → If CSI is known at transmitter.

Open loop - CSI is unknown at transmitter :-

Alamouti Scheme :-

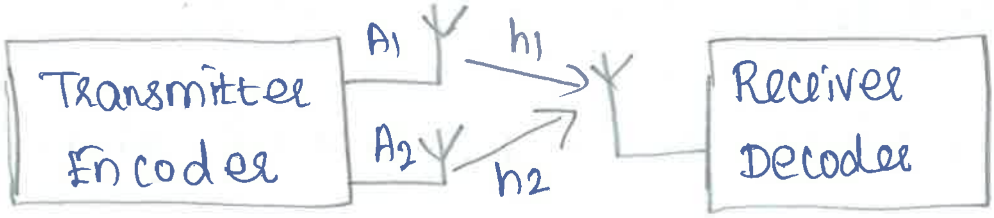
* The transmitter has no CSIT and hence we assume that there are only two-antennas which shares the transmit power equally.

* It is a 2×1 transmit diversity.

2 ↓ Transmitter, 1 → Receiver

* Alamouti's scheme is used in two-antenna transmit diversity.

* CSIR is known in this technique.



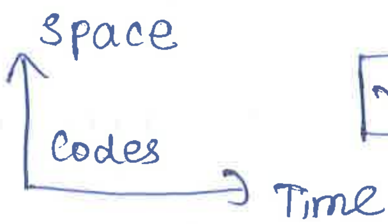
a) 2×1 transmit diversity

* Transmitter sends multiple signals (or) replica of the same signals. These signals interfere at the receiver but if coded properly, the receiver can recover the signal.

* Simplest implementation is using orthogonal space time block codes (or) Alamouti codes.

* The Alamouti code for 2 Transmit antennas is,

$$\begin{array}{ccc} & A_1 & A_2 \\ \text{Time} \quad 0 & \multirow{2}{*}{$\left[\begin{array}{cc} s_1 & s_2 \\ -s_2^* & s_1^* \end{array}\right]$} & \\ 1 & & \end{array}$$



$$\boxed{r(t) = \sqrt{.5}\,(h_1+h_2)\,s(t)}$$ ↳ Received signal

* The received symbol over the first symbol period is,

$$\boxed{y_1 = h_1 s_1 + h_2 s_2 + n_1}$$

* The received symbol over the second symbol period is,

$$\boxed{y_2 = -h_1 s_2^* + h_2 s_1^* + n_2}$$

In general,

(ie)

$$\boxed{y = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}} \Rightarrow \boxed{y = H_A S + n}$$

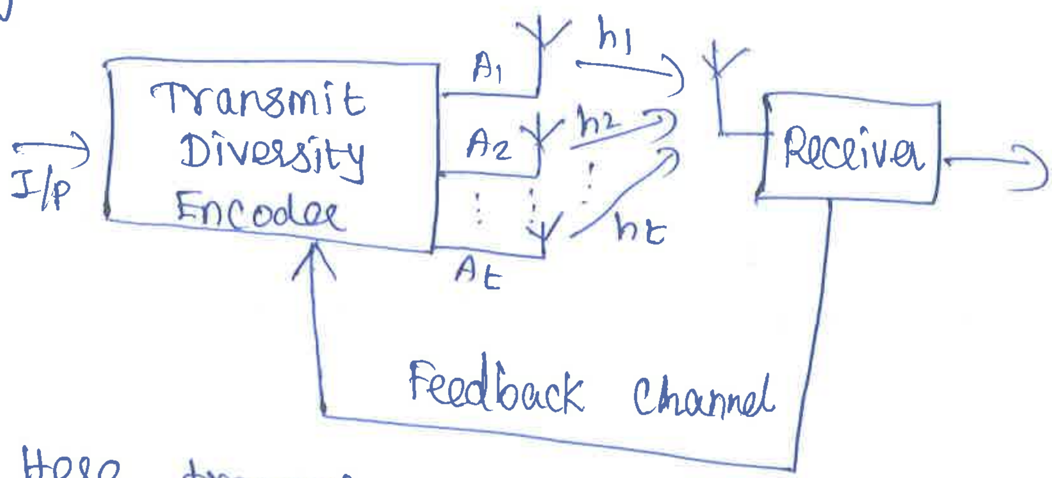
* The output SNR for 2x1 Alamouti scheme is,

