1. (2pt) In this question, you will construct your own Bayes net, and do some inference on it. You go to a casino to play the following game.
   - You flip two coins, Coin 1 and Coin 2.
   - If both coins come up heads, then you win.
   - If at least one coin comes up tails, then you lose.
   - With probability .4, the casino is fair.
   - If the casino is fair, then each coin has a probability of .5 of coming up heads.
   - If the casino is not fair, then each coin has a probability of .3 of coming up heads.
   - With probability .1, the casino is bankrupt.
   - If you win, and the casino is not bankrupt, the probability that you get paid is .8.
   - If you win, and the casino is bankrupt, the probability that you get paid is .2.
   - If you lose, the probability that you get paid is 0.

   1) (0.5pt) Draw the Bayesian network for this problem. You should assume independence unless a dependence is explicitly given above. Your Bayes net should contain at least the following (binary) variables: CasinoFair, Coin1Heads, Coin2Heads, CasinoBankrupt, GetPaid. (You can add Win if you like.) Give the conditional probability tables.
   2) (0.5pt) Write down the expression for the joint probability distribution as a product of conditional probabilities.
   3) (0.5pt) What is the Markov Blanket of variable Coin2Heads?
   4) (0.5pt) Both of your coins came up heads! What is the probability that you will get paid? That is, what is P(GetPaid = true|Coin1Heads = true ∧ Coin2Heads = true)? Derive the posterior distribution using Bayesian rules.

2. Calculate the entropy in bits for each of the following random variables:
   (i) Pixel values in an image whose possible grey values are all the integers from 0 to 255 with uniform probability.
   (ii) Humans classified according to whether they are, or are not, mammals.
   (iii) Gender in a tri-sexed insect population whose three genders occur with probabilities 1/4, 1/4, and 1/2.
   (iv) A population of persons classified by whether they are older, or not older, than the population's median age.

3. (2pt) Let p(x, y) be as shown in the table below.

X\Y	0	1	2
0	1/12	1/6	1/12
1	1/6	1/6	1/6
2	0	1/12	1/12

   Find
   (a) H(X), H(Y),
   (b) H(X,Y)
   (c) H(Y|X)
   (d) I(X;Y)
   (e) Draw a Venn diagram for the quantities in (a) through (d)

4. (1pt) We have a dataset in the following table where A, B denote attributes and Y denotes labels. We want to build a decision tree to classify them according to Y.

Y	A	B
−	1	0
−	1	0
+	1	0
+	1	0
+	1	1
+	1	1
+	1	1
+	1	1

Which attribute should be selected for the next split? Give your explanation.

1) A
2) B
3) A or B (tie)
4) Neither