$\therefore \omega_i = \omega_c + K V_m \underbrace{\cos \omega_m t}$

$max = +1$

$min = -1$

$$\boxed{\begin{aligned} \omega_{max} &= \omega_c + K V_m \\ &\text{and} \\ \omega_{min} &= \omega_c - K V_m \end{aligned}}$$

$$J_n(m_f) = \left(\frac{m_f}{2}\right)^n \left[\frac{1}{n!} + \frac{(m_f/2)^2}{1!(n+1)!} + \frac{(m_f/2)^4}{2!(n+2)!} + \cdots\right]$$

By using Bessel function,

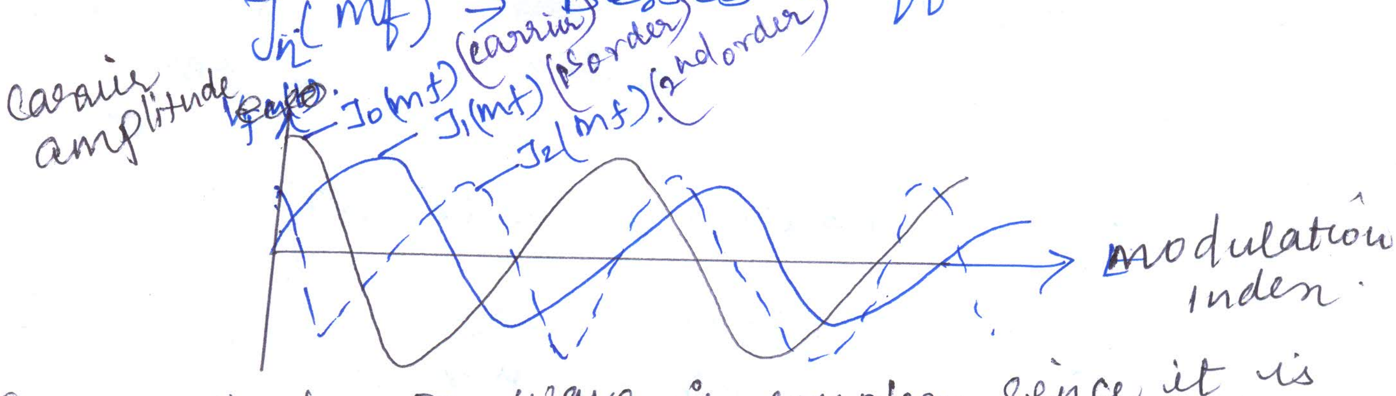
$$V_{FM}(t) = V_c \sin(\omega_c t + m_f \sin \omega_m t)$$

$$= V_c \left[\sin \omega_c t \cos(m_f \sin \omega_m t) + \cos \omega_c t \sin(m_f \sin \omega_m t)\right].$$

$$V_{FM}(t) = V_c \left[J_0(m_f) \sin \omega_c t\right] + V_c \left[J_1(m_f) (\sin(\omega_c + \omega_m)t - \sin(\omega_c - \omega_m)t\right] + V_c \left[J_2(m_f) (\sin(\omega_c + 2\omega_m)t + (\sin(\omega_c - 2\omega_m)t\right]$$

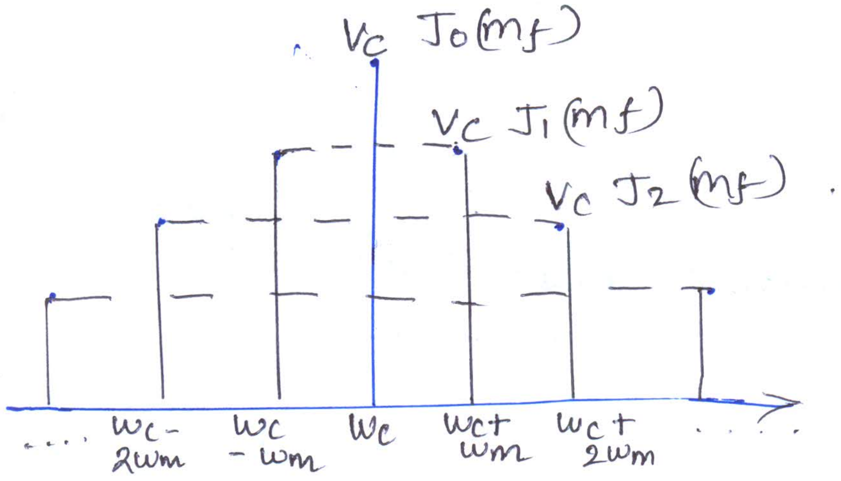
$V_{FM}(t)$ = Carrier + Infinite no/- of Sidebands.