$$\boxed{\gamma = \frac{\sum_{i=1}^{2} |h_i|^2 E_s}{2 N_o}}$$

* It is the only orthogonal space time block code that can achieve its full diversity gain without needing to sacrifice its data rate.

Closed loop transmit diversity - CSI is known at transmitter:

* CSIT is known from the feeback of the system.



* Here transmit diversity system has 't' transmit antennas and one receiver antenna.
* Let s(t) denote the transmitted signal with total energy per symbol E_s.
* The signal is multiplied by a complex gain,

$$\boxed{\alpha_i = a_i e^{-j\theta_i}} \quad (0 \leq a_i \leq 1)$$

and then sent through the i^{th} antenna.

* This complex multiplication performs both co-phasing and weighting relative to the channel gains.

* Hence the received signal is,

$$\boxed{r(t) = \sum_{i=1}^{M} a_i r_i \, s(t)}$$

* To maximize the SNR value, a_i is set as,

$$\boxed{a_i = \frac{r_i}{\sqrt{\sum_{i=1}^{M} r_i^2}}}$$

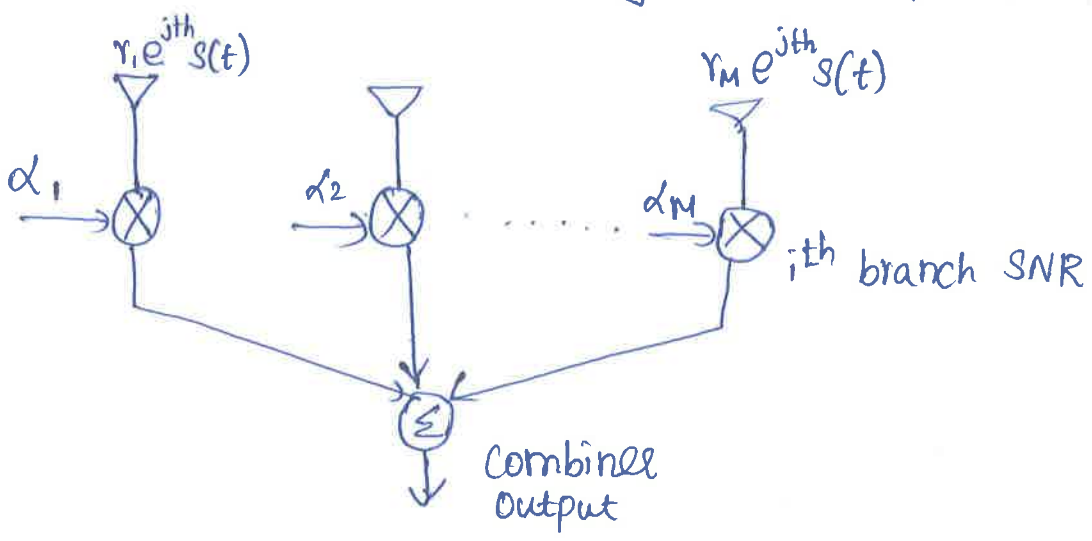
The resulting SNR is given by,

$$\boxed{\gamma_{\Sigma} = \frac{E_s}{N_o} \sum_{i=1}^{M} r_i^2}$$

* Thus, the transmit diversity when the channel gains are known to the transmitter is similar to receiver diversity with Maximal Ratio combining (MRC).

Explain Receiver diversity in detail.

