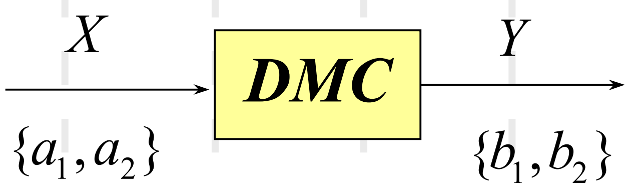
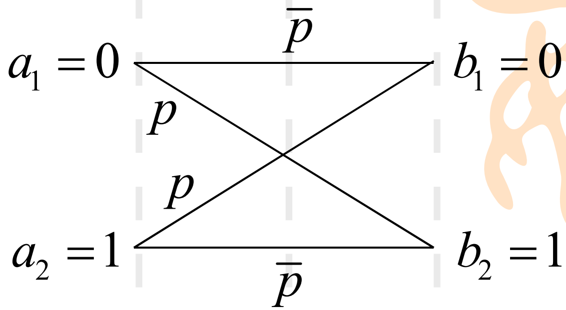
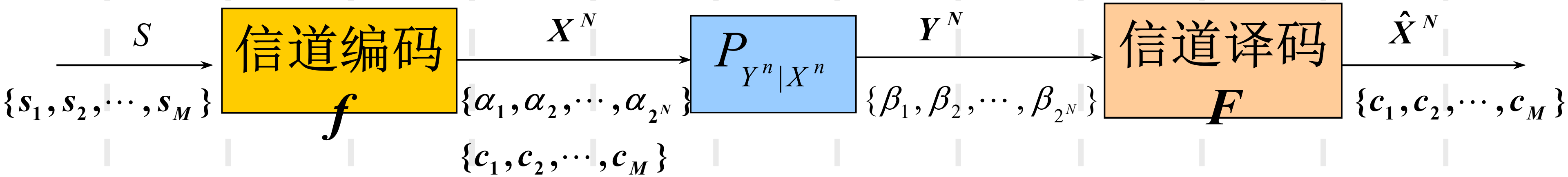
4、最小（汉明）距离译码规则





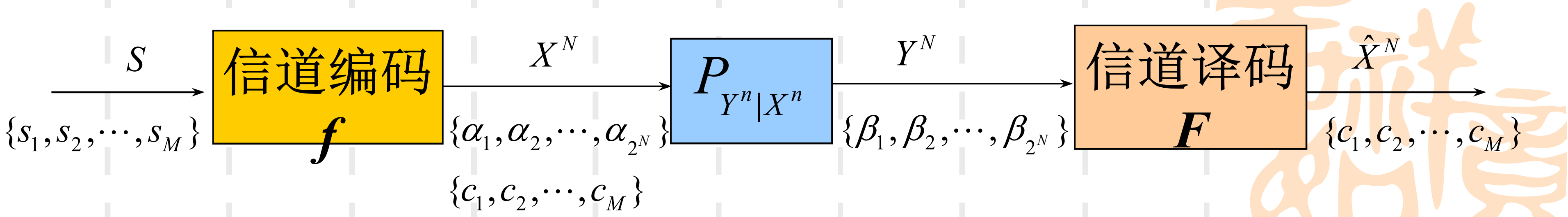


$$\{s_1,s_2,\cdots,s_M\}\xrightarrow{f}\{c_1,c_2,\cdots,c_M\}\xRightarrow{N\text{ 次扩展信道}}\{\beta_1,\beta_2,\cdots,\beta_{2^N}\}\xrightarrow{F}\{c_1,c_2,\cdots,c_M\}$$

极大似然译码规则：

$$\boldsymbol{F}:\begin{cases}\boldsymbol{F}(\beta_j)=\boldsymbol{c}_j^*\in\boldsymbol{C}, & \beta_j\in\boldsymbol{B}^N\\ \boldsymbol{P}(\beta_j\,|\,\boldsymbol{c}_j^*)\geq\boldsymbol{P}(\beta_j\,|\,\boldsymbol{c}_i), & \boldsymbol{c}_i\in\boldsymbol{C}\subset\boldsymbol{A}^N\end{cases}$$

与汉明距离
有何联系？



记： $c_i = a_{i_1} a_{i_2} \cdots a_{i_N}$

$\beta_j = b_{j_1} b_{j_2} \cdots b_{j_N}$

$$P(\beta_j \mid c_i) = P(b_{j_1} b_{j_2} \cdots b_{j_N} \mid a_{i_1} a_{i_2} \cdots a_{i_N}) = P(b_{j_1} \mid a_{i_1}) P(b_{j_2} \mid a_{i_2}) \cdots P(b_{j_N} \mid a_{i_N})$$

设码距$D(c_i,\beta_j)$

$$p(\beta_j \mid c_i) = p^{D(c_i,\beta_j)} \bar{p}^{[N-D(c_i,\beta_j)]} = p^{D(c_i,\beta_j)} \frac{(1-p)^N}{(1-p)^{D(c_i,\beta_j)}} = (1-p)^N \left(\frac{p}{1-p} \right)^{D(c_i,\beta_j)}$$

假设 $\bar{p} > p$

$\Downarrow$

$p/(1-p) < 1$

$\Longrightarrow$ $D(c_i, \beta_j) \downarrow \quad \Rightarrow P(\beta_j \mid c_i) \uparrow$

最小（汉明）距离译码规则：

$$F: \begin{cases} F(\beta_j) = c_j^* \in C, \quad \beta_j \in B^N \\ D(c_j^*, \beta_j) = \min\left[D(c_i, \beta_j) \right], \quad c_i \in C \subset A^N \end{cases}$$

几点说明

- 最小距离译码规则可在一般信道中采用，但不一定与极大似然译码规则等价；
- 对于二元对称信道，若正确概率大于错误概率，则最小距离译码规则与极大似然译码规则等价，并且当输入等概时是最佳的。

汉明距离与平均差错率

前提：二元对称信道输入等概。

译码函数：$F(\beta_j)=c_j^* \in C$，　$\beta_j \in B^n$

平均差错率：

$$P_e = 1-\frac{1}{M}\sum_j P\left[\beta_j \mid c_j^*\right] = 1-\frac{1}{M}\sum_j p^{D(c_j^*,\beta_j)}\,\overline{p}^{[N-D(c_j^*,\beta_j)]}$$

信源符号数量