31　2　7

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1.1

$\boldsymbol{q} \in \mathbb{R}^{n_q}$ $\boldsymbol{q}$

$$n_q$$
$$\boldsymbol{e}(\boldsymbol{q}) : \mathbb{R}^{n_q} \to \mathbb{R}^{n_e} \qquad\qquad \boldsymbol{q}$$

$$\boldsymbol{e}(\boldsymbol{q}) = \boldsymbol{0} \tag{1}$$

$\boldsymbol{e}(\boldsymbol{q})$ 6
(1)

(1)   (1)
[1]

$$\min_{\boldsymbol{q}} \ F(\boldsymbol{q}) \tag{2a}$$

$$\text{where} \ \ F(\boldsymbol{q}) \stackrel{\text{def}}{=} \frac{1}{2}\|\boldsymbol{e}(\boldsymbol{q})\|^2 \tag{2b}$$

$\boldsymbol{q}_{min}$ $\boldsymbol{q}_{max}$

$$\min_{\boldsymbol{q}} \ F(\boldsymbol{q}) \quad \text{s.t.} \ \ \boldsymbol{q}_{min} \le \boldsymbol{q} \le \boldsymbol{q}_{max} \tag{3}$$

$\boldsymbol{q}_{min}, \boldsymbol{q}_{max}$ (3)

[2]

$$\min_{\boldsymbol{q}} \ F(\boldsymbol{q}) \tag{4a}$$

$$\text{s.t.} \ \ \boldsymbol{A}\boldsymbol{q} = \bar{\boldsymbol{b}} \tag{4b}$$

$$\boldsymbol{C}\boldsymbol{q} \ge \bar{\boldsymbol{d}} \tag{4c}$$

$\Delta\boldsymbol{q}_k^*$ $\boldsymbol{q}_{k+1} = \boldsymbol{q}_k + \Delta\boldsymbol{q}_k^*$ (4)   [3]

$$\min_{\Delta\boldsymbol{q}_k} \ F(\boldsymbol{q}_k) + \nabla F(\boldsymbol{q}_k)^T\Delta\boldsymbol{q}_k + \frac{1}{2}\Delta\boldsymbol{q}_k^T\nabla^2 F(\boldsymbol{q}_k)\Delta\boldsymbol{q}_k \tag{5a}$$

$$\text{s.t.} \ \ \boldsymbol{A}\Delta\boldsymbol{q}_k = \bar{\boldsymbol{b}} - \boldsymbol{A}\boldsymbol{q}_k \tag{5b}$$

$$\boldsymbol{C}\Delta\boldsymbol{q}_k \ge \bar{\boldsymbol{d}} - \boldsymbol{C}\boldsymbol{q}_k \tag{5c}$$

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[1] $\boldsymbol{W}$ $\|\boldsymbol{e}(\boldsymbol{q})\|_{\boldsymbol{W}}^2 = \boldsymbol{e}(\boldsymbol{q})^T\boldsymbol{W}\boldsymbol{e}(\boldsymbol{q}) = \boldsymbol{e}(\boldsymbol{q})^T\boldsymbol{S}^T\boldsymbol{S}\boldsymbol{e}(\boldsymbol{q}) = \|\boldsymbol{S}\boldsymbol{e}(\boldsymbol{q})\|^2$ $\boldsymbol{S}$
(2b)
[2]   (3)

$$\boldsymbol{q}_{min} \le \boldsymbol{q} \le \boldsymbol{q}_{max}$$
$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix}\boldsymbol{q} \ge \begin{pmatrix} \boldsymbol{q}_{min} \\ -\boldsymbol{q}_{max} \end{pmatrix}$$

[3]   (5a) $F(\boldsymbol{q})$ $\boldsymbol{q}_k$

18
Numerical optimization, S. Wright and J. Nocedal, Springer Science, vol. 35, 1999, `http://www.xn--vjq503akpco3w.top/literature/Nocedal_Wright_Numerical_optimization_v2.pdf`.

$\nabla F(\boldsymbol{q}_k), \nabla^2 F(\boldsymbol{q}_k)$ $\qquad$ $F(\boldsymbol{q}_k)$ $\qquad$ [4]

$$\nabla F(\boldsymbol{q}) = \left(\frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}\right)^T \boldsymbol{e}(\boldsymbol{q}) \tag{6a}$$

$$= \boldsymbol{J}(\boldsymbol{q})^T \boldsymbol{e}(\boldsymbol{q}) \tag{6b}$$

$$\nabla^2 F(\boldsymbol{q}) = \sum_{i=1}^{m} e_i(\boldsymbol{q}) \nabla^2 e_i(\boldsymbol{q}) + \left(\frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}\right)^T \frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}} \tag{6c}$$

$$\approx \left(\frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}\right)^T \frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}} \tag{6d}$$

$$= \boldsymbol{J}(\boldsymbol{q})^T \boldsymbol{J}(\boldsymbol{q}) \tag{6e}$$

$e_i(\boldsymbol{q}) \ (i = 1, 2, \cdots, m)$ $\quad$ $\boldsymbol{e}(\boldsymbol{q})$ $\quad i$ $\qquad$ (6c) $\qquad$ (6d) $\qquad$ $\boldsymbol{e}(\boldsymbol{q})$

$$\boldsymbol{J}(\boldsymbol{q}) \stackrel{\text{def}}{=} \frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}} \in \mathbb{R}^{n_e \times n_q} \quad \boldsymbol{e}(\boldsymbol{q})$$

(6a) , $\quad$ (6d) $\qquad$ (5a) $\qquad$ [5]

$$\frac{1}{2}\boldsymbol{e}_k^T \boldsymbol{e}_k + \boldsymbol{e}_k^T \boldsymbol{J}_k \Delta \boldsymbol{q}_k + \frac{1}{2}\Delta \boldsymbol{q}_k^T \boldsymbol{J}_k^T \boldsymbol{J}_k \Delta \boldsymbol{q}_k \tag{7a}$$

$$= \frac{1}{2}\|\boldsymbol{e}_k + \boldsymbol{J}_k \Delta \boldsymbol{q}_k\|^2 \tag{7b}$$

$\boldsymbol{e}_k \stackrel{\text{def}}{=} \boldsymbol{e}(\boldsymbol{q}_k), \boldsymbol{J}_k \stackrel{\text{def}}{=} \boldsymbol{J}(\boldsymbol{q}_k)$

$$(5)$$

$$\min_{\Delta \boldsymbol{q}_k} \ \frac{1}{2}\Delta \boldsymbol{q}_k^T \boldsymbol{J}_k^T \boldsymbol{J}_k \Delta \boldsymbol{q}_k + \boldsymbol{e}_k^T \boldsymbol{J}_k \Delta \boldsymbol{q}_k \tag{8a}$$

$$\text{s.t.} \ \ \boldsymbol{A}\Delta \boldsymbol{q}_k = \boldsymbol{b} \tag{8b}$$

$$\boldsymbol{C}\Delta \boldsymbol{q}_k \geq \boldsymbol{d} \tag{8c}$$

$$\boldsymbol{b} = \bar{\boldsymbol{b}} - \boldsymbol{A}\boldsymbol{q}_k \tag{9}$$

$$\boldsymbol{d} = \bar{\boldsymbol{d}} - \boldsymbol{C}\boldsymbol{q}_k \tag{10}$$

1.2

(2a) $\qquad$ $\hat{F}(\boldsymbol{q})$

$$\hat{F}(\boldsymbol{q}) = F(\boldsymbol{q}) + F_{reg}(\boldsymbol{q}) \tag{11}$$

$$\text{where} \ \ F_{reg}(\boldsymbol{q}) = \frac{1}{2}\boldsymbol{q}^T \bar{\boldsymbol{W}}_{reg}\boldsymbol{q} \tag{12}$$

$\hat{F}(\boldsymbol{q})$

$$\nabla \hat{F}(\boldsymbol{q}) = \nabla F(\boldsymbol{q}) + \nabla F_{reg}(\boldsymbol{q}) \tag{13a}$$

$$= \boldsymbol{J}(\boldsymbol{q})^T \boldsymbol{e}(\boldsymbol{q}) + \bar{\boldsymbol{W}}_{reg}\boldsymbol{q} \tag{13b}$$

$$\nabla^2 \hat{F}(\boldsymbol{q}) = \nabla^2 F(\boldsymbol{q}) + \nabla^2 F_{reg}(\boldsymbol{q}) \tag{13c}$$

$$\approx \boldsymbol{J}(\boldsymbol{q})^T \boldsymbol{J}(\boldsymbol{q}) + \bar{\boldsymbol{W}}_{reg} \tag{13d}$$

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[4] (5a) $\quad \nabla^2 F(\boldsymbol{q}_k)$ $\qquad$ $\boldsymbol{q}_k$ $\qquad$ $F(\boldsymbol{q}_k)$

[5] (7b)

Feasible pattern generation method for humanoid robots, F. Kanehiro et al., Proceedings of the 2009 IEEE-RAS International Conference on Humanoid Robots, pp. 542-548, 2009.

(8)

$$\min_{\Delta \boldsymbol{q}_k} \frac{1}{2} \Delta \boldsymbol{q}_k^T \left(\boldsymbol{J}_k^T \boldsymbol{J}_k + \bar{\boldsymbol{W}}_{reg} \right) \Delta \boldsymbol{q}_k + \left(\boldsymbol{J}_k^T \boldsymbol{e}_k + \bar{\boldsymbol{W}}_{reg} \boldsymbol{q}_k \right)^T \Delta \boldsymbol{q}_k \tag{14a}$$

$$\text{s.t.} \quad \boldsymbol{A} \Delta \boldsymbol{q}_k = \boldsymbol{b} \tag{14b}$$

$$\boldsymbol{C} \Delta \boldsymbol{q}_k \geq \boldsymbol{d} \tag{14c}$$

1.3

Gauss-Newton   Levenberg-Marquardt   (14a)

[6]

$$\min_{\Delta \boldsymbol{q}_k} \frac{1}{2} \Delta \boldsymbol{q}_k^T \left(\boldsymbol{J}_k^T \boldsymbol{J}_k + \bar{\boldsymbol{W}}_{reg} + \lambda \boldsymbol{I} \right) \Delta \boldsymbol{q}_k + \left(\boldsymbol{J}_k^T \boldsymbol{e}_k + \bar{\boldsymbol{W}}_{reg} \boldsymbol{q}_k \right)^T \Delta \boldsymbol{q}_k \tag{15a}$$

$$\text{s.t.} \quad \boldsymbol{A} \Delta \boldsymbol{q}_k = \boldsymbol{b} \tag{15b}$$

$$\boldsymbol{C} \Delta \boldsymbol{q}_k \geq \boldsymbol{d} \tag{15c}$$

[7] λ

$$\lambda = \lambda_r F(\boldsymbol{q}_k) + w_r \tag{16}$$

$\lambda_r \quad w_r$

1.4

$$\boldsymbol{W}_{reg} \quad \overset{\text{def}}{=} \quad \bar{\boldsymbol{W}}_{reg} + \lambda \boldsymbol{I} \tag{17a}$$

$$\boldsymbol{v}_{reg} \quad \overset{\text{def}}{=} \quad \bar{\boldsymbol{W}}_{reg} \boldsymbol{q}_k \tag{17b}$$

(15)

$$\min_{\Delta \boldsymbol{q}_k} \frac{1}{2} \Delta \boldsymbol{q}_k^T \left(\boldsymbol{J}_k^T \boldsymbol{J}_k + \boldsymbol{W} \right) \Delta \boldsymbol{q}_k + \left(\boldsymbol{J}_k^T \boldsymbol{e}_k + \boldsymbol{v}_{reg} \right)^T \Delta \boldsymbol{q}_k \tag{18a}$$

$$\text{s.t.} \quad \boldsymbol{A} \Delta \boldsymbol{q}_k = \boldsymbol{b} \tag{18b}$$

$$\boldsymbol{C} \Delta \boldsymbol{q}_k \geq \boldsymbol{d} \tag{18c}$$

2   4   *** -configuration-task   (18)

:config-vector	get $\boldsymbol{q}$
:set-config	set $\boldsymbol{q}$
:task-value	get $\boldsymbol{e}(\boldsymbol{q})$
:task-jacobian	get $\boldsymbol{J}(\boldsymbol{q}) \overset{\text{def}}{=} \frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}$
:config-equality-constraint-matrix	get $\boldsymbol{A}$
:config-equality-constraint-vector	get $\boldsymbol{b}$
:config-inequality-constraint-matrix	get $\boldsymbol{C}$
:config-inequality-constraint-vector	get $\boldsymbol{d}$
:regular-matrix	get $\boldsymbol{W}_{reg}$
:regular-vector	get $\boldsymbol{v}_{reg}$

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[6]   (trust region)
[7] Levenberg-Marquardt   ,   ,   , vol. 29, no. 3, pp. 269-277, 2011.

1.5

$$J(q) \stackrel{\text{def}}{=} \frac{\partial e(q)}{\partial q}$$

2

q

$e(q)$

A, b, C, d

2.1 q 2.2 q

3   2

4   4.1

4.2   B

3

5   5.1   jskeus

5.2

5.4

2

2.1

instant-configuration-task [class]

:super	**propertied-object**
:slots	(_robot-env robot-environment instance)

(_theta-vector $\boldsymbol{\theta}$ [rad] [m])

(_wrench-vector $\hat{\boldsymbol{w}}$ [N] [Nm])

(_torque-vector $\boldsymbol{\tau}$ [Nm])

(_phi-vector $\boldsymbol{\phi}$ [rad] [m])

(_num-kin $N_{kin} := |\mathcal{T}^{kin\text{-}trg}| = |\mathcal{T}^{kin\text{-}att}|$)

(_num-contact $N_{cnt} := |\mathcal{T}^{cnt\text{-}trg}| = |\mathcal{T}^{cnt\text{-}att}|$)

(_num-variant-joint $N_{var\text{-}joint} := |\mathcal{J}_{var}|$)

(_num-invariant-joint $N_{invar\text{-}joint} := |\mathcal{J}_{invar}|$)

(_num-drive-joint $N_{drive\text{-}joint} := |\mathcal{J}_{drive}|$)

(_num-posture-joint $N_{posture\text{-}joint} := |\mathcal{J}_{posture}|$)

(_num-external $N_{ex} :=$ number of external wrenches)

(_num-collision $N_{col} :=$ number of collision check pairs)

(_dim-theta $dim(\boldsymbol{\theta}) = N_{var\text{-}joint}$)

(_dim-wrench $dim(\hat{\boldsymbol{w}}) = 6N_{cnt}$)

(_dim-torque $dim(\boldsymbol{\tau}) = N_{drive\text{-}joint}$)

(_dim-phi $dim(\boldsymbol{\phi}) = N_{invar\text{-}joint}$)

(_dim-variant-config $dim(\boldsymbol{q}_{var})$)

(_dim-invariant-config $dim(\boldsymbol{q}_{invar})$)

(_dim-config $dim(\boldsymbol{q})$)

(_dim-task $dim(\boldsymbol{e})$)

(_kin-scale-mat-list K_{kin})

(_target-posture-scale $k_{posture}$)

(_norm-regular-scale-max k_{max})
(_norm-regular-scale-offset k_{off})
(_torque-regular-scale k_{trq})
(_variant-joint-list $\mathcal{J}_{var}$)
(_invariant-joint-list $\mathcal{J}_{invar}$)
(_drive-joint-list $\mathcal{J}_{drive}$)
(_kin-target-coords-list $\mathcal{T}^{kin\text{-}trg}$)
(_kin-attention-coords-list $\mathcal{T}^{kin\text{-}att}$)
(_contact-target-coords-list $\mathcal{T}^{cnt\text{-}trg}$)
(_contact-attention-coords-list $\mathcal{T}^{cnt\text{-}att}$)
(_variant-joint-angle-margin margin of $\boldsymbol{\theta}$ [deg] [mm])
(_invariant-joint-angle-margin margin of $\boldsymbol{\phi}$ [deg] [mm])
(_delta-linear-joint trust region of linear joint configuration [mm])
(_delta-rotational-joint trust region of rotational joint configuration [deg])
(_contact-constraint-list list of contact-constraint instance)
(_posture-joint-list $\mathcal{J}_{posture}$)
(_posture-joint-angle-list $\bar{\boldsymbol{\theta}}^{trg}$)
(_external-wrench-list $\{\boldsymbol{w}_1^{ex}, \boldsymbol{w}_2^{ex}, \cdots, \boldsymbol{w}_{N_{ex}}^{ex}\}$)
(_external-coords-list $\{T_1^{ex}, T_2^{ex}, \cdots, T_{N_{ex}}^{ex}\}$)
(_collision-pair-list list of bodyset-link or body pair)
(_collision-distance-margin-list list of collision distance margin)
(_only-kinematics? whether to consider only kinematics or not)
(_variant-task-jacobi buffer for $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}}$)
(_invariant-task-jacobi buffer for $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}}$)
(_task-jacobi buffer for $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}}$)
(_collision-theta-inequality-constraint-matrix buffer for $\boldsymbol{C}_{col,\theta}$)
(_collision-phi-inequality-constraint-matrix buffer for $\boldsymbol{C}_{col,\phi}$)
(_collision-inequality-constraint-vector buffer for $\boldsymbol{C}_{col}$)

$\boldsymbol{q}^{(l)}$ $\boldsymbol{e}^{(l)}(\boldsymbol{q}^{(l)})$

l $\boldsymbol{x}^{(l)}$

$\boldsymbol{x}$ " " "instant"

$\boldsymbol{q}$ $\boldsymbol{e}(\boldsymbol{q})$ $\frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}$

$\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{C}, \boldsymbol{d}$

-

  robot-environment robot-environment

  variant-joint-list $\mathcal{J}_{var}$

  invariant-joint-list $\mathcal{J}_{invar}$ ( )

  drive-joint-list $\mathcal{J}_{drive}$ ( )

-

  kin-target-coords-list $\mathcal{T}^{kin\text{-}trg}$

kin-attention-coords-list $\mathcal{T}^{kin\text{-}att}$

kin-scale-mat-list K_{kin}

-

contact-target-coords-list $\mathcal{T}^{cnt\text{-}trg}$

contact-attention-coords-list $\mathcal{T}^{cnt\text{-}att}$

contact-constraint-list

-                                     (   )

posture-joint-list $\mathcal{J}_{posture}$

posture-joint-angle-list $\bar{\boldsymbol{\theta}}^{trg}$

target-posture-scale $k_{posture}$

-                (   )

collision-pair-list   bodyset-link   body

collision-distance-margin   (   )

collision-distance-margin-list   (   )

-       (   )

external-wrench-list   (   )

external-coords-list   (   )

-

norm-regular-scale-max k_{max}

norm-regular-scale-offset k_{off}

torque-regular-scale k_{trq}

$\boldsymbol{q}$

$$\boldsymbol{q} := \begin{pmatrix} \boldsymbol{\theta}^T & \hat{\boldsymbol{w}}^T & \boldsymbol{\tau}^T & \boldsymbol{\phi}^T \end{pmatrix}^T \tag{19}$$

$\boldsymbol{\theta} \in \mathbb{R}^{N_{var\text{-}joint}}$ [rad] [m]

$\hat{\boldsymbol{w}} \in \mathbb{R}^{6N_{cnt}}$ [N] [Nm]

$\boldsymbol{\tau} \in \mathbb{R}^{N_{drive\text{-}joint}}$ [Nm] [N]

$\boldsymbol{\phi} \in \mathbb{R}^{N_{invar\text{-}joint}}$ [rad] [m]

$\hat{\boldsymbol{w}}$

$$\hat{\boldsymbol{w}} = \begin{pmatrix} \boldsymbol{w}_1^T & \boldsymbol{w}_2^T & \cdots & \boldsymbol{w}_{N_{cnt}}^T \end{pmatrix}^T \tag{20}$$

$$= \begin{pmatrix} \boldsymbol{f}_1^T & \boldsymbol{n}_1^T & \boldsymbol{f}_2^T & \boldsymbol{n}_2^T & \cdots & \boldsymbol{f}_{N_{cnt}}^T & \boldsymbol{n}_{N_{cnt}}^T \end{pmatrix}^T \tag{21}$$

$\boldsymbol{e}(\boldsymbol{q})$

$$\boldsymbol{e}(\boldsymbol{q}) := \begin{pmatrix} \boldsymbol{e}^{kinT}(\boldsymbol{q}) & \boldsymbol{e}^{eom\text{-}transT}(\boldsymbol{q}) & \boldsymbol{e}^{eom\text{-}rotT}(\boldsymbol{q}) & \boldsymbol{e}^{trqT}(\boldsymbol{q}) & \boldsymbol{e}^{postureT}(\boldsymbol{q}) \end{pmatrix}^T \tag{22}$$

$\boldsymbol{e}^{kin}(\boldsymbol{q}) \in \mathbb{R}^{6N_{kin}}$ [rad] [m]

$\boldsymbol{e}^{eom\text{-}trans}(\boldsymbol{q}) \in \mathbb{R}^3$ [N]

$\boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{q}) \in \mathbb{R}^3$ [Nm]

$\boldsymbol{e}^{trq}(\boldsymbol{q}) \in \mathbb{R}^{N_{drive\text{-}joint}}$ [rad] [m]

$$\boldsymbol{e}^{posture}(\boldsymbol{q}) \in \mathbb{R}^{N_{posture\text{-}joint}} \qquad \text{[rad] [m]}$$

:init *&key* *(name)* [method]

      (robot-env)

      (variant-joint-list (send robot-env :variant-joint-list))

      (invariant-joint-list (send robot-env :invariant-joint-list))

      (drive-joint-list (send robot-env :drive-joint-list))

      (only-kinematics?)

      (kin-target-coords-list)

      (kin-attention-coords-list)

      (contact-target-coords-list)

      (contact-attention-coords-list)

      (variant-joint-angle-margin 3.0)

      (invariant-joint-angle-margin 3.0)

      (delta-linear-joint)

      (delta-rotational-joint)

      (contact-constraint-list (send-all contact-attention-coords-list :get :contact-constraint))

      (posture-joint-list)

      (posture-joint-angle-list)

      (external-wrench-list)

      (external-coords-list)

      (collision-pair-list)

      (collision-distance-margin 0.01)

      (collision-distance-margin-list)

      (kin-scale 1.0)

      (kin-scale-list)

      (kin-scale-mat-list)

      (target-posture-scale 0.001)

      (norm-regular-scale-max (if only-kinematics? 0.001 1.000000e-05))

      (norm-regular-scale-offset 1.000000e-07)

      (torque-regular-scale 1.000000e-04)

      &allow-other-keys

    Initialize instance

:robot-env [method]

    return robot-environment instance

:variant-joint-list [method]

    return $\mathcal{J}_{var}$

:invariant-joint-list [method]

    return $\mathcal{J}_{invar}$

:drive-joint-list [method]

    return $\mathcal{J}_{drive}$

:only-kinematics? [method]

    return whether to consider only kinematics or not

:theta [method]

     return $\boldsymbol{\theta}$

:wrench [method]

     return $\hat{\boldsymbol{w}}$

:torque [method]

     return $\boldsymbol{\tau}$

:phi [method]

     return $\boldsymbol{\phi}$

:num-kin [method]

     return $N_{kin} := |\mathcal{T}^{kin\text{-}trg}| = |\mathcal{T}^{kin\text{-}att}|$

:num-contact [method]

     return $N_{cnt} := |\mathcal{T}^{cnt\text{-}trg}| = |\mathcal{T}^{cnt\text{-}att}|$

:num-variant-joint [method]

     return $N_{var\text{-}joint} := |\mathcal{J}_{var}|$

:num-invariant-joint [method]

     return $N_{invar\text{-}joint} := |\mathcal{J}_{invar}|$

:num-drive-joint [method]

     return $N_{drive\text{-}joint} := |\mathcal{J}_{drive}|$

:num-posture-joint [method]

     return $N_{target\text{-}joint} := |\mathcal{J}_{target}|$

:num-external [method]

     return $N_{ex} :=$ number of external wrench

:num-collision [method]

     return $N_{col} :=$ number of collision check pairs

:dim-variant-config [method]

$$
\begin{aligned}
dim(\boldsymbol{q_{var}}) \quad &:= \quad dim(\boldsymbol{\theta}) + dim(\hat{\boldsymbol{w}}) + dim(\boldsymbol{\tau}) \tag{23} \\
&= \quad N_{var\text{-}joint} + 6N_{cnt} + N_{drive\text{-}joint} \tag{24}
\end{aligned}
$$

     return $dim(\boldsymbol{q_{var}})$

:dim-invariant-config [method]

     return $dim(\boldsymbol{q_{invar}}) := dim(\boldsymbol{\phi}) = N_{invar\text{-}joint}$

:dim-config [method]

     return $dim(\boldsymbol{q}) := dim(\boldsymbol{q_{var}}) + dim(\boldsymbol{q_{invar}})$

:dim-task [method]

$$
\begin{aligned}
dim(\boldsymbol{e}) \quad &:= \quad dim(\boldsymbol{e}^{kin}) + dim(\boldsymbol{e}^{eom\text{-}trans}) + dim(\boldsymbol{e}^{eom\text{-}rot}) + dim(\boldsymbol{e}^{trq}) + dim(\boldsymbol{e}^{posture}) \tag{25} \\
&= \quad 6N_{kin} + 3 + 3 + N_{drive\text{-}joint} + N_{posture\text{-}joint} \tag{26}
\end{aligned}
$$

return $dim(\boldsymbol{e})$

:variant-config-vector [method]

$$\text{return } \boldsymbol{q_{var}} := \begin{pmatrix} \boldsymbol{\theta} \\ \hat{\boldsymbol{w}} \\ \boldsymbol{\tau} \end{pmatrix}$$

:invariant-config-vector [method]

return $\boldsymbol{q_{invar}} := \boldsymbol{\phi}$

:config-vector [method]

$$\text{return } \boldsymbol{q} := \begin{pmatrix} \boldsymbol{q_{var}} \\ \boldsymbol{q_{invar}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\theta} \\ \hat{\boldsymbol{w}} \\ \boldsymbol{\tau} \\ \boldsymbol{\phi} \end{pmatrix}$$

:set-theta *theta-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{\theta}$.

:set-wrench *wrench-new &key (relative? nil)* [method]

Set $\hat{\boldsymbol{w}}$.

:set-torque *torque-new &key (relative? nil)* [method]

Set $\boldsymbol{\tau}$.

:set-phi *phi-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{\phi}$.

:set-variant-config *variant-config-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{q_{var}}$.

:set-invariant-config *invariant-config-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{q_{invar}}$.

:set-config *config-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{q}$.