* Here one transmit antenna and many receive antennas are used.
* In receivery diversity, the independent fading ~~channels~~ Paths associated with multiple antennas are combined.
* combining techniques are linear.
* The output of the combiner is just a weighted sum of the different fading paths (or) branches.
* If weights of all the paths α_i are zero, except one Path means, only one path is passed.
* If more than one path α_i are non-zero means then the combiner adds together multiple paths.



a) Linear combiner

* Each Path is weighted by a different value so it requires co-phasing. co-phasing means the phase θ_i of the i^{th} branch is removed through multiplication by $\alpha_i = a_i e^{-j\theta_i}$

* This phase removal requires coherent detection.
* without co-phasing the resultant output exhibits fading.
* The main purpose of diversity is to coherently combine the independent fading paths so that the effects of fading are mitigated.

$$\boxed{\alpha_\Sigma = \Sigma_i a_i r_i}$$

Combined output = [original transmitted signal $s(t)$] × [random complex amplitude term].

* The received SNR is,

$$\boxed{\gamma_\Sigma = \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{N_0 \sum_{i=1}^{M} a_i^2}}$$

Array Gain:

* Increase in SNR in the absence of fading is called array gain. Array gain (A_g) is defined as the increase in the average combined SNR $\bar{\gamma}_\Sigma$ over the average branch SNR $\bar{\gamma}$.

$$\boxed{A_g = \frac{\bar{\gamma}_\Sigma}{\bar{\gamma}}}$$

Diversity Gain:

* The average probability of error is given as,

$$\boxed{P_s = c\,\bar{\gamma}^{-M}}$$

$M \rightarrow$ diversity order
$c \rightarrow$ constant.

Types of combining Technique :-

→ selection → Threshold → Maxial ratio combining

→ Equal gain coimbining.

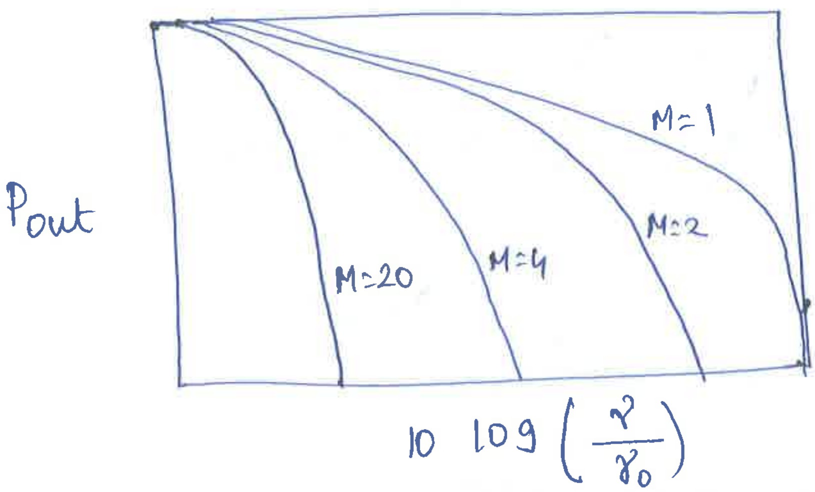
Selection combining :-

* The receiver selects the signal with the largest instantaneous power.
* Here only one branch is used at a time.
* Selection combining requires only one receiver that is Switched into active antenna branch.
* The outage Probability of the selection combiner is,

$$\boxed{P_{out}(\gamma_0) = \left[1 - e^{-\gamma_0/\bar{\gamma}}\right]^M}$$

* The average SNR of the combiner output is,

$$\boxed{\gamma_\Sigma = \bar{\gamma} \sum_{i=1}^{M} \frac{1}{i}}$$



a) outage probability of selection combining.

Threshold Combining :-

* Each of the branches are scanned in sequential order and best SNR is selected.

* Threshold γ_T is fixed above that active branch SNR.

