Algorithms Design I

Prologue

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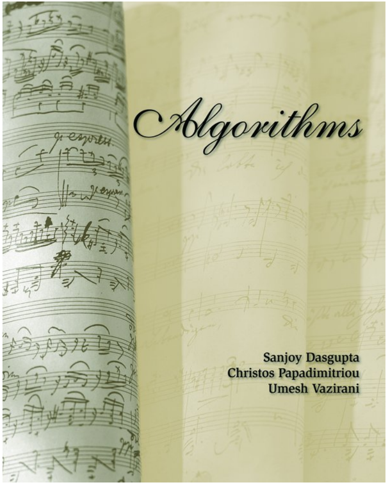
Reference Book

Textbook

Algorithms

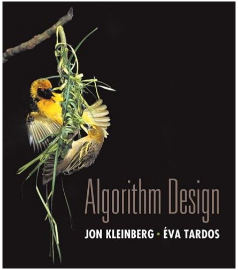
- Sanjoy Dasgupta
- San Diego Christos Papadimitriou
- Umesh Vazirani
- McGraw-Hill, 2007.

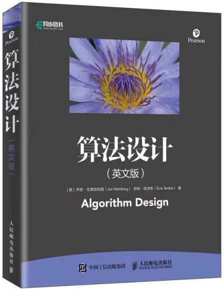


Reference book

Algorithm Design

- Jon Kleinberg, Éva Tardos
- Addison-Wesley, 2005.

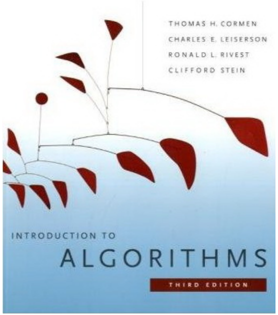




Reference Book

Introduction to Algorithms

- Thomas H. Cormen
- Charles E. Leiserson
- Ronald L. Rivest
- Clifford Stein
- The MIT Press (3rd edition), 2009.



Scoring Policy

30% Homework.

- Six assignments.
- Each one is 5pts.
- Work out individually.
- Each assignment will be evaluated by A, B, C, D, F (Excellent(5), Good(5), Fair(4), Delay(3), Fail(0))

10% Project.

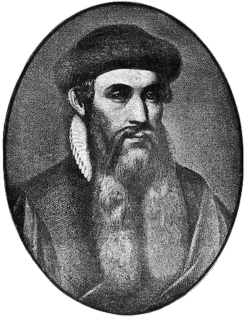
- A comprehensive report.
- To be announced shortly.

60% Final exam.

Any Questions?

Two Things Change the World

Johann Gutenberg



Johann Gutenberg (1398 - 1468)

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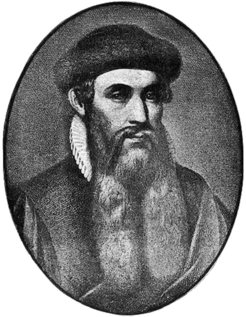

Johann Gutenberg (1398 - 1468)

In 1448 in the German city of Mainz a goldsmith named Johann Gutenberg discovered a way to print books by putting together movable metallic pieces.

Two Ideas Changed the World

Because of the typography, literacy spread, the Dark Ages ended, the human intellect was liberated, science and technology triumphed, the Industrial Revolution happened.

Many historians say we owe all this to typography.

Others insist that the key development was not typography, but algorithms.

Decimal System

Gutenberg would write the number 1448 as $MCDXLVIII$.

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The decimal system was invented in India around AD 600. Using only 10 symbols, even very large numbers were written down compactly, and arithmetic is done efficiently by elementary steps.

Al Khwarizmi

Al Khwarizmi (780 - 850)

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In the 12th century, Latin translations of his work on the Indian numerals, introduced the decimal system to the Western world. (Source: Wikipedia)

Algorithms

Al Khwarizmi laid out the basic methods for

- adding,
- multiplying,
- dividing numbers,
- extracting square roots,
- calculating digits of π.