:kin-target-coords-list [method]

$$T_m^{kin\text{-}trg} = \{\boldsymbol{p}_m^{kin\text{-}trg}, \boldsymbol{R}_m^{kin\text{-}trg}\} \quad (m = 1, 2, \cdots, N_{kin}) \tag{27}$$

return $\mathcal{T}^{kin\text{-}trg} := \{T_1^{kin\text{-}trg}, T_2^{kin\text{-}trg}, \cdots, T_{N_{kin}}^{kin\text{-}trg}\}$

:kin-attention-coords-list [method]

$$T_m^{kin\text{-}att} = \{\boldsymbol{p}_m^{kin\text{-}att}, \boldsymbol{R}_m^{kin\text{-}att}\} \quad (m = 1, 2, \cdots, N_{kin}) \tag{28}$$

return $\mathcal{T}^{kin\text{-}att} := \{T_1^{kin\text{-}att}, T_2^{kin\text{-}att}, \cdots, T_{N_{kin}}^{kin\text{-}att}\}$

:contact-target-coords-list [method]

$$T_m^{cnt\text{-}trg} = \{\boldsymbol{p}_m^{cnt\text{-}trg}, \boldsymbol{R}_m^{cnt\text{-}trg}\} \quad (m = 1, 2, \cdots, N_{cnt}) \tag{29}$$

return $\mathcal{T}^{cnt\text{-}trg} := \{T_1^{cnt\text{-}trg}, T_2^{cnt\text{-}trg}, \cdots, T_{N_{cnt}}^{cnt\text{-}trg}\}$

:contact-attention-coords-list [method]

$$T_m^{cnt\text{-}att} = \{\boldsymbol{p}_m^{cnt\text{-}att}, \boldsymbol{R}_m^{cnt\text{-}att}\} \quad (m = 1, 2, \cdots, N_{cnt}) \tag{30}$$

return $\mathcal{T}^{cnt\text{-}att} := \{T_1^{cnt\text{-}att}, T_2^{cnt\text{-}att}, \cdots, T_{N_{cnt}}^{cnt\text{-}att}\}$

:contact-constraint-list [method]
return list of contact-constraint instance

:wrench-list [method]
return $\{\boldsymbol{w}_1, \boldsymbol{w}_2, \cdots, \boldsymbol{w}_{N_{cnt}}\}$

:force-list [method]
return $\{\boldsymbol{f}_1, \boldsymbol{f}_2, \cdots, \boldsymbol{f}_{N_{cnt}}\}$

:moment-list [method]
return $\{\boldsymbol{n}_1, \boldsymbol{n}_2, \cdots, \boldsymbol{n}_{N_{cnt}}\}$

:external-wrench-list [method]
return $\{\boldsymbol{w}_1^{ex}, \boldsymbol{w}_2^{ex}, \cdots, \boldsymbol{w}_{N_{ex}}^{ex}\}$

:external-force-list [method]
return $\{\boldsymbol{f}_1^{ex}, \boldsymbol{f}_2^{ex}, \cdots, \boldsymbol{f}_{N_{ex}}^{ex}\}$

:external-moment-list [method]
return $\{\boldsymbol{n}_1^{ex}, \boldsymbol{n}_2^{ex}, \cdots, \boldsymbol{n}_{N_{ex}}^{ex}\}$

:mg-vec [method]
return $m\boldsymbol{g}$

:cog *&key (update? t)* [method]
return $\boldsymbol{p}_G(\boldsymbol{q})$

:kinematics-task-value *&key (update? t)* [method]

$$\boldsymbol{e}^{kin}(\boldsymbol{q}) = \boldsymbol{e}^{kin}(\boldsymbol{\theta}, \boldsymbol{\phi}) \tag{31}$$

$$= \begin{pmatrix} \boldsymbol{e}_1^{kin}(\boldsymbol{\theta}, \boldsymbol{\phi}) \\ \boldsymbol{e}_2^{kin}(\boldsymbol{\theta}, \boldsymbol{\phi}) \\ \vdots \\ \boldsymbol{e}_{N_{kin}}^{kin}(\boldsymbol{\theta}, \boldsymbol{\phi}) \end{pmatrix} \tag{32}$$

$$\boldsymbol{e}_m^{kin}(\boldsymbol{\theta}, \boldsymbol{\phi}) = K_{kin} \begin{pmatrix} \boldsymbol{p}_m^{kin\text{-}trg}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{p}_m^{kin\text{-}att}(\boldsymbol{\theta}, \boldsymbol{\phi}) \\ \boldsymbol{a}\left(\boldsymbol{R}_m^{kin\text{-}trg}(\boldsymbol{\theta}, \boldsymbol{\phi})\boldsymbol{R}_m^{kin\text{-}att}(\boldsymbol{\theta}, \boldsymbol{\phi})^T\right) \end{pmatrix} \in \mathbb{R}^6 \quad (m = 1, 2, \cdots, N_{kin}) \tag{33}$$

$$\boldsymbol{a}(\boldsymbol{R}) \qquad\qquad \boldsymbol{R}$$

return $\boldsymbol{e}^{kin}(\boldsymbol{q}) \in \mathbb{R}^{6N_{kin}}$

:eom-trans-task-value *&key (update? t)* [method]

$$
\begin{aligned}
\boldsymbol{e}^{eom\text{-}trans}(\boldsymbol{q}) &= \boldsymbol{e}^{eom\text{-}trans}(\hat{\boldsymbol{w}}) & (34)\\
&= \sum_{m=1}^{N_{cnt}} \boldsymbol{f}_m - m\boldsymbol{g} + \sum_{m=1}^{N_{ex}} \boldsymbol{f}_m^{ex} & (35)
\end{aligned}
$$

return $\boldsymbol{e}^{eom\text{-}trans}(\boldsymbol{q}) \in \mathbb{R}^3$

:eom-rot-task-value *&key (update? t)* [method]

$$
\begin{aligned}
\boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{q}) &= \boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}, \boldsymbol{\phi}) & (36)\\
&= \sum_{m=1}^{N_{cnt}} \left\{ \left(\boldsymbol{p}_m^{cnt\text{-}trg}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{p}_G(\boldsymbol{\theta}, \boldsymbol{\phi}) \right) \times \boldsymbol{f}_m + \boldsymbol{n}_m \right\}\\
&\quad + \sum_{m=1}^{N_{ex}} \left\{ (\boldsymbol{p}_m^{ex}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{p}_G(\boldsymbol{\theta}, \boldsymbol{\phi})) \times \boldsymbol{f}_m^{ex} + \boldsymbol{n}_m^{ex} \right\} & (37)
\end{aligned}
$$

return $\boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{q}) \in \mathbb{R}^3$

:torque-task-value *&key (update? t)* [method]

$$
\begin{aligned}
\boldsymbol{e}^{trq}(\boldsymbol{q}) &= \boldsymbol{e}^{trq}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}, \boldsymbol{\tau}, \boldsymbol{\phi}) & (38)\\
&= \boldsymbol{\tau} + \sum_{m=1}^{N_{cnt}} \boldsymbol{\tau}_m^{cnt}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{\tau}^{grav}(\boldsymbol{\theta}, \boldsymbol{\phi}) + \sum_{m=1}^{N_{ex}} \boldsymbol{\tau}_m^{ex}(\boldsymbol{\theta}, \boldsymbol{\phi}) & (39)\\
&= \boldsymbol{\tau} + \sum_{m=1}^{N_{cnt}} \boldsymbol{J}_{drive\text{-}joint,m}^{cnt\text{-}att}(\boldsymbol{\theta}, \boldsymbol{\phi})^T \boldsymbol{w}_m - \boldsymbol{\tau}^{grav}(\boldsymbol{\theta}, \boldsymbol{\phi}) + \sum_{m=1}^{N_{ex}} \boldsymbol{J}_{drive\text{-}joint,m}^{ex}(\boldsymbol{\theta}, \boldsymbol{\phi})^T \boldsymbol{w}_m^{ex} & (40)
\end{aligned}
$$

$\boldsymbol{\tau}_m^{cnt}(\boldsymbol{\theta}, \boldsymbol{\phi}) \qquad m \qquad\qquad\qquad\qquad \boldsymbol{w}_m \qquad\qquad\qquad \boldsymbol{\tau}_m^{grav}(\boldsymbol{\theta}, \boldsymbol{\phi})$

return $\boldsymbol{e}^{trq}(\boldsymbol{q}) \in \mathbb{R}^{N_{drive\text{-}joint}}$

:posture-task-value *&key (update? t)* [method]

$$
\begin{aligned}
\boldsymbol{e}^{posture}(\boldsymbol{q}) &= \boldsymbol{e}^{posture}(\boldsymbol{\theta}) & (41)\\
&= k_{posture} \left(\bar{\boldsymbol{\theta}}^{trg} - \bar{\boldsymbol{\theta}} \right) & (42)
\end{aligned}
$$

$\bar{\boldsymbol{\theta}}^{trg}, \bar{\boldsymbol{\theta}} \qquad\qquad\qquad \mathcal{J}_{posture}$

return $\boldsymbol{e}^{posture}(\boldsymbol{q}) \in \mathbb{R}^{N_{posture\text{-}joint}}$

:task-value *&key (update? t)* [method]

$$
\text{return } \boldsymbol{e}(\boldsymbol{q}) := \begin{pmatrix} \boldsymbol{e}^{kin}(\boldsymbol{q}) \\ \boldsymbol{e}^{eom\text{-}trans}(\boldsymbol{q}) \\ \boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{q}) \\ \boldsymbol{e}^{trq}(\boldsymbol{q}) \\ \boldsymbol{e}^{posture}(\boldsymbol{q}) \end{pmatrix} = \begin{pmatrix} \boldsymbol{e}^{kin}(\boldsymbol{\theta}, \boldsymbol{\phi}) \\ \boldsymbol{e}^{eom\text{-}trans}(\hat{\boldsymbol{w}}) \\ \boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}, \boldsymbol{\phi}) \\ \boldsymbol{e}^{trq}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}, \boldsymbol{\tau}, \boldsymbol{\phi}) \\ \boldsymbol{e}^{posture}(\boldsymbol{\theta}) \end{pmatrix}
$$

:kinematics-task-jacobian-with-theta [method]

$$\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\theta}} \;=\; \begin{pmatrix} \frac{\partial \boldsymbol{e}_1^{kin}}{\partial \boldsymbol{\theta}} \\ \frac{\partial \boldsymbol{e}_2^{kin}}{\partial \boldsymbol{\theta}} \\ \vdots \\ \frac{\partial \boldsymbol{e}_{N_{kin}}^{kin}}{\partial \boldsymbol{\theta}} \end{pmatrix} \tag{43}$$

$$\frac{\partial \boldsymbol{e}_m^{kin}}{\partial \boldsymbol{\theta}} \;=\; K_{kin} \left\{ \boldsymbol{J}_{\theta,m}^{kin\text{-}trg}(\boldsymbol{\theta},\boldsymbol{\phi}) - \boldsymbol{J}_{\theta,m}^{kin\text{-}att}(\boldsymbol{\theta},\boldsymbol{\phi}) \right\} \quad (m = 1,2,\cdots,N_{kin}) \tag{44}$$

return $\dfrac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\theta}} \in \mathbb{R}^{6N_{kin} \times N_{var\text{-}joint}}$

:kinematics-task-jacobian-with-phi [method]

$$\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\phi}} \;=\; \begin{pmatrix} \frac{\partial \boldsymbol{e}_1^{kin}}{\partial \boldsymbol{\phi}} \\ \frac{\partial \boldsymbol{e}_2^{kin}}{\partial \boldsymbol{\phi}} \\ \vdots \\ \frac{\partial \boldsymbol{e}_{N_{kin}}^{kin}}{\partial \boldsymbol{\phi}} \end{pmatrix} \tag{45}$$

$$\frac{\partial \boldsymbol{e}_m^{kin}}{\partial \boldsymbol{\phi}} \;=\; K_{kin} \left\{ \boldsymbol{J}_{\phi,m}^{kin\text{-}trg}(\boldsymbol{\theta},\boldsymbol{\phi}) - \boldsymbol{J}_{\phi,m}^{kin\text{-}att}(\boldsymbol{\theta},\boldsymbol{\phi}) \right\} \quad (m = 1,2,\cdots,N_{kin}) \tag{46}$$

return $\dfrac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\phi}} \in \mathbb{R}^{6N_{kin} \times N_{invar\text{-}joint}}$

:eom-trans-task-jacobian-with-wrench [method]

$$\frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \hat{\boldsymbol{w}}} \;=\; \begin{pmatrix} \frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \boldsymbol{f}_1} & \frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \boldsymbol{n}_1} & \cdots & \frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \boldsymbol{f}_{N_{cnt}}} & \frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \boldsymbol{n}_{N_{cnt}}} \end{pmatrix} \tag{47}$$

$$=\; \begin{pmatrix} \boldsymbol{I}_3 & \boldsymbol{O}_3 & \cdots & \boldsymbol{I}_3 & \boldsymbol{O}_3 \end{pmatrix} \tag{48}$$

return $\dfrac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \hat{\boldsymbol{w}}} \in \mathbb{R}^{3 \times 6N_{cnt}}$

:eom-rot-task-jacobian-with-theta [method]

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\theta}} \;=\; \sum_{m=1}^{N_{cnt}} \left\{ -[\boldsymbol{f}_m \times] \left(\boldsymbol{J}_{\theta,m}^{cnt\text{-}trg}(\boldsymbol{\theta},\boldsymbol{\phi}) - \boldsymbol{J}_{G\theta}(\boldsymbol{\theta},\boldsymbol{\phi}) \right) \right\}$$
$$+ \sum_{m=1}^{N_{ex}} \left\{ -[\boldsymbol{f}_m^{ex} \times] \left(\boldsymbol{J}_{\theta,m}^{ex}(\boldsymbol{\theta},\boldsymbol{\phi}) - \boldsymbol{J}_{G\theta}(\boldsymbol{\theta},\boldsymbol{\phi}) \right) \right\} \tag{49}$$

$$=\; \left[\left(\sum_{m=1}^{N_{cnt}} \boldsymbol{f}_m + \sum_{m=1}^{N_{ex}} \boldsymbol{f}_m^{ex} \right) \times \right] \boldsymbol{J}_{G\theta}(\boldsymbol{\theta},\boldsymbol{\phi})$$
$$- \sum_{m=1}^{N_{cnt}} [\boldsymbol{f}_m \times] \boldsymbol{J}_{\theta,m}^{cnt\text{-}trg}(\boldsymbol{\theta},\boldsymbol{\phi}) - \sum_{m=1}^{N_{ex}} [\boldsymbol{f}_m^{ex} \times] \boldsymbol{J}_{\theta,m}^{ex}(\boldsymbol{\theta},\boldsymbol{\phi}) \tag{50}$$

$\sum_{m=1}^{N_{cnt}} \boldsymbol{f}_m + \sum_{m=1}^{N_{ex}} \boldsymbol{f}_m^{ex} = m\boldsymbol{g}$ \qquad eom-trans-task

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\theta}} \;=\; [m\boldsymbol{g} \times] \boldsymbol{J}_{G\theta}(\boldsymbol{\theta},\boldsymbol{\phi}) - \sum_{m=1}^{N_{cnt}} [\boldsymbol{f}_m \times] \boldsymbol{J}_{\theta,m}^{cnt\text{-}trg}(\boldsymbol{\theta},\boldsymbol{\phi}) - \sum_{m=1}^{N_{ex}} [\boldsymbol{f}_m^{ex} \times] \boldsymbol{J}_{\theta,m}^{ex}(\boldsymbol{\theta},\boldsymbol{\phi}) \tag{51}$$

return $\dfrac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\theta}} \in \mathbb{R}^{3 \times N_{var\text{-}joint}}$

:eom-rot-task-jacobian-with-wrench [method]

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \hat{\boldsymbol{w}}} \;=\; \begin{pmatrix} \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{f}_1} & \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{n}_1} & \cdots & \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{f}_{N_{cnt}}} & \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{n}_{N_{cnt}}} \end{pmatrix} \tag{52}$$

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{f}_m} \;=\; \left[\left(\boldsymbol{p}_m^{cnt\text{-}trg}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{p}_G(\boldsymbol{\theta}, \boldsymbol{\phi}) \right) \times \right] \quad (m = 1, 2, \cdots, N_{cnt}) \tag{53}$$

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{n}_m} \;=\; \boldsymbol{I}_3 \quad (m = 1, 2, \cdots, N_{cnt}) \tag{54}$$

return $\dfrac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \hat{\boldsymbol{w}}} \in \mathbb{R}^{3 \times 6 N_{cnt}}$

:eom-rot-task-jacobian-with-phi [method]

$$\begin{aligned}
\frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\phi}} \;=&\; \sum_{m=1}^{N_{cnt}} \left\{ -[\boldsymbol{f}_m \times] \left(\boldsymbol{J}_{\phi,m}^{cnt\text{-}trg}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{J}_{G\phi}(\boldsymbol{\theta}, \boldsymbol{\phi}) \right) \right\} \\
&+ \sum_{m=1}^{N_{ex}} \left\{ -[\boldsymbol{f}_m^{ex} \times] \left(\boldsymbol{J}_{\phi,m}^{ex}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{J}_{G\phi}(\boldsymbol{\theta}, \boldsymbol{\phi}) \right) \right\} \tag{55} \\
=&\; \left[\left(\sum_{m=1}^{N_{cnt}} \boldsymbol{f}_m + \sum_{m=1}^{N_{ex}} \boldsymbol{f}_m^{ex} \right) \times \right] \boldsymbol{J}_{G\phi}(\boldsymbol{\theta}, \boldsymbol{\phi}) \\
&- \sum_{m=1}^{N_{cnt}} [\boldsymbol{f}_m \times] \boldsymbol{J}_{\phi,m}^{cnt\text{-}trg}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \sum_{m=1}^{N_{ex}} [\boldsymbol{f}_m^{ex} \times] \boldsymbol{J}_{\phi,m}^{ex}(\boldsymbol{\theta}, \boldsymbol{\phi}) \tag{56}
\end{aligned}$$

$\sum_{m=1}^{N_{cnt}} \boldsymbol{f}_m + \sum_{m=1}^{N_{ex}} \boldsymbol{f}_m^{ex} = m\boldsymbol{g}$ eom-trans-task

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\phi}} \;=\; [m\boldsymbol{g} \times] \boldsymbol{J}_{G\phi}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \sum_{m=1}^{N_{cnt}} [\boldsymbol{f}_m \times] \boldsymbol{J}_{\phi,m}^{cnt\text{-}trg}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \sum_{m=1}^{N_{ex}} [\boldsymbol{f}_m^{ex} \times] \boldsymbol{J}_{\phi,m}^{ex}(\boldsymbol{\theta}, \boldsymbol{\phi}) \tag{57}$$

return $\dfrac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\phi}} \in \mathbb{R}^{3 \times N_{invar\text{-}joint}}$

:torque-task-jacobian-with-theta [method]

$$\frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\theta}} \;=\; \sum_{m=1}^{N_{cnt}} \frac{\partial \boldsymbol{\tau}_m^{cnt}}{\partial \boldsymbol{\theta}} - \frac{\partial \boldsymbol{\tau}^{grav}}{\partial \boldsymbol{\theta}} + \sum_{m=1}^{N_{ex}} \frac{\partial \boldsymbol{\tau}_m^{ex}}{\partial \boldsymbol{\theta}} \tag{58}$$

return $\dfrac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\theta}} \in \mathbb{R}^{N_{drive\text{-}joint} \times N_{var\text{-}joint}}$

:torque-task-jacobian-with-wrench [method]

$$\frac{\partial \boldsymbol{e}^{trq}}{\partial \hat{\boldsymbol{w}}} \;=\; \begin{pmatrix} \dfrac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{w}_1} & \dfrac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{w}_2} & \cdots & \dfrac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{w}_{N_{cnt}}} \end{pmatrix} \tag{59}$$

$$\frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{w}_m} \;=\; \boldsymbol{J}_{drive\text{-}joint,m}^{cnt\text{-}att}(\boldsymbol{\theta}, \boldsymbol{\phi})^T \quad (m = 1, 2, \cdots, N_{cnt}) \tag{60}$$

return $\dfrac{\partial \boldsymbol{e}^{trq}}{\partial \hat{\boldsymbol{w}}} \in \mathbb{R}^{N_{drive\text{-}joint} \times 6 N_{cnt}}$

:torque-task-jacobian-with-phi [method]

$$\frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\phi}} \quad = \quad \sum_{m=1}^{N_{cnt}} \frac{\partial \boldsymbol{\tau}_m^{cnt}}{\partial \boldsymbol{\phi}} - \frac{\partial \boldsymbol{\tau}_m^{grav}}{\partial \boldsymbol{\phi}} + \sum_{m=1}^{N_{ex}} \frac{\partial \boldsymbol{\tau}_m^{ex}}{\partial \boldsymbol{\phi}} \tag{61}$$

return $\frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\phi}} \in \mathbb{R}^{N_{drive\text{-}joint} \times N_{invar\text{-}joint}}$

:torque-task-jacobian-with-torque [method]

$$\frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\tau}} \quad = \quad \boldsymbol{I}_{N_{drive\text{-}joint}} \tag{62}$$

return $\frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\tau}} \in \mathbb{R}^{N_{drive\text{-}joint} \times N_{drive\text{-}joint}}$

:posture-task-jacobian-with-theta *&key (update? nil)* [method]

$$\left(\frac{\partial \boldsymbol{e}^{posture}}{\partial \boldsymbol{\theta}}\right)_{i,j} \quad = \quad \begin{cases} -k_{posture} & (\mathcal{J}_{posture,i} = \mathcal{J}_{var,j}) \\ 0 & \text{otherwise} \end{cases} \tag{63}$$

return $\frac{\partial \boldsymbol{e}^{posture}}{\partial \boldsymbol{\theta}} \in \mathbb{R}^{N_{posture\text{-}joint} \times N_{var\text{-}joint}}$

:variant-task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} \quad = \quad \begin{matrix} 6N_{kin} \\ 3 \\ 3 \\ N_{drive\text{-}joint} \\ N_{posture\text{-}joint} \end{matrix} \begin{pmatrix} \overset{N_{var\text{-}joint}}{\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\theta}}} & \overset{6N_{cnt}}{\frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \hat{\boldsymbol{w}}}} & \overset{N_{drive\text{-}joint}}{} \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \hat{\boldsymbol{w}}} & \\ \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \hat{\boldsymbol{w}}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\tau}} \\ \frac{\partial \boldsymbol{e}^{posture}}{\partial \boldsymbol{\theta}} & & \end{pmatrix} \tag{64}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} \in \mathbb{R}^{(6N_{kin}+3+3+N_{drive\text{-}joint}+N_{posture\text{-}joint}) \times (N_{var\text{-}joint}+6N_{cnt}+N_{drive\text{-}joint})}$

:invariant-task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \quad = \quad \begin{matrix} 6N_{kin} \\ 3 \\ 3 \\ N_{drive\text{-}joint} \\ N_{posture\text{-}joint} \end{matrix} \begin{pmatrix} \overset{N_{invar\text{-}joint}}{\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\phi}}} \\ \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\phi}} \\ \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\phi}} \\ \\ \end{pmatrix} \tag{65}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \in \mathbb{R}^{(6N_{kin}+3+3+N_{drive\text{-}joint}+N_{posture\text{-}joint}) \times N_{invar\text{-}joint}}$

:task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} \quad = \quad \begin{pmatrix} \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} & \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \end{pmatrix} \tag{66}$$

$$= \quad \begin{array}{c} \\ 6N_{kin} \\ 3 \\ 3 \\ N_{drive\text{-}joint} \\ N_{posture\text{-}joint} \end{array} \begin{array}{ccc} N_{var\text{-}joint} & 6N_{cnt} & N_{drive\text{-}joint} & N_{invar\text{-}joint} \end{array} \begin{pmatrix} \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\theta}} & & & \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\phi}} \\ & \frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \hat{\boldsymbol{w}}} & & \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \hat{\boldsymbol{w}}} & & \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\phi}} \\ \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \hat{\boldsymbol{w}}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\tau}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\phi}} \\ \frac{\partial \boldsymbol{e}^{posture}}{\partial \boldsymbol{\theta}} & & & \end{pmatrix} \tag{67}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} \in \mathbb{R}^{(6N_{kin}+3+3+N_{drive\text{-}joint}+N_{posture\text{-}joint}) \times (N_{var\text{-}joint}+6N_{cnt}+N_{drive\text{-}joint}+N_{invar\text{-}joint})}$

:theta-max-vector *&key (update? nil)* [method]

return $\boldsymbol{\theta}_{max} \in \mathbb{R}^{N_{var\text{-}joint}}$

:theta-min-vector *&key (update? nil)* [method]

return $\boldsymbol{\theta}_{min} \in \mathbb{R}^{N_{var\text{-}joint}}$

:delta-theta-limit-vector *&key (update? nil)* [method]

get trust region of $\boldsymbol{\theta}$

return $\Delta \boldsymbol{\theta}_{limit}$

:theta-inequality-constraint-matrix *&key (update? nil)* [method]

$$\begin{cases} \boldsymbol{\theta}_{min} \leq \boldsymbol{\theta} + \Delta \boldsymbol{\theta} \leq \boldsymbol{\theta}_{max} \\ -\Delta \boldsymbol{\theta}_{limit} \leq \Delta \boldsymbol{\theta} \leq \Delta \boldsymbol{\theta}_{limit} \quad (\text{if} \quad \Delta \boldsymbol{\theta}_{limit} \quad \text{is set}) \end{cases} \tag{68}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \Delta \boldsymbol{\theta} \geq \begin{pmatrix} \boldsymbol{\theta}_{min} - \boldsymbol{\theta} \\ -(\boldsymbol{\theta}_{max} - \boldsymbol{\theta}) \\ -\Delta \boldsymbol{\theta}_{limit} \\ -\Delta \boldsymbol{\theta}_{limit} \end{pmatrix} \tag{69}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\boldsymbol{\theta}} \Delta \boldsymbol{\theta} \geq \boldsymbol{d}_{\boldsymbol{\theta}} \tag{70}$$

return $\boldsymbol{C}_{\boldsymbol{\theta}} := \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \in \mathbb{R}^{4N_{var\text{-}joint} \times N_{var\text{-}joint}}$

:theta-inequality-constraint-vector *&key (update? t)* [method]

return $\boldsymbol{d}_{\boldsymbol{\theta}} := \begin{pmatrix} \boldsymbol{\theta}_{min} - \boldsymbol{\theta} \\ -(\boldsymbol{\theta}_{max} - \boldsymbol{\theta}) \\ -\Delta \boldsymbol{\theta}_{limit} \\ -\Delta \boldsymbol{\theta}_{limit} \end{pmatrix} \in \mathbb{R}^{4N_{var\text{-}joint}}$

:wrench-inequality-constraint-matrix *&key (update? t)* [method]

$\boldsymbol{w} \in \mathbb{R}^6$

$$\boldsymbol{C}_w \boldsymbol{w} \geq \boldsymbol{d}_w \tag{71}$$

N_{cnt} $\hat{\boldsymbol{w}}$

$$\boldsymbol{C}_{w,m}(\boldsymbol{w}_m + \Delta\boldsymbol{w}_m) \geq \boldsymbol{d}_{w,m} \quad (m = 1, 2, \cdots, N_{cnt}) \tag{72}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{w,m}\Delta\boldsymbol{w}_m \geq \boldsymbol{d}_{w,m} - \boldsymbol{C}_{w,m}\boldsymbol{w}_m \quad (m = 1, 2, \cdots, N_{cnt}) \tag{73}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{C}_{w,1} & & & \\ & \boldsymbol{C}_{w,2} & & \\ & & \ddots & \\ & & & \boldsymbol{C}_{w,N_{cnt}} \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{w}_1 \\ \Delta\boldsymbol{w}_2 \\ \vdots \\ \Delta\boldsymbol{w}_{N_{cnt}} \end{pmatrix} \geq \begin{pmatrix} \boldsymbol{d}_{w,1} - \boldsymbol{C}_{w,1}\boldsymbol{w}_1 \\ \boldsymbol{d}_{w,2} - \boldsymbol{C}_{w,2}\boldsymbol{w}_2 \\ \vdots \\ \boldsymbol{d}_{w,N_{cnt}} - \boldsymbol{C}_{w,N_{cnt}}\boldsymbol{w}_{N_{cnt}} \end{pmatrix} \tag{74}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\hat{w}}\Delta\hat{\boldsymbol{w}} \geq \boldsymbol{d}_{\hat{w}} \tag{75}$$

$$\text{return } \boldsymbol{C}_{\hat{w}} := \begin{pmatrix} \boldsymbol{C}_{w,1} & & & \\ & \boldsymbol{C}_{w,2} & & \\ & & \ddots & \\ & & & \boldsymbol{C}_{w,N_{cnt}} \end{pmatrix} \in \mathbb{R}^{N_{wrench\text{-}ineq} \times dim(\hat{\boldsymbol{w}})}$$

:wrench-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d}_{\hat{w}} := \begin{pmatrix} \boldsymbol{d}_{w,1} - \boldsymbol{C}_{w,1}\boldsymbol{w}_1 \\ \boldsymbol{d}_{w,2} - \boldsymbol{C}_{w,2}\boldsymbol{w}_2 \\ \vdots \\ \boldsymbol{d}_{w,N_{cnt}} - \boldsymbol{C}_{w,N_{cnt}}\boldsymbol{w}_{N_{cnt}} \end{pmatrix} \in \mathbb{R}^{N_{wrench\text{-}ineq}}$$

:torque-max-vector *&key (update? nil)* [method]

$\quad\quad$ return $\boldsymbol{\tau}_{max} \in \mathbb{R}^{N_{drive\text{-}joint}}$

:torque-min-vector *&key (update? nil)* [method]

$\quad\quad$ return $\boldsymbol{\tau}_{min} \in \mathbb{R}^{N_{drive\text{-}joint}}$

:torque-inequality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{\tau}_{min} \leq \boldsymbol{\tau} + \Delta\boldsymbol{\tau} \leq \boldsymbol{\tau}_{max} \tag{76}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \Delta\boldsymbol{\tau} \geq \begin{pmatrix} \boldsymbol{\tau}_{min} - \boldsymbol{\tau} \\ -(\boldsymbol{\tau}_{max} - \boldsymbol{\tau}) \end{pmatrix} \tag{77}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\boldsymbol{\tau}}\Delta\boldsymbol{\tau} \geq \boldsymbol{d}_{\boldsymbol{\tau}} \tag{78}$$

$$\text{return } \boldsymbol{C}_{\boldsymbol{\tau}} := \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \in \mathbb{R}^{2N_{drive\text{-}joint} \times N_{drive\text{-}joint}}$$

:torque-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d}_{\boldsymbol{\tau}} := \begin{pmatrix} \boldsymbol{\tau}_{min} - \boldsymbol{\tau} \\ -(\boldsymbol{\tau}_{max} - \boldsymbol{\tau}) \end{pmatrix} \in \mathbb{R}^{2N_{drive\text{-}joint}}$$

:phi-max-vector *&key (update? nil)* [method]

$\quad\quad$ return $\boldsymbol{\phi}_{max} \in \mathbb{R}^{N_{invar\text{-}joint}}$

:phi-min-vector *&key (update? nil)* [method]

$\quad\quad$ return $\boldsymbol{\phi}_{min} \in \mathbb{R}^{N_{invar\text{-}joint}}$

:delta-phi-limit-vector *&key (update? nil)* [method]

$\quad\quad$ get trust region of $\boldsymbol{\phi}$

$\quad\quad$ return $\Delta\boldsymbol{\phi}_{limit}$

:phi-inequality-constraint-matrix *&key (update? nil)* [method]

$$\begin{cases} \boldsymbol{\phi}_{min} \leq \boldsymbol{\phi} + \Delta\boldsymbol{\phi} \leq \boldsymbol{\phi}_{max} \\ -\Delta\boldsymbol{\phi}_{limit} \leq \Delta\boldsymbol{\phi} \leq \Delta\boldsymbol{\phi}_{limit} \quad (\text{if} \quad \Delta\boldsymbol{\phi}_{limit} \text{ is set}) \end{cases} \tag{79}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \Delta\boldsymbol{\phi} \geq \begin{pmatrix} \boldsymbol{\phi}_{min} - \boldsymbol{\phi} \\ -(\boldsymbol{\phi}_{max} - \boldsymbol{\phi}) \\ -\Delta\boldsymbol{\phi}_{limit} \\ -\Delta\boldsymbol{\phi}_{limit} \end{pmatrix} \tag{80}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\phi}\Delta\boldsymbol{\phi} \geq \boldsymbol{d}_{\phi} \tag{81}$$

$$\text{return } \boldsymbol{C}_{\phi} := \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \in \mathbb{R}^{4N_{invar\text{-}joint} \times N_{invar\text{-}joint}}$$

:phi-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d}_{\phi} := \begin{pmatrix} \boldsymbol{\phi}_{min} - \boldsymbol{\phi} \\ -(\boldsymbol{\phi}_{max} - \boldsymbol{\phi}) \\ -\Delta\boldsymbol{\phi}_{limit} \\ -\Delta\boldsymbol{\phi}_{limit} \end{pmatrix} \in \mathbb{R}^{4N_{invar\text{-}joint}}$$