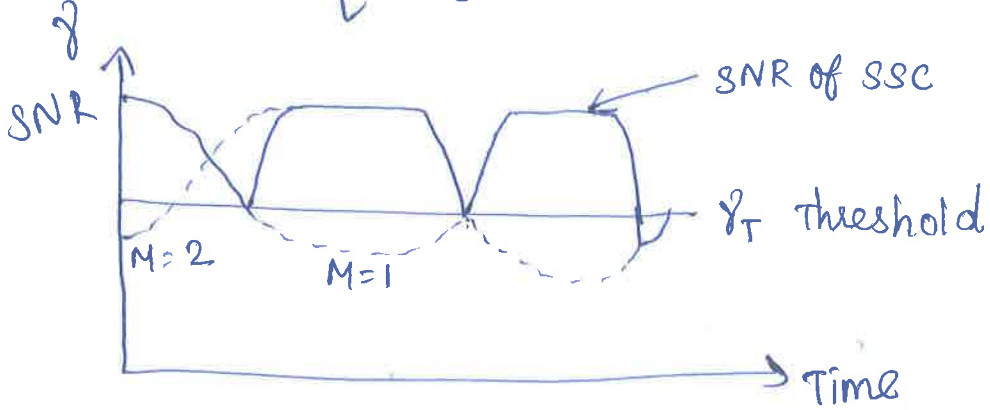
* Active branch is monitored, if it falls below a certain threshold, then the receiver switches to another branch.

* If the diversity scheme has only $M=2$ branches, this method is called switching and stay combining (SSC).

$\gamma_i \Rightarrow$ SNR of the i^{th} branch.

* For two branch diversity with independent and identically distributed branch statistics, the cumulative distribution function of the combiner output is,

$$P_{\gamma_\Sigma}(\gamma) = \begin{cases} P_{\gamma_1}(\gamma_T)\ P_{\gamma_2}(\gamma) & \text{for } \gamma < \gamma_T \\ P(\gamma_T \le \gamma_2 \le \gamma) + P_{\gamma_1}(\gamma_T)\ P_{\gamma_2}(\gamma) & \text{for } \gamma \geq \gamma_T \end{cases}$$



a) SNR of SSC technique.

Maximal-Ratio-combining :- (MRC)

* In MRC, the output is a weighted sum of all branches.

* The signals are co-phased and so complex amplitude

$\alpha_i = a_i e^{j\theta}$.

$\theta_i \Rightarrow$ Phase of the incoming signal on the i^{th} branch.

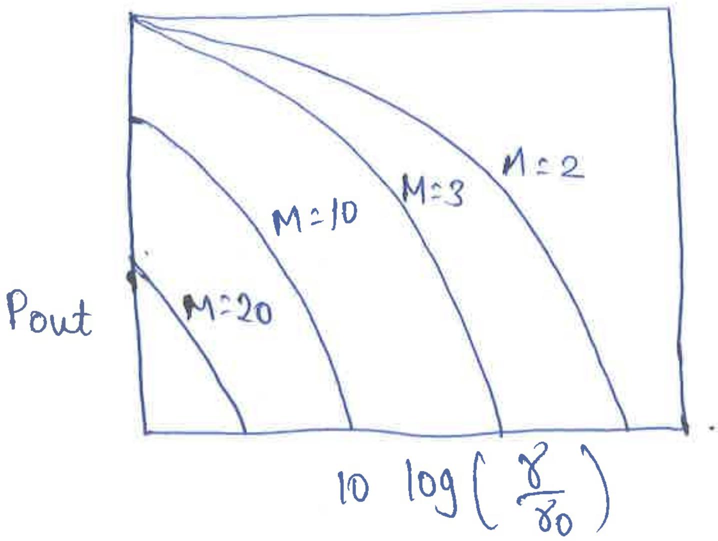
* Envelope of combiner output,

$$\boxed{r = \sum_{i=1}^{M} a_i r_i}$$

To maximize SNR, optimal weight is,

$$\boxed{a_i^2 = \frac{r_i^2}{N_0}}$$

The output SNR of combiner $\Rightarrow \boxed{\gamma_\Sigma = \sum_{i=1}^{M} \gamma_i}$



* MRC Probability distribution.