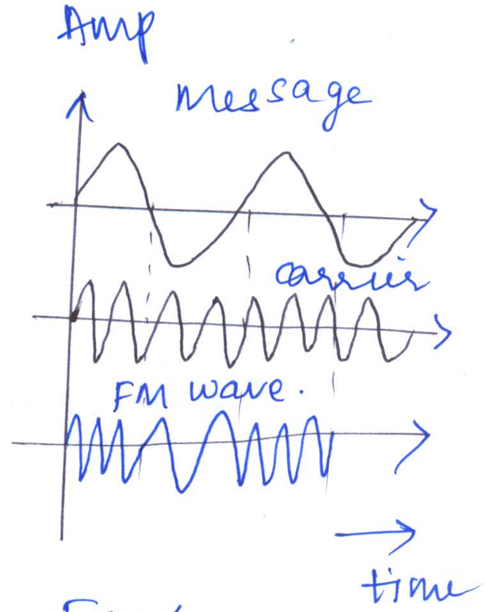
$J_n(m_f) \to$ Bessels Coefficient.



Expression for FM wave is complex since it is sine of sine function. So solve the equation by using Bessels function.

Frequency Spectrum:





* Transmission Bandwidth of FM:

i) Ideally / Theoretically it is infinite, because of infinite no/- of side bands

ii) Practically, BW = $2 f_m \times$ no/- of significant sidebands.

$$= 2 f_m \cdot m_f$$

$$\text{or } BW = 2\omega_m \cdot m_f$$

$$= 2\omega_m \cdot \frac{\Delta\omega}{\omega_m}$$

$$= 2\Delta\omega \text{ or } 2\Delta f \text{ (Hertz)}$$

(radians)

* Carsons rule | Rule of Thumb:

It states that the BW of FM is twice the sum of the deviation & highest modulating frequency

$$BW = 2[\delta + f_m(max)]$$

$$= 2(\Delta f + f_m(max)).$$

Emphirical relation (ω)

$$BW = 2(\Delta\omega + \omega_m)$$

$$= 2\Delta\omega(1 + \omega_m/\Delta\omega)$$

$$= 2\Delta\omega(1 + 1/m_f)$$

PROPERTIES:

1: Narrow band FM (NBFM): $m_f < 1$.
NBFM is the FM wave with smaller bandwidth. The modulation index of NBFM is small as compared to one radian. Hence it contains C, LSB, USB (carrier, LSB, USB)

2: Wide band FM (WBFM): $m_f > 1$.
WBFM contains carrier, infinite number of sideband located around the carrier. FM has infinite BW, so it is named as WBFM.

3: Constant average power: The envelope of FM wave always has a constant magnitude. $P_t = \frac{V_c^2}{2R}$. $\boxed{P_t = \frac{1}{2} V^2}$ $R = 1\,\Omega$

Percent Modulation of FM: defined as the ratio of actual freq deviation produced by modulating signal to the max. allowable frequency deviation. $(\delta / \Delta f)$.

Deviation ratio: δ / f_m :
Ratio of maximum deviation to the maximum modulating frequency.

K_f → deviation sensitivity → represents i/p − o/p transfer function of modulators. $\left(\frac{\text{rad/sec}}{\text{volt}}\right)$.

PHASE MODULATION:

Def :- It is defined as the process by which changing the phase of the carrier signal inaccordance with the instantaneous amplitude of the message.

Let the modulating signal be $V_m(t) = V_M \cos\omega_m t$ ——— (1)

" carrier " $V_c(t) = V_c \sin(\omega_c t + \theta)$ ——— (2)

θ - phase angle of carrier.