:variant-config-inequality-constraint-matrix *&key (update? nil)* [method]

$$\begin{cases} \boldsymbol{C}_{\theta}\Delta\boldsymbol{\theta} \geq \boldsymbol{d}_{\theta} \\ \boldsymbol{C}_{\hat{w}}\Delta\hat{\boldsymbol{w}} \geq \boldsymbol{d}_{\hat{w}} \\ \boldsymbol{C}_{\tau}\Delta\boldsymbol{\tau} \geq \boldsymbol{d}_{\tau} \end{cases} \tag{82}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{C}_{\theta} & & \\ & \boldsymbol{C}_{\hat{w}} & \\ & & \boldsymbol{C}_{\tau} \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\theta} \\ \Delta\hat{\boldsymbol{w}} \\ \Delta\boldsymbol{\tau} \end{pmatrix} \geq \begin{pmatrix} \boldsymbol{d}_{\theta} \\ \boldsymbol{d}_{\hat{w}} \\ \boldsymbol{d}_{\tau} \end{pmatrix} \tag{83}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{var}\Delta\boldsymbol{q}_{var} \geq \boldsymbol{d}_{var} \tag{84}$$

$$\text{return } \boldsymbol{C}_{var} := \begin{pmatrix} \boldsymbol{C}_{\theta} & & \\ & \boldsymbol{C}_{\hat{w}} & \\ & & \boldsymbol{C}_{\tau} \end{pmatrix} \in \mathbb{R}^{N_{var\text{-}ineq} \times dim(\boldsymbol{q}_{var})}$$

:variant-config-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d}_{var} := \begin{pmatrix} \boldsymbol{d}_{\theta} \\ \boldsymbol{d}_{\hat{w}} \\ \boldsymbol{d}_{\tau} \end{pmatrix} \in \mathbb{R}^{N_{var\text{-}ineq}}$$

:invariant-config-inequality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{C}_{\phi}\Delta\boldsymbol{\phi} \geq \boldsymbol{d}_{\phi} \tag{85}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{invar}\Delta\boldsymbol{q}_{invar} \geq \boldsymbol{d}_{invar} \tag{86}$$

$$\text{return } \boldsymbol{C}_{invar} := \boldsymbol{C}_{\phi} \in \mathbb{R}^{N_{invar\text{-}ineq} \times dim(\boldsymbol{q}_{invar})}$$

:invariant-config-inequality-constraint-vector *&key (update? t)* [method]

return $\boldsymbol{d}_{invar} := \boldsymbol{d}_\phi \in \mathbb{R}^{N_{invar\text{-}ineq}}$

:config-inequality-constraint-matrix *&key (update? nil)* [method]
$\qquad\qquad\qquad\qquad\qquad$ *(update-collision? nil)*

$$\begin{cases} \boldsymbol{C}_{var}\Delta\boldsymbol{q}_{var} \geq \boldsymbol{d}_{var} \\ \boldsymbol{C}_{invar}\Delta\boldsymbol{q}_{invar} \geq \boldsymbol{d}_{invar} \\ \boldsymbol{C}_{col}\begin{pmatrix} \Delta\boldsymbol{q}_{var} \\ \Delta\boldsymbol{q}_{invar} \end{pmatrix} \geq \boldsymbol{d}_{col} \end{cases} \tag{87}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{C}_{var} & \\ & \boldsymbol{C}_{invar} \\ \hline & \boldsymbol{C}_{col} \end{pmatrix}\begin{pmatrix} \Delta\boldsymbol{q}_{var} \\ \Delta\boldsymbol{q}_{invar} \end{pmatrix} \geq \begin{pmatrix} \boldsymbol{d}_{var} \\ \boldsymbol{d}_{invar} \\ \boldsymbol{d}_{col} \end{pmatrix} \tag{88}$$

$$\Leftrightarrow \quad \boldsymbol{C}\Delta\boldsymbol{q} \geq \boldsymbol{d} \tag{89}$$

return $\boldsymbol{C} := \begin{pmatrix} \boldsymbol{C}_{var} & \\ & \boldsymbol{C}_{invar} \\ \hline & \boldsymbol{C}_{col} \end{pmatrix} \in \mathbb{R}^{N_{ineq} \times dim(\boldsymbol{q})}$

:config-inequality-constraint-vector *&key (update? t)* [method]
$\qquad\qquad\qquad\qquad\qquad$ *(update-collision? nil)*

return $\boldsymbol{d} := \begin{pmatrix} \boldsymbol{d}_{var} \\ \boldsymbol{d}_{invar} \\ \boldsymbol{d}_{col} \end{pmatrix} \in \mathbb{R}^{N_{ineq}}$

:variant-config-equality-constraint-matrix *&key (update? nil)* [method]
$\qquad$ return $\boldsymbol{A}_{var} \in \mathbb{R}^{0 \times dim(\boldsymbol{q}_{var})}$ (no equality constraint)

:variant-config-equality-constraint-vector *&key (update? t)* [method]
$\qquad$ return $\boldsymbol{b}_{var} \in \mathbb{R}^0$ (no equality constraint)

:invariant-config-equality-constraint-matrix *&key (update? nil)* [method]
$\qquad$ return $\boldsymbol{A}_{invar} \in \mathbb{R}^{0 \times dim(\boldsymbol{q}_{invar})}$ (no equality constraint)

:invariant-config-equality-constraint-vector *&key (update? t)* [method]
$\qquad$ return $\boldsymbol{b}_{invar} \in \mathbb{R}^0$ (no equality constraint)

:config-equality-constraint-matrix *&key (update? nil)* [method]
$\qquad$ return $\boldsymbol{A} \in \mathbb{R}^{0 \times dim(\boldsymbol{q})}$ (no equality constraint)

:config-equality-constraint-vector *&key (update? t)* [method]
$\qquad$ return $\boldsymbol{b} \in \mathbb{R}^0$ (no equality constraint)

:torque-regular-matrix *&key (update? nil)* [method]
$\qquad\qquad\qquad\qquad\qquad$ *(only-variant? nil)*

$$F_{tau}(\boldsymbol{q}) \;=\; \left\|\frac{\boldsymbol{\tau}}{\boldsymbol{\tau}_{max}}\right\|^2 \quad (\qquad\qquad\qquad) \tag{90}$$
$$\;=\; \boldsymbol{\tau}^T\bar{\boldsymbol{W}}_\tau\boldsymbol{\tau} \tag{91}$$

$$\bar{\boldsymbol{W}}_{\tau} := \begin{pmatrix} \frac{1}{\tau_{max,1}^2} & & & \\ & \frac{1}{\tau_{max,2}^2} & & \\ & & \ddots & \\ & & & \frac{1}{\tau_{max,N_{drive\text{-}joint}}^2} \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{\tau}) \times dim(\boldsymbol{\tau})} \tag{92}$$

only-variant? is true:

$$\boldsymbol{W}_{\tau} := \begin{matrix} & \begin{matrix} dim(\boldsymbol{\theta}) & dim(\hat{\boldsymbol{w}}) & dim(\boldsymbol{\tau}) \end{matrix} \\ \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\boldsymbol{\tau}) \end{matrix} & \begin{pmatrix} & & \\ & & \\ & & \bar{\boldsymbol{W}}_{\tau} \end{pmatrix} \end{matrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var}) \times dim(\boldsymbol{q}_{var})} \tag{93}$$

otherwise:

$$\boldsymbol{W}_{\tau} := \begin{matrix} & \begin{matrix} dim(\boldsymbol{\theta}) & dim(\hat{\boldsymbol{w}}) & dim(\boldsymbol{\tau}) & dim(\boldsymbol{\phi}) \end{matrix} \\ \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\boldsymbol{\tau}) \\ dim(\boldsymbol{\phi}) \end{matrix} & \begin{pmatrix} & & & \\ & & & \\ & & \bar{\boldsymbol{W}}_{\tau} & \\ & & & \end{pmatrix} \end{matrix} \in \mathbb{R}^{dim(\boldsymbol{q}) \times dim(\boldsymbol{q})} \tag{94}$$

return $\boldsymbol{W}_{\tau}$

:torque-regular-vector *&key (update? t)* [method]
          (only-variant? nil)

$$\bar{\boldsymbol{v}}_{\tau} \quad := \quad \bar{\boldsymbol{W}}_{\tau} \boldsymbol{\tau} \tag{95}$$

$$= \begin{pmatrix} \frac{\tau_1}{\tau_{max,1}^2} \\ \frac{\tau_2}{\tau_{max,2}^2} \\ \vdots \\ \frac{\tau_{dim(\boldsymbol{\tau})}}{\tau_{max,dim(\boldsymbol{\tau})}^2} \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{\tau})} \tag{96}$$

only-variant? is true:

$$\boldsymbol{v}_{\tau} := \begin{matrix} & \begin{matrix} 1 \end{matrix} \\ \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\boldsymbol{\tau}) \end{matrix} & \begin{pmatrix} \\ \\ \bar{\boldsymbol{v}}_{\tau} \end{pmatrix} \end{matrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var})} \tag{97}$$

otherwise:

$$\boldsymbol{v}_{\tau} := \begin{matrix} & \begin{matrix} 1 \end{matrix} \\ \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\boldsymbol{\tau}) \\ dim(\boldsymbol{\phi}) \end{matrix} & \begin{pmatrix} \\ \\ \bar{\boldsymbol{v}}_{\tau} \\ \end{pmatrix} \end{matrix} \in \mathbb{R}^{dim(\boldsymbol{q})} \tag{98}$$

return $\boldsymbol{v}_{\tau}$

:torque-ratio [method]

$$\text{return } \frac{\boldsymbol{\tau}}{\boldsymbol{\tau}_{max}} := \begin{pmatrix} \frac{\tau_1}{\tau_{max,1}} \\ \frac{\tau_2}{\tau_{max,2}} \\ \vdots \\ \frac{\tau_{N_{drive\text{-}joint}}}{\tau_{max,N_{drive\text{-}joint}}} \end{pmatrix}$$

:regular-matrix [method]

$$\boldsymbol{W}_{reg} := \min(k_{max}, \|\boldsymbol{e}\|^2 + k_{off})\boldsymbol{I} + \boldsymbol{W}_\tau \tag{99}$$

$$\text{return } \boldsymbol{W}_{reg} \in \mathbb{R}^{dim(\boldsymbol{q}) \times dim(\boldsymbol{q})}$$

:regular-vector [method]

$$\boldsymbol{v}_{reg} := \boldsymbol{v}_\tau \tag{100}$$

$$\text{return } \boldsymbol{v}_{reg} \in \mathbb{R}^{dim(\boldsymbol{q})}$$

:update-collision-inequality-constraint [method]

1   2 $\boldsymbol{p}_1, \boldsymbol{p}_2$ 1   2 $\boldsymbol{p}_1, \boldsymbol{p}_2$

d_{margin}

$$\boldsymbol{d}_{12}^T(\boldsymbol{p}_1 - \boldsymbol{p}_2) \geq d_{margin} \tag{101}$$

$$\text{where} \quad \boldsymbol{d}_{12} = \boldsymbol{p}_1 - \boldsymbol{p}_2 \tag{102}$$

$\Delta\boldsymbol{q}$

$$\boldsymbol{d}_{12}^T \{(\boldsymbol{p}_1 + \Delta\boldsymbol{p}_1) - (\boldsymbol{p}_2 + \Delta\boldsymbol{p}_2)\} \geq d_{margin} \tag{103}$$

$$\text{where} \quad \Delta\boldsymbol{p}_1 = \boldsymbol{J}_{\theta,1}\Delta\boldsymbol{\theta} + \boldsymbol{J}_{\phi,1}\Delta\boldsymbol{\phi} \tag{104}$$

$$\Delta\boldsymbol{p}_2 = \boldsymbol{J}_{\theta,2}\Delta\boldsymbol{\theta} + \boldsymbol{J}_{\phi,2}\Delta\boldsymbol{\phi} \tag{105}$$

$$\boldsymbol{J}_{\theta,i} = \frac{\partial\boldsymbol{p}_i}{\partial\boldsymbol{\theta}}, \quad \boldsymbol{J}_{\phi,i} = \frac{\partial\boldsymbol{p}_i}{\partial\boldsymbol{\phi}} \quad (i = 1, 2) \tag{106}$$

$$\boldsymbol{d}_{12}^T \{(\boldsymbol{p}_1 + \boldsymbol{J}_{\theta,1}\Delta\boldsymbol{\theta} + \boldsymbol{J}_{\phi,1}\Delta\boldsymbol{\phi}) - (\boldsymbol{p}_2 + \boldsymbol{J}_{\theta,2}\Delta\boldsymbol{\theta} + \boldsymbol{J}_{\phi,2}\Delta\boldsymbol{\phi})\} \geq d_{margin} \tag{107}$$

$$\Leftrightarrow \quad \boldsymbol{d}_{12}^T(\boldsymbol{J}_{\theta,1} - \boldsymbol{J}_{\theta,2})\Delta\boldsymbol{\theta} + \boldsymbol{d}_{12}^T(\boldsymbol{J}_{\phi,1} - \boldsymbol{J}_{\phi,2})\Delta\boldsymbol{\phi} \geq -(\boldsymbol{d}_{12}^T(\boldsymbol{p}_1 - \boldsymbol{p}_2) - d_{margin}) \tag{108}$$

$$\Leftrightarrow \quad \boldsymbol{c}_{col,var}^T\Delta\boldsymbol{\theta} + \boldsymbol{c}_{col,invar}^T\Delta\boldsymbol{\phi} \geq d_{col} \tag{109}$$

$$\text{where} \quad \boldsymbol{c}_{col,var}^T = \boldsymbol{d}_{12}^T(\boldsymbol{J}_{\theta,1} - \boldsymbol{J}_{\theta,2}) \tag{110}$$

$$\boldsymbol{c}_{col,invar}^T = \boldsymbol{d}_{12}^T(\boldsymbol{J}_{\phi,1} - \boldsymbol{J}_{\phi,2}) \tag{111}$$

$$d_{col} = -(\boldsymbol{d}_{12}^T(\boldsymbol{p}_1 - \boldsymbol{p}_2) - d_{margin}) \tag{112}$$

i $\boldsymbol{c}_{col,var,i}^T, \boldsymbol{c}_{col,invar,i}^T, d_{col,i}$ $i =$

$1, 2, \cdots, N_{col}$

$$\begin{pmatrix} \boldsymbol{C}_{col,\theta} & \boldsymbol{C}_{col,\phi} \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\theta} \\ \Delta\boldsymbol{\phi} \end{pmatrix} \geq \boldsymbol{d}_{col} \tag{113}$$

$$\boldsymbol{C}_{col,\theta} := \begin{pmatrix} \boldsymbol{c}_{col,var,1}^T \\ \vdots \\ \boldsymbol{c}_{col,var,N_{col}}^T \end{pmatrix} \in \mathbb{R}^{N_{col} \times dim(\boldsymbol{\theta})} \tag{114}$$

$$\boldsymbol{C}_{col,\phi} := \begin{pmatrix} \boldsymbol{c}_{col,invar,1}^T \\ \vdots \\ \boldsymbol{c}_{col,invar,N_{col}}^T \end{pmatrix} \in \mathbb{R}^{N_{col} \times dim(\boldsymbol{\phi})}, \tag{115}$$

$$\boldsymbol{d}_{col} := \begin{pmatrix} d_{col,1} \\ \vdots \\ d_{col,N_{col}} \end{pmatrix} \in \mathbb{R}^{N_{col}} \tag{116}$$

update inequality matrix $\boldsymbol{C}_{col,\theta}, \boldsymbol{C}_{col,\phi}$ and inequality vector $\boldsymbol{d}_{col}$ for collision avoidance

:collision-theta-inequality-constraint-matrix [method]
    return $\boldsymbol{C}_{col,\theta} \in \mathbb{R}^{N_{col} \times dim(\boldsymbol{\theta})}$

:collision-phi-inequality-constraint-matrix [method]
    return $\boldsymbol{C}_{col,\phi} \in \mathbb{R}^{N_{col} \times dim(\boldsymbol{\phi})}$

:collision-inequality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{C}_{col} := N_{col} \begin{array}{c} \begin{array}{cccc} dim(\boldsymbol{\theta}) & dim(\hat{\boldsymbol{w}}) & dim(\boldsymbol{\tau}) & dim(\boldsymbol{\phi}) \end{array} \\ \left(\begin{array}{cccc} \boldsymbol{C}_{col,\theta} & \boldsymbol{O} & \boldsymbol{O} & \boldsymbol{C}_{col,\phi} \end{array} \right) \end{array} \tag{117}$$

return $\boldsymbol{C}_{col} \in \mathbb{R}^{N_{col} \times dim(\boldsymbol{q})}$

:collision-inequality-constraint-vector *&key (update? nil)* [method]
    return $\boldsymbol{d}_{col} \in \mathbb{R}^{N_{col}}$

:update-viewer [method]
    Update viewer.

:print-status [method]
    Print status.

2.2

trajectory-configuration-task [class]

   :super **propertied-object**
   :slots   (_instant-config-task-list list of instant-config-task instance)
        (_num-instant-config-task L)
        (_dim-variant-config $dim(\boldsymbol{q}_{var})$)
        (_dim-invariant-config $dim(\boldsymbol{q}_{invar})$)

(_dim-config $dim(\boldsymbol{q})$)

(_dim-task $dim(\boldsymbol{e})$)

(_norm-regular-scale-max k_{max})

(_norm-regular-scale-offset k_{off})

(_adjacent-regular-scale k_{adj})

(_torque-regular-scale k_{trq})

(_task-jacobi buffer for $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}}$)

$\boldsymbol{q}$ $\boldsymbol{e}(\boldsymbol{q})$

" " "trajectory"

$\boldsymbol{q}$ $\boldsymbol{e}(\boldsymbol{q})$ $\frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}$

$\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{C}, \boldsymbol{d}$

-

  instant-config-task-list instant-configuration-task

-

  norm-regular-scale-max k_{max}

  norm-regular-scale-offset k_{off}

  adjacent-regular-scale k_{adj}

  torque-regular-scale k_{trq}

  $\boldsymbol{q}$

$$\boldsymbol{q} := \begin{pmatrix} \boldsymbol{q}_{var}^{(1)T} & \boldsymbol{q}_{var}^{(2)T} & \cdots & \boldsymbol{q}_{var}^{(L)T} & \boldsymbol{q}_{invar}^{T} \end{pmatrix}^{T} \tag{118}$$

$$\boldsymbol{q}_{invar} := \boldsymbol{q}_{invar}^{(1)} = \boldsymbol{q}_{invar}^{(2)} = \cdots = \boldsymbol{q}_{invar}^{(L)} \tag{119}$$

$\boldsymbol{q}_{var}^{(l)}, \boldsymbol{q}_{invar}^{(l)}$ $(l = 1, 2, \cdots, L)$ l

$\boldsymbol{e}(\boldsymbol{q})$

$$\boldsymbol{e}(\boldsymbol{q}) := \begin{pmatrix} \boldsymbol{e}^{(1)T}(\boldsymbol{q}_{var}^{(1)}, \boldsymbol{q}_{invar}) & \boldsymbol{e}^{(2)T}(\boldsymbol{q}_{var}^{(2)}, \boldsymbol{q}_{invar}) & \cdots & \boldsymbol{e}^{(L)T}(\boldsymbol{q}_{var}^{(L)}, \boldsymbol{q}_{invar}) \end{pmatrix}^{T} \tag{120}$$

$\boldsymbol{e}^{(l)}(\boldsymbol{q}_{var}^{(l)}, \boldsymbol{q}_{invar})$ $(l = 1, 2, \cdots, L)$ l

:init *&key (name)* [method]

   (instant-config-task-list)

   (norm-regular-scale-max 1.000000e-04)

   (norm-regular-scale-offset 1.000000e-07)

   (adjacent-regular-scale 0.005)

   (torque-regular-scale 0.001)

  Initialize instance

:instant-config-task-list [method]

  return instant-config-task-list

:dim-variant-config [method]

return $dim(\boldsymbol{q_{var}}) := \sum_{l=1}^{L} dim(\boldsymbol{q}_{var}^{(l)})$

:dim-invariant-config [method]

return $dim(\boldsymbol{q_{invar}}) := dim(\boldsymbol{q}_{invar}^{(l)}) \ (l = 1, 2, \cdots, L \qquad)$

:dim-config [method]

return $dim(\boldsymbol{q}) := dim(\boldsymbol{q_{var}}) + dim(\boldsymbol{q_{invar}})$

:dim-task [method]

return $dim(\boldsymbol{e}) := \sum_{l=1}^{L} dim(\boldsymbol{e}^{(l)})$

:variant-config-vector [method]

$$\text{return } \boldsymbol{q_{var}} := \begin{pmatrix} \boldsymbol{q}_{var}^{(1)} \\ \boldsymbol{q}_{var}^{(2)} \\ \vdots \\ \boldsymbol{q}_{var}^{(L)} \end{pmatrix}$$

:invariant-config-vector [method]

$$\text{return } \boldsymbol{q_{invar}} := \boldsymbol{q}_{invar}^{(l)} \ (l = 1, 2, \cdots, L \qquad)$$

:config-vector [method]

$$\text{return } \boldsymbol{q} := \begin{pmatrix} \boldsymbol{q_{var}} \\ \boldsymbol{q_{invar}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{q}_{var}^{(1)} \\ \boldsymbol{q}_{var}^{(2)} \\ \vdots \\ \boldsymbol{q}_{var}^{(L)} \\ \boldsymbol{q_{invar}} \end{pmatrix}$$

:set-variant-config *variant-config-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{q_{var}}$.

:set-invariant-config *invariant-config-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{q_{invar}}$.

:set-config *config-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{q}$.

:task-value *&key (update? t)* [method]

$$\text{return } \boldsymbol{e}(\boldsymbol{q}) := \begin{pmatrix} \boldsymbol{e}^{(1)}(\boldsymbol{q}_{var}^{(1)}, \boldsymbol{q_{invar}}) \\ \boldsymbol{e}^{(2)}(\boldsymbol{q}_{var}^{(2)}, \boldsymbol{q_{invar}}) \\ \vdots \\ \boldsymbol{e}^{(L)}(\boldsymbol{q}_{var}^{(L)}, \boldsymbol{q_{invar}}) \end{pmatrix}$$

:variant-task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} = \begin{pmatrix} \frac{\partial \boldsymbol{e}^{(1)}}{\partial \boldsymbol{q}_{var}^{(1)}} & & & \boldsymbol{O} \\ & \frac{\partial \boldsymbol{e}^{(2)}}{\partial \boldsymbol{q}_{var}^{(2)}} & & \\ & & \ddots & \\ \boldsymbol{O} & & & \frac{\partial \boldsymbol{e}^{(L)}}{\partial \boldsymbol{q}_{var}^{(L)}} \end{pmatrix} \tag{121}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} \in \mathbb{R}^{dim(\boldsymbol{e}) \times dim(\boldsymbol{q}_{var})}$

:invariant-task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} = \begin{pmatrix} \frac{\partial \boldsymbol{e}^{(1)}}{\partial \boldsymbol{q}_{invar}} \\ \frac{\partial \boldsymbol{e}^{(2)}}{\partial \boldsymbol{q}_{invar}} \\ \vdots \\ \frac{\partial \boldsymbol{e}^{(L)}}{\partial \boldsymbol{q}_{invar}} \end{pmatrix} \tag{122}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \in \mathbb{R}^{dim(\boldsymbol{e}) \times dim(\boldsymbol{q}_{invar})}$

:task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} \quad = \quad \begin{pmatrix} \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} & \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \end{pmatrix} \tag{123}$$

$$= \quad \begin{pmatrix} \frac{\partial \boldsymbol{e}^{(1)}}{\partial \boldsymbol{q}_{var}^{(1)}} & & & \boldsymbol{O} & \frac{\partial \boldsymbol{e}^{(1)}}{\partial \boldsymbol{q}_{invar}} \\ & \frac{\partial \boldsymbol{e}^{(2)}}{\partial \boldsymbol{q}_{var}^{(2)}} & & & \frac{\partial \boldsymbol{e}^{(2)}}{\partial \boldsymbol{q}_{invar}} \\ & & \ddots & & \\ \boldsymbol{O} & & & \frac{\partial \boldsymbol{e}^{(L)}}{\partial \boldsymbol{q}_{var}^{(L)}} & \frac{\partial \boldsymbol{e}^{(L)}}{\partial \boldsymbol{q}_{invar}} \end{pmatrix} \tag{124}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} \in \mathbb{R}^{dim(\boldsymbol{e}) \times dim(\boldsymbol{q})}$

:variant-config-inequality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{C}_{var} := \begin{pmatrix} \boldsymbol{C}_{var}^{(1)} & & & \boldsymbol{O} \\ & \boldsymbol{C}_{var}^{(2)} & & \\ & & \ddots & \\ \boldsymbol{O} & & & \boldsymbol{C}_{var}^{(L)} \end{pmatrix} \tag{125}$$

return $\boldsymbol{C}_{var} \in \mathbb{R}^{N_{var\text{-}ineq} \times dim(\boldsymbol{q}_{var})}$

:variant-config-inequality-constraint-vector *&key (update? t)* [method]

$$\boldsymbol{d}_{var} := \begin{pmatrix} \boldsymbol{d}_{var}^{(1)} \\ \boldsymbol{d}_{var}^{(2)} \\ \vdots \\ \boldsymbol{d}_{var}^{(L)} \end{pmatrix} \tag{126}$$

return $\boldsymbol{d}_{var} \in \mathbb{R}^{N_{var\text{-}ineq}}$

:invariant-config-inequality-constraint-matrix *&key (update? nil)*   [method]

$$\boldsymbol{C}_{invar} := \boldsymbol{C}_{invar}^{(l)} \ (l = 1, 2, \cdots, L \qquad) \tag{127}$$

return $\boldsymbol{C}_{invar} \in \mathbb{R}^{N_{invar\text{-}ineq} \times dim(\boldsymbol{q}_{invar})}$

:invariant-config-inequality-constraint-vector *&key (update? t)*   [method]

$$\boldsymbol{d}_{invar} := \boldsymbol{d}_{invar}^{(l)} \ (l = 1, 2, \cdots, L \qquad) \tag{128}$$

return $\boldsymbol{d}_{invar} \in \mathbb{R}^{N_{invar\text{-}ineq}}$

:config-inequality-constraint-matrix *&key (update? nil)*   [method]
(update-collision? nil)

$$\boldsymbol{C} := \begin{pmatrix} \boldsymbol{C}_{var} & \\ & \boldsymbol{C}_{invar} \\ \hline & \boldsymbol{C}_{col} \end{pmatrix} \in \mathbb{R}^{N_{ineq} \times dim(\boldsymbol{q})} \tag{129}$$

return $\boldsymbol{C} \in \mathbb{R}^{N_{ineq} \times dim(\boldsymbol{q})}$

:config-inequality-constraint-vector *&key (update? t)*   [method]
(update-collision? nil)

$$\boldsymbol{d} := \begin{pmatrix} \boldsymbol{d}_{var} \\ \boldsymbol{d}_{invar} \\ \boldsymbol{d}_{col} \end{pmatrix} \tag{130}$$

return $\boldsymbol{d} \in \mathbb{R}^{N_{ineq}}$

:variant-config-equality-constraint-matrix *&key (update? nil)*   [method]

$$\boldsymbol{A}_{var} := \begin{pmatrix} \boldsymbol{A}_{var}^{(1)} & & & \boldsymbol{O} \\ & \boldsymbol{A}_{var}^{(2)} & & \\ & & \ddots & \\ \boldsymbol{O} & & & \boldsymbol{A}_{var}^{(L)} \end{pmatrix} \tag{131}$$

return $\boldsymbol{A}_{var} \in \mathbb{R}^{N_{var\text{-}eq} \times dim(\boldsymbol{q}_{var})}$

:variant-config-equality-constraint-vector *&key (update? t)*   [method]

$$\boldsymbol{b}_{var} := \begin{pmatrix} \boldsymbol{b}_{var}^{(1)} \\ \boldsymbol{b}_{var}^{(2)} \\ \vdots \\ \boldsymbol{b}_{var}^{(L)} \end{pmatrix} \tag{132}$$

return $\boldsymbol{b}_{var} \in \mathbb{R}^{N_{var\text{-}eq}}$

:invariant-config-equality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{A}_{invar} := \boldsymbol{A}_{invar}^{(l)} \ (l = 1, 2, \cdots, L \qquad) \tag{133}$$

return $\boldsymbol{A}_{invar} \in \mathbb{R}^{N_{invar\text{-}eq} \times dim(\boldsymbol{q}_{invar})}$

:invariant-config-equality-constraint-vector *&key (update? t)* [method]

$$\boldsymbol{b}_{invar} := \boldsymbol{b}_{invar}^{(l)} \ (l = 1, 2, \cdots, L \qquad) \tag{134}$$

return $\boldsymbol{b}_{invar} \in \mathbb{R}^{N_{invar\text{-}eq}}$

:config-equality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{A} := \begin{pmatrix} \boldsymbol{A}_{var} & \\ & \boldsymbol{A}_{invar} \end{pmatrix} \in \mathbb{R}^{N_{eq} \times dim(\boldsymbol{q})} \tag{135}$$

return $\boldsymbol{A} \in \mathbb{R}^{N_{eq} \times dim(\boldsymbol{q})}$

:config-equality-constraint-vector *&key (update? t)* [method]

$$\boldsymbol{b} := \begin{pmatrix} \boldsymbol{b}_{var} \\ \boldsymbol{b}_{invar} \end{pmatrix} \tag{136}$$

return $\boldsymbol{b} \in \mathbb{R}^{N_{eq}}$

:update-collision-inequality-constraint [method]
update inequality matrix $\boldsymbol{C}_{col,\theta}^{(l)}, \boldsymbol{C}_{col,\phi}^{(l)}$ and inequality vector $\boldsymbol{d}_{col}^{(l)}$ for collision avoidance $(l = 1, 2, \cdots, L)$

:collision-inequality-constraint-matrix *&key (update? nil)* [method]

$$\hat{\boldsymbol{C}}_{col,\theta}^{(l)} \ := \ N_{col}^{(l)} \begin{array}{ccc} dim(\boldsymbol{\theta}^{(l)}) & dim(\hat{\boldsymbol{w}}^{(l)}) & dim(\boldsymbol{\tau}^{(l)}) \\ \left(\boldsymbol{C}_{col,\theta}^{(l)} \right. & \boldsymbol{O} & \left. \boldsymbol{O} \right) \end{array} \tag{137}$$

$$\boldsymbol{C}_{col} \ := \ \begin{pmatrix} \hat{\boldsymbol{C}}_{col,\theta}^{(1)} & & & & \boldsymbol{C}_{col,\phi}^{(1)} \\ & \hat{\boldsymbol{C}}_{col,\theta}^{(2)} & & & \boldsymbol{C}_{col,\phi}^{(2)} \\ & & \ddots & & \vdots \\ & & & \hat{\boldsymbol{C}}_{col,\theta}^{(L)} & \boldsymbol{C}_{col,\phi}^{(L)} \end{pmatrix} \tag{138}$$

return $\boldsymbol{C}_{col} \in \mathbb{R}^{N_{col} \times dim(\boldsymbol{q})}$

