



CS 2100: Data Structures & Algorithms 1

Red-Black Trees (brief) & Tree Applications

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Spring 2022

Friendly Reminders

- Masks are **required** at all times during class (University Policy)
- If you forget your mask (or mask is lost/broken), I have a few available
 - **Just come up to me at the start of class and ask!**
- No eating or drinking in the classroom, please
- Our lectures will be **recorded** (see Collab) – please allow 24-48 hrs to post
- If you feel **unwell**, or think you are, **please stay home**
 - *We will work with you!*
 - At home: eye mask instead! **Get some rest** 😊

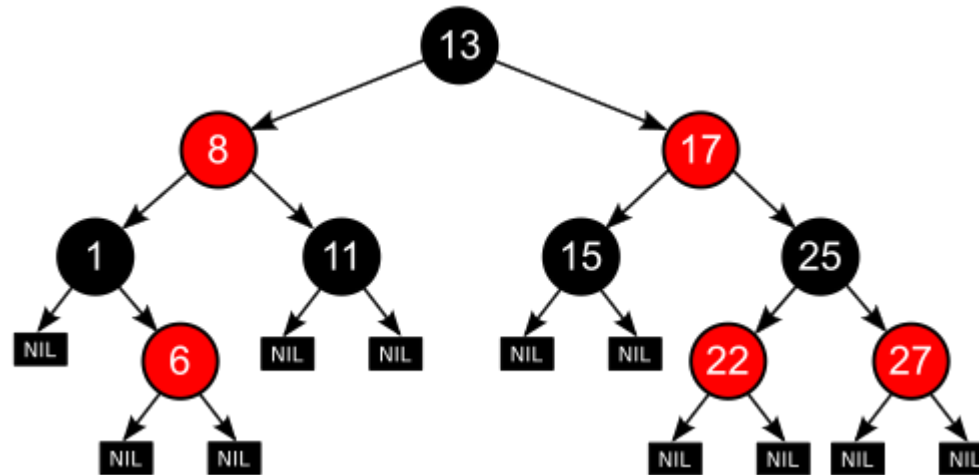


Red-Black Trees

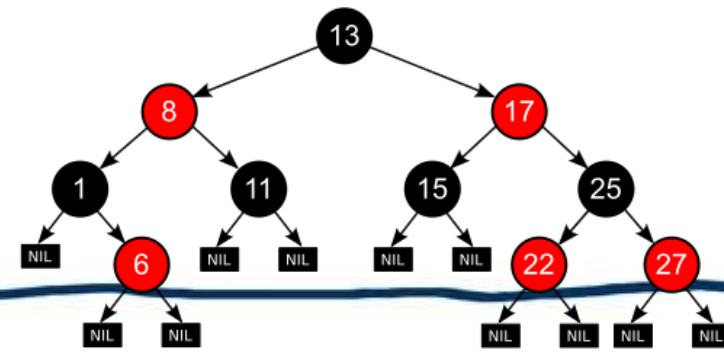
(Brief)

Red-Black Trees

- Each node has a color attribute, which is either (wait for it...) red or black ☺
- Animation site examples are [HERE](#) and [HERE](#). (*All copyright remains with original author(s) as applicable*). There are more out there, you can find, even one by [Daniel Liang](#).



Red-Black Tree Properties



All of these properties must hold for a red-black tree

- A node is either **red** or **black**
- The **root** is **black**
- All **leaves** are **black**
 - The leaves may be the NULL children
- Both **children** of every **red** node are **black**
 - Therefore, a **black** node is the only possible parent for a **red** node
- Every simple path from a node to any descendant leaf contains the **same number of black nodes**
 - Counting or not counting the NULL **black** nodes; it doesn't make a difference as long as you are *consistent*

Red-Black Tree Operations

Insert

- Insert the node as for a **normal BST**
 - And color it **red**
- 5 possible cases:
 1. The new node is the **root** node
 2. The new node's **parent** is **black**
 3. Both the **parent** and **uncle (aunt?)** are **red**
 4. **Parent** is **red**, **uncle/aunt** is **black**, new node is the **right** child of parent
 5. **Parent** is **red**, **uncle/aunt** is **black**, new node is the **left** child of parent

Delete / Remove

- Do a **normal BST remove**
- Find **next highest/lowest value**, put its value in the node to be deleted, **remove** that highest/lowest node
 - Note that that node won't have 2 children!
- We replace the node to be deleted with its **left** child
 - This child is N, its sibling is S, its parent is P
- There are 6 possible cases! (See next slide)

Red-Black Tree: Removal Cases

- A total of 6 cases!
 1. N is the new **root**
 2. S is **red**
 3. P, S, and S's children are **black**
 4. S and S's children are **black**, but P is **red**
 5. S is **black**, S's left child is **red**, S's right child is **black**, and N is the left child of its parent
 6. S is **black**, S's right child is **red**, and N is the left child of parent P
- We won't see them in detail, though, but you can find details on the Wiki
 - https://en.wikipedia.org/wiki/Red%E2%80%93black_tree

Why Red-Black Trees vs. AVL Trees?

- **AVL trees** are **more rigidly balanced** than **red-black trees**
 - Thus, more rotations are required during the operations in the worst case
- Time-critical applications will see a performance boost
- Functional programming languages used red-black trees for associative arrays (hashes)
 - The tree can be a persistent data structure
 - A data structure that retains a "memory" of its mutations

Tree Applications

Some examples and concluding thoughts on Trees

When Are Trees Not Good To Use?

