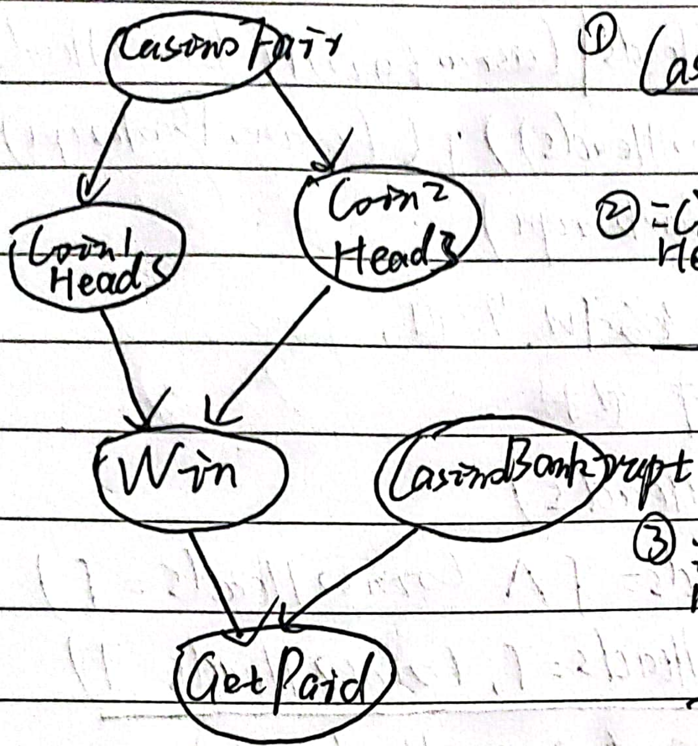
3.19：概率推理部分作业(一)：

1、图：



条件概率表：

①

CasinoFair	T	F
P	0.4	0.6

②：

Coin1 Heads \ Fair	T	F
T	0.5	0.3
F	0.5	0.7

③

Coin2 Heads \ Fair	T	F
T	0.5	0.3
F	0.5	0.7

④：

Coin1 Heads	Coin2 Heads	P(Win=True)	P(Win=F)
T	T	1	0
T	F	0	1
F	T	0	1
F	F	0	1

⑤：

CasinoBankrupt	T	F
P	0.1	0.9

(1) =

Win	Casino Bankrupt	P(GetPaid=T)	P(GetPaid=F)
T	T	0.2	0.8
T	F	0.8	0.2
F	T	0	1
F	F	0	1

(2) 联合概率分布表达式为：

$P(CasinoFair, Coin1Heads, Coin2Heads, Win, CasinoBankrupt, GetPaid)$
$= P(CasinoFair)\, P(Coin1Heads \mid CasinoFair)\, P(Coin2Heads \mid Fair)\, P(Win \mid (Coin1Heads, Coin2Heads))\, P(CasinoBankrupt)\, P(GetPaid \mid Win, CasinoBankrupt)$.

(3) 根据(1)中的贝叶斯网络可以知道，

Coin2Head 的马尔科夫毯中包括：

(CasinoFair, Win, Coin1Heads).

(4) $P(GetPaid=T \mid Coin1Heads=T \wedge Coins2Heads=T)$

$$= \frac{P(GetPaid=T, Coins1Heads=T, Coins2Heads=T)}{P(Coins1Head=T, Coins2Heads=T)}$$

$$= \frac{P(GetPaid=T, Win=True)}{P(Win=True)}$$

~~$P(GetPaid=T, Win=T) = P(GetPaid=T, Win=T \mid CasinoBankrupt=F)\, P(CasinoBankrupt=F) + P(GetPaid=T, Win=T \mid CasinoBankrupt=T)\, P(CasinoBankrupt=T)$~~

~~而~~

$P(Win=T) = P(Win=T \mid CasinoFair=T)P(CasinoFair=T)$
$+P(Win=T \mid CasinoFair=F)P(CasinoFair=F)$

$= 0.4\times 0.5\times 0.5 + 0.6\times 0.3\times 0.3 = 0.154$.

$P(GetPaid=T \mid Win=T, CasinoBankrupty=F) = 0.8$

$\therefore \begin{cases} \dfrac{P(G=T, W=T, CB=F)}{P(W=T, CB=F)} = 0.8 \\ \dfrac{P(G=T, W=T, CB=T)}{P(W=T, CB=T)} = 0.2 \end{cases}$

$\therefore P(G=T, W=T, CB=F) = 0.8\,P(W=T)P(CB=F) = 0.11088$

$P(G=T, W=T, CB=T) = \cancel{0.8}\,0.2\,P(W=T)P(CB=T) = \cancel{0.01232}\ \cancel{0.02}\ 0.00308$.