* At high SNR, MRC achieves full diversity order.

Equal gain combining :- [EGC]

* EGC is a simple technique. It co-phases the signals on each branch and then combines them with equal weighting.

$$\boxed{\alpha_i = e^{-\theta_i}}$$

Combiner output SNR = $\gamma_\Sigma = \frac{1}{N_o M}\left(\sum_{i=1}^{M} \gamma_i\right)^2$

* In EGC, the branch weights are all set to unity. ~~otherwise~~ the combining technique is similar to MRC except that the weighting circuits are omitted. (set to unity in EGC).

Explain the MIMO channel capacity.

* Shannon capacity of a MIMO channel equals the maximum data rate that can be transmitted over the channel with small error probability.

* Channel capacity depends on information known about the channel gain matrix or its distribution at the transmitter (or) receiver.

Types:-

1) Static channel capacity
2) Fading channel capacity.

Static channel capacity:-

* The capacity of a MIMO channel is an extension of SISO channel.

* The capacity in terms of mutual information between the channel input vector x and output vector y as,

$$C = \max_{P(x)} I(X;Y) = \max_{P(x)} \left[H(Y) - H(Y/X) \right]$$

where, $H[y] \Rightarrow$ entropy of output vector

$H\left[\frac{y}{x}\right] \Rightarrow$ entropy of (y/x)

Here $H\left[\frac{Y}{X}\right] = H(n)$, the entropy in the noise.

* This noise n has fixed entropy, independent of the channel input. So, maximizing mutual information is equivalent to maximizing the entropy in y.

* covariance matrix associated with MIMO channel output is,

$$\boxed{R_y = E[yy^H] = HR_xH^H + I_{Mr}}$$

$R_x \Rightarrow$ covariance matrix on the input vector x.

$R_y \Rightarrow$ covariance matrix of the output vector

* For all random vectors with a given covariance matrix R_y, the entropy of y is maximized when 'y' is a zero-mean, circularly symmetric complex Gaussian random vector. [ZMCSCG].

* But y is ZMCSCG only if input x is ZMCSCG.

* Thus we have,

$$H(Y) = B \log_2 \det[\pi e R_y] \text{ and}$$

$$H(n) = B \log_2 \det[\pi e I_{Mr}]$$

Resulting Mutual Information is,

$$\boxed{I(X:Y) = B \log_2 \det[I_{Mr} + HR_xH^H]}$$

where, $B \rightarrow$ Bandwidth

$I_{Mr} \rightarrow$ receive multi antenna mutual information

det $\rightarrow$ determinant.

* The MIMO capacity is achieved by maximizing the mutual information over all input covariance matrix R_x, satisfying the power constraint. (14)

$$\boxed{C = \max_{R_x : Tr(R_x) = \rho} \left[B \log_2 \det \left[I_{M_r} + H R_x H^H \right] \right]} \rightarrow (X)$$

Channel known at transmitter : Water Filling :-

* In MIMO parallel decomoposition technique, the capacity of the channel equals the sum of capacities on each of the independent parallel channels with transmit power optimally allocated between the channels.

* Substituting the matrix of SVD in (X), we get, the MIMO capacity with CSIT and CSIR as,

$$\boxed{C = \max \sum_{i=1}^{R_H} B \log_2 (1 + \sigma_i^2 P_i)}$$

where $R_H \Rightarrow$ number of non-zero singular values σ_i^2 of H.

* Since the MIMO channel decomposes into R_H parallel channels, it has R_H degrees of freedom.

