Mathematics Methods for Computer Science

Instructor: Xubo Yang

SJTU-SE DALAB

Lecture VII

Linear Algebra Review

What Have We Done?

$$A\vec{x} = \vec{b}$$

Gaussian Elimination

- Codifies the typical approach taken on paper
- **Phases:** Forward substitution, back substitution (pivoting)
- **Elimination matrices:** Notational convenience, algorithmically slow!

LU Factorization

- $O(n^3)$ time to compute
- Allows for solving linear systems via forward/backward substitution ($O(n2)$ time)
- Might not exist - need pivots (e.g. LUP)

Cholesky Factorization: LL^T

For symmetric, positive definite matrices

QR Factorization

- R is **upper triangular**
- Q has **orthonormal columns**
- **Many algorithms:** Gram-Schmidt, Householder, Givens
- **Least-squares** w/o squaring condition

Diagonalizability: $D = X^{-1}AX$

- Diagonalizable iff there is a full eigenspace
- **Spectral theorem:** symmetric/Hermitian $\Rightarrow$ full, orthogonal eigenbasis
- **Computation:** Variations of power method
- **Note:** $AX = XD$ (usually $AX \neq DX$!!)

Singular Value Decomposition

$$A = U\Sigma V^T$$

Variational Approach

Define energy measuring something desirable and minimize it.

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Lagrange multipliers!

Regularization

Improves conditioning of ill-posed problems

E.g., Tikhonov regularization.

$$E(\vec{x}) = ||A\vec{x} - \vec{b}||_2^2 + \alpha||\vec{x}||_2^2$$

Multiple formulations.

Connection to truncated SVD.

Look for Special Structure

- Symmetric
- Positive definite
- Sparse
- Normal equations
- Square
- Full rank
- Block
- Triangular

Reduce to Known Algorithm

Show that a specific problem is equivalent to:

- Least squares (curve fitting)
- Eigenvectors (ODEs, embedding)
- Factorization (metric learning)
- SVD (principal components analysis)

Stability and Conditioning

Complement algorithmic analysis with understanding quality of output

Julia

Useful for study.

Advice

Draw matrix pictures.

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Draw matrix pictures.

Experiment.

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Draw matrix pictures.

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Ask for help