

# Mathematical Modeling of Shot Put Throwing

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## Abstract

There are multiple impacts on the result of the Shot Put competition, e.g. the speed of the ball, the throwing degrees, height, etc. Different factors can cause different results. This article used the kinetic principle and the mathematics model in the 2017 Athletics World Championships Women's Shot Put Final which includes Strike Speed, Strike Height, Strike Height Relative to Body Height, Distance Beyond Toe, Strike Tilt, etc., and created a regression model to analyze key points on the result of the Shot Put competition.

## Keywords

Shot Put Competition, Regression, Kinetic Trace, Mathematics Model

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## 1 Introduction

The ideal routine of the throwing trace is a parabola (as projectile motion). However, the trace isn't a parabola because of the air residence.

Without air residence, the trace will be like in 1.

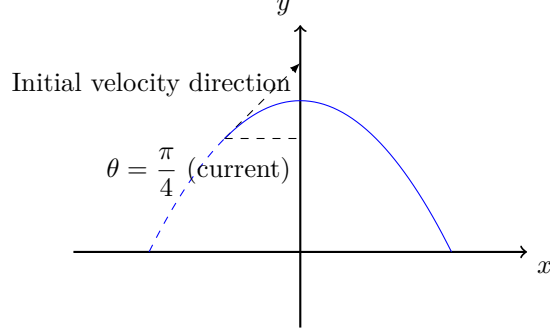


Figure 1: Ideal trace

However, because of the following reasons,

- Air residence,
- Distortion of the throwing direction,
- Start position,
- ...

The trace in 1 is inaccurate.

Therefore, the article will develop a mathematical model for the trajectory of the shot put that includes the effect of air resistance. As for the distortion of the throwing direction, the article may use the 3D transforms with Euler angles to adjust the result.

## 2 Parameters

Some parameters are included, while some parameters are ignored.

### 2.1 Parameters in Kinetic Model

In the kinetic analysis, we don't bring the exact data. Some definitions are following the table 1.

explain	parameter	e.g. value
gravitational acceleration	$g$	9.787m/s <sup>2</sup>
shot put mass	$m$	4kg
atmospheric drag coefficient	$k$	N/A
throw speed	$v$	N/A <sup>1</sup>
throwing angle	$\theta$	37.0°
throwing height	$h$	2.08m
tilt angle left and right	$\varphi$	-21°
horizontal displacement of the shot put <sup>2</sup>	$s$	19.94m

<sup>1</sup> Actually, the speed is a vector, and the  $v_x$ ,  $v_y$  not only depends on the value of the absolute value, but also depends on the throwing angle. Expressing with complex number is also OK, one of the example is  $13.24e^{\cos 37.0^\circ + i \sin 37.0^\circ}$  m/s, whose  $v_y$  is 7.944m/s, and  $v_x$  is 10.592m/s.

<sup>2</sup> including exceeds toe board distance.

Table 1: Parameters of the Mathematical Model

We also assume the following conditions:

1. The velocity is small, so  $F = -kv$  should be used instead of  $F = -kv^2$ .

2. The horizontal velocity is not reduced to 0 due to air resistance.
3. There is no influence of field forces other than the gravity field, which is assumed to be uniformly strong.

## 2.2 Parameters in the Regression Model

### 2.3 Ignore

Because some parameters are useless, we ignore following parameters:

1. Front and rear tilt angles,
2. Strike height relative to body height,
3. Best round,
4. ...

## 3 Mathematical Model with Kinetic Analysis

### 3.1 Force Analysis

During the movement of a shot put ball, it is subjected to 2 forces: gravity and air resistance. The force of gravity is always vertically downward, however, air resistance changes with the direction of velocity.

The exciting thing is that we can break down air resistance in 2.

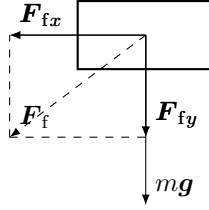


Figure 2: Force Situation

We define air resistance as  $\mathbf{F}_f = -k\mathbf{v}$ . So, the resistance vertically is  $\mathbf{F}_{fy} = -k\mathbf{v}_y$ , and horizontal direction is the same.

### 3.2 Vertical Motion

There are 2 parts of the vertical motion, **ascend** and **descend**. However, we can regard them as a directed motion with the help of vectors.

Defining the direction of vertically downward is the positive one, so  $\mathbf{v}_0$ , which is the primal velocity, is negative horizontally and vertically. Meanwhile, the acceleration of gravity,  $g$ , is positive vertically.

Via Newton's Second Theorem, we can know the force situation vertically:

$$ma = mg - kv_y,$$

That is to say,

$$m \frac{dv_y}{dt} = mg - kv_y \quad (1)$$

Separate parametric variables and take integrals on both sides.

$$\int \frac{dv_y}{mg - kv_y} = \int \frac{dt}{m} \quad (2)$$

Then we can get

$$-\frac{1}{k} \ln (mg - kv_y) = \frac{t}{m} + C \quad (3)$$

Also bring in the initial velocity (vertical):  $v_y|_{t=0} = v_0 \sin \theta$  into the  $C$ , we can get the equation of vertical velocity:

$$v_y = \frac{mg}{k} - \left( \frac{mg}{k} - v_0 \sin \theta \right) e^{-\frac{k}{m}t} \quad (4)$$

Since we have defined the positive direction, we can simply integral the velocity to calculate the value of displacement:

$$\int_0^t v_y dt = h \quad (5)$$

And the final equation of  $t$  is

$$\boxed{\frac{t}{m} \left( \frac{mg}{k} - v_0 \sin \theta \right) e^{-\frac{k}{m}t} + v_0 \sin \theta = h} \quad (6)$$

We can solve this equation by methods such as Newton's method or Newton-like methods. After receiving  $t$ , we can get the horizontal displacement.