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These procedures were precise, unambiguous, mechanical, efficient, correct.

They were algorithms, a term coined to honor the wise man after the decimal system was finally adopted in Europe, many centuries later.

What Is An Algorithm

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An abstract recipe, prescribing a process which may be carried out by a human, a computer or by other means.

Any well-defined computational procedure that makes some value, or set of values, as input and produces some value, of set of values, as output. An algorithm is thus a finite sequence of computational steps that transform the input into the output.

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A program is

- an implementation of an algorithm, or algorithms.
- A program does not necessarily terminate.

program不一定终止，但是一个算法一定会终止。

Fibonacci Algorithm

Leonardo Fibonacci

Leonardo Fibonacci (1170 - 1250)

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Fibonacci helped the spread of the decimal system in Europe, primarily through the publication in the early 13th century of his Book of Calculation, the Liber Abaci. (Source: Wikipedia)

Fibonacci Sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots$$

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Formally,

$$F_n = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F_{n-1} + F_{n-2} & \text{if } n > 1 \end{cases}$$

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Q: What is F_{100} or F_{200}?

An Exponential Algorithm

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```
FIBO1(n)
```

a nature number n;

if $n = 0$ **then** `return(`*0*`);`

if $n = 1$ **then** `return(`*1*`);`

`return(`$\texttt{FIBO1}(n-1)$ + $\texttt{FIBO1}(n-2)$`);`

Three Questions about An Algorithm

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2. How much time does it take, as a function of n?
3. Can we do better?

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The first question is trivial, as this algorithm is precisely Fibonacci's definition of F_n

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Let $T(n)$ be the number of computer steps needed to compute `FIBO1`(n)

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这里的"+3"是因为要先执行两个if(均失败),之后还要对n-1与n-2两种情况做一个额外的加法。这里还认为加法需要常数时间。

It is easy to shown, for all $n \in \mathbb{N}$,

$$T(n) \geq F_n$$

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It is easy to shown, for all $n \in \mathbb{N}$,

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It is exponential to n.

递归树，对于F(n)，其递归树的高度为n-1层，故为2^(n-1)。为啥是n-1层？因为每一次递归都有F(n-1)项，所以会从n一直减到1.

Why Exponential Is Bad?

$$T(200) \geq F_{200} \geq 2^{138} \approx 2.56 \times 10^{42}$$

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In 2022, the fastest is Frontier, 1.102×10^{18} per second.

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Just one more number every year!

上述里说明了，一个算法不光需要正确性，还需要一个合适的时间开销！
注：一般而言，多项式时间的算法计算机都是可以接受的，只不过有很多多项式时间算法都是可以优化的。

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Such is the curse of exponential time.

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Now we know $\texttt{FIB1}(n)$ is correct and inefficient, so can we do better?

An Polynomial Algorithm


```
FIBO2(n)
a nature number n;

if n = 0 then return(0);
create an array f[0...n];
f[0] = 0; f[1] = 1;
for i = 2 to n do
    f[i] = f[i-1] + f[i-2];
end
return(f[n]);
```

在认为加法花费常数项时间的情况下，该优化之后的算法时间复杂度为O(n)。

动态规划！

An Analysis

The correctness of `FIBO2` is trivial.

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How long does it take?

The inner loop consists of a single computer step and is executed $n - 1$ times. Therefore the number of computer steps used by FIBO2 is linear in n.

A More Careful Analysis

We count the number of basic computer steps executed by each algorithm and regard these basic steps as taking a constant amount of time.

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The n-th Fibonacci number is about $0.694n$ bits long, and this can far exceed 32 as n grows.

Arithmetic operations on arbitrarily large numbers cannot possibly be performed in a single, constant-time step.

A More Careful Analysis

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The addition of two n-bit numbers takes time roughly proportional to n (next lecture).

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The number of steps taken by `FIBO2` is proportional to n^2, and still polynomial in n.

多项式的

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Q: Can we do better?

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- Exercise 0.4

Big-O Notation

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Expressing running time in terms of basic computer steps is already a simplification. The time taken by one such step depends crucially on the particular processor, etc.

Accounting for these architecture-specific details is too complicated and yields a result that does not generalize from one computer to the next.

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The Coefficient 7 in the leading term is also left out, and just say that the algorithm takes time $O(n^3)$ (pronounced big oh of n^3).

Big-O Notation

$f(n)$ and $g(n)$ are the running times of two algorithms on inputs of size n.

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大0表示法

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- Let $f(n)$ and $g(n)$ be functions from positive integers to positive reals.
- $f = O(g)$ if there is a constant $c > 0$ such that $f(n) \leq c \cdot g(n)$.

$f = O(g)$ is very loose analog of "$f \leq g$". It differs from the usual notion of $\leq$ because of the constant c, so that for instance $10n = O(n)$.

Why Disregard the Constant?

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We are choosing between two algorithms: One takes $f_1(n) = n^2$ steps, while the other takes $f_2(n) = 2n + 20$ steps.

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- If $n \leq 5$, then $f_1(n) \leq f_2(n)$.
- If $n > 5$, then $f_1(n) > f_2(n)$.

f_2 scales much better as n grows, and therefore it is superior.

Why Disregard the Constant?

This superiority is captured by the big-O notion: $f_2 = O(f_1)$.

$$\frac{f_2(n)}{f_1(n)} = \frac{2n+20}{n^2} \leq 22$$

for all $n \in \mathbb{N}$.

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On the other hand, $f_1 \neq O(f_2)$, since the ratio

$$\frac{f_1(n)}{f_2(n)} = \frac{n^2}{2n+20}$$

can get arbitrarily large.

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In order to stay focused on the big picture, we treat functions as equivalent if they differ only by multiplicative constants.

Why Disregard the Constant?

Recall $f_1(n) = n^2$ and $f_2(n) = 2n + 20$, suppose we have a third algorithm which uses $f_3(n) = n + 1$ steps.

Is this better than f_2?

Certainly, but only by a constant factor.

The discrepancy between f_2 and f_3 is tiny compared to the huge gap between f_1 and f_2.

In order to stay focused on the big picture, we treat functions as equivalent if they differ only by multiplicative constants.

$f_2 = O(f_3)$ and $f_3 = O(f_2)$.

Other Similar Notations

Just as $O(\cdot)$ is an analog of $\leq$, we also define analogs of $\geq$ and $=$ as follows,

- $f = \Omega(g)$ means $g = O(f)$.
- $f = \Theta(g)$ means $f = \Omega(g)$ and $f = O(g)$.

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Recall $f_1(n) = n^2$, $f_2(n) = 2n + 20$, and $f_3(n) = n + 1$, then

- $f_2 = \Theta(f_3)$ and $f_1 = \Omega(f_2)$

老师上课的思考题：为什么归并排序的时间复杂度一定要是O(NlogN),而不能写为其O的O(N^2)之类的呢？因为NlogN的O实际上应该是θ，但是一般我们只关注平均性能，所以一般使用O就行了。因为对于一个算法而言，其是可以存在一个最好情况和最坏情况的。但是当N足够大时，平均性能更有意义。

Some Simple Rules

Multiplicative constants can be omitted: $14n^2$ becomes n^2.

n^a dominates n^b if $a > b$, for instance, n^2 dominates n

a^n dominates b^n if $a > b$, for instance, 3^n dominates 2^n

Any exponential dominates any polynomial: 3^n dominates n^5

Any polynomial dominates any logarithm: n dominates $(\log n)^3$.

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Any polynomial dominates any logarithm: n dominates $(\log n)^3$. This also means, for example, n^2 dominates $n \log n$.