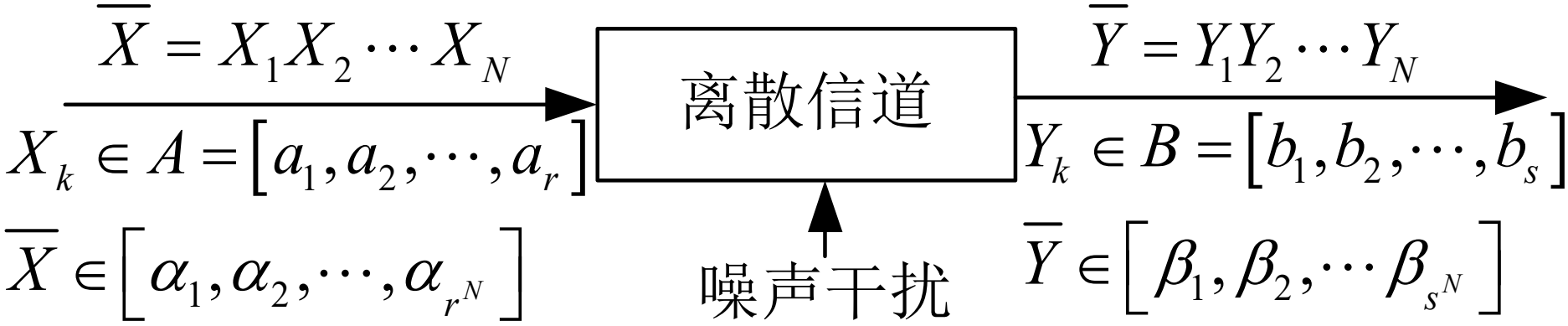
3.6 扩展信道及其信道容量

3.6.1 扩展信道的数学模型



N次扩展信道的模型↑

把输入$\overline{X}$(也记为X^N)和输出$\overline{Y}$(也记为Y^N)

都分别当作一个新的随机变量——

联合随机变量，它们的取值集合分别为A^N和B^N：

$$\overline{\mathrm{X}} \in A^N = \left\{\alpha_1, \alpha_2, \cdots, \alpha_{r^N}\right\}$$

$$\alpha_h = \left(a_{h1}, a_{h2}, \cdots a_{hN}\right) \quad a_{hi} \in A = \left\{a_1, a_2, \cdots, a_r\right\}$$

$$\overline{Y} \in B^N = \left\{\beta_1, \beta_2, \cdots, \beta_{s^N}\right\}$$

$$\beta_l = \left(b_{l1}, b_{l2}, \cdots, b_{lN}\right) \quad b_{lj} \in B = \left\{b_1, b_2, \cdots b_s\right\}$$

转移概率集合为

$$P_{\overline{Y}|\overline{X}} = \left\{P(\beta_l \mid \alpha_h) \mid h = 1, 2, \cdots, r^N; l = 1, 2, \cdots, s^N\right\}$$

数学模型可记为： $\left\{\overline{X}, P_{\overline{Y}|\overline{X}}, \overline{Y}\right\}$

信道是**DMC**的充要条件：

$$P(\beta_l \mid \alpha_h) = P(b_{l1}b_{l2}\cdots b_{lN} \mid a_{h1}a_{h2}\cdots a_{hN})$$

$$= \prod_{k=1}^{N} P(b_{lk} \mid a_{hk}) \quad \text{对于任意N均成立。}$$

例3.11 求**BSC**的**2**次扩展信道数学模型

解：单符号**BSC**的输入和输出符号集分别为：

$$A=\{a_1,a_2\};\quad B=\{b_1,b_2\}$$

2次扩展信道的输入和输出符号集分别为

$$A^2=\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\}=\{a_1a_1,a_1a_2,a_2a_1,a_2a_2\}$$

$$B^2=\{\beta_1,\beta_2,\beta_3,\beta_4\}=\{b_1b_1,b_1b_2,b_2b_1,b_2b_2\}$$

计算转移概率，

$$P(\beta_1\mid\alpha_1)=P(b_1b_1\mid a_1a_1)$$

$$=P(b_1\mid a_1)P(b_1\mid a_1)=\overline{p}^{2}$$

$$P(\beta_2\mid\alpha_1)=P(b_1b_2\mid a_1a_1)$$

$$=P(b_1\mid a_1)P(b_2\mid a_1)=\overline{p}\,p$$

$$P(\beta_4 \mid \alpha_1) = P(b_2 b_2 \mid a_1 a_1)$$
$$= P(b_2 \mid a_1) P(b_2 \mid a_1) = p^2$$
$$\vdots \qquad \vdots \qquad \vdots$$

可以得到2次扩展信道的转移概率矩阵：

$$\left[P_{\overline{Y}|\overline{X}} \right] = \begin{bmatrix} \overline{p}^2 & \overline{p}\,p & \overline{p}\,p & p^2 \\ \overline{p}\,p & \overline{p}^2 & p^2 & \overline{p}\,p \\ \overline{p}\,p & p^2 & \overline{p}^2 & \overline{p}\,p \\ p^2 & \overline{p}\,p & \overline{p}\,p & \overline{p}^2 \end{bmatrix}$$