- *Trees are fast* -- so when would we **not** want to use them?
 - When the **items do not have a sorted order**
 - A list of todo tasks
 - When we want **less complexity**
 - A stack or a queue
 - When we want an $\Theta(1)$ operation on retrieves
 - Vector `get()`
 - When we want an $\Theta(1)$ time for all operations
 - **Hash tables** can (almost) achieve that

Application Of Tress: Programs

- Any program can be represented as a tree; consider the following program (no external source code):

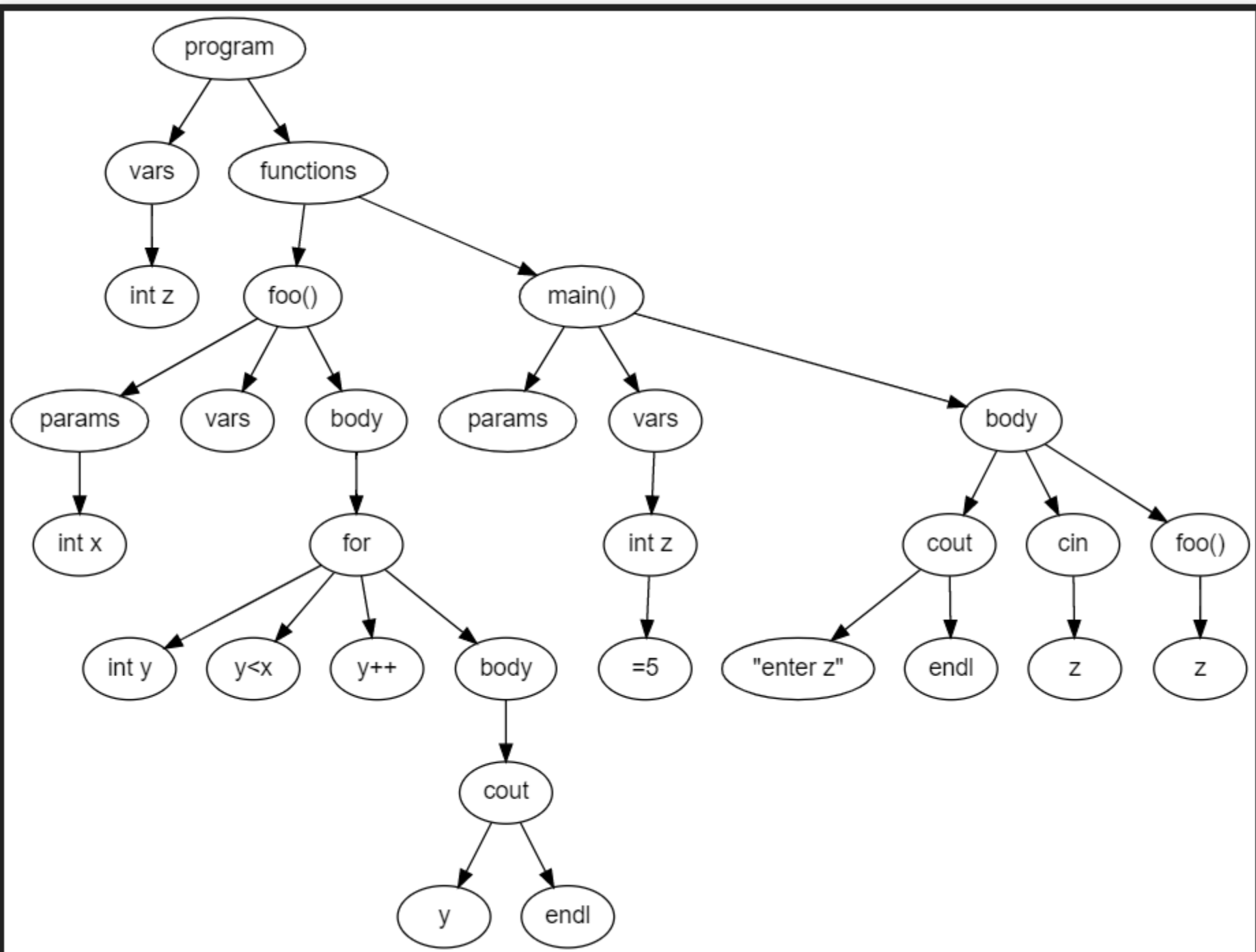
```
int z;

int foo (int x) {
    for ( int y = 0; y < x; y++ )
        cout << y << endl;
}

int main() {
    int z = 5;
    cout << "enter x" << endl;
    cin >> z;
    foo(z);
}
```

- Note that there are two `int z` declarations; this will be relevant shortly

The Program Tree



Notes on The Program Tree

- Called an "abstract syntax tree" or a "parse tree"
- Each **node** can be a different type
 - Having different properties and different number of children
 - A for loop node has four children (for init, for expression, for update, body)
 - A function node has at least three children (parameters, variables, body)
 - (we are ignoring other possible children of a function node here)
 - A body node has a variable number children
- A **compiler** will build such a tree in memory
 - And traverse it many times
 - For example, to figure out which 'z' is used in the main() function
 - Or to do **code generation**
 - Each node has an overridden method to generate the code for that node
 - Or to do **type checking**
 - Or to do **code optimization**

Comparing Two Programs

- What if we read in two programs...
 - ... and build parse trees for each
 - ... and compare their structure?
- We would be able to compare the two programs while ignoring such things as:
 - Function / method order
 - Variable renaming
 - Different comments

Measure of Structural Similarity

- "*A System for Detecting Software Plagiarism*" ([website](http://theory.stanford.edu/~aiken/moss/) - <http://theory.stanford.edu/~aiken/moss/>)
 - The paper the site is based on can be found [here](http://theory.stanford.edu/~aiken/publications/papers/sigmod03.pdf)
(<http://theory.stanford.edu/~aiken/publications/papers/sigmod03.pdf>)
- It will load up all the programs for a class
- And do all n^2 comparisons
- And display the most similar programs