:collision-inequality-constraint-vector *&key (update? nil)* [method]

$$\boldsymbol{d}_{col} \ := \ \begin{pmatrix} \boldsymbol{d}_{col}^{(1)} \\ \boldsymbol{d}_{col}^{(2)} \\ \vdots \\ \boldsymbol{d}_{col}^{(L)} \end{pmatrix} \tag{139}$$

return $\boldsymbol{d}_{col} \in \mathbb{R}^{N_{col}}$

:adjacent-regular-matrix *&key (update? nil)* [method]

$$F_{adj}(\boldsymbol{q}) \quad = \quad \sum_{l=1}^{L-1} \|\boldsymbol{\theta}_{l+1} - \boldsymbol{\theta}_l\|^2 \tag{140}$$

$$= \quad \boldsymbol{q}^T \boldsymbol{W}_{adj} \boldsymbol{q} \tag{141}$$

$$\bar{\boldsymbol{I}}_{adj} \quad := \quad \begin{matrix} & dim(\boldsymbol{\theta}^{(l)}) & dim(\hat{\boldsymbol{w}}^{(l)}) & dim(\boldsymbol{\tau}^{(l)}) \\ dim(\boldsymbol{\theta}^{(l)}) \\ dim(\hat{\boldsymbol{w}}^{(l)}) \\ dim(\boldsymbol{\tau}^{(l)}) \end{matrix} \begin{pmatrix} \boldsymbol{I} & & \\ & & \\ & & \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var}^{(l)}) \times dim(\boldsymbol{q}_{var}^{(l)})} \tag{142}$$

$$\bar{\boldsymbol{W}}_{adj} \quad := \quad \begin{pmatrix} \bar{\boldsymbol{I}}_{adj} & -\bar{\boldsymbol{I}}_{adj} & & & \\ -\bar{\boldsymbol{I}}_{adj} & 2\bar{\boldsymbol{I}}_{adj} & -\bar{\boldsymbol{I}}_{adj} & & \boldsymbol{O} \\ & & \ddots & & \\ & & -\bar{\boldsymbol{I}}_{adj} & 2\bar{\boldsymbol{I}}_{adj} & -\bar{\boldsymbol{I}}_{adj} \\ \boldsymbol{O} & & & -\bar{\boldsymbol{I}}_{adj} & \bar{\boldsymbol{I}}_{adj} \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var}) \times dim(\boldsymbol{q}_{var})} \tag{143}$$

$$\boldsymbol{W}_{adj} \quad := \quad \begin{pmatrix} \bar{\boldsymbol{W}}_{adj} & \\ & \boldsymbol{O} \end{pmatrix} \tag{144}$$

return $\boldsymbol{W}_{adj} \in \mathbb{R}^{dim(\boldsymbol{q}) \times dim(\boldsymbol{q})}$

:adjacent-regular-vector *&key (update? t)* [method]

$$\boldsymbol{v}_{adj} \quad := \quad \boldsymbol{W}_{adj} \boldsymbol{q} \tag{145}$$

return $\boldsymbol{v}_{adj} \in \mathbb{R}^{dim(\boldsymbol{q})}$

:torque-regular-matrix *&key (update? nil)* [method]

$$\bar{\boldsymbol{W}}_{\tau} \quad := \quad \begin{pmatrix} \boldsymbol{W}_{\tau}^{(1)} & & & \boldsymbol{O} \\ & \boldsymbol{W}_{\tau}^{(2)} & & \\ & & \ddots & \\ \boldsymbol{O} & & & \boldsymbol{W}_{\tau}^{(L)} \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var}) \times dim(\boldsymbol{q}_{var})} \tag{146}$$

$$\boldsymbol{W}_{\tau} \quad := \quad \begin{pmatrix} \bar{\boldsymbol{W}}_{\tau} & \\ & \boldsymbol{O} \end{pmatrix} \tag{147}$$

return $\boldsymbol{W}_{\tau} \in \mathbb{R}^{dim(\boldsymbol{q}) \times dim(\boldsymbol{q})}$

:torque-regular-vector *&key (update? t)* [method]

$$\bar{\boldsymbol{v}}_{\tau} \quad := \quad \begin{pmatrix} \boldsymbol{v}_{\tau}^{(1)} \\ \boldsymbol{v}_{\tau}^{(2)} \\ \vdots \\ \boldsymbol{v}_{\tau}^{(L)} \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var})} \tag{148}$$

$$\boldsymbol{v}_{\tau} \quad := \quad \begin{pmatrix} \bar{\boldsymbol{v}}_{\tau} \\ \boldsymbol{0} \end{pmatrix} \tag{149}$$

return $\boldsymbol{v}_\tau \in \mathbb{R}^{dim(\boldsymbol{q})}$

:regular-matrix [method]

$$\boldsymbol{W}_{reg} := \min(k_{max}, \|\boldsymbol{e}\|^2 + k_{off})\boldsymbol{I} + \boldsymbol{W}_{adj} + \boldsymbol{W}_\tau \tag{150}$$

return $\boldsymbol{W}_{reg} \in \mathbb{R}^{dim(\boldsymbol{q}) \times dim(\boldsymbol{q})}$

:regular-vector [method]

$$\boldsymbol{v}_{reg} := \boldsymbol{v}_{adj} + \boldsymbol{v}_\tau \tag{151}$$

return $\boldsymbol{v}_{reg} \in \mathbb{R}^{dim(\boldsymbol{q})}$

:update-viewer [method]

Update viewer.

:print-status [method]

Print status.

:play-animation *&key* *(robot-env)* [method]
(loop? t)

Play motion.

:generate-robot-state-list *&key* *(robot-env)* [method]
(joint-name-list (send-all (send robot-env :robot :joint-list) :name))
(root-link-name (send (car (send robot-env :robot :links)) :name))
(step-time 0.004)
(divide-num 100)
(limb-list (list :rleg :lleg :rarm :larm))

Generate and return robot state list.

3

sqp-optimization [class]

:super **propertied-object**

:slots (_config-task instance of configuration-task)
(_qp_retval buffer for QP return value)
(_qp-status buffer for QP status)
(_qp-int-status QP status)
(_task-value buffer for task value $\boldsymbol{e}(\boldsymbol{q})$)
(_task-jacobian buffer for task jacobian $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}}$)
(_dim-config-buf-matrix matrix buffer)
(_convergence-check-func function to check convergence)
(_failure-callback-func callback function of failure)

(_pre-process-func pre-process function)
(_post-process-func post-process function)
(_i buffer for iteration count)
(_status status of sqp optimization)
(_no-visualize? whether to supress visualization)
(_no-print? whether to supress print)

instant-configuration-task　　　　trajectory-configuration-task　　instance (　　configuration-
task　　　)　　　　　　　　　configuration-task　　　　　$\|\boldsymbol{e}(\boldsymbol{q})\|^2$
　　　　　　$\boldsymbol{q}$

:init *&key* *(config-task)*　　　　　　　　　　　　　　　　　　　　　　[method]
　　　　　(convergence-check-func)
　　　　　(failure-callback-func)
　　　　　(pre-process-func)
　　　　　(post-process-func)
　　　　　(no-visualize?)
　　　　　(no-print?)

　　　Initialize instance

:config-task　　　　　　　　　　　　　　　　　　　　　　　　　　　　[method]
　　　　Return configuration-task instance

:optimize *&key* *(loop-num 100)*　　　　　　　　　　　　　　　　　　[method]
　　　　　　(loop-num-min)
　　　　　　(update-viewer-interval 1)
　　　　　　(print-status-interval 10)

　　　Optimize

　　　In each iteration, do following:

　　　　1. check convergence
　　　　2. call pre-process function
　　　　3. print status
　　　　4. solve QP and update configuration
　　　　5. call post-process function

　　　Solve following QP:

$$\min_{\Delta\boldsymbol{q}^{(k)}} \frac{1}{2}\Delta\boldsymbol{q}^{(k)T}\boldsymbol{W}\Delta\boldsymbol{q}^{(k)} + \boldsymbol{v}^T\Delta\boldsymbol{q}^{(k)} \tag{152}$$

$$\text{s.t.} \quad \boldsymbol{A}\Delta\boldsymbol{q}^{(k)} = \boldsymbol{b} \tag{153}$$

$$\boldsymbol{C}\Delta\boldsymbol{q}^{(k)} \geq \boldsymbol{d} \tag{154}$$

$$\text{where} \quad \boldsymbol{W} = \left(\frac{\partial\boldsymbol{e}(\boldsymbol{q}^{(k)})}{\partial\boldsymbol{q}^{(k)}}\right)^T \left(\frac{\partial\boldsymbol{e}(\boldsymbol{q}^{(k)})}{\partial\boldsymbol{q}^{(k)}}\right) + \boldsymbol{W}_{reg} \tag{155}$$

$$\boldsymbol{v} = \left(\frac{\partial\boldsymbol{e}(\boldsymbol{q}^{(k)})}{\partial\boldsymbol{q}^{(k)}}\right)^T \boldsymbol{e}(\boldsymbol{q}^{(k)}) + \boldsymbol{v}_{reg} \tag{156}$$

and update configuration:

$$\boldsymbol{q}^{(k+1)} = \boldsymbol{q}^{(k)} + \Delta\boldsymbol{q}^{(k)*} \tag{157}$$

:iteration [method]

Return iteration index.

:status [method]

Return status of sqp optimization.

4

4.1

robot-object-environment [class]

            :super   **robot-environment**
            :slots   (_obj $\mathcal{O}$)
                       (_obj-with-root-virtual $\hat{\mathcal{O}}$)

     1.
     2.
     3.

    robot $\mathcal{R}$ (cascaded-link   )
    object $\mathcal{O}$ (cascaded-link   )
    contact-list $\{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_{N_C}\}$ (2d-planar-contact   )

       R $\hat{\mathcal{R}}$
                   O $\hat{\mathcal{O}}$

:init *&key* *(robot)* [method]
      (object)
      (contact-list)
      (root-virtual-mode :6dof)
      (root-virtual-joint-class-list)
      (root-virtual-joint-axis-list)

Initialize instance

:object *&rest args* [method]

return $\mathcal{O}$

:object-with-root-virtual *&rest args* [method]

return $\hat{\mathcal{O}}$

instant-manipulation-configuration-task [class]

:super   **instant-configuration-task**

:slots   (_robot-obj-env robot-object-environment instance)

        (_wrench-obj-vector $\hat{\boldsymbol{w}}_{obj}$ [N] [Nm])

        (_num-contact-obj $N_{cnt\text{-}obj} := |\mathcal{T}^{cnt\text{-}trg\text{-}obj}|$)

        (_num-act-react $N_{act\text{-}react} := |\mathcal{P}^{act\text{-}react}|$)

        (_dim-wrench-obj $dim(\hat{\boldsymbol{w}}_{obj}) = 6N_{cnt\text{-}obj}$)

        (_contact-target-coords-obj-list $\mathcal{T}^{cnt\text{-}trg\text{-}obj}$)

        (_contact-constraint-obj-list list of contact-constraint instance for object)

        (_act-react-pair-list $\mathcal{P}^{act\text{-}react}$)

$$\boldsymbol{q}^{(l)} \qquad \boldsymbol{e}^{(l)}(\boldsymbol{q}^{(l)})$$

$$l \qquad \boldsymbol{x}^{(l)}$$

$$\boldsymbol{x} \qquad\qquad\qquad\qquad \text{``} \quad \text{''} \quad \text{``instant''}$$

$$\boldsymbol{q} \qquad\qquad \boldsymbol{e}(\boldsymbol{q}) \qquad\qquad \frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}$$

$$\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{C}, \boldsymbol{d}$$

instant-configuration-task

-

 robot-object-environment   robot-object-environment

-

 contact-target-coords-obj-list $\mathcal{T}^{cnt\text{-}trg\text{-}obj}$

 contact-constraint-obj-list

-

 act-react-pair-list $\mathcal{P}^{act\text{-}react}$

$$\boldsymbol{q}$$

$\boldsymbol{\theta} \in \mathbb{R}^{N_{var\text{-}joint}}$ [rad] [m]

$\hat{\boldsymbol{w}} \in \mathbb{R}^{6N_{cnt}}$ [N] [Nm]

$\hat{\boldsymbol{w}}_{obj} \in \mathbb{R}^{6N_{cnt\text{-}obj}}$ [N] [Nm]

$\boldsymbol{\tau} \in \mathbb{R}^{N_{drive\text{-}joint}}$ [Nm] [N]

$\boldsymbol{\phi} \in \mathbb{R}^{N_{invar\text{-}joint}}$ [rad] [m]

$$\hat{\boldsymbol{w}}$$

$$\hat{\boldsymbol{w}} \quad = \quad \begin{pmatrix} \boldsymbol{w}_1^T & \boldsymbol{w}_2^T & \cdots & \boldsymbol{w}_{N_{cnt}}^T \end{pmatrix}^T \tag{158}$$

$$= \quad \begin{pmatrix} \boldsymbol{f}_1^T & \boldsymbol{n}_1^T & \boldsymbol{f}_2^T & \boldsymbol{n}_2^T & \cdots & \boldsymbol{f}_{N_{cnt}}^T & \boldsymbol{n}_{N_{cnt}}^T \end{pmatrix}^T \tag{159}$$

$$\boldsymbol{e}(\boldsymbol{q})$$

$$\boldsymbol{e}^{kin}(\boldsymbol{q}) \in \mathbb{R}^{6N_{kin}} \qquad \qquad [\text{rad}]\ [\text{m}]$$

$$\boldsymbol{e}^{eom\text{-}trans}(\boldsymbol{q}) \in \mathbb{R}^{3} \qquad \qquad [\text{N}]$$

$$\boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{q}) \in \mathbb{R}^{3} \qquad \qquad [\text{Nm}]$$

$$\boldsymbol{e}^{eom\text{-}trans\text{-}obj}(\boldsymbol{q}) \in \mathbb{R}^{3} \qquad \qquad [\text{N}]$$

$$\boldsymbol{e}^{eom\text{-}rot\text{-}obj}(\boldsymbol{q}) \in \mathbb{R}^{3} \qquad \qquad [\text{Nm}]$$

$$\boldsymbol{e}^{trq}(\boldsymbol{q}) \in \mathbb{R}^{N_{drive\text{-}joint}} \qquad \qquad [\text{rad}]\ [\text{m}]$$

$$\boldsymbol{e}^{posture}(\boldsymbol{q}) \in \mathbb{R}^{N_{posture\text{-}joint}} \qquad \qquad [\text{rad}]\ [\text{m}]$$

:init *&rest args &key*   *(contact-target-coords-obj-list)*   [method]
                                             (contact-constraint-obj-list)
                                             (act-react-pair-list)
                                             &allow-other-keys

    Initialize instance

:robot-obj-env   [method]
    return robot-object-environment instance

:wrench-obj   [method]
    return $\hat{\boldsymbol{w}}_{obj}$

:num-contact-obj   [method]
    return $N_{cnt\text{-}obj} := |\mathcal{T}^{cnt\text{-}trg\text{-}obj}|$

:dim-variant-config   [method]

$$dim(\boldsymbol{q_{var}}) \quad := \quad dim(\boldsymbol{\theta}) + dim(\hat{\boldsymbol{w}}) + dim(\hat{\boldsymbol{w}}_{obj}) + dim(\boldsymbol{\tau}) \tag{160}$$

$$= \quad N_{var\text{-}joint} + 6N_{cnt} + 6N_{cnt\text{-}obj} + N_{drive\text{-}joint} \tag{161}$$

    return $dim(\boldsymbol{q_{var}})$

:dim-task   [method]

$$dim(\boldsymbol{e}) \quad := \quad dim(\boldsymbol{e}^{kin}) + dim(\boldsymbol{e}^{eom\text{-}trans}) + dim(\boldsymbol{e}^{eom\text{-}rot}) + dim(\boldsymbol{e}^{eom\text{-}trans\text{-}obj})$$
$$+ dim(\boldsymbol{e}^{eom\text{-}rot\text{-}obj}) + dim(\boldsymbol{e}^{trq}) + dim(\boldsymbol{e}^{posture}) \tag{162}$$

$$= \quad 6N_{kin} + 3 + 3 + 3 + 3 + N_{drive\text{-}joint} + N_{posture\text{-}joint} \tag{163}$$

    return $dim(\boldsymbol{e})$

:variant-config-vector   [method]

$$\text{return } \boldsymbol{q_{var}} := \begin{pmatrix} \boldsymbol{\theta} \\ \hat{\boldsymbol{w}} \\ \hat{\boldsymbol{w}}_{obj} \\ \boldsymbol{\tau} \end{pmatrix}$$

:config-vector [method]

$$\text{return } \boldsymbol{q} := \begin{pmatrix} \boldsymbol{q_{var}} \\ \boldsymbol{q_{invar}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\theta} \\ \hat{\boldsymbol{w}} \\ \hat{\boldsymbol{w}}_{obj} \\ \boldsymbol{\tau} \\ \boldsymbol{\phi} \end{pmatrix}$$

:set-wrench-obj *wrench-obj-new &key (relative? nil)* [method]

Set $\hat{\boldsymbol{w}}_{obj}$.

:set-variant-config *variant-config-new &key (relative? nil)* [method]
(apply-to-robot? t)

Set $\boldsymbol{q_{var}}$.

:contact-target-coords-obj-list [method]

$$T_m^{cnt\text{-}trg\text{-}obj} = \{\boldsymbol{p}_m^{cnt\text{-}trg\text{-}obj}, \boldsymbol{R}_m^{cnt\text{-}trg\text{-}obj}\} \quad (m = 1, 2, \cdots, N_{cnt\text{-}obj}) \tag{164}$$

$$\text{return } \mathcal{T}^{cnt\text{-}trg\text{-}obj} := \{T_1^{cnt\text{-}trg\text{-}obj}, T_2^{cnt\text{-}trg\text{-}obj}, \cdots, T_{N_{cnt\text{-}obj}}^{cnt\text{-}trg\text{-}obj}\}$$

:contact-constraint-obj-list [method]

return list of contact-constraint instance for object

:wrench-obj-list [method]

return $\{\boldsymbol{w}_{obj,1}, \boldsymbol{w}_{obj,2}, \cdots, \boldsymbol{w}_{obj,N_{obj}}\}$

:force-obj-list [method]

return $\{\boldsymbol{f}_{obj,1}, \boldsymbol{f}_{obj,2}, \cdots, \boldsymbol{f}_{obj,N_{cnt\text{-}obj}}\}$

:moment-obj-list [method]

return $\{\boldsymbol{n}_{obj,1}, \boldsymbol{n}_{obj,2}, \cdots, \boldsymbol{n}_{obj,N_{cnt\text{-}obj}}\}$

:mg-obj-vec [method]

return $m_{obj}\boldsymbol{g}$

:cog-obj *&key (update? t)* [method]

return $\boldsymbol{p}_{Gobj}(\boldsymbol{q})$

:eom-trans-obj-task-value *&key (update? t)* [method]

$$\boldsymbol{e}^{eom\text{-}trans\text{-}obj}(\boldsymbol{q}) = \boldsymbol{e}^{eom\text{-}trans\text{-}obj}(\hat{\boldsymbol{w}}_{obj}) \tag{165}$$

$$= \sum_{m=1}^{N_{cnt\text{-}obj}} \boldsymbol{f}_{obj,m} - m_{obj}\boldsymbol{g} \tag{166}$$

return $\boldsymbol{e}^{eom\text{-}trans\text{-}obj}(\boldsymbol{q}) \in \mathbb{R}^3$

:eom-rot-obj-task-value *&key (update? t)* [method]

$$\boldsymbol{e}^{eom\text{-}rot\text{-}obj}(\boldsymbol{q}) \;=\; \boldsymbol{e}^{eom\text{-}rot\text{-}obj}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}_{obj}, \boldsymbol{\phi}) \tag{167}$$

$$= \sum_{m=1}^{N_{cnt\text{-}obj}} \left\{ \left(\boldsymbol{p}_m^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{p}_{Gobj}(\boldsymbol{\theta}, \boldsymbol{\phi}) \right) \times \boldsymbol{f}_{obj,m} + \boldsymbol{n}_{obj,m} \right\} \tag{168}$$

return $\boldsymbol{e}^{eom\text{-}rot\text{-}obj}(\boldsymbol{q}) \in \mathbb{R}^3$

:task-value *&key (update? t)* [method]

$$\text{return } \boldsymbol{e}(\boldsymbol{q}) := \begin{pmatrix} \boldsymbol{e}^{kin}(\boldsymbol{q}) \\ \boldsymbol{e}^{eom\text{-}trans}(\boldsymbol{q}) \\ \boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{q}) \\ \boldsymbol{e}^{eom\text{-}trans\text{-}obj}(\boldsymbol{q}) \\ \boldsymbol{e}^{eom\text{-}rot\text{-}obj}(\boldsymbol{q}) \\ \boldsymbol{e}^{trq}(\boldsymbol{q}) \\ \boldsymbol{e}^{posture}(\boldsymbol{q}) \end{pmatrix} = \begin{pmatrix} \boldsymbol{e}^{kin}(\boldsymbol{\theta}, \boldsymbol{\phi}) \\ \boldsymbol{e}^{eom\text{-}trans}(\hat{\boldsymbol{w}}) \\ \boldsymbol{e}^{eom\text{-}rot}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}, \boldsymbol{\phi}) \\ \boldsymbol{e}^{eom\text{-}trans\text{-}obj}(\hat{\boldsymbol{w}}_{obj}) \\ \boldsymbol{e}^{eom\text{-}rot\text{-}obj}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}_{obj}, \boldsymbol{\phi}) \\ \boldsymbol{e}^{trq}(\boldsymbol{\theta}, \hat{\boldsymbol{w}}, \boldsymbol{\tau}, \boldsymbol{\phi}) \\ \boldsymbol{e}^{posture}(\boldsymbol{\theta}) \end{pmatrix}$$

:eom-trans-obj-task-jacobian-with-wrench-obj [method]

$$\frac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} \;=\; \begin{pmatrix} \dfrac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \boldsymbol{f}_{obj,1}} & \dfrac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \boldsymbol{n}_{obj,1}} & \cdots & \dfrac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \boldsymbol{f}_{obj,N_{cnt\text{-}obj}}} & \dfrac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \boldsymbol{n}_{obj,N_{cnt\text{-}obj}}} \end{pmatrix} \tag{169}$$

$$= \begin{pmatrix} \boldsymbol{I}_3 & \boldsymbol{O}_3 & \cdots & \boldsymbol{I}_3 & \boldsymbol{O}_3 \end{pmatrix} \tag{170}$$

return $\dfrac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} \in \mathbb{R}^{3 \times 6 N_{cnt\text{-}obj}}$

:eom-rot-obj-task-jacobian-with-theta [method]

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\theta}} \;=\; \sum_{m=1}^{N_{cnt\text{-}obj}} \left\{ -[\boldsymbol{f}_{obj,m} \times] \left(\boldsymbol{J}_{\theta,m}^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{J}_{Gobj\theta}(\boldsymbol{\theta}, \boldsymbol{\phi}) \right) \right\} \tag{171}$$

$$= \left[\left(\sum_{m=1}^{N_{cnt\text{-}obj}} \boldsymbol{f}_{obj,m} \right) \times \right] \boldsymbol{J}_{Gobj\theta}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \sum_{m=1}^{N_{cnt\text{-}obj}} [\boldsymbol{f}_{obj,m} \times] \boldsymbol{J}_{\theta,m}^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta}, \boldsymbol{\phi}) \tag{172}$$

$\sum_{m=1}^{N_{cnt\text{-}obj}} \boldsymbol{f}_{obj,m} = m_{obj} \boldsymbol{g}$ eom-trans-obj-task

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\theta}} \;=\; [m_{obj} \boldsymbol{g} \times] \boldsymbol{J}_{Gobj\theta}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \sum_{m=1}^{N_{cnt\text{-}obj}} [\boldsymbol{f}_{obj,m} \times] \boldsymbol{J}_{\theta,m}^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta}, \boldsymbol{\phi}) \tag{173}$$

return $\dfrac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\theta}} \in \mathbb{R}^{3 \times N_{var\text{-}joint}}$

:eom-rot-obj-task-jacobian-with-wrench-obj [method]

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} \;=\; \begin{pmatrix} \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{f}_{obj,1}} & \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{n}_{obj,1}} & \cdots & \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{f}_{obj,N_{cnt\text{-}obj}}} & \dfrac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{n}_{obj,N_{cnt\text{-}obj}}} \end{pmatrix} \tag{174}$$

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{f}_{obj,m}} \;=\; \left[\left(\boldsymbol{p}_m^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta}, \boldsymbol{\phi}) - \boldsymbol{p}_{Gobj}(\boldsymbol{\theta}, \boldsymbol{\phi}) \right) \times \right] \quad (m = 1, 2, \cdots, N_{cnt\text{-}obj}) \tag{175}$$

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{n}_{obj,m}} \;=\; \boldsymbol{I}_3 \quad (m = 1, 2, \cdots, N_{cnt\text{-}obj}) \tag{176}$$

return $\dfrac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} \in \mathbb{R}^{3 \times 6 N_{cnt\text{-}obj}}$

:eom-rot-obj-task-jacobian-with-phi [method]

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\phi}} \;\; = \;\; \sum_{m=1}^{N_{cnt\text{-}obj}} \left\{ -[\boldsymbol{f}_{obj,m} \times] \left(\boldsymbol{J}_{\phi,m}^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta},\boldsymbol{\phi}) - \boldsymbol{J}_{Gobj\phi}(\boldsymbol{\theta},\boldsymbol{\phi}) \right) \right\} \tag{177}$$

$$= \;\; \left[\left(\sum_{m=1}^{N_{cnt\text{-}obj}} \boldsymbol{f}_{obj,m} \right) \times \right] \boldsymbol{J}_{Gobj\phi}(\boldsymbol{\theta},\boldsymbol{\phi}) - \sum_{m=1}^{N_{cnt\text{-}obj}} [\boldsymbol{f}_{obj,m} \times] \boldsymbol{J}_{\phi,m}^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta},\boldsymbol{\phi}) \tag{178}$$

$$\sum_{m=1}^{N_{cnt\text{-}obj}} \boldsymbol{f}_{obj,m} = m_{obj}\boldsymbol{g} \qquad \text{eom-trans-obj-task}$$

$$\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\phi}} \;\; = \;\; [m_{obj}\boldsymbol{g} \times] \boldsymbol{J}_{Gobj\phi}(\boldsymbol{\theta},\boldsymbol{\phi}) - \sum_{m=1}^{N_{cnt\text{-}obj}} [\boldsymbol{f}_{obj,m} \times] \boldsymbol{J}_{\phi,m}^{cnt\text{-}trg\text{-}obj}(\boldsymbol{\theta},\boldsymbol{\phi}) \tag{179}$$

return $\frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\phi}} \in \mathbb{R}^{3 \times N_{invar\text{-}joint}}$

:variant-task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} \;\; = \;\; \begin{matrix} 6N_{kin} \\ 3 \\ 3 \\ 3 \\ 3 \\ N_{drive\text{-}joint} \\ N_{posture\text{-}joint} \end{matrix} \begin{pmatrix} \overset{N_{var\text{-}joint}}{\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\theta}}} & \overset{6N_{cnt}}{\frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \hat{\boldsymbol{w}}}} & \overset{6N_{cnt\text{-}obj}}{} & \overset{N_{drive\text{-}joint}}{} \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \hat{\boldsymbol{w}}} & & \\ & & \frac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} & \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\theta}} & & \frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} & \\ \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \hat{\boldsymbol{w}}} & & \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\tau}} \\ \frac{\partial \boldsymbol{e}^{posture}}{\partial \boldsymbol{\theta}} & & & \end{pmatrix} \tag{180}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} \in \mathbb{R}^{(6N_{kin}+3+3+3+3+N_{drive\text{-}joint}+N_{posture\text{-}joint}) \times (N_{var\text{-}joint}+6N_{cnt}+6N_{cnt\text{-}obj}+N_{drive\text{-}joint})}$

:invariant-task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \;\; = \;\; \begin{matrix} 6N_{kin} \\ 3 \\ 3 \\ 3 \\ 3 \\ N_{drive\text{-}joint} \\ N_{posture\text{-}joint} \end{matrix} \begin{pmatrix} \overset{N_{invar\text{-}joint}}{\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\phi}}} \\ \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\phi}} \\ \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\phi}} \\ \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\phi}} \\ \\ \end{pmatrix} \tag{181}$$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \in \mathbb{R}^{(6N_{kin}+3+3+3+3+N_{drive\text{-}joint}+N_{posture\text{-}joint}) \times N_{invar\text{-}joint}}$

:task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} = \begin{pmatrix} \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{var}} & \frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}_{invar}} \end{pmatrix} \tag{182}$$

$$= \begin{array}{c} 6N_{kin} \\ 3 \\ 3 \\ 3 \\ 3 \\ N_{drive\text{-}joint} \\ N_{posture\text{-}joint} \end{array} \begin{pmatrix} \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\theta}} & & & & \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{\phi}} \\ & \frac{\partial \boldsymbol{e}^{eom\text{-}trans}}{\partial \hat{\boldsymbol{w}}} & & & \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \hat{\boldsymbol{w}}} & & & \frac{\partial \boldsymbol{e}^{eom\text{-}rot}}{\partial \boldsymbol{\phi}} \\ & & \frac{\partial \boldsymbol{e}^{eom\text{-}trans\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} & & \\ \frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\theta}} & & \frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \hat{\boldsymbol{w}}_{obj}} & & \frac{\partial \boldsymbol{e}^{eom\text{-}rot\text{-}obj}}{\partial \boldsymbol{\phi}} \\ \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\theta}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \hat{\boldsymbol{w}}} & & \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\tau}} & \frac{\partial \boldsymbol{e}^{trq}}{\partial \boldsymbol{\phi}} \\ \frac{\partial \boldsymbol{e}^{posture}}{\partial \boldsymbol{\theta}} & & & & \end{pmatrix} \tag{183}$$

with column headers $N_{var\text{-}joint}$, $6N_{cnt}$, $6N_{cnt\text{-}obj}$, $N_{drive\text{-}joint}$, $N_{invar\text{-}joint}$

return $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} \in \mathbb{R}^{(6N_{kin}+3+3+3+3+N_{drive\text{-}joint}+N_{posture\text{-}joint}) \times (N_{var\text{-}joint}+6N_{cnt}+6N_{cnt\text{-}obj}+N_{drive\text{-}joint}+N_{invar\text{-}joint})}$

