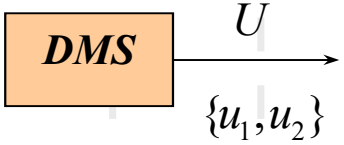
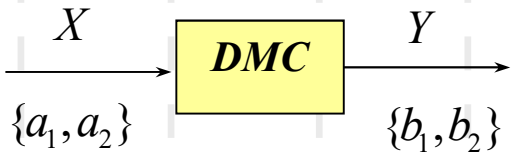
5.3 平均差错率与信道编码

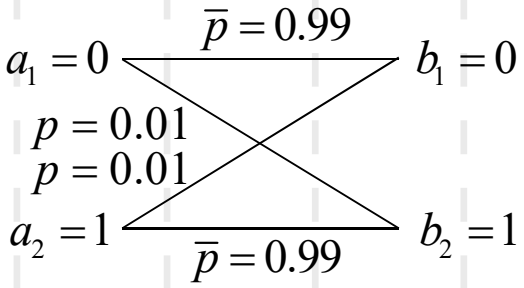
- P_e与译码规则有关，即使选择最佳译码规则，也只能使P_e有限地减小，难以满足信息传递系统的高可靠性要求。
- 要进一步降低P_e，必须在传送之前进行信道编码。

例：



$$\begin{bmatrix} U \\ P_U \end{bmatrix} = \begin{bmatrix} u_1 = 0 & u_1 = 1 \\ 0.5 & 0.5 \end{bmatrix}$$





二元信源的消息个数：M=2

熵：H(U)=logM=1 bit/符号

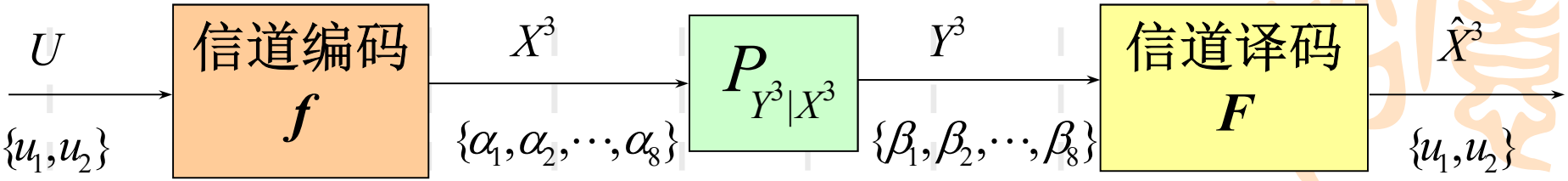
由于信道输入等概，这时极大似然译码规则是最佳的。

无信道编码时：

$$[P_{XY}] = \begin{matrix} \\ a_1 \\ a_2 \end{matrix} \begin{matrix} \begin{matrix} b_1 & b_2 \end{matrix} \\ \begin{bmatrix} \underline{0.99} & 0.01 \\ 0.01 & \underline{0.99} \end{bmatrix} \end{matrix} \qquad F: \begin{cases} F(b_1) = a_1 \\ F(b_2) = a_2 \end{cases}$$

$$P_e = 1 - \frac{1}{r}\sum_{j=1}^{s} P\left[b_j \mid F(b_j)\right] = 1 - \frac{1}{2}(0.99 + 0.99) = 10^{-2}$$

1、“简单重复”编码



$$u_1 = 0 \xrightarrow{f} \alpha_1 = 000$$
$$\alpha_2 = 001$$
$$\alpha_3 = 010$$
$$\alpha_4 = 011$$
$$\alpha_5 = 100$$
$$\alpha_6 = 101$$
$$\alpha_7 = 110$$
$$u_2 = 1 \xrightarrow{f} \alpha_8 = 111$$

$$\Longrightarrow$$

$$\left.\begin{aligned}\beta_1 &= 000\\ \beta_2 &= 001\\ \beta_3 &= 010\\ \beta_5 &= 100\end{aligned}\right\} \xrightarrow{F} \alpha_1 = 000$$

$$\left.\begin{aligned}\beta_4 &= 011\\ \beta_6 &= 101\\ \beta_7 &= 110\\ \beta_8 &= 111\end{aligned}\right\} \xrightarrow{F} \alpha_8 = 111$$

$$[P_{Y^3|X^3}]=\begin{matrix}\beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 & \beta_7 & \beta_8\end{matrix}$$
$$\begin{bmatrix}\bar{p}^3 & \bar{p}^2p & \bar{p}^2p & \bar{p}p^2 & \bar{p}^2p & \bar{p}p^2 & \bar{p}p^2 & p^3 \\ p^3 & \bar{p}p^2 & \bar{p}p^2 & \bar{p}^2p & \bar{p}p^2 & \bar{p}^2p & \bar{p}^2p & \bar{p}^3\end{bmatrix}\begin{matrix}\alpha_1\\ \alpha_8\end{matrix}$$

$$F(\beta_1)=\alpha_1 \quad F(\beta_2)=\alpha_1 \quad F(\beta_3)=\alpha_1 \quad F(\beta_4)=\alpha_8$$
$$F(\beta_5)=\alpha_1 \quad F(\beta_6)=\alpha_8 \quad F(\beta_7)=\alpha_8 \quad F(\beta_8)=\alpha_8$$

$$P_e=1-\frac{1}{r}\sum_{l=1}^{8}P\left[\beta_l \mid F(\beta_l)\right]=1-\frac{1}{2}(\bar{p}^3+\bar{p}^2p+\bar{p}^2p+\bar{p}^2p+\bar{p}^2p+\bar{p}^2p+\bar{p}^2p+\bar{p}^3)\approx 3\times10^{-4}$$

信道编码之后的信息率：$R=\frac{H(U)}{N}=\frac{\log M}{N}=\frac{\log 2}{3}=\frac{1}{3}$ bit/符号

信道编码前后比较

无编码

$N=1$

$P_e=10^{-2}$

$$R=\frac{\log M}{N}=\frac{\log 2}{1}=1 \text{ bit/符号}$$

“重复2次”编码

$N=3$

$P_e=3\times10^{-4}$

$$R=\frac{\log M}{N}=\frac{\log 2}{3}=\frac{1}{3} \text{ bit/符号}$$

“重复”编码 的其它结果

$N=5, P_e=10^{-5}$, $R=\frac{1}{5}$ bit/符号

$N=7, P_e=4\times10^{-7}$, $R=\frac{1}{7}$ bit/符号

$N=9, P_e=10^{-9}$, $R=\frac{1}{9}$ bit/符号

结论：

随着“重复”次数的增加，P_e下降，R 也跟着下降。