

### 3.5.4 一般DMC信道容量解的充要条件

定理(一般DMC信道容量解的充要条件): 一般DMC $\{X, P_{Y|X}, Y\}$ , 其平均互信息量 $I(X;Y)$ 在输入分布为 $P_X^* = \{P^*(a_1), P^*(a_2), \dots, P^*(a_r)\}$ 时取最大值的充要条件是

$$I(a_i; Y) \big|_{P_X = P_X^*} = C \quad \text{当 } P^*(a_i) > 0 \text{ 时}$$

$$I(a_i; Y) \big|_{P_X = P_X^*} \leq C \quad \text{当 } P^*(a_i) = 0 \text{ 时}$$

式中:

$$I(a_i; Y) = \sum_{j=1}^s P(b_j | a_i) \log \frac{P(b_j | a_i)}{P(b_j)}$$

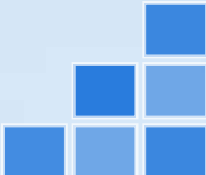
$$\begin{aligned} I(X; Y) &= \sum_{i=1}^r \sum_{j=1}^s P(a_i) P(b_j | a_i) \log \frac{P(b_j | a_i)}{P(b_j)} \\ &= \sum_{i=1}^r P(a_i) \sum_{j=1}^s P(b_j | a_i) \log \frac{P(b_j | a_i)}{P(b_j)} \\ &= \sum_{i=1}^r P(a_i) I(a_i; Y) \end{aligned}$$

例：设信道转移矩阵如下，求新的容量和最佳输入分布。

$$[P_{Y|X}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-\delta & \delta \\ 0 & \delta & 1-\delta \end{bmatrix}$$

解： 
$$I(a_i; Y) = \sum_{j=1}^s P(b_j | a_i) \log \frac{P(b_j | a_i)}{P(b_j)} = C$$

$$\begin{cases} I(a_1; Y) = \log \frac{1}{P(b_1)} = C \\ I(a_2; Y) = (1-\delta) \log \frac{1-\delta}{P(b_2)} + \delta \log \frac{\delta}{P(b_3)} = C \\ I(a_3; Y) = \delta \log \frac{\delta}{P(b_2)} + (1-\delta) \log \frac{1-\delta}{P(b_3)} = C \\ P(b_1) + P(b_2) + P(b_3) = 1 \end{cases} \Rightarrow \begin{cases} C = \log(1 + 2^{[1-h_2(\delta)]}) = \log[1 + 2\delta^\delta(1-\delta)^{(1-\delta)}] \\ P(b_1) = 2^{-C} = \frac{1}{1 + 2\delta^\delta(1-\delta)^{(1-\delta)}} \\ P(b_2) = P(b_3) = 2^{-[C+h_2(\delta)]} = \frac{\delta^\delta(1-\delta)^{(1-\delta)}}{1 + 2\delta^\delta(1-\delta)^{(1-\delta)}} \end{cases}$$


$$[P_{Y|X}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-\delta & \delta \\ 0 & \delta & 1-\delta \end{bmatrix}$$

设最佳输入分布为  $P_X^* = \{P^*(a_1), P^*(a_2), P^*(a_3)\}$

则

$$\begin{cases} P(b_1) = P^*(a_1) \\ P(b_2) = (1-\delta)P^*(a_2) + \delta P^*(a_3) \\ P(b_3) = \delta P^*(a_2) + (1-\delta)P^*(a_3) \end{cases}$$



$$\begin{cases} P^*(a_1) = \frac{1}{1 + 2\delta^\delta(1-\delta)^{(1-\delta)}} \\ P^*(a_2) = P^*(a_3) = \frac{\delta^\delta(1-\delta)^{(1-\delta)}}{1 + 2\delta^\delta(1-\delta)^{(1-\delta)}} \end{cases}$$