:wrench-obj-inequality-constraint-matrix &key (update? t)　　　　　　　　　　[method]
$$\boldsymbol{w}_{obj} \in \mathbb{R}^6$$

$$\boldsymbol{C}_{w_{obj}} \boldsymbol{w}_{obj} \geq \boldsymbol{d}_{w_{obj}} \tag{184}$$

$N_{cnt\text{-}obj}$　　　　　　　　　　　　　　　　　　　$\hat{\boldsymbol{w}}_{obj}$

$$\boldsymbol{C}_{w_{obj},m}(\boldsymbol{w}_{obj,m} + \Delta \boldsymbol{w}_{obj,m}) \geq \boldsymbol{d}_{w_{obj},m} \quad (m = 1, 2, \cdots, N_{cnt\text{-}obj}) \tag{185}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{w_{obj},m} \Delta \boldsymbol{w}_{obj,m} \geq \boldsymbol{d}_{w_{obj},m} - \boldsymbol{C}_{w_{obj},m} \boldsymbol{w}_{obj,m} \quad (m = 1, 2, \cdots, N_{cnt\text{-}obj}) \tag{186}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{C}_{w_{obj},1} & & & \\ & \boldsymbol{C}_{w_{obj},2} & & \\ & & \ddots & \\ & & & \boldsymbol{C}_{w_{obj},N_{cnt\text{-}obj}} \end{pmatrix} \begin{pmatrix} \Delta \boldsymbol{w}_{obj,1} \\ \Delta \boldsymbol{w}_{obj,2} \\ \vdots \\ \Delta \boldsymbol{w}_{obj,N_{cnt\text{-}obj}} \end{pmatrix} \geq \begin{pmatrix} \boldsymbol{d}_{w_{obj},1} - \boldsymbol{C}_{w_{obj},1} \boldsymbol{w}_{obj,1} \\ \boldsymbol{d}_{w_{obj},2} - \boldsymbol{C}_{w_{obj},2} \boldsymbol{w}_{obj,2} \\ \vdots \\ \boldsymbol{d}_{w_{obj},N_{cnt\text{-}obj}} - \boldsymbol{C}_{w_{obj},N_{cnt\text{-}obj}} \boldsymbol{w}_{obj,N_{cnt\text{-}obj}} \end{pmatrix} \tag{187}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\hat{w}_{obj}} \Delta \hat{\boldsymbol{w}}_{obj} \geq \boldsymbol{d}_{\hat{w}_{obj}} \tag{188}$$

return $\boldsymbol{C}_{\hat{w}_{obj}} := \begin{pmatrix} \boldsymbol{C}_{w_{obj},1} & & & \\ & \boldsymbol{C}_{w_{obj},2} & & \\ & & \ddots & \\ & & & \boldsymbol{C}_{w_{obj},N_{cnt\text{-}obj}} \end{pmatrix} \in \mathbb{R}^{N_{wrench\text{-}obj\text{-}ineq} \times 6N_{cnt\text{-}obj}}$

:wrench-obj-inequality-constraint-vector &key (update? t)　　　　　　　　　　[method]

return $\boldsymbol{d}_{\hat{w}_{obj}} := \begin{pmatrix} \boldsymbol{d}_{w_{obj},1} - \boldsymbol{C}_{w_{obj},1} \boldsymbol{w}_{obj,1} \\ \boldsymbol{d}_{w_{obj},2} - \boldsymbol{C}_{w_{obj},2} \boldsymbol{w}_{obj,2} \\ \vdots \\ \boldsymbol{d}_{w_{obj},N_{cnt\text{-}obj}} - \boldsymbol{C}_{w_{obj},N_{cnt\text{-}obj}} \boldsymbol{w}_{obj,N_{cnt\text{-}obj}} \end{pmatrix} \in \mathbb{R}^{N_{wrench\text{-}obj\text{-}ineq}}$

:variant-config-inequality-constraint-matrix &key (update? nil)　　　　　　　　　　[method]

$$\begin{cases} \boldsymbol{C}_\theta \Delta\boldsymbol{\theta} \geq \boldsymbol{d}_\theta \\ \boldsymbol{C}_{\hat{w}} \Delta\hat{\boldsymbol{w}} \geq \boldsymbol{d}_{\hat{w}} \\ \boldsymbol{C}_{\hat{w}_{obj}} \Delta\hat{\boldsymbol{w}}_{\boldsymbol{obj}} \geq \boldsymbol{d}_{\hat{w}_{obj}} \\ \boldsymbol{C}_\tau \Delta\boldsymbol{\tau} \geq \boldsymbol{d}_\tau \end{cases} \tag{189}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{C}_\theta & & & \\ & \boldsymbol{C}_{\hat{w}} & & \\ & & \boldsymbol{C}_{\hat{w}_{obj}} & \\ & & & \boldsymbol{C}_\tau \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\theta} \\ \Delta\hat{\boldsymbol{w}} \\ \Delta\hat{\boldsymbol{w}}_{obj} \\ \Delta\boldsymbol{\tau} \end{pmatrix} \geq \begin{pmatrix} \boldsymbol{d}_\theta \\ \boldsymbol{d}_{\hat{w}} \\ \boldsymbol{d}_{\hat{w}_{obj}} \\ \boldsymbol{d}_\tau \end{pmatrix} \tag{190}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{var} \Delta\boldsymbol{q}_{var} \geq \boldsymbol{d}_{var} \tag{191}$$

$$\text{return } \boldsymbol{C}_{var} := \begin{pmatrix} \boldsymbol{C}_\theta & & & \\ & \boldsymbol{C}_{\hat{w}} & & \\ & & \boldsymbol{C}_{\hat{w}_{obj}} & \\ & & & \boldsymbol{C}_\tau \end{pmatrix} \in \mathbb{R}^{N_{var\text{-}ineq} \times dim(\boldsymbol{q}_{var})}$$

:variant-config-inequality-constraint-vector &key (update? t)   [method]

$$\text{return } \boldsymbol{d}_{var} := \begin{pmatrix} \boldsymbol{d}_\theta \\ \boldsymbol{d}_{\hat{w}} \\ \boldsymbol{d}_{\hat{w}_{obj}} \\ \boldsymbol{d}_\tau \end{pmatrix} \in \mathbb{R}^{N_{var\text{-}ineq}}$$

:act-react-equality-constraint-matrix &key (update? nil)   [method]

$$\hat{\boldsymbol{w}}_{i(m)} + \hat{\boldsymbol{w}}_{obj,j(m)} = \boldsymbol{0} \quad (m = 1, 2, \cdots, N_{act\text{-}react}) \tag{192}$$

$$\Leftrightarrow \quad \boldsymbol{A}_{act\text{-}react,robot,m}\hat{\boldsymbol{w}} + \boldsymbol{A}_{act\text{-}react,obj,m}\hat{\boldsymbol{w}}_{obj} = \boldsymbol{0} \quad (m = 1, 2, \cdots, N_{act\text{-}react}) \tag{193}$$

$$\text{where} \quad \boldsymbol{A}_{act\text{-}react,robot,m} = \begin{pmatrix} \boldsymbol{O}_6 & \boldsymbol{O}_6 & \cdots & \overset{i(m)}{\boldsymbol{I}_6} & \cdots & \boldsymbol{O}_6 & \boldsymbol{O}_6 \end{pmatrix} \in \mathbb{R}^{6 \times 6N_{cnt}} \tag{194}$$

$$\boldsymbol{A}_{act\text{-}react,obj,m} = \begin{pmatrix} \boldsymbol{O}_6 & \boldsymbol{O}_6 & \cdots & \overset{j(m)}{\boldsymbol{I}_6} & \cdots & \boldsymbol{O}_6 & \boldsymbol{O}_6 \end{pmatrix} \in \mathbb{R}^{6 \times 6N_{cnt\text{-}obj}} \tag{195}$$

$$\Leftrightarrow \quad \boldsymbol{A}_{act\text{-}react,robot}\hat{\boldsymbol{w}} + \boldsymbol{A}_{act\text{-}react,obj}\hat{\boldsymbol{w}}_{obj} = \boldsymbol{0} \tag{196}$$

$$\text{where} \quad \boldsymbol{A}_{act\text{-}react,robot} = \begin{pmatrix} \boldsymbol{A}_{act\text{-}react,robot,1} \\ \vdots \\ \boldsymbol{A}_{act\text{-}react,robot,N_{act\text{-}react}} \end{pmatrix} \in \mathbb{R}^{6N_{act\text{-}react} \times 6N_{cnt}} \tag{197}$$

$$\boldsymbol{A}_{act\text{-}react,obj} = \begin{pmatrix} \boldsymbol{A}_{act\text{-}react,obj,1} \\ \vdots \\ \boldsymbol{A}_{act\text{-}react,obj,N_{act\text{-}react}} \end{pmatrix} \in \mathbb{R}^{6N_{act\text{-}react} \times 6N_{cnt\text{-}obj}} \tag{198}$$

$$\Leftrightarrow \quad \boldsymbol{A}_{act\text{-}react} \begin{pmatrix} \hat{\boldsymbol{w}} \\ \hat{\boldsymbol{w}}_{obj} \end{pmatrix} = \boldsymbol{0} \in \mathbb{R}^{6N_{act\text{-}react}} \tag{199}$$

$$\text{where} \quad \boldsymbol{A}_{act\text{-}react} = \begin{pmatrix} \boldsymbol{A}_{act\text{-}react,robot} & \boldsymbol{A}_{act\text{-}react,obj} \end{pmatrix} \in \mathbb{R}^{6N_{act\text{-}react} \times (6N_{cnt} + 6N_{cnt\text{-}obj})} \tag{200}$$

$$\Leftrightarrow \quad \boldsymbol{A}_{act\text{-}react} \begin{pmatrix} \hat{\boldsymbol{w}} + \Delta\hat{\boldsymbol{w}} \\ \hat{\boldsymbol{w}}_{obj} + \Delta\hat{\boldsymbol{w}}_{obj} \end{pmatrix} = \boldsymbol{0} \tag{201}$$

$$\Leftrightarrow \quad \boldsymbol{A}_{act\text{-}react} \begin{pmatrix} \Delta\hat{\boldsymbol{w}} \\ \Delta\hat{\boldsymbol{w}}_{obj} \end{pmatrix} = \boldsymbol{b}_{act\text{-}react} \tag{202}$$

$$\text{where} \quad \boldsymbol{b}_{act\text{-}react} = -\boldsymbol{A}_{act\text{-}react} \begin{pmatrix} \hat{\boldsymbol{w}} \\ \hat{\boldsymbol{w}}_{obj} \end{pmatrix} \tag{203}$$

$i(m), j(m)$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad m$

return $\boldsymbol{A}_{act\text{-}react} \in \mathbb{R}^{6N_{act\text{-}react} \times (6N_{cnt} + 6N_{cnt\text{-}obj})}$

:act-react-equality-constraint-vector *&key (update? t)* [method]

return $\boldsymbol{b}_{act\text{-}react} \in \mathbb{R}^{6N_{act\text{-}react}}$

:variant-config-equality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{A}_{act\text{-}react} \begin{pmatrix} \Delta\hat{\boldsymbol{w}} \\ \Delta\hat{\boldsymbol{w}}_{obj} \end{pmatrix} = \boldsymbol{b}_{act\text{-}react} \tag{204}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{O} & \boldsymbol{A}_{act\text{-}react} & \boldsymbol{O} \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{\theta} \\ \Delta\hat{\boldsymbol{w}} \\ \Delta\hat{\boldsymbol{w}}_{obj} \\ \Delta\boldsymbol{\tau} \end{pmatrix} = \boldsymbol{b}_{act\text{-}react} \tag{205}$$

$$\Leftrightarrow \quad \boldsymbol{A}_{var}\Delta\boldsymbol{q}_{var} = \boldsymbol{b}_{var} \tag{206}$$

return $\boldsymbol{A}_{var} := \begin{pmatrix} \boldsymbol{O} & \boldsymbol{A}_{act\text{-}react} & \boldsymbol{O} \end{pmatrix} \in \mathbb{R}^{6N_{act\text{-}react} \times dim(\boldsymbol{q}_{var})}$

:variant-config-equality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{b}_{var} := \boldsymbol{b}_{act\text{-}react} \in \mathbb{R}^{6N_{act\text{-}react}}$$

:invariant-config-equality-constraint-matrix *&key (update? nil)* [method]

$$\text{return } \boldsymbol{A}_{invar} \in \mathbb{R}^{0 \times dim(\boldsymbol{q}_{invar})} \text{ (no equality constraint)}$$

:invariant-config-equality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{b}_{invar} \in \mathbb{R}^0 \text{ (no equality constraint)}$$

:config-equality-constraint-matrix *&key (update? nil)* [method]

$$\boldsymbol{A}_{var}\Delta\boldsymbol{q}_{var} = \boldsymbol{b}_{var} \tag{207}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{A}_{var} & \boldsymbol{O} \end{pmatrix} \begin{pmatrix} \Delta\boldsymbol{q}_{var} \\ \Delta\boldsymbol{q}_{invar} \end{pmatrix} = \boldsymbol{b}_{var} \tag{208}$$

$$\Leftrightarrow \quad \boldsymbol{A}\Delta\boldsymbol{q} = \boldsymbol{b} \tag{209}$$

$$\text{return } \boldsymbol{A} := \begin{pmatrix} \boldsymbol{A}_{var} & \boldsymbol{O} \end{pmatrix} \in \mathbb{R}^{N_{eq} \times dim(\boldsymbol{q})}$$

:config-equality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{b} := \boldsymbol{b}_{var} \in \mathbb{R}^{N_{eq}}$$

:torque-regular-matrix *&key (update? nil)* [method]
(only-variant? nil)

$$F_{tau}(\boldsymbol{q}) = \left\| \frac{\boldsymbol{\tau}}{\boldsymbol{\tau}_{max}} \right\|^2 \quad (\qquad\qquad\qquad) \tag{210}$$

$$= \boldsymbol{\tau}^T \bar{\boldsymbol{W}}_\tau \boldsymbol{\tau} \tag{211}$$

$$\bar{\boldsymbol{W}}_\tau := \begin{pmatrix} \frac{1}{\tau_{max,1}^2} & & & \\ & \frac{1}{\tau_{max,2}^2} & & \\ & & \ddots & \\ & & & \frac{1}{\tau_{max,N_{drive\text{-}joint}}^2} \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{\tau}) \times dim(\boldsymbol{\tau})} \tag{212}$$

only-variant? is true:

$$\boldsymbol{W}_\tau := \begin{matrix} & \begin{matrix} dim(\boldsymbol{\theta}) & dim(\hat{\boldsymbol{w}}) & dim(\hat{\boldsymbol{w}}_{obj}) & dim(\boldsymbol{\tau}) \end{matrix} \\ \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\hat{\boldsymbol{w}}_{obj}) \\ dim(\boldsymbol{\tau}) \end{matrix} & \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \bar{\boldsymbol{W}}_\tau \end{pmatrix} \end{matrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var}) \times dim(\boldsymbol{q}_{var})} \tag{213}$$

otherwise:

$$\boldsymbol{W}_\tau := \begin{matrix} & \begin{matrix} dim(\boldsymbol{\theta}) & dim(\hat{\boldsymbol{w}}) & dim(\hat{\boldsymbol{w}}_{obj}) & dim(\boldsymbol{\tau}) & dim(\boldsymbol{\phi}) \end{matrix} \\ \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\hat{\boldsymbol{w}}_{obj}) \\ dim(\boldsymbol{\tau}) \\ dim(\boldsymbol{\phi}) \end{matrix} & \begin{pmatrix} & & & & \\ & & & & \\ & & & & \\ & & & \bar{\boldsymbol{W}}_\tau & \\ & & & & \end{pmatrix} \end{matrix} \in \mathbb{R}^{dim(\boldsymbol{q}) \times dim(\boldsymbol{q})} \tag{214}$$

return $\boldsymbol{W}_\tau$

:torque-regular-vector *&key (update? t)*   [method]
       (only-variant? nil)

$$\bar{\boldsymbol{v}}_\tau \;\; := \;\; \bar{\boldsymbol{W}}_\tau \boldsymbol{\tau} \tag{215}$$

$$= \begin{pmatrix} \frac{\tau_1}{\tau_{max,1}^2} \\ \frac{\tau_2}{\tau_{max,2}^2} \\ \vdots \\ \frac{\tau_{dim(\boldsymbol{\tau})}}{\tau_{max,dim(\boldsymbol{\tau})}^2} \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{\tau})} \tag{216}$$

only-variant? is true:

$$\boldsymbol{v}_\tau := \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\hat{\boldsymbol{w}}_{obj}) \\ dim(\boldsymbol{\tau}) \end{matrix} \begin{pmatrix} 1 \\ \\ \\ \bar{\boldsymbol{v}}_\tau \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{q}_{var})} \tag{217}$$

otherwise:

$$\boldsymbol{v}_\tau := \begin{matrix} dim(\boldsymbol{\theta}) \\ dim(\hat{\boldsymbol{w}}) \\ dim(\hat{\boldsymbol{w}}_{obj}) \\ dim(\boldsymbol{\tau}) \\ dim(\boldsymbol{\phi}) \end{matrix} \begin{pmatrix} 1 \\ \\ \\ \bar{\boldsymbol{v}}_\tau \\ \\ \end{pmatrix} \in \mathbb{R}^{dim(\boldsymbol{q})} \tag{218}$$

return $\boldsymbol{v}_\tau$

:collision-inequality-constraint-matrix *&key (update? nil)*   [method]

$$\boldsymbol{C}_{col} := N_{col} \begin{pmatrix} \overset{dim(\boldsymbol{\theta})}{\boldsymbol{C}_{col,\theta}} & \overset{dim(\hat{\boldsymbol{w}})}{\boldsymbol{O}} & \overset{dim(\hat{\boldsymbol{w}}_{obj})}{\boldsymbol{O}} & \overset{dim(\boldsymbol{\tau})}{\boldsymbol{O}} & \overset{dim(\boldsymbol{\phi})}{\boldsymbol{C}_{col,\phi}} \end{pmatrix} \tag{219}$$

return $\boldsymbol{C}_{col} \in \mathbb{R}^{N_{col} \times dim(\boldsymbol{q})}$

:update-viewer   [method]
   Update viewer.

:print-status   [method]
   Print status.

4.2 B

4.2.1 B

B

B

$$b_{i,0}(t) \quad \overset{\text{def}}{=} \quad \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{220}$$

$$b_{i,n}(t) \quad \overset{\text{def}}{=} \quad \frac{t - t_i}{t_{i+n} - t_i} b_{i,n-1}(t) + \frac{t_{i+n+1} - t}{t_{i+n+1} - t_{i+1}} b_{i+1,n-1}(t) \tag{221}$$

t_i

B

t_s, t_f B

$n < m$

$$t_n = t_s \tag{222}$$

$$t_m = t_f \tag{223}$$

$t_i \; (0 \leq i \leq n + m)$

$$
\begin{aligned}
t_i &= \frac{i - n}{m - n}(t_f - t_s) + t_s \tag{224} \\
&= hi + \frac{mt_s - nt_f}{m - n} \tag{225}
\end{aligned}
$$

$$h \overset{\text{def}}{=} \frac{t_f - t_s}{m - n} \tag{226}$$

(225)   (220) ,   (221)   B

$$
\begin{aligned}
b_{i,0}(t) &= \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{227} \\
b_{i,n}(t) &= \frac{(t - t_i)b_{i,n-1}(t) + (t_{i+n+1} - t)b_{i+1,n-1}(t)}{nh} \tag{228}
\end{aligned}
$$

n B m

B

(227) ,   (228)   B $b_{i,n}(t)$

$$\sum_{i=0}^{m-1} b_{i,n}(t) = 1 \quad (t_s \leq t \leq t_f) \tag{229}$$

$$0 \leq b_{i,n}(t) \leq 1 \quad (i = 0, 1, \cdots, m - 1, \; t_s \leq t \leq t_f) \tag{230}$$

B

B

$$\dot{\boldsymbol{b}}_n(t) = \frac{d\boldsymbol{b}_n(t)}{dt} = \boldsymbol{D}\boldsymbol{b}_{n-1}(t) \tag{231}$$

$$\boldsymbol{b}_n(t) \stackrel{\text{def}}{=} \begin{pmatrix} b_{0,n}(t) \\ b_{1,n}(t) \\ \vdots \\ b_{m-1,n}(t) \end{pmatrix} \in \mathbb{R}^m \tag{232}$$

$$\boldsymbol{D} \stackrel{\text{def}}{=} \frac{1}{h} \begin{pmatrix} 1 & -1 & & & \boldsymbol{O} \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & -1 \\ \boldsymbol{O} & & & & 1 \end{pmatrix} \in \mathbb{R}^{m \times m} \tag{233}$$

k

$$\boldsymbol{b}_n^{(k)}(t) = \frac{d^{(k)}\boldsymbol{b}_n(t)}{dt^{(k)}} = \boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \tag{234}$$

B

j $\qquad\qquad$ $\theta_j(t)$

$$\theta_j(t) \stackrel{\text{def}}{=} \sum_{i=0}^{m-1} p_{j,i} b_{i,n}(t) = \boldsymbol{p}_j^T \boldsymbol{b}_n(t) \in \mathbb{R} \quad (t_s \leq t \leq t_f) \tag{235}$$

$$\boldsymbol{p}_j = \begin{pmatrix} p_{j,0} \\ p_{j,1} \\ \vdots \\ p_{j,m-1} \end{pmatrix} \in \mathbb{R}^m, \quad \boldsymbol{b}_n(t) = \begin{pmatrix} b_{0,n}(t) \\ b_{1,n}(t) \\ \vdots \\ b_{m-1,n}(t) \end{pmatrix} \in \mathbb{R}^m \tag{236}$$

$\boldsymbol{p}_j$ $\qquad$ $\boldsymbol{b}_n(t)$ $\qquad\qquad\qquad$ $\boldsymbol{p}_j$ $\qquad\qquad\qquad\qquad$ $\boldsymbol{p}_j$

$j = 1, 2, \cdots, N_{joint}$

$$\boldsymbol{\theta}(t) \stackrel{\text{def}}{=} \begin{pmatrix} \theta_1(t) \\ \theta_2(t) \\ \vdots \\ \theta_{N_{joint}}(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{p}_1^T \boldsymbol{b}_n(t) \\ \boldsymbol{p}_2^T \boldsymbol{b}_n(t) \\ \vdots \\ \boldsymbol{p}_{N_{joint}}^T \boldsymbol{b}_n(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{p}_1^T \\ \boldsymbol{p}_2^T \\ \vdots \\ \boldsymbol{p}_{N_{joint}}^T \end{pmatrix} \boldsymbol{b}_n(t) = \boldsymbol{P}\boldsymbol{b}_n(t) \in \mathbb{R}^{N_{joint}} \tag{237}$$

$$\boldsymbol{P} \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{p}_1^T \\ \boldsymbol{p}_2^T \\ \vdots \\ \boldsymbol{p}_{N_{joint}}^T \end{pmatrix} \in \mathbb{R}^{N_{joint} \times m} \tag{238}$$

(237)

$$\boldsymbol{\theta}(t) = \begin{pmatrix} \theta_1(t) \\ \theta_2(t) \\ \vdots \\ \theta_{N_{joint}}(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{b}_n^T(t)\boldsymbol{p}_1 \\ \boldsymbol{b}_n^T(t)\boldsymbol{p}_2 \\ \vdots \\ \boldsymbol{b}_n^T(t)\boldsymbol{p}_{N_{joint}} \end{pmatrix} = \begin{pmatrix} \boldsymbol{b}_n^T(t) & & & \boldsymbol{O} \\ & \boldsymbol{b}_n^T(t) & & \\ & & \ddots & \\ \boldsymbol{O} & & & \boldsymbol{b}_n^T(t) \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \vdots \\ \boldsymbol{p}_{N_{joint}} \end{pmatrix} = \boldsymbol{B}_n(t)\boldsymbol{p} \in \mathbb{R}^{N_{joint}} \quad (239)$$

$$\boldsymbol{B}_n(t) \overset{\text{def}}{=} \begin{pmatrix} \boldsymbol{b}_n^T(t) & & & \boldsymbol{O} \\ & \boldsymbol{b}_n^T(t) & & \\ & & \ddots & \\ \boldsymbol{O} & & & \boldsymbol{b}_n^T(t) \end{pmatrix} \in \mathbb{R}^{N_{joint} \times mN_{joint}}, \quad \boldsymbol{p} \overset{\text{def}}{=} \begin{pmatrix} \boldsymbol{p}_1 \\ \boldsymbol{p}_2 \\ \vdots \\ \boldsymbol{p}_{N_{joint}} \end{pmatrix} \in \mathbb{R}^{mN_{joint}} \quad (240)$$

B

(237) (231)

$$\dot{\boldsymbol{\theta}}(t) = \boldsymbol{P}\dot{\boldsymbol{b}}_n(t) \tag{241}$$

$$= \boldsymbol{P}\boldsymbol{D}\boldsymbol{b}_{n-1}(t) \tag{242}$$

$$= \begin{pmatrix} \boldsymbol{p}_1^T \\ \vdots \\ \boldsymbol{p}_{N_{joint}}^T \end{pmatrix} \boldsymbol{D}\boldsymbol{b}_{n-1}(t) \tag{243}$$

$$= \begin{pmatrix} \boldsymbol{p}_1^T \boldsymbol{D}\boldsymbol{b}_{n-1}(t) \\ \vdots \\ \boldsymbol{p}_{N_{joint}}^T \boldsymbol{D}\boldsymbol{b}_{n-1}(t) \end{pmatrix} \tag{244}$$

$$= \begin{pmatrix} \boldsymbol{b}_{n-1}^T(t)\boldsymbol{D}^T \boldsymbol{p}_1 \\ \vdots \\ \boldsymbol{b}_{n-1}^T(t)\boldsymbol{D}^T \boldsymbol{p}_{N_{joint}} \end{pmatrix} \tag{245}$$

$$= \begin{pmatrix} \boldsymbol{b}_{n-1}^T(t)\boldsymbol{D}^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \boldsymbol{b}_{n-1}^T(t)\boldsymbol{D}^T \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_1 \\ \vdots \\ \boldsymbol{p}_{N_{joint}} \end{pmatrix} \tag{246}$$

$$= \begin{pmatrix} \boldsymbol{b}_{n-1}^T(t) & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \boldsymbol{b}_{n-1}^T(t) \end{pmatrix} \begin{pmatrix} \boldsymbol{D}^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \boldsymbol{D}^T \end{pmatrix} \begin{pmatrix} \boldsymbol{p}_1 \\ \vdots \\ \boldsymbol{p}_{N_{joint}} \end{pmatrix} \tag{247}$$

$$= \boldsymbol{B}_{n-1}(t)\hat{\boldsymbol{D}}_1\boldsymbol{p} \tag{248}$$

$$\hat{\boldsymbol{D}}_1 = \begin{pmatrix} \boldsymbol{D}^T & & & \boldsymbol{O} \\ & \boldsymbol{D}^T & & \\ & & \ddots & \\ \boldsymbol{O} & & & \boldsymbol{D}^T \end{pmatrix} \in \mathbb{R}^{mN_{joint} \times mN_{joint}} \tag{249}$$

$$k$$

$$
\boldsymbol{\theta}^{(k)}(t) \;=\; \frac{d^{(k)}\boldsymbol{\theta}(t)}{dt^{(k)}} \tag{250}
$$

$$
=\; \boldsymbol{P}\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \tag{251}
$$

$$
=\; \boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \boldsymbol{p} \tag{252}
$$

$$
\hat{\boldsymbol{D}}_k \;=\; \begin{pmatrix} (\boldsymbol{D}^k)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & (\boldsymbol{D}^k)^T \end{pmatrix} = (\hat{\boldsymbol{D}}_1)^k \in \mathbb{R}^{mN_{joint} \times mN_{joint}} \tag{253}
$$

$$(251) \qquad\qquad (252)$$

$$\boldsymbol{\theta} \in \mathbb{R}^{N_{joint}} \qquad\qquad\qquad \boldsymbol{r} \in \mathbb{R}^6 \qquad\qquad \boldsymbol{f}(\boldsymbol{\theta})$$

$$
\boldsymbol{r} = \boldsymbol{f}(\boldsymbol{\theta}) \tag{254}
$$

$$(239)$$

$$
\boldsymbol{r}(t) = \boldsymbol{f}(\boldsymbol{\theta}(t)) = \boldsymbol{f}(\boldsymbol{B}_n(t)\boldsymbol{p}) \tag{255}
$$

$$l = 1, 2, \cdots, N_{tm} \qquad\qquad t_l \qquad\qquad\qquad\qquad \boldsymbol{r}_l$$
$$t_l$$

$$
\boldsymbol{e}(\boldsymbol{p},\boldsymbol{t}) \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{e}_1(\boldsymbol{p},\boldsymbol{t}) \\ \boldsymbol{e}_2(\boldsymbol{p},\boldsymbol{t}) \\ \vdots \\ \boldsymbol{e}_{N_{tm}}(\boldsymbol{p},\boldsymbol{t}) \end{pmatrix} = \begin{pmatrix} \boldsymbol{r}_1 - \boldsymbol{f}(\boldsymbol{\theta}(t_1)) \\ \boldsymbol{r}_2 - \boldsymbol{f}(\boldsymbol{\theta}(t_2)) \\ \vdots \\ \boldsymbol{r}_{N_{tm}} - \boldsymbol{f}(\boldsymbol{\theta}(t_{N_{tm}})) \end{pmatrix} = \begin{pmatrix} \boldsymbol{r}_1 - \boldsymbol{f}(\boldsymbol{B}_n(t_1)\boldsymbol{p}) \\ \boldsymbol{r}_2 - \boldsymbol{f}(\boldsymbol{B}_n(t_2)\boldsymbol{p}) \\ \vdots \\ \boldsymbol{r}_{N_{tm}} - \boldsymbol{f}(\boldsymbol{B}_n(t_{N_{tm}})\boldsymbol{p}) \end{pmatrix} \in \mathbb{R}^{6N_{tm}} \tag{256}
$$

$$
\boldsymbol{e}_l(\boldsymbol{p},\boldsymbol{t}) \;\stackrel{\text{def}}{=}\; \boldsymbol{r}_l - \boldsymbol{f}(\boldsymbol{\theta}(t_l)) = \boldsymbol{r}_l - \boldsymbol{f}(\boldsymbol{B}_n(t_l)\boldsymbol{p}) \in \mathbb{R}^6 \;\; (l = 1, 2, \cdots, N_{tm}) \tag{257}
$$

$$
\boldsymbol{t} \;\stackrel{\text{def}}{=}\; \begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_{N_{tm}} \end{pmatrix} \in \mathbb{R}^{N_{tm}} \tag{258}
$$

$$\boldsymbol{p} \qquad\qquad \boldsymbol{t}$$

$$
F(\boldsymbol{p},\boldsymbol{t}) \;\stackrel{\text{def}}{=}\; \frac{1}{2}\|\boldsymbol{e}(\boldsymbol{p},\boldsymbol{t})\|^2 \tag{259}
$$

$$
=\; \frac{1}{2}\sum_{l=1}^{N_{tm}} \|\boldsymbol{r}_l - \boldsymbol{f}(\boldsymbol{\theta}(t_l))\|^2 \tag{260}
$$

$$
=\; \frac{1}{2}\sum_{l=1}^{N_{tm}} \|\boldsymbol{r}_l - \boldsymbol{f}(\boldsymbol{B}_n(t_l)\boldsymbol{p})\|^2 \tag{261}
$$

l について $\boldsymbol{e}_{tol,l} \geq \boldsymbol{0} \in \mathbb{R}^6$ とし、 $\tilde{\boldsymbol{e}}_l(\boldsymbol{p},\boldsymbol{t})$ を

$$\tilde{e}_{l,i}(\boldsymbol{p},\boldsymbol{t}) \stackrel{\text{def}}{=} \begin{cases} e_{l,i}(\boldsymbol{p},\boldsymbol{t}) - e_{tol,l,i} & e_{l,i}(\boldsymbol{p},\boldsymbol{t}) > e_{tol,l,i} \\ e_{l,i}(\boldsymbol{p},\boldsymbol{t}) + e_{tol,l,i} & e_{l,i}(\boldsymbol{p},\boldsymbol{t}) < -e_{tol,l,i} \quad (i=1,2,\cdots,6) \\ 0 & \text{otherwise} \end{cases} \tag{262}$$

