

互信息量及其性质

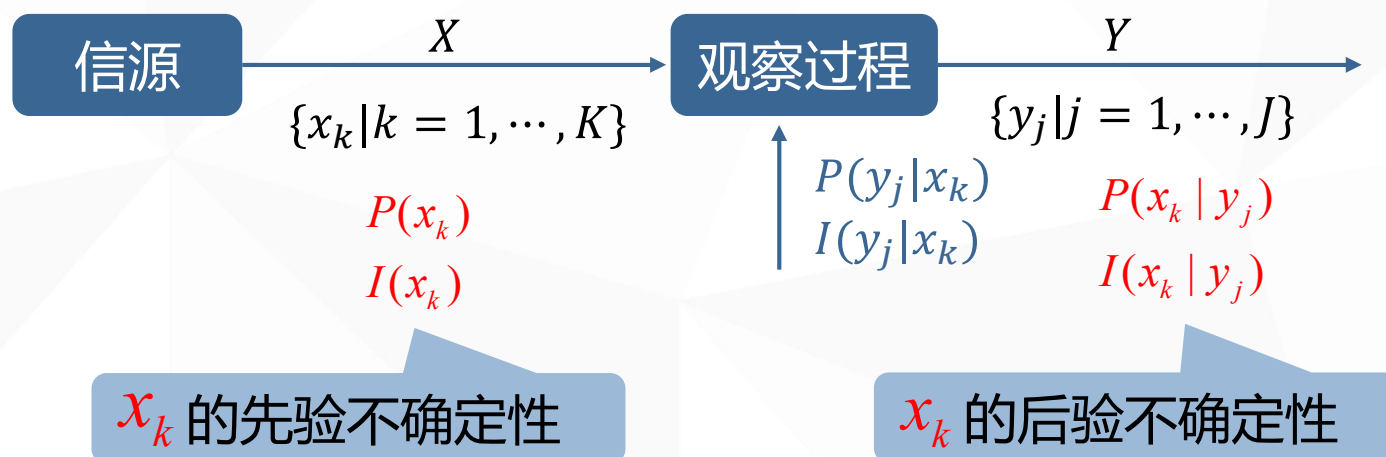
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»» 信息的度量方法



$$\begin{aligned} & \text{从 } y_j \text{ 中获得的关于 } x_k \text{ 的信息 } I(x_k; y_j) \\ &= x_k \text{ 的先验不确定性} - x_k \text{ 的后验不确定性} \\ &= I(x_k) - I(x_k|y_j) \end{aligned}$$

»» 互信息量的概率表达式

$I(x_k; y_j)$: 互信息量, 事件信息

$$\begin{aligned} I(x_k; y_j) &= I(x_k) - I(x_k|y_j) = [-\log P(x_k)] - [-\log P(x_k|y_j)] \\ &= \log \frac{P(x_k|y_j)}{P(x_k)} = \log \frac{P(x_k, y_j)}{P(x_k)P(y_j)} \end{aligned}$$



$$\begin{aligned} I(y_j; x_k) &= I(y_j) - I(y_j|x_k) = [-\log P(y_j)] - [-\log P(y_j|x_k)] \\ &= \log \frac{P(y_j|x_k)}{P(y_j)} = \log \frac{P(x_k, y_j)}{P(x_k)P(y_j)} \end{aligned}$$

»» 实在信息

$$I(x_k; y_j) = I(x_k) - I(x_k|y_j)$$

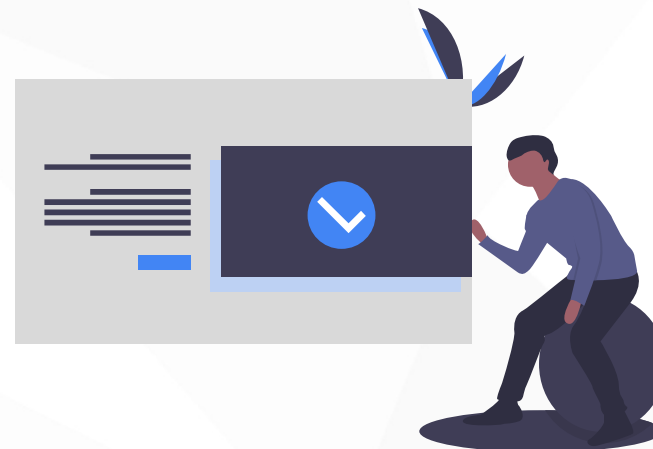
$$I(x_k|y_j) = 0 \Rightarrow I(x_k; y_j) = I(x_k) - 0 = I(x_k)$$