Power constraint $\boxed{\rho = \frac{P}{\sigma^2}}$ $P \Rightarrow$ Power.

* Interms of power allocation P_i to the i^{th} parallel channel, we have,

$$\boxed{C = \max \sum_{i=1}^{R_H} B \log_2 \left(1 + \frac{P_i \gamma_i}{P} \right)}$$

where $\gamma_i = \frac{\sigma_i^2 P}{\sigma^2}$ is the SNR associated with the i^{th} channel at full power.

* At high SNRs, the channel capacity increases linearly with the number of degrees of freedom in channel.

* Solving the optimization leads to a water-filling power allocation for the MIMO channel.

$$\frac{P_i}{P} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma_i} & , \gamma_i \geq \gamma_0 \\ 0 & , \gamma_i < \gamma_0 \end{cases}$$

where $\gamma_0 \rightarrow$ cutoff value.

* The resulting capacity is then,

$$\boxed{C = \sum_{i: \gamma_i > \gamma_0} B \log \left(\frac{\gamma_i}{\gamma_0} \right)}$$

Channel unknown at transmitter:
[uniform power Allocation].

* If the receiver knows the channels and if the transmitter does not know it, then uniform power allocation is done.

* without channel information, the transmitter cannot optimize its power allocation (or) input covariance structure across antennas. So equal power is allocated to each transmit antenna, resulting in input covariance matrix equal to scaled identity matrix.

(ie) $R_x = \left[\frac{P}{M_t}\right] I_{M_t}$.

* For an M- transmit, M_r - receive antenna system, the mutual information is given by,

$$\boxed{I(x;y) = B \log_2 \det\left[I_{M_r} + \frac{\rho}{M_t} H H^H\right]}$$

Using SVD of H,

$$\boxed{I(x;y) = \sum_{i=1}^{R_H} B \log_2\left(1 + \frac{\gamma_i}{M_t}\right)}$$

where, $\gamma_i = \frac{\sigma_i^2 P}{\sigma^2}$;

Hence the MIMO capacity in the absence of CSIT is given by,

$$\boxed{C = MB \log_2 (1 + \rho)}$$

where $M = \min(M_t, M_r)$.

Capacity of Fading channels:-

* The channel gain matrix, if experiences flat fading, the gains vary with time.

→ Channel known at transmitter (Water-filling)

→ Channel unknown at transmitter (Ergodic capacity & capacity with outage)

→ No CSI at transmitter (or) receiver.

CSIR and CSIT known:-

* With CSIT and CSIR, the channel optimizes its transmission energy for each fading channel realization.

* The capacity of such channel is the average of capacities associated with each channel realization with power optimally allocated. This average capacity is called ergodic capacity of the channel, given by,

$$C = E_H\left[\max_{R_X : Tr(R_X) = P} B \log_2 \det\left[I_{M_r} + H R_x H^H\right]\right]$$

$$C = E_H\left[\max_{P_i : \sum_i P_i \le \bar{P}} \sum_i B \log_2\left(1 + \frac{P_i \gamma_i}{\bar{P}}\right)\right] \rightarrow (1)$$

where, $\gamma_i = \frac{\sigma_i^2 \bar{P}}{\sigma^2}$;

(1) ⇒ short term power constraint where $P = \bar{P}$;

* A less restrictive Power Constraint is a long term Power constraint given by, $E_H[P_H] \leq \bar{P}$.

$$\therefore \quad C = \max E_H\left[\max B \log_2 \det\left[I_{M_r} + H R_x H^H\right]\right]$$

$$C = \max E_H\left[\max \sum_i B \log_2\left(1 + \frac{P_i \gamma_i}{P_H}\right)\right]$$

where $\gamma_i = \frac{\sigma_i^2 P_H}{\sigma^2}$;

* The short-term power constraint gives rise to water filling in space across the antennas where as long term power constraint allows for 2D water filling across both time and space.