### 3.3 Horizontal Motion

The horizontal motion of the shot put is decelerate linear motion. However, the deceleration changes with the velocity.

Through Newton's Second Theorem, we can also get the horizontal force situation:

$$ma = kv_x$$

And the initial velocity (horizontal) is  $v_0 \cos \theta$ . The methodology is consistent with the above, so it will not be repeated. After solving this differential equation, we can get the velocity:

$$v_x = v_0 \cos \theta e^{-\frac{k}{m}t} \quad (7)$$

Then if we know the motion time  $t$ , we can get the horizontal distance of the shot put, which is the *score*:

$$\boxed{s = \int_0^t v_x dt = \frac{-mv_0 \cos \theta}{k} e^{-\frac{k}{m}t} - \frac{-mv_0 \cos \theta}{k}} \quad (8)$$

It is important to note that since the magnitude of the air resistance is related to the velocity, differential equations must be used instead of the ordinary integral method. Since the equations in the vertical direction cannot be solved algebraically and directly, no simple and explicit algebraic relation among parameters above can be given.

So, the trace of the shot put is in the figure 3. That's a bit of an exaggeration.

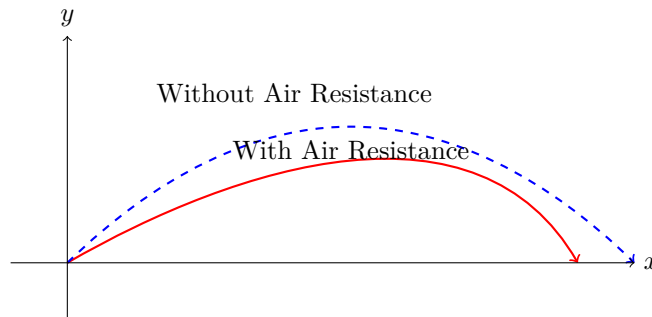


Figure 3: The Trace of the Shot Put with Air Residence

## 4 Neural Network of Impacts

Through PyTorch, we can create a neural network with regression to evaluate impacts of parameters.

## 5 Model Analysis

The mathematical model uses differential equations to solve the question.

### 5.1 Data Brought into the Analysis

Using the formula 4, we can get the vertical velocity of the shot put. Using **Geogebra**, we can get the  $v_y - t$  graph in the figure 4. Data is used following the table ??.

Parameter	Value	Unit
$v_0$	-12	m/s
$\theta$	37.0	°
$h$	2.08	m
$k$	0.00215 <sup>1</sup>	N · s/m
$m$	4	kg

<sup>1</sup> The value of  $k$  is following the formula:  $k = \frac{1}{2}C_D A \rho$ , where  $C_D$  is the drag coefficient,  $A$  is the cross-sectional area, and  $\rho$  is the air density.

We regard the shot put as a sphere whose  $r$  is 0.10m, and the air density is 1.225kg/m<sup>3</sup>. So, the value of  $k$  is 0.00215N · s/m.

The value of  $k$  is not accurate, but it is enough for the model.

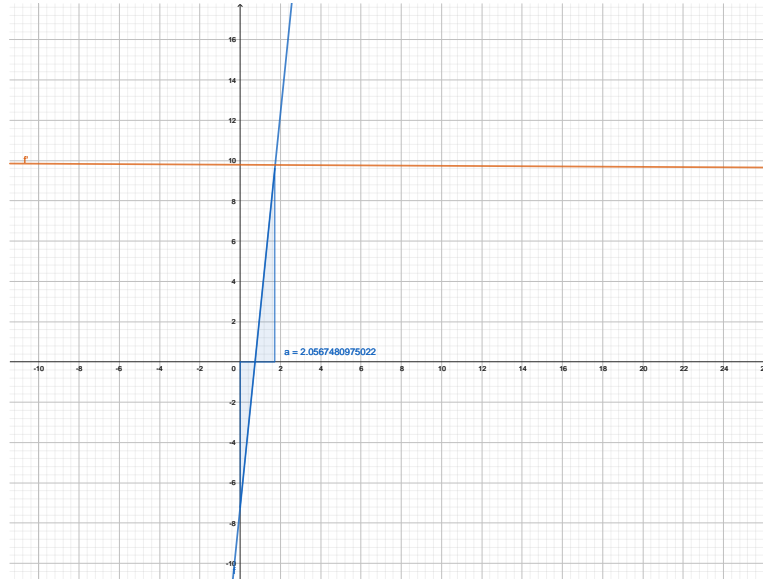


Figure 4: The  $v_y - t$  Graph

The blue one is the graph, and the orange one is the derivative of the blue one. It seems that the velocity is decreasing little by little.

When  $t \approx 1.76$ s, the integral of the velocity, which is the displacement, is 2.08m, which is the throwing height.

Then, we can bring  $t$  into the horizontal equation to get the horizontal displacement, which is the score.

## 5.2 Example: Gong's Shot Put in 2017

We selected Gong Lijiao's shot put in 2017 as an example. The data is in the attachment.

We set the  $k = 0.00215$ , and received the result of the horizontal displacement with the model, which returns 19.62599m, which is close to the real data.

Through `calculate/newton.py` provided in the attachment, we use `scipy` to calculate the approx value of  $t$ , which is 1.856988167816502s.

Then we can get the horizontal displacement, which is 19.62599m, which is close to the real data (1.6% error).

If we add *FB Truck Lean* into the release angle, the error will be reduced to 0.8% (19.78m).

## 5.3 Features

The mathematical model considers the air residence, which improves the accuracy of the model significantly. Ease of calculation through decomposition of motion.