$\tilde{e}_{l,i}(\boldsymbol{p},\boldsymbol{t})$ は $\tilde{\boldsymbol{e}}_l(\boldsymbol{p},\boldsymbol{t})$ の i 番目の要素である。 $e_{l,i}(\boldsymbol{p},\boldsymbol{t})$ は $\boldsymbol{e}(\boldsymbol{p},\boldsymbol{t})$ の i 番目の要素である。

(261) を Gauss-Newton 法 や Levenberg-Marquardt 法 で解く。
(256) を用いる。

$\boldsymbol{e}_l(\boldsymbol{p},\boldsymbol{t})$ を $\boldsymbol{p}$ で偏微分すると

$$\frac{\partial \boldsymbol{e}_l(\boldsymbol{p},\boldsymbol{t})}{\partial \boldsymbol{p}} = \frac{\partial}{\partial \boldsymbol{p}}\{\boldsymbol{r}_l - \boldsymbol{f}(\boldsymbol{B}_n(t_l)\boldsymbol{p})\} \tag{263}$$

$$= -\frac{\partial}{\partial \boldsymbol{p}}\boldsymbol{f}(\boldsymbol{B}_n(t_l)\boldsymbol{p}) \tag{264}$$

$$= -\left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}(t_l)}\frac{\partial \boldsymbol{\theta}}{\partial \boldsymbol{p}} \tag{265}$$

$$= -\boldsymbol{J}(\boldsymbol{\theta}(t_l))\frac{\partial}{\partial \boldsymbol{p}}\{\boldsymbol{B}_n(t_l)\boldsymbol{p}\} \tag{266}$$

$$= -\boldsymbol{J}(\boldsymbol{\theta}(t_l))\boldsymbol{B}_n(t_l) \tag{267}$$

$\boldsymbol{\theta}(\boldsymbol{p};t) = \boldsymbol{B}_n(t)\boldsymbol{p}$ である。

$$\boldsymbol{J} \stackrel{\text{def}}{=} \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}} \tag{268}$$

$\boldsymbol{e}_l(\boldsymbol{p},\boldsymbol{t})$ を $\boldsymbol{t}$ で偏微分すると

$$\frac{\partial \boldsymbol{e}_l(\boldsymbol{p},\boldsymbol{t})}{\partial t_l} = \frac{\partial}{\partial t_l}\{\boldsymbol{r}_l - \boldsymbol{f}(\boldsymbol{P}\boldsymbol{b}_n(t_l))\} \tag{269}$$

$$= -\frac{\partial}{\partial t_l}\boldsymbol{f}(\boldsymbol{P}\boldsymbol{b}_n(t_l)) \tag{270}$$

$$= -\left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}(t_l)}\frac{\partial \boldsymbol{\theta}}{\partial t_l} \tag{271}$$

$$= -\boldsymbol{J}(\boldsymbol{\theta}(t_l))\frac{\partial}{\partial t_l}\{\boldsymbol{P}\boldsymbol{b}_n(t_l)\} \tag{272}$$

$$= -\boldsymbol{J}(\boldsymbol{\theta}(t_l))\boldsymbol{P}\dot{\boldsymbol{b}}_n(t_l) \tag{273}$$

$$= -\boldsymbol{J}(\boldsymbol{\theta}(t_l))\boldsymbol{P}\boldsymbol{D}\boldsymbol{b}_{n-1}(t_l) \tag{274}$$

$\boldsymbol{\theta}(\boldsymbol{p};t) = \boldsymbol{P}\boldsymbol{b}_n(t)$ である。

$$
\boldsymbol{e}_{sv}(\boldsymbol{p}, \boldsymbol{t}) \quad \overset{\text{def}}{=} \quad \dot{\boldsymbol{\theta}}(t_s) \tag{275}
$$

$$
= \quad \boldsymbol{B}_{n-1}(t_s)\hat{\boldsymbol{D}}_1 \boldsymbol{p} \tag{276}
$$

$$
= \quad \boldsymbol{PDb}_{n-1}(t_s) \tag{277}
$$

$$
\boldsymbol{e}_{fv}(\boldsymbol{p}, \boldsymbol{t}) \quad \overset{\text{def}}{=} \quad \dot{\boldsymbol{\theta}}(t_f) \tag{278}
$$

$$
= \quad \boldsymbol{B}_{n-1}(t_f)\hat{\boldsymbol{D}}_1 \boldsymbol{p} \tag{279}
$$

$$
= \quad \boldsymbol{PDb}_{n-1}(t_f) \tag{280}
$$

$$
\boldsymbol{e}_{sa}(\boldsymbol{p}, \boldsymbol{t}) \quad \overset{\text{def}}{=} \quad \ddot{\boldsymbol{\theta}}(t_s) \tag{281}
$$

$$
= \quad \boldsymbol{B}_{n-2}(t_s)\hat{\boldsymbol{D}}_2 \boldsymbol{p} \tag{282}
$$

$$
= \quad \boldsymbol{PD}^2 \boldsymbol{b}_{n-2}(t_s) \tag{283}
$$

$$
\boldsymbol{e}_{fa}(\boldsymbol{p}, \boldsymbol{t}) \quad \overset{\text{def}}{=} \quad \ddot{\boldsymbol{\theta}}(t_f) \tag{284}
$$

$$
= \quad \boldsymbol{B}_{n-2}(t_f)\hat{\boldsymbol{D}}_2 \boldsymbol{p} \tag{285}
$$

$$
= \quad \boldsymbol{PD}^2 \boldsymbol{b}_{n-2}(t_f) \tag{286}
$$

$$
\frac{\partial \boldsymbol{e}_{sv}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} \quad = \quad \boldsymbol{B}_{n-1}(t_s)\hat{\boldsymbol{D}}_1 \tag{287}
$$

$$
\frac{\partial \boldsymbol{e}_{fv}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} \quad = \quad \boldsymbol{B}_{n-1}(t_f)\hat{\boldsymbol{D}}_1 \tag{288}
$$

$$
\frac{\partial \boldsymbol{e}_{sa}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} \quad = \quad \boldsymbol{B}_{n-2}(t_s)\hat{\boldsymbol{D}}_2 \tag{289}
$$

$$
\frac{\partial \boldsymbol{e}_{fa}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} \quad = \quad \boldsymbol{B}_{n-2}(t_f)\hat{\boldsymbol{D}}_2 \tag{290}
$$

$$
\frac{\partial \boldsymbol{e}_{sv}(\boldsymbol{p}, \boldsymbol{t})}{\partial t_s} \quad = \quad \boldsymbol{PD}\frac{\partial \boldsymbol{b}_{n-1}(t_s)}{\partial t_s} = \boldsymbol{PD}^2 \boldsymbol{b}_{n-2}(t_s) \tag{291}
$$

$$
\frac{\partial \boldsymbol{e}_{fv}(\boldsymbol{p}, \boldsymbol{t})}{\partial t_f} \quad = \quad \boldsymbol{PD}\frac{\partial \boldsymbol{b}_{n-1}(t_f)}{\partial t_f} = \boldsymbol{PD}^2 \boldsymbol{b}_{n-2}(t_f) \tag{292}
$$

$$
\frac{\partial \boldsymbol{e}_{sa}(\boldsymbol{p}, \boldsymbol{t})}{\partial t_s} \quad = \quad \boldsymbol{PD}^2\frac{\partial \boldsymbol{b}_{n-2}(t_s)}{\partial t_s} = \boldsymbol{PD}^3 \boldsymbol{b}_{n-3}(t_s) \tag{293}
$$

$$
\frac{\partial \boldsymbol{e}_{fa}(\boldsymbol{p}, \boldsymbol{t})}{\partial t_f} \quad = \quad \boldsymbol{PD}^2\frac{\partial \boldsymbol{b}_{n-2}(t_f)}{\partial t_f} = \boldsymbol{PD}^3 \boldsymbol{b}_{n-3}(t_f) \tag{294}
$$

(235)

$$
\boldsymbol{p}_j \le \theta_{max,j}\boldsymbol{1}_m \tag{295}
$$

B ((229) , (230)) $\boldsymbol{1}_m \in \mathbb{R}^m$ 1

m

$$\theta_j(t) \quad = \quad \sum_{i=0}^{m-1} p_{j,i} b_{i,n}(t) \tag{296}$$

$$\leq \quad \sum_{i=0}^{m-1} \theta_{max,j} b_{i,n}(t) \tag{297}$$

$$= \quad \theta_{max,j} \sum_{i=0}^{m-1} b_{i,n}(t) \tag{298}$$

$$= \quad \theta_{max,j} \tag{299}$$

$\theta_{min,j} \mathbf{1}_m \leq \boldsymbol{p}_j \qquad\qquad \theta_{min,j} \leq \theta_j(t)$

$\qquad j \qquad\qquad\qquad\qquad \theta_{max,j}, \theta_{min,j}$

$$\theta_{min,j} \mathbf{1}_m \leq \boldsymbol{p}_j \leq \theta_{max,j} \mathbf{1}_m \ (j = 1, 2, \cdots, N_{joint}) \tag{300}$$

$$\hat{\boldsymbol{E}} \boldsymbol{\theta}_{min} \leq \boldsymbol{p} \leq \hat{\boldsymbol{E}} \boldsymbol{\theta}_{max} \tag{301}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \boldsymbol{p} \geq \begin{pmatrix} \hat{\boldsymbol{E}} \boldsymbol{\theta}_{min} \\ -\hat{\boldsymbol{E}} \boldsymbol{\theta}_{max} \end{pmatrix} \tag{302}$$

$$\hat{\boldsymbol{E}} \overset{\text{def}}{=} \begin{pmatrix} \mathbf{1}_m & & & \mathbf{0}_m \\ & \mathbf{1}_m & & \\ & & \ddots & \\ \mathbf{0}_m & & & \mathbf{1}_m \end{pmatrix} \in \mathbb{R}^{m N_{joint} \times N_{joint}} \tag{303}$$

$$\begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \Delta \boldsymbol{p} \geq \begin{pmatrix} \hat{\boldsymbol{E}} \boldsymbol{\theta}_{min} - \boldsymbol{p} \\ -\hat{\boldsymbol{E}} \boldsymbol{\theta}_{max} + \boldsymbol{p} \end{pmatrix} \tag{304}$$

$(235) \qquad (231)$

$$\dot{\theta}_j(t) \quad = \quad \boldsymbol{p}_j^T \dot{\boldsymbol{b}}_n(t) = \boldsymbol{p}_j^T \boldsymbol{D} \boldsymbol{b}_{n-1}(t) = (\boldsymbol{D}^T \boldsymbol{p}_j)^T \boldsymbol{b}_{n-1}(t) \in \mathbb{R} \ \ (t_s \leq t \leq t_f) \tag{305}$$

$$\ddot{\theta}_j(t) \quad = \quad \boldsymbol{p}_j^T \ddot{\boldsymbol{b}}_n(t) = \boldsymbol{p}_j^T \boldsymbol{D}^2 \boldsymbol{b}_{n-2}(t) = ((\boldsymbol{D}^2)^T \boldsymbol{p}_j)^T \boldsymbol{b}_{n-2}(t) \in \mathbb{R} \ \ (t_s \leq t \leq t_f) \tag{306}$$

$\qquad j \qquad\qquad\qquad\qquad\qquad v_{max,j}, a_{max,j}$

$$-v_{max,j} \mathbf{1}_m \leq \boldsymbol{D}^T \boldsymbol{p}_j \leq v_{max,j} \mathbf{1}_m \ (j = 1, 2, \cdots, N_{joint}) \tag{307}$$

$$-a_{max,j} \mathbf{1}_m \leq (\boldsymbol{D}^2)^T \boldsymbol{p}_j \leq a_{max,j} \mathbf{1}_m \ (j = 1, 2, \cdots, N_{joint}) \tag{308}$$

$$-\hat{E}v_{max} \leq \hat{D}_1 p \leq \hat{E}v_{max} \tag{309}$$

$$\Leftrightarrow \quad \begin{pmatrix} \hat{D}_1 \\ -\hat{D}_1 \end{pmatrix} p \geq \begin{pmatrix} -\hat{E}v_{max} \\ -\hat{E}v_{max} \end{pmatrix} \tag{310}$$

$$-\hat{E}a_{max} \leq \hat{D}_2 p \leq \hat{E}a_{max} \tag{311}$$

$$\Leftrightarrow \quad \begin{pmatrix} \hat{D}_2 \\ -\hat{D}_2 \end{pmatrix} p \geq \begin{pmatrix} -\hat{E}a_{max} \\ -\hat{E}a_{max} \end{pmatrix} \tag{312}$$

$$\begin{pmatrix} \hat{D}_1 \\ -\hat{D}_1 \end{pmatrix} \Delta p \geq \begin{pmatrix} -\hat{E}v_{max} - \hat{D}_1 p \\ -\hat{E}v_{max} + \hat{D}_1 p \end{pmatrix} \tag{313}$$

$$\begin{pmatrix} \hat{D}_2 \\ -\hat{D}_2 \end{pmatrix} \Delta p \geq \begin{pmatrix} -\hat{E}a_{max} - \hat{D}_2 p \\ -\hat{E}a_{max} + \hat{D}_2 p \end{pmatrix} \tag{314}$$

$$t_s \leq t_l \leq t_f \quad (l = 1, 2, \cdots, N_{tm}) \tag{315}$$

$$\Leftrightarrow \quad t_s \mathbf{1} \leq t \leq t_f \mathbf{1} \tag{316}$$

$$\Leftrightarrow \quad \begin{pmatrix} I \\ -I \end{pmatrix} t \geq \begin{pmatrix} t_s \mathbf{1} \\ -t_f \mathbf{1} \end{pmatrix} \tag{317}$$

$$\begin{pmatrix} I \\ -I \end{pmatrix} \Delta t \geq \begin{pmatrix} t_s \mathbf{1} - t \\ -t_f \mathbf{1} + t \end{pmatrix} \tag{318}$$

$$\tag{319}$$

$$t_l \leq t_{l+1} \quad (l = 1, 2, \cdots, N_{tm} - 1) \tag{320}$$

$$\Leftrightarrow \quad -t_l + t_{l+1} \geq 0 \quad (l = 1, 2, \cdots, N_{tm} - 1) \tag{321}$$

$$\Leftrightarrow \quad D_{tm} t \geq \mathbf{0} \tag{322}$$

$$D_{tm} = \begin{pmatrix} -1 & 1 & & & O \\ & -1 & 1 & & \\ & & & \ddots & \\ O & & & -1 & 1 \end{pmatrix} \in \mathbb{R}^{(N_{tm}-1) \times N_{tm}} \tag{323}$$

$$D_{tm} \Delta t \geq -D_{tm} t \tag{324}$$

$$
\begin{aligned}
F_{sqr,k}(\boldsymbol{p}) &= \int_{t_s}^{t_f} \left\| \boldsymbol{\theta}^{(k)}(t) \right\|^2 dt & (325)\\
&= \int_{t_s}^{t_f} \left\| \boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \boldsymbol{p} \right\|^2 dt & (326)\\
&= \int_{t_s}^{t_f} \left(\boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \boldsymbol{p} \right)^T \left(\boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \boldsymbol{p} \right) dt & (327)\\
&= \boldsymbol{p}^T \left\{ \int_{t_s}^{t_f} \left(\boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \right)^T \boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k dt \right\} \boldsymbol{p} & (328)\\
&= \boldsymbol{p}^T \boldsymbol{H}_k \boldsymbol{p} & (329)
\end{aligned}
$$

$$
\boldsymbol{H}_k = \int_{t_s}^{t_f} \left(\boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \right)^T \boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k dt \tag{330}
$$

$$
\boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k = \begin{pmatrix} \boldsymbol{b}_{n-k}^T(t) & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \boldsymbol{b}_{n-k}^T(t) \end{pmatrix} \begin{pmatrix} (\boldsymbol{D}^k)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & (\boldsymbol{D}^k)^T \end{pmatrix} \tag{331}
$$

$$
= \begin{pmatrix} \boldsymbol{b}_{n-k}^T(t)(\boldsymbol{D}^k)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \boldsymbol{b}_{n-k}^T(t)(\boldsymbol{D}^k)^T \end{pmatrix} \tag{332}
$$

$$
= \begin{pmatrix} \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T \end{pmatrix} \tag{333}
$$

$$
\left(\boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \right)^T \boldsymbol{B}_{n-k}(t) = \begin{pmatrix} \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T \end{pmatrix}^T \begin{pmatrix} \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T \end{pmatrix} \tag{334}
$$

$$
= \begin{pmatrix} \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)\left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)\left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t) \right)^T \end{pmatrix} \tag{335}
$$

$$
(t_f - t_s)
$$

$$
\begin{aligned}
F_{duration}(\boldsymbol{t}) &= \left| t_1 - t_{N_{tm}} \right|^2 & (336)\\
&= \boldsymbol{t}^T \begin{pmatrix} 1 & -1 \\ & \\ -1 & 1 \end{pmatrix} \boldsymbol{t} & (337)
\end{aligned}
$$

$$t_s = t_1 \qquad\qquad t_f = t_{N_{tm}} \qquad\qquad\qquad \boldsymbol{t}$$

4.2.2   B

bspline-configuration-task [class]

:super **propertied-object**

:slots (robot robot instance)

(control-vector $\boldsymbol{p}$)

(timing-vector $\boldsymbol{t}$)

(num-kin $N_{kin} := |\mathcal{T}^{kin\text{-}trg}| = |\mathcal{T}^{kin\text{-}att}|$)

(num-joint $N_{joint} := |\mathcal{J}|$)

(num-control-point N_{ctrl})

(num-timing N_{tm})

(bspline-order B-spline order, n)

(dim-control-vector $dim(\boldsymbol{p})$)

(dim-timing-vector $dim(\boldsymbol{t})$)

(dim-config $dim(\boldsymbol{q})$)

(dim-task $dim(\boldsymbol{e})$)

(num-collision $N_{col} :=$ number of collision check pairs)

(stationery-start-finish-task-scale k_{stat})

(first-diff-square-integration-regular-scale $k_{sqr,1}$)

(second-diff-square-integration-regular-scale $k_{sqr,2}$)

(third-diff-square-integration-regular-scale $k_{sqr,3}$)

(motion-duration-regular-scale $k_{duration}$)

(norm-regular-scale-max $k_{max,p}$)

(norm-regular-scale-offset $k_{off,p}$)

(timing-norm-regular-scale-max $k_{max,t}$)

(timing-norm-regular-scale-offset $k_{off,t}$)

(joint-list $\mathcal{J}$)

(start-time t_s)

(finish-time t_f)

(kin-time-list $\{t_1^{kin\text{-}tm}, t_2^{kin\text{-}tm}, \cdots, t_{N_{kin}}^{kin\text{-}tm}\}$)

(kin-variable-timing-list list of bool. t for variable timing.)

(kin-target-coords-list $\mathcal{T}^{kin\text{-}trg}$)

(kin-attention-coords-list $\mathcal{T}^{kin\text{-}att}$)

(kin-pos-tolerance-list list of position tolerance $e_{tol,pos}$ [m])

(kin-rot-tolerance-list list of rotation tolerance $e_{tol,rot}$ [rad])

(joint-angle-margin margin of $\boldsymbol{\theta}$ [deg] [mm])

(collision-pair-list list of bodyset-link or body pair)

(keep-timing-order? whether to keep order of timing $\boldsymbol{t}$ or not)

(bspline-matrix buffer for $\boldsymbol{B}_n(t)$)

(_diff-mat buffer for $\boldsymbol{D}^k$)
(_diff-mat-list buffer for $\{\boldsymbol{D}^1, \boldsymbol{D}^2, \cdots, \boldsymbol{D}^K\}$)
(_extended-diff-mat-list buffer for $\{\hat{\boldsymbol{D}}_1, \hat{\boldsymbol{D}}_2, \cdots, \hat{\boldsymbol{D}}_K\}$)
(_task-jacobi buffer for $\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}}$)
(_regular-mat buffer for $\boldsymbol{W}_{reg}$)
(_regular-vec buffer for $\boldsymbol{v}_{reg}$)

B $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \boldsymbol{q} \qquad\qquad \boldsymbol{e}(\boldsymbol{q})$

$\boldsymbol{q}$ $\qquad\qquad\qquad\qquad\qquad \boldsymbol{e}(\boldsymbol{q})$ $\qquad\qquad\qquad\qquad\qquad \frac{\partial \boldsymbol{e}(\boldsymbol{q})}{\partial \boldsymbol{q}}$

$\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{C}, \boldsymbol{d}$

-

robot

joint-list $\mathcal{J}$

- B

start-time t_s B

finish-time t_f B

num-control-point N_{ctrl}

bspline-order n B

-

kin-target-coords-list $\mathcal{T}^{kin\text{-}trg}$

kin-attention-coords-list $\mathcal{T}^{kin\text{-}att}$

kin-time-list $\{t_1^{kin\text{-}tm}, t_2^{kin\text{-}tm}, \cdots, t_{N_{kin}}^{kin\text{-}tm}\}$

kin-variable-timing-list $\qquad\qquad\qquad\qquad$ (t) $\qquad$ (nil)

$\qquad$ t $\qquad\qquad\qquad\qquad\qquad \boldsymbol{t} \qquad N_{tm}$

$\qquad\qquad \boldsymbol{q}$

$$\boldsymbol{q} := \begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{t} \end{pmatrix} \tag{338}$$

$\boldsymbol{p} \in \mathbb{R}^{N_{ctrl} N_{joint}}$ $\qquad$ (B $\qquad\qquad\qquad\qquad\qquad$) [rad] [m]

$\boldsymbol{t} \in \mathbb{R}^{N_{tm}}$ $\qquad\qquad$ ($\qquad\qquad\qquad\qquad\qquad$) [sec]

$\quad \boldsymbol{e}(\boldsymbol{q})$

$$\boldsymbol{e}(\boldsymbol{q}) := \begin{pmatrix} \boldsymbol{e}^{kin}(\boldsymbol{q}) \\ \boldsymbol{e}^{stat}(\boldsymbol{q}) \end{pmatrix} \in \mathbb{R}^{6N_{kin}+4N_{joint}} \tag{339}$$

$\boldsymbol{e}^{kin}(\boldsymbol{q}) \in \mathbb{R}^{6N_{kin}}$ $\qquad\qquad$ [rad] [m]

$\boldsymbol{e}^{stat}(\boldsymbol{q}) \in \mathbb{R}^{4N_{joint}}$ $\qquad\qquad\qquad$ [rad][rad/s][rad/s^2][m][m/s][m/s^2]

:init *&key* *(name)* [method]
   (robot)
   (joint-list (send robot :joint-list))
   (start-time 0.0)
   (finish-time 10.0)
   (num-control-point 10)
   (bspline-order 3)
   (kin-time-list)
   (kin-variable-timing-list (make-list (length kin-time-list) :initial-element nil))
   (kin-target-coords-list)
   (kin-attention-coords-list)
   (kin-pos-tolerance-list (make-list (length kin-time-list) :initial-element 0.0))
   (kin-rot-tolerance-list (make-list (length kin-time-list) :initial-element 0.0))
   (joint-angle-margin 3.0)
   (collision-pair-list)
   (keep-timing-order? t)
   (stationery-start-finish-task-scale 0.0)
   (first-diff-square-integration-regular-scale 0.0)
   (second-diff-square-integration-regular-scale 0.0)
   (third-diff-square-integration-regular-scale 0.0)
   (motion-duration-regular-scale 0.0)
   (norm-regular-scale-max 1.000000e-05)
   (norm-regular-scale-offset 1.000000e-07)
   (timing-norm-regular-scale-max 1.000000e-05)
   (timing-norm-regular-scale-offset 1.000000e-07)

  Initialize instance

:robot [method]
  return robot instance

:joint-list [method]
  return $\mathcal{J}$

:num-kin [method]
  return $N_{kin} := |\mathcal{T}^{kin\text{-}trg}| = |\mathcal{T}^{kin\text{-}att}|$

:num-joint [method]
  return $N_{joint} := |\mathcal{J}|$

:num-control-point [method]
  return N_{ctrl}

:num-timing [method]
  return N_{tm}

:num-collision [method]
  return $N_{col} :=$ number of collision check pairs

:dim-config [method]
  return $dim(\boldsymbol{q}) := dim(\boldsymbol{p}) + dim(\boldsymbol{t}) = N_{ctrl}N_{joint} + N_{tm}$

:dim-task [method]

return $dim(\boldsymbol{e}) := dim(\boldsymbol{e}^{kin}) + dim(\boldsymbol{e}^{stat}) = 6N_{kin} + 4N_{joint}$

:control-vector [method]

return control vector $\boldsymbol{p}$

:timing-vector [method]

return timing vector $\boldsymbol{t}$

:config-vector [method]

return $\boldsymbol{q} := \begin{pmatrix} \boldsymbol{p} \\ \boldsymbol{t} \end{pmatrix}$

:set-control-vector *control-vector-new &key (relative? nil)* [method]

Set $\boldsymbol{p}$.

:set-timing-vector *timing-vector-new &key (relative? nil)* [method]

Set $\boldsymbol{t}$.

:set-config *config-new &key (relative? nil)* [method]

Set $\boldsymbol{q}$.

:bspline-vector *tm &key (order-offset 0)* [method]

$$\boldsymbol{b}_n(t) \quad := \quad \begin{pmatrix} b_{0,n}(t) \\ b_{1,n}(t) \\ \vdots \\ b_{N_{ctrl}-1,n}(t) \end{pmatrix} \in \mathbb{R}^{N_{ctrl}} \tag{340}$$

return $\boldsymbol{b}_n(t)$

:bspline-matrix *tm &key (order-offset 0)* [method]

$$\boldsymbol{B}_n(t) := \begin{pmatrix} \boldsymbol{b}_n^T(t) & & & \boldsymbol{O} \\ & \boldsymbol{b}_n^T(t) & & \\ & & \ddots & \\ \boldsymbol{O} & & & \boldsymbol{b}_n^T(t) \end{pmatrix} \in \mathbb{R}^{N_{joint} \times N_{ctrl}N_{joint}} \tag{341}$$

return $\boldsymbol{B}_n(t)$

:differential-matrix *&key (diff-order 1)* [method]

$$\boldsymbol{D} \quad := \quad \frac{1}{h} \begin{pmatrix} 1 & -1 & & & \boldsymbol{O} \\ & 1 & -1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & -1 \\ \boldsymbol{O} & & & & 1 \end{pmatrix} \in \mathbb{R}^{N_{ctrl} \times N_{ctrl}} \tag{342}$$

return $\boldsymbol{D}^k$

:extended-differential-matrix *&key (diff-order 1)* [method]

$$
\hat{\boldsymbol{D}}_k \quad := \quad \begin{pmatrix} (\boldsymbol{D}^k)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & (\boldsymbol{D}^k)^T \end{pmatrix} \in \mathbb{R}^{N_{ctrl}N_{joint} \times N_{ctrl}N_{joint}} \tag{343}
$$

return $\hat{\boldsymbol{D}}_k$

:bspline-differential-matrix *tm &key (diff-order 1)* [method]
return $\boldsymbol{B}_{n-k}(t)\hat{\boldsymbol{D}}_k \in \mathbb{R}^{N_{joint} \times N_{ctrl}N_{joint}}$

:control-matrix [method]

$$
\boldsymbol{P} := \begin{pmatrix} \boldsymbol{p}_1^T \\ \boldsymbol{p}_2^T \\ \vdots \\ \boldsymbol{p}_{n_{joint}}^T \end{pmatrix} \in \mathbb{R}^{N_{joint} \times N_{ctrl}} \tag{344}
$$

return $\boldsymbol{P}$

:theta *tm* [method]
return $\boldsymbol{\theta}(t) = \boldsymbol{B}_n(t)\boldsymbol{p}$ [rad][m]

:theta-dot *tm &key (diff-order 1)* [method]
return $\boldsymbol{\theta}^{(k)}(t) = \dfrac{d^{(k)}\boldsymbol{\theta}(t)}{dt^{(k)}} = \boldsymbol{P}\boldsymbol{D}^k\boldsymbol{b}_{n-k}(t)$ [rad/s^k][m/s^k]

:theta-dot-numerical *tm &key (diff-order 1)* [method]
(delta-time 0.05)
return $\boldsymbol{\theta}^{(k)}(t) = \dfrac{d^{(k)}\boldsymbol{\theta}(t)}{dt^{(k)}} = \dfrac{\boldsymbol{\theta}^{(k-1)}(t+\Delta t) - \boldsymbol{\theta}^{(k-1)}(t)}{\Delta t}$ [rad/s^k][m/s^k]

:apply-theta-to-robot *tm* [method]
apply $\boldsymbol{\theta}(t)$ to robot.