According to def, phase is changed,

$$\theta \propto V_m(t)$$

$$\theta = K V_m(t)$$

$$= K V_m \cos\omega_m t$$

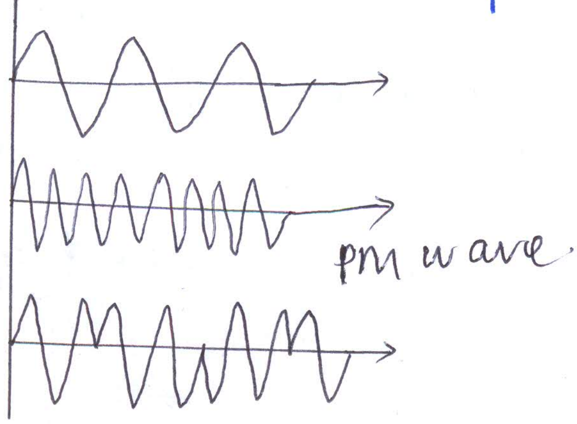
After phase modulation,

$$V_{PM}(t) = V_c \sin(\omega_c t + \theta)$$

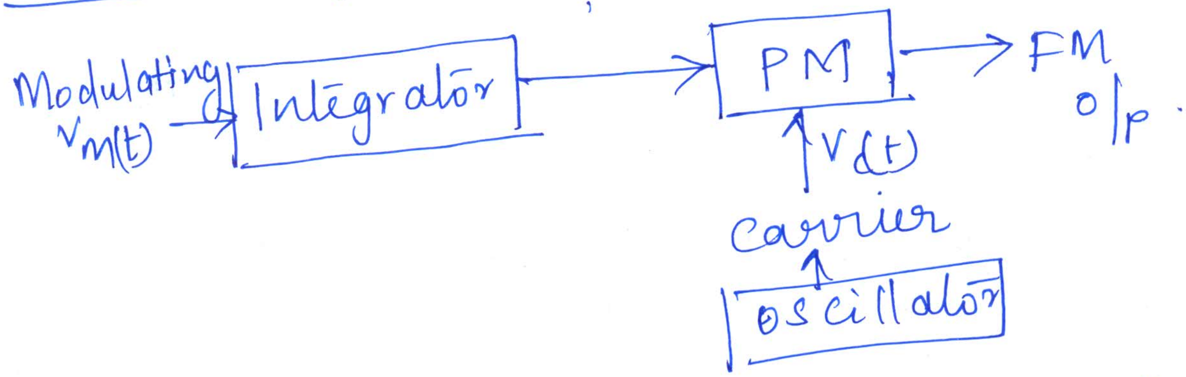
$$= V_c \sin(\omega_c t + K V_m \cos\omega_m t)$$

$$= V_c \sin(\omega_c t + m_p \cos\omega_m t).$$

m_p - modulation index of PM.



Conversion of PM to FM:



FM wave can be obtained by integrating modulating signal before applying it to the modulator.

$$V_m(t) = V_m \cos w_m t \quad \text{— (1)}$$

$$\int V_m(t) = \int V_m \cos w_m t$$

$$= \frac{V_m}{w_m} \sin w_m t \quad \text{— (2)}$$

After PM, $\theta \propto V_m(t)$

$$\theta = K V_m(t)$$

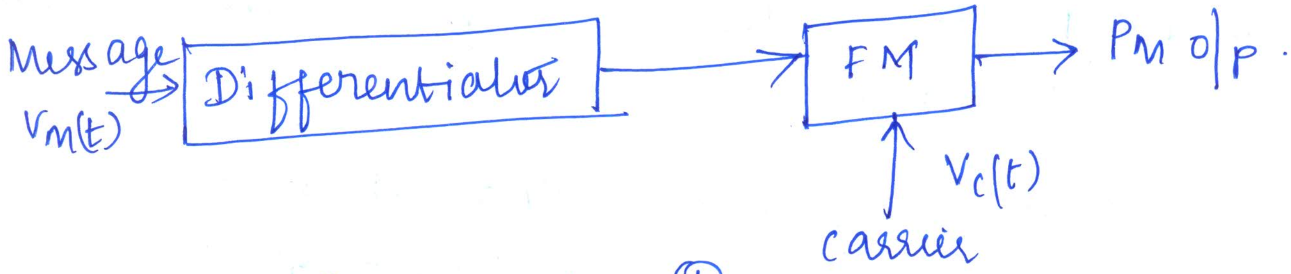
$$= K \int V_m(t) = K \frac{V_m}{w_m} \sin w_m t.$$

Instantaneous value of modulated voltage be, $V_{FM}(t) = V_c \sin(w_c t + \theta)$

$$= V_c \sin\left(w_c t + \frac{K V_m}{w_m} \sin w_m t\right)$$

Fm o/p. $= V_c \sin(w_c t + m_f \sin w_m t)$.

Conversion of FM to PM:



$V_m(t) = v_m \cos \omega_m t$ — (1)

$\frac{d}{dt} v_m(t) = v_m \omega_m (-\sin \omega_m t)$

$= -v_m \omega_m \sin \omega_m t$.

After FM, $\omega_i = \omega_c + \left(\frac{d}{dt} V_m(t)\right) . K$.

$= \omega_c - K V_m \omega_m \sin \omega_m t$.

$\phi_i = \int \omega_i \, dt$

$= \int (\omega_c - K v_m \omega_m \sin \omega_m t) \, dt$

$= \omega_c t + K v_m \frac{\omega_m}{\omega_m} \cos \omega_m t$

$= \omega_c t + K v_m \cos \omega_m t$.

Instantaneous signal voltage after modulation is $V_{pm}(t) = v_c \sin \phi_i$

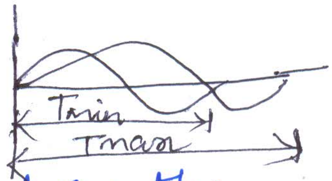
$v_{pm}(t) = v_c \sin(\omega_c t + m_p \cos \omega_m t)$

$m_p \Rightarrow$ mod. Index of PM.

Definition :

① Instantaneous Frequency deviation : It is defined as the first time derivative of the instantaneous phase deviation.

② Instantaneous frequency : It is the precise frequency of the carrier at a given instant of time.



③ Frequency deviation : Δf.

In FM, the deviation is defined as the amount by which the carrier frequency is varied from its unmodulated value. Magnitude of frequency deviation is proportional to the amplitude of the modulating signal.

④ Phase deviation : $(\Delta\theta)$: The relative angular displacement of the carrier phase in radians in respect to the reference phase. The change in the carrier phase produces a corresponding change in frequency.

⑤ Instantaneous phase deviation : It is the instantaneous change in the phase of the carrier at a given instant of time & indicates how much phase of the carrier is changing with respect to its reference phase.