Channel unknown at transmitter:-

→ CSIR is known here. But CSIT is unknown.

Ergodic capacity:-

* It defines the Maximum rate, averaged over all channel realizations.

$$C = E_H\left[B \log_2 \det\left[I_{M_r} + \frac{\rho}{M_t} H H^H\right]\right]$$

where $M = \min(M_t, M_r)$.

* comparing the capacity of channel of SISO fading channel with 4X4 MIMO channel,

Ergodic capacity of MIMO channel = 4 x SISO channel.

* If the channel gain matrix is unknown at the transmitter, then power is allocated according to mean (or) covariance.

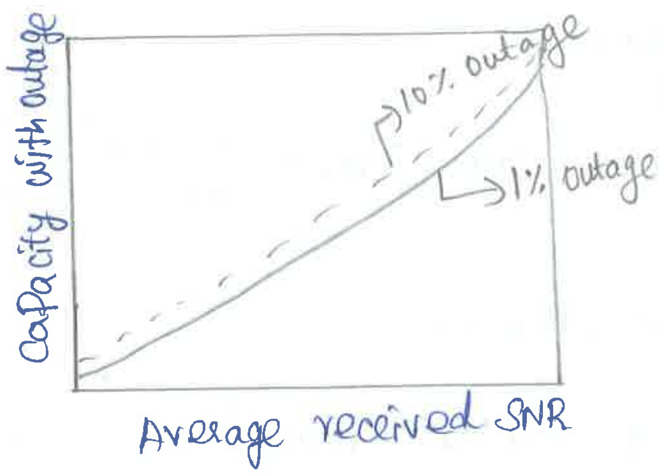
Capacity with outage :-

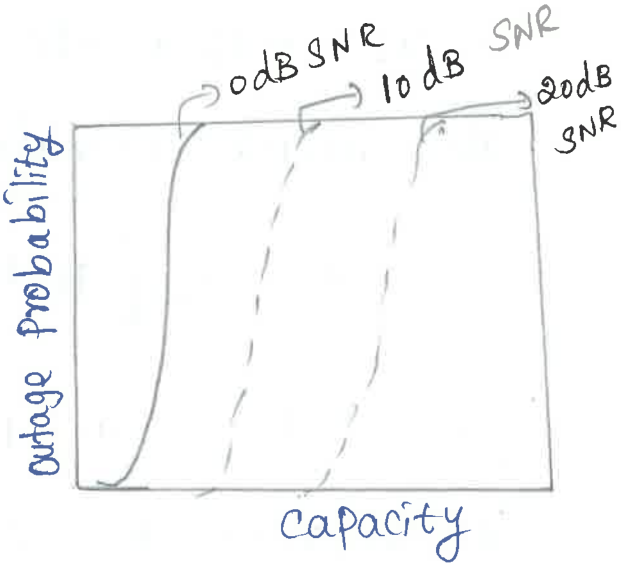
* Capacity with outage applies to slowly varying channels, where the channel matrix 'H' is constant over a relatively long transmission time and then changes to a new value.

* Since CSI is unknown at the transmitter, it has to fix a transmission rate (R).

* For each value of R, there will be an outage Probability which equals the probability that the transmitted data will not be received correctly.

* The outage capacity depends on the probability distribution.





No CSI at the Transmitter or receiver:-

* when there is no CSI at either the transmitter (or) receiver, it is difficult to obtain channel capacity.

* For an independent and identically distributed zero mean spatially white (i.i.d ZMSW) block fading channel, increasing the no. of transmit antennas, does not increase the capacity.

Based on SNR:-

→ At low SNR, capacity is limited by noise and grows linearly with the no. of channel degrees of freedom.

→ At moderate to high SNR, capacity is limited by estimation error and its growth is also linear in the number of channel degrees of freedom.

→ At high SNR, there is no multiplexing gain associated with multiple antennas for slowly varying channels without transmitter (or) receiver CSI.