:kin-target-coords-list [method]

$$
T_m^{kin\text{-}trg} = \{\boldsymbol{p}_l^{kin\text{-}trg}, \boldsymbol{R}_l^{kin\text{-}trg}\} \quad (l = 1, 2, \cdots, N_{kin}) \tag{345}
$$

return $\mathcal{T}^{kin\text{-}trg} := \{T_1^{kin\text{-}trg}, T_2^{kin\text{-}trg}, \cdots, T_{N_{kin}}^{kin\text{-}trg}\}$

:kin-attention-coords-list [method]

$$
T_m^{kin\text{-}att} = \{\boldsymbol{p}_l^{kin\text{-}att}, \boldsymbol{R}_l^{kin\text{-}att}\} \quad (l = 1, 2, \cdots, N_{kin}) \tag{346}
$$

return $\mathcal{T}^{kin\text{-}att} := \{T_1^{kin\text{-}att}, T_2^{kin\text{-}att}, \cdots, T_{N_{kin}}^{kin\text{-}att}\}$

:kin-start-time [method]
return $t_s^{kin} := t_1^{kin\text{-}tm}$

:kin-finish-time [method]

return $t_f^{kin} := t_{N_{kin}}^{kin\text{-}tm}$

:motion-duration [method]

return $(t_{N_{kin}}^{kin\text{-}tm} - t_1^{kin\text{-}tm})$

:kinematics-task-value *&key (update? t)* [method]

$$\boldsymbol{e}^{kin}(\boldsymbol{q}) = \boldsymbol{e}^{kin}(\boldsymbol{p},\boldsymbol{t}) \tag{347}$$

$$= \begin{pmatrix} \boldsymbol{e}_1^{kin}(\boldsymbol{p},\boldsymbol{t}) \\ \boldsymbol{e}_2^{kin}(\boldsymbol{p},\boldsymbol{t}) \\ \vdots \\ \boldsymbol{e}_{N_{kin}}^{kin}(\boldsymbol{p},\boldsymbol{t}) \end{pmatrix} \tag{348}$$

$$\boldsymbol{e}_l^{kin}(\boldsymbol{p},\boldsymbol{t}) = \begin{pmatrix} \boldsymbol{p}_l^{kin\text{-}trg} - \boldsymbol{p}_l^{kin\text{-}att}(\boldsymbol{p},\boldsymbol{t}) \\ \boldsymbol{a}\left(\boldsymbol{R}_l^{kin\text{-}trg}\boldsymbol{R}_l^{kin\text{-}att}(\boldsymbol{p},\boldsymbol{t})^T\right) \end{pmatrix} \in \mathbb{R}^6 \quad (l=1,2,\cdots,N_{kin}) \tag{349}$$

$$\boldsymbol{a}(\boldsymbol{R}) \qquad\qquad \boldsymbol{R}$$

return $\boldsymbol{e}^{kin}(\boldsymbol{q}) \in \mathbb{R}^{6N_{kin}}$

:stationery-start-finish-task-value *&key (update? t)* [method]

$$\boldsymbol{e}^{stat}(\boldsymbol{q}) = \boldsymbol{e}^{stat}(\boldsymbol{p},\boldsymbol{t}) \tag{350}$$

$$= \begin{pmatrix} \boldsymbol{e}_{sv}^{stat}(\boldsymbol{p},\boldsymbol{t}) \\ \boldsymbol{e}_{fv}^{stat}(\boldsymbol{p},\boldsymbol{t}) \\ \boldsymbol{e}_{sa}^{stat}(\boldsymbol{p},\boldsymbol{t}) \\ \boldsymbol{e}_{fa}^{stat}(\boldsymbol{p},\boldsymbol{t}) \end{pmatrix} \tag{351}$$

$$\boldsymbol{e}_{sv}^{stat}(\boldsymbol{p},\boldsymbol{t}) := \dot{\boldsymbol{\theta}}(t_s^{kin}) \tag{352}$$

$$\boldsymbol{e}_{fv}^{stat}(\boldsymbol{p},\boldsymbol{t}) := \dot{\boldsymbol{\theta}}(t_f^{kin}) \tag{353}$$

$$\boldsymbol{e}_{sa}^{stat}(\boldsymbol{p},\boldsymbol{t}) := \ddot{\boldsymbol{\theta}}(t_s^{kin}) \tag{354}$$

$$\boldsymbol{e}_{fa}^{stat}(\boldsymbol{p},\boldsymbol{t}) := \ddot{\boldsymbol{\theta}}(t_f^{kin}) \tag{355}$$

return $\boldsymbol{e}^{stat}(\boldsymbol{q}) \in \mathbb{R}^{4N_{joint}}$

:task-value *&key (update? t)* [method]

return $\boldsymbol{e}(\boldsymbol{q}) := \begin{pmatrix} \boldsymbol{e}^{kin}(\boldsymbol{q}) \\ k_{stat}\boldsymbol{e}^{stat}(\boldsymbol{q}) \end{pmatrix} \in \mathbb{R}^{6N_{kin}+4N_{joint}}$

:kinematics-task-jacobian-with-control-vector [method]

(267) $\boldsymbol{e}^{kin}$ $\boldsymbol{p}$

$$\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{p}} = \begin{pmatrix} \frac{\partial \boldsymbol{e}_1^{kin}}{\partial \boldsymbol{p}} \\ \frac{\partial \boldsymbol{e}_2^{kin}}{\partial \boldsymbol{p}} \\ \vdots \\ \frac{\partial \boldsymbol{e}_{N_{kin}}^{kin}}{\partial \boldsymbol{p}} \end{pmatrix} \tag{356}$$

$$\frac{\partial \boldsymbol{e}_l^{kin}}{\partial \boldsymbol{p}} = -\boldsymbol{J}^{kin\text{-}att}(\boldsymbol{\theta}(t_l^{kin\text{-}tm}))\boldsymbol{B}_n(t_l^{kin\text{-}tm}) \quad (l=1,2,\cdots,N_{kin}) \tag{357}$$

$$\text{return } \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{p}} \in \mathbb{R}^{6N_{kin} \times N_{ctrl}N_{joint}}$$

:kinematics-task-jacobian-with-timing-vector [method]

(274) $\boldsymbol{e}^{kin}$ $\boldsymbol{t}$

$$\frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{t}} = \begin{pmatrix} \frac{\partial \boldsymbol{e}_1^{kin}}{\partial \boldsymbol{t}} \\ \frac{\partial \boldsymbol{e}_2^{kin}}{\partial \boldsymbol{t}} \\ \vdots \\ \frac{\partial \boldsymbol{e}_{N_{kin}}^{kin}}{\partial \boldsymbol{t}} \end{pmatrix} \tag{358}$$

$\frac{\partial \boldsymbol{e}_l^{kin}}{\partial \boldsymbol{t}}$ $\quad i \quad$ $\left[\frac{\partial \boldsymbol{e}_l^{kin}}{\partial \boldsymbol{t}} \right]_i \in \mathbb{R}^6$ $\quad (i = 1, 2, \cdots, N_{tm})$

$$\left[\frac{\partial \boldsymbol{e}_l^{kin}}{\partial \boldsymbol{t}} \right]_i = \begin{cases} -\boldsymbol{J}^{kin\text{-}att}(\boldsymbol{\theta}(t_l^{kin\text{-}tm}))\boldsymbol{PD}\boldsymbol{b}_{n-1}(t_l^{kin\text{-}tm}) & t_l^{kin\text{-}tm} \text{ and } t_i \text{ is identical} \\ \boldsymbol{0} & \text{otherwise} \end{cases} \tag{359}$$

$$\text{return } \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{t}} \in \mathbb{R}^{6N_{kin} \times N_{tm}}$$

:stationery-start-finish-task-jacobian-with-control-vector [method]

(287) , (288) , (289) , (290) $\boldsymbol{e}^{stat}$ $\boldsymbol{p}$

$$\frac{\partial \boldsymbol{e}^{stat}}{\partial \boldsymbol{p}} = \begin{pmatrix} \frac{\partial \boldsymbol{e}_{sv}^{stat}}{\partial \boldsymbol{p}} \\ \frac{\partial \boldsymbol{e}_{fv}^{stat}}{\partial \boldsymbol{p}} \\ \frac{\partial \boldsymbol{e}_{sa}^{stat}}{\partial \boldsymbol{p}} \\ \frac{\partial \boldsymbol{e}_{fa}^{stat}}{\partial \boldsymbol{p}} \end{pmatrix} \tag{360}$$

$$\frac{\partial \boldsymbol{e}_{sv}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} = \boldsymbol{B}_{n-1}(t_s^{kin})\hat{\boldsymbol{D}}_1 \tag{361}$$

$$\frac{\partial \boldsymbol{e}_{fv}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} = \boldsymbol{B}_{n-1}(t_f^{kin})\hat{\boldsymbol{D}}_1 \tag{362}$$

$$\frac{\partial \boldsymbol{e}_{sa}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} = \boldsymbol{B}_{n-2}(t_s^{kin})\hat{\boldsymbol{D}}_2 \tag{363}$$

$$\frac{\partial \boldsymbol{e}_{fa}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{p}} = \boldsymbol{B}_{n-2}(t_f^{kin})\hat{\boldsymbol{D}}_2 \tag{364}$$

$$\text{return } \frac{\partial \boldsymbol{e}^{stat}}{\partial \boldsymbol{p}} \in \mathbb{R}^{4N_{joint} \times N_{ctrl}N_{joint}}$$

:stationery-start-finish-task-jacobian-with-timing-vector [method]

(291) , (292) , (293) , (294) $\boldsymbol{e}^{stat}$ $\boldsymbol{t}$

$$\frac{\partial \boldsymbol{e}^{stat}}{\partial \boldsymbol{t}} = \begin{pmatrix} \frac{\partial \boldsymbol{e}_{sv}^{stat}}{\partial \boldsymbol{t}} \\ \frac{\partial \boldsymbol{e}_{fv}^{stat}}{\partial \boldsymbol{t}} \\ \frac{\partial \boldsymbol{e}_{sa}^{stat}}{\partial \boldsymbol{t}} \\ \frac{\partial \boldsymbol{e}_{fa}^{stat}}{\partial \boldsymbol{t}} \end{pmatrix} \tag{365}$$

$$\frac{\partial \boldsymbol{e}_x^{stat}}{\partial \boldsymbol{t}} \quad i \qquad\qquad \left[\frac{\partial \boldsymbol{e}_x^{stat}}{\partial \boldsymbol{t}}\right]_i \in \mathbb{R}^{N_{joint}} \qquad\qquad (x \in \{sv, fv, sa, fa\}, i = 1, 2, \cdots, N_{tm})$$

$$\left[\frac{\partial \boldsymbol{e}_{sv}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{t}}\right]_i = \begin{cases} \boldsymbol{P}\boldsymbol{D}^2\boldsymbol{b}_{n-2}(t_s^{kin}) & t_s^{kin} \text{ and } t_i \text{ is identical} \\ \boldsymbol{0} & \text{otherwise} \end{cases} \tag{366}$$

$$\left[\frac{\partial \boldsymbol{e}_{fv}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{t}}\right]_i = \begin{cases} \boldsymbol{P}\boldsymbol{D}^2\boldsymbol{b}_{n-2}(t_f^{kin}) & t_f^{kin} \text{ and } t_i \text{ is identical} \\ \boldsymbol{0} & \text{otherwise} \end{cases} \tag{367}$$

$$\left[\frac{\partial \boldsymbol{e}_{sa}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{t}}\right]_i = \begin{cases} \boldsymbol{P}\boldsymbol{D}^3\boldsymbol{b}_{n-3}(t_s^{kin}) & t_s^{kin} \text{ and } t_i \text{ is identical} \\ \boldsymbol{0} & \text{otherwise} \end{cases} \tag{368}$$

$$\left[\frac{\partial \boldsymbol{e}_{fa}^{stat}(\boldsymbol{p}, \boldsymbol{t})}{\partial \boldsymbol{t}}\right]_i = \begin{cases} \boldsymbol{P}\boldsymbol{D}^3\boldsymbol{b}_{n-3}(t_f^{kin}) & t_f^{kin} \text{ and } t_i \text{ is identical} \\ \boldsymbol{0} & \text{otherwise} \end{cases} \tag{369}$$

return $\dfrac{\partial \boldsymbol{e}^{stat}}{\partial \boldsymbol{t}} \in \mathbb{R}^{4N_{joint} \times N_{tm}}$

:task-jacobian [method]

$$\frac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} = \begin{matrix} \\ 6N_{kin} \\ 4N_{joint} \end{matrix} \overset{\displaystyle N_{ctrl}N_{joint} \qquad N_{tm}}{\begin{pmatrix} \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{p}} & \frac{\partial \boldsymbol{e}^{kin}}{\partial \boldsymbol{t}} \\ k_{stat}\frac{\partial \boldsymbol{e}^{stat}}{\partial \boldsymbol{p}} & k_{stat}\frac{\partial \boldsymbol{e}^{stat}}{\partial \boldsymbol{t}} \end{pmatrix}} \tag{370}$$

return $\dfrac{\partial \boldsymbol{e}}{\partial \boldsymbol{q}} = \mathbb{R}^{(6N_{kin}+4N_{joint}) \times (N_{ctrl}N_{joint}+N_{tm})}$

:theta-max-vector *&key (update? nil)* [method]

return $\boldsymbol{\theta}_{max} \in \mathbb{R}^{N_{joint}}$

:theta-min-vector *&key (update? nil)* [method]

return $\boldsymbol{\theta}_{min} \in \mathbb{R}^{N_{joint}}$

:theta-inequality-constraint-matrix *&key (update? nil)* [method]

(301)

$$\hat{\boldsymbol{E}}\boldsymbol{\theta}_{min} \leq \boldsymbol{p} + \Delta\boldsymbol{p} \leq \hat{\boldsymbol{E}}\boldsymbol{\theta}_{max} \tag{371}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \Delta\boldsymbol{p} \geq \begin{pmatrix} \hat{\boldsymbol{E}}\boldsymbol{\theta}_{min} - \boldsymbol{p} \\ -\hat{\boldsymbol{E}}\boldsymbol{\theta}_{max} + \boldsymbol{p} \end{pmatrix} \tag{372}$$

$$\Leftrightarrow \quad \boldsymbol{C_\theta}\Delta\boldsymbol{p} \geq \boldsymbol{d_\theta} \tag{373}$$

$$\hat{\boldsymbol{E}} := \begin{pmatrix} \boldsymbol{1}_{N_{ctrl}} & & & \boldsymbol{0}_{N_{ctrl}} \\ & \boldsymbol{1}_{N_{ctrl}} & & \\ & & \ddots & \\ \boldsymbol{0}_{N_{ctrl}} & & & \boldsymbol{1}_{N_{ctrl}} \end{pmatrix} \in \mathbb{R}^{N_{ctrl}N_{joint} \times N_{joint}} \tag{374}$$

$\boldsymbol{1}_{N_{ctrl}} \in \mathbb{R}^{N_{ctrl}} \qquad 1 \qquad N_{ctrl}$

return $\boldsymbol{C_\theta} := \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \in \mathbb{R}^{2N_{ctrl}N_{joint} \times N_{ctrl}N_{joint}}$

:theta-inequality-constraint-vector *&key (update? t)* [method]

return $\boldsymbol{d_\theta} := \begin{pmatrix} \hat{\boldsymbol{E}}\boldsymbol{\theta}_{min} - \boldsymbol{p} \\ -\hat{\boldsymbol{E}}\boldsymbol{\theta}_{max} + \boldsymbol{p} \end{pmatrix} \in \mathbb{R}^{2N_{ctrl}N_{joint}}$

:velocity-max-vector *&key (update? nil)* [method]

$$\text{return } \boldsymbol{v}_{max} \in \mathbb{R}^{N_{joint}}$$

:velocity-inequality-constraint-matrix *&key (update? nil)* [method]

(309)

$$-\hat{\boldsymbol{E}}\boldsymbol{v}_{max} \leq \hat{\boldsymbol{D}}_1(\boldsymbol{p} + \Delta\boldsymbol{p}) \leq \hat{\boldsymbol{E}}\boldsymbol{v}_{max} \tag{375}$$

$$\Leftrightarrow \quad \begin{pmatrix} \hat{\boldsymbol{D}}_1 \\ -\hat{\boldsymbol{D}}_1 \end{pmatrix} \Delta\boldsymbol{p} \geq \begin{pmatrix} -\hat{\boldsymbol{E}}\boldsymbol{v}_{max} - \hat{\boldsymbol{D}}_1\boldsymbol{p} \\ -\hat{\boldsymbol{E}}\boldsymbol{v}_{max} + \hat{\boldsymbol{D}}_1\boldsymbol{p} \end{pmatrix} \tag{376}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\dot{\boldsymbol{\theta}}}\Delta\boldsymbol{p} \geq \boldsymbol{d}_{\dot{\boldsymbol{\theta}}} \tag{377}$$

$$\text{return } \boldsymbol{C}_{\dot{\boldsymbol{\theta}}} := \begin{pmatrix} \hat{\boldsymbol{D}}_1 \\ -\hat{\boldsymbol{D}}_1 \end{pmatrix} \in \mathbb{R}^{2N_{ctrl}N_{joint} \times N_{ctrl}N_{joint}}$$

:velocity-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d}_{\dot{\boldsymbol{\theta}}} := \begin{pmatrix} -\hat{\boldsymbol{E}}\boldsymbol{v}_{max} - \hat{\boldsymbol{D}}_1\boldsymbol{p} \\ -\hat{\boldsymbol{E}}\boldsymbol{v}_{max} + \hat{\boldsymbol{D}}_1\boldsymbol{p} \end{pmatrix} \in \mathbb{R}^{2N_{ctrl}N_{joint}}$$

:acceleration-max-vector *&key (update? nil)* [method]

$$\text{return } \boldsymbol{a}_{max} \in \mathbb{R}^{N_{joint}}$$

:acceleration-inequality-constraint-matrix *&key (update? nil)* [method]

(311)

$$-\hat{\boldsymbol{E}}\boldsymbol{a}_{max} \leq \hat{\boldsymbol{D}}_2(\boldsymbol{p} + \Delta\boldsymbol{p}) \leq \hat{\boldsymbol{E}}\boldsymbol{a}_{max} \tag{378}$$

$$\Leftrightarrow \quad \begin{pmatrix} \hat{\boldsymbol{D}}_2 \\ -\hat{\boldsymbol{D}}_2 \end{pmatrix} \Delta\boldsymbol{p} \geq \begin{pmatrix} -\hat{\boldsymbol{E}}\boldsymbol{a}_{max} - \hat{\boldsymbol{D}}_2\boldsymbol{p} \\ -\hat{\boldsymbol{E}}\boldsymbol{a}_{max} + \hat{\boldsymbol{D}}_2\boldsymbol{p} \end{pmatrix} \tag{379}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\ddot{\boldsymbol{\theta}}}\Delta\boldsymbol{p} \geq \boldsymbol{d}_{\ddot{\boldsymbol{\theta}}} \tag{380}$$

$$\text{return } \boldsymbol{C}_{\ddot{\boldsymbol{\theta}}} := \begin{pmatrix} \hat{\boldsymbol{D}}_2 \\ -\hat{\boldsymbol{D}}_2 \end{pmatrix} \in \mathbb{R}^{2N_{ctrl}N_{joint} \times N_{ctrl}N_{joint}}$$

:acceleration-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d}_{\ddot{\boldsymbol{\theta}}} := \begin{pmatrix} -\hat{\boldsymbol{E}}\boldsymbol{a}_{max} - \hat{\boldsymbol{D}}_2\boldsymbol{p} \\ -\hat{\boldsymbol{E}}\boldsymbol{a}_{max} + \hat{\boldsymbol{D}}_2\boldsymbol{p} \end{pmatrix} \in \mathbb{R}^{2N_{ctrl}N_{joint}}$$

:control-vector-inequality-constraint-matrix *&key (update? nil)* [method]

$$\begin{cases} \boldsymbol{C}_{\boldsymbol{\theta}}\Delta\boldsymbol{p} \geq \boldsymbol{d}_{\boldsymbol{\theta}} \\ \boldsymbol{C}_{\dot{\boldsymbol{\theta}}}\Delta\boldsymbol{p} \geq \boldsymbol{d}_{\dot{\boldsymbol{\theta}}} \\ \boldsymbol{C}_{\ddot{\boldsymbol{\theta}}}\Delta\boldsymbol{p} \geq \boldsymbol{d}_{\ddot{\boldsymbol{\theta}}} \end{cases} \tag{381}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{C}_{\boldsymbol{\theta}} \\ \boldsymbol{C}_{\dot{\boldsymbol{\theta}}} \\ \boldsymbol{C}_{\ddot{\boldsymbol{\theta}}} \end{pmatrix} \Delta\boldsymbol{p} \geq \begin{pmatrix} \boldsymbol{d}_{\boldsymbol{\theta}} \\ \boldsymbol{d}_{\dot{\boldsymbol{\theta}}} \\ \boldsymbol{d}_{\ddot{\boldsymbol{\theta}}} \end{pmatrix} \tag{382}$$

$$\Leftrightarrow \quad \boldsymbol{C}_{\boldsymbol{p}}\Delta\boldsymbol{p} \geq \boldsymbol{d}_{\boldsymbol{p}} \tag{383}$$

$$\text{return } \boldsymbol{C}_{\boldsymbol{p}} := \begin{pmatrix} \boldsymbol{C}_{\boldsymbol{\theta}} \\ \boldsymbol{C}_{\dot{\boldsymbol{\theta}}} \\ \boldsymbol{C}_{\ddot{\boldsymbol{\theta}}} \end{pmatrix} \in \mathbb{R}^{N_{p\text{-}ineq} \times dim(\boldsymbol{p})}$$

:control-vector-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d_p} := \begin{pmatrix} \boldsymbol{d_\theta} \\ \boldsymbol{d_{\dot\theta}} \\ \boldsymbol{d_{\ddot\theta}} \end{pmatrix} \in \mathbb{R}^{N_{p\text{-}ineq}}$$

:timing-vector-inequality-constraint-matrix *&key (update? nil)* [method]

(316)　　　　　　　　　　　B

$$t_s \boldsymbol{1} \leq \boldsymbol{t} + \Delta \boldsymbol{t} \leq t_f \boldsymbol{1} \tag{384}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \end{pmatrix} \Delta \boldsymbol{t} \geq \begin{pmatrix} t_s \boldsymbol{1} - \boldsymbol{t} \\ -t_f \boldsymbol{1} + \boldsymbol{t} \end{pmatrix} \tag{385}$$

(322)

$$\boldsymbol{D}_{tm}(\boldsymbol{t} + \Delta \boldsymbol{t}) \geq \boldsymbol{0} \tag{386}$$

$$\Leftrightarrow \quad \boldsymbol{D}_{tm} \Delta \boldsymbol{t} \geq -\boldsymbol{D}_{tm} \boldsymbol{t} \tag{387}$$

$$\boldsymbol{D}_{tm} = \begin{pmatrix} -1 & 1 & & & \boldsymbol{O} \\ & -1 & 1 & & \\ & & & \ddots & \\ \boldsymbol{O} & & & -1 & 1 \end{pmatrix} \in \mathbb{R}^{(N_{tm}-1)\times N_{tm}} \tag{388}$$

$$\begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{D}_{tm} \end{pmatrix} \Delta \boldsymbol{t} \geq \begin{pmatrix} t_s \boldsymbol{1} - \boldsymbol{t} \\ -t_f \boldsymbol{1} + \boldsymbol{t} \\ -\boldsymbol{D}_{tm} \boldsymbol{t} \end{pmatrix} \Leftrightarrow \quad \boldsymbol{C_t} \Delta \boldsymbol{p} \geq \boldsymbol{d_t} \tag{389}$$

$$\text{return } \boldsymbol{C_t} := \begin{pmatrix} \boldsymbol{I} \\ -\boldsymbol{I} \\ \boldsymbol{D}_{tm} \end{pmatrix} \in \mathbb{R}^{(3N_{tm}-1)\times dim(\boldsymbol{t})}$$

:timing-vector-inequality-constraint-vector *&key (update? t)* [method]

$$\text{return } \boldsymbol{d_t} := \begin{pmatrix} t_s \boldsymbol{1} - \boldsymbol{t} \\ -t_f \boldsymbol{1} + \boldsymbol{t} \\ -\boldsymbol{D}_{tm} \boldsymbol{t} \end{pmatrix} \in \mathbb{R}^{(3N_{tm}-1)}$$

:config-inequality-constraint-matrix *&key (update? nil)* [method]
　　　　　　　　　　　　　　　(update-collision? nil)

$$\begin{cases} \boldsymbol{C_p} \Delta \boldsymbol{p} \geq \boldsymbol{d_p} \\ \boldsymbol{C_t} \Delta \boldsymbol{t} \geq \boldsymbol{d_t} \end{cases} \tag{390}$$

$$\Leftrightarrow \quad \begin{pmatrix} \boldsymbol{C_p} & \\ & \boldsymbol{C_t} \end{pmatrix} \begin{pmatrix} \Delta \boldsymbol{p} \\ \Delta \boldsymbol{t} \end{pmatrix} \geq \begin{pmatrix} \boldsymbol{d_p} \\ \boldsymbol{d_t} \end{pmatrix} \tag{391}$$

$$\Leftrightarrow \quad \boldsymbol{C} \Delta \boldsymbol{q} \geq \boldsymbol{d} \tag{392}$$

$$\text{return } \boldsymbol{C} := \begin{pmatrix} \boldsymbol{C_p} & \\ & \boldsymbol{C_t} \end{pmatrix} \in \mathbb{R}^{N_{ineq}\times dim(\boldsymbol{q})}$$

:config-inequality-constraint-vector *&key (update? t)* [method]
$$(update\text{-}collision?\ nil)$$

$$\text{return } \boldsymbol{d} := \begin{pmatrix} \boldsymbol{d_p} \\ \boldsymbol{d_t} \end{pmatrix} \in \mathbb{R}^{N_{ineq}}$$

:config-equality-constraint-matrix *&key (update? nil)* [method]
return $\boldsymbol{A} \in \mathbb{R}^{0 \times dim(\boldsymbol{q})}$ (no equality constraint)

:config-equality-constraint-vector *&key (update? t)* [method]
return $\boldsymbol{b} \in \mathbb{R}^0$ (no equality constraint)

:square-integration-regular-matrix *&key (diff-order 1)* [method]
$$(delta\text{-}time\ (/\ (\text{-}\ \_finish\text{-}time\ \_start\text{-}time)\ 100.0))$$

(329)

$$F_{sqr,k}(\boldsymbol{p}) = \int_{t_s}^{t_f} \left\| \boldsymbol{\theta}^{(k)}(t) \right\|^2 dt \tag{393}$$

$$= \boldsymbol{p}^T \boldsymbol{H}_{sqr,k} \boldsymbol{p} \tag{394}$$

$$\boldsymbol{H}_{sqr,k} = \int_{t_s}^{t_f} \left(\boldsymbol{B}_{n-k}(t) \hat{\boldsymbol{D}}_k \right)^T \boldsymbol{B}_{n-k}(t) \hat{\boldsymbol{D}}_k dt \tag{395}$$

$$= \int_{t_s}^{t_f} \begin{pmatrix} \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t)\right)\left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t)\right)^T & & \boldsymbol{O} \\ & \ddots & \\ \boldsymbol{O} & & \left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t)\right)\left(\boldsymbol{D}^k \boldsymbol{b}_{n-k}(t)\right)^T \end{pmatrix} dt \tag{396}$$

return $\boldsymbol{H}_{sqr,k} \in \mathbb{R}^{dim(\boldsymbol{p}) \times dim(\boldsymbol{p})}$

:first-differential-square-integration-regular-matrix *&key (delta-time (/ (- _finish-time _start-time) 100.0))* [method]
return $\boldsymbol{H}_{sqr,1} \in \mathbb{R}^{dim(\boldsymbol{p}) \times dim(\boldsymbol{p})}$

:second-differential-square-integration-regular-matrix *&key (delta-time (/ (- _finish-time _start-time) 100.0))* [method]
return $\boldsymbol{H}_{sqr,2} \in \mathbb{R}^{dim(\boldsymbol{p}) \times dim(\boldsymbol{p})}$

:third-differential-square-integration-regular-matrix *&key (delta-time (/ (- _finish-time _start-time) 100.0))* [method]
return $\boldsymbol{H}_{sqr,3} \in \mathbb{R}^{dim(\boldsymbol{p}) \times dim(\boldsymbol{p})}$

:control-vector-regular-matrix [method]

$$\boldsymbol{W}_{reg,p} := \min(k_{max,p}, \|\boldsymbol{e}\|^2 + k_{off,p})\boldsymbol{I} + k_{sqr,1}\boldsymbol{H}_{sqr,1} + k_{sqr,2}\boldsymbol{H}_{sqr,2} + k_{sqr,3}\boldsymbol{H}_{sqr,3} \tag{397}$$

return $\boldsymbol{W}_{reg,p} \in \mathbb{R}^{dim(\boldsymbol{p}) \times dim(\boldsymbol{p})}$

:control-vector-regular-vector [method]

$$\boldsymbol{v}_{reg,p} := (k_{sqr,1}\boldsymbol{H}_{sqr,1} + k_{sqr,2}\boldsymbol{H}_{sqr,2} + k_{sqr,3}\boldsymbol{H}_{sqr,3})\boldsymbol{p} \tag{398}$$

return $\boldsymbol{v}_{reg,p} \in \mathbb{R}^{dim(\boldsymbol{p})}$

:motion-duration-regular-matrix [method]
(337)

$$
\begin{aligned}
F_{duration}(\boldsymbol{t}) &= |t_1 - t_{N_{tm}}|^2 \tag{399}\\
&= \boldsymbol{t}^T \begin{pmatrix} 1 & & -1 \\ & & \\ -1 & & 1 \end{pmatrix} \boldsymbol{t} \tag{400}\\
&= \boldsymbol{t}^T \boldsymbol{H}_{duration}\boldsymbol{t} \tag{401}
\end{aligned}
$$

return $\boldsymbol{H}_{duration} \in \mathbb{R}^{dim(\boldsymbol{t})\times dim(\boldsymbol{t})}$

:timing-vector-regular-matrix [method]

$$\boldsymbol{W}_{reg,t} := \min(k_{max,t}, \|\boldsymbol{e}\|^2 + k_{off,t})\boldsymbol{I} + k_{duration}\boldsymbol{H}_{duration} \tag{402}$$

return $\boldsymbol{W}_{reg,t} \in \mathbb{R}^{dim(\boldsymbol{t})\times dim(\boldsymbol{t})}$

:timing-vector-regular-vector [method]

$$\boldsymbol{v}_{reg,t} := k_{duration}\boldsymbol{H}_{duration}\boldsymbol{t} \tag{403}$$

return $\boldsymbol{v}_{reg,t} \in \mathbb{R}^{dim(\boldsymbol{t})}$

:regular-matrix [method]

$$\boldsymbol{W}_{reg} := \begin{pmatrix} \boldsymbol{W}_{reg,p} & \\ & \boldsymbol{W}_{reg,t} \end{pmatrix} \tag{404}$$

return $\boldsymbol{W}_{reg} \in \mathbb{R}^{dim(\boldsymbol{q})\times dim(\boldsymbol{q})}$

:regular-vector [method]

$$\boldsymbol{v}_{reg} := \begin{pmatrix} \boldsymbol{v}_{reg,p} \\ \boldsymbol{v}_{reg,t} \end{pmatrix} \tag{405}$$

return $\boldsymbol{v}_{reg} \in \mathbb{R}^{dim(\boldsymbol{q})\times dim(\boldsymbol{q})}$

:update-collision-inequality-constraint [method]
Not implemented yet.