从 y_j 中得到了 x_k 的全部信息



x_k 含有的实在信息

在数值上等于 $I(x_k)$



»» 实在信息



例题 甲在一 8×8 的方格棋盘上随意放入一个棋子，在乙看来棋子落入的位置是不确定的。

- (1) 若甲告知乙棋子落入方格的行号，这时乙得到了多少信息量？
- (2) 若甲将棋子落入方格的行号和列号都告知乙，这时乙得到了多少信息量？

解 棋格按顺序编号 $Z = \{z_l | l = 1, 2, \dots, 64\}$

棋格行号 $X = \{x_k | k = 1, 2, \dots, 8\}$

棋格列号 $Y = \{y_j | j = 1, 2, \dots, 8\}$

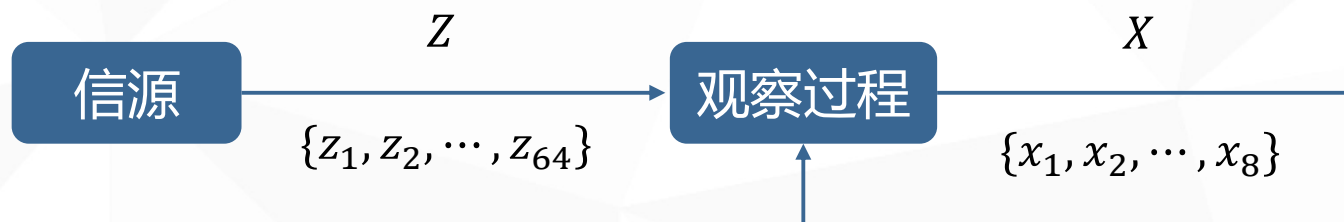
$$P(z_l) = \frac{1}{64} \quad l = 1, \dots, 64$$

$$\left. \begin{array}{l} P(z_l | x_k) = \frac{1}{8} \\ P(z_l | x_k y_j) = 1 \end{array} \right\} \quad l = 1, \dots, 64; \quad k = 1, \dots, 8; \quad j = 1, \dots, 8$$

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例题 (1) 告知行号, 乙得到的信息量:



$$\begin{aligned} I(z_l; x_k) &= I(z_l) - I(z_l|x_k) = -\log P(z_l) - [-\log P(z_l|x_k)] \\ &= -\log \frac{1}{64} - [-\log \frac{1}{8}] \\ &= 6 - 3 = 3 \quad \text{bit/符号} \end{aligned}$$

»» 实在信息



例题 (2) 既告知行号又告知列号, 乙得到的信息量:



$$\begin{aligned} I(z_l; x_k y_j) &= I(z_l) - I(z_l | x_k y_j) = -\log P(z_l) - [-\log P(z_l | x_k y_j)] \\ &= -\log \frac{1}{64} - [-\log 1] \\ &= 6 - 0 = 6 \quad \text{bit/符号} \end{aligned}$$

»» 互信息量的性质

1 **互易性:** $I(x_k; y_j) = I(y_j; x_k)$

证明:

$$\begin{aligned} I(y_j; x_k) &= I(y_j) - I(y_j|x_k) = \log \frac{P(y_j|x_k)}{P(y_j)} \\ &= \log \frac{P(x_k, y_j)}{P(x_k)P(y_j)} = \log \frac{P(x_k|y_j)}{P(x_k)} = I(x_k; y_j) \end{aligned}$$

2 **独立变量的互信息量为0: 若 x_k 、 y_j 相互独立, 则**

$$I(x_k; y_j) = I(y_j; x_k) = 0$$

$$I(x_k; y_j) = I(x_k) - I(x_k|y_j) = I(x_k) - I(x_k) = 0$$

$$I(y_j; x_k) = I(y_j) - I(y_j|x_k) = I(y_j) - I(y_j) = 0$$

»» 互信息量的性质

3 互信息量可正可负

$$I(x_k; y_j) = I(x_k) - I(x_k|y_j)$$

- ◆ 若为正值，通过接收 y_j 判断是否发送 x_k 的不确定性变小，能够正常通信；
- ◆ 若为负值，意味着传输中的问题，如信道噪声、干扰等，收到 y 判断是否发送 x_k 的不确定性更大。

4 互信息量不可能大于符号的实在信息

$$I(x_k; y_j) = I(y_j; x_k) \leq \begin{cases} I(x_k) \\ I(y_j) \end{cases}$$

»» 条件互信息量

记三元联合概率空间为

$$[XYZ, P_{XYZ}] = [(x_k, y_j, z_l), P(x_k, y_j, z_l) | k \in I_X, j \in I_Y, l \in I_Z]$$

在 z_l 出现的条件之下, x_k 与 y_j 之间的互信息量为

$$I(x_k; y_j | z_l) = I(x_k | z_l) - I(x_k | y_j z_l)$$

$$\begin{aligned} I(x_k; y_j | z_l) &= -\log P(x_k | z_l) + \log P(x_k | y_j z_l) \\ &= \log \frac{P(x_k | y_j z_l)}{P(x_k | z_l)} = \log \frac{P[(x_k, y_j) | z_l]}{P(x_k | z_l) P(y_j | z_l)} \\ &= I(y_j | z_l) - I(y_j | x_k z_l) \\ &= I(y_j; x_k | z_l) \end{aligned}$$

»» 条件互信息量

x_k 与 $(y_j z_l)$ 之间的互信息量为:

$$I(x_k; y_j z_l) = I(x_k; y_j) + I(x_k; z_l | y_j) \quad (\text{可加性})$$

证明:

$$\begin{aligned} I(x_k; y_j z_l) &= \log \frac{P(x_k | y_j z_l)}{P(x_k)} = \log \frac{P(x_k | y_j z_l) P(x_k | y_j)}{P(x_k) P(x_k | y_j)} \\ &= \log \frac{P(x_k | y_j)}{P(x_k)} + \log \frac{P(x_k | y_j z_l)}{P(x_k | y_j)} \\ &= I(x_k; y_j) + I(x_k; z_l | y_j) \end{aligned}$$

A background network diagram consisting of numerous nodes (dots) connected by thin lines, forming a complex web. The nodes are colored in shades of blue and grey, and the lines are thin and light blue. The overall shape of the network is roughly triangular, with the base at the bottom and the apex at the top.

感谢观看！

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