1

1）

max	$2x_1+x_2$
subject to	$x_1+2x_2 \leqslant 4$
	$-x_1+x_2 \leqslant 1$
	$4x_1+2x_2 \leqslant 11$
	$x_1,x_2 \geqslant 0$

或

maximize	$\begin{pmatrix}2\\1\end{pmatrix}^T x$
subject to	$\begin{bmatrix}1 & 2\\-1 & 1\\4 & 2\end{bmatrix} x \preccurlyeq \begin{bmatrix}4\\1\\11\end{bmatrix}$
	$\boldsymbol{x} \geqslant 0$

2）令 $y_1\times[x_1+2x_2\leqslant 4]$， $y_2\times[-x_1+x_2\leqslant 1]$， $y_3\times[4x_1+2x_2\leqslant 11]$，得到$(y_1-y_2+4y_3)x_1+(2y_1+y_2+2y_3)x_2\leqslant 4y_1+y_2+11y_3$。考虑原始目标 $z=2x_1+x_2\leqslant(y_1-y_2+4y_3)x_1+(2y_1+y_2+2y_3)x_2$，得到对偶问题：

minimize	$4y_1+y_2+11y_3$
subject to	$y_1-y_2+4y_3\geqslant 2$
	$2y_1+y_2+\mathbf{2}y_3\geqslant 1$
	$y_1,y_2,y_3\geqslant 0$

2.

（1）GP (2) LP (3) SDP (4) QP

3.

The Lagrangian function is

$L(x,y,z,\lambda,v) = x^2+9y^2+z^2+\lambda_1(1-xy)+\lambda_2(-z)+v(x^2+y^2-4)$

KKT:

$1-xy\leq 0$,
$-z\leqslant 0$,
$x^2+y^2=4$,
$\lambda_1,\lambda_2\geq 0$，
$\lambda_1(1-xy)=0$,
$\lambda_2 z=0$,
$2x-\lambda_1 y+2vx=0$
$18y-\lambda_1 x+2vy=0$
$2z-\lambda_2=0$

4.

(i)

Solution. Clearly

$$\nabla f = \begin{pmatrix} 2x_1 + 2 \\ 4x_2 \end{pmatrix} \quad \Rightarrow \quad \nabla f(\mathbf{x}^0) = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Therefore

$$\nabla f(\mathbf{x}^0) \cdot \mathbf{d}^0 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} = -2 < 0.$$

Hence $\mathbf{d}^0$ is a descent direction at $\mathbf{x}^0$.

(ii)

$$\nabla \mathrm{f}^2(\mathrm{x}) = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}$$

So

$$x_1 = x_0 - [\nabla \mathrm{f}^2(x_0)]^{-1} \nabla \mathrm{f}(x_0)$$
$$= (0,\ 1) - \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix} / 8 * \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$
$$= (-1,\ 0)$$

5.

(1) maximum flow: 13

(2) minimum cut: {s、a、b、c、d}/{t} 或者{s、b、c、d}/{a、t}