:update-viewer *&key (trajectory-delta-time (/ (- _finish-time _start-time) 10.0))* [method]
Update viewer.

:print-status [method]

Print status.

:print-motion-information [method]

Print motion information.

:play-animation *&key (robot)* [method]
$\qquad\qquad$ *(delta-time (/ (- _finish-time _start-time) 100.0))*
$\qquad\qquad$ *(only-motion-duration? t)*

Print motion animation.

:plot-theta-graph *&key (joint-id nil)* [method]
$\qquad\qquad$ *(divide-num 200)*
$\qquad\qquad$ *(plot-numerical? nil)*
$\qquad\qquad$ *(only-motion-duration? t)*
$\qquad\qquad$ *(dat-filename /tmp/bspline-configuration-task-plot-theta-graph.dat)*
$\qquad\qquad$ *(dump-pdf? nil)*
$\qquad\qquad$ *(dump-filename (ros::resolve-ros-path package://eus_qp/optmotiongen/logs/bspline-configu*

Plot graph.

:generate-angle-vector-sequence *&key (divide-num 100)* [method]

Generate angle-vector-sequence.

get-bspline-knot *i n m x_min x_max h* [function]

$$
\begin{aligned}
t_i &= \frac{i-n}{m-n}(t_f - t_s) + t_s & (406)\\
&= hi + \frac{mt_s - nt_f}{m-n} & (407)
\end{aligned}
$$

return knot t_i for B-spline function

bspline-basis-func *x i n m x_min x_max &optional (n-orig n) (m-orig m)* [function]

$$
\begin{aligned}
b_{i,0}(t) &= \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1}\\ 0 & \text{otherwise} \end{cases} & (408)\\
b_{i,n}(t) &= \frac{(t-t_i)b_{i,n-1}(t) + (t_{i+n+1} - t)b_{i+1,n-1}(t)}{nh} & (409)
\end{aligned}
$$

return B-spline function value $b_{i,n}(t)$.

5

5.1

joint [class]

$\qquad\qquad$:super $\quad$ **propertied-object**
$\qquad\qquad$:slots $\quad$ (parent-link)

(child-link)
(joint-angle)
(min-angle)
(max-angle)
(default-coords)
(joint-velocity)
(joint-acceleration)
(joint-torque)
(max-joint-velocity)
(max-joint-torque)
(joint-min-max-table)
(joint-min-max-target)

:child-link *&rest args* [method]

Returns child link of this joint. If any arguments is set, it is passed to the child-link.

Override to support the case that child-link is cascaded-link instantiate. Return the root link of child cascaded-link instantiate in that case.

:axis-vector [method]

Return joint axis vector. Represented in world coordinates.

return $\boldsymbol{a}_i \in \mathbb{R}^3$

:pos [method]

Return joint position. Represented in world coordinates.

return $\boldsymbol{p}_i \in \mathbb{R}^3$

bodyset-link [class]

    :super   **bodyset**
    :slots   (rot)
        (pos)
        (parent)
        (descendants)
        (worldcoords)
        (manager)
        (changed)
        (geometry::bodies)
        (joint)
        (parent-link)
        (child-links)
        (analysis-level)
        (default-coords)
        (weight)
        (acentroid)
        (inertia-tensor)
        (angular-velocity)

(angular-acceleration)

(spacial-velocity)

(spacial-acceleration)

(momentum-velocity)

(angular-momentum-velocity)

(momentum)

(angular-momentum)

(force)

(moment)

(ext-force)

(ext-moment)

:mg [method]

return $mg = \|m\boldsymbol{g}\|$

:mg-vec [method]

return $m\boldsymbol{g}$

cascaded-link [class]

:super	**cascaded-coords**	
:slots	(rot)	
	(pos)	
	(parent)	
	(descendants)	
	(worldcoords)	
	(manager)	
	(changed)	
	(links)	
	(joint-list)	
	(bodies)	
	(collision-avoidance-links)	
	(end-coords-list)	

:calc-jacobian-from-joint-list *&key (union-joint-list)* [method]

(move-target)

(joint-list (mapcar #'(lambda (mt) (send-all (send self :link-list (send mt :pa

(transform-coords (mapcar #'(lambda (mt) (make-coords)) move-target))

(translation-axis (mapcar #'(lambda (mt) t) move-target))

(rotation-axis (mapcar #'(lambda (mt) t) move-target))

union-joint-list list of all joints considered in jacobian. column num of jacobian is same with length of
union-joint-list.

move-target list of move-target.

joint-list list of joint-list which is contained in each chain of move-target.

transform-coords list of transform-coords of each move-target.

translation-axis list of translation-axis of each move-target.

rotation-axis list of rotation-axis of each move-target.

Get jacobian matrix from following two information: (1) union-joint-list and (2) list of move-target. One recession compared with :calc-jacobian-from-link-list is that child-reverse is not supported. (Only not implemented yet because I do not need such feature in current application.)

:calc-cog-jacobian-from-joint-list *&key (union-joint-list)*   [method]
         (update-mass-properties t)
         (translation-axis :z)

union-joint-list list of all joints considered in jacobian. column num of jacobian is same with length of union-joint-list.

Get CoG jacobian matrix from union-joint-list.

:find-link-route *to &optional from*   [method]
  Override to support the case that joint does not exist between links. Change from (send to :parent-link) to (send to :parent).

5.2

2d-planar-contact   [class]

      :super   **cascaded-link**
      :slots   (_contact-coords T_{cnt})
         (_contact-pre-coords $T_{cnt\text{-}pre}$)

:init *&key (name contact)*   [method]
    (contact-pre-offset 100)
  Initialize instance

:contact-coords *&rest args*   [method]
  return $T_{cnt} := \{\boldsymbol{p}_{cnt}, \boldsymbol{R}_{cnt}\}$

:contact-pre-coords *&rest args*   [method]
  return $T_{cnt\text{-}pre} := \{\boldsymbol{p}_{cnt\text{-}pre}, \boldsymbol{R}_{cnt\text{-}pre}\}$

:set-from-face *&key (face)*   [method]
     (margin 150.0)
  set coords and min/max joint angle from face.

robot-environment   [class]

:super	**cascaded-link**
:slots	(_robot $\mathcal{R}$)
	(_robot-with-root-virtual $\hat{\mathcal{R}}$)
	(_root-virtual-joint-list list of root virtual joint)
	(_contact-list $\{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_{N_C}\}$)
	(_variant-joint-list $\mathcal{J}_{var}$)
	(_invariant-joint-list $\mathcal{J}_{invar}$)
	(_drive-joint-list $\mathcal{J}_{drive}$)

1.

2.

robot $\mathcal{R}$	(cascaded-link	)
contact-list $\{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_{N_C}\}$	(2d-planar-contact	)

R $\hat{\mathcal{R}}$

:init *&key (robot)* [method]
   (contact-list)
   (root-virtual-mode :6dof)
   (root-virtual-joint-class-list)
   (root-virtual-joint-axis-list)
   Initialize instance

:dissoc-root-virtual [method]
   dissoc root virtual parent/child structure.

:init-pose [method]
   set zero joint angle.

:robot *&rest args* [method]
   return $\mathcal{R}$

:robot-with-root-virtual *&rest args* [method]
   return $\hat{\mathcal{R}}$

:contact-list *&rest args* [method]
   return $\{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_{N_C}\}$

:contact *name &rest args* [method]
   return $\mathcal{C}_i$

:variant-joint-list *&optional (jl :nil)* [method]
   return $\mathcal{J}_{var}$

:invariant-joint-list *&optional (jl :nil)* [method]
> return $\mathcal{J}_{invar}$

:drive-joint-list *&optional (jl :nil)* [method]
> return $\mathcal{J}_{drive}$

:root-virtual-joint-list [method]
> return list of root virtual joint

5.3 irteus   inverse-kinematics

cascaded-link [class]

:super	**cascaded-coords**
:slots	(rot)
	(pos)
	(parent)
	(descendants)
	(worldcoords)
	(manager)
	(changed)
	(links)
	(joint-list)
	(bodies)
	(collision-avoidance-links)
	(end-coords-list)

:inverse-kinematics-optmotiongen *target-coords &key* *(stop 50)* [method]
(link-list)
(move-target)
(debug-view)
(revert-if-fail t)
(transform-coords target-coords)
(translation-axis (cond ((atom move-target) t) (t (make-li
(rotation-axis (cond ((atom move-target) t) (t (make-list
(thre (cond ((atom move-target) 1) (t (make-list (length n
(rthre (cond ((atom move-target) (deg2rad 1)) (t (make-li
(collision-avoidance-link-pair :nil)
(collision-distance-limit 10.0)
(obstacles)
(min-loop)
(root-virtual-mode :fix)
(joint-angle-margin 0.0)
(posture-joint-list)

(posture-joint-angle-list)
(target-posture-scale 0.001)
(pre-process-func)
(post-process-func)
&allow-other-keys

Solve inverse kinematics problem with sqp optimization. ;; target-coords, move-target, rotation-axis, translation-axis ;; -¿ both list and atom OK. target-coords : The coordinate of the target that returns coordinates. Use a list of targets to solve the IK relative to multiple end links simultaneously. Function is not available to target-coords. link-list : List of links to control. When the target-coords is list, this should be a list of lists. move-target : Specify end-effector coordinate. When the target-coords is list, this should be list too. stop : Maximum number for IK iteration. Default is 50. debug-view : Set t to show debug message and visualization. Use :no-message to just show the irtview image. Default is nil. revert-if-fail : Set nil to keep the angle posture of IK solve iteration. Default is t, which return to original position when IK fails. translation-axis : :x :y :z for constraint along the x, y, z axis. :xy :yz :zx for plane. Default is t. rotation-axis : Use nil for position only IK. :x, :y, :z for the constraint around axis with plus direction. When the target-coords is list, this should be list too. Default is t. thre : Threshold for position error to terminate IK iteration. Default is 1 [mm]. rthre : Threshold for rotation error to terminate IK iteration. Default is 0.017453 [rad] (1 deg).

cascaded-link [class]

:super	**cascaded-coords**	
:slots	(rot)	
	(pos)	
	(parent)	
	(descendants)	
	(worldcoords)	
	(manager)	
	(changed)	
	(links)	
	(joint-list)	
	(bodies)	
	(collision-avoidance-links)	
	(end-coords-list)	

:inverse-kinematics-trajectory-optmotiongen *target-coords-list &key* *(stop 50)* [method]
(move-target-list)
(debug-view)
(revert-if-fail t)
(transform-coords-list :nil)
(translation-axis-list :nil)
(rotation-axis-list :nil)
(thre 1.0)
(rthre (deg2rad 1))
(thre-list :nil)

(rthre-list :nil)

(collision-avoidance-link-pair :nil)

(collision-distance-limit 10.0)

(obstacles)

(min-loop)

(root-virtual-mode :fix)

(root-virtual-joint-invariant? nil)

(joint-angle-margin 0.0)

(posture-joint-list (make-list (length targe

(posture-joint-angle-list (make-list (length

(adjacent-regular-scale 0.0)

(pre-process-func)

(post-process-func)

&allow-other-keys

Solve inverse kinematics problem with sqp optimization. target-coords-list : The coordinate of the target that returns coordinates. Use a list of targets to solve the IK relative to multiple end links simultaneously. Function is not available to target-coords. move-target-list : Specify end-effector coordinate. When the target-coords is list, this should be list too. stop : Maximum number for IK iteration. Default is 50. debug-view : Set t to show debug message and visualization. Use :no-message to just show the irtview image. Default is nil. revert-if-fail : Set nil to keep the angle posture of IK solve iteration. Default is t, which return to original position when IK fails. translation-axis-list : :x :y :z for constraint along the x, y, z axis. :xy :yz :zx for plane. Default is t. rotation-axis-list : Use nil for position only IK. :x, :y, :z for the constraint around axis with plus direction. When the target-coords is list, this should be list too. Default is t. thre : Threshold for position error to terminate IK iteration. Default is 1 [mm]. rthre : Threshold for rotation error to terminate IK iteration. Default is 0.017453 [rad] (1 deg).

robot-model [class]

:super	**cascaded-link**	
:slots	(rot)	
	(pos)	
	(parent)	
	(descendants)	
	(worldcoords)	
	(manager)	
	(changed)	
	(links)	
	(joint-list)	
	(bodies)	
	(collision-avoidance-links)	
	(end-coords-list)	
	(larm-end-coords)	
	(rarm-end-coords)	
	(lleg-end-coords)	
	(rleg-end-coords)	
	(head-end-coords)	

<div align="center">

(torso-end-coords)

(larm-root-link)

(rarm-root-link)

(lleg-root-link)

(rleg-root-link)

(head-root-link)

(torso-root-link)

(larm-collision-avoidance-links)

(rarm-collision-avoidance-links)

(larm)

(rarm)

(lleg)

(rleg)

(torso)

(head)

(force-sensors)

(imu-sensors)

(cameras)

(support-polygons)

</div>

:limb *limb method &rest args* [method]

   Extend to support to call :inverse-kinematics-optmotiongen.

contact-ik-arg [class]

   :super   **cascaded-link**

   :slots   (_contact-coords T_{cnt})

   inverse-kinematics-optmotiongen   *target-coords, translation-axis, rotation-axis, transform-coords*

:init *&key (target-coords)* [method]

      (translation-axis)

      (rotation-axis)

      (transform-coords)

   Initialize instance

:contact-coords *&rest args* [method]

   return $T_{cnt} := \{\boldsymbol{p}_{cnt}, \boldsymbol{R}_{cnt}\}$

ik-arg-axis->axis-list *ik-arg-axis* [function]

   Convert translation-axis / rotatoin-axis to axis list.

generate-contact-ik-arg-from-rect-face *&key (rect-face)* [function]

                    (margin (or (send rect-face :get :margin) 0))

   Generate contact-ik-arg instance from rectangle face.

generate-contact-ik-arg-from-line-segment *&key (line-seg)*   [function]
$$(margin\ (or\ (send\ line\text{-}seg\ :get\ :margin)\ 0))$$

    Generate contact-ik-arg instance from line segment.

axis->index *axis*   [function]
axis->sgn *axis*   [function]

5.4

get-link-jacobian-for-contact-torque *&key (robot)*   [function]
$$(drive\text{-}joint\text{-}list)$$
$$(contact\text{-}coords)$$
$$(contact\text{-}parent\text{-}link)$$

contact-coords m *contact-coords* $\boldsymbol{r}_m \in \mathbb{R}^6$ *drive-joint-list* $\boldsymbol{\psi} \in \mathbb{R}^{N_{drive\text{-}joint}}$ $\boldsymbol{J}_m$

$$\dot{\boldsymbol{r}}_m \;=\; \boldsymbol{J}_m \dot{\boldsymbol{\psi}} \tag{410}$$

$$=\; \sum_{i=1}^{N_{drive\text{-}joint}} \boldsymbol{j}_m^{(i)} \dot{\psi}_i \tag{411}$$

$$\boldsymbol{j}_m^{(i)} \;=\; \begin{pmatrix} \boldsymbol{a}_{\psi_i} \times (\boldsymbol{p}_m - \boldsymbol{p}_{\psi_i}) \\ \boldsymbol{a}_{\psi_i} \end{pmatrix} \tag{412}$$

$$\boldsymbol{J}_m \;=\; \begin{pmatrix} \boldsymbol{j}_m^{(1)} & \boldsymbol{j}_m^{(2)} & \cdots & \boldsymbol{j}_m^{N_{drive\text{-}joint}} \end{pmatrix} \tag{413}$$

$\boldsymbol{a}_{\psi_i}, \boldsymbol{p}_{\psi_i} \in \mathbb{R}^3$ i

$return\ \boldsymbol{J}_m \in \mathbb{R}^{6 \times N_{drive\text{-}joint}}$

get-contact-torque *&key (robot)*   [function]
$$(drive\text{-}joint\text{-}list)$$
$$(wrench\text{-}list)$$
$$(contact\text{-}target\text{-}coords\text{-}list)$$
$$(contact\text{-}attention\text{-}coords\text{-}list)$$

$$\boldsymbol{\tau}^{cnt}$$

$$\boldsymbol{\tau}^{cnt} = \sum_{m=1}^{N_{cnt}} \boldsymbol{J}_m^T \boldsymbol{w}_m \tag{414}$$

$\boldsymbol{w}_m$ m

$return\ \boldsymbol{\tau}^{cnt} \in \mathbb{R}^{N_{drive\text{-}joint}}$

get-contact-torque-jacobian *&key (robot)*   [function]
$$(joint\text{-}list)$$
$$(drive\text{-}joint\text{-}list)$$
$$(wrench\text{-}list)$$
$$(contact\text{-}target\text{-}coords\text{-}list)$$
$$(contact\text{-}attention\text{-}coords\text{-}list)$$

(413) $\quad \boldsymbol{J}_m$

$$\boldsymbol{J}_m = \begin{pmatrix} \boldsymbol{j}_{x,m}^T \\ \boldsymbol{j}_{y,m}^T \\ \boldsymbol{j}_{z,m}^T \\ \boldsymbol{j}_{R,m}^T \\ \boldsymbol{j}_{P,m}^T \\ \boldsymbol{j}_{Y,m}^T \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial r_{x,m}}{\partial \boldsymbol{\psi}}\right)^T \\ \left(\frac{\partial r_{y,m}}{\partial \boldsymbol{\psi}}\right)^T \\ \left(\frac{\partial r_{z,m}}{\partial \boldsymbol{\psi}}\right)^T \\ \left(\frac{\partial r_{R,m}}{\partial \boldsymbol{\psi}}\right)^T \\ \left(\frac{\partial r_{P,m}}{\partial \boldsymbol{\psi}}\right)^T \\ \left(\frac{\partial r_{Y,m}}{\partial \boldsymbol{\psi}}\right)^T \end{pmatrix} \tag{415}$$

(414)

$$\boldsymbol{\tau}^{cnt} = \sum_{m=1}^{N_{cnt}} \boldsymbol{J}_m^T \boldsymbol{w}_m \tag{416}$$

$$= \sum_{m=1}^{N_{cnt}} \begin{pmatrix} \boldsymbol{j}_{x,m} & \boldsymbol{j}_{y,m} & \boldsymbol{j}_{z,m} & \boldsymbol{j}_{R,m} & \boldsymbol{j}_{P,m} & \boldsymbol{j}_{Y,m} \end{pmatrix} \begin{pmatrix} f_{x,m} \\ f_{y,m} \\ f_{z,m} \\ n_{x,m} \\ n_{y,m} \\ n_{z,m} \end{pmatrix} \tag{417}$$

$$= \sum_{m=1}^{N_{cnt}} \left(\boldsymbol{j}_{x,m} f_{x,m} + \boldsymbol{j}_{y,m} f_{y,m} + \boldsymbol{j}_{z,m} f_{z,m} + \boldsymbol{j}_{R,m} n_{x,m} + \boldsymbol{j}_{P,m} n_{y,m} + \boldsymbol{j}_{Y,m} n_{z,m} \right) \tag{418}$$

$$= \sum_{m=1}^{N_{cnt}} \left(\frac{\partial r_{x,m}}{\partial \boldsymbol{\psi}} f_{x,m} + \frac{\partial r_{y,m}}{\partial \boldsymbol{\psi}} f_{y,m} + \frac{\partial r_{z,m}}{\partial \boldsymbol{\psi}} f_{z,m} + \frac{\partial r_{R,m}}{\partial \boldsymbol{\psi}} n_{x,m} + \frac{\partial r_{P,m}}{\partial \boldsymbol{\psi}} n_{y,m} + \frac{\partial r_{Y,m}}{\partial \boldsymbol{\psi}} n_{z,m} \right) \tag{419}$$

$joint\text{-}list \qquad\qquad \boldsymbol{\theta} \in \mathbb{R}^{N_{joint}} \quad drive\text{-}joint\text{-}list \qquad\qquad \boldsymbol{\psi} \in \mathbb{R}^{N_{drive\text{-}joint}}$

$\frac{\partial \boldsymbol{\tau}^{cnt}}{\partial \boldsymbol{\theta}}$

$$\frac{\partial \boldsymbol{\tau}^{cnt}}{\partial \boldsymbol{\theta}} = \sum_{m=1}^{N_{cnt}} \left(\frac{\partial \boldsymbol{J}_m}{\partial \boldsymbol{\theta}} \right)^T \boldsymbol{w}_m \tag{420}$$

$$= \sum_{m=1}^{N_{cnt}} \left(f_{x,m} \frac{\partial^2 r_{x,m}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} + f_{y,m} \frac{\partial^2 r_{y,m}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} + f_{z,m} \frac{\partial^2 r_{z,m}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} + n_{x,m} \frac{\partial^2 r_{R,m}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} + n_{y,m} \frac{\partial^2 r_{P,m}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} + n_{z,m} \frac{\partial^2 r_{Y,m}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} \right) \tag{421}$$

$$= \left[\sum_{m=1}^{M} \begin{pmatrix} f_{x,m} & f_{y,m} & f_{z,m} & n_{x,m} & n_{y,m} & n_{z,m} \end{pmatrix} \begin{pmatrix} \frac{\partial^2 r_{x,m}}{\partial \psi_i \partial \theta_j} \\ \frac{\partial^2 r_{y,m}}{\partial \psi_i \partial \theta_j} \\ \frac{\partial^2 r_{z,m}}{\partial \psi_i \partial \theta_j} \\ \frac{\partial^2 r_{R,m}}{\partial \psi_i \partial \theta_j} \\ \frac{\partial^2 r_{P,m}}{\partial \psi_i \partial \theta_j} \\ \frac{\partial^2 r_{Y,m}}{\partial \psi_i \partial \theta_j} \end{pmatrix} \right]_{i=1,\cdots,N_{drive\text{-}joint}, j=1,\cdots,N_{joint}} \tag{422}$$

$return \ \frac{\partial \boldsymbol{\tau}^{cnt}}{\partial \boldsymbol{\theta}} \in \mathbb{R}^{N_{drive\text{-}joint} \times N_{joint}}$

get-link-jacobian-for-gravity-torque *&key* *(robot)* [function]
(drive-joint-list)
(gravity-link)

gravity-link k gravity-link $\boldsymbol{p}_{cog,k} \in \mathbb{R}^3$ drive-joint-list $\boldsymbol{\psi} \in \mathbb{R}^{N_{drive\text{-}joint}}$ $\boldsymbol{J}_{cog,k}$

$$\dot{\boldsymbol{p}}_{cog,k} = \boldsymbol{J}_{cog,k}\dot{\boldsymbol{\psi}} \tag{423}$$

$$= \sum_{i=1}^{N_k} \boldsymbol{j}_{cog,k}^{(i)}\dot{\psi}_i \tag{424}$$

$$\boldsymbol{j}_{cog,k}^{(i)} = \begin{cases} \boldsymbol{a}_{\psi_i} \times (\boldsymbol{p}_{cog,k} - \boldsymbol{p}_{\psi_i}) & (1 \le i \le N_k) \\ \boldsymbol{0}_3 & (N_k + 1 \le i \le N_{drive\text{-}joint}) \end{cases} \tag{425}$$

$\boldsymbol{a}_{\psi_i}, \boldsymbol{p}_{\psi_i} \in \mathbb{R}^3$ i gravity-link
$1, \cdots, N_k$ gravity-link $N_k + 1, \cdots, N_{drive\text{-}joint}$

$$\boldsymbol{J}_{cog,k} = \begin{pmatrix} \boldsymbol{j}_{cog,k}^{(1)} & \cdots & \boldsymbol{j}_{cog,k}^{(N_k)} & \boldsymbol{j}_{cog,k}^{(N_k+1)} & \cdots & \boldsymbol{j}_{cog,k}^{N_{drive\text{-}joint}} \end{pmatrix} \tag{426}$$

$$= \begin{pmatrix} \boldsymbol{j}_{cog,k}^{(1)} & \cdots & \boldsymbol{j}_{cog,k}^{(N_k)} & \boldsymbol{0} & \cdots & \boldsymbol{0} \end{pmatrix} \tag{427}$$

return $\boldsymbol{J}_{cog,k} \in \mathbb{R}^{3 \times N_{drive\text{-}joint}}$

get-gravity-torque &key (robot) [function]
(drive-joint-list)
(gravity-link-list)

$\boldsymbol{\tau}^{grav}$ P111 (3.3.22)

$$\boldsymbol{\tau}^{grav} = \left(\sum_{k=1}^{N_{gravity\text{-}link}} m_k \boldsymbol{J}_{cog,k}^T \right) \boldsymbol{g} \tag{428}$$

m_k k

return $\boldsymbol{\tau}^{grav} \in \mathbb{R}^{N_{drive\text{-}joint}}$

get-gravity-torque-jacobian &key (robot) [function]
(joint-list)
(drive-joint-list)
(gravity-link-list)

(427) $\boldsymbol{J}_{cog,k}$

$$\boldsymbol{J}_{cog,k} = \begin{pmatrix} \boldsymbol{j}_{cog,x,k}^T \\ \boldsymbol{j}_{cog,y,k}^T \\ \boldsymbol{j}_{cog,z,k}^T \end{pmatrix} = \begin{pmatrix} \left(\frac{\partial p_{cog,x,k}}{\partial \boldsymbol{\psi}}\right)^T \\ \left(\frac{\partial p_{cog,y,k}}{\partial \boldsymbol{\psi}}\right)^T \\ \left(\frac{\partial p_{cog,z,k}}{\partial \boldsymbol{\psi}}\right)^T \end{pmatrix} \tag{429}$$

$$(428)$$

$$\boldsymbol{\tau}^{grav} \quad = \quad \left(\sum_{k=1}^{N_{gravity\text{-}link}} m_k \boldsymbol{J}_{cog,k}^{T} \right) \boldsymbol{g} \tag{430}$$

$$= \quad \sum_{k=1}^{N_{gravity\text{-}link}} m_k \begin{pmatrix} \boldsymbol{j}_{cog,x,k} & \boldsymbol{j}_{cog,y,k} & \boldsymbol{j}_{cog,z,k} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix} \tag{431}$$

$$= \quad \sum_{k=1}^{N_{gravity\text{-}link}} m_k g \boldsymbol{j}_{cog,z,k} \tag{432}$$

$$= \quad \sum_{k=1}^{N_{gravity\text{-}link}} m_k g \frac{\partial p_{cog,z,k}}{\partial \boldsymbol{\psi}} \tag{433}$$

joint-list $\qquad\qquad \boldsymbol{\theta} \in \mathbb{R}^{N_{joint}} \quad$ *drive-joint-list* $\qquad\qquad \boldsymbol{\psi} \in \mathbb{R}^{N_{drive\text{-}joint}}$

$\frac{\partial \boldsymbol{\tau}^{grav}}{\partial \boldsymbol{\theta}}$

$$\frac{\partial \boldsymbol{\tau}^{grav}}{\partial \boldsymbol{\theta}} \quad = \quad \sum_{k=1}^{N_{gravity\text{-}link}} m_k g \frac{\partial^2 p_{cog,z,k}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} \tag{434}$$

$$\frac{\partial^2 p_{cog,z,k}}{\partial \boldsymbol{\psi} \partial \boldsymbol{\theta}} \quad = \quad \left[\frac{\partial^2 p_{cog,z,k}}{\partial \psi_i \partial \theta_j} \right]_{i=1,\cdots,N_{drive\text{-}joint},j=1,\cdots,N_{joint}} \tag{435}$$

$$\frac{\partial \boldsymbol{\tau}^{grav}}{\partial \boldsymbol{\theta}} \quad = \quad \left[\sum_{k=1}^{N_{gravity\text{-}link}} m_k g \frac{\partial^2 p_{cog,z,k}}{\partial \psi_i \partial \theta_j} \right]_{i=1,\cdots,N_{drive\text{-}joint},j=1,\cdots,N_{joint}} \tag{436}$$

return $\frac{\partial \boldsymbol{\tau}^{grav}}{\partial \boldsymbol{\theta}} \in \mathbb{R}^{N_{drive\text{-}joint} \times N_{joint}}$