$\therefore P(GetPaid=T \mid Coin1Heads=T \wedge Coins2Heads=T)$

$= 0.74$

2. (i) $H_1 = \sum_{i=1}^{256} p_i \log\frac{1}{p_i} = 256\times\frac{1}{256}\cdot\log_2 256 = 8$ bits.

(ii) $H_2 = \cancel{\sum} p_1\log\frac{1}{p_1} + p_2\log\frac{1}{p_2}$
$= 2\times\frac{1}{2}\log 2 = 1$ bits.

(iii) $H_3 = \frac{1}{4}\log 4 + \frac{1}{4}\log 4 + \frac{1}{2}\log 2 = 1 + 0.5 = \cancel{1.5}\ 1.5$ bits

(iv) $H_4 = \frac{1}{2}\log 2 + \frac{1}{2}\log 2 = 1$ bits

3. 解：(a) ∵ 由题知：X分布为：

X	0	1	2
P	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Y分布为：

Y	0	1	2
P	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{3}$

$H(X) = \frac{1}{3}\log_2 3 + \frac{1}{2}\log_2 2 + \frac{1}{6}\log_2 6 = \frac{1}{2} + \frac{1}{3}\log_2 3$
$+ \frac{1}{6}\log_2 3 + \frac{1}{6} = \frac{2}{3} + \frac{1}{2}\log_2 3 \approx 1.46$ bits.

$H(Y) = \frac{1}{4}\log_2 4 + \frac{5}{12}\log_2 \frac{12}{5} + \frac{1}{3}\log_2 3.$

$= \cancel{\frac{1}{2}} \; 1.55$ bits

(b): $H(X,Y) = \frac{1}{12}\log_2 12 + \frac{1}{6}\log_2 6 + \frac{1}{12}\log_2 12$

$+ \frac{1}{6}\log_2 6 + \frac{1}{6}\log_2 6 + \frac{1}{6}\log_2 6 + \frac{1}{12}\log_2 12 + \frac{1}{12}\log_2 12$

$= 2.92$ bits.

(c): $H(Y|X) = H(X,Y) - H(X)$

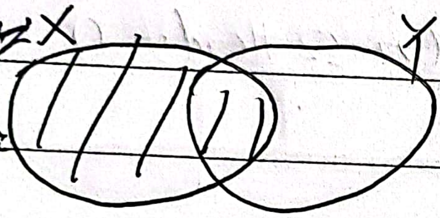
$= 1.46$ bits.

(d): $I(X;Y) = H(X) + H(Y) - H(X,Y)$

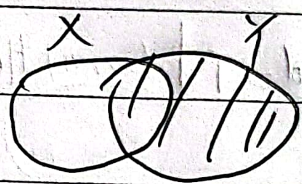
$= 0.09$ bits.

(e):

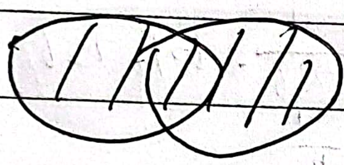
(a): ~~H(X)~~:



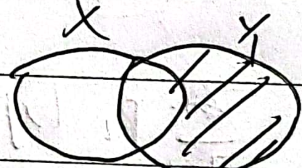
$H(Y)$:



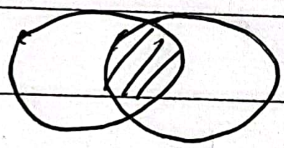
(b):



(c):



(d):



二、解：① 从table中可知 A的取值与Y的取值无关，

故 $I(A;Y)=0$，即相互信息为0。

② B分布为：

B	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

，Y分布为：

Y	−	+
P	$\frac{1}{4}$	$\frac{3}{4}$

B, Y 联合分布为：

Y \ B	0	1
−	$\frac{1}{4}$	0
+	$\frac{1}{4}$	$\frac{1}{2}$

$\therefore H(B) = \frac{1}{2} \times 2\log_2 2 = 1$ bit, ~~H(Y)~~

$H(Y) = \frac{1}{4}\log_2 4 + \frac{3}{4}\log_2 \frac{4}{3} = \frac{1}{2} + \frac{3}{2} - \frac{3}{4}\log_2 3.$

$= 0.811$ bits

$H(B,Y) = \frac{1}{4}\log_2 4 + \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2 = 1 + \frac{1}{2} = 1.5$ bits.

$\therefore I(B;Y) = H(B) + H(Y) - H(B,Y) = 0.311 > 0.$

$\therefore$ 下一次